Chapter 1

WHAT IS THE VAN HIELE THEORY?

He (Thomas Hobbes) was 40 years old before he looked on Geometry; which happened accidentally. Being in a Gentleman’s Library, Euclid’s Elements lay open, and 'twas the 47 El. libri I. He read the Proposition. By G—, sayd he this is impossible! So he reads the Demonstration of it, which referred him back to another, which he also read. Et sic deinceps [and so on] that at last he was demonstratively convinced of that trueth. This made him in love with Geometry.


Introduction

How to develop one’s thinking, how to extend one’s capacity to reason are topics which have occupied the minds of many philosophers, psychologists and educators. Over the past two-hundred years in Western civilisation, two contrasting theories of education have been the focus of debate. Dominant in education until well into this century has been the established traditional model based largely on the philosophies of the early Greek educationists. In this model, the authority of teachers is stressed and their role is seen as instilling in pupils a required body of set subject matter. Emphasis is on preparation for the future rather than an enrichment of the present. Directly challenging this perspective is the model of the progressivists in which the child’s interests and needs are regarded as the main factor in deciding what should be taught. Practical knowledge is given a place in the curriculum, and activity methods and learning by discovery replace formal instruction as the dominant educative process.

Clearly, there are pragmatic issues to be resolved between the extremes of these points of view. What appears to be necessary is an appropriate balance which draws on the strengths of both models. Those who would be considered to support such a balance include two Dutch mathematics educators, the husband and wife team of Pierre van Hiele and the late Dina van Hiele-Geldof. Like the traditionalists, the van Hieles have defined the nature of the subject matter. However, the subject matter is defined in line with how students’ understandings develop. Like the progressivists, the van Hieles focused their work on the notion that rather than being the authoritative conveyer of subject matter, the role of the instructor should be to help students gain and progress in their understanding.
It is the research and life work of the van Hiel's referred to as the van Hiele theory, which contains the psychological and mathematical ideas which form the basis of this research. This chapter provides a framework for the thesis by detailing components of the van Hiele theory. It is divided into three sections. The first section, referred to as the theoretical perspective, details how the van Hieles developed their theory and provides an overview of its key elements. The section headed, levels and phases, describes the two main operational aspects of the theory. Finally, the conclusion draws together the major ideas identified.

**Theoretical Perspective**

This section is developed in three parts, namely, the early views of the van Hieles, the influence of Piaget, and the significance of the notions of structure and insight to the theory.

**Early views**

The foundational ideas of the van Hiele theory of teaching and learning geometry came from two aspects, namely, the van Hieles' personal experiences as teachers of geometry, and their doctoral research under the guidance of H. Freudenthal and M. J. Langeveld. As they became more experienced as teachers, they had grown to understand that the learning of facts could not be the purpose of teaching mathematics, and, more importantly, that the development of insight ought to be the purpose. However, this view was in conflict with schooling at the time, where it was considered best that the teacher taught facts and methods, even if the students did not understand them. Pierre van Hiele expressed this:

> In the old view, every topic should be built from truths that are sure. In mathematics, such truths are called axioms ... and they are strengthened by definitions. With the principles and the definitions the whole pattern of the topic is given. This is an authoritative construction: the pupil has nothing to say about it.  

(1986, p.233)

Pierre van Hiele explained that his initial goal was to contribute to the improvement of teaching. The contrast between traditional teaching and his proposed method of learning is explained in *The Child's Thought and Geometry*:

> Finally, the student has learned to apply a system of relations that has been offered to him ready-made, he has learned to apply it in certain situations specifically designed for it. But he has not learned how to construct such a system himself in a domain which is still unstructured. If, on the other hand, we were to succeed in ensuring as a result of our teaching that the students are capable of constructing for themselves a deductive relational system in a new domain, we would have produced the optimal mathematical training.  

(1959, in Fuys, Geddes & Tischler 1984, p.245)
As a consequence, their research career began with these notions as their philosophical background and, in a practical way, through their enrolment in companion PhDs at the University of Utrecht. Pierre’s research study was concerned with the theoretical framework, and the significance of insight in leading to the development of understanding. Dina’s work focused on the didactics necessary in leading students from one level to the next. Shortly after completing their joint theses, Dina van HiLE-Geldof died, and Pierre van Hiele was left to continue their research on his own.

Being a teacher of mathematics, van Hiele drew mainly upon geometrical examples to illustrate his theory of mathematics education. He saw geometry as “a perfect conceptual system, where things rigorously followed from each other” (Freudenthal 1973, p.401), and used it to demonstrate the pedagogical application of the theory. More recently, van Hiele (1986) expanded his ideas slightly, interpreting the theory in relation to other fields of learning, such as algebra, fractions, and the natural sciences. However, he still maintained a strong geometrical focus. The combination of views and research of both Pierre van Hiele and Dina van Hiele-Geldof is now referred to collectively as the van Hiele Theory, and provides a framework to guide research, and from which instruction can be planned and evaluated. However, the theoretical ideas behind the theory did not emerge spontaneously, but evolved from the writings, and were in the tradition, of Piaget.

The influence of Piaget

Piaget’s writings have had a wide-ranging influence on much educational research. Concerning the teaching of geometry, his thoughts are explained in his paper entitled Les structures mathématiques et les structures opératoires de l’intelligence (1955, in Wirszup 1976, p.76):

Traditional geometry instruction begins too late and then takes up the concept of measurement right away, thus omitting the qualitative phase of transforming spatial operations into logical ones. This is to say that instruction is realized in a sequence corresponding to the historical development of geometry ... . But the development of geometric operations in children actually proceeds in the opposite direction -- from the qualitative to the quantitative.

The crucial discoveries made by Piaget in the teaching of geometry inspired, among others, the thinking of van Hiele, and the research conducted at the USSR Academy of Pedagogical Sciences (Wirszup 1976). In the latter case, Soviet educators of the early 1960s, experiencing acute dissatisfaction with students’ poor results in geometry, looked for reasons. Research led to the development of an innovative new Soviet geometry curriculum resulting from the influence of Piaget and from the psychological and mathematical ideas of the van Hieles
(Pyshkalo 1968, in Wirszup 1976, p.91). They were able to weave an intricate web bonding innovative geometric pedagogy, the fundamental principles discovered by Piaget, and the van Hiele theory.

In his search for clarification of insight, van Hiele (1986, p.5) acknowledged that “an important part of the roots of my work can be found in the theories of Piaget”. For example, Piaget’s introduction of the notion of levels was the catalyst for van Hiele, leading to the setting up of a theory of levels of learning:

It was significant that Piaget first introduced levels: the experimental person at the lower level not understanding the leader at the higher level ... . The learning material supplied by Piaget consisted of whole structures, whereas the other psychologists always occupied themselves with elements.  

(p.6)

While van Hiele believed that “Piaget’s thesis that insight very often arises from an action is true” (p.108), he did not concur with Piaget on all points. There were many important differences between their thinking, and this resulted in the development of ideas along a different line to that of Piaget. For example, van Hiele (pp.5-6) felt that Piaget’s two levels were insufficient in geometry. He noted also that Piaget did not recognise the need to stimulate children to move from one level to the next and, consequently, did not consider how this might be achieved. Piaget appears to have disagreed with the suggestion that children did not always understand his questions because the language he used was not that of the children. Van Hiele suggested that this lack of recognition of the significance of the learning process led Piaget to overlook the very important role language plays in the movement between levels. Failing to acknowledge the significance of the learning process led also to Piaget not seeing that structures of a higher level resulted from the making explicit, and the studying of, the rules governing the lower structure. In Piaget’s theory, children are born with the higher structure and have only to become aware of it. In van Hiele’s theory, the rules of the lower level become the new structure of the higher level. It is this notion of structure that is a central aspect of the van Hiele theory.

Hence, it can be seen that the van Hiele theory developed as a result of the challenge offered by Piaget’s writings. Significantly, it was Piaget’s notion of levels of thinking that stimulated the direction of van Hiele’s thinking. However, van Hiele’s opposition to many of Piaget’s other ideas led to his identifying many of the features of the van Hiele theory, for example, the role of language, the movement between levels, and the significance of structure.
Structure and insight

Many of van Hiele’s (1986) ideas of structure are borrowed from Gestalt theory. He stressed that if one is to achieve understanding, the learning of structures is a superior goal to the learning of facts. “In a structure, facts have sense; if a part of a structure is forgotten, the remaining part facilitates recall of the lost one” (p.viii). Piaget and van Hiele both found that a structure has the characteristic of having a totality, that it can be experienced as a whole, and that it should be seen as being more than the sum of its elements.

Structures, i.e., given things obeying certain laws, are classified by van Hiele as being either strong or feeble (pp.19-20). Strong structures have a rigidity which is controlled by the rule of the structure. Such structures can be extended in only one way, and so can be continued with great certainty, without making mistakes. If it is a strong structure it will be possible, usually, to superimpose a mathematical pattern onto it. By contrast, if the rule is not given, or if it is difficult to distinguish the rule, a structure is feeble. For example, it is not easy to determine the pattern made by fallen leaves, nor to identify the structure of the pictures of a painter, especially if one only has access to the painter’s early works. Language also plays a significant role in the development of the structure within a level. For example, at the first level, the visual perception of structures can be improved by the use of words which support accurate thinking about structures. “Gradually, the language develops to form the background of a new structure, the structure of the second level, the descriptive structure” (pp.83-84).

Van Hiele (1986, p.28) identified four important properties or features from Gestalt psychologists which he believed govern structure. These are:

1. It is possible to extend a structure. Whoever knows a part of the structure also knows the extension of it. The extension of a structure is subjected to the same rules as the given part of it.

2. A structure may be seen as a part of a finer structure. The original structure is not affected by this. The rules of the game are not changed, they are only enlarged. In this way it is possible to have more details take part in the building up of the structure.

3. A structure may be seen as a part of a more-inclusive structure. This more-inclusive structure also has more rules. Some of them define the original structure.

4. A given structure may be isomorphic with another structure. In this case the two structures are defined by rules that correspond with each other. So if you have studied the given structure, you also know how the other structure is built up.
He illustrated these four properties by applying them to the concept ‘human skeleton’. Having become acquainted with the human skeleton, the structure of the concept can be extended by our becoming aware that we have such a skeleton. Second, the structure is made finer by giving it more detail, such as naming the parts (bones). This involves the construction of a language. The third property is illustrated when the human skeleton is compared to the skeletons of other animals. Isomorphism can be seen directly in this comparison, or globally if the support structure of the skeleton is compared to the support structure of a building, i.e., the supportive system of the bones is compared to the supportive system of the building, the pillars, beams and other elements. Isomorphism is also demonstrated mathematically if one compares the sets of rules defining the two operations, addition and multiplication. The structure of the rules, the existence of an identity, the commutative and the associative properties, are the same.

