CHAPTER ONE

INTRODUCTION

1.0 Chapter Overview

Starting with an outline of school algebra activities, the introductory chapter sets the scene and establishes the context for the investigation of teachers' use of cognitive digital technological tools for the purpose of enhancing their students' understanding of algebra concepts. As part of the context for the inquiry, Kieran's (2007) Generational, Transformational, and Global/meta-level (GTG) model (Figure 1.1) was used to categorise the school algebra activities within which these tools are used by both teachers and students. The GTG Model (Kieran, 2007) was chosen for two reasons; firstly, it is a synthesis of several researchers' (e.g., Bell, 1996; Kaput, 1995; Lee, 1997, Rojano, 2004; Smith, 2003; Star, 2005; Sutherland, 2004) perspectives from their investigations of the nature of algebraic activity. Secondly, it accounts for nearly all secondary school algebra activities. The description of the GTG model is followed by an outline of the focus of the study in terms of its research questions and statements of the study's rationale and significance. Also included within this chapter is a brief historical background outlining the efforts undertaken in Australia to integrate digital technology into the teaching and learning of mathematics within Australian schools from the time, in the 1980s, when Texas Instruments (TI) invented the calculator in the United States.

1.1 The focus of the investigation

The purpose of this inquiry was to identify and describe the roles and concerns of teachers when they use digital cognitive tools to enhance their students' understanding of school algebra concepts. The hope, for the researcher, was that the inquiry would contribute to teachers' confidence and competence (i.e., teachers' pedagogical technology knowledge) in using digital technological tools to teach school algebra.

1.2 The school algebra context

School algebra comprises generational, transformational and global/meta-level algebraic activity. Kieran's (2004, 2007) GTG model (Figure 1.1) is a useful structuring device for conceptualising these three principal activities.

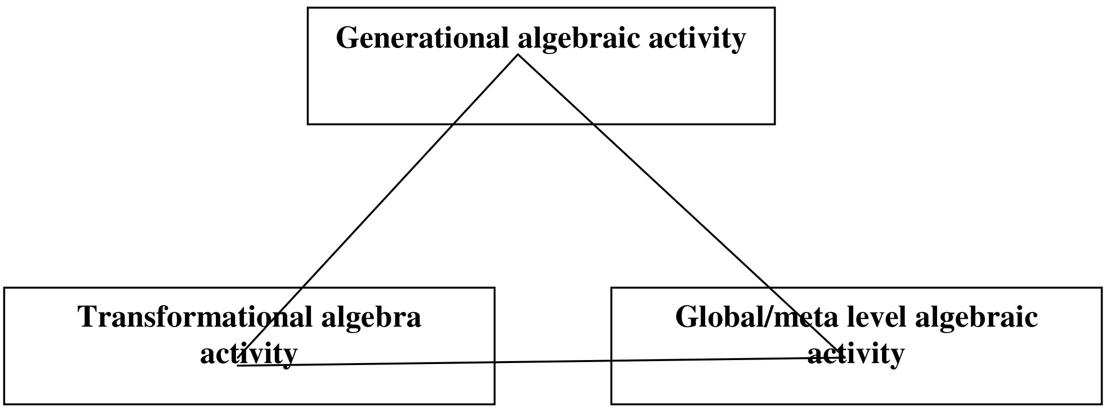

Figure 1.1: Kieran's GTG model of algebraic activities (Kieran, 2004, 2007)

Generational algebraic activity is expressed through the forming of the expressions and equations that are the objects of algebra. Its focus is the representation (and interpretation) of situations, properties, patterns, and relations and much of the initial meaning-making of algebra is considered to be situated in this sphere of algebraic activity. Transformational algebraic activity is expressed through the collecting of like terms, factoring, expanding, substituting, adding and multiplying polynomials expressions, exponentiation with polynomials, solving equations, simplifying expressions and working with equivalent expressions and equations. Global/meta-level activities are those activities for which algebra is used as a tool but which are not exclusive to algebra such as problem solving, modelling, noticing structure, studying change, generalising, analysing relationships, justifying, proving, and predicting. All three activities (generational, transformational and global/meta-level) constitute the core of algebra (Kieran, 2007) and are essential ingredients of the junior and senior mathematics courses at the secondary school level in New South Wales, Australia. For a long time, algebra was a paper-and-pencil activity with the focus primarily on transformational work (processes of algebra). The arrival of electronic technology from the mid-1980s onward led to a greater interest in the generational and global/meta-level algebraic activity. Reflecting on the crucial roles of technology and of the algebra teacher in the teaching and learning of algebra, Kieran (2004) noted:

> I see the future of algebra teaching and learning as one geared toward giving meaning not only to the objects of algebra but also to its manipulative processes, and this with the help of technology … The algebra teacher has a crucial role to play both in bringing algebraic representations to the fore and in making their manipulation by students a venue for epistemic growth (p. 32).

Through exploratory case studies, teachers' use of cognitive technology in algebra to enhance student understanding of algebraic concepts, was investigated by addressing the following research questions:

- **RQ1**: What roles do mathematics teachers adopt as they integrate cognitive digital technological environments in algebra lessons?

- **RQ2**: Why do teachers use cognitive digital technological environments the way they do when teaching algebra?

- **RQ3**: How do teachers incorporate cognitive digital technological environments in their teaching of algebra?

- **RQ4:** How can shortcomings in the pedagogical technology knowledge of in-service teachers be addressed for enhanced student understanding of algebra?

The inquiry involved collecting data from four research participants, all of whom were qualified teachers of mathematics to 12-17 year-old students in secondary schools in the New England/North West region of New South Wales. The researcher used semi-structured interviews for data collection with the intention of obtaining in-depth information about teachers' practices, concerns and needs in relation to their teaching of generational and transformational algebraic activity in cognitive digital technological environments. The purpose for the researcher was to identify areas that might inform the professional development and training needs (or pedagogical technology knowledge) of in-service mathematics teachers in secondary schools in New South Wales. The assumption was that both algebraic objects (arising out of generational activity) and the processes that manipulate these objects (transformational activity) can be made meaningful for students by enhancing the pedagogical technology knowledge of these teachers.

1.3 Rationale for the Study

Teachers are not likely to successfully integrate cognitive digital technology in their teaching if they experience limitations in their conceptualizations of algebra and also in their pedagogical technology knowledge (PTK) (Thomas & Chinnappan, 2008). Cognitive technological tools can be used to store, manipulate, and retrieve information, and they have the capability not only of engaging students in instructional activities to increase their learning, but also of helping them solve complex problems to enhance cognitive skills (Jonassen & Reeves, 1996; Newby, Stepich, Lehman, & Russell, 2000). Use of this technology, however, requires a high level of understanding and reflection by teachers, because tools can enhance or inhibit students' development of robust understandings of mathematical concepts (Heid & Blume, 2008). The findings of several researchers reveal that teachers are generally under-prepared to integrate technology into their instruction in meaningful ways (Guin, Ruthven, & Trouche, 2005; Strudler & Wetzel, 1999; Schrum, 1999; Willis & Mehlinger, 1996).

1.4 The significance of the study

The study is significant for a number of reasons: firstly, there is an increasing availability of digital technological environments to algebra teachers in New South Wales schools with the curricular requirement (Board of Studies NSW, 2008) that they be used in the teaching of certain aspects of school algebra. For example, in the Stage 5.2 Mathematics Years 7-10 Syllabus (Board of Studies, 2008), 12-16 year-old students are expected to 'use graphics calculators and spreadsheet software to plot pairs of lines and read off the point of intersection' (p. 91).

Secondly, there is a need to encourage the growth in the actual classroom use of cognitive digital technology through widening and making available the theoretical base for its successful application. While Australia is at the high index level on the Index of Availability of School Resources for Mathematics Instruction (ASRMI) in the 2007 Trends in International Mathematics and Science Study (TIMSS), with 55% of the 2007 eighth grade students having a high ASMRI (IEA, 2007), the research community is concerned with the quality of learning that eventuates from its use and the teacher characteristics that produce the flexibility to integrate a variety of technologies in mathematics teaching and learning (Thomas & Chinnappan, 2008).

Thirdly, while past research in the transformational activities of algebra tended to focus on the nature of manipulative processes used in expression simplicity (e.g. Davis, Jockush, & McNight, 1978; Sleeman, 1984) and in equation solving (e.g., Bell, O'Brien, & Shiu, 1980; Kieran, 1988; Whitman, 1976), the field has recently experienced a change that includes attention to the collaborative work involving algebra teachers with their intimate knowledge of the classroom setting in conjunction with a deep awareness of the mathematical and pedagogical affordances of digital tools (Kieran & Yerushalmy, 2004). Fourthly, in the words of Heid and Blume (2008), 'algebra occupies an undeniable central role in school mathematics programs……no other mainstream school mathematics topic is more likely to be profoundly affected by the growing presence of technologies in schools' (p. 55). Lastly, there is a need to address concerns about the direction of research which were raised by the Board of Studies NSW as long ago as 1999. The Board (Goodyer, 1999) noted that:

> much of the research was concerned with student use of the new technologies, rather than more fundamental curriculum and pedagogical issues. There was a clear need for more research in a variety of areas and a requirement to direct these issues to the attention of universities and other research institutions to enable them to be incorporated into research agendas, including the crucial issue of teacher preparation … (p. 9)

1.5 Background to technology integration into mathematics

Nearly twenty years after the invention of the handheld calculator in the United States by Texas Instruments (TI) and almost at the same time as the mass production of the microcomputer there were urgent calls for schools to teach students to cope with technology and to incorporate 'new' technology into the curriculum and teaching methods. The preface to the 1985 Report of the Australian Education Council Task Force on Education and Technology was emphatic about what needed to be done (Australian Educational Council, 1985):

> The education system must adapt to new demands and possibilities in such a way as to enable Australians to exploit the benefits and opportunities offered by new technologies, while ensuring that social needs are met and the potential of individuals is maximised in a time of rapid change, uncertainty and limited resources.

Besides the challenge of limited resources in terms of insufficiency in the number of technological tools available, there was the challenge to teachers to revolutionise their approaches when using technology in their classrooms as can be inferred from the following Australian Education Council (1985) statement:

> Educators are being asked not simply to add on new information, techniques and knowledge, but to make radical changes in lesson content and classroom teaching practice (p. 66).

By the 1990s, there were mathematics curriculum policies directed at promoting the use of technology to aid students' learning and understanding of mathematics (Australian Education Council, 1991). These efforts manifested and continue to manifest as various state and territory mathematics curriculum statements and syllabuses that permit, encourage or expect the use of technologies such as computers, scientific calculators, graphics calculators or calculators with computer algebra systems (CAS). In 1999, Ministers of Education for the states, territories and the federal government, through the Adelaide Declaration, re-emphasised not only the expectation that students were to develop high standards of knowledge and skills in the key learning area of mathematics; they also declared, in one of the eight general goals for learning, that students were to be confident, creative and productive users of new technologies and to understand their impact on society (Ministerial Council on Education, Employment, Training and Youth Affairs, 1999). The implied necessity for teachers to be in a position to cause this to be is obvious. More recently, and in a more explicit reference to what is expected of excellent teachers, the Australian Association of Mathematics Teachers (2006) stated that such teachers are able to use technologies to make a positive difference to the learning outcomes, both cognitive and affective, of the students they teach.

1.5.1 Technology and algebra learning and teaching.

It has been suggested by Heid and Blume (2008) that no other mainstream school mathematics topic is as likely as algebra to be deeply affected by the growing presence of technology in schools. Technological tools not only change the nature of the tasks that are presented to students, they change the nature of the teaching as well. To take advantage of the potential of technology would seem to require not only tasks that are designed to push students beyond the limits of their current algebraic thinking and encourage further development of that thinking, but also approaches to teaching that facilitate such growth in students (Kieran, 2007). Concern with how technology is used by and with students highlights the fact that teachers exercise a critical role in the technology-based algebra learning experiences of students. Nevertheless, although there is a widespread belief in Australian educational circles and in society at large that students' learning will be enhanced by engaging with digital technologies (e.g., computers, software and calculators), this is not always the case; there are situations when technology will limit opportunities for conceptualising in algebra. Although several factors are responsible for such situations, this inquiry limits itself to addressing one factor; teachers' roles, concerns and their pedagogical technology knowledge with respect to digital technology integration with algebra for enhanced students understanding of algebra concepts.

1.6 Conclusion: Overview of the Research

The increasing availability of digital tools in secondary schools has focused the attention of educators on the role played by teachers' knowledge and practises in producing robust understandings of algebra concepts that are required of numerate students. In the introductory chapter, the researcher has provided justification for investigating the way teachers use digital technology in the teaching and learning of school algebra: the potential of this technology is yet to be fully realised in secondary schools in New South Wales.

While there are many issues influencing teachers' use of technology in an algebra classroom, the inquiry focuses on the quality of the knowledge-base required for the integration of digital technology in the teaching and learning of algebra. Through the identification and description of teachers' roles and concerns in technology-enabled algebra classrooms, the study has, as its ultimate aim, the enhancement of student understanding of concepts in algebra.

To achieve this aim, Chapter Two provides the theoretical framework for the investigation, beginning with the identifying and description of the different roles played by teachers as they integrate technology in a mathematics lesson. This is followed by a discussion of factors related to the teaching of algebra that foster and impede the implementation of technology-rich mathematics education in relation to algebra learning by students. Teachers' concerns when they use technology and models identifying modes of their use of this technology are discussed. Finally, the chapter also discusses the theoretical perspectives that provide the base for the integration of technology in mathematics teaching and learning.

Chapter Three details the methodological approach underpinning the research. After outlining paradigm considerations, the section provides some insight into the epistemological approach used in the inquiry. A discussion on why the case study research strategy operating within an interpretive and subjective paradigm best suits this research is provided, followed by reasons for choosing the multiple case study design variant of case study research for this investigation. A description is given of the recruitment process for the four high school teachers who participated in this research. A discussion of the process of obtaining the research data from them concludes this chapter.

Chapter Four, the results chapter, starts by highlighting the research questions followed by a description of the basis for choosing the schools that were selected for research sites. An explanation is given for using focused interviews to collect data for the investigation. The four research questions were reformulated into ten questions that the researcher posed, in a fluid way as suggested by Rubin and Rubin (1995) to the teacher participants in the investigation. The chapter ended with the presentation and analysis of the collected data.

CHAPTER TWO

LITERATURE REVIEW

2.0 Introduction

In the literature review, the literature relevant to teachers' use of digital technological tools to enhance student understanding of algebraic concepts is explored for each of the four research questions. It covers all the main theories, aspects, concepts and existing research related to this study. The chapter is divided into four main areas. Section 2.1 provides an overview of technology use in school algebra. Section 2.2 addresses the literature in relation to the roles that mathematics teachers adopt as they integrate cognitive digital technological environments in algebra lessons. Section 2.3 reviews the literature on why teachers use cognitive technological environments the way they do, i.e., on the factors influencing teachers' approaches to technology when teaching algebra concepts. Section 2.4 explores the literature on how teachers incorporate cognitive technology in their algebra lessons. The section also examines the theories that have been advanced to explain technology integration in mathematics. Section 2.5 looks at what has been done to address shortcomings in the pedagogical technology knowledge (PTK) of teachers for enhanced student understanding of algebra. Section 2.6 discusses the theoretical frameworks on the roles of the digital tools themselves, given that the potentialities of these tools influence the way the teachers use them. The last part, Section 2.7, separately discusses the theoretical frameworks on digital tool use in mathematics classrooms based on the writings of Vygotsky with their sociocultural emphases in light of the increasing attention given to sociocultural perspectives (with their acknowledgement of the complex, dynamic, and contextualised nature of learning in social situations) from the 1980s onwards (Streetland, 1985).

2.1 Overview of digital technology use in school mathematics

It is recognised by several Australasian researchers that digital technologies (such as computers and graphics calculators) have the potential to transform mathematics teaching and learning (Arnold, 2004; Forster, Flynn, Frid, & Sparrow, 2004; Goos & Cretchley, 2004). Despite the availability of these technologies in schools and the formulation of policies to ensure their use for the teaching and learning of mathematics, they still play only a marginal role in mathematics classrooms (Norton & Cooper, 2001). Clearly, while the availability of digital tools and the presence of an enabling mathematics curriculum policy framework are necessary conditions for students to fully exploit the opportunities provided by these tools, they are not sufficient on their own to cause this to be. For student understanding to be enhanced, the pedagogical implications of using digital technology in mathematics need to be addressed. This can be and, indeed has been done, at the 'whole of mathematics' level or at the level of each of the major areas of school mathematics. For this inquiry, the literature is examined in relation to teachers' use of digital technology when teaching algebra concepts. In particular, research into teacher's roles, concerns, factors influencing the way they use technology is reviewed followed by an examination of some of the theoretical frameworks informing its use.

2.2 Teachers' roles in technology-based algebra classrooms

Enriched understanding of algebra with technology has its basis not just in the skilful development of technology but also in how that technology is used by teachers. The finding from research, however, is that integrating digital technology in a mathematics classroom is a complex undertaking for teachers (Lumb, Monaghan, & Mulligan, 2000; Monaghan, 2005). This was the conclusion arrived at in a study by Monaghan (2005) of 13 mathematics teachers from 7 high schools in England in which the teachers provided their own accounts as they used the following digital technology tools: spreadsheets (*Excel*), graphical packages (*Omnigraph*), calculators, algebra software (*Derive*) and geometry systems (*Geometers' Sketchpad*). While this study focused on teachers' activities, another study was needed that extended beyond their activities to how they viewed teachers' roles in a different setting.

Another study, carried out much earlier in high schools in Melbourne (Asp, Dowsey & Stacey, 1992) had teachers' roles as they used computers in their mathematics classrooms as its focus. The researchers too had found that integrating these computers was not a simple exercise. Nevertheless, they also concluded that when the teachers received support from each other and from the researchers, there were adjustments in their roles. For example, there was a move towards a more relaxed classroom management style once the teacher changed their views of the context of their teaching. While in this study each school was provided with only two computers, this enquiry goes further to study of teachers' roles in a situation of increased availability, diversity and regularity of use of digital technology tools.

Other researchers have also examined teachers' roles in technology-using mathematics classrooms, including the description and identification of these roles (Fraser, Burkhardt, Coupland, Phillips, Pimm, and Ridgway, 1988; Farrell, 1996; Heid, Sheets, & Matras, 1990). Those educators who have largely focussed on the role of the digital tool in the teaching and learning of mathematics have implicitly defined the role of the teacher (e.g., Goos, Galbraith, Renshaw & Geiger, 2001, 2003; Doerr & Zangor, 2000). Guin and Trouche (1999) and also Trouche (2004) have argued that rather than identify each and every role played by the teacher, it is more important to have a holistic view of what they did. These researchers view the teachers' role holistically as being that of externally steering students 'instrumental geneses'. Instrumental genesis is discussed in more detail in Section 2.6.6.2.

2.2.1 Typologies of teacher roles

Several researchers have attempted to capture the roles and responsibilities of teachers using digital technology in their mathematics classrooms. Examples of these researchers include Fraser, Burkhardt, Coupland, Phillips, Pimm, and Ridgway (1988), Farrell (1996), and Heid, Sheets, and Matras (1996).

Farrell (1996) used videotapes of six pre-calculus classrooms to determine the extent to which the teachers in these classrooms assumed Manager, Task Setter, Explainer, Counsellor, Fellow Investigator, and Resource roles when using graphing technologies. Farrell adapted these roles for teachers from the Systematic Classroom Analysis Notation (SCAN). SCAN (Beeby, Burkhardt, & Fraser, 1979; Fraser, Burkhardt, Coupland, Phillips, Pimm & Ridgway, 1988) is a tool that can be used for classroom observations. A shift was observed in both students' and teachers' roles towards that of consultant and fellow investigator, accompanied by a similar movement away from teacher exposition towards planned or informal group work. A shortcoming in this approach is the description of the role of the teacher as a learner from or with the students. In this case the students take on the role of the explainer. This approach to the impact of digital technologies on classroom practice seems to be a more mature analysis of teacher learning because it asks 'how' technology is used and how this changes the teacher's role instead of merely contrasting the teaching of a particular mathematics topic with or without technology as evidenced in the study by Farrell (1996).

In contrast to the work of Farrell (1996), Heid, Sheets and Matras (1990) developed their list of roles from observations of teachers who were using a combination of graphing utilities, table generators, curve fitters, and symbolic manipulators in high school algebra classrooms. Farrell (1996) explicitly considered how teachers' roles manifested themselves in the graphing calculator-using pre-calculus classrooms and how these manifestations compared with those of the same teachers in non-technology-using classrooms. Heid and colleagues on the other hand described teacher responsibilities and challenges as well as roles in classrooms in which teachers and students used both a graphing program and a symbolic manipulator. Zbiek and Hollebrands (2008) have combined the roles as described by Farrell (1996) and those from Heid et al (1990) to produce a list in which there is evidence of overlap among some categories and also in which some aspects are missing, possibly as a result of the state of the technology and technology use in mathematics teaching at the time of these studies.

2.2.1.1 Teacher as Allocator of time

The teacher as an allocator of time, while presented as a responsibility and challenge in the work of Heid et al (1990), can be recast as a role on the ground that responsibilities and challenges are often enacted through roles in the classroom. Here the teacher is working with the time requirements of the school as well as orchestrating time for accommodating the needs of individual students.

2.2.1.2 Teacher as Facilitator or Catalyst

The teacher facilitates the introduction of a new problem or real-world context and the discussion of various solutions so that the lesson reaches an appropriate close (Heid et al, 1990). There are, however, different conceptions of teaching. The ‘facilitator’ role seems a more appropriate metaphor for the role of the teacher within a radical constructivist paradigm. In contrast, the social constructionist metaphor would be that of ‘mediator’ (Lerman, 1994). In another study (Doerr & Zangor, 2000) where the focus was primarily on the roles of the graphing calculator (the Texas Instruments TI 82/83), the teacher was observed assuming the role of facilitator and catalyst. The study was aimed at, among other things, a consideration of the relationship of teachers’ roles, knowledge, and beliefs to students’ graphing calculator use. The teacher’s encouragement of student control of calculator use in the classroom inspired student initiative and student-led discussion. While the other roles of the teacher are not mentioned, it is likely that they were managed well enough to lead to the pre-eminence of the facilitator and catalyst roles.

