SEDIMENT TRANSPORT INDUCED BY RAINDROPS IMPACTING SHALLOW FLOWS

By

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A thesis submitted for the Degree of Doctor of Philosophy

of the University of New England

April, 1992

ABSTRACT

A theory for RIFT, the transport process that results from raindrops impacting shallow flows, is presented. The theory relies on the observation that, after being lifted from the soil surface, a particle moves downstream a distance that depends on flow velocity and the time the particle remains suspended in the flow. The theory indicates that sediment transport rates increase linearly with rainfall intensity and flow velocity when entrainment by flow is absent.

Laboratory experiments where sand was eroded by rain-impacted flow provide support for the theory. In addition to influencing the downstream motion of particles detached by raindrop impacts, surface-water flows absorb raindrop energy. Consequently, interactions between flow depth and drop size were also examined through the laboratory experiments. This resulted in a mathematical model of RIFT that accounts for the interactions between raindrops and flow on the sediment transport rate. When applied to experimental data, the model showed that the time-averaged effect of rainfall on sediment transport by rain-impacted flow is independent of the manner by which the rain is applied.

Particle size was also varied in the experiments. Particles having similar fall velocities in water but different densities were transported at different rates. The differences were more the result of differences in the masses of material lifted into the flow by a drop impact than differences in the distances the particles travelled after being disturbed.

On soil surfaces, pre-detached particles stored on the surface between impacts protect the soil matrix. Using a numerical model of RIFT, the protective effect was shown to vary in time, with particle size and with the difficulty experienced by drops in detaching soil particles from the soil matrix.

Flow depths and velocities are seldom measured but the RIFT theory provides a mechanism for developing models using more commonly measured parameters. An analysis of field experiments using an alternative to the more commonly used interrill erosion model indicates that actual interrill erodibilities may differ significantly from those determined using the previous model. An alternative to the EI_{30} index in the USLE also results from the RIFT theory.

CERTIFICATE

I certify that the substance of this thesis has not already been submitted for any degree and is not being currently submitted for any degree.

I certify that any help received in preparing this thesis, and all sources used, have been acknowledged in this thesis.

[redacted] 8/4/92

PREFACE

The work reported in this thesis results from a long term interest in erosion resulting from raindrop impact. Initially, when I joined the CSIRO Division of Soils 1969, I worked on splash erosion and acoustic impact as a measure of the erosive power of rainfall. When the raintower facilities of the Division became fully operational in the mid 1970s, as part of the Sediment Transport Group, I was involved in performing some initial laboratory experiments on sediment transport by rain-impacted flow before undertaking experiments with runoff and soil loss plots under natural rainfall. Erosion by rain-impacted flow was the major cause of the erosion in these field experiments and it soon became obvious that internationally there was a lack of knowledge on this erosion process. Most of mathematical models of rainfall erosion produced to date fail to account for sediment transport by rain-impacted flow because of this. The work reported here is a contribution towards resolving this problem.

Most of the work reported here was done at the Canberra Laboratories of the CSIRO Division of Soils. In regard to this work, the assistance of Colin McLachlan, Joseph Kemei and Neville Carrigy who helped from time to time with some of the experiments is gratefully acknowledged. Some of the statistical procedures used in this thesis result from work done by Dr. Jeff Wood (I.N.R.E. Biometric Unit, CSIRO). The assistance of Dr. David Smiles, Dr. Pat Walker, Dr. John Williams in providing an administrative climate which allowed the work to proceed is also gratefully acknowledged. The quality of the figures in this thesis is a testament to the skills of the Division's Drawing Office in Adelaide. Also, a special vote of thanks is extended to John Hutka who undertook much of the work in developing the raintower facilities back in the mid 1970s. Without the development of such facilities, much of the work reported here would not have been possible.

The work covering drops with low impact velocities was done at the Department of Resource Engineering, University of New England with the assistance of a CSIRO/UNE co-operative grant. This work required the development of new facilities at the university. The support of the workshop staff of the Department of Resource Engineering, in particular David Sauer and

Trevor Stace, in developing these facilities is gratefully acknowledge. My thanks are also given to Prof. John Burton (Dept. Res. Eng., UNE), Dr. Alf Cass (Dept. Agron. and Soils, UNE) and Prof. Ian Moore (CRES, Australian National University) who acted as supervisors for the Ph.D program.