Frequently, the first and fourth properties can be seen directly, whereas, it is usual for the second and third properties to require study. If education is pupil-centred, i.e., it is directed at the development of insight, pupils should be stimulated to discover the second and third properties of the structure they are studying, i.e., the details of the finer structure and the extension of rules and comparisons in the more inclusive structure.

Once one has perceived how structures work, an understanding of the nature of structures develops, and of the innate truth of the statement “Structure is what structure does” (p.5) emerges. This is essential if insight is to be achieved. Van Hiele (p.76) argued that insight develops from discursive thinking based on perception of a strong structure. If the arrangement of a perceived structure is able to be explained, if someone else can learn to see the structure, then the structure is a strong structure and the perception is based on insight. Van Hiele (pp.4-5) explained that he became convinced that the learning of facts could not be the purpose of teaching mathematics, instead he believed that insight might be gained as the result of perception of a structure.

The development of insight ought to be the purpose of teaching. Insight is present when persons act on account of a mental structure and in accordance with the structure they perceive (p.24). “Thinking is preceded by a viewing of the situation, a phase in which one ‘undergoes’ the situation. In this phase that precedes thinking, insight is already possible” (p.116). “Insight exists when a person acts in a new situation adequately and with intention” (p.24). Van Hiele summarised this meaning of insight under three points:
1. Insight can be observed when there has been an adequate action in a new situation.

2. Insight can be ascertained when there has been action on the strength of an established structure from which the answers to new questions can be read.

3. The best examples of insight happen unexpectedly; they are not brought about by planning.

(p.154)

Insight was seen by both Piaget and van Hiele as developing from the perception of a structure, and that it was important to perceive the structure as a whole. However, van Hiele took the significance of structure further, theorising that insight results from the perception of structure, leading to the achievement of understanding. In addition, he argued that students need a catalyst if they are to progress from one level to the next.

Levels and Phases

Three main aspects of the van Hiele theory are discussed in this section, namely: (i) the sequence of levels of insight are defined; (ii) the properties of the levels are discussed; and (iii) the phases of learning, which enable students to progress through the levels, are detailed.

The levels

The initial conception offered by the van Hieles was of five hierarchical levels. The levels are of cognitive development since “the levels are situated not in the subject matter but in the thinking of man” (1986, p.41). Originally the van Hieles numbered their five levels as basic and 1 to 4. However, in his later writings, Structure and Insight: A Theory of Mathematics Education (1986), Pierre van Hiele adopted a 1 to 5 scale, as do the majority of researchers in recent studies. The levels are described by the van Hieles in several papers in both general and behavioural terms. They are also described by the van Hieles’ mentor, Freudenthal (1973), and by several researchers, such as Wirszup (1976), Hoffer (1981), Usiskin (1982), Fuys, Geddes and Tischler (1985), and Shaughnessy and Burger (1985). A summary of these descriptions, using the notation 1 to 5, is given below.

Level 1

At this level, perception is visual only. A figure is seen as a total entity and of a specific shape. The square may be visualised as having four equal sides meeting at right angles, however, there is no overt appreciation of these properties. The figure can be reproduced in different orientations provided care has been taken by a teacher to present the figure in
varying positions. A square is seen as completely different to a rectangle or a rhombus, or to any other quadrilateral. Figures can be identified by name. Shape is all important.

Level 2
When Level 2 has been reached, a figure presents as a totality of its properties. It is the properties of the figure that determine its external form. A student can describe the drawing of the figure according to its properties, and can identify a poorly drawn figure. Properties, however, remain distinct. Relations between properties are not perceived. Definitions may be recited, but it is done without understanding. A figure can be analysed through measurement. A rectangle and a parallelogram may both be identified as having equal and parallel opposite sides, however, there is no understanding that a rectangle belongs to the class of parallelograms.

Level 3
The third level is attained when the student can perceive the significance of the properties established in the previous level, order them logically, recognise relationships between them, and operate with these relationships both within figures and between related figures. Class inclusion is perceived, the square is seen to be a rectangle. Simple deduction results as properties evolve one from another following experimentation and exploration. For example, the equality of angles and linear segments can be deduced from congruent triangles. Also, the investigation of the equality (using algebraic values) of interior alternate angles formed by two parallel lines can lead the student to perceive that the angle sum of a triangle is 180°. There is comprehension of the essence of geometry. A quadrilateral tree can be developed. Attempts at definitions frequently contain too many conditions. The role of the definition and the possibility of being able to construct formal proofs are still beyond the comprehension of the students.

Level 4
When students are thinking at Level 4 they can manipulate the interrelationships developed at Level 3 with logic. They understand the significance of ‘necessary and sufficient’ conditions and can thus construct definitions. The relationships between groups of theorems, and between theorems and their converses can be seen. The essence of axioms is understood. Proof of a construction can be initiated, alternative yet sufficient definitions can be constructed, and the need to justify relationships can be seen, allowing attention to be directed towards the symmetry of those relationships. Relationships become the object of thought.
Level 5

At Level 5, the essence of geometry is perceived, as is the necessity for rigor. Abstraction becomes paramount as the concrete nature of objects, relations and interpretations recedes. Thus symbols cease to hold their original significance. Students have reached the scientific insight and can investigate non-Euclidean geometry, such as projective geometry. They can seek out missing axioms, study the foundations of a theory, establish foundations of a new theory and build on these foundations with a deductive system.

Overall, the five levels represent a hierarchy of development that has a natural logic and a simple character that has an appeal to many researchers and teachers (Usiskin 1982, pp.6-7). Also, because the van Hieles developed their theory while working with secondary school students, they wrote much less about Levels 1 and 5 than they wrote about the middle three levels. Van Hiele (1986, p.41) acknowledged that initially, they did not perceive the importance of Level 1. This might have been because geometry was not taught in the primary schools in the Netherlands. The oversight led to their labelling as the first level of thinking, that which is concerned with the identification of properties. Additionally, the van Hieles recognised thinking at Level 5 as being generally for those who have advanced well beyond secondary schooling. Van Hiele (p.47) reflected on the first article, written jointly with his wife in 1955, “You see that we did not try to describe levels higher than the fourth. Those levels are much more difficult to discern than Levels 2, 3, and 4. Moreover, it has turned out that such levels are easily too highly valued. And what are we going to do with such levels? In school we have to deal with Levels 2, 3, and 4.”

Since his initial description of five levels in 1955, van Hiele acknowledged, in Structure and Insight (1986), that his views have changed a little. While the change in his views appears to have developed over a period of some years, his intentions are not always clear in his writings. Usiskin (1982, p.14) noted that during his trip through the United States in 1980, “van Hiele disavowed belief in a fifth level, thus changing a view he had expressed some years before”. At times in Structure and Insight, van Hiele refers to there being more than two or three levels, sometimes five or more. On other occasions he refers to an alternative set of three levels of thinking (1986, pp.53 and 84-86). The existence of this latter model of only three levels was confirmed in personal communications in 1994 with Dr van Hiele at The Hague and again at the University of New England, Armidale. In these discussions he indicated that the first or visual level corresponded approximately with Level 1, the middle or descriptive level with Levels 2 and 3, and the theoretical level with Levels 4 and higher. Pegg and Davey’s (in press) description of the levels of the alternative model is based on van Hiele’s descriptors (1986, pp.53-57):
Visual level: decisions are guided by a visual network.
Descriptive level: the elements and relations are described.
Theoretical level: deductive coherence is prominent, geometry generated according to Euclid is considered.

This interpretation is supported by van Hiele’s explanation of how language relates to the three levels. He remarked that the language of the first level makes it possible to speak of visual observations, that logical or other relations are part of the language of the second level, and that the language of the third level has a much more abstract character.

However, not all writers see the break-down into three levels as having the same composition as in the above model. For example, in the three-level model reported by Fuys, Geddes, and Tischler (1988, in Clements and Battista 1992, p.431), the new descriptive level corresponds solely to the original Level 2, and the new theoretical level corresponds with the original Levels 3, 4 and 5. In favour of this interpretation one can argue that all stages of proof, including the very early use of congruency in proving equality of corresponding sides and angles (Level 3), are grouped within the one level. This leaves the simple recognition and description of properties to stand alone as the second level.

If the composition of the original levels is examined with regard to the understanding of geometry, the interpretation by the Pegg and Davey model is, perhaps, the more logical. Intrinsic to the new descriptive level, as defined by Pegg and Davey, is the total development concerned with recognition of shapes as the bearers of properties together with the relations connecting the properties. The overall understanding of the properties then becomes extrinsic to the abstract thinking of the theoretical level in which the student develops an ability to construct proof, leading to an understanding of the nature of proof. Formal analysis is intrinsic to the new theoretical level.

The Pegg and Davey interpretation is supported by researchers who have demonstrated the jump in the learning curve between learning to relate the properties of shapes and the actual constructions of proof of those relations. For example, the above interpretation corresponds with the structure of Biggs and Collis’ (1991) SOLO Taxonomy (Structure of the Observed Learning Outcome) in which students’ responses are evaluated according to the demonstrated quality of learning. In the SOLO evaluation, responses showing appreciation of properties are categorised as belonging to the concrete-symbolic mode. The next higher mode, the formal mode, includes responses demonstrating logical thinking and proof construction. It is significant that both the levels of the van Hiele theory and the modes of the
SOLO Taxonomy demonstrate some similarity in their hierarchical structure (Pegg & Davey, in press).

Although there is continuing debate concerning the number and the composition of the levels, there is consensus of opinion that the levels exist, that they are hierarchical, and that they measure cognitive development. Additionally, the definition of the nature of the subject matter in line with the development of students’ understanding is not contested. While most writers are using the five-level model, and the 1 to 5 nomenclature, support for the three-level model is growing, and is acknowledged by van Hiele himself.

**Properties of the levels**

Pierre van Hiele (1959, in Fuys et al 1984, p.246) in further clarifying the sequence of levels of thought, identified the following four main properties:

a. At each level there appears in an extrinsic way that which was intrinsic at the preceding level.

b. Each level has its own linguistic symbols and its own system of relations connecting these signs. A relation which is “correct” at one level can reveal itself to be incorrect at another. Think, for example, of the relation between a rectangle and a square.

c. Two people who reason at different levels cannot understand each other. This is what often happens between teacher and student. Neither of them can manage to follow the thought process of the other and their dialogue can only proceed if the teacher tries to form an idea of the student’s thinking and to conform to it.

d. The maturation which leads to a higher level happens in a special way. Several stages can be revealed in it (this maturation must be considered above all as a process of apprenticeship and not as ripening of a biological sort). It is thus possible and desirable that the teacher aids and accelerates it. The aim of the art of teaching is precisely to face the question of knowing how these phases are passed through, and how help can effectively be given to the student.