2.2.1.3 Teacher as Collaborator or Fellow Investigator

As a collaborator (Heid et al, 1990) or fellow investigator (Farrell, 1996), the teacher is unfamiliar with both the problem and the solution and therefore is a true participant in mathematical learning. This role, like that of the facilitator, is also premised on the conception of a teacher as a co-constructor of knowledge with the students.

2.2.1.4 Teacher as a Counsellor

As a counsellor (Farrell, 1996), the teacher is familiar with the problem and is able to advise and assist students when they ask for teacher input. This role includes playing the devil's advocate as well as providing encouragement or serving as a stimulator or diagnostician. In Farrell's (1996) study, a role was coded either as present or not present during each segment. The number of segments containing any instance of the role was compared as a percentage of the total number of segments. The role of counsellor occurred relatively infrequently during nontechnology-use segments (19%) as compared to during technology-use segments (43%). A similar finding was made by Rochowitz (1996). The comparisons were based on codes of 5-minute segments of videotape. A role was coded either as present or not present during each segment. Farrell (1996) compared the number of segments containing any instance of the role as a percentage of the total number of segments. This role appears to overlap with that of teacher as 'technical assistant' as described by Heid et al (1990).

2.2.1.5 Teacher as an Evaluator

Presented as a responsibility and a challenge by Heid and colleagues (1990), the teacher as an evaluator of student learning uses informal and formal assessments of different types to describe individual students' emerging understandings with and without technology. Given the different proficiencies with which students use technology, the evaluation may need to be adjusted to cover assessments at different levels of the use of technology instead of merely being concerned with contrasting the teaching and learning of particular mathematics topics with and without technology.

2.2.1.6 Teacher as an Explainer

Here (Farrell, 1996) the teacher demonstrates, establishes the context, focuses the classroom direction, and serves as a rule giver and knowledge source. In relation to the use of a particular digital tool, the teacher explains how it is used to the students, with or without the use of an example. For a tool with a key pad, such a graphics calculator, the teacher will explain the use of particular keys and key combinations to solve a problem.

2.2.1.7 Teacher as Manager

In this role, the teacher serves as tactical manager, director, and authoritarian. In Farrell's (1996) study the teachers functioned as managers in nearly 100% of the 5-minute segments over which the comparisons of roles were coded. This 100% percent presence of the manager role of the teacher may have been so pervasive as to be ignored or not be noticed by Heid and colleagues (1990). The other reason for the omission could be that the manager role addresses matters linked to general class management rather than instructional and learning issues.

2.2.1.8 Teacher as Planner and Conductor

As a planner and conductor of classroom lessons (Heid et al, 1990), the teacher plans and implements with-technology and without-technology activities, and chooses among whole-class, small-group, or individual settings as needed. This includes selection, chronology, and creation of curriculum materials and technology tools. In Farrell's (1996) study, this role was observed more in technology-using classrooms than in technology-free settings. Other researchers have documented the roles of teachers in technology- using mathematics classrooms. Stallings (1995) reported the role of planner in a study of a first-year algebra teacher who saw her role of planner (and also implementer) as far more intense when technology was involved than when it was not. The teacher also believed that she became better at planning for technology use as she gained experience. In some cases the roles that the teachers desired to take on contrasted with those they found themselves in. A teacher using a function probe (Confrey, 1991) wished to be a counsellor but ended up acting more as a technical assistant.

2.2.1.9 Teacher as a Resource

The teacher as resource is described by Farrell (1996) as representing a system to be explored and functions as a giver of factual information. Farrell's study found a low incidence of the resource role when technology was in use. It was found only in 5% of 5-minute segments of a videotaped lesson (when technology was in use) and 3% (when technology was not in use). In each of the 5-minute segments over which observations for roles were made, a role was coded as either present or not present during each segment. The difference between the resource role and the technical assistant role (noted by Heid et al, 1990 and described below) is that in the former the teacher merely possesses information to share while in the latter the students have to ask an appropriate collection of questions in order to obtain relevant information.

2.2.1.10 Teacher as Task Setter

The teacher's role in this case is to engage students in mathematical work through questioning and decision making and also by setting the examples and strategies (Farrell, 1996). The teacher provides the work for them to do in class or as home work using a specified digital tool. The students may refer to the examples that the teacher set or ask for help as they attempt the tasks given to them. The work is then checked to find whether it is correct or not

2.2.1.11 Teacher as Technical Assistant

Teachers as technical assistants help students with hardware and software difficulties (Heid et al, 1990). This means the teachers must be problem solvers through determining the nature of the technology-related difficulty and generating and choosing from among possible solutions.

2.2.1.12 Technology as master, servant, partner and extension of self

Another example of a study about the role of the teacher in a mathematics mediated classroom, albeit an inferred one, is that by Goos, Galbraith, Renshaw and Geiger (2001, 2003). In their study of graphic calculator and computer use by students and teachers in five senior secondary mathematics classrooms in Victoria (Australia) they identified the following roles for the technology and hence inferred roles for the teacher and the student. The first role is that of technology as a master—in which the teachers and students may be subservient to the technology if their knowledge and usage are limited to a narrow range of operations over which they have technical competence. The teacher and students then become servant to the technology.

Secondly, the technology can be viewed as a servant, making the teacher and student to have a sort of mastery role over it. It is a role characterised by a fast use of technology and its reliable replacement for mental or pen and paper calculations, but lacking in creative applications for it that are designed to change the nature of the activities. An example of such application would be the use of a graphics calculator for the purpose of producing answers to routine exercises (Goos & et al, 2001, 2003). The third role is that of technology as partner, in which a rapport has developed between the user and the technology, leading to a creative use of the technology to increase the power students and teachers have over their learning and teaching respectively. The creative partnership between the teacher and technology was illustrated for the technology as partner mode using the example of students being invited to present and examine alternative mathematical conjectures using the overhead projector. An even larger number of possibilities for the student are now available with the increasing availability of the interactive whiteboard in secondary school in Australia with the flexibility provided by its interactivity.

Despite the reference by Goos and et al (2001, 2003) to almost a human like rapport between the user and the technology in this partner mode, however, they could also have categorised it as a special case of technology as a servant with a now much more skilled master. The fourth mode is that of technology as extension of self, with the users incorporating the technological expertise as an integral part of their mathematical repertoire.

An example of this mode of technology, from a teacher's perspective, is the writing of courseware to support an integrated teaching program. The teacher then plays a master or servant role depending on the whether the program has been created to play servant role or a master role.

In light of the different teacher roles outlined above in technology-based mathematics classrooms, it would be interesting to find what roles the teachers perceive themselves to be taking on in the setting of a New South Wales secondary school mathematics classroom, conflicts between these roles and also the differences in the blends and depths to which individual teachers assume these roles in such a setting.

2.3 Factors influencing the way algebra is taught in digital technology environments

Although there is a strong political will for the integration of digital technology in algebra, there is weak implementation in the classrooms (Assude, Buteau & Forgasz, 2009). The need does still exist, therefore, to realise the potential of technology in the teaching and learning of algebra (Thomas & Chinnappan, 2008). An examination of the literature reveals some of the factors related to the teaching of algebra that foster and impede the implementation of technology-rich mathematics education with respect to algebra learning by students. The factors are considered here are different but not independent levels of analysis which have many intersections.

2.3.1 Social, political, economic and cultural factors

Artefacts such as digital technological tools are fundamental constituents of culture (Coles, 1996). The things people do in their everyday settings such as teachers using familiar technology in an algebra classroom context involve a multitude of coordinated artefacts which mediate their attitudes and beliefs, their social interactions and their actions on the non-human world. If new artefacts such as digital technologies enter the classroom in other than a peripheral form, then there is likely to be tension, until a new culture is established in which digital practices are coordinated with established artefacts and routines. As an example, some researchers have found that the increasing introduction of multi-representational technologies such as graphing calculators (sometimes accompanied by the implementation of curricula involving related new approaches to school algebra) has caused teachers to experience tension between familiar methods and the use of technology or between the curriculum and their personal educational conceptions (e.g., Chazan, 1999; Haimes, 1996; Lloyd, 1999; Slavit, 1996).

2.3.1.1 The political and economic factors

At the political level, there is a strong will for integrating digital technologies technology in the official (planned) curriculum for secondary schools. For example, Lynch (2006) found that in Australia as in the UK and the USA, governments have promoted the integration of computer use across the school curriculum since the 1980s. In the late 1990s the Mexican Ministry of Education sponsored a national project called EMAT (Teaching Mathematics with Technology) which had a large government-sponsored campaign that included many advertisements on radio and television claiming that computers improve children’s learning

(Ursini & Sacristán, 2006). This claim was made without reference to the way these computers were used.

In New South Wales, a major driver of computer-based technology for schools was the Labor Government's *Computers in Schools Policy (CSIP) 1995-99* (Harriman, 2002). The government aimed at getting teachers to use computer-based technology to improve learning in all curriculum areas, with the primary focus being to normalise the use of this technology as an educational tool (Harriman, 2002). This aim lies behind such statements as, 'Technology is a useful tool for students when graphing and comparing graphs of relationships' in the introduction to the 'Patterns and Algebra' strand in the Years 7-10 Mathematics syllabus document prepared by the Board of Studies NSW (2002, p. 77). To achieve its goals, the NSW government made significant infrastructural investments that led to improved access to computer-based technologies in every NSW government school. However, Cuban, Kirkpatrick and Peck (2001) found through a study of teachers and students in two high schools in the United States, that access to technological equipment and software seldom led to widespread teacher and student use.

Aware that more needs to be done besides the provision of the technological equipment, the NSW Government, through the Department of Education and Training (DET) has engaged in collaborative projects with universities in NSW and other research organisations to determine what practices enhance student learning in relation to the use of computer-based technologies. An example of such collaborative engagement is the 'Enhancing Learning Using New Technologies' project (Orlando, 2009). Also known as the *Effects Project*, this study was an Australian Research Council Strategic Partnership with Industry Research

Scheme between the Faculty of Education, University of Technology, Sydney and the Curriculum Support Directorate, NSW Department of Education and Training (DET) (Orlando, 2009). The project focussed on seven case-study schools between 2001 and 2003 (four high schools and three primary schools). A mass of data on teachers' usage of Information and Communication Technologies (ICT) was collected from 110 teacher interviews and 71 classroom observations. The study was broad-based in the sense that it was looking at the overall usage of ICT in the Key Learning Areas (KLAs) in schools. This means that opportunities exist for studies whose focus is on the use of technology and not just ICT, but other technologies such as graphics calculators, spreadsheets, and other cognitive digital technologies within particular strands in a KLA such as within the 'Patterns and Algebra' strand with the Mathematics KLA.

2.3.1.2 The social and cultural factors

In addition to the political and economic factors, social and cultural factors also influence the push to integrate digital technologies in the mathematics curriculum. In relation to the social and cultural factors influencing the use of technologies in schools, the President of the Board of Studies NSW noted that:

> Most chroniclers of social and economic change are agreed on one thing: the information and communication technologies (ICT) are having and will continue to have a profound impact on us all. As the twentieth century comes to a close, the zeitgeist is clearly dominated by the information revolution with its emphasis on digitization and instant communication and retrieval of knowledge … We need to work towards a better understanding of the direction of the information and communication technology revolution so that we can be effective managers of the change in our sector (Stanley, 1999; p. 1).

In other words, the technology culture was out there in the society and the mission of the education sector entailed being ready to effectively embrace it. Stanley (1999) also implied that the use of technology such as ICT (which are only part of the digital technologies used in algebra learning) was social and the integration of these technologies in schools was an answer to the social needs created by the presence of the technology. This educational justification for the influence of the social and cultural factors in technology integration appears to be a way to implicate the education community despite studies that have questioned this. For example, Cuban, Kirkpatrick and Peck (2001) raised doubts about computers having any real meaning for learning and Thomas (2006) after a longitudinal study of teacher use of computers in mathematics wondered whether current teacher use of technology is qualitatively and quantitatively sufficient to promote any benefits.

2.3.2 The mathematical and epistemological factors

Advanced mathematical constructs such as those encountered in the teaching and learning of algebra (e.g., numbers, pro-numerals, terms, expressions, equations, inequations, and functions) are totally inaccessible to our senses, being capable only of being seen by the mind's eye (Artigue, 2002; Sfard, 1991). Put differently, algebraic constructs are abstract (or non-ostensive) entities. A student may think that the number or pro-numeral written down or a line representing a function drawn on paper or the board when studying algebra is 'the real thing'. To counter this, the teacher needs to carefully emphasize that the sign on the paper is but one among many possible representations of some abstract entity, which by itself cannot be seen or touched. In a technological world, important objects of algebra like functions and variables take on new meanings as they are no longer seen as mere abstract notions in the classroom (especially in the context of exploring real-world phenomena) (Goos, Stillman & Vale, 2007). The functions concept in particular, being multi-faceted (Lloyd & Wilson, 1998) cannot be fully understood within a single representation environment. According to a study by Even (1998), being able to make links between representations (Algebraic, Graphical and Numerical) (Fig. 2) is crucial to understanding the underlying concepts of functions.

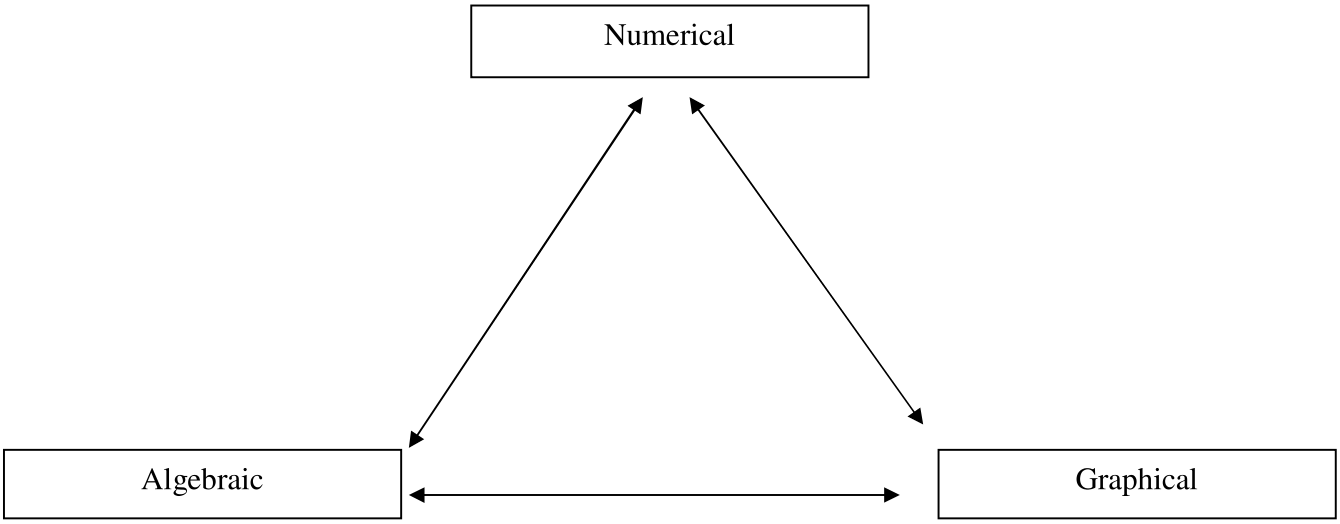

Figure 2.1: The links between function representations

Functions are fundamental to mathematics. The word 'function' expresses the idea that knowledge of one fact tells us another. In mathematics, the most important functions are those in which knowledge of one number tells us another number. Modern approaches to algebra regard functions as very important for the study of school algebra. According to Kieran (2007):

> … reformist algebra programs tend to give a great deal of weight to functions, to various ways of representing functional situations, and to the solution of "real-world" problems by methods other than manual symbolic manipulation, such as technology supported methods.(p. 709).

A characteristic of functions is that they can have a tabular, graphical, or algebraic representation (Figure 2). The logic underpinning the multirepresentational activity represented by the arrows in Figure 2 for algebra objects and processes is that technological tools ‘can step in and assist with problem solving when students do not have the required manipulative skills’ (Kieran, 2007). The other reason for multiple representations as shown in Figure 2 is that they provide a choice from which a student can create meaning about an algebraic process or object, that is, to develop the ability to ‘see’ abstract ideas behind the symbols (Sfard & Linchevski, 1994). The opportunity to coordinate objects and actions within two different representations, such as the graphical and the letter-symbolic or algebraic, is considered by many to be crucial in meaning making in algebra (e.g., Fey, 1989; Romberg, Fennema, & Carpenter, 1993; Yerushalmy & Schwarz, 1993).

Digital technological tools, such as graphics calculators, can make available these three different representations of algebraic objects. For example, studies by Confrey (1994) and also Confrey and Maloney (2008) have shown that spreadsheets (numeric and symbolic) can make tabular representations of functions a viable entry point for students, and tabular representations of a function can turn attention to a recursive definition of a function and a covariational approach to a function. In turn, the simultaneous availability of multiple representations of a concept can facilitate the development of meaning for the concept. Figure 2.2 below shows the symbolic, graphical and numerical representations of the family of functions represented by $y = x^2 \pm c$ for the TI 92 graphics calculator.

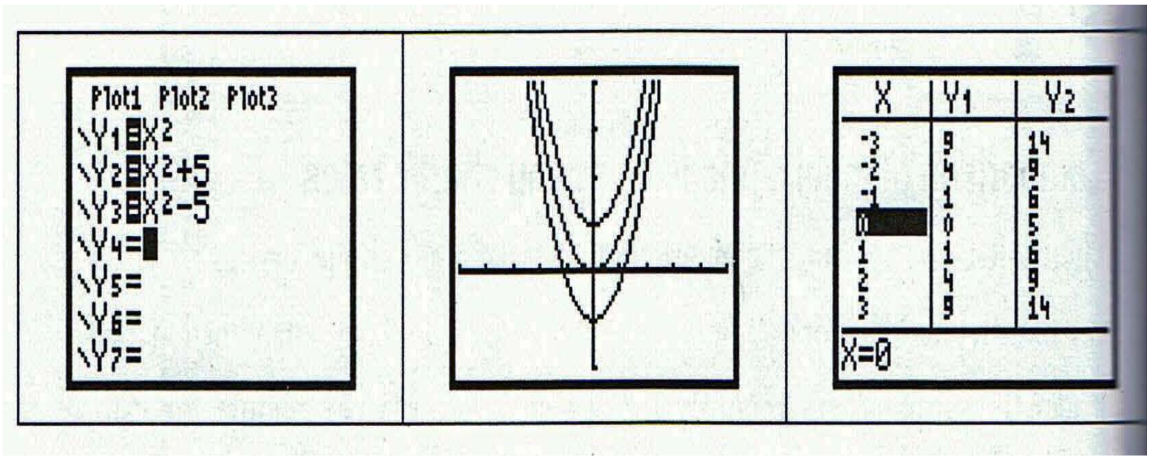

Figure 2.2: Symbolic, graphical and tabular representations of a family of functions represented by the general equation $y = x^2 \pm c$ (Goos, Stillman & Vale, 2007).

Schwarz and Hershkowitz (1999) showed that the zooming, scaling, and scrolling capacity of multiple representational tools and the activity of transforming influence students' reasoning about the whole-parts relations among different graphical representations of the same function. Once students identify a feature of a representation that matches with their thinking, technology can afford opportunity for exploration. Teachers need to be aware, however, that in algebra, as in other areas of mathematics, the capability of technology to link representations dynamically and to change them rapidly suggests that technologically produced representations may have different representations to the user than those produced by hand (Heid & Blume, 2008). Ainley (2000) pointed out that the dynamic and interactive features of computer–generated graphs support the user's control of the appearance of the graph and make the computer-based tools more likely candidates as technical tools or instructional devices than graphs that are generated by hand. The availability of multiple representations, however, is not a sufficient condition for algebra learning to occur. It is the actions taken on those representations that count (with the added advantage of the fact that potential actions are more varied in a technological setting). Additionally, the representations must be simultaneously present, and not sequential.

A study by Yerushalmy and Schwarz (1993) found that the sequential learning of the symbolic and graphical representations did not promote the tendency of the learner to move between them, because each of the representations presented to the learner a separate symbol system devoid of mutual and constructive interaction. Additionally, despite function-graphing technologies being able to provide students with the opportunities to make these links and to develop rich conceptual schemas, the students do not necessarily make the links merely by using the technology (Goos, Stillman & Vale, 2007). Even with the best designed tasks, there is always a place for teacher monitoring and possible intervention. For example, Arcavi (2003) reported that students do not always notice what an expert (the teacher) would expect in a graphing software environment. They may instead notice irrelevancies such as the graphs starting at the bottom of the screen which, according to Arcavi (2003), 'are automatically dismissed or unnoticed by the expert's vision' (p.36). The 'experts' too can have problems. For example Larbode (2007) and Hoyles and Noss (2003) found that some teachers are not aware of the role of multiple representations of algebraic concepts. In light of the findings from these studies, it is worthwhile to investigate: (a) NSW high school teachers' understanding of the mathematical and epistemological base that informs their use of multiple representational technology; (b) the ways in which these teachers monitor their students and the kind of interventions they make when students are learning algebraic constructs with multiple representational technology such as graphics calculators, and (c) their knowledge of students' mathematical thinking..