SUMMARY

Rain-impacted flows often dominate the sheet and inter-rill erosion environment, but the factors that influence sediment transport by these flows have been studied little. Many modern models of rainfall erosion ignore the contribution rain-impacted flows make to the movement of soil material over the soil surface. In this thesis, a theoretical basis for investigating the factors that influence sediment transport by rain-impacted flow is presented in Chapter 2. The effects of a number of these factors are studied experimentally in Chapter 3. Then the application of the theory to modelling the erosion of soil surfaces by rain-impacted flow is demonstrated in Chapters 4 and finally, some suggestions for further study are presented in Chapter 5.

When a soil particle is detached from the soil matrix by a raindrop impacting a shallow flow, the particle may move downstream across an arbitrary boundary in one of 4 modes. The particle may move aerially by drop splash, but if it fails to be lifted above the water surface, or falls back into the water upstream of the boundary, it may then move in one of 3 modes that are associated with the flow. It may, if it is small or of low density, move as suspended load. If it is larger or of higher density, it will fall back to the soil surface before the boundary. If the flow has sufficient velocity, the particle may then be entrained by the flow. If not, then the subsequent downstream movement can only occur under the stimulus of a external force. Raindrops impacting shallow flows can provide that force.

The downstream movement of soil particles that relies on repeated stimulation by raindrop impact is termed Raindrop Induced Flow Transport (RIFT). The theory for RIFT presented in this thesis relies on the observation that, after being lifted from the soil surface as a result of a drop impact, a particle travels a distance (x_p) downstream that depends on the time the particle remains suspended in the flow (t_p) and the velocity of the flow (u). The distance travelled controls the extent of a zone, called the active zone, in which all drop impacts cause soil material to pass across the boundary. As a result, the sediment transport rate (q_{SR}) across the boundary is given by the product of the frequency (f) of the drop impacts in this zone and the mass of material (D) each drop lifts into the flow. The sediment transport rate can

be expressed as a function of rainfall intensity (R), raindrop size (d) particle size (p) and flow velocity by

$$q_{sR}\ (p,d) = \frac{6\ R_d\ t'_{pd}\ u\ D_{pd}}{\pi\ d^3} \qquad (2.20)$$

where t'_{pd} is the effective average duration of the suspension of p sized particles induced by the impact of drops of size d.

On soil surfaces, pre-detached particles are stored on the soil surface between impacts and, as a result, drop impacts may lift both pre-detached particles and particles from the soil matrix. Pre-detached particles sitting on the surface are lifted first and particles from the soil matrix are lifted only if there is excess energy left after this process. The pre-detached particles provide a degree of protection (H), with the result that

$$D_{pd} = H.D_{pd.D} + (1-H)D_{pd.M} \qquad (2.24)$$

where $D_{pd.D}$ is the value of D_{pd} obtained when the soil matrix is completely protected and $D_{pd.M}$ is the value of D_{pd} obtained when no pre-detached material exists. The need to store particles on the soil surface during the transport process results in the development of a layer of pre-detached particles on the soil surface. Through the use of a numerical model of the RIFT processes, the temporal and spatial variability of this layer is demonstrated in Section 2.4.

Available data, together with new data collected during this study (Chapter 3), confirm that Eq. 2.20 can account for effects of R and u on q_{sR}. These data also show that q_{sR} is influenced by particle size, density and flow depth (h). Apart from drop size and particle size, factors such as drop velocity, drop shape, particle density, and flow depth influence D_{pd} and t'_{pd}. The data collected during the study show that,

$$D_{pd}t'_{pd} = k_0(1 - \beta h) \qquad \text{for } h_1<h<h_2 \quad (3.5)$$

where $h_1 \approx 4$ mm and $h_2 \approx 3d$, k_0 is the intercept on the "y" axis projected by the linear equation

$$D_{pd}t'_{pd} = k_0 - b_2h \qquad (3.6),$$

and β is the inverse of the projected intercept on the "x" axis. Both β and k_0 vary with drop size and velocity but β varies independently of the characteristics of the eroding surface. k_0 is influenced by both the drop and the surface characteristics and k_0 decreases in value with particle fall velocity to a power less than 0.5 when particle size varies.