Thus, students cannot function adequately at one level without having mastered the previous levels. They can “be pushed to a higher level too early and aided by means of algorithms to simulate this higher level” (Freudenthal 1973, p.416). If the language of instruction is on a level higher than that of the student, there is a communication problem. In addition to the four main properties listed above, there are several features associated with the levels. These are detailed in the next chapter, together with a discussion of the related research. Included in the features are: a) how students’ perceptions change at different levels; b) how students are on different levels for different concepts; c) how student cognitive growth takes time and
involves a restructuring in thought processes as one moves from level to level; and, d) how the implications of level reduction and of rote learning are associated with the van Hiele levels.

The phases
While the van Hieles believed that cognitive development in geometry can be promoted through instruction, to move a student from one level to the next requires that the teacher assist the student directly in the processes of exploration and reflection. These processes take time, and the teacher must not only be able to recognize that a student is progressing through the levels, but also be prepared to allow for the time needed by the student to grow to the next level.

Dina van Hiele-Geldof (1958, in Fuys et al 1984, p.217) stressed the importance of the teacher placing the emphasis on how the learning takes place, i.e., on why, rather than on what is done. To help achieve this, the van Hieles defined five phases in teaching procedures which take students through to a necessary thinking crisis, and hence lead them from one level to the next. The phases are approximately sequential (Usiskin 1982, p.6) but not discrete. The phases, according to Dina van Hiele (1958, in Fuys et al 1984, pp.217-223) are:

Information or Inquiry: This phase allows the pupil to discover the intrinsic ordering in the material that is presented to him. For example, the knowledge of shapes is developed through manipulation of material objects. This brings the pupils to purposeful action and perception.

Directed orientation: The pupil is brought into a new phase of learning through conversations. Manipulation is prominent. He now looks at figures being folded, for example, but in the process of doing so, he purposefully looks at what equalities of parts of the figure are being revealed. The pupil thus demonstrates a disposition towards exploration in being willing to carry out the assigned operations.

Explicitation: The results of the manipulations of material objects are now expressed in words. Equalities that have been observed are enumerated. Subjective experiences are exchanged, thus becoming objectified. In this way the figures acquire geometric properties, the theorems are expressed. The role of the teacher here consists of introducing the necessary technical terms.

Free orientation: By comparing symbols with one another, by searching for similarities and differences, the pupils orient themselves in the domain of symbols. For example, by working with figures, the pupils finally will recognize the figures by some of their properties. Others will begin perceiving a square as a rhombus because the square possesses all the properties of the rhombus. During free orientation the teacher appeals to the inventive ability of his pupils.
Integration: Free orientation finally leads to being oriented in the domain of the symbols. The symbols then possess field characteristics. The operation “to fit” is reversible and associative. The manipulations are understood, there is insight into the operation. When integration has taken place, the pupil is able to operate with the figures as a totality of properties.

Only at the end of the fifth phase, van Hiele noted (1986, p.177), does one see “ordinary learning” when memorising and the setting of rules occur. Van Hiele summarised:

At the close of this fifth phase a new level of thought is attained. The student has at his disposal a system of relations which are related to the whole of the domain explored. This new domain of thought, which has acquired its own intuition, is substituted for the previous domain of thought which had a completely different intuition.

(1959, p.247)

It might be wondered how one can know that a student is progressing through levels of thinking, and that a student has moved to a higher level of thinking. Van Hiele explained (1955, p.289, in van Hiele 1986, p.39):

You can say somebody has attained a higher level of thinking when a new order of thinking enables him, with regard to certain operations, to apply these new operations on new objects. The attainment of the new level cannot be effected by teaching, but still by a suitable choice of exercises the teacher can create a situation for the pupil favourable to the attainment of the higher level of thinking.

The van Hieles noticed that the movement to a new level of thought results from a maturation process associated with progress through the five phases. In their description of progression through the levels, they (van Hiele & van Hiele-Geldof 1958, pp.75-76, inWirszup 1976, p.79), observed a discontinuity in the learning process:

The discontinuities are ... jumps in the learning curve, [and] these jumps reveal the presence of levels. The learning process has stopped; later on it will start itself once again. In the meantime, the pupil seems to have “matured.” The teacher does not succeed in further explanation of the subject. He and ... the other students who have reached the new level seem to speak a language which cannot be understood by the pupils who have not yet reached the new level. They might accept the explanation of the teacher, but the subject taught will not sink into their minds. The pupil himself feels helpless; perhaps he can imitate certain actions, but he has no view or his own activity until he has reached the new level. At that time the learning process will take on a more continuous character. Routines will be formed and an algorithmic skill will be acquired as the prerequisites to a new jump which may lead to a still higher level.
Thus, the identification of how students move from one level to the next supports the notion of discreteness of the levels and the cognitive nature of the levels. Also, it has helped clarify some of the properties of the levels, for example, the importance of language and the intrinsic/extrinsic feature. In defining the five teaching phases necessary to lead students from one level to the next, the van Hieles have defined the role of the teacher, and, also, identified the utmost importance of this role. In a guided learning process, the task of the teacher is not to impart knowledge, but rather to promote thought and discussion, and, hence, a situation is created effecting an accelerated development. Dina van Hiele-Geldof reported “Nobody will dispute the necessity of providing instruction. It is exactly by means of guided learning processes that the child can grow faster and reach a developmental level he could not attain without instruction” (1958, in Fuys et al 1984, pp.217-223).

Eventually, after working though the levels, Pierre van Hiele concluded:

Then indeed we have found a superstructure governing the problems of our topic. But this superstructure has been built up by our own working and puzzling and by discussion with other people also struggling with the material. So, if we have problems, we should not seek a closed theory to solve them: closed theories usually suit only a very limited domain.

(1986, p.234)

Conclusion

Over the centuries several educationists, for example, Rousseau, Dewey, and Piaget, have argued that there was need for reform in the educational system. Their writings have challenged traditional pedagogy’s focus on subject matter, proposing that the focus should be on the needs of students, and on how they learn.

Within this broad framework is the work of the husband and wife team of Pierre van Hiele and Dina van Hiele-Geldof and their theoretical perspective for teaching and learning which has been the subject of this chapter. Geometry was used to expound their theory. Often the question was asked: “Why was it that so many children who mastered most subjects got nowhere in their study of geometry?” (Wirszup 1976, p.75). In the past, explained van Hiele (1959, in Fuys et al 1984, pp.244-245), the teacher presented the students with the information which the teacher possessed. The students learned this information and how to operate it by applying routines and memorised algorithms which were often not understood. Everything seemed in order. However, the student had only learned to apply a system of relations that had been offered ready-made.

The van Hiele theory offers an alternative system in which the teaching process addresses the development of students’ thought processes, i.e., the focus is changed from the subject matter
to the student and how he/she is thinking. Students’ understanding of a topic undergoes several changes as they grow through their studies. These changes fall into a series of categories, a hierarchical series of levels of thinking. The synthesis of the levels, the phases, and the perception of insight is known as the van Hiele Theory. Progression from one level to the next is not the result of natural development. It results from the student being within an appropriate learning environment. The series of five teaching phases is seen as one way to promote the development of insight and progression to a higher level.

The van Hiele model of teaching and learning was slow in gaining international attention in the West. It was not until approximately twenty years ago that intense interest in the van Hiele theory of development in reasoning was aroused in the United States by Wirszup (1976), and later by Hoffer (1981). Crowley (1987, p.1) explained:

It was not until the 1970s that a North American, Izaak Wirszup (1976), began to write and speak about the model. At about the same time, Hans Freudenthal, the van Hieles’ professor from Utrecht, called attention to their works in his titanic book, *Mathematics as an Educational Task* (1973). During the past decade there has been increased North American interest in the van Hieles’ contributions. This has been particularly enhanced by the 1984 (Geddes, Fuys, & Tischler) translations into English of some of the major works of the couple.

The interest generated in the van Hiele theory from the presentation of Wirszup’s paper (1976) at the Research Workshop sponsored by The Georgia Center for the Study of Learning and Teaching Mathematics has led to extensive research in several countries. In the United States, three major projects were supported by grants from the National Science Foundation (Burger & Shaughnessy 1986; Fuys, Geddes, & Tischler 1985; and Usiskin 1982). In addition, there has been research directed at the levels themselves as well as on properties and features of the levels. Research has also been designed to apply knowledge of the levels and their properties. The next chapter considers this research.
Chapter 2

RESEARCH ON THE LEVEL ASPECT OF THE VAN HIELE THEORY

In the preceding chapter it was shown how a desire to improve their teaching outcomes led two mathematics educators, Pierre van Hiele and Dina van Hiele-Geldof, to develop a theory of mathematics learning (van Hiele 1986, p.vii). An overview of the key elements in the theory, and a description of the two main operational aspects of the theory, the levels and the phases, were described and discussed.

In this chapter, research from Australia and overseas, relevant to the important properties and features from the van Hiele theory, is considered. Included in the reported research are the findings from the three major projects from the United States in the early 1980s, namely, the University of Chicago project (Usiskin 1982), the Brooklyn College project directed by Dorothy Geddes, (Fuys, D., Geddes, D. & Tischler, R. 1985), and the Oregon project (Burger & Shaughnessy 1986). To assist in this process, extensive use has been made of the comprehensive summaries of research on aspects of the theory, by Clements and Battista (1991, 1992), Hoffer (1983), Pegg (1992), and Pegg and Davey (in press).

A modification of the structure used by Pegg (1992) is applied to assist with the organisation of the chapter. This chapter considers first, the research concerning the levels, second, research directed at properties and features of the levels, and, third, research designed to use knowledge of the levels and their properties. Finally, Mayberry’s (1981, 1983) work is discussed in relation to the direction of this study.

Research Concerning the Levels

To many researchers, there is no doubt of the existence of the levels. Hoffer (1983, p.224) asserted “It is folly to ask whether thought levels as proposed by van Hiele actually exist. That they are being used as descriptive frames of reference guarantees their existence.” Soviet mathematics educators, after much intensive research and experimentation, verified the validity of the assertions and principles governing van Hiele’s theory on levels in the teaching of geometry (Wirszup 1976, p.77).

Van Hiele’s concept of levels provides a theoretical framework for developing a network of relations during a process of learning, based on the acquisition of insight. The notion of the
levels, and many of their properties and features are mainly dependent on a hierarchical structure, each level demanding a 'higher' insight than the preceding one (van Hiele 1957, in Fuys, Geddes & Tischler 1984, p.238).