2.3.3 School or institutional factors

While the integration of digital technologies in the teaching of algebra is implemented at the social and epistemological levels and some mathematical practices of reference are indicated at the epistemological level, the school or institution provides the environment for the implemented curriculum. The differences at the school level are likely to arise because of variations in approach towards technology use from one school to another. The material factors in a school are an essential condition for using digital technologies: graphics calculators, computers, accessibility to the computer lab if it exists, funds for purchasing software, technical assistance, and professional development opportunities (Assude, Buteau & Forgasz, 2009). NSW Government and Non-Government high schools have not suffered the same high level of funding constraints in relation to the availing of these resources as has happened elsewhere like in the poorer developing countries.

There are also cultural factors at the school and especially at the mathematics faculty level and during professional development activities relating to the different ways that teachers work together. Using digital technologies successfully will be fostered if the algebra teacher is not isolated but treated inclusively as a member of a community of users of technology in the mathematics faculty.

Although the availability of the technological resources is necessary in a school before they can be used, it is not a sufficient condition. Thomas (2006) found that in some countries there are pedagogical materials ready for teachers to use the technology but the technology is not available and in others (e.g., France) the technology is available but there are no pedagogical materials. The assessment factors and requirements at the school level also impact on the extent to which it is used in class

. For example, Forgasz, Griffith and Tan (2006) reported on two studies in which teachers' views of graphics calculators use were examined. In one of these studies, a comparison was made between the views of Victorian teachers and those held by teachers from Singapore. According to the findings of this study:

> Mandating technology tool use in an assessment program, as was the case in Victoria, plays an important part in explaining the extent of their use by teachers, and may also account for the Victorian teachers' preference for graphics calculators over computers. (p. 4).

The second study revealed that, in general, teachers in Victoria 'believed that graphics calculators have had a positive impact on their teaching and on students' learning outcomes and that the curriculum has been enriched' (p. 5). In light of these findings, it is worthwhile to interview algebra teachers in New South Schools about their views on the impact (on student understanding) of mandating the school-wide use of certain digital technologies, like CAS (Computer Algebra Systems).

2.3.4 The classroom and didactical factors

In a mathematics classroom where digital tools are not being used for the teaching and learning, the normal interrelationships are between the teacher, the students and mathematical knowledge. The resulting complex educational system has been described by Steinbring (2005) as a didactic triangle. When 'technology' is added to the didactic triangle to create a fourth vertex, the didactic triangle is transformed into a 3D tetrahedron, creating three new triangular faces, each face illustrating possible inter-relationships among student, teacher, mathematical knowledge and technology as shown in Figure 2.3 below.

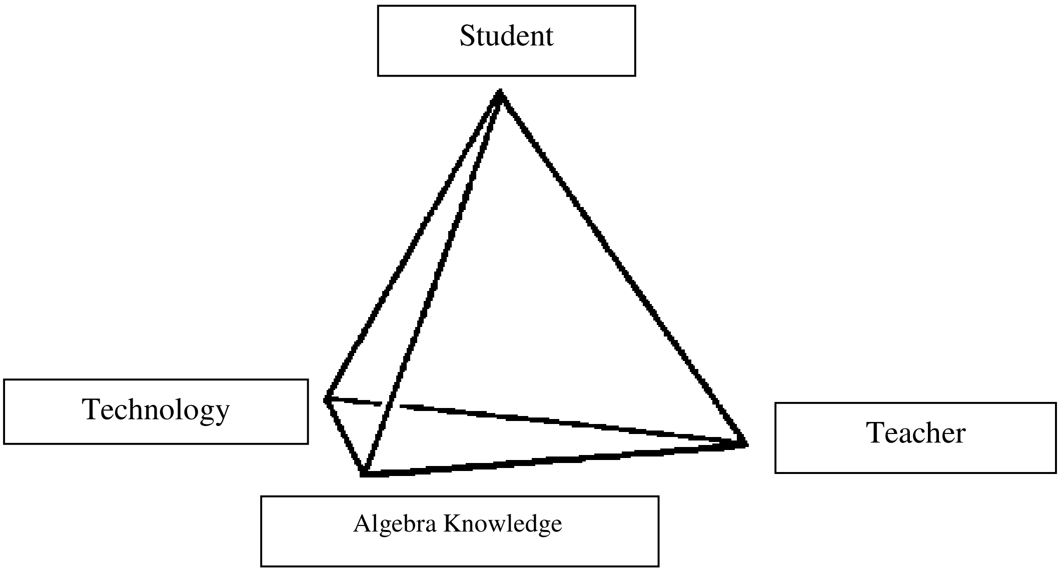

Figure 2.3: The didactic tetrahedron. Adapted from Olive and Makar (2009).

In a study of 13 high school mathematics teachers from seven schools in England, Monaghan (2004) explored their lesson planning, use of written materials, the areas in which they experienced tension and classroom interactions as they moved to significant use of technology during one school year. The teachers' activities were viewed through the lens of Saxe's (1991) Four Parameter Model. The study highlighted the complexity of the integration of technology into mathematics classroom practice regardless of whether teachers found it an 'easy' transformation of practice or not. Another finding was that the emergent goals for the teacher using technology were sometimes not the planned goals, with the study suggesting the need to train teachers to appreciate the dialectics of emergent goals in order for them to appreciate the dynamics of practice. Referring specifically to the social interactions aspect of technology use, Kieran (2007) found that the use of certain forms of digital technology as didactical tools occasioned mathematical discussions that do not normally occur in algebra classrooms. At the same time, students do experience problems with finding suitable language with which to talk about algebraic objects and processes, as well as their own algebraic thinking, within the classroom discussions (Kieran & Drijvers, 2006). According to Kieran (2007), the role of the teacher is of crucial importance if such discussions are to be fruitful. It would be interesting to find out teachers views on the role of language and discourse in algebra-with-technology classroom interactions.

On the use of written materials in algebra learning Kieran and Drijvers (2006) have suggested that the insertion of cognitive digital technology into the algebra classroom does not remove the need for paper-and-pencil algebraic techniques. This in turn suggests a need for research concerning the impact of classroom discussion and the teacher's management of this discussion on algebraic growth and understanding among students when digital environments are integrated with algebra learning and teaching. It is worthwhile to find out teacher's views on what kind of discussions they believe are most beneficial, how best to encourage students to present their ideas, with what kind of tasks, with which students, and at what specific moments.

2.4 Teachers' interventions and concerns

Although cognitive technologies are a powerful addition to the algebra classroom (in the sense that they may enhance student understanding of algebraic concepts), they can only become real learning tools when supported by appropriate teacher intervention; that is, teacher intervention that comprehends the nature of the technology and the way students think in algebra, and also acknowledges the complexity of technology integration in the teaching and learning of algebra. Zbiek and Hollebrands (2008) have suggested that teachers using or intending to use technology move along a learning continuum to reach the position of integrating technology extensively and well in their teaching of algebra. As they go through this developmental process they have concerns and exhibit particular ways of drawing technology into their practice

2.4.1 Teachers' concerns

Teaching with technology is an innovation that elicits both affect and action. Concerns are the affective aspect of implementing technology in teaching. It has been found that teachers' concerns generally change as they become more knowledgeable about the use of technology (Zbiek & Hollebrands, 2008). Halls and Hord (1987, 2001) addressed the issue of teachers' concerns by developing the Concerns-Based Adoption Model (CBAM) in which they described the stages through which teachers pass when they encounter innovations. Hall and Hord's (1987, 2001) Stages of Concern are 1.personal concerns, 2. management concerns and 3.technological concerns. These stages are of particular use in helping to organise research insights about a teacher's technology use.

2.4.1.1 Personal concerns

Teachers using technology in mathematics have expressed personal concerns about how the technology would affect them personally as teachers in their own classrooms. For example, teachers using a variety of computer software packages with elementary and middle school students in New Zealand were concerned about how to arrange the classroom and place themselves physically in the room (Thomas, Tyrell, & Bullock, 1996). McDougall (1996), in a study of four teachers and found that one of them exhibited a high degree of fear about revealing to students her lack of complete knowledge of the mathematical relationships the students were discovering. It is clear that for this teacher, there was fear about changing the role of the teacher from one who was viewed as the sole mathematical authority to one who was a co-constructor of knowledge. The second concern for teachers is the difference (when using technology) between the expected and transformed mathematics and the lack of preparedness on the part of the teacher for this transformation (Monaghan, 2004).

Monaghan (2004) reported such a situation in relation to a teacher who was using the spreadsheet *Excel* software tool. This teacher's lesson planning focussed on mathematical ideas not particularly concerned with technology, but in practice, 'techno-mathematical' ideas became the focus of the lessons. For example, in one of her lessons focused on ratio, she set an investigation where three children were left a sum of money in a will to be distributed over a number of years in the ratio of their ages. The students', and subsequently her, foci or emergent goals in this lesson were on filling the spreadsheet cells correctly, not only with the correct equation but also using a suitable cell format. She commented after the lesson that she was unhappy with this focus on 'cell arithmetic'. This example is an instance of tool use transforming mathematics, with the problem arising of the teacher being unprepared for this transformation. A similar finding (in which the mathematical focus of the lesson had to be put aside in order to focus on how to perform the technical operation) emerged from a study by Lumb, Monaghan and Mulligan (2000) for a teacher using the algebra software tool called *Derive* in the process of finding $\frac{dy}{dx}$ for $y = xsinx$.

The students were expected to display the by-hand process of $\frac{dy}{dx} = sinx\frac{dy}{dx}(x) + x\frac{dy}{dx}(sinx)$, but among the many ways of differentiating with *Derive* there was none that involved the essential (to the teacher) intermediate step of $sinx\frac{dy}{dx}(x) + x\frac{dy}{dx}(sinx)$. It is an aim of this study to explore the personal concerns that the selected teachers have relating to the use of technology in their algebra lessons and the differences between what they intend to use the technology for and the transformed algebra that emerges together with the associated tension.

2.4.1.2 Management Concerns

Several management concerns arise for teachers using or intending to use technology. One such concern is the issue of having control of and managing technology-enabled classroom environments. For example, Lampert (1993) found that high school teachers who adopted a guided-discovery approach when using certain technologies felt unsure of which students were learning what, in a situation where students were able to use the technology to pursue their own correct or incorrect conjectures. It is possible that the need of these teachers to know which conjectures the students were pursuing so that they could manage the classrooms may have influenced the ways in which they structured technology-based activities. It could also be that those teachers who perceived a greater need for control might have created activities that were less open-ended and more directive in terms of how technology should be used.

The issue of how the technology is allocated, for example whether the teachers use a room that has computers for every lesson or where they use a computer lab for only a few of the lessons is a management concern. Monaghan (2004) found that teachers with and without computers in their classes planned the use of, and spontaneously used, computers in different ways. Teachers who did not have computers in their class mainly planned 'all or nothing' computer-based lessons, i.e., all the lesson on the computer or no use of the computer at all. This seriously affected their lesson planning. It was also found that teachers using computer rooms without a board for writing on generally devoted the last part of the lesson prior to the computer-based lesson to explaining what was going to be done with the computers in the next lesson.

The extent to which allocation and privileging of resources in the classrooms is a management concern may be related to concerns about consequences that are related to their goals for student learning (Zbiek & Hollerbrands, 2008). Tharp, Fitzsimmons, and Brown Ayers (1997) found, for example, that teachers who held rule-based conceptions of mathematics teaching and learning were more likely than their peers to use a highly structured approach to the use of the graphing calculator in precalculus. Rule-based high school teachers interviewed by Akujobi (1995) were more likely than their colleagues to reserve any use of computer software for remediation or for drill and practice. On the other hand, in a large scale survey of middle and high school teachers' use of computers in the US state of Missouri it was reported that teachers rarely used the computers for purposes other than drill and practice (Manoucherhri, 1999). In terms of specific technologies, teachers' views of graphing technologies may influence the ways in which they use them. Jost (1992) found that teachers who perceived the graphing calculator as a computational tool stressed content oriented goals and viewed learning as listening. Teachers who viewed graphing calculators as instructional tools stressed student-centred goals, using interactive teaching approaches, and viewed learning as student-centred. However, the ways in which teachers use technology in their classrooms does not seem directly related to how they may have experienced technology as a learner (Fleener, 1995). Zbiek (1995) reported that although a teacher of first–year secondary school algebra enjoyed and valued open-ended explorations with technology in her own learning she did not feel comfortable allowing her students to engage in such tasks. Similar results were reported by Timmerman (1998) and also by Olive and Steffe (1994).

2.4.1.3 Technology concerns

From a study by Hall and Hord (1987, 2001), concerns about how the technology affected teachers personally and concerns about management seemed to take priority over, and perhaps to precede, concerns about learning. Thomas, Tyrrell and Bullock (1996) reported that it was not until the elementary, middle, and high school teachers they studied were familiar with the technology and had created ways to incorporate it into their teaching that teachers focused their concerns on how technology could be used to engage students in exploring mathematics. Findings from several research studies (e,g., Beaudin & Bowers, 1997; Tharp, Fitzsimmons & Brown Ayers, 1997; Wiske & Houde, 1993) suggest that there may exist stages through which teachers pass or a level of comfort and familiarity with the technology and curriculum before they are able to focus on students and what they are learning.

Basing some of the interview questions on the categorisation of concerns by Wiske and Houde (1993) would be particularly interesting for algebra because they developed their categorisations based on an examination of the nature and progression of teachers' concerns as teachers incorporated the Geometric Supposer (a geometry application that allows students to develop mini-programs that draw shapes). Using Hall and Hord's (1987) Stages of Concern about innovations in teaching, Wiske and Houde (1993) categorised the concerns into four clusters: Unrelated Concerns, Self Concerns, Task Concerns, and Impact Concerns.

Unrelated concerns are not related to teaching or learning, self concerns are focused on the teacher using the technology to gain personal understandings of what it can and cannot do. These concerns also revolve around teachers' beliefs about whether they posses the knowledge needed to use the technology in the classroom and how its use will affect them personally. Task concerns are focused around selecting appropriate tasks and managing the use of technology in the classroom. Impact concerns are focused on understanding what impact the technology will have on students, determining how to collaborate with others who are using technology, and thinking about how the use of technology may affect the curriculum. Impact concerns seem to consider the effects of technology broadly by considering technology's impact on students, on other teachers, and on the curriculum. This seems to suggest a recognisable pattern to teachers concerns. For this investigation, the researcher will be interested in determining whether such a pattern of concerns (excluding the unrelated concerns category) applies to technology used in the teaching and learning of algebra.

2.4.2 Modes of use of technology

Teachers move along a continuum from nontechnology stances to incorporating technology extensively and well in their teaching of mathematics. A number of studies provide insights into what teachers do with technology on their own and in their classrooms as they move along the continuum. Hall and Hord (1987, 2001), for example, developed a model alongside their Stages of Concern which includes Levels of Use. The other one is the PURIA model of Beaudin and Bowers (1997).

2.4.2.1 Hall and Hord's Levels of Use model

The five different levels of use in this model are: Mechanical, Routine, Refinement, Integration, and Renewal (Hall & Hord, 1987, 2001). In general, the use of an innovation begins with a mechanical use for which the person may closely follow a user guide to complete specific activities and may not have long-range plans for how the innovation may be used next week or next month. An example is the use of a graphing calculator. At this first level a teacher may try a set of graphing-calculator lessons using prescriptive handouts without considering how or if at all the calculators will be used in future lessons. After employing the innovation for sometime the person may develop some patterns and routines for using it. At this level, the teacher may establish patterns for distributing and collecting the calculators and methods for performing mathematical tasks such as graphing or curve fitting. The teacher may have longer-range plans for calculator use and these plans may be stable or unchanging.

At the next level, users may refine their use of the innovation with a focus on their clients (students in the teacher's case). For example, the graphing-calculator lessons may be revised to match the local students' needs. At the level of integration, teachers may recommend the innovation to their colleagues so that through a collective effort they can extend the positive effects of the innovation to a larger population of students. The graphing calculator lessons become a part of what other teachers use. The last level, renewal, occurs when teachers make major changes or modifications in their use of the innovation to improve outcomes. Using Levels of Use, however, seems not to be useful when discussing the intertwined components of learning to teach mathematics with technology.

2.4.2.2 The Play, Use, Recommend, Incorporate, Assess (PURIA) model

The PURIA (Play, Use, Recommend, Incorporate, and Assess) model (Beaudin & Bowers, 1997) provides a perspective that allows for the explicit consideration of teachers' needs to learn the technology, to learn to do mathematics with technology, to use the technology with students, and to attend to student learning as a guide for innovation. Table 2.1 shows in the left column the stages by Beaudin and Bowers (1997) and in the right column their description by Zbiek and Hollebrands (2008). While the original version of the PURIA modes by Beaudin and Bowers (1997) were intended for CAS (Computer Algebra Systems) use, the version in the table below was extended by Zbiek and Hollebrands (2008) to technology beyond CAS.

Table 2.1: The elaborated and extended PURIA Model (Zbiek and Hollebrands (2008)

PURIA Mode	**Nature of Activity during the Mode**
Plays with the technology	Uses technology for no clear mathematical purpose
Uses technology as personal tool	Uses technology in doing mathematics of one's own design. May be using it as a learner of mathematics but not using it in a classroom setting and not using it with students
Recommends technology to others	Recommends use to a student, a peer, or a small group of students or peers. This likely is not in a formal classroom setting and it is not an integrated part of instruction
Incorporates technology into classroom instruction	Integrates the technology into classroom instruction. This occurs to varying degrees
Assesses students' use of technology	Examines how students use the technology and what they learn from using it.

The extension of the original PURIA modes by Zbiek and Hollebrands (2008) reflect the teacher becoming familiar with the technology as a tool for doing mathematics in the Play and Use Modes. The growth during these modes includes the transition of the technology as the developer's tool into the teacher's instrument for doing mathematics, a crucial aspect of learning to use technology captured by the notion of instrumental genesis (Guin & Trouche, 1999; Trouche, 2000). In the Incorporate and Assess modes, the teacher's attention turns, implicitly or consciously, toward the use of the technology as a pedagogical tool, including the development of instrumental orchestrations (Trouche, 2000) or elaborated plans regarding use of technology in the social dimensions of classrooms. The Recommends mode seems marked by a transition between mathematical and pedagogical aspects of the technology. The first two modes suggest that teachers use the technology, playfully at first and then in a more structured manner, for their own personal use prior to using it with students. After teachers are knowledgeable about the tool, they may share ideas with colleagues or recommend that their students use it in limited ways outside of main classroom activities. This initial limited use may provide teachers with the confidence needed to incorporate the use of the tool into classroom practice. After the technology has been incorporated, the teacher may feel ready to assess what the students are learning with the technology involved (Zbiek & Hollebrands, 2008).

A study by Sarama, Clements, and Henry (1998) and another by Pagnucco (1994) have noted the difficulty of moving from the Play to Incorporate modes, even in encouraging environments. The literature also highlights the critical role of the Recommend mode as teachers worked together formally and informally or as they worked with individual students or with small groups of students (Stallings, 1995; Almekbel, 2000; Van Netta, 2000). In particular, the data from Van Netta's (2000) study implicitly suggests an absence of Recommend mode while reporting a negative classroom experience with technology. It is also possible for some teachers to progress very little within a mode. Some studies reported little change over time in the ways in which teachers used technology (Kendall & Stacey, 2001) while other studies reported great improvements (Piliero, 1994; Tharp, Fitzsimmons, & Brown Ayers, 1997; Thomas, Tyrrell, & Bullock, 1996; Zbiek, 1995).

2.5 Addressing PTK shortcomings: From PCK to PTK

One of the factors influencing teachers' use of technology in the classroom is their pedagogical content knowledge (PCK) (Shulman, 1986). Pedagogical content knowledge in the specific case of algebra refers to understanding the algebra ideas that are involved in a particular topic and how these relate to the principles and techniques required to teach and learn it, including the appropriate structuring of content and relevant classroom discourse and activities (Chinnappan & Thomas, 2003; Cooney, 1999; Shulman, 1986; Simon, 1995).

2.5.1 Pedagogical Technology Knowledge (PTK)

While PCK provides insight into transformations that teachers need to make with their mathematics subject-matter knowledge so that that knowledge can be learned by their students, it fails to fully capture the integration of knowledge of technology with subject-matter knowledge by teachers of mathematics in contexts where learning is supported with technological aids (Thomas & Chinnappan, 2008). As a result of this inadequacy, the notion of pedagogical technical knowledge (PTK) was developed by Thomas and Hong (2005).

When applied to algebra, PTK includes the principles, conventions, and techniques required to teach algebra through technology (Hong & Thomas, 2006; Thomas & Hong, 2005). It partly comprises teachers' perspectives on the algebra and the technology, and partly their familiarity with it. For PTK to become a valuable tool for teachers of school algebra, certain conditions need to be met. For example, according to Thomas and Chinnappan (2008), PTK requires a new mindset on the part of the teachers; a mindset characterised by a shift of mathematical focus to a broader perspective of the implications of the technology for the learning of algebra. Teachers need to appreciate that technological tools may be employed in teaching in qualitatively different ways, and that students may form a variety of different relationships with them depending on the teacher's directional emphasis or, according to Kendal and Stacey (1999, 2001), depending on 'teacher privileging'. Kendal and Stacey concluded, from observing different approaches to the teaching of algebra by two teachers, that teacher privileging shapes student preference so that they follow the teacher's lead. In other words, privileging can have a positive impact on students' uptake of technology in exploring algebra. The benefits of privileging also provide justification for the argument that teachers who have built up a sound store of PTK are more likely to feel comfortable in accessing tools of technology in designing algebra learning experiences.

2.6 Theoretical frameworks on the roles of digital tools

The theories in the first part of the literature review explicitly address teachers' roles and concerns when they use digital technology in mathematics and the way they use technology. In the second part, Section 2.6, theoretical frameworks about the role of the digital technology tools are discussed. This examination is relevant to the inquiry because the way the teachers use the tools in the teaching of algebra is inextricably linked to the capabilities of these tools.