In this study, coal was used to examine the movement of particles of a density similar to that of aggregates. The experiments show that Eq. 3.5 applies not only to sand but also to coal but, when the particles have similar fall velocities, q_{sR} for coal particles greatly exceeded the values for sand. Eq. 3.5 is also valid for erosion of soil surfaces by rain-impacted flows where a wide range of particle size and density are present.

It follows from the combination of Eqs. 2.20 and 3.5 that the effects of rain (r), flow and soil (s) on q_{sR} can be represented by an equation of the form

$$q_{sR}[s,r] = k_s \text{ R u } f[h,r] \qquad (3.21)$$

where k_s is the susceptibility of the soil to erosion by rain-impacted flow and $f[h,r]$ is a function that accounts for the interaction between raindrop size and flow depth. Analysis of the data from the experiments presented here, together with the data from Moss and Green (1983), indicates that raindrop size has a non-significant influence on q_{sR} when medium-to-large drops travelling at or close to their terminal velocities impact flows shallower than about 4 mm. Evidently, this effect results from the water surface restricting the height to which particles are lifted in the flow when these high energy drops impact shallow flow because drop size influences q_{sR} when small drops travelling at terminal velocity, and medium-to-large drops travelling at subterminal velocity, impact flows shallower than 4 mm. On the basis of the apparent constraint placed on q_{sR} by the height of the water surface, the q_{sR}-h relationship observed for 5.1 mm drops travelling at terminal velocity,

$$q_{sR}[s,r] = 0.001553\ k_s \text{ R u h } \exp(5.7975 - 0.1881h),$$

provides a mechanism for determining the upper limit of q_{sR} for flows shallower than about 20 mm. The above equation results from

$$k_s f[h,d] = h \exp(5.7975 - 0.1881h) \qquad (3.26a)$$

and the observation that, for 0.2 mm sand used in the experiments, k_s = 644 kg.s m^{-3} (Table 3.7).

In that, at some critical depth (h_c), the $f[h,d]$ to h relationship departs from Eq. 3.26a, Eq. 3.26a is applies when $h \leq h_c$, and, for 0.2 mm sand,

$$k_s f[h,d] = h \exp(5.7975 - 0.1881h_c - b'_d(h-h_c)) \qquad (3.26b)$$

where

$$b'_d = \exp(0.77749 - 0.48251\ d) \qquad (3.27)$$

applies when $h < h_c$. Together, Eqs. 3.21 ,3.26 and 3.27 provide a mechanism for estimating the effect of drop size - flow depth interaction for rain with non-uniform drop-size distributions, and a mechanism for separating the erosivity and erodibility components in erosion by rain-impacted flow. Using this mechanism, the assumption that the time averaged effect of rainfall on sediment transport by rain-impacted flow is independent of whether rain is applied as a pulse, as often the case with field rainfall simulators, or applied as a continuous stream, as in the case of natural rainfall, was found to be valid (Section 4.2). However, the effect of pulsed rainfall on variations in the susceptibility of surfaces to erosion lies outside the scope of this study.

While factors such as flow depth (h) and velocity (u) directly affect q_{sR}, they are seldom measured. It is well known that sediment discharge is given by the product of flow discharge (q_w) and sediment concentration (c). Thus

$$q_{sR} = q_w\ c_R \qquad (4.5)$$

where c_R is the sediment concentration resulting from the raindrops impacting the flow. Since flow discharge is the product of flow depth and velocity, it follows from Eq. 3.21 that

$$c_R[s,r] = k_s \; R \; f[h,r] \; h^{-1} \qquad (4.6)$$

and

$$q_{sR} = k_s \; q_w \; R \; f[h,r] \; h^{-1} \qquad (4.7).$$

Considering q_{sR} in terms of flow discharge thus eliminates the need to consider flow velocity but the effect of flow depth remains to be accounted for. However, it is also well known that slope gradient (S) influences flow depth and velocity and, as a result, Eq. 4.7 can be rewritten as

$$q_{sR} = k_1 \; q_w \; R \; f[S] \qquad (4.8)$$

where $f[S]$ is a function that accounts for the effect of slope gradient on q_{sR} and k_1 is a coefficient influenced by variations in soil characteristics and also by variations in flow depth that are not accounted for directly by $f[S]$. Analysis of data from Meyer and Harmon (1989) shows that, for S<30%,

$$f[S] = S \qquad (4.9)$$

but other functions that include an interaction between soil and slope gradient may be more appropriate (Section 5.2).