The concept of levels of understanding raises a number of questions. Three questions stand out and are addressed in this section.

1. Do the levels represent discrete stages of major knowledge reorganisation, i.e., can the levels be properly described as stages. If so, what is meant by being 'at' a level?
2. How many levels are there?
3. Can a student’s level of development be evaluated? If so, are the levels topic specific?

Background
Before considering these questions it is valuable to consider the four criteria for levels as described by Steffe and Cobb (1988, in Clements & Battista 1992, p.433), namely:

1. **Constancy**: some property, state, or activity remains constant throughout each stage;
2. **Incorporation**: the earlier stage must become incorporated in the next;
3. **Order invariance**: the stages must emerge developmentally in a constant order;
4. **Integration**: the structural properties that define a given stage must form an integrated whole.

**Constancy** suggests that to be at a level, a student should exhibit some reliable behaviours indicative of that level, and be able to undertake activities directed at that level. While there is some debate whether students need display all behavioural characteristics to be ‘at’ a level, it is clear that a majority of key characteristics should be discernible. Hence, although it is necessary for students to exhibit reliable behaviour, thus indicating reasoning at a level, it is not sufficient to say that students are not at a level because they have not been exposed to certain characteristics.

Transition from one level to the next depends on the **incorporation** of the knowledge of the earlier stage into the next stage at the new, higher level. This involves a restructuring of the knowledge of the earlier level, so that the knowledge becomes the object of thought of the
next level, i.e., an external accessory to the new learning at the higher level. Hence, if a series of levels exists, the incorporation of prior knowledge into the learning of the next level predisposes the levels to have a hierarchical nature, i.e., the levels will have a sequential and constant order.

A hierarchical learning structure implies that there is an order invariance within the levels, i.e., that they always occur in the same order of succession. For levels to have constancy in representing discrete stages of knowledge, and for the knowledge of the earlier stage to be incorporated into the next, it is essential that a hierarchical structure can be demonstrated as a feature of the levels.

Behavioural characteristics that define a given level should demonstrate integration, as well as showing constancy to the definition of the level, i.e., there should be a network of relations between the characteristics regarding the degree of insight defined at the level. This constancy is whole, whether or not the characteristics are considered independent of the curriculum, or of the teaching strategies used. Hence, the behavioural characteristics exhibited by students assigned to the same level should be consistent. Discreteness of the levels is dependent on the integrated nature of the behavioural characteristics.

If the van Hiele levels do satisfy Steffe and Cobb’s (1988) four criteria above, i.e., the behavioural characteristics of the levels need to be defined, constant and integrated; and the levels need to be hierarchical, with each successive level incorporating the earlier levels, then the levels do represent discrete stages of major knowledge reorganisation (Clements & Battista 1992, p.433).

Following the discussion below of the research addressing the three questions (outlined earlier) concerning the features of the levels, the relevance of Steffe and Cobb’s four criteria to the van Hiele levels is considered.

**Do the levels represent discrete stages of major knowledge reorganisation, and, if so, what is meant by being ‘at’ a level?**

If the creation of a series of levels demands that each level has a ‘higher’ insight than the preceding one, then the components of each level need to be identified and classified. The progression from one level to the next involves the elimination of some of the concepts of the earlier level, and their replacement with newly identified concepts. Van Hiele (1957, in Fuys, Geddes & Tischler 1984, p.238) describes the development of the concepts or structures of the new level:
At first these structures will be undifferentiated but they are likely to lose their original form when analysed. The result will be a new ‘higher’ structure, embodying the classifying principles of the original ones. ... This is an entirely new process of thought: we call it the ‘transition to a higher level of thinking.’

Hence, the learning or knowledge able to be displayed by a student at the new level will differ from the knowledge displayed by that student at the preceding level in that the earlier knowledge will have been reorganised. This means that each level can be identified by the insight displayed by a student, i.e., the insight displayed by a student at a level is defined, and hence, represents discrete knowledge dependent on the integrated nature of the behavioural characteristics of that level. A definition needs to have a consistent interpretation, i.e., it needs to show constancy and totality. Students can therefore be described as being ‘at’ a level when they display reliable behaviour that indicates understanding of the characteristics of that van Hiele level. Such definitions of the van Hiele levels in behavioural terms were compiled by Mayberry (1981, pp.47-49). To produce the working definitions of the levels, Mayberry first examined articles by van Hiele for pertinent statements. However, there is need for finer detailing concerning what is meant by a student being ‘at’ a level. For example, for Level 2, how many and which properties need to be recognised by a student to qualify as being ‘at’ Level 2?

The concept of the theory is not without controversy, not all writers agreeing the levels are defined, and, hence, that progression from one level to the next can be described. Schoenfeld (1986, p.250) maintained that “the theory does not provide a determinist, structuralist view of a fixed progression of cleanly divided ‘stages’ through which individuals must pass.” He considered that the phases of learning which provide the means of transition between levels, while they represent common sense, are not theoretical phases. They are “lacking a detailed explanation of cognitive processes that underlie competent performance” (p.252). Clements and Battista (1992, p.433) suggested that there should be some research into the van Hiele theory that is structured so as to simultaneously test alternate hypotheses about characteristics of the levels.

**How many levels are there?**

There is still some controversy regarding the number of levels. Schoenfeld (1986, p.250), in acknowledging this, considered that “determining the precise number of levels (Are there four, five, or six?) is not a central issue with regard to the theory.” However, Clements and Battista (1992, p.431) who described van Hiele’s recent characterisation of a three-level model, cautioned that “if levels can be changed and combined, their hypothesized discrete, hierarchical psychological nature must be questioned.”
Concerning the number of levels, there needs to be consideration of the following two questions. First, does the difficulty in testing for Level 5, observed by Usiskin (1982), imply that the level may not exist? Mayberry (1981, p.51) treated Level 5 differently to the first four levels, i.e., the questions were made referent-free, and not put into concept categories. Level 5 was not included by Gutiérrez, Jaime and Fortuny (1991) in their alternative evaluation method, as they found measuring this level to be unsuccessful. If the three-level model is adopted, the original van Hiele Level 5, together with Level 4 become the new third level, and, hence, the question becomes trivial.

Second, since several of the students tested failed to demonstrate recognition of concepts (Level 1) (Mayberry 1981; Senk 1989; Usiskin 1982), is there a need for an earlier level, say Level 0, below the levels identified by the van Hieles? Mayberry (1983) found an unexpected 13% of the response pattern quintuples showing students unable to reach Level 1 criteria for certain concepts. She surmised that “perhaps there is a zero level before figure recognition and discrimination occur” (p.67). 26% of Us:skin’s (1982) students who began the year at Level 0, remained at Level 0 at the end of the year. Such stability, argued Senk (1989) indicates the existence of Level 0. This question remains pertinent whether the three-level model is adopted, or whether the four/five-level model is retained.

While there appears to be evidence in favour of defining the highest level as containing all formal understanding of geometry, there is research indicating the existence of thinking more primitive than van Hiele’s Level 1. Clements and Battista (1992, p.429) define this pre-recognition Level 0:

At the pre-recognition level, children perceive geometric shapes, but perhaps because of a deficiency in perceptual activity, may attend to only a subset of a shape’s visual characteristics. They are unable to identify many common shapes. They may distinguish between figures that are curvilinear and those that are rectilinear but not among figures in the same class; that is, they may differentiate between a square and a circle, but not between a square and a triangle.

There is need for consensus concerning the number of levels in the van Hiele Theory lest argument on the notion leads to argument about the central issues with regard to the theory, and, hence, to criticism of the nature of the theory.

**Can a student’s level of development be evaluated?**

Many researchers have shown that the van Hiele levels of thinking in students can be determined empirically, whether by interview or by written test (Burger & Shaughnessy 1986; Fuys et al 1985; Hoffer 1981; Mayberry 1981; Senk 1982; Usiskin 1982). Knowing a
student’s van Hiele level in geometry is useful in describing the student’s stage of development in that concept. Being able to determine whether students have changed their van Hiele level of thinking after a period of study is often a requirement for investigation into some facet of the levels. For example, Usiskin (1982), designed a multiple-choice test across several geometric topics. On applying the test, he found that the majority of secondary students (about 75%) could be assigned a van Hiele level. Chaiyasang (1988) used Usiskin’s test to determine the effect on student learning of the new geometry program commenced several years earlier, in 1978 in Thailand. He found that he was able to allocate levels to 90% of the students tested, and hence, he was able to evaluate the effectiveness of the new course in developing the geometric understanding of the students. Similarly, other researchers have used various tests for determining levels as a tool in their investigation into some of the properties and features of the van Hiele theory (Bobango 1987; Crowley 1987; Denis 1987; Gutiérrez, Jaime, & Fortuny 1991; Nasser 1990).

While there appears to be consensus that students’ van Hiele levels can be determined, the structure of the assessment instrument needs also to be investigated. Senk (1989) suggested that assessment instruments should be topic specific. Mayberry (1981) who designed her interview items to investigate this notion, found that only two of the nineteen students reached a consensus across topics, supporting also the notion that tests need to be topic specific. That Mayberry’s questionnaire takes the different levels exhibited by students for different concepts into consideration was a factor in Denis’ (1987, p.47) decision to used the Mayberry test in preference to Usiskin’s test in her study. Additionally, there is need to consider as well, the method of assessment. Interviews have been shown to be reliable indicators of van Hiele levels of thinking (Burger & Shaughnessy 1986; Gutiérrez & Jaime 1987; Mayberry 1981), but they are very time consuming. Although Usiskin’s multiple choice test has been used in over 100 studies, other investigators (e.g., Crowley 1990; Pegg 1992; Usiskin & Senk 1990; Wilson P. S. 1990) have questioned the test for its validity and reliability. In particular, Pegg (1992, p.23) suggested that, in assuming that the students will be at the same level for different concepts, the test is basically flawed. Crowley (1990) indicated that paper and pencil testing needs to be further refined and evaluated, while Gutiérrez, Jaime, Shaughnessy and Burger (1991, p.116) consider paper and pencil open-ended questions followed by short targeted interviews the most promising possibility. Usiskin and Senk (1990, p.244) raised the issue of the conflict between keeping a test brief and focusing on the reliability of the test. Clearly, more investigation needs to be carried out into the development of a reliable written test that can be used to assess students’ van Hiele levels.
Conclusion
Although Schoenfeld (1986) raised issues concerning the validity of the theory, and Clements and Battista (1992) would like to see research designed to test alternate hypotheses about characteristics, most research supports the notion that the van Hiele levels of understanding satisfy the four criteria as outlined by Steffe and Cobb (1988). The hierarchical nature of the levels means that the levels represent a developmental pathway where each given level is attainable only after going through the lower levels in order. Additionally, knowledge acquired at one level becomes incorporated in the learning of the next level. Each level is definable in terms of reliable descriptors, the descriptors forming an integrated totality. To be at a level, a student must display behaviour consistent with the definition of the level. Even though tests have been developed which can be used to determine the level at which a student is working, further research is necessary to investigate the number of levels, whether they are topic specific, and what types of tests give the most reliable results.