More than one theoretical framework has been adapted for this study because no single theoretical framework can explain all phenomena in the complex setting of learning algebra (or any other branch of mathematics) in a technology-rich environment (Schoenfeld, 2002). Different theoretical frameworks offer different windows on it, and each view on the landscape can be sound and valuable. In fact, the different perspectives are more likely to be complementary, with each one of them contributing to the whole picture.

2.6.1 Technology as amplifier and organiser

Pea (1987, p. 91) described cognitive technologies as media that help 'transcend the limitations of the mind in thinking, learning, problem-solving activities'. He distinguished two functions of cognitive tools: the amplifier function and the organiser function. The former refers to the amplification of possibilities, for example by investigating many cases of similar situations at high speed. The complexity of the examples can be amplified as well, thus allowing for a wider range of examples with an increasing variation. In this way, information technology makes accessible insights that otherwise would be hard for students to gain.

Although some (e.g. Heid, 2002) have claimed that the amplifier function of cognitive technology has little or no effect on the curriculum, it was believed by others that it may be used to invite algebraic generalisation (Mason, 1996). For the algebra teacher, the best way to make use of the amplifying and reorganizing capabilities of cognitive technological tools is by first becoming aware of the two-way reorganisation possibilities afforded by this technology—cognitive tools affect teachers and students, but also that teachers and students affect the technology (Pea, 1987). In particular, teachers affect the technology both by the way they decide on the appropriate ways of using them and how, in refining educational goals, they change the technology to provide a better fit with these goals. In the case of learners, cognitive re-organisation occurs when their interaction with technology as a new semiotic system qualitatively transforms their thinking; for example, the use of spreadsheets and graphing software can alter the traditional privileging of algebraic over graphical or numerical reasoning (Goos, Galbraith, Renshaw & Geiger, 2003).

2.6.2 Tutor, Tool, Tutee

A new framework emerged with the arrival and increasing proliferation of the microcomputer. In this framework, educational computing activity was classified according to three modes or roles of the computer: tutor, tool, and tutee (Taylor, 1980). To function as a *tutor*, “the computer presents some subject material, the student responds, the computer evaluates the response and, from the results of the evaluation, determines what to present next’ (p. 3). An online tutorial program *HOTmaths* (2010), which offers tutorials for upper primary and junior high school students in Australia, is an example of a program acting in tutor mode. To function as a *tool*, the computer requires, according to Taylor (1980), much less in the way of expert programming than is required for the computer as a tutor and can be used in a variety of ways (e.g. as a calculator). The third mode of educational computing activity, that *tutee* was described by Taylor as follows: "To use the computer as tutee is to tutor the computer; for that the student or teacher doing the tutoring must learn to program, to talk to the computer in a language it understands" (p. 4). The rationale behind this mode of computing was that the human tutors would learn what they were trying to teach the computer and hence would gain new insights into their thinking through learning to program.

2.6.3 White Box-Black Box

Put forward by Buchberger (1989), the White Box-Black Box (WBBB) notion is a theoretical idea that focused on the interaction between the knowledge of the learner and the characteristics of the technological tool. According to Buchberger (1989), the technology is being used as a white box when the students are aware of the mathematics they are asking the technology to carry out; otherwise the technology is being used as a black box. He argued that the use of symbolic manipulation software as a black box can be disastrous for students when they are initially learning some new area of mathematics (a usage similar to the tool mode within the Tutor-Tool-Tutee framework). However, other researchers (e.g., Heid, 1988; Berry, Graham, & Watkins, 1994) showed that students can develop conceptual understanding with symbolic manipulation software before mastering by-hand manipulation techniques.

2.6.4 Representation

Cognitive technological tools have representational potential that can be exploited in the teaching and learning of algebraic concepts. Researchers have, however, found it difficult to come up with a coherent and unifying theoretical frame for representation (Kaput, 1987). Kaput (1987) proposed that a concept of representation ought to describe the following five components; the represented entity, the representing entity; those particular aspects of the represented entity that are being represented, those aspects of the representing entity that are doing the representing; and the correspondence between the two entities. This framework served as a basis for conceptualizing several representational studies involving the three representations of the tabular, the graphical, and the symbolic (e.g., Schwarz & Bruckheimer, 1988).

One example of a multiple representational tool that can appropriately serve as a bridge to algebra and algebraic symbols (if embedded in a suitable sequence of activities) are the spreadsheet environments (Kieran & Yerushalmy, 2004). One of their main advantages is that they allow for an integrated use of numerical, graphical, and algebraic representations. A problematic area with spreadsheets is that the difference between spreadsheet formulas and algebraic expressions is sometimes more than syntactic and can cause some conceptual difficulties. In particular, there is a lack of transparency in spreadsheet tables, with the formulas being 'hidden' behind the resulting numbers, leading students to encounter both cognitive and technical difficulties in monitoring their work (Tabach & Friedlander, 2004). Generally, however, new technologies offer the opportunity of exploiting the coordination of multiple representations of algebraic concepts-both within a given cognitive technological environment and between a given cognitive technological environment and the traditional paper-and-pencil environment (Kieran & Yerushalmy, 2004).

2.7 Theoretical frameworks based on the theoretical writing of Vygotsky

While constructivism and its Piagetian roots provided the underpinnings of the theoretical elaborations that emerged in research related to technology use in the decades up to 1980s (elaborations that focused primarily on the cognitive aspects of learning), the theoretical writing of Vygotsky with its sociocultural emphasis began to receive attention within the international mathematics education community from the 1980s onwards (Streefland, 1985). The steady growth in the development of sociocultural perspectives was reflected, for example, in the 1995 PME (Psychology of Mathematics Education) Conference where presentations were devoted to Vygotskian theory (Meira & Carraher, 1995).

Sociocultural theories view learning as the product of interactions with other people and with material and representational tools offered by the learning environment. The sociocultural perspective offers rich insights into conditions affecting the innovative use of technology in school mathematics because it acknowledges the complex, dynamic, and contextualized nature of learning in social situations. Examples of frameworks based upon or which borrow heavily from Vygotskian theory include Valsiner's (1997) zone theory, instrumentation (Rabardel, 2001) and Webbing and Situated Abstraction (Noss & Hoyles, 1996). The next section discusses Valsiner's Zone Theory.

2.7.1 Valsiner's Zone Theory

Recently adapted by Goos and Bennison (2008) in their investigation of teacher's use of technology in secondary mathematics classrooms in Queensland to apply to interactions between teachers, students, technology, and the teaching-learning environment, Valsiner's (1997) zone theory was originally designed as an explanatory structure in the field of child development. It is a framework which extends Vygotsky's (1978) concept of the Zone of Proximal Development (ZPD). The ZPD has been defined as the gap between the learner's present capabilities and the higher level of performance that could be achieved with appropriate assistance-to incorporate the social setting and the goals and actions of participants.

Valsiner (1997) described two additional zones; the Zone of Free Movement (ZFM) and the Zone of Promoted Action (ZPA). The ZFM structures an individual's access to different areas of the environment, the availability of different objects within an accessible area, and the ways the individual is permitted to or enabled to act with accessible objects in accessible areas. The ZPA represents the efforts of a more experienced or knowledgeable person to promote the development of new skills. In terms of teachers' professional learning, the ZFM can be interpreted as constraints within the school environment, such as student characteristics, access to resources and teaching materials, and curriculum and assessment requirements, while the ZPA represents opportunities to learn from pre-service teacher education, colleagues in the school setting, and professional development (Goos & Bennison, 2008).

The different kinds of knowledge and experience identified by previous research on technology use by mathematics teachers (Fine & Fleener, 1994; Forgasz & Prince, 2001, Manouchehri, 1999; Simonsen & Dick, 1997; Wallen, Williams, & Garner, 2003) represent elements of a teacher's ZPD, ZFM, AND ZPA (Goos & Bennison, 2008). They include a range of factors influencing uptake and implementation such as skill and previous experience in using technology, time and opportunities to learn (pre-service education, professional development), access to hardware and software, availability of appropriate teaching materials, technical support, institutional culture, knowledge of how to integrate technology into mathematics teaching, beliefs about the role of technology in learning, and beliefs about mathematics and how it is learned. This is summarized in Table 2.2 below:

Table 2.2: Factors affecting technology usage (Goos & Bennison, 2008).

Valsiner's Zones	**Elements of the Zones**
Zone of Proximal Development(ZPD)	Skill/experience in working with technology Pedagogical knowledge (technology integration) Pedagogical beliefs (technology; mathematics)
Zone of Free Movement(ZFM)	Access to hardware, software, teaching materials; Support from colleagues (including technical support); Institutional culture Curriculum and assessment requirements Students (perceived abilities, motivation, behaviour)
Zone of Promoted Action(ZPA)	Pre-service education (university program); Practicum and beginning teacher experience; Professional development

The goal for this study is to investigate the extent to which these elements apply to algebra teaching and learning with technology through an in-depth study of a few algebra teachers' institutional and actual technological tools use experiences. This will add to the knowledge and experiences identified from a number of studies (in mathematics as a whole) on teachers' and students' use of cognitive technologies in Australia (Forgasz, 2002; Routitsky & Tobin, 1998; Tobin, Routitsky, & Jones, 1999; and Loong, 2003).

2.7.2 Instrumental approaches

Instrumental approaches in the use of technological tools for the learning and teaching of mathematics have emerged from dissatisfaction with the overly optimistic conclusion of earlier researchers about the potential of these tools (Trouche, 2003). According to Penglase and Arnold (1996), many authors believed that graphics calculators have the potential to revolutionize mathematics education, both in the way it is taught and the content and emphases of curricula, which led to much enthusiastic comment about the capabilities, potential and implications of this technology. Two weaknesses (or illusions) can be discerned from this research; the illusion of a naturally positive contribution to learning (the technological environment allows one to see and hence to understand) and the illusion of a positive contribution to teaching (the technological environment takes care of the technical part of the task leaving the learner to concentrate on the conceptual part).

Later research exposed these illusions. For example, Hershkowitz and Kieran (2001) noted that technological environments could induce students to reach *false representatives* (those that do not represent the critical properties of the mathematical entity at all) or to interweave representatives together in a non-meaningful, algorithmic fashion. Guin and Trouche (2002) (cited in Trouche, 2003) also noted the possible confusion between a mathematical object and the representations a calculator gives of it as part of the didactic phenomena appearing in earlier studies of students' work. These phenomena call into question the assertion that possibilities of visualization and manipulation of mathematical objects afforded by technological tools necessarily lead to a better comprehension of concepts (Hoyles, 2001). They also expose the neglect by earlier researches of the extent to which the tools were at the students' disposal and that to which the students were familiar with these tools (Penglase & Arnold, 1996).

Following the study of a large amount of research from several countries, Lagrange, Artigue, Larbode, and Trouche (2001) showed that the influence of tools on student activity and the knowledge constructed can be accounted for through new approaches which present the relationship between perception and conceptualization in a more dialectic way. In particular, they distinguished two dimensions that run through recent research on the use of technological tools in mathematics teaching and learning; the institutional dimension and the instrumental dimension. The institutional dimension investigates the extent to which content to be taught as well as tasks and procedures or techniques are affected by the institution in which they are taught (Chevallard, 1999; cited in Trouche, 2003)

2.7.2.1 The instrumental dimension

The instrumental dimension of cognitive technologies distinguishes an artefact and the instrument which a human being is able to build from this artefact (Rabardel, 2002). There are several instrumental approaches to the use of cognitive technological tools in teaching and learning. Only one, as presented by Verillon and Rabardel (1995) in relationship with mathematics education, is discussed in this study. Verillon and Rabardel's (1995) instrumental approach is based on two key elements:

(a) the pre-structuring of both the *action* modes and the *cognitive* modes of the user of an artefact through the constraints it imposes on the user and;

(b) the long process through which the artefact becomes an instrument called instrumental genesis. This process is linked to the characteristics of the artefact and those of the subject. It is important to note that the notion of artefact is quite a wide one, because although it is often a physical object, it is not always one. As an example of an artefact which is a non-physical object, Bueno-Ravel and Gueudet (2007) have described a scenario for using computer algebra in algebra teaching that is made available in a digital working space for professional development and which teachers can use to shape their teaching of algebra. In general, an instrument is the result of a meaningful relationship developing between the artefact and the user for a specific type of task. In very simple terms, an instrument can be defined as: Instrument = Artefact + Schemes and Techniques, for a given type of task. The process of an artefact becoming an instrument, called *instrumental genesis*, is an ongoing, non-trivial and time-consuming evolution, during which a bilateral relationship between the artefact and the user is established.

The student's knowledge guides the way the tool is used and, in a sense, shapes the tool in a process called *instrumentalization*, while the affordances and constraints of the tool influence the student's problem-solving strategies and the corresponding emergent conceptions in a process called *instrumentation*. The dual nature of instrumentation and instrumentalization within instrumental genesis comes down to the student's thinking being shaped by the artefact while also shaping the artefact (Hoyles & Noss, 2003).

The difference between instrumentalization and instrumentation can be illustrated using the Texas Instrument (TI-84) graphics calculator by a student. The menu option 'Calculate Intersect' of a TI-84 calculator is an artefact that calculates the coordinates of intersection points of graphs. On the one hand, this artefact enriches the student's view of solving equations with a graphical representation (Drijvers, Kieran & Mariotti, 2010). On the other hand, the artefact may limit the student's conception of solutions, as the results are restricted to rounded-off decimal values instead of exact values. If the student were to program the calculator so that it gives the solutions in the form of radicals it would be an example of instrumentalization.

The transforming of an artefact into an instrument (instrumental genesis) involves the development of schemes and techniques. Schemes and techniques are viewed differently depending on the instrumental approach under consideration. The cognitive ergonomy approach uses Vergnaud's (1996, cited in Trouche, 2003) definition of a scheme as an invariant organisation of behaviour for a given class of situations (in other words, it is a more or less stable way to deal with specific situations or tasks). Since schemes are parts of instruments, they are often called instrumentation schemes. Two types of schemes can be observed within instrumentation schemes; schemes of instrumented action and utilization schemes. Since schemes cannot be observed directly (Drijvers, et al, 2010), the focus is placed instead on the observable instrumented techniques. Instrumented techniques are defined as a more or less stable sequence of interactions between the user and the artefact with a particular goal. Of particular significance for this study is the essential idea in both schemes and techniques that technical and conceptual aspects co emerge and are closely related (Trouche, 2003).

2.7.2.2 Orchestration

Instrumental genesis has a social dimension-students develop mental schemes within the context of the classroom community. A process of collective instrumental genesis is therefore taking place in parallel with the individual geneses. Trouche (2004) introduced the notion of *instrumental orchestration* to describe the process of collective instrumental genesis and the management of the individual instruments by the teacher in the collective learning process. An instrumental orchestration is the intentional and systematic organisation of the various artefacts available in a computerised learning environment by the teacher for a given mathematical situation, in order to guide students' instrumental geneses (Trouche, 2004).

An instrumental orchestration is defined by didactic configurations (i.e., arrangements of the artifactual environment according to the various stages of the mathematical situation) and exploitation modes of these configuration modes (Brousseau, 1998). It is worthwhile noting that teachers' instrumental geneses take place while they (the teachers) are orchestrating students' collective instrumentation, because the artefacts in use by the students are artefacts that the teachers use when teaching. At the same time, teachers have artefacts that are exclusive to them such as their own electronic resources, teaching experiences, and teaching scenarios-making instrumentation theory a double-layered and fruitful tool in teacher training on orchestrating technology (Bueno-Ravel & Gueudet, 2007).

Instrumental approaches to tool use in mathematics have been used, for example, in the study of the use of spreadsheets in mathematics learning (Haspekian, 2005). Like other frameworks, instrumental approaches have their inadequacies (such as the imbalance between the cognitive ergonomics and the anthropological approaches). According to Drijvers et al (2010), however, combining instrumental approaches with other theoretical perspectives can be a fruitful avenue for research on the integration of technology in mathematics learning and teaching. Limiting itself to high school algebra, this study has adapted aspects from instrumental approaches, together with others from other frameworks, to investigate interactions between teachers, students, technology, and the teaching-learning environment.

2.7.3 Mediation and Semiotic Mediation

Mediation and semiotic mediation were first elaborated by Vygotsky (1978). Because of its epistemological nature, any immediate relationship with algebra (or any other area of mathematics) is impossible; any relation must pass through a mediation process. In other words, ideal, immaterial, nonperceivable, entities (including algebra objects such as terms, expressions and equations), can only be thought of and shared through their materialization in a concrete perceivable entity, generally referred to as a representation (Drijvers et al, 2010). Figure 5 below illustrates a model of the learning process (adapted from Jones, 2000), inspired by a socio-cultural approach that focuses on the use of an artefact and is expressed in terms of mediation. In this model, artefacts (such as the cognitive technological tools used in the teaching and learning of school algebra) are considered to only to be a means to accomplish a concrete action, such as a calculator to compute multiplication, but are also considered to be a means for learning.

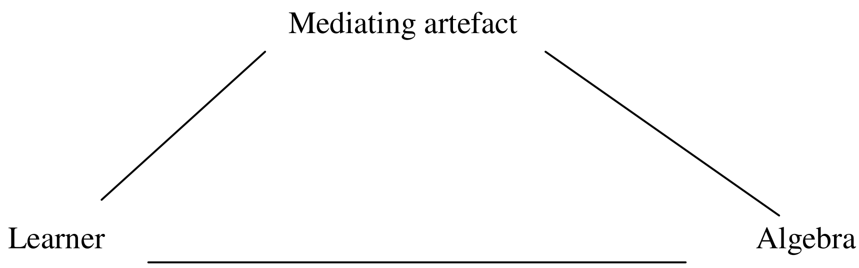

Figure 2.4: A model of the process of mediation by an artefact (adapted from Jones, 2000)

Semiotic perspectives (which focus on the role of signs and symbols and their use or interpretation) can contribute to the study of the integration of technologies into mathematics teaching and learning (Sàenz-Ludlow & Presmeg, 2006). The determination of the semiotic potential of an artefact constitutes a basic element for designing any pedagogical plan centred on the use of a given artefact. Since algebra is rich in signs and symbols, semiotic approaches can be relevant to studies involving the teaching and learning of algebra. Being ideal, immaterial and non-perceivable, algebraic objects and processes (e.g., terms, numbers, and expressions) can be thought of and shared or they acquire existence only through their materialization in a concrete perceivable entity, generally referred to as representation (Drijvers et al, 2010). In other words, mediation processes are necessary for a teacher or pupils to have any relation with algebra objects and processes. The mediating potential of any artefact resides in the double semiotic link that such an artefact has with both the meanings emerging from its use for accomplishing a task, and the mathematical meanings evoked by that use, as recognised by an expert in mathematics (the teacher, in the case of school algebra).

When a semiotic process (using signs, words, gestures, drawings) takes place in the classroom, social interaction may assume a common goal oriented to teaching/learning algebra. Both pupils and the teacher may be involved in the evolution of signs referring to personal meanings. According to Cobb, Wood, and Yackel (1993), the teacher's action will be at both the cognitive and meta-cognitive levels (i.e., both fostering the evolution of meanings and guiding pupils to be aware of their mathematical status). In acting at the meta-cognitive level, the teacher's role becomes crucial: taking the educational goal of introducing pupils to algebra culture, the teacher plays the role of cultural mediator. Purposely, the teacher bridges the individual and the social perspectives and makes the artefact function as a semiotic mediator and not simply as a mediator. It is the awareness of the semiotic potential of an artefact (both in terms of mathematical meanings and in terms of personal meanings) which allows the teacher to use the artefact as a tool of semiotic mediation. Stating this differently, any artefact will be referred to as a tool of semiotic mediation as long as it is (or it is conceived to be) intentionally used by the teacher to mediate a mathematical context through a designed didactical intervention (Bartolini Bussi & Mariotti, 2002).

2.8 Conclusion

Researchers recognise the potential of cognitive technologies in enhancing the teaching and learning of mathematics, of which algebra is an essential constituent. They are also aware, however, of the difficulty for teachers of integrating technology into mathematics. Given the centrality of the teacher in ensuring that maximum exploitation is made of the potential of cognitive digital tools, the literature has been interrogated for studies into teacher-related technology integration issues (roles, concerns, influences, shortcomings and theoretical frameworks). The examination of the literature has revealed the efforts of researchers to describe teacher's roles, concerns, factors influencing their use of digital technology and ways in which they use this technology. Seen through the lenses provided by the various theoretical frameworks, deficiencies in these teacher-related issues impact on teachers' pedagogical technology knowledge. Examination of a number of studies has shown that it is worthwhile to conduct a study to address shortcomings in teachers' pedagogical technology knowledge in relation to the integration of cognitive digital technologies in school algebra.

CHAPTER THREE

RESEARCH DESIGN AND METHODOLOGY

3.0 Introduction

This chapter presents the details of the methodology of the inquiry. It begins with a brief outline of the context of the research design followed by a discussion of paradigm considerations and the justification for selecting the case study research strategy within an interpretive paradigm.

Following the presentation of the research questions, the other components of the research design are discussed within the context of this inquiry. The components are: the rationale for choosing the intrinsic multiple-case study design, the unit of analysis, the logic linking the data to the rationale, and the criteria for interpreting the findings of the study. The issue of how the credibility of the inquiry in terms of its reliability and validity was established is then discussed. Finally, the chapter discusses the selection of candidates for the cases studies, the pilot case study and the process of gaining access to research data.

3.1 Definition and purpose

The research design detailed in this chapter is the logical plan that links the data collected and the conclusions drawn to the study's research questions (Denzin & Lincoln, 1994; Yin, 2003). Its purpose was to guide the investigator in the process of collecting, analysing, and interpreting data (Nachmias & Nachmias, 1992)

3.1.1 Paradigm considerations

Whether they believe so or not, all researchers have biases and theoretical perspectives (Schoenfeld, 2002). Not only do the researchers' framing assumptions shape what they attend to in their research, they also affect the scope and robustness of the findings. In all empirical research paradigm issues are always present even if only implicitly (Grix, 2004; Punch, 2003). According to Guba and Lincoln (1994), 'a paradigm is a basic belief system or world view that guides the investigator, not only in the choices of method but in ontologically and epistemologically fundamental ways' (p. 105).