Equation 4.8 is comparable to a widely used model that uses R^2 rather than R q_w, and K_i rather than k_1. Analysis of 18 cropland soils used in the USDA Water Erosion Prediction Project (WEPP) in terms of the two models indicates that the susceptibility of some soils to erosion by rain-impacted flow may differ substantially from the values used in WEPP (Section 4.1).

The dynamic depositional layer (DDL), the layer that results from the need to store particles on the soil surface between drop impacts when RIFT operates and entrainment by flow is absent, has a major influence on sediment discharge through the term H in Eq. 2.24. While the development and effect of the DDL is demonstrated using a numerical model of the RIFT processes in Section 2.4, the modelling concepts used in that section are impractical for field sized areas. In Chapter 5, an alternative approach is suggested. This approach considers the particle uplift and deposition events separately during an element of time, and the protective effect of the DDL to be absolute (H=1)

when the mass of the DDL in the active zone associated with any particle is greater or equal to the maximum mass RIFT can transport from that zone during that element of time. Otherwise, H is given by the ratio of the mass of the DDL in the active zone and the maximum mass RIFT can transport from that zone during that element of time. While the model produces reasonable results under a set of arbitrary conditions, the concepts need to be evaluated under more realistic circumstances.

Just as the consideration of the product of flow discharge and sediment concentration provides an alternative interrill erosion model to the one used in WEPP, so too does this product provide an alternative to the EI_{30} index in the Universal Soil Loss Equation (USLE) and the Revised Universal Soil Loss Equation (RUSLE). In this case, the product of flow discharge, or its surrogate, the excess rate of rainfall, and the rate of expenditure of rain kinetic energy may be considered as a more processed orientated index than E_{30}. Also, the use of such an index would enable factors, such as antecedent soil moisture, that have important impacts on soil losses from individual rain storms but which are currently ignored in the USLE and RUSLE, to be taken into account. While soil erodibilities would need to be re-evaluated if EI_{30} is replaced by this product, many of the algorithms and procedures for determining the effect of other factors, such as vegetative cover and changes in soil organic matter, that are currently part of the USLE and RUSLE could be retained. Under these circumstances, the model could perhaps fill the void between the USLE and WEPP models that will almost certainly exist in many parts of the world as a result of the extensive data requirements of WEPP.

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surface during a series of experiments made by the author using the apparatus shown in Fig. 3.4 under artificial rain produced by 23 g by 1.25 inch hypodermic needles in the raintower described by Walker *et al.* (1977). Each data point represents the mean of two measurements of 10 minutes duration separated by approximately 10 minutes for rain where d = 2.7 mm, Rd = 64 mm h^{-1} (nominal), and drop fall = 11.2 m.

The relationship between the term $q_{s:R}/Ru$ and flow depth (h) for the various rains in the experiments with 0.2 mm sand.

Figure 4.11. 4.26

The relationship between the observed values of $k_s f[h,r]$ and those predicted from Eqs.3. 25-3.29 for 0.2 mm sand.

LIST OF TABLES

and the closest containers. Row 4 is the row closest to the downstream end of the target.

Table 5.1. 5.8

Spatial variations in particle size distribution (expressed proportions of the total mass) and the protective effect of the DDL (H) produced by the model in Appendix III

LIST OF SYMBOLS

α_i Ratio of concentration of ith size class next to bed to mean concentration over entire depth

β A coefficient ($=b2k_0^{-1}$)

β_d A coefficient dependent on d

χ ratio of the terminal velocity of fall of a particle in water (v_p) to its average velocity of fall through a flow.

a Detachability coefficient

a_S a coefficient influenced by soil characteristics

a_D a for deposited layer

a_M a for soil matrix

a_X Either a_D or a_M

a_D* Value of aD below critical flow depth (h*)

a_1 Coefficient

b A power

b_1 Coefficient

b_2 Coefficient in $D_{pd}t'_{pd}$ to h relationship

b_s a coefficient influenced by soil characteristics

b'_d A coefficient dependent on d

c Sediment concentration

c_i Sediment concentration of ith size class

c_R Sediment concentration produced by RIFT

C_e Fraction of soil unprotected from drop impact

d Drop size (mm)

d_i Sediment deposition rate for ith size class

D_{pd} Mass of p sized particles lifted by a d sized drop

D_{pdz} Mass of p sized particles lifted to height z by a d sized drop

DDL Dynamic depositional layer resulting from the need for storage

of material on the surface between drop impacts

e_i Rainfall detachment rate for ith size class

$f[h,d]$ Function accounting for the interaction between drop size and flow depth (0-1)