Research Directed at Properties and Features
Several properties and features are associated with the van Hiele levels. These are considered in this section under the following headings, discreteness, hierarchy, intrinsic versus extrinsic, language and levels, differences in perception at different levels, different levels for different concepts, time for cognitive development, crisis in thinking and level reduction, and rote learning.

Discreteness
When levels are described as discrete, it indicates that there is a discontinuity in the learning process, namely, that growth moves through a level plateau before there is a jump to a new level. The van Hieles observed such jumps in the learning curve. These are periods when learning seemed to have stopped, later to recommence. These ‘jumps’ were taken as indicators of the presence of levels, and that the levels were discrete. Van Hiele (1986, p.49) explained “The most distinctive property of the levels is their discontinuity, the lack of coherence between their networks of relations.” Explaining this comment, he (1959, in Fuys, Geddes & Tischler 1984, p.247) stated:

At the close of the fifth phase (of teaching) ... the student has at his disposal ... a new domain of thought, which has acquired its own intuition, (and) is substituted for the previous domain of thought which had a completely different intuition.

The discreteness of the levels has been commented on by several researchers, but does not appear to have been the subject of specifically targeted research. Nevertheless, there has not
been unanimous support for the idea. Russian research, as identified in Clements and Battista (1991, p.223) supports the notion of the levels being discrete, as does Hoffer (1983, p.225):

There are those who say that learning is continuous. In reality, learning is quite discontinuous. It is teaching that should be continuous.

Mayberry supported this notion also. Her investigation into the hierarchical nature of the van Hiele levels of pre-service teachers is based on this assumption, that there exists a discontinuity between the levels. She also believed that the levels are static, and that a student works in only one level at any one time. This notion was incorporated into the design of her study where a set of tasks was designed to the operational definition of each level. These tasks were used to determine the van Hiele levels of understanding of a sample of pre-service teachers. This allowed Mayberry to hypothesise (1981, p.10) that a student who conformed to the van Hiele theory would display the following characteristics.

He would fail not only to answer correctly but also fail to understand the intent of a question which required thought above the attained level. In terms of test results a subject who could be described as a perfect level \( n \) would respond correctly to all questions at and below level \( n \) but incorrectly to all questions above level \( n \).

The results of Mayberry’s study did not indicate any contradiction to the notion that the levels are discrete. This may be a consequence of the nature of the design of her research, i.e., that each test question was designed to measure understanding in only one level, and assessment of the response was restricted to whether the student displayed the desired level of understanding.

However, other researchers (Burger & Shaughnessy 1986; Usiskin 1982) saw that the levels are not discrete, that, realistically, they are of a more dynamic nature. Usiskin (1982, p.80), in his finding that students in the process of transition from one level to the next are difficult to classify reliably, appears to be the first to have expressed doubt. The discreteness of the levels is also questioned by Burger and Shaughnessy (1986, p.45). Their study did not detect discontinuity between the levels. They considered that the occasional difficulty reviewers had in deciding between levels while making level assignments was evidence in support of the notion of the dynamic nature of the levels. i.e., of levels having a more continuous nature, since students may move back and forth between levels quite a few times while they are in transition from one level to the next.

This notion of the levels being, to some extent, continuous has mixed support from the Geddes project team (Fuys et al 1985), and the work of Pegg and Davey (1989). The Geddes
team (Fuys et al. 1985, pp.233-234) found that performance of some students suggested that they were at a plateau for a particular level, and could not progress to the next level without a time pause. On the other hand, other students seemed to be in transition between Levels 1 and 2, dealing with familiar shapes in terms of properties (Level 2), yet lapsing to Level 1 when confronted by unfamiliar shapes. Pegg and Davey (1989, p.26) in their study clarifying level descriptions for students understanding of some basic 2-D geometric shapes, supported the notion that the levels are not necessarily discrete. They found many students using two levels of thought in their descriptions, i.e., the use of global images in addition to the properties identified.

The work of Gutiérrez, Jaime and Fortuny (1991) went further. They not only assumed a more dynamic notion of the levels, they used this notion to develop a method of evaluating the capacity of a student to use each van Hiele level. Their research showed that although most students indicate a dominant level of thinking in their responses, a large number of them clearly reflect in their answers, the presence of other levels. For example, a student may be assessed as having a grade component for Level 1 of 100%; Level 2, 88%; Level 3, 55%; and Level 4, 10%. Not all students demonstrate similar movement through the levels. There appears to be some critical proportion of acquisition of a lower level that is necessary before exploration of the next level is begun, this proportion varying for different students. While not supporting the notion of discreteness of the levels, the method of evaluation developed by Gutiérrez et al reinforces the hierarchy of the levels, the degree of acquisition of a lower level usually being more complete than the degree of acquisition of a higher level.

Overall, there appears support in the literature for both views. On the one hand the van Hiele levels represent discrete entities, while on the other, growth through the levels is continuous. Research, such as that of the Geddes project, Burger and Shaughnessy, Pegg and Davey, and the Gutiérrez team, supports both views. Results indicate that while some students appear to show discontinuity in their learning pattern, others display a more continuous growth curve, either moving between levels when in transition, or using two or more levels at the one time.

**Hierarchy**

Van Hiele considered the hierarchical nature of the levels, i.e., the organisation of successive levels such that one incorporates a higher level of reasoning than the other, to be essential to his concept of the structure of the levels. Hierarchical structure is intrinsic to one of Steffe and Cobb’s (1988) criteria for levels, _order invariance_: the stages must emerge developmentally in a constant order. In his early writings, van Hiele indicated the importance of this criterion on several occasions. He (van Hiele 1955, pp.289-290, in van Hiele 1986,
p.40) referred to the need for students to attain a ‘higher’ level, and the unlikeliness of their falling back to a ‘lower’ level, later explaining, that “each succeeding structural form demands a ‘higher’ insight than the preceding one” (1957, in Fuys, Geddes & Tischler 1984, p.238). He (1959, in Fuys, Geddes & Tischler 1984, p.250) again referred to the hierarchical structure as resulting in the “transfer from one level of thought to a higher level” at the conclusion of the fifth phase of teaching.

The hierarchical nature of the van Hiele learning levels is taken as inherent to the theory by many researchers (e.g., Usiskin 1982, p.4; Chaiyasang 1988, p.8). Silfverberg (1984) considered that the transition from one level to another presupposed the mastery of activities pertaining to the previous level. That hierarchical structure of the levels is important is emphasised in that many of the characteristics of the levels evolve from the feature, e.g., the intrinsic/extrinsic nature, the significance of language at each level, the differences in perception for different levels, and the need for a crisis in thinking before a student can progress to the next higher level. The importance of the hierarchical nature of the levels is emphasised in De Villiers’ outline of the strengths of the van Hiele theory,

The most important aspect of the van Hiele theory is the distinction of five levels in the mastery of geometry, and the hypothesis that they form a learning hierarchy.

(1987, p.3)

The notion of hierarchy underpins also the interpretation of results made by several researchers. Burger and Shaughnessy (1986, p.42) confirmed the hierarchical nature of the levels, as did both de Villiers (1987, p.10), in his investigation into moving students from partition to hierarchical classification, and Davey and Pegg (1989) in their work on students’ skills associated with angles and parallelism. The learner cannot achieve one level without having passed through the previous levels (Fuys 1985, p.449), and will have mastered large chunks of the prior levels in order to function adequately at one of the advanced levels (Hoffer 1981, p.14). Mason (1989, p.14), in discussing the results of her investigation into geometric understanding and misconceptions among gifted students, noted that students need experiences of Levels 2 and 3 to provide a foundation for their Level 4 reasoning. The hierarchical nature of the levels is also supported by the investigation into the learning and understanding of congruence, carried out by Nasser (1990, p.302). She found that, in general, students performing at a certain level were successful in tasks demanding a lower level performance.

Data analysis procedures designed to investigate the hierarchical nature of the levels has indicated consistently that the levels are hierarchical. Guttman’s scalogram analysis was used
by Mayberry (1983) to show that her tasks, in representing the levels, did form a hierarchy (reproducibility 0.97). A similar result (reproducibility 0.98) was achieved when Denis (1987), in her study, used the Mayberry items on a sample of Puerto Rican students. Again, the hierarchical nature of the first four van Hiele levels has been confirmed in the investigations of other researchers, namely, Gutiérrez and Jaime (1987), Smith (1989), and Wilson, M. (1990). Gutiérrez and Jaime (1987) found that almost all students showed a lesser degree of acquisition for each successively higher van Hiele level, while the two paper-and-pencil tests compared by Smith (1989), both gave acceptable reproducibility coefficients. Smith omitted the highest level, Level 5 from his study, citing Usiskin’s conclusion (1982, p.79) that “level 5 either does not exist or is not testable”. In comparison, Wilson, M. (1990) included results for all five levels when he re-analysed Usiskin’s data using the Rasch model. The analysis confirmed the hierarchical nature of the first four levels, but not the fifth level, supporting Usiskin’s conclusion concerning the fifth level.

While the results of several studies support the notion of hierarchy of the levels, no published attempt has been found which analyses the small number of students who failed to fit the notion of hierarchy in studies by Gutiérrez et al. (1991), Mayberry (1981), and Usiskin (1982). Generally, researchers have taken this characteristic as inherent to the van Hiele theory. Nevertheless, a few researchers have indicated the need for further research into the issue of hierarchy. Mayberry (1981, p.93), for example, stated that if additional research further validates the hierarchical nature of the van Hiele levels, the geometry curriculum in schools should be revised to take these levels into account. De Villiers (1987, p.20) suggested that the levels

should not so much be viewed as prescriptive in regard to a learning and teaching hierarchy, but merely as descriptive of the results and outcomes of our present teaching strategies and curricula.

A warning against allowing the hierarchical nature of the levels to dominate the teaching/discovery process is sounded by De Block-Docq (1994, p.188), in the analysis of her replication (in part) of van Hiele-Geldof’s research. De Block-Docq suggested there is danger if the attainment of the levels is allowed to become the principle purpose of teaching and hierarchy is enforced too rigidly. This can result in a loss of the underlying importance of the maturation of mathematical thinking, and the importance of the resolution of problems in large coherent contexts.