At the core of every paradigm are four concepts as highlighted in Figure 6: epistemology, ontology, methodology and axiology (Denzin & Lincoln, 2003). The ontological assumptions give rise to epistemological assumptions, which, in turn, give rise to methodological considerations (Hitchcock & Hughes, 1995). Methodological considerations, in turn, give rise to issues of instrumentation and data collection (Hitchcock & Hughes, 1995). While it is the exception rather than the rule for reference to be made to the ontological and epistemological nature of a research project, its methodology will always be stated because of its closeness to research practice (Sarantakos, 2005).

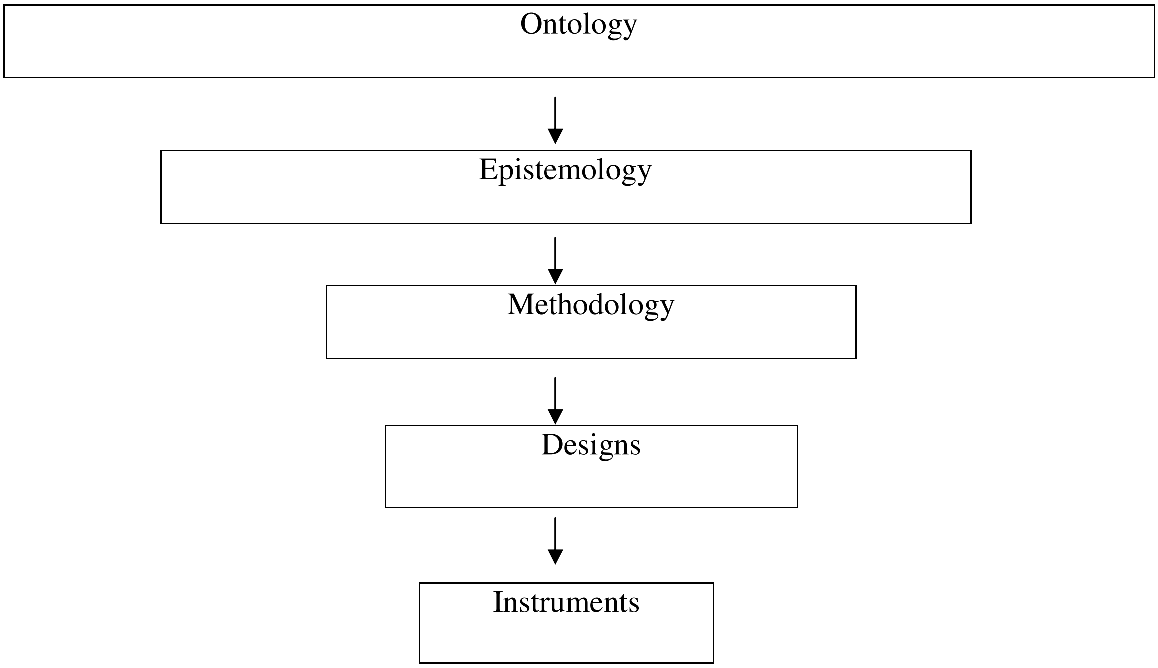

Figure 3.1: The parameters that guide research

The purpose of this study was to investigate teachers' use of technology to enhance student understanding of algebra in high schools in New South Wales. A number of essential epistemological and methodological choices needed to be made by the researcher in undertaking this investigation, including the context and setting in which the data was to be collected and the trustworthiness of the data, considerations of the way reality is perceived by individuals, and the researcher/participant partnerships (Guba & Lincoln, 1985). The criterion for assessing the trustworthiness of the data was the data's credibility described in terms of its validity and reliability (Silverman, 2005).

3.1.2 The research strategy

Selection of the research strategy for the inquiry was based on a consideration of the following: the type of research questions posed, the extent of control that the researcher had over the actual behavioural events on which the data collection was based, and the degree of focus on contemporary as opposed to historical events (Cohen, Manion, & Morrison, 2002; Yin, 2003). For this inquiry, the cases study research strategy was found to be the most appropriate.

3.1.3 Case Study Research

A case study can be defined as an empirical inquiry that investigates a contemporary phenomenon within a real-life context, especially when the boundaries between the phenomenon and context are not clearly evident (Hitchcock & Hughes, 1995; Stake, 2005; Yin, 2003). Characteristic of the case study inquiry is that it copes with the technically distinctive situation in which there will be many more variables than data points. The case study strategy benefits from the prior development of theoretical propositions to guide data collection and analysis. A distinguishing feature of case studies as suggested by Sturman (1999) is that human systems such as that of the algebra teacher, the students and technology for use in the teaching and learning of algebra within the classroom have a wholeness or integrity to them rather than being a loose connection of traits, which necessitates their in-depth investigation. Furthermore, contexts are unique and dynamic; hence case studies investigate and report the complex dynamic and unfolding interactions of events, human relationships and other factors in a unique instance. The integration of digital technology into school algebra is one example of a complex and dynamic area that lends itself readily to a case study investigation. According to Yerushalmy and Chazan (2002),

> School algebra is a complicated curricular arena to describe, one that is undergoing change … mathematics educators do not seem to be in broad agreement about how one might conceptualize this curricular area. In particular, technological innovations seem to have fractured whatever agreements may have existed with regard to school algebra....In terms of understanding the impact of technology on teaching and learning,

> it seems important to have a more nuanced understanding of the role of technology in supporting the teaching and learning of algebra (pp.725-727).

The case study research strategy was selected by the researcher because: (a) the research questions posed were largely of the ‘how?’ and ‘why?’ kind (Yin, 2003); (b) the single ‘What?’question among the set of research questions could also be investigated using the case study research strategy because of the exploratory nature of the inquiry (Yin, 2003); (c) the inquiry was directed at a contemporary and not a historical event, i.e., the current use of digital technology by teachers to enhance their students’ learning of school algebra in high schools in the New England/North-West region of New South Wales, Australia (Yin, 2003); and (d) the investigator had no control over the behaviours investigated (Yin, 2003).

3.1.4 Types of case study and the case study design selected for this study

There are several typologies of case studies, with aspects of each type being of relevance to this investigation. Stake (2005) has, for example, identified three types of case study. The first of these, the intrinsic case study, is undertaken primarily because one wants a better understanding of this particular case and not because this case represents other cases or because it illustrates a particular trait or problem, but instead because, ‘in all its particularity and ordinariness, this case itself is of interest’ (Stake, 2005, p. 445). The second is the instrumental case study, in which a particular case is examined mainly to provide insight into an issue or to redraw a generalisation. According to Stake (2005), in a instrumental case study,

> the case is of secondary interest, it plays a supportive role, and it facilitates our understanding of something else. The case is still looked at in depth, its contexts scrutinised and its ordinary activities detailed, but all because this helps us pursue the external interest. The case may be seen as typical of other cases or not. Here the choice of case is made to advance understanding of that other interest. We simultaneously have several interests (p. 445).

Commenting about these two types of case study, Stake (2005) has emphasised that there is no hard-and-fast line distinguishing intrinsic case study from instrumental, but rather that between them there is a zone of combined purpose. If, however, there is even less interest in one particular case, a number of cases may be studied jointly in order to investigate a phenomenon, population or general condition. The third type of study in Stake's (2005) typology, is the multiple case study or collective case study. According to Stake, multiple case study 'is instrumental case study extended to several cases' (pp. 445-446). Since a number of cases are to be investigated in this enquiry, the researcher will use a multiple case study approach. In particular, the researcher will study the use of technological tools by teachers to enhance student understanding of algebra and extend the study to four schools. There are common characteristics already known to the researcher about the schools. They are all run by the Catholic Schools Office in Armidale, New South Wales, Australia and all follow the same mathematics curricula as all the other high schools in New South Wales. The researcher is unaware of any other information in relation to whether or not they manifest some common characteristic. The schools have been chosen because of the belief that understanding what happens during the teacher's use of technology in algebra teaching and learning will lead to better understanding, and perhaps better theorizing about a still larger collection of cases.

The second typology of case study was categorised by White (1992). In this classification, social science case work was categorised according to three purposes; for identity, for explanation or to control. White's (1992) classification applies to this study to the extent that it seeks to identify and explain how teachers use cognitive technology to enhance the understanding of algebra by their students. Yin's (2003) typology of case study research identifies single-and multiple-case studies while cautioning against distinguishing sharply between them because, in reality, they are but two variants of case study designs. Using a 2 x 2 matrix, Yin (2003) has identified four basic designs for case studies; two single-case designs (type 1 and type 2) and two multiple-case designs (type 3 and type 4) (see Figure 3.2 below).

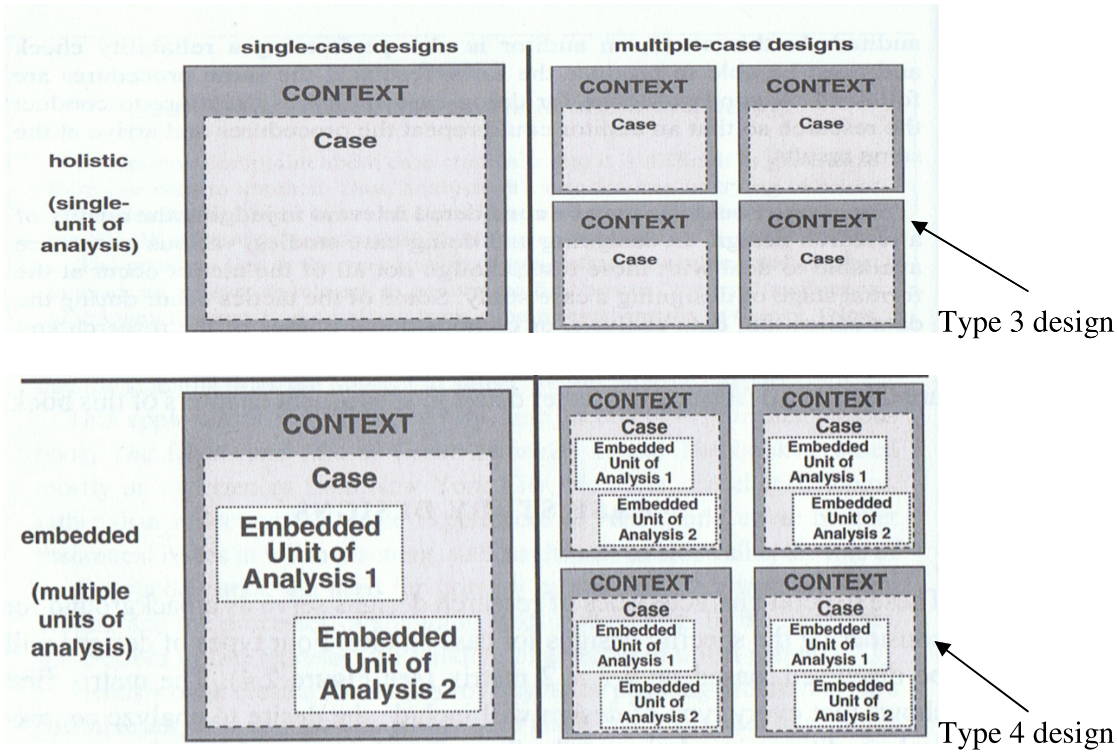

Figure 3.2: Basic types of designs for case studies (Yin, 2003).

In all design types, the dotted lines ‘indicate that the boundaries between the ‘case’ and the context are not likely to be sharp (Yin, 2003, p. 39). Using Yin’s (2003) classification as shown in Figure 3.2 above, this study will follow the holistic multiple-case design (type 3 design).The reason for this is that the evidence from multiple cases is often considered more compelling, with the overall study being regarded as more robust that with single-case designs (Herriott & Firestone, 1983). Yin (2003) has argued that

> …although all designs can lead to successful case studies, when you have choice, multiple-case designs may be preferred over single-case designs. Even if you can only do a “two-case” study, your chances of doing a good case study will be better than using a single-case design. Single-case designs are vulnerable if only because you will have to put “all your eggs in one basket.” More important, the analytic benefits from having two (or more) cases may be substantial (p. 53).

For this investigation, the holistic design has been chosen because no logical subunits to those of the four schools selected could be identified (Yin, 2003). The holistic approach, also called the global approach, has a major problem in that it can allow the researcher to avoid examining any specific phenomenon with operational detail. To counter this, the researcher ensured that the entire case study was not conducted at an abstract level and lacking any clear measures or data. Another major advantage of multiple-case studies, which the researcher took advantage of, was the opportunity to follow a replication logic similar to what happens with multiple experiments (Hersen & Barlow, 1976). The four cases studied were selected so as to predict similar results and give a literal replication. The random sampling logic, commonly used in surveys, does not apply in this study because it would have required an operational enumeration of the entire universe or pool of potential respondents and then a statistical procedure for selecting a specific subset of respondents to be surveyed (Yin, 2003).

3.2 Components of the research design

The multiple case study design selected for this study comprises the following five components (Yin, 2003); the research questions, the rationale, the unit of analysis, the logic linking the data to the rationale and the criteria for interpreting the findings.

3.2.1 The research questions

The following research questions listed previously in Chapter One provide a focus for this study:

1. What roles do mathematics teachers adopt as they integrate cognitive digital technological environments in algebra lessons?

2. Why do teachers use digital technological environments the way they do when teaching algebra?

3. How do teachers incorporate cognitive digital technological environments in their teaching of algebra?

4. How can shortcomings in the pedagogical technology knowledge of in-service teachers be addressed for enhanced students understanding of algebra?

3.2.2 Rationale for using a case study strategy

A multiple case study strategy has been selected to investigate and answer these questions. Yin (2003), when presenting the relevant situations for different research strategies has suggested the following three conditions to justify using a case study strategy: 'how' and 'why' forms of research questions and 'what' questions for an exploratory study, the study does not require the control of behavioural events and the focus of the study is on contemporary events. Research question 1 is a 'what' type of question and, this being an exploratory study, any of the five research strategies, experiment, survey, archival analysis, history, and case study can be used. The case study strategy is the most suitable approach on the grounds that the question seeks to investigate 'a contemporary phenomenon within its real life context' (Yin, 2003, p.13). Secondly, the investigator has no control over the issue being studied. Research question 2 is a 'why' type of question and hence it is suited to a case study investigation. The same reason, the examination of contemporary events in which relevant behaviours cannot be manipulated, is used to justify a case study strategy for the 'how' type research questions 3 and 4 above. According to Yin (2003),

> "how" and "why" questions are more explanatory and likely to lead to the use of case studies" (p. 6).

This is because such questions deal with operational links needing to be traced over time, rather than mere frequencies or incidences.

3.2.2.1 Rationale for a multiple case study design

The researcher interviewed mathematics teachers in four secondary schools, with each teacher's use of technology when teaching algebra treated as a single case. Using Stake's (2005) typology of case studies, each of these cases was taken to be an instrumental case study because the use of technology in the teaching of mathematics was examined mainly to provide insight into the issue of the use of this technology in the specific area of algebra teaching and learning. The different cases were studied jointly and, for this reason, the investigation became a multiple or collective case study (Stake, 2005). In other words, instrumental study with the examination of a particular case to provide insight into an issue, was extended to the four cases. Referring to multiple case study, Stake (2005) notes that:

> Individual cases in the collection may or may not be known in advance to manifest some common characteristic. They may be similar or dissimilar, with redundancy and variety both important. They are chosen because it is believed that understanding them will lead to better understanding, and perhaps better theorizing (p. 446).

There are certain aspects were common to all the four cases under investigation which were known in advance. Most important among them is the fact that all the four schools follow the New South Wales (NSW) Board of Studies Curriculum Framework for mathematics and will follow the Australian Curriculum, Assessment and Reporting Agency (ACARA) (2010) framework once it is implemented. In relation to technology, the NSW Board of Studies (2003) framework recognises its use as one of the key competencies incorporated into the objectives, outcomes and content of the K-10 mathematics syllabus. There is therefore an expectation for students to know why and when they have to use certain technologies and also to evaluate the effectiveness of its application (NSW Board of Studies, 2003). Specifically, the syllabus document (NSW Board of studies, 2003) states that, 'computer software as well as scientific and graphics calculators can be used to facilitate teaching and learning' (p. 10).

This places a responsibility on mathematics teachers to assist students to learn how to use technology and to determine whether the use of the technology is appropriate or counterproductive. While the content in the 'Patterns and Algebra' strand taught to students in all four schools is the same because they all follow the same syllabuses, the approaches to the use of technology within schools or among teachers within a school, may or may not be similar. The possibility of non-uniformity in regard to the use of digital technology made it worthwhile area to investigate in each school. It was also interesting to study how different teachers responded to suggestions about the use of particular technologies when teaching certain algebraic concepts. For example in the 5.2 stage of the 7-10 Mathematics syllabus (NSW Board of Studies, 2003), it is suggested to teachers that they use ‘graphics calculators and spreadsheet software to plot pairs of lines and read off the point of intersection’ (p.90). This raises questions like, ‘do teachers use any of these tools?’. ‘How do teachers use them?’. Such information is taken into account when answering the research questions for this inquiry.

While the use of, for example, a survey strategy could have provided data about the number and types of cognitive technological tools that each mathematics faculty possesses (i.e., answer primarily ‘what’ questions), it is unlikely to satisfactorily bring out data that can answer ‘how’ questions posed in the study. The case study approach is therefore a more comprehensive method for gathering rich data because one can answer how, why, and what questions effectively through it.

3.2.3 The unit of analysis

The major entity being analysed in a study comprises its unit of analysis (Cuban, 2002). As an example, Yin (2003) has suggested that an individual can be the primary unit of analysis if the individual is the case being studied. In other words, there is a link between determining what the ‘case’ is and the study’s unit of analysis. Since determining what the ‘case’ is in a study is problematic (Stake, 2005), the specification of the unit of analysis is not always straightforward. The difficulty lies with the fact that not everything is a case. For example, a medical doctor may be a case but his/her doctoring may not be a case on the ground that it lacks the specificity or boundedness associated with a case. According to Yin

(2003), to overcome this problem and select the appropriate unit of analysis, researchers must accurately specify their primary research questions because there is a relationship between the researcher's definition of the initial research questions and the definition of the study's unit of analysis. Given the focus on the mathematics teacher in this inquiry, the unit of analysis for the inquiry was the mathematics teacher in a technology-enabled algebra classroom.

3.2.4 Linking the data to the rationale

The rationale for the study is the basis of its original objectives and, in turn, it reflects the study's research questions and the review of the literature. How the data collected is linked to the rationale for the study is a critical component of a case study research design. It foreshadows the data analysis steps, being part of the foundation for the analysis.

3.2.4.1 The inquiry's rationale

Teachers are unlikely to successfully integrate technology in their teaching if they experience limitations in their pedagogical technology knowledge (PTK) and also in their conceptualizations of algebra. Cognitive technological tools can store, manipulate, and retrieve information, and have the capability not only of engaging students in instructional activities to increase their learning, but also of helping them solve complex problems to enhance cognitive skills (Jonassen & Reeves, 1996; Newby, Stepich, Lehman, & Russell, 2000). The use of cognitive tools, however, requires a high level of understanding and reflection by teachers, because tools can enhance or inhibit students' development of robust understandings of mathematical concepts (Heid & Blume, 2008). A number of researchers have noted that teachers are generally under-prepared to integrate technology into their instruction in meaningful ways (Guin, Ruthven, & Trouche, 2005; Strudler & Wetzel, 1999; Schrum, 1999; Willis & Mehlinger, 1996).

It was therefore the purpose of this inquiry to identify and describe ways in which teachers can enhance students' understandings of algebraic concepts as they (the teachers) integrate cognitive digital technology in their teaching of algebraic transformation. The inquiry aimed to achieve this within the context of finding links between teachers' skills in integrating cognitive technological tools with school algebra, their content knowledge/understandings of algebra and students' conceptualizations of algebraic transformations. Several ways are available for linking the data to this rationale for the study. A special form of pattern matching (Campbell, 1975; Trochim, 1989) called explanation building was used in this inquiry to achieve this. Pattern matching involves building an explanation about the case and identifying a set of causal links. The explanations are a result of a series of iterations starting with the initial theoretical statement followed by comparing findings of an initial case, revising the statement, comparing details of the case, revising and then comparing with other additional cases. Using this approach, case study data collected was examined by building explanations about the cases (Yin, 2003).

3.2.5 Credibility of the research

The credibility of any inquiry rests upon how it addresses reliability and validity. For this inquiry, the reliability of interview schedules was addressed by ensuring that each respondent understood the question in the same way so that the responses of all respondents could be coded without any possibility of uncertainty (Silverman, 2005). This was done in two ways: through pre-testing of the interview schedule and, secondly, by aiming for low-inference descriptors through audio-recording all interviews and getting the researcher and not another person to carefully transcribe the recordings. According to Seale (1999), the use of low-inference descriptors involves:

> recording verbatim accounts of what people say, for example, rather than researchers' reconstruction of what a person said, which would allow researchers' personal perspectives to influence the reporting (p. 148).

Validity, on the other hand, was achieved through the use of the constant comparative method—by inspecting and comparing all (audio-recording) of one case with another through the use of a simple table (See Appendix 1).

3.2.6 Selection of candidates for case studies

The selection of candidates for this study was through purposive sampling, which is a non-probability approach to obtaining a sample used in qualitative research to select information rich cases for an in depth study (Cohen et al, 2002; Schofield, 1993). Information rich sources make it possible for a researcher to "gain a great deal of information about issues of central importance to the research from relatively small samples or numbers of participants" (Patton, p. 169). All four informants are four-year trained teachers with qualifications to teach mathematics at the secondary school level. They all regularly use at least one form of cognitive digital technology in their mathematics lessons. The selection of the case study informants was also based on a long standing knowledge of the schools by the researcher. Table 3.1 is a summary of the criteria used to select participants and schools for the study.