$f[S]$ Function accounting for the effect of slope gradient in a interrill erosion model

f_d Frequency of impacts of drop size d within x_{pz}-0.5x_{cz} upstream of a boundary

f_p $D_{pd.M}$-to-$D_{pd.D}$ ratio

f_s Factor accounting for departures from $f[S]=S$

F Distance of drop fall

F_d Spatially averaged impact frequency of d sized drops

h Flow depth (mm)

H Coefficient of protection of matrix soil by deposit

h_{0d} Flow depth for no RIFT projected for drop size d by linear q_{sR} to h relationship

h_c The critical flow depth where the depth of flow does not constrain the height to which particles are lifted

I Number of sediment size classes

K Coefficient in relationship between T and R

k_0 Constant in linear $D_{pd}t'_{pd}$ to h relationship

k_{0d} Constant influenced by d

k_p Coefficient varying with particle size

k_s Coefficient dependent on soil properties

k_1 Coefficient depending on k_s and a number of other factors

k'_{pd} Constant dependent on particle and drop size

k'_d Factor accounting for drop size contributions to k'_{pd}

k'_p Factor accounting for particle size contributions to k'_{pd}

L Loss of sand relative to loss at position 17

L_m Mean of L

L_x Length of a plane surface

M_{Dz} Mass of particles lifted to height z

N Number of drop sizes

n Variable in summation equation

p A power in e_i to R relationship

p particle size (mm)

p_m Drop momentum

P_{max} Peak pressure exerted at the point on the bed below the centre of a drop impact

P_{pD} Proportion of p sized particles in DDL

P_{pM} Proportion of p sized particles in soil matrix

Q Rain induced discharge per unit area

q_s Rate sediment transported by flow across a unit width of a boundary

q_{so} Mass of material transported across the downstream boundary of an element

q_{soD} Mass of material from DDL transported across the downstream boundary of an element

q_{soM} Mass of material soil matrix transported across the downstream boundary of an element

q_{sR} q_s for sediment transported by RIFT

q_w Rate at which water is discharged across a unit width of a boundary

R Rainfall intensity

R_d Intensity of rain of drop size d

r_i Sediment entrainment rate for ith size class

S Slope gradient

T Rate at which sediment is transported across a boundary

T_A Rate at which splash transports sediment across a boundary

T_B Rate at which bed load is transported across a boundary

T_F Rate at which rain-impacted flow transports sediment across a boundary

T_S Rate at which suspended load is transported across a boundary

T_R Rate at which RIFT transports sediment across a boundary

t_p Time p sized particle remains suspended following disturbance by a drop impact

t_{pz} Time p sized particle remains suspended after being lifted to a height z

t'_{pd} Average time p sized particle remains suspended following disturbance by a drop impact

u Flow velocity

v_i Mean fall velocity in water for particles in size class i

v_p Fall velocity in water for particles of size p

v_d Fall velocity in air for drops of size d

w A power in the relationship between T and R

W_f Width of flow

X Parameter related to stress applied by impacting drop

X_c Critical value of X below which particles are not lifted into the flow by a drop impact

x Distance

x_{cz} Diameter of particle cloud at height z

x_p Distance travelled by a particle of size p

x_{pz} Distance travelled by a particle of size p lifted to height z

x'_{pd} Average distance travelled by a particle of size p disturbed by the impact of a d sized drop

z Height from bed

LIST OF EQUATIONS

$$T_F = T_S + T_B + T_R \tag{1.1}$$

$$T = T_F + T_A = T_S + T_B + T_R + T_A \tag{2.1}$$

$$T = K\ R^w \tag{2.2}$$

$$q_s = q_w c \tag{2.3}$$

$$\frac{\partial}{\partial x}(q_w c_i) + \frac{\partial}{\partial t}(h c_i) = e_i - d_i + r_i \tag{2.4}$$