**Intrinsic versus extrinsic**

When a notion is essential to the learning, it is said to be intrinsic to that learning. At this point in the learning, the student is moving through the phases towards attainment of a level.
Once the notion is understood and the knowledge is owned, i.e., the student has reached the level, it becomes an external accessory to the new learning (towards the next higher level) and is now described as being extrinsic. The intrinsic-extrinsic nature of the acquisition of knowledge is an important feature of the van Hiele theory, essential to its hierarchical nature, and the second of Steffe and Cobb’s (1988) criteria for levels. Van Hiele explained:

> At each level there appears in an extrinsic way that which was intrinsic at the preceding level. At the [first] level, figures were in fact also determined by their properties, but someone thinking at this level is not aware of these properties.

(1959, in Fuys, Geddes & Tischler 1984, p.246)

The intrinsic-extrinsic feature has been explained by various researchers in different ways. Coxford (1978, p.330) expressed the notion in terms of structures; “If the goal is to help a person internalize a particular mathematical structure, then it seems reasonable to first allow the learner to internalize the concepts to be structured.” In a similar vein, Mayberry (1981, p.1) described this feature as the object of perception at one level of insight becoming the object of study at the next level. This interpretation was incorporated into the design of her interview tasks. Hoffer (1983, p.206) supported this interpretation, arguing that the objects studied at Level \( n \) consist of extensions of the objects at Level \( n-1 \), i.e., at Level \( n-1 \) a student will study certain limited versions of objects, making explicit statements about the objects; at Level \( n \), the objects that were explicitly stated previously now become the subject of study.

During their project, Fuys et al (1985) developed further the notion that objects of study at one level are an extension of the objects of study at the previous level. This notion was implicit to their exploration of a student’s ‘potential level’ of thinking, the level a student has the potential to reach as he/she responds to prompting in an interview situation. The team felt that measurement of a student’s van Hiele level of thinking, as determined solely by prior learning experiences, may not accurately assess the student’s ability to think, especially if the student had little or no prior learning experiences in the topic. This concept of ‘potential’ level represents an upper bound of Vygotsky’s (1976) notion of a zone of proximal development. A student’s level, as routinely assessed at interview, is basically what can be offered spontaneously. That the van Hiele level of knowledge students display in a test or interview situation is dependent on prior learning experiences, and, hence, may not be a true measure of their ability to reason, is also supported by the arguments of other researchers (e.g., de Villiers 1987; Mayberry 1981; Pegg 1992). They argue that the van Hiele levels of thinking determined for students may not be independent of the curriculum. If students have not been exposed to certain notions, for example, class inclusion, this lack of experience may limit the extension of their knowledge in that area, thus impeding the growth of knowledge.
which is intrinsic at one level, i.e., not promoting the development of knowledge to become
extrinsic at the next level.

Van Hiele (1986, p.96) summarised concisely the notions of the intrinsic/extrinsic feature in
his statement “At each level one is explicitly busy with the internal ordering of the previous
level.” The incorporation and restructuring of knowledge of the earlier level into the next
level is essential to the notion of hierarchy of the levels, and results from progression through
a series of learning phases. However, attainment of the higher level is not necessarily
dependent on mastery of all characteristics, but rather, of the mastery of particular
characteristics. Additionally, progression to the next level can be hindered by the curriculum
and certain teaching practices.

**Language and levels**

As students move through phases of the learning process, it is important that they
conceptualise their newly acquired knowledge in their own language. Van Hiele (1986, p.97)
explained, “At this time it becomes clear how important it is that the pupils exchange views
about the new things they have discovered.” He (p.79) continued, “If in a learning process an
individual has learned to understand a structure by direct contact with reality, he has to learn
the language for it, giving him the ability to exchange views about it with other people.” The
use of language is significant for three reasons. First, the oral expression and communication
to others of the ideas about structures, which they have recently explored, is instrumental in
the clarification, realisation and confirmation of their new knowledge. Second, the
discussing and listening to others, i.e., the sharing of ideas, leads to an extension of their
insight. Third, language is specific to a student’s level of development and the use of
relevant language is essential for development within the level.

Oral expression of ideas is particularly important during the explicitation phase as the student
moves towards attainment of the next level. Without the availability and use of appropriate
language, students cannot verbally express and exchange the ideas they have been exploring,
the teacher cannot communicate with the student, and the textbook does not provide
purposeful material for study. Too often the teacher speaks a different language to the
students, with the teacher using the language of a higher level than is understood by the
students. A classroom background is used by van Hiele (1959, in Fuys, Geddes & Tischler
1984, p.246) to illustrate this:
Two people who reason at two different levels cannot understand each other. This is what often happens between teacher and student. Neither of them can manage to follow the thought process of the other and their dialogue can only proceed if the teacher tries to form for himself an idea of the students’ thinking and to conform to it. ... A true dialogue must be established at the level of the students. For this to happen, the teacher must often, after class, ask himself about the responses of this students and strive to understand their meaning.

Several researchers refer to instances in which the level of the language of instruction differed from the level of the language of the learner. For example, Usiskin (1982, p.5) cited a student remarking “I can follow a proof when you do it in class, but I can’t do it at home.” This student is probably at Level 3, and the teacher, at Level 4, has failed to adapt his/her language to the level of the student, leaving the student unable to understand fully what is being said.

It is not only the language of the teacher that is significant. Usiskin (1982) commented on the importance of the level at which a textbook is written matching the level of the students for whom it is designed, if it is to benefit the students. That US textbook materials of the early 1980s provided students with little opportunity to make progress was remarked on by Fuys et al (1985). They noted that the material may actually impede progress by concentrating on inappropriate content and using inappropriate language. Fuys et al found also that some students have poor geometric vocabulary, using either non-standard language, or standard language imprecisely. This lack of vocabulary not only limited the students’ progress, but also was an indicator of their level of geometric reasoning.

A strong linkage between the use of certain words, word combinations, and the various levels at which students were operating was noticed by Burger and Shaughnessy (1986). This word-level linkage is illustrated in the varying interpretations students have of the word ‘angle’. Davey and Pegg (1991, pp.4-7) recorded students at Piagetian Levels I and II using the words ‘corner’ and ‘point’ in preference to ‘angle’, while students at Piagetian Level III described an angle as ‘where two straight lines meet at a corner’, ‘round like a circle, all the way would be 360°’, ‘a sharp bend in a line’, ‘the area, the distance or the length between the lines’. Similar interpretations of the word ‘angle’ were recorded by Wilson, P. S. (1990, p.43) in her report on inconsistencies in students’ ideas related to their definitions. She felt that the problems of a restricted vocabulary parallels the problems associated with a lack of experience, interfering with the students’ reasoning. Similar difficulties with vocabulary were also found by Hoffer (1983, p.215), who commented on students having gross misconceptions or totally incorrect ideas regarding their meanings, even though familiar with geometric words. For example, some students interpret the word ‘triangle’ as solely meaning an equilateral triangle in the ‘textbook position’, i.e., with a base parallel to the lines of a
Pegg and Davey (1989 p.26) observed that certain words, for example, flat, top and bottom, corners, even, were constantly used by students, yet seldom appeared in teachers’ language, and that this language of students “transcended state borders and can be compared directly with student responses in the U.S.” They noted that “this supports van Hiele’s proposition that people on a certain level share a similar language.”

The importance of students experiencing language as they progress through each level was recognised by Crowley (1987) in her paper on van Hiele-based activities. She encouraged students to articulate consciously, in their own language, what otherwise might be vague and undeveloped ideas by providing them opportunities to describe, to ask, to report, to argue, to explain and to discuss. Wilson, P. S. (1990, p.45) supports this in her observation that a restricted vocabulary interfered with growth in students’ reasoning. The particularly important part that language plays in a student’s understanding of class inclusion is recorded by De Villiers (1987) in his summary of Malan’s (1986) study. For example, it was found that “students’ difficulty with hierarchical classification often lay with the meaning of the word ‘is’ in the question ‘Is a square a rectangle?’ They seemed to interpret it as meaning ‘equivalent to’ or ‘is the same as’, which of course is not what we mean by it, namely, ‘is a subset of’” (De Villiers 1987, p.17). Shaughnessy and Burger (1985) emphasised the importance of teachers remembering that students’ concepts underlying language may be vastly different from what teachers think they are, and that frustration and discouragement often result when students are unable to understand the language of the teacher. For students, summarised Clements & Battista, (1992, p.433), the limitations associated with imprecise language “is a critical factor in (their) progressing through the levels.”

The effect of the use of inappropriate language is not confined to the learning situation in the classroom. If the language of test questions is of a level different to that of the students, through their inability to understand the wording completely, they may not display all their knowledge in their responses. In discussing the limitations of her study, Mayberry (1981, pp.97-98) remarked that since test questions are interpreted by students according to their language level, had the wording of the actual questions in her test been different there is the possibility that a different set of responses may have resulted. Clements and Battista (1991, p.226) observed that, if a question used wording such as ‘explain’ and ‘provided that’, while students on higher levels understood these words, and acknowledged the need to justify their reasoning, students on a lower level did not understand the words, and, hence, listed only the indicated facts.
Hence, language is intricately involved in the teaching processes and in the development of thinking. Teaching and assessment can become ineffective if the language of the teacher, the textbooks, and the test materials are not the language of the level of the students. Additionally, if the students are not given adequate opportunity during the process of learning to verbally express their ideas, listen to others, and to have discussion, their development can be hampered, and there may not be any progression in their attainment of new levels of thinking.

**Differences in perception at different levels**

Structures and objects are interpreted by students according to their degree or level of understanding. With cognitive development, van Hiele (1957, in Fuys, Geddes & Tischler 1984, p.238) remarked that the analysis of perceptions “leads to new forms of identification and thus to new structures.” An interpretation that is ‘correct’ at one level can be incorrect at another (1959, in Fuys, Geddes & Tischler 1984, p.246). Furthermore, “one can only express oneself clearly in mathematics when one uses symbols belonging to one’s own level” (p.248). The same symbols (including linguistic symbols) can appear at successive levels, but with different meanings or content (van Hiele, 1986, p.61). This is supported by Silfverberg (1984) who observed that at a level a certain property becomes a signal. For example, at Level 1, the signal for a rectangle is its shape being oblong.

Initially, symbols have a strong visual element. As this visual element lessens in significance, the symbols acquire a verbal image. For example, to a student thinking at Level 1, the word ‘square’ indicates a gestalt shape, whereas to a student thinking at Level 2, the term suggests a quadrilateral to which specific properties are attached (Burger & Shaughnessy 1986; Fuys et al 1988). To a student working at Level 3, the square may be seen as part of the family of rectangles. As symbols begin to influence the orientation of the thinking, they act as a signal, determining the direction of the thinking. At each level, the interpretation of a concept is specific to, and different from, that at other levels. Action developed in this context makes access to the next level possible (van Hiele, 1959, in Fuys, Geddes & Tischler 1984, p.251).