Table 3.1: Criteria for selection of the participants and schools for the study

Teacher

- Four-year qualified teacher
- Qualified to teach mathematics at the secondary level
- Teaches junior secondary and/or senior secondary classes

School

- School run by the Catholic Schools Office (CSO), Diocese of Armidale, NSW
- Director (CSO) permission to undertake study in Diocese of Armidale Schools
- Principal and informant permission to undertake study in the schools
- Secondary school within 3 hours of road travel by the researcher

Codes (Table 3.2) were used for both the informants and schools for confidentiality purposes.

Table 3.2: Codes used for informants and schools

Teacher 1 (T1) Secondary School A	Teacher 2(T2) Secondary School B
Teacher 3(T3) Secondary School C	Teacher 4(T4) Secondary School D

3.2.7 The Pilot Case Study

As part of the final preparation for data collection, a pilot case study was conducted. It was carried out to help the researcher to 'refine data collection plans with respect to both the content of the data and the procedures to be followed' (Yin, 2003, p. 79). Unlike a pre-test in which the data collection plan is used as the final plan, the pilot test is a more formative procedure which assists the researcher to develop relevant lines of questions. For this study, the pilot case was chosen based on the following criteria: the informant at the site was the first to accept to be interviewed, and the site was geographically convenient, being only a couple of hours drive away for the researcher. The questions for the structured interview were piloted and, following discussions between the researcher and the study's co-investigator (principal supervisor), adjustments were made to give greater emphasis to certain questions specified and discussed in Chapter 4-Results.

3.2.8 The process of the inquiry

The following section details the sequence of activities undertaken to establish the research project, obtain official approval to conduct the research, develop rapport in the case study sites, undertake the data collection and trustworthiness process, analyse data and write up the research.

The key informants in this research were chosen because they regularly used technology (including cognitive technological tools) in their mathematics classrooms. Accessibility and prior knowledge of some of the key participants and schools enabled a trusting teacher-researcher relationship necessary to obtain sound qualitative research related to the teachers' classroom practices in relation to their use of technology during their algebra lessons. The researcher spent about 50 minutes with each teacher, with about 30 minutes of this used for actual interviews. To maintain confidentiality, codes comprising letters and numbers were used for each of the informants interviewed for the study and their schools (Table 3.2).

3.2.9 The process of gaining access to research data.

The process of gaining access to research data was multifaceted and occurred at a number of levels during the research process. Figure 3.3 below depicts the various levels at which access had to be sought.

UNIVERSITY OF NEW ENGLAND

UNE, Faculty of the Professions, School of Education accepted the Research Proposal and allowed the study to proceed. The Human Research Ethics Committee (HREC) approved the research project 0n 07/07/2010 (Appendix 3). Full documentation of the Ethics Policy of UNE can be found on http://www.une.edu.au/research-services/researchdevelopmentintegrity/ethics/human-ethics/hrec.php (accessed on 14-11-2010).

CATHOLIC SCHOOLS OFFICE, DIOCESE OF ARMIDALE, NSW

Permission to approach principals and conduct research in schools run by the Catholic Schools Office (CSO), Diocese of Armidale, approved on 6 September, 2010 on condition that a summary report of findings and/or recommendations to the schools be forwarded to the CSO as soon as practicable after results are to hand (Appendix 2).

Teachers	**Mathematics Faculty Coordinators**	**Principals**
-Information about the research project provided to them through the Information Sheet for Participants (Appendix 4). -Permission sought and Consent Forms for Participants signed to interview them and to audiotape conversations (Appendix 5).	Informed about research project through sending them the Information Sheet for Participants (Appendix 4). Those willing to participate in the project were (to be interviewed and audiotape the conversations), like the other teachers, given Consent Forms for Participants to sign.	Principal approval to interview teachers in each school was sought through adding the request to the Information Sheet for Participants (Appendix 6) and through sending them copies of permissions from the HREC, CSO, Armidale.

Figure 3.3: The process of gaining access to research data.

3.2.10 Time and duration of data collection

Table 3.3 below summarizes the approximate amount of time and duration of the various phases involved in data collection, transcription, and preparation of data and participant member checking.

Table 3.3: Time frame

	T1-School A	**T2-School B**	**T3-School C**	**T4-School D**	**Total**
Key participant interviews	October-November 2010 30 minutes	October-November 2010 30 minutes	October-November 2010 30 minutes	October-November 2010 30 minutes	2hrs
Transcribing the digital audio	2 hours	1 hour 30 min.	1 hour 30 min.	I hour 30 min.	6hrs 30min.

3.3 Chapter Summary

The case study research protocol within an interpretive paradigm was chosen for this inquiry on the basis of findings by researchers (Hitchcock & Hughes, 1995; Stake, 2005; Yin, 2003) that it was the best approach for the investigation of a contemporary phenomenon within its real-life context—the phenomenon in this case being the current use by teachers of cognitive digital technology to enhance student understanding of algebra in secondary schools.

After providing the definition and purpose of the research design, Section 3.1 discussed paradigm considerations within the context of the fact that all researchers bring a world view to their research and therefore must deal with paradigm issues. The section also provided the rationale for the researcher's choice of the case study research strategy for the study. The different typologies of case studies were identified in order to locate the case study variant for this inquiry. Three typologies, by Stake (2005), White (1992) and Yin (2003) were used to categorise this inquiry as being at the same time a multiple case study, an instrumental case study and as having the capacity to identify issues for a researcher.

Section 3.2 identified and described the components of the research design beginning with the research questions, followed by the rationale for selecting a case study design and then for selecting the multiple case study design variant, the unit of analysis. In addition, the section linked the data to be collected to the rationale for the study before discussing how the credibility of the study in terms of its reliability and validity was established. The selection of participants was discussed, including the criteria for selecting them, justification for the use of purposive sampling and the important role played by the researcher's knowledge of the schools from which the participants were selected. The way the pilot case study was conducted was also described together with the reasons for the researcher's selection of the site for this study. Finally, the process of gaining access to research data was outlined, including all the permission stages through which the researcher had to go before finally getting to interview the teachers. Section 3.3, which was the last section, provided an idea of the time and duration of the data collection for the inquiry.

CHAPTER FOUR

RESULTS

4.0 Introduction

This chapter presents the data and analysis of the data for the research investigation. Section 4.1 discusses the research context of each research informants including the selection process. The section also introduces the coding used for both the participants and their schools to ensure that confidentiality was maintained.

Section 4.2 addresses the data collection method (interviews), highlighting why this method was used in relation to its appropriateness for the cases study research strategy used in the investigation. Also discussed is the means by which the voices of the participants were captured followed by a presentation of the list of questions that guided the researcher to collect data during the interviews. The data collected from each of the participants is then presented as a 'thick description' (Lincoln & Guba, 1985; Punch, 2003). A thick description involves stating both the observation and its context because human behaviour only has meaning to an observer who is aware of the context in which the behaviour is taking place (Ryle, 2002). The collected data is analysed in the last section of the chapter, Section 4.3.

4.1 The school context of the participants

The method of selection of participants in this study is best described as purposive sampling (Cohen, Manion & Morrison, 2002). The researcher handpicked the cases to be included in the study on the basis of a judgement of their typicality. The aim was to select mathematics teachers who had at least five years of experience in the role. The schools that were selected for the research sites had certain common characteristics. For example, they were all administered by the Catholic Diocese of Armidale through the Catholic Schools Office and, like the state schools, they followed the New South Wales Board of Studies mathematics curricula. The Armidale Catholic Schools Office (CSO) schools were selected by the researcher primarily because he anticipated that they would be easier for him to access for data collection as he is an employee of the CSO. They were also all within three hours driving distance from the researcher's town, and hence were not too costly to visit. In general, a working knowledge of the Catholic Schools Office in Armidale, the schools, personal contacts within schools and teacher peers known to the researcher were the main criteria for the selection of the participants and the schools. All the schools are located in the New England/North-West region of New South Wales in urban areas each with a population of at least 10 000 residents.

For purposes of this research the school sites are referred to as A, B, C, D and individual pseudonyms were assigned to all teacher participants (Teacher 1 or T1-School A; Teacher 2 or T2-School B; Teacher 3 or T3-School C; Teacher 4 or T4-School D) (Table 3.2) to preserve their anonymity. As a condition for getting permission from the Director, CSO, Armidale to access CSO schools, the researcher was required to maintain the privacy of the school and that of any school personnel involved in the study (See Appendix 5). Three of the schools had a Year 7 - 12 structure, while one had a K-10 structure. All the interviewed teachers obtained their teaching qualifications at universities in Australia.

4.2 Data Collection/ Interviews

Data collection was achieved by interviewing four mathematics teachers from each of the selected schools. All the interviewees were qualified to teach mathematics in New South Wales schools and they had, as a minimum, an undergraduate teaching qualification with mathematics as one of their teaching areas. The data was collected between October and November, 2010. Interviews were used on the ground that they are one of the most important sources of case study information (Yin, 2003). According to Yin (2003, p. 92),

> Interviews are an essential source of case study evidence because most case studies are about human affairs. These human affairs should be reported and interpreted through the eyes of specific interviewees, and well-informed respondents can provide important insights into a situation

Focused interviews, in which respondents were interviewed for short periods of time using a certain set of open-ended questions derived from the case study protocol in a conversational manner, were conducted by the investigator (Merton, Fiske, & Kendall, 1990). All interviews were captured using a digital audio recording device. The audio records were then transcribed. In this section, the data collected from the interviews with all four teachers is presented in line with the suggestion by Yin (2003) that an investigator should display and present evidence separate from any interpretation. The interviewees were asked the same questions while ensuring that the needs of the investigator's line of enquiry were satisfied, friendly and in a non-threatening manner (Becker, 1998). In these interviews, ‘why’ questions were not posed directly, to avoid creating defensiveness on the informant’s part; they were instead posed as ‘how’ questions. In general, relying on the guidelines by Rubin and Rubin (1995), the actual stream of questions was fluid rather than rigid. The questioning style advocated by Rubin and Rubin (1995) was followed with all the interviewees/respondents by the investigator. Eleven questions, constructed in such a way as to cover the study’s four research questions, were posed to each of the participants. Each case study includes the responses to all the questions (Appendix 1) and information about how long each of the participants had taught mathematics and which classes they taught in 2010.

4.2.1 Case Study: Teacher 1(T1)-School A

Teacher 1 (T1) has taught high school mathematics for 15 years since completing a four-year undergraduate degree. He teaches 4 classes (Years 7, 8, 9, 10). The cognitive digital tools that he regularly uses are scientific calculators and those provided within the interactive whiteboard environment. He also uses online maths tutors like *hotmaths* (www.hotmaths.com.au) and *mathsonline* (www.mathsonline.com.au). The first of these two clearly contains cognitive technology that transcends the limitations of the mind (Pea, 1987) in the form of widgets that students can use to solve a number of problems in algebra such as adding or subtracting algebraic expressions. According to Pea (1987):

> A common feature of.....cognitive technologies is that they make external the intermediate products of thinking (e.g., outputs of component steps in solving a complex algebraic equation), which can then be analysed, reflected upon, and discussed. Transient and private thought processes subject to the distortions and limitations of attention and memory are “captured” and embodied in a communicable medium that persists, providing material

records that can become objects of analysis in their own right-conceptual building blocks rather than shifting sands (p. 91).

Figure 4.1 below is an example of a widget in the *HOTmaths* tutor program in which students click on a button to learn how to add or subtract algebraic expressions.

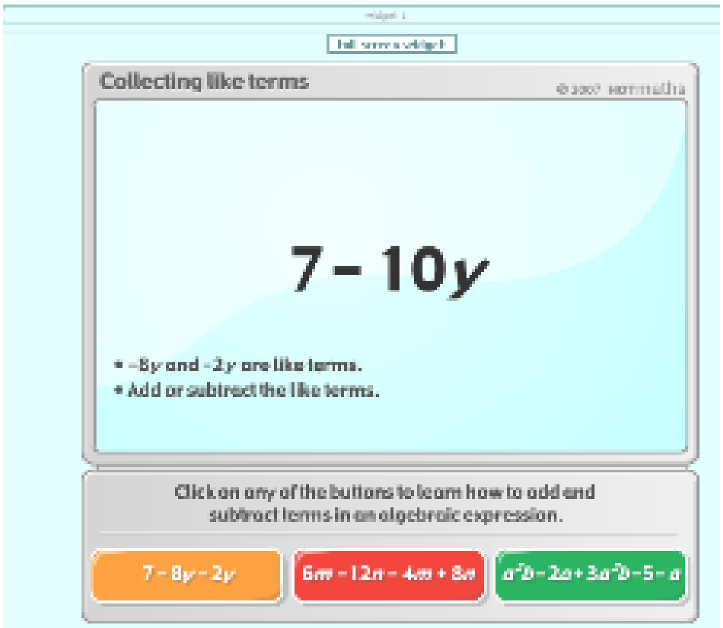

Figure 4.1: Example of a widget from *HOTmaths*

T1 finds the interactive whiteboard environment very useful when teaching different algebraic concepts especially presenting ‘patterns’ which is a generational algebraic activity (Kieran (2007). The others are factoring, expanding, substituting, adding and multiplying polynomial expressions, exponentiation with polynomials, solving equations, simplifying expressions and working with equivalent expressions and equations all of which comprise transformational algebraic activity (Kieran, 2007). When the investigator listed nine global meta/level algebraic activities (Kieran, 2007), T1 selected four of them as capable of being addressed within the interactive whiteboard environment; problem solving, modelling, noticing structure, and generalising. The reason T1 gives for the usefulness of the interactive whiteboard environment is that it presents an opportunity for bringing out any other tool that he needs when teaching algebra or any other area of mathematics.

In response to question 3, T1 learns how to use these tools at locally held workshops, at conference presentations (the Mathematics Association of New South Wales), and through discussions with colleagues in his school. T1 goes through any work at home or at school involving the tool before presenting it to the students, so as to be ready for their questions. Although often limited by time, T1 always gives the students the opportunity to use the tools on their own during lessons and then sets aside time to respond to any queries regarding any aspect of the tool that they experienced difficulties with. T1 uses digital tools for '85% to 90%' of the time in the class and views the teacher's role in a technology-enabled classroom as that of a facilitator. However, T1 is aware that other teachers, for different reasons, do no more than demonstrate the use of tools to the students.

T1 has never used graphics calculators or felt any need for them, despite being aware that some models are permitted by the New South Wales Board of Studies for use by students. T1 has not had to reflect about other approaches other than that of a facilitator available to teachers when using digital tools and has no personal concerns (Hall & Hord, 2001) relating to the use of digital technology in general in the school. As for technology concerns (Hall & Hord, 2001), T1 said that he did have any, believing that the opportunities are there for any teacher who wishes to learn how to use any piece of technology- and he has used them to the maximum. Technology concerns, according to the Concerns-Based Adoption Model (Hall & Hord, 2001), refer to shortcomings in the teachers' pedagogical technology knowledge (PTK) with respect to the use of certain tools, whether the teacher is confident when using them or not. According to T1, shortcomings in in-service teachers' PTK are largely of their own making. T1 also believes that with *mathsonline*, he has achieved much with the students in the Year 9 and Year 10 classes doing the 5.3 strand of the NSW Mathematics syllabus. T1 has no management concerns about having control of and managing technology-enabled classroom environments (Hall & Hord, 2001). T1 believes that workshops held at local levels can address any shortcomings in the teachers' pedagogical technology knowledge (PTK) in all areas of mathematics and not just algebra.

4.2.2 Case Study: Teacher 2 –School B.

After obtaining an undergraduate degree, T2 taught in other schools before moving to School B. Altogether T2 has taught for over 30 years, starting at the primary school level before moving to the secondary school level. T2 was not familiar with any other digital tools before she started teaching mathematics. With much effort and over a long period of time, T2 learnt how to use the graphics calculator (one of the Casio models) and eventually was comfortable using it to teach mathematics to students in the senior classes. T2 uses it mainly to teach solving equations and the sketching of graphs. The school has one set of graphics calculators that students can use at school. Senior students are encouraged to buy their own equipment. All students have a scientific calculator. T2 has access to computers and interactive whiteboards. In addition, T2 has taught all classes and all strands of mathematics from Years 7 through to Year 12. T2 has learnt how to use tools from colleagues, by teaching herself at home and also at school. T2 also asserts that, 'teachers must learn to use technology and to teach using it as this is the age of digital technology'. Having been given a list of possible roles for a teacher using technology in the classroom by the investigator, T2 described her role as that of a facilitator and also as that of a fellow investigator, with the acknowledgement that in many instances in this role, the students' skills were far ahead of hers. As a facilitator, T2 allows the students to play with the technology, but in the junior years (Years 7 to Year 9) she controls the extent to which they can do this due to time limitations. T2 expressed personal concerns with regard to the use of some types of cognitive digital devices like graphics calculators as she feels that she has a lot to learn. Personal concerns, when interpreted using the Concerns-Based Adoption Model (Hall and Hord, 2001), refer to any fears or lack of confidence expressed by a teacher with regard to the use of technological devices in a classroom environment. Through T2's many years of teaching she felt that she did not have any management concerns in relation to having control of and managing technology-enabled classroom environments. T2 strongly felt that teachers should make use of every opportunity to learn to use technology, including those who have been in service for a very long time like herself. T2 has also used the graphics calculator for aspects of transformational algebraic activity (expanding, substituting, exponentiation with polynomials, solving equations) and global/ meta-level algebraic activity (word problem solving, modelling) while for studying change (another global/meta-level algebraic activity) she uses tools within the interactive whiteboard environment—especially when she is introducing differentiation in Years 11 and 12.

4.2.3. Case Study: Teacher 3 (T3) - School C

T3 loves using different technological tools in his mathematics teaching. He has taught mathematics at the secondary school level for nearly 10 years and has used different digital tools for as long as he can remember. He teaches mathematics to students in Years 7, 8, 9, 11, and 12. He finds using digital tools relatively easy compared to the other teachers in the school as he worked in the Information and Communications Technology (ICT) industry for about 8 years prior to taking up the current teaching position. Nevertheless, T3 takes the time to experiment with certain cognitive digital technologies like graphics calculators prior to using them with students in the classroom. T3 strongly advocates for teachers to go on training courses to improve their pedagogical technical knowledge with respect to digital tools that they are not familiar with. He feels that, 'in-service teachers need lots of exposure, by going through lots of problems with a tool, before they attempted to use it in their lessons. However, T3 believes that some teachers will never understand how to use some tools no matter how much they are taught. According to him, 'some teachers are wired to not be able to understand how some technologies work'. This is interesting as it raises the possibility that this teacher might hold the same view with regard to some of the students that he teaches. For algebra lessons in the senior classes, he uses the graphics calculator as much as possible-with the students encouraged to manipulate the calculator as much as possible. The activities for which he uses the graphics calculator and the interactive whiteboard are: solving equations, modelling, problem solving, expanding and substituting (which are all transformational algebraic activities-Kieran (2007)). He spends time ensuring that students learn the function represented by each of the keys before he lets them use the calculator for solving problems. While T3 suggests that he does not have management concerns in relation to controlling and managing technology-enabled classroom environments, he has some professional concerns. While most computers were up to date in some of the classrooms, some of them were unusable, which he found very frustrating. He felt that there was a need for consistency in terms of hardware and software for all computers. T3 was worried that he might upset some authority figures, and therefore he was hesitant to ask for the computers to be repaired or for software to be purchased, especially in a situation where the faculty coordinator appeared uncomfortable with using digital technology. He wondered why those in authority were hesitant to ask parents to purchase graphics calculators but more willing to ask them to buy materials for other subject areas such as Food Technology. To T3, there appeared to be a cultural problem when it came to convincing 'gate keepers' about the value of students using digital tools.

After the investigator listing and describing the roles of a teacher when students use digital tools for T3, T3 felt that his role was that of a facilitator, which he understands to mean making students to understand the concept of a tool before they are taught how to use it. T3's main use of digital technology in algebra was in 'graphing things' as he put it, and in the solving of equations. Form a list of possible algebraic activities in which a teacher can use digital technology provided by the investigator, T3 selected the following: solving equations, factoring, expanding, simplifying expressions, and exponentiation with polynomials, e.g. $(x + 2)^2$. The main digital tools that T3 uses are the interactive whiteboard and the graphics calculators (Casio models).

4.2.4 Case Study: Teacher 4 (T4)- School D

T4 has taught mathematics for less than 2 years in New South Wales. He taught in U.K. high schools prior to taking up his present position. In 2009 he taught students in Years 11 and 12. He presently teaches students in Years 7-10 only. T4 has no problems using the digital tools available in the school to teach algebra, although he believes that the integration of digital tools with mathematics is a problem for many teachers.T4 uses digital tools for all mathematics lessons except when he wishes to solve a problem using a pencil and paper approach. It is also his experience that most algebra topics can be taught with the graphics calculator to students in the senior classes. He has found the graphics calculator even more useful when it is manipulated in an interactive whiteboard environment. One of the strengths of this combination, according to T4, is the opportunity it provides to focus the attention of the students to only one aspect of any algebra concept that he is teaching. There are also times when T4 has asked students to connect their laptops to the whiteboard so that their work can be seen by the others. When using digital technology to teach mathematics, T4 views himself primarily as a facilitator and fellow investigator with the students. He selected these roles from Farrell's (1996) typology of teachers' roles shown to him by the researcher. He demonstrates these roles in two ways. Firstly, he provides students with the opportunity to play with digital tools if new ways of using them need to be learned. This is done to address students' curiosity and novelty of tools to them. Secondly, instead of revealing all the steps about how to use a digital tool to solve a problem, he seeks as much of their input as possible, especially at the beginning of learning a new concept. He does this by presenting a problem on the interactive whiteboard and then asking a student to try and solve it while the others watch. With respect to his views on the need for the professional development on the integration of digital tools into mathematics for in-service teachers, T4 believes that opportunities for further training are already available, but most teachers lack the motivation to take advantage of them. He also said that he does not have any personal, technology or management concerns in relation to his use of digital tools in mathematics.