$$e_i = a\ C_e \frac{R^p}{I} \tag{2.5}$$

$$d_i = v_i c_i \tag{2.6}$$

$$c_i = \frac{a\ C_e R^p}{I(Q + v_i)} \tag{2.7}$$

$$a = H\ a_D + (1-H)\ a_M \tag{2.8}$$

$$a_X = a_{X*}\ (h_* h^{-1})^b \tag{2.9}$$

$$c_i = \frac{a_{D*}\ h_*^b\ R^p}{h^b \sum_{i=1}^{I} \alpha_i v_i} \tag{2.10}$$

$$t_p = h\ v_p^{-1} \tag{2.11}$$

$$x_p = t_p\ u \tag{2.12}$$

$$t_p \approx t_{pz} = z\ v_p^{-1} \tag{2.13}$$

$$M_{Dz}\ [p,d] = f_d\ D_{pdz} \tag{2.14}$$

$$M_{Dz}\ [p,d] = F_d\ x_{pz}\ D_{pdz}\ W_f \tag{2.15}$$

$$M_{Dz}\ [p,d] = \frac{6\ R_d\ x_{pz}\ D_{pdz}\ W_f}{\pi\ d^3} \tag{2.16}$$

$$M_{Dz}\ [p,d] = \frac{6\ R_d\ t_{pz}\ u\ D_{pdz}\ W_f}{\pi\ d^3} \tag{2.17}$$

$$M_D\ [p,d] = \frac{6\ R_d\ t'_{pd}\ u\ D_{pd}\ W_f}{\pi\ d^3} \tag{2.18}$$

$$t'_{pd} = \frac{\int_0^{zh} (t_{pz}\ D_{pdz})\ dz}{D_{pd}} \tag{2.19}$$

$$q_{sR}\ [p,d] = \frac{6\ R_d\ t'_{pd}\ u\ D_{pd}}{\pi\ d^3} \tag{2.20}$$

$$q_w = hu \tag{2.21}$$

$$c_R\ [p,d] = \frac{q_{sR}\ [p,d]}{q_w} = \frac{6\ R_d\ t'_{pd}\ D_{pd}}{\pi\ d^3 h} \tag{2.22}$$

$$c_i\ (d) = \frac{6\ R_d\ t'_{pd}\ D_{pd}}{\pi\ d^3\ h} \tag{2.23}$$

$$D_{pd} = H.D_{pd.D} + (1-H)D_{pd.M} \tag{2.24}$$

$$q_{sR}\ (p,d) = F_d\ x'_{pd}\ D_{pd} \tag{2.25}$$

$$D_{pd}t'_{pd} = \frac{q_{sR}[p,d]\ \pi\ d^3}{6\ R_d\ u} \tag{3.1a}$$

$$D_{pd}t'_{pd} = \frac{c_R[p,d]\ \pi\ d^3\ h}{6\ R_d} \tag{3.1b}$$

$$D_{pd}t'_{pd} = a_1\ d^{3.48} \tag{3.2}$$

$$D_{pd}t'_{pd} = 0.372\ h^{b_1} \tag{3.3}$$

$$b_1 = 9.59\ \log_{10}(d) - 10.29 \tag{3.4}$$

$$D_{pd}t'_{pd} = k_0(1 - \beta h) \tag{3.5}$$

$$D_{pd}t'_{pd} = k_0 - b_2 h \tag{3.6}$$

$$\beta = \frac{b_2}{k_0} \tag{3.7}$$

$$\beta = 0.175 - 0.0260\ d \tag{3.8}$$

$$\log k_0 = -0.507 - 0.616 \log p + 2.24 \log d \tag{3.9}$$

$$\log k_0 = -1.301 - 0.465 \log v_p + 2.24 \log d \tag{3.10}$$

$$k_0 = 2.67 v_d - 8.07 \tag{3.11}$$

$$\beta = 0.197 - 0.0118 v_d \tag{3.12}$$

$$\beta = 0.0852 - 0.00442 v_d \tag{3.13}$$

$$\log k_0 = 7.543 - 0.4652 \log v_p + 0.7025 \log p_m \tag{3.14}$$

$$\beta = 0.1104 - 0.001166\ p_m \tag{3.15}$$

$$x'_p = \frac{\sum_{n=1}^{15} (L_n . x_{pn})}{L_m} \tag{3.16}$$

$$D_{pd}t'_{pd} = k'_{pd}\ (1 - \beta_d h) \tag{3.17}$$

$$D_{pd}t'_{pd} = k'_p k'_d\ (1 - \beta_d h) \tag{3.18}$$

$$q_{sR}\ [p,d] = \frac{6\ R_d\ u\ k'_p k'_d [1 - \beta_d h]}{\pi\ d^3} \tag{3.19}$$