The notion that students at different levels think differently has been supported by many studies. Shaughnessy and Burger (1985, p.425) discussed the notion that the role of definitions at different levels is misunderstood. A student’s definition may be vastly different to that of the teacher. While the teacher is writing a careful (Level 4) definition of a rectangle on the board, the student (at Level 2) may be thinking about all the properties that the teacher has left out. Students will distort what they ‘see’, interpreting it according to what they
understand (Coxford 1978, p.323). Burger and Shaughnessy (1986, pp.46-47) observed also that students at different levels use different problem-solving techniques. They also noted that students assigned to the same level behave consistently, and this consistency contributes to the operational characterisation of the level.

Piaget and Inhelder (1967, p.33) claimed that “a student’s inability to recognise inconsistencies is related to cognitive development; that is, dependent on maturation.” The van Hiele theory, in contrast, explains the apparent inconsistencies as being dependent on lack of experience rather than maturation. Adding to this, Wilson, P. S. (1990, p.32) commented that because a student and a teacher are working at different levels, they may not share the same understanding of an idea. A student’s definition that appears inconsistent to the teacher may be consistent within the confines of that student’s level of reasoning. For example, students might be consistent in their understanding that a square is a rectangle if they do not have the concept of rigidity of a shape, i.e., their understanding is that a shape can be stretched in any direction so as to assume the properties of another shape. This reasoning would appear inconsistent to a teacher looking for the definition that a square has four equal sides. Wilson (p.45) warned that “teachers and researchers need to listen to students. What may seem inconsistent may actually be consistent when viewed with the students’ definition, universe of instances, and guidelines gathered from his or her experience.”

The interpretation of material by students according to their level of understanding affects also their method of responding to a task. Clements and Battista (1992, p.430) found that, whereas students at higher levels tended to justify their responses, “students at lower levels believed that they should respond to a task on paper exactly as it appeared (for example, changing its orientation was not allowed).” However, this issue can be confused when the context of a question is not made clear. Van Hiele-Geldof (1957, in Fuys, Geddes & Tischler 1984, p.212) found that the question “What do you see?” is meaningful only if the student knows the context in which the question has to be answered. De Villiers (1987, p.22) observed that response errors result when the intention of a question is not known. For example, a question asking for identification of all rectangles in a set of quadrilaterals, if set amongst other low level questions, may elicit a Level 1 response from a person normally working at Level 3 or higher. Mayberry (1981) found only three out of nineteen students gave a Level 3 response to such an item whereas eleven students achieved criteria for Level 3 or higher. Pegg (1992, p.24) maintained that it is not sufficient to say that students are not at Level 3 because they have failed to recognise class inclusion, a Level 3 perception. In assigning students to a level of thinking according to the students’ interpretation of a symbol
such as the linguistic symbol ‘rectangle’, it is important to consider the context in which that symbol has been used.

Regarding the function of symbols in the overall progression through the levels, van Hiele (1957, in Fuys, Geddes & Tischler 1984, p.240) stated:

Provided the process of teaching lasts long enough, the symbols used in it will progressively lose their original significance until finally their only function will be that of junctions in a network of relations.

In summary, research supports the notion that symbols are closely related to the content of the relevant level, taking on different meanings at different levels of understanding. Teachers need to listen to students, and to communicate with them at their level, using appropriate interpretations of the symbols. Teachers also need to recognise that what can appear to them as inconsistencies, are usually consistent within a framework of the student’s level of thinking. Correspondingly, it is also essential that test items be constructed and phrased appropriately, poorly constructed questions frequently result in response errors. There is need for the development of guidelines concerning the framing of test questions appropriate to the level being investigated, leading to a greater degree of reliability in results.

Different levels for different concepts

The heart of the van Hiele theory lies in the significance that it is possible to think and reason at different levels in each scientific discipline (van Hiele, 1959, in Fuys, Geddes & Tischler 1984, p.251). Once a level has been attained for one discipline, it will take less time for that level to be reached for other related disciplines. However, it is not only between disciplines that levels can differ. Researchers have shown that within the one discipline, students can be found to be at different levels for different concepts. Across the seven geometric concepts tested, Mayberry (1981), found that students more commonly exhibited different levels of insight. Seventeen of the nineteen students tested, failed to show consensus of levels across the seven concepts. This finding that students do not necessarily demonstrate the same level of thinking in all geometric areas was supported by Mason (1989) in her analysis of clinical interviews of gifted middle-school students. Burger and Shaughnessy (1986) took Mayberry’s findings into consideration when designing their research, limiting their investigation to the 2-dimensional shapes, quadrilaterals and triangles. They still found some students exhibited different preferred van Hiele levels of reasoning on different tasks. For similar reasons, Denis (1987), in using the Mayberry questions to investigate the relationship between the Piagetian stage of cognitive development and van Hiele levels of geometric thought among Puerto Rican adolescents, selected items pertaining to the four concepts with
the highest rate of reproducibility. Silfverberg (1984) considered that the problem of the levels being concept-bound to be sufficiently significant as to render the placement of students according to their van Hiele level of little use. He concluded that there is need for further clarification of the content of each level and the relation of the concepts to one another.

Students can be working at different levels, not only within the broader outlines of concepts, but also within features of these concepts. For example, Davey and Pegg (1989) found that some students were very familiar and competent at dealing with and recognising right angles, but these characteristics were not reflected in their general knowledge of angles. They also found (Pegg & Davey 1989) that students were able to respond at higher levels on questions related to more familiar shapes, such as square and rectangle, than less familiar ones, such as parallelogram and rhombus.

Usiskin (1982) offered an alternative point of view. He found the van Hiele theory to have three appealing characteristics: elegance, comprehensiveness, and wide applicability. For him, the theory has a simple structure, and it not only applied to geometry but to all mathematical understanding. From this Usiskin assumed that students would be at the same level for different concepts, hence he designed his widely used multiple-choice test across a number of topic areas in geometry. This assumption, Pegg (1992, p.4) asserted, leads to a basic flaw in the test.

There is some debate whether the notion of students working at different levels for different concepts applies across all five van Hiele levels, whether this applies to only the first four levels, or whether the different levels reflect the outcomes of factors, such as teaching strategies and curriculum. Although Mayberry (1981, p.51) used seven concepts in the design for her assessment instrument when testing the first four van Hiele levels, she felt that questions for the fifth level were referent free, hence unable to be put into concept categories. Senk (1989, p.320) suggested that since the degree to which a student reasons at different levels is dependant on many factors, such as prior experience and similarity of concepts, research attempting to determine the van Hiele levels of thinking should use instruments that are content specific.

Fuys et al (1985, p.189) studied the effect of the amount of experience of students had with different topics on the level of understanding displayed. They compared students’ performances across the three concepts, properties of figures, angle sums, and area. When studying a new concept, students were found to lapse frequently to Level 1 thinking, but with
instruction, were quickly able to move to the higher level of thinking they had reached on a similar concept. The researchers (p.238) claimed this suggests that a student’s ‘potential’ level of thinking remains constant across concepts. While there does not appear to be any research into the constancy of a student’s ‘potential’ level of thinking, the idea does support the notion that it takes less time to reach a level for one concept if that level has already been attained for another concept. However, this may not apply to all levels.

It appears, in summary, that most researchers accept that students frequently are at different levels, not only for different disciplines, but also for different concepts and even for different features within a concept. However, the degree to which these differences affect an instrument being used to determine a student’s working level not only needs to be taken into account by researchers, but also needs further investigation.

**Time for cognitive development**

To move students from one level to the next requires interaction between the teacher, students and subject matter. Van Hiele (1986, p.50) commented that transition from one level to the next is not a natural process; it takes place under the influence of a teaching-learning program, and, again, time is necessary for the development of the students’ capacity to acquire knowledge. This section comments on the effect of time on cognitive growth in various situations. These include the time needed to lead students through the teaching phases by courses designed by the van Hieles (and by other researchers), the need to attend to the aspects of concepts, and limitations to cognitive growth.

It takes time to explore and reflect on ideas, and hence, growth is necessarily gradual (van Hiele-Geldof, 1958, in Fuys, Geddes & Tischler 1984, p.225). The teacher creates a situation of generated tasks in which students are encouraged to explore and reflect on a subject as they progress through the five teaching phases. Learning is achieved, and students attain a new level of thought.

The time required in the acquisition of understanding needs to be acknowledged and taken into account. Van Hiele-Geldof (1957, in Fuys, Geddes & Tischler 1984) wrote of seventy lessons being needed to move twelve-year-olds from Level 1 to Level 3. In her replication of van Hiele-Geldof’s lessons, De Block-Docq (1992, in De Block-Docq 1994), found she required a similar amount of time. This need for time was emphasised by Freudenthal (1973, p.402) when explaining that geometric deduction cannot be imposed upon the learner; it needs to be taught as a reinvention. Van Hiele (1986) spoke of it as taking “nearly two years
of continual education to have pupils experience the intrinsic value of deduction, and (that) still more time is necessary to understand the intrinsic meaning of this concept” (p.64).

This need to allow time for student growth has since been written about by several researchers, e.g., Bobango (1987), Olive and Scally (1982), Teppo (1991), Usiskin (1982), and Wirszup (1976). The then new experimental Soviet geometry eight-year curriculum was reported by Wirszup (1976, pp.94-95) as allowing ninety-five hours over the first three years of school to move students from visual recognition in grade 1 to being ready “to begin studying semiductive geometry (at Level III) in grade 4.” Similarly, in the USA, if geometry were studied continuously, Usiskin (1982, p.6) maintained that the time needed to take students from Level 1 to Level 3 would amount to half a year of lessons. Teppo (1991, pp.212-213) also emphasised the considerable amount of time needed for the total development of students from Level 1 to Level 2 understanding in her series of activities in symmetry that lead students through the five teaching phases. Olive and Scally (1982, p.125) designed an eighteen-week geometry course for ninth-graders using the computer program LOGO as a teaching tool. The reflective processing thus encouraged over the eighteen-week period was considered necessary for the construction of the higher-order concepts involved in formal geometry. Also using the computer as a teaching tool, Bobango (1987) created sixteen microcomputer lessons for her investigation into the possibility of raising students’ levels of geometric thought by means of phase-based instruction.

The time required for developing understanding in any one concept can be broken down into the time needed for exploring each feature of the concept. For example, the learning of properties of a rhombus requires the exploration of the separate features, the sides, angles, diagonals, symmetry, etc. Fifteen significant aspects that occur in geometric study were listed by van Hiele (1957, in Fuys, Geddes & Tischler 1984, pp.238-239). Each aspect needs to be given proper consideration in compiling a syllabus, and, hence, each separate aspect takes time to work through. Overall, commented Coxford (1968, p.330), the development of a cognitive structure is continuous, perhaps requiring a period of several years.