4.3 Analysis of the results

This section discusses the actions taken to examine the case study evidence (summarised in Table 4.1) obtained through interviewing the 4 teachers. Although the analysis of case study evidence is especially difficult due to the absence of well defined strategies and techniques, every case study, according to Yin (2003), should nevertheless strive to have a general analytic strategy. For this enquiry, explanation building (Yin, 2003) was used along with case description as the general analytic strategy. Explanation building uses pattern matching logic (Trochim, 1989) to analyse case study data by building an explanation about the case.

Table 4.1: Summary of data collected from participants.

	Teacher 1 (T1) School A	**Teacher 2 (T2) School B**	**Teacher 3 (T3) School C**	**Teacher 4(T4) School D**
Years of teaching	15	35	8	2
Year levels taught in 2010	7,8,9,10	7,8,10,11,12	7,8,9,10,11,12	7,8,9,10
Tools used when teaching algebra	Interactive Whiteboards, Scientific calculators, *Hotmaths* widgets	Interactive Whiteboards Scientific Calculators, Graphics Calculators	Interactive Whiteboards, Scientific Calculators Graphing software	Interactive Whiteboards, Scientific Calculators
How knowledge of tool use was acquired	Help from peers at school and at workshops, conferences, self teaching	Help from peers at school and at workshops, conferences, self teaching	Self taught (was IT expert before changing to teaching)	As a student at school and at uni, Workshops and conferences, peer support
Problem with any Level of Use?	No	Yes, with the graphics calculator	No	No
Concerns related to the use of technology: Personal Concerns? Technology concerns? Management Concerns?	Said that he had no concerns	Had personal, technology and management concerns in relation to graphics calculator use	Had management and technology concerns in relation to lack of uniformity of mathematical software on computers	Said that he had no concerns
Teacher views role as:	Facilitator	Facilitator and fellow investigator (with students)	Facilitator	Facilitator
Recommended assistance for in-service teachers	Workshops, conferences	Workshops, conferences, students	Workshops, conferences, colleagues	Workshops, conferences

4.3.1 Participants' use of interactive whiteboards

All the informants reported using the interactive whiteboards and expressed their satisfaction not only with the visualisation of algebra objects and processes that it afforded them but also the opportunity it provided for their manipulation of those objects and processes. The capacity to be linked to laptops further meant that information that originally that had to be copied by hand on a traditional non-interactive whiteboard could now be manipulated and brought to the student's visual attention quickly and efficiently for those teachers who reported having good skills in the use of digital tools (T1, T3 and T4). Mentioned by teachers, T2 and T3, and of relevance to the teaching and learning of algebra concepts (through generational or transformational or global/meta-level activities), was the interactive whiteboards particular advantage in facilitating the drawing of graphs using special software such as *graph plotter* and *autograph*. All the participants said that they did not have any concerns, be they personal, management or technological concerns in relation to the use of interactive whiteboards.

4.3.2 Informants' use of scientific calculators

Scientific calculators were used, by all participants, for the transformational algebraic activity of substituting in an equation or an inequality or in the solving of simple equations (equations in one variable). They afforded limited visualisation for whole class instruction unless they were linked to an interactive whiteboard. All the participants said that they did not have any personal, technological or management concerns (explained in Chapter Two) in relation to the use of scientific calculators.

4.3.3 Informants' use of graphics calculators

Only two teachers, T2 and T3, used graphics calculators in 2010. This was because their mathematics faculties had decided to use them for the teaching and learning of mathematics for the senior students and Year 9 and Year 10 students doing advanced courses in mathematics. T2 and T3 used graphics calculators for solving problems in all areas of algebra. They also made use of the calculator's multi-representational capacity including the ability to present an algebraic object in tabular, numerical, and graphical forms. The teachers found it especially useful when linked to the interactive whiteboard.T3 was more confident in doing this. T2 said she had personal concerns in the sense of not having full confidence in herself when using them in class and technological concerns in the sense of being still a learner in relation to how to use them to solve problems. T3, on the other hand, reported not having any personal or technological concerns when using graphics calculators in the classroom. He, however, there was frustration with some senior students not having these calculators and also with his school not being keen to encourage the parents of these students to purchase them. T3 clearly had a management concern in relation to the use of graphics calculators. T2, where the use of the graphics calculator was concerned, was confident only with the mechanical, routine and refinement levels of its use (See Section 2.4.2 in Chapter Two). T1 and T4 did not use graphics calculators in their lessons.

4.3.4 Informants' use of tutor programs and graphing software

To help students understand algebra concepts, participants used widgets in *HOTmaths* (www.hotmaths.com.au) tutor program such as that shown in Figure 9. In the widget, a button is clicked to get the result of a transformational algebra activity. For example, in Figure 9, pressing a button causes the display of the result of adding or subtracting algebraic expressions. T1 said *HOTmaths* was one of the most exciting programs in his mathematics teaching. All the participants said they used graphing software regularly in their algebra work. T1, T2 and T4 used *Autograph* to draw graphs of algebra functions while T3 used *graph plotter* for the same purpose. All the participants reported that the use of this software was enhanced by the use of the interactive whiteboard in the sense that they could effectively focus the whole class on it.

4.3.5 How the informants viewed their role (s) when using technology

All the informants, on being asked to state their role when conducting lessons in which they and the students were using digital technology, replied that they were facilitators. In the classification of the roles of teachers using graphing technology by Heid, Sheets and Matras (1990), the facilitator role refers to what the teacher does when introducing a new problem or a new real-world context. For all the interview participants in the study, the facilitator role is meaningful only when viewed as being a reference to all their activities/roles in a technology-enabled algebra lesson. T2 said that, in addition to being a facilitator, she was a fellow investigator with the students when graphics calculators were used. She clarified her fellow investigator role as referring to the fact that both she and the students were still learning how to use the graphics calculator. Using the classification of roles of teachers using graphing technology (such as the graphics calculator) identified by Heid et al (1990), T2's self identification as 'a fellow investigator' was a reference to a weakness in her technical assistant role.

4.3.6 Informants' concerns when using technology

Reference to concerns here is based on Hall and Hord's (1987, 2001) Stages of Concerns in their Concerns-Based Adoption Model (CBAM) described in Chapter Two. Besides T1 and T4, the other two teachers said they had no concerns (personal, management or technological) when using technology in algebra lessons. T1 had personal, management and technological concerns while T3 had only management and technological concerns.

4.3.7 Informants' suggestions of how in-service teachers can be supported

All the teachers who were interviewed had attended a workshop or conference in which they had learnt something about how to use technology in mathematics. T2, for example, acknowledged that she benefited greatly from her attendance of the cluster workshops conducted by CASIO representatives on the effective use of their graphics calculators by mathematics teachers. All interviewees had also attended at least one conference run by the Mathematical Association of New South Wales during which they had participated in at least one 'how to use technology in mathematics' session. This explains their suggestion of workshops and conferences as sources of training for in-service teachers on technology integration in mathematics. Peer support within the mathematics faculty was mentioned only by T3, although the researcher's experience is that all teachers of mathematics rely on this method of learning how to use technology. Teacher reliance on student support for the use of a particular technology was mentioned only by T2.

4.4 Chapter Summary

Section 4.1 outlined the school context of the study participant's schools, showing the characteristics that were common to the schools. The codes used for the teachers who participated in the study and their schools were also presented in this section. Section 4.2 provided the rationale for the use of interviews for this inquiry. Next, the means by which the interviews were captured (digital audio recording) was mentioned, followed by a listing of the 11 questions that comprised the interview questions and that the respondents were asked by the researcher. The data were also presented, separately for each respondent, in this section in the form of rich descriptions and also summarised in Table 4.1.

Section 4.3 comprised the analysis of the results presented in Section 4.2. With case description as the general analytic strategy, the researcher used the explanation building analytic strategy (a special form of pattern matching logic) to analyse the data. Given that the focus of the inquiry was teachers' use of digital technology was carried out starting with a reference to each of the digital technology tools that the respondents, in their responses during the interviews, said that they used when teaching algebra to their students.

CHAPTER 5

DISCUSSION OF RESULTS

5.0 Introduction

The inquiry's aim was to identify and describe the roles of mathematics teachers' when they use digital technology to enhance their students' understanding of algebra. This chapter discusses the results of the inquiry in relation to the theoretical frameworks introduced in the review of the literature. Underlying the discussion is the understanding that the inquiry, being a case study, is meaningful only in relation to analytic and not statistical generalisation (Yin, 2003).

The discussion in Section 5.1 links digital technology use, algebraic activities and teachers' roles. Section 5.2 addresses the rationale for teachers' choice of approaches to the use of digital technology during algebra lessons. Section 5.3 examines how teachers use the technology while Section 5.4 discusses how shortcomings in teachers' pedagogical technology knowledge can be addressed to enhance student understanding of algebra. The last section, 5.5 links the results of the inquiry to the theories presented in the literature review.

5.1 The teacher's role in a technology-enabled algebra classroom

The investigator presented to the informants a list of the possible roles that a teacher might adopt when using technology in a mathematics classroom; allocator of time, catalyst and facilitator, collaborator, counsellor, evaluator, explainer, manager, planner and conductor, resource, task setter, and technical assistant. The informants selected the 'facilitator' role, arguing that this encompassed the other roles. According to Heid, Sheets and Matras (1990), however, the facilitator role has a narrower focus than the broad one that the interviewed teachers had in mind. The teacher facilitates the introduction of a new problem or real-world context and the discussion of various solutions so that the lesson can be closed appropriately (Heid, Sheets and Matras (1990).

When asked for clarification of their understanding of the facilitator role, T2 and T3 associated it with the setting of boundaries for student activities in a technology-enabled mathematics lesson. This view, however, is closer to the 'manager' role (Farrell, 1996) in which the teacher serves as tactical manager, director and authoritarian–which is more to do with class management than with instructional and learning issues. T2 also saw herself as a 'fellow investigator' (Farrell, 1996) or 'collaborator' (Heid et al, 1990) most probably because she viewed herself as not being very good with some of the technologies, having taken to them late in her 30-year teaching career. It is possible that the use of newer technologies such as digital tools in her mathematics teaching was challenging T2 to change her existing practices (Zbiek & Hollebrands, 2008). Unlike the other three teachers, T2 did not use any digital tools as a student and therefore, as a teacher of technology-enabled mathematics lessons, started lower in the continuum from a nontechnology stance to incorporating technology extensively and appropriately in her teaching of mathematics.

Of the four teachers T1 had the least variety in terms of the number of technologies that he used in his classes. He did not use any 'advanced' cognitive technology like graphics calculators. This implies that his role as 'Technical Assistant' was not as involving as it was for the other three. According to Heid et al (1990), the 'Technical Assistant' role involves helping students with hardware and software difficulties—the teacher must be a problem solver who determines the nature of any technology-related difficulty before generating and choosing from among possible solutions. T3, with his background in Information and Communication Technologies, was most comfortable with this role. T4 was unfamiliar with use of the graphics calculator, but he was not worried about this, possibly because he 'did not find using any new technology' challenging if he had to use it.

5.2 Teachers' choice of approaches to the use of technology

The question of why the teachers used cognitive technological environments the way they did when teaching algebra was approached by breaking it up into three questions:

1. How did you learn to use the tools that you use in algebra lessons?
2. Describe how you have used a digital tool outside an algebra classroom setting;
3. Describe other ways (different to yours) in which a particular tool you are using is used by other teachers.

Teachers' choice of approaches to the use of digital technology is influenced by the type, location and number of digital tools available in the school. Informants T2, T3 and T4 felt that, although their views were sought prior to purchasing digital tools, they did not have much influence on determining the type and number of digital tools bought by the school. They had even less influence in relation to purchase and location decisions for digital tools such as interactive whiteboards which they shared with teachers from other faculties. T1, being a mathematics faculty coordinator, has a greater influence in determining which tools are to be purchased for use by teachers in the faculty. For example, he is always aware of how much money the faculty has or can get to purchase items for the faculty's use. All the interviewed teachers have access to computers with software that can be used for teaching algebra, for example graphing software. However, T3 noted that there are inconsistencies between the programs that the teachers have on their laptops, and what is available for students. There were also inconsistencies between software on computers in different rooms at the school with teachers getting updated software, but not the student computers. Only T1 mentioned that he regularly tries out a digital tool at home before coming to use it at school. This is not always an advantage as there may be a need to consult a colleague at school for help when difficulties are encountered. The digital tools that he found most appropriate for algebra use in his practice were the 'widgets' in *HOTmaths* tutor program (Figure 4.1), and the scientific calculator for use when solving substitutions problems, and the interactive whiteboard for setting up patterns.

All the respondents believed that there are different ways in which tools are being used or that other teachers were using them much more extensively and intensively, especially when it came to the graphics calculator and the interactive whiteboard. T2 believed strongly that students were even better skilled in the use of some of the tools than she and other teachers were, and so she relied on them sometimes to find solutions to some problems. T3 appeared to be aware of many more software tools than the other respondents. He was keen on learning how to use different technologies and used some of his out of school time doing so, to become as efficient as possible. He was also willing to help his colleagues at school but he was sensitive to those who were afraid to reveal their weaknesses in using some technologies.

5.3 How teachers incorporate technology into algebra

Some digital tools, like the graphics calculator and software such as *autograph*, can be used by teachers and students for the multiple representation of mathematics objects and processes (See Figure 2.1 and also Section 2.3.2). Such digital tools can be linked to the interactive whiteboard so that all students can focus on one activity on the screen in the classroom. The informants reported using the interactive whiteboard to present equations, expressions, calculations, graphs, models. With students in the senior years and students doing the advanced course in Years 9 and 10, teacher T2 and T3 used graphic calculators to solve nearly all algebra problems. T3 was keen to see that the students know as much of what the graphics calculator can do before he lets them use it to solve textbook problems. T2 and T3 combined the multiple presentational capabilities of both the graphics calculator and the interactive whiteboard to teach algebra to both senior and junior students in their schools. T2, when introducing an algebra topic, first explained the concepts to the student using the interactive whiteboard alone before she used any other tool. She felt that it is important that students know the purpose for which they are using the tool before they use it. T4 started using tools right from the start of the lesson without having to explain how the tool worked first. This could be explained by the fact that he does not use advanced technologies like the graphics calculator, which require more explanation before many students can use them effectively. T4 found the less complicated scientific calculator, which was his main digital tool, easier to use for solving problems in algebra.

5.4 Overcoming teacher's shortcomings in the use of technology

When learning to integrate digital technology into mathematics, teachers move along a continuum from nontechnology stances to incorporating technology extensively in their teaching of mathematics. In this investigation, all the informants admitted to having shortcomings in their use of digital technology. However, they were all not at the same level on this technology integration continuum. Using Hall and Hord's (2001) Levels of Use model (See Section 2.4.2.1), it is possible to identify the approximate positions of the informants in relation to their technology integration capabilities. On this model, the lowest level of use is the mechanical level while the highest is the renewal level.

At the mechanical level, the teacher closely follows a user-guide to complete specific activities to the highest level. At the highest level, teachers are able to make major changes or modifications in their use of the digital tool to improve outcomes. The investigator sought from respondents how they overcame shortcomings in their use of digital technology when they taught algebra. T1 relied, for his knowledge, on cluster workshops and conferences such as those provided by the Mathematical Association of New South Wales to improve his knowledge of how to use technology in mathematics in general. He believed that this should be the way to go for his colleagues and other teachers who may be experiencing problems with the use of digital tools in the teaching not just of algebra but also other areas of school mathematics. T2 believed that, in addition to workshops, teachers in a faculty should assist each other more and those who are weak in certain areas should not hesitate to seek help from the others. Sometimes, according to T2, a teacher should ask for a student's help as most students are quick at learning how to use digital technologies. In saying this, she was unconsciously encouraging the 'Recommend' level of use in the PURIA (Play, Use, Recommend, Incorporate, Assess) model suggested by Beaudin and Bowers (1997). T3 felt that although learning to use technology is possible for in-service teachers, there are some whom he felt were incapable of learning how to use certain types of technology, no matter how much explaining is done to them. For T4, once one learns how to use technology in earlier days, then one should find it easy to learn how to use newer technologies.

The suggestions of the informants about what should be done to improve their use of digital tools correspond with the informants' concerns. Zbiek and Hollebrands (2008) have defined concerns as the affective aspect of implementing technology in teaching. As the teacher moves up the continuum of ability in the use of technology, there is change in their concerns. Hall and Hord (2001) suggested a Stages of Concern model for teachers encountering an innovation such as the first time use of a particular piece of digital technology with school algebra. The three stages of concern—personal, management and technology—have been experienced to different extents by the four informants. T2's responses suggested that she has experienced more of each of the three concerns, and T1 and T3 have experienced the least number of personal, management or technology concerns in relation to their use of digital technology.

5.5 Linking the results and theories on technology integration

The adapting of more than one theoretical framework to this inquiry is based on the suggestion by Schoenfeld (2002) that no single theoretical framework can explain all phenomena in the complex setting of learning algebra in a digital technology rich learning environment.

5.5.1 The use of the interactive whiteboard

The popularity of the use of the interactive whiteboard among respondents can be explained by several theories. Using Buchberger's (1989) White Box-Black Box idea, the widespread use of the interactive whiteboard by serving as a conduit through which a teacher can explain the working of another digital tool, say a graphics calculator, means that the interactive whiteboard is serving to reduce the black box use of the graphics calculator by the teacher. In simpler algebra work, scientific calculators can be projected through the interactive whiteboard to highlight the keys the teacher is pressing if the students have a similar calculator. The multirepresentational (Schwarz & Bruckheimer, 1988) capacity of the interactive whiteboard, that is, the capacity to simultaneously present a magnified version of tabular, graphical and symbolic representations of algebra objects and processes, can also not be ignored by teachers of algebra. In instrumentation approaches (Rabardel, 2002), the interactive whiteboard provides a background against which the teacher can orchestrate (manipulate) virtual digital tools to guide students' instrumental geneses when such tools are being used to learn algebra. Finally, using Vigotsky's (1978) idea of mediation, the interactive whiteboard serves a crucial role as a mediating artefact for the teacher and the students to represent algebra objects and processes, they would otherwise not have access to, given that these objects and processes are ideal, immaterial and non-perceivable.

5.5.2 The use/non-use of the graphics calculator

For the teacher who uses the graphics calculator, it can be argued that one of its main uses is as an amplifier and an organiser (Pea, 1987). It has an amplifier function in the sense that it can help quickly investigate many similar cases. For example, the solving of equations or inequalities at all levels of high school algebra. According to Pea (1987), problems for the algebra teacher in relation to exploiting the amplifier and organiser capabilities of a tool such as a graphics calculator have their origin in the teachers' lack of awareness of the two-way reorganisation possibilities afforded by the digital tools. The teacher who had problems using the technology, T2, according to Pea (1987) most likely had a low ability in relation to affecting the technology in relation to deciding the appropriate way of using it for the goal of getting students to learn algebra with the tool.

It can also be argued that the graphics calculator has a high potential for use as a black box, i.e., it can be used with the students not being aware of mathematics they are asking the technology to carry out (Buchberger, 1989). Buchberger argued that the use of symbolic manipulation software such as is contained in the graphics calculator can be disastrous for students when they are initially learning some new area of mathematics. On the other hand, research by Heid (1988) and also by Berry, Graham, and Watkins (1994) showed that students can develop conceptual understanding with symbolic manipulation software before mastering by-hand manipulation techniques. Teachers with problems in the use of graphics calculators, in the context of this research finding, are likely to deny some students an enhanced understanding of algebra concepts.

The graphics calculator played a significant role in the teaching and learning of algebra for two of the teachers interviewed. The instrumental approaches framework (Rabardel, 2002; Trouche, 2004) can be used to explain their abilities in relation to their use of the graphics calculator. For one of them, T3, the process of his own instrumental genesis was at an advanced level. Instrumental genesis is the process of an artefact becoming an instrument. It is an ongoing, non-trivial and time-consuming evolution, during which a bilateral relationship between the artefact and the user is established. T2 reported that she still experienced some problems when using graphics calculators. This means T2 is possibly at a lower level than T3 in relation to her instrumental genesis, and therefore was likely to experience greater orchestration (the management of the collective instrumental geneses of her students) issues with her students in their learning of algebra.

Finally, the representational capacity of the graphics calculator, with its ability to simultaneously present symbolic, graphical and tabular representations of algebraic objects and processes (Figure 3) gives enormous advantage to those teachers, such as T3, who know how to use this aspect of these calculators. The advantage for students learning algebra is that it provides a choice so that students who learn algebra concepts more easily when they are presented in a particular form will concentrate on this form of the algebra object and may use their being comfortable with this form (e.g. graphical) to study the other forms (tabular and symbolic).

5.5.3 Graphing software.

Graphing software is embedded in at least one of the digital tools that the teachers said that they used in their classrooms. It is particularly relevant to algebra learning as high school students are required to draw graphical representations of certain functions. Informant T3, for example, reported that he used *Graph Plotter* software in the classroom. The graphs were magnified through the interactive whiteboard. It is clear that if the teacher did the explainer role (Farrell, 1996) well while using graphing software, there can be enhanced understanding of algebra concepts. The software also plays a representational role as the graph drawn also has the original equation beside it. The drawing of several graphs in the same picture which is afforded by graphing software can highlight to students the nature of certain types of graphs and enhance the 'White Box' or reduce the 'Black Box' use of this software, possibly leading to better understanding of algebra.