$$c_R\ [p,d] = \frac{6\ R_d\ \ k'_p k'_d [h^{-1} - \beta_d]}{\pi\ d^3} \tag{3.20}$$

$$q_{sR}[p,d] = k_s\ R\ u\ f[h,d] \tag{3.21}$$

$$f[h,d] \approx 1-\beta_d[h-h_x] \qquad , h_x<h<3d \tag{3.22}$$

$$\beta_d = 0.3119 - 0.0507d \qquad , r^2=0.986 \tag{3.23}$$

$$k_s\ f[h,d] = \frac{q_{sR}[p,d]}{R\ u} \tag{3.24}$$

$$h_c = 1.74768 + 2.88237\ \log(d) \tag{3.25}$$

$$k_s f[h,d] = h\ \exp[5.7975 - 0.1881h] \qquad , h\leq h_c \tag{3.26a}$$

$$k_s f[h,d] = h\ \exp[5.7975 - 0.1881h_c - b'_d(h - h_c)] \qquad , h>h_c \tag{3.26b}$$

$$b'_d = \exp[0.76649 - 0.48251\ d] \tag{3.27}$$

$$f[h,r] = \frac{\sum_{n=1}^{N} (f[h,d]R_d)_n}{\sum_{n=1}^{N} (R_d)_n} \tag{3.29}$$

$$k_s = \frac{q_{sR}[s,r]}{R\ u\ f[h,r]} \tag{3.30}$$

$$D_i = K_i\ R^2\ S_f \tag{4.1}$$

$$S_f = 1.05 - 0.85\ \exp\{-4\ \sin\ (\phi)\} \tag{4.2}$$

$$D_i = q_{sR}\ L_x^{-1} \tag{4.3}$$

$$K_i = \frac{q_{sR}}{L_x\ R^2[1 - 0.85\ \exp\{-4\ \sin\ (\phi)\}]} \tag{4.4}$$

$$q_{sR} = q_w \; c_R \tag{4.5}$$

$$c_R[s,r] = k_s \; R \; f[h,r] \; h^{-1} \tag{4.6}$$

$$q_{sR} = k_s \; q_w \; R \; f[h,r] \; h^{-1} \tag{4.7}$$

$$q_{sR} = k_1 \; q_w \; R \; f[S] \tag{4.8}$$

$$f[S] = S \tag{4.9}$$

$$q_{sR} = k_1 \; q_w \; R \; S \tag{4.10}$$

$$k_1 = \frac{q_{sR}}{R q_w S} \tag{4.11}$$

$$q_{sR} = k_1 \; R c_w S \; f_s \tag{4.12}$$

$$f_s = \frac{q_{sR}}{k_1 R S c_w} \tag{4.13}$$

$$f_s = 0.259 + 0.000049 \; (1.197^{[90-S]}) \qquad \text{for } S>38\% \tag{4.14}$$

$$f_s = \frac{c_R}{k_1 R S} \tag{4.15}$$

$$S_L = 277.18 - 149.16 D_{wsa} + 19.41 D_{wsa}^2 \qquad , r^2=0.957 \tag{4.16}$$

$$Q = 9.02 + 0.918 D_{wsa} - 1.2385 D_{wsa}^2 \qquad , r^2=0.956 \tag{4.17}$$

$$\frac{q_{sR}}{Ru} = k_s f[s,r] \tag{4.18}$$

$$q_{so}[p] = q_{soD}[p] + q_{soM}[p] \tag{5.1}$$

$$q_{soD}[p] = H \; P_{pD} \; k_p \; R \; u \; f[h,r] \; W_f \tag{5.2}$$

$$q_{soM}[p] = f_p \; (1-H) \; P_{pM} \; k_p \; R \; u \; f[h,r] \; W_f \tag{5.3}$$

$$f[S] = a_S + b_S S \tag{5.4}$$

$$f[S] = \exp\ (a_S + b_S S) \tag{5.5}$$