However, the desired cognitive development is not always achieved as students progress through a designed course. The Brooklyn College Project (Fuys, 1982) demonstrated that there was a resultant growth in geometric thinking in students, provided sufficient time was allowed for suitable instruction. The study was designed, in part, to determine sixth-grade students’ ‘potential’ level (the level demonstrated after instruction). Instructional models were developed to be used in six-to-eight forty-five-minute interview sessions, and instructional material was designed to develop topics that would move students towards the
next level. Growth in thinking was indicated in students’ responses as they moved through the learning experience in the interviews. All except the three weakest students demonstrated some growth in their geometric thinking during the period of the interviews. The researchers commented that three students failed to progress because “the interview schedule did not permit the time needed to carefully develop topics with these 3 students” (p.118), suggesting that the program used for the stronger students was possibly too rapid for the weaker students. This finding supports van Hiele’s statement (1959, in Fuys, Geddes & Tischler 1984, p.251) that “when one directs instruction too rapidly towards a mathematical relational system ... one risks losing mathematics forever.”

The time expended, no matter how long, can be unproductive, and the desired learning not achieved, if the instruction is not at a level compatible with the students. Usiskin’s report (1982) and Senk’s analysis (1989, pp.318-319) showed that students’ ability to succeed in a one-year proof-writing course in high school was linked to their entering van Hiele level. Very weak students with a low van Hiele level did not show any progress. This suggests that time itself is not sufficient for cognitive development, that instruction must be suited to the student’s level of understanding if the time spent is to be productive.

Understanding cannot be imposed upon the learner. To lead students through some learning cycle, such as the five phases, from one level to the next involves an appreciable amount of time. Not only cannot the process be hastened, the time spent can be unproductive if the instruction is not at a level appropriate to the student, or if the time spent is insufficient.

**Crisis in thinking and level reduction**

As students work through the last of the five teaching phases leading toward the higher level of thought, they should acquire an overview of the whole domain of thought that they have been exploring, thus initiating a crisis of thinking (van Hiele, 1986, pp.40-45). Progression of students to a higher level is not acquired without facing such a crisis in thinking. Writings which explore the manner in which a student may reach a crisis in thinking, ways of avoiding such a crisis, and the nature of level reduction are reviewed in this section.

A higher level of thinking is not reached through instruction from written materials alone. Direct instruction, exploration and reflection by the students lead them to the crisis in thinking which must be faced before a higher level can be reached. However, achievement of the next higher level is not automatic. In a personal communication, van Hiele indicated to Clements and Battista (1992, p.431) that “students cannot be forced to think at a higher level, but will themselves determine when the moment to go to the higher level has come.” If the
teacher and the students are relating to one another, the students’ reactions will show the teacher how, and to what extent, they are absorbing the subject matter, and the teacher will know how to bring about a further increase of insight (van Hiele, 1957, in Fuys, Geddes & Tischler 1984, p.241). The interaction between the teacher and students is important in supplying students with a necessary and sufficient amount of help so they can achieve the maturation essential for growth to the next level. At each level, students must become aware of what is expected, so they can perform competently and intentionally.

Malan (1986), in his investigation in which he attempted to lead children who were still partition classifiers to being hierarchical classifiers, showed also that students cannot be forced into a crisis of thinking. Of the eight students in this category, only three were able to be led towards hierarchical thinking. The sample transcriptions of the remaining students show a firm commitment to partition classification, suggesting a lack of readiness to progress to the higher level of classification.

Development of thought cannot occur if the crisis is avoided. Students must be allowed to integrate their knowledge into a coherent network, otherwise they will not face the crisis of thinking necessary for a new level of thought to be attained. Teachers need to be aware of certain strategies that can help avoid the crisis in thinking, and, hence, inhibit growth. These include the learning of rules by rote (discussed later), and the application of algorithms that are not understood. This latter point leads to students simulating a higher level of thought, referred to by van Hiele (1986) as level reduction. If the teacher offers an explanation, he/she applies a reduction of level, i.e., a structure at a higher level has been re-interpreted at a lower level. The students will have learned the structure rather than have surmounted a crisis of thinking. Hence the students have remained at the lower level.

Level reduction is brought about by transforming structures of the theoretical level with the help of a system of signs before a network of relations has been formed, thus making the structures more visible (van Hiele, 1986, pp.40-44). An example of this is seen when the symbols F or Z are used to help students ‘understand’ corresponding and alternate angles. Such a procedure can be counterproductive in that it removes the stimulus for students to seek insight. Fuys (1985, p.461) reported that the Brooklyn Project found that US textbook material on geometry for Grades 4 to 6 may impede progress by not only concentrating on Level 1, but also reducing much of the Level 2 and 3 material, thus providing very little opportunity to make progress. In particular, the textbooks provided very little Level 2 experience. Mason (1989, p.12) observed that textbooks frequently illustrated figures in the
prototype position, i.e., with sides parallel to the sides of the page, thus failing to help students have insight into the different orientations and sizes of figures.

Pegg and Davey (1989, p.19) were concerned that prompting during an interview could have a ‘level reducing’ quality that could lead to artificially high performances by students. Hence, their procedure when investigating the level descriptors of the van Hiele theory, differed from other studies (e.g., Fuys et al 1985) in that there was not a teaching role, lest the associated prompting may have caused some students to function at a much higher level than they could achieve on their own.

Hence, if students’ cognitive development is to grow, then they must face a crisis in thinking before they can reach a higher level of understanding. However, this cannot be forced upon them before they are ready. Level reduction, the simulation of a higher level, provides a method of avoiding a crisis, does not promote cognitive growth, and can impede development in understanding.

Rote learning
The role of rote learning in assisting or hindering the development of knowledge is unclear. While rote learning, when self-initiated, can promote growth in understanding, it can also, when imposed upon students, lead to the avoidance of a crisis in thinking. In this section, after exploring van Hiele’s ideas on the value of rote learning, the observations of other researchers are noted.

Van Hiele directed that the value of rote learning lies in its correct inclusion in the five teaching phases, and that it is not be considered a substitute for cognitive development. Teaching students by having them memorise facts, i.e., having them apply routine algorithms without understanding, is a form of level reduction and may not promote development of ability to reason. For example, he communicated to Mayberry (1981, p.53) that the learning of a definition by heart does not require any application of reasoning, and, therefore, is not an indication of any level of thinking. Van Hiele (1986) observed that rote learning occurs when the teacher has neglected to let the students discover how to explore new domains of thought by themselves. However, rather than discard it completely, he emphasised the importance of ‘ordinary learning’ or memorisation. At the end of integration, the fifth phase in the teaching cycle during which the child has procured a survey of the various thinking paths, van Hiele (p.177) noted that the student should memorise the necessary rules, such as tables of multiplication.
Several researchers have written about causes leading to, and effects resulting from, rote learning. For example, if a teacher is using language at a higher level than that of the students, Bobango (1987, p.6) commented that the resulting lack of communication may lead to students memorising facts and proofs instead of gaining an understanding of concepts and logical reasoning. Again, Fuys (1985, p.458) observed that students frequently knew information, such as the angle sum of a triangle or area rules for rectangles and triangles, by rote rather than by inductive or deductive explanations. Vygotsky (1934/1986, p.228) summarised:

Direct teaching of concepts is impossible and fruitless. A teacher who tries to do this usually accomplishes nothing but empty verbalism, a parrotlike repetition of words by the child, simulating a knowledge of the corresponding concepts but actually covering up a vacuum.

Clements and Battista (1991, p.229) found that “after being taught about the properties of squares and rectangles, if asked why they say that squares are special kinds of rectangles, many first grade students say simply “because the teacher told us”.” Stating that squares are a special kind of rectangle is not an indication that these students have mastered the class inclusion of Level 3 thinking, but that they have rote learned that statement. This leads to the observation that progress depends on the methods of instruction received. While some methods enhance progress, others retard or even prevent it. In particular, if learning is to occur, teaching must be relevant to the level of the students. “One can train young children in the arithmetic of fractions without telling them what fractions mean ... or geometric relationships like a square is a rectangle. In situations like these ... understanding has not occurred” Crowley (1987, p.4).

**Overview of properties**

In general, there is a consensus among researchers concerning most of the properties and features of the van Hiele levels. The main aspects of the findings of the researchers will be viewed in relation to Steffe and Cobb’s (1988) four criteria for the existence of levels. The four criteria are constancy, incorporation, order invariance, and integration.

Whereas a few writers have indicated the need for research into hierarchy, researchers have generally supported the hierarchical nature of the levels, thus agreeing that there is order invariance in the levels. Closely associated with the notion of hierarchy is the intrinsic/extrinsic nature of acquisition of knowledge. The incorporation and accompanying restructuring of the knowledge of the earlier level into a new, higher level, i.e. the intrinsic knowledge of one level becoming extrinsic to the learning of the next, reinforces the hierarchical structure of the levels, ensuring a sequential and constant order. On these two
features depend the other features, i.e., the discreteness of the behavioural characteristics for each level, the differences in language used and perceptions at different levels, different levels for different concepts, and the features associated with growth between levels, i.e., the need for time, crisis in thinking, and level reduction.

While some researchers consider the levels to be discrete, i.e., that there is a discontinuity in the learning curve between the levels, recent research tends to support the notion of the levels being of a more continuous nature, i.e., that students are able to display some degree of mastery of more than one van Hiele level. However, neither of these views conflict with Steffe and Cobb’s (1988) criteria. If the behavioural definitions of the levels are constant, and their characteristics are integrated, then they reflect discrete stages of knowledge, i.e., the levels are discrete in their behavioural expectations. However, this discreteness of the behavioural expectations does not prevent students from exploring the next higher level, once they have mastered a majority of key characteristics, thus allowing for a more continuous nature in the learning.

If the behavioural definitions of the levels are constant and their characteristics have an integrated nature, then several features will display changes as students move from one level to the next higher level. Researchers have demonstrated this in the use of, and different interpretation in language by students at different levels of understanding. Such students have also been shown to interpret symbols and structures differently for different levels, i.e., they have different perceptions at different levels. It is important for teachers to recognise these two features least they use language or perception of a concept that is at a level different to that of the students, and, hence, there is a break in communication between students and the teacher.

Associated with these notions of changes in language and perception with different levels is the feature that students have been found to be at different levels for different concepts, on occasions being at a higher level for more familiar concepts. This indicates that the integrated characteristics in a behavioural definition are specific to each concept. However, research shows also that once a student achieves a new level for one concept, similarities between