5.5.4 *HOTmaths* program

Respondent T1 reported widespread use of this tutor program in his junior high school mathematics classes. Using the Tutor, Tool, Tutee framework developed by Taylor (1980), the *HOTmath* program functions as a tool when it is used, for example, as a calculator. A teacher, such as T1, using this program is likely to encounter this tool aspect of it when using widgets such as that in Figure 4.1 to solve algebra problems. Using the instrumental approaches framework (Rabardel, 2002; Trouche, 2004), a teacher's use of, say, the graphics calculator through the interactive whiteboard to teach algebra will involve an instrumental orchestration process. Instrumental orchestration is the intentional and systematic organisation of the different artefacts in a computerised learning environment to guide students' instrumental geneses.

5.5.5 Teachers’ pedagogical technology knowledge

All the respondents reported the need for professional development in relation to the use of digital tools for themselves and for all the other teachers who were experiencing problems with the use of digital technology in mathematics. Respondents T3 and T4 felt that they had sufficient technical ability to be able to use most pedagogical tools. T1 was comfortable with those that he was using. T2 felt strongly that teachers needed to continue getting as informed as much as possible about all the available digital tools. In addition, T2 was appreciative of manufacturers who took time to train teachers how to use their tools effectively. Of all the theories in the literature review, only Valsiner’s Zone Theory (Valsiner, 1997) directly focused the professional training needs of teachers in relation to their pedagogical technology knowledge. Like some of the other theories in the review of the literature, it is based on the writings of Vygotsky, and it extends Vygotsky’s (1978) concept of the Zone of Proximal Development (ZPD). The ZPD has been defined as the gap between the learner’s present capabilities and the higher level of performance that could be achieved with appropriate assistance. A key element of the ZPD is the pedagogical technology knowledge of teachers in relation to technology integration in mathematics. Valsiner (1997) extended the ZPD by incorporating two additional zones, the ZFM (Zone of Free Movement) and ZPA (Zone of Promoted Action) (Table 2) all of which are relevant when addressing teachers’ professional needs in the context of their integrating digital technology tools into the teaching of algebra concepts.

5.6 Conclusion

To address the pedagogical technology knowledge (PTK) of teachers in relation to their use of digital technology in their teaching of algebra is a complex undertaking. Underlying this complexity are the many factors that come into play once a mathematics teacher starts to integrate knowledge of technology with subject-matter knowledge. Meaningful contribution to teachers' PTK can only come about once attention is paid to such issues as the teachers' capacity to use the digital tool, the roles played by the teacher in a technology-enabled classroom, the concerns arising from shortcomings in the quality of teachers' integration knowledge base, and an understanding of the workings of the available digital tools. Such is the complexity of the integration of digital technology into school algebra that no single theoretical framework can satisfactorily account for all phenomena in this multifaceted area. Consequently, more than one framework must be sought and examined for their explanatory power in order to build the quality of the knowledge base required to enhance student understanding of algebra concepts in a technology-enabled algebra classroom.

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS

6.0 Introduction

The final chapter answers the investigation's research questions within the context of the evidence collected. In addition, it discusses the implications and limitations of the study before concluding with recommendations for further research. Sections 6.1, 6.2, 6.3, and 6.4 address the first, second, third and fourth research questions respectively. Section 6.5 discusses the implications of the study while Section 6.6 addresses its limitations. While Section 6.7 provides recommendations for further studies, Section 6.8 concludes the chapter with the overall summary conclusion.

6.1 What roles do mathematics teachers adopt as they integrate cognitive digital technological environments in algebra lessons?

All the informants saw their role primarily as that of facilitators of their students' learning experiences when cognitive digital technological environments are integrated with school algebra activities. Only informant T2 mentioned two roles: facilitator and fellow investigator with the students. When asked to elaborate on their understanding of the facilitator role, they said it encompassed all the classroom activities of teachers when they integrated digital technology in a mathematics lesson. They argued that concern with individual activities was pointless. The teacher being conscious of getting the students to understand the concepts being discussed was more important. However, if their pedagogical technology knowledge is to be improved in relation to how they play the facilitator role, then the teachers need to be conscious of the individual activities that they are engaged in when integrating technology into algebra. Secondly, they need to know how well or how poorly they are carrying out each activity. Some of the activities are: allocator of time, catalyst, counsellor, evaluator, explainer, manager, planner and conductor, resource, task setter and technical assistant.

A balance needs to be established between the roles so that the performance of some roles does not overshadow the others. For example, it is possible in algebra for the teacher to spend more time on the ‘explainer’ activity than, say, the ‘task setter’ activity. A second example is that of time allocation. As time allocators, teachers need to be conscious of how they allocate time to different activities during the lesson because they have to work within the time requirements of the school as well as accommodate the needs of individual students.

Playing the collaborator role effectively includes admitting that they are as much learners as the students. This may be difficult in instances where the teacher’s managerial approach is authoritarian instead of, say, that of a tactical manager with respect to class management style. Informant T3 viewed the technical assistant role as a very important one. It would be interesting to find out, through another data collection method, such as non-participant observation, how technically competent teachers like T3 allocate time to the other roles they are expected to play in a technology-enabled algebra classroom.

6.2 Why do teachers use cognitive digital technology environments the way they do when teaching algebra?

For some of the informants, such as T3 and T4, the learning experiences prior to taking up their current teaching positions were the primary influence on why they used digital technology the way they did in their mathematics classrooms. For T1 and T2, learning how to use digital technology has been mainly through the opportunities availed through attendance at workshops and conferences such as those conducted by the Mathematical Association of New South Wales. T2 reported that she benefitted greatly from workshops conducted at School B by CASIO on how to use CASIO graphics calculators. T1, T2 and T4 teachers also mentioned learning from demonstrations by their peers in the mathematics faculty. T3 benefitted greatly from having been an IT specialist and trainer in different working environments prior to taking up the current teaching position. T3 was relied upon by the other teachers in the school for their knowledge of how to use mathematical software that they had difficulties with.T2 was the only one of the four informants who admitted to being assisted by students on some occasions when teaching algebra using a graphics calculator. This can be an excellent way of learning if there is good rapport between the students and the teacher or if the teacher is not self conscious about being helped by students in areas of difficulty. The other method used by all informants was that of self teaching.T1 reported doing this on a regular basis while at home. At those times when the person who can help is too busy, teachers may be left with no other option that to learn how to use the tool themselves. Since it is possible to fall into time wasting habits/approaches with a digital tool, it may be necessary for teachers to raise their efficiency levels by learning from those more knowledgeable than them.

6.3 How do teachers incorporate cognitive digital technological environments in their teaching of algebra?

Informants T2 and T3 reported linking a graphics calculator to the interactive whiteboard in the classroom through the laptop when teaching algebra to students in the senior classes and to those doing the advanced mathematics courses in the junior classes. One of the uses of this arrangement was to focus all the students in the class to the relationship between the algebraic, graphical and tabular representations of a function. The visualisation afforded by the interactive whiteboard was combined with the multi-presentational capacity of the graphics calculator. T2 and T3 believed that this greatly assisted the students to understand functions in algebra. T1 and T4 also linked scientific calculators to the interactive whiteboard via their laptops, so that the students could easily see the keys that needed to be pressed to solve a problem in algebra. According to T2, this use of the interactive whiteboard for teaching algebra saved much time in the lesson as the teacher did not have to move around showing individual students which key to press on the calculator as was the case prior to the installation of interactive whiteboards.

The informants were also using the digital tools for computational purposes in algebra. Among their advantages in algebra learning were the high speed of computation, the amplifier effect and, when used correctly, the improved accuracy of computation compared to when no tool is used. For all the informants there was a widespread use of digital tools as computational tools for global/meta-level algebraic activities such as problem-solving (See Figure 1.1). T1 found that some 'widgets' in the *HOTmath* tutor program could be used to solve problems in algebra and was using them for this purpose in the classroom. According to T1, this saved much time as there was no need to write long equations several times on the screen of the interactive whiteboard.

6.4 How can shortcomings in the pedagogical technology knowledge of in-service teachers be addressed for enhanced student understanding of algebra?

T1 and T2 felt strongly that there was a need for teachers to regularly attend professional development workshops if they were to improve the quality of their knowledge base for integrating digital technology with algebraic activities. The other respondents felt that many teachers lack the motivation to attend such workshops. The Mathematical Association of New South Wales was viewed as a significant provider of in-service training in the use of digital technology for teaching and learning mathematics.

This training, by adding to a teacher's skills 'bank' most likely helps to reduce tension between familiar sources and methods and the use of cognitive technological environments. For the informants, training presents a real possibility for reducing the 'black box' use of tools. Students taught by teachers who are well trained in the use of tools and in how to integrate them in mathematics are likely to benefit from the rich usage of these tools.

Shortcomings in pedagogical technology knowledge were highlighted most by T2, who has taught mathematics for over 30 years. T2 was however, very willing to learn, even from her students. This willingness appears to suggest that some long serving teachers can become avid learners of the new possibilities afforded by digital technological environments for algebra learning.

6.5 Implications of the study

The study has implications for a number of issues related to the integration of digital technology in algebra teaching and learning.

6.5.1 Implications for educators and policy makers

As the use of digital technology grows in secondary schools, its use is also growing in mathematics classrooms. With algebra being a key ingredient of all secondary mathematics curricula, it is important that educators and other stakeholders start looking closely at what is happening in the classrooms where technology is being used with algebra as technology has the capacity to facilitate learning when used appropriately by teachers and students or to constrain learning when used inappropriately.

6.5.2 Implications for school managers

It is not enough for schools to avail digital technology to their mathematics faculties. There is a need to place as much importance on teachers' pedagogical technology knowledge (PTK) in relation to the use of this technology as on the purchasing of the tools. Teachers need to be encouraged to attend workshops and/or school managers need to show a greater willingness to spend money on bringing experts into schools, for example, manufacturers of the tools, and university educators in order to raise the quality of teachers PTK for improved algebra teaching and learning.

6.6 Limitations of the Study

There were limitations to the study in relation to the size of the sample, triangulation and resource limitations. In relation to the sample size, the study would have benefitted from a larger sample than four schools and four teachers. With the small 'sample', the researcher took Yin's (2003) suggestion that, '… in doing a case study, your goal is not to enumerate frequencies (statistical generalisation)'. The study was also not as strong as it should have been had the researcher used methodological triangulation along with the data triangulation. Data triangulation was achieved because 'data was collected from multiple sources but aimed at corroborating the same phenomenon' (Yin, 2003, p. 99). Use of methodological triangulation would have improved the study's findings through the development of converging lines of enquiry. Finally, the researcher did not have the time and finances to travel to all the schools whose teachers he would have wished to interview. The large geographical spread of the schools also limited how much time the researcher spent with each teacher. The study would also have benefited from the researcher interviewing the students taught by the teachers that he interviewed. This would have provided the students' perspectives of their experiences regarding the use of digital technology in their algebra lessons.

6.7 Recommendations

A number of recommendations can be drawn from this exploratory case study investigation of teachers' use of cognitive digital technological tools in the teaching of algebra.

6.7.1 Further Research

There is a need to carry out a closer investigation of what is happening in the classroom with respect to the use of technology in algebra through having a number of participant and non-participant classroom observations of a large number of high school mathematics teachers. Longitudinal studies would be particularly useful in mapping out what is happening in the classrooms and finding out linkages, and the nature of such linkages, between the use of cognitive technological tools and students' performance in algebra. Lastly, investigating the use of digital tools by teachers to enhance student understanding of algebra, can be carried out with a much larger sample than that for this study to facilitate better generalization from the data.

6.7.2 Encouragement of professional development

There is a need to encourage as many teachers as possible to attend conferences of professional associations such as the Mathematical Association of New South Wales where they learn, often from their peers, how to use tools differently, how to use a tool they have never used before, when to use certain tools and so many other issues related to tool use. Teachers who are knowledgeable in the use of certain tools should also attend these conferences for the purpose of presenting to the other teachers. To facilitate teacher participation at such conferences, school membership to professional associations should be encouraged by mathematics faculty coordinators in schools. University teachers also become available at these conferences and are an excellent resource for teachers who attend their presentations.

While professional associations are best suited for helping in-service teachers, tertiary teacher training institutions should continue to lay the foundations for the excellent integration of technology with mathematics for pre-service teachers. Mathematics education courses should contain a strong element of theory related to technology integration in mathematics teaching.

Finally, cluster workshops, probably the least expensive alternative (whether managed by professional associations or education authorities) should be run on a regular basis, with mathematics teachers encouraged to participate as learners or presenters, besides discussing local difficulties experienced with the use of specific tools. The workshops should not stop at merely addressing tool use in mathematics in general but should move to discussing tool use in specific areas of mathematics, such as algebra. Given the importance of algebra as a component of high school mathematics courses in New South Wales and in Australia as a whole, it is important that tool use by teachers in algebra teaching and learning is taken seriously by mathematics teachers and those responsible for their training (in the case of pre-service teachers) and their professional development (for the in-service teachers).

6.8 Conclusion

Overall, the study has highlighted the importance of raising the quality of teachers' knowledge base for integrating digital technology with knowledge of algebra if students' understandings of algebra concepts are to be enhanced. While there have been increases in the availability of digital tools in secondary schools, there have needs to be a corresponding increase in the level of teachers' ability to use them effectively for teaching algebra.

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APPENDICES

Appendix 1: Interview Questions

Question
1. How long have you taught mathematics at the secondary school level?
2. Which year levels do you presently teach?
3. Prior to using a digital tool in the mathematics/algebra classroom, how did you learn how to use it? (How did the teacher acquire his/her pedagogical technology knowledge relating to the use of a particular tool?)
4. Describe how you have used a digital tool outside an algebra classroom setting on your own.
5. Describe how you have used a digital tool when teaching an algebra concept.
6. Once you are proficient in the use of a digital tool, describe how you have got your students to use it in an algebra lesson.
7. What do you view your role to be as you integrate digital technological environments in algebra lessons? Describe this view of your role.
8. Are you aware of any other ways (different to yours) in which a particular digital tool (say a graphing calculator) is used by other teachers?
9. In light of question 5 (in the interview, this was posed as 'in light of the previous question') why have you chosen to use your particular approach for using the digital tool?
10. Describe any concerns you have in relation to the incorporation of technology in algebra lessons: (a) Personal Concerns? (b) Technology Concerns? (c) Management Concerns?
11. How can shortcomings in the pedagogical technology knowledge of in-service teachers be addressed for enhanced student understanding of algebra?

Appendix 2: Information Sheet for the participants

une
University of New England

School of Education
University of New England
Armidale NSW 2351
Australia
Phone 61 2 6773 4221
Fax 61 2 6773 2445
education@une.edu.au
www.une.edu.au

13-9-2010

To the Principal,

Dear Mr/ Mrs

I would like to request for permission to interview mathematics teachers in your school for a research project.

This is the information that will be given to the Mathematics teachers in your school who will be participating in the research project that I will be conducting. I have obtained permission from both the Ethics Committee at UNE and the Director, Catholic Schools Office, Armidale. I have enclosed copies of these for your perusal. I have also enclosed a consent form for participants. If you grant me the permission, please discuss with and make the Mathematics Coordinator aware of my request.

INFORMATION SHEET for PARTICIPANTS
Research Project: Investigating Teachers' Technology Use to Enhance Student Understanding of Algebra

I wish to invite you to participate in my research on above topic. The details of the study follow and I hope you will consider being involved. I am conducting this research project for my Master of Education (Honours) at the University of New England. *My supervisors are Associate Professor Stephen Tobias and Dr. Pep Serow of University of New England. Associate Professor Tobias can be contacted by email at stobias@une.edu.au or by phone on 02 67732573. Dr Serow can be contacted by email at **pserow2**@une.edu.au or by phone on 02 67732378.*

Aim of the Study:

The study aims to identify and describe ways in which teachers enhance student understanding of algebraic concepts as they (the teachers) integrate digital technological environments in their teaching of algebra.

Time Requirements: 30 minutes

Interviews:
I would like to have face to face interviews which will last for approximately 30 minutes. There will be a series of open-ended questions that allow you to explore your views and practices related to your teaching of Algebra in digital technological environments. These

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Armidale NSW 2351
Australia
Phone 61 2 6773 4221
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education@une.edu.au
www.une.edu.au

interviews will be audio recorded using digital recording. Following the interview, a transcript will be posted to you if you wish to see one.

Participation is completely voluntary. You may withdraw from the project at any time and there will be no disadvantage if you decide not to participate or withdraw at any time.

It is unlikely that this research will raise any personal or upsetting issues but if it does you may wish to contact your local Community Health Centre

The digital files will be kept in a locked filing cabinet within the School of Education at UNE. The transcriptions will be kept in the same manner for five(5) years following thesis submission and then destroyed.

Research Process:
It is anticipated that this research will be completed by the end of November, 2010. The results may also be presented at conferences or written up in journals without any identifying information.

This project has been approved by the Human Research Ethics Committee of the University of New England (Approval No. HE10/118, Valid to 07/07/2011)

Should you have any complaints concerning the manner in which this research is conducted, please contact the Research Ethics Officer at the following address:
Research Services
University of New England
Armidale, NSW 2351.
Telephone: (02) 6773 3449 Facsimile (02) 6773 3543
Email: ethics@une.edu.au

Thank you for considering this request and I look forward to further contact with you.

Regards

M. Masige

Michael Masige

Appendix 3: Consent Form for Participants

University of New England

School of Education
University of New England
Armidale NSW 2351
Australia
Phone 61 2 6773 4221
Fax 61 2 6773 2445
education@une.edu.au
www.une.edu.au

Consent Form for Participants **13-9-2010**

Research Project: *Investigating Teachers' Use Technology Use to Enhance Student Understanding of Algebra*

I, ..., have read the information contained in the Information Sheet for Participants and any questions I have asked have been answered to my satisfaction. Yes/No

I agree to participate in this activity, realising that I may withdraw at any time. Yes/No

I agree that research data gathered for the study may be published using a pseudonym Yes/No

I agree to the interview being audiotape recorded and transcribed. Yes/No

....
Participant Date

....
Researcher Date

Appendix 4: Approval letter from HREC Committee of UNE

une
University of New England

Ethics Office
Research Development & Integrity
Research Division
Armidale NSW 2351
Australia
Phone 02 6773 3449
Fax 02 6773 3543
jo-ann.sozou@une.edu.au
www.une.edu.au/research-services

HUMAN RESEARCH ETHICS COMMITTEE

MEMORANDUM TO:	A/Prof S Tobias, Dr P Serow & Mr M Masige School of Education

This is to advise you that the Human Research Ethics Committee has approved the following:

PROJECT TITLE:	Investigating teachers' technology use to enhance student understanding of algebra.
APPROVAL No.:	HE10/118
COMMENCEMENT DATE:	07/07/2010
APPROVAL VALID TO:	07/07/2011
COMMENTS:	Nil. Conditions met in full.

The Human Research Ethics Committee may grant approval for up to a maximum of three years.
For approval periods greater than 12 months, researchers are required to submit an application for renewal at each twelve-month period. All researchers are required to submit a Final Report at the completion of their project. The Progress/Final Report Form is available at the following web address:

The *NHMRC National Statement on Ethical Conduct in Research Involving Humans* requires that researchers must report immediately to the Human Research Ethics Committee anything that might affect ethical acceptance of the protocol. This includes adverse reactions of participants, proposed changes in the protocol, and any other unforeseen events that might affect the continued ethical acceptability of the project.

In issuing this approval number, it is required that all data and consent forms are stored in a secure location for a minimum period of five years. These documents may be required for compliance audit processes during that time. If the location at which data and documentation are retained is changed within that five year period, the Research Ethics Officer should be advised of the new location.

Jo Sozou.

Jo-Ann Sozou
Secretary

07/07/2010

A09/2596

Appendix 5: Approval letter from the Director, CSO, Armidale, NSW

CATHOLIC SCHOOLS OFFICE

P.O. Box 636 ✠ 125 Barney Street ✠ Armidale NSW 2350 ✠ Phone (02) 6772 7388 ✠ Fax (02) 6772 9285
ABN 90 702 916 776 • Email cso@arm.catholic.edu.au

6th September 2010.

Mr Michael Masige
P.O. Box 294
GUNNEDAH NSW 2380

Dear Michael,

Further to your request to conduct research schools in the Diocese of Armidale, I am pleased to advise that approval has been granted.

The approval allows you to approach school principals and seek their involvement in your project, *Investigating Teachers' Technology Use to Enhance Student Understanding of Algebra.*

It should be understood that it is the prerogative of the principal to decline your invitation to be involved in this study or to withdraw from involvement at any time.

The privacy of the school and that of any school personnel involved in your study must, of course, be preserved at all times.

When your research has been completed, please forward a summary report of the findings and/or recommendations to the school as soon as practicable after results are to hand.

It is necessary that you provide a copy of this letter to the principal when seeking their involvement in this study.

I wish you well in this undertaking.

Yours sincerely,

D. K. Barnes
for

John M. Mula
Diocesan Director of Catholic Schools

Appendix 6: Letter of Support from St, Mary's College Principal

St Mary's College
Founded by the Sisters of Mercy 1879

PO Box 730
GUNNEDAH NSW 2380

Ph: (02) 6742 2124
Fax: (02) 6742 0188

12th March 2010

Mr John Mula
Diocesan Director
Catholic Schools Office
PO Box 636
ARMIDALE NSW 2350

Dear John,

I enclose an application from Michael Masige to conduct research in Mathematics Education in schools in our diocese as part of his M.Ed. (Hons) program through UNE.

Michael has been on the staff of St Mary's College since 2003. He is dedicated to the teaching profession and is one of the most conscientious and thoughtful teachers I have ever worked with. He is also committed to his own professional development and has been undertaking some tertiary course for almost all the time I have known him. I am confident that his research will be of benefit not only to Michael personally, but also to the Mathematics faculty at St Mary's College and to the Mathematics teaching community as a whole.

Michael proposes to take leave without pay for the purposes of conducting his research.

I am very happy to endorse Michael's proposal to you.

Yours sincerely,

A. J. Moran.

Tony Moran
Principal

"Virtue is the way of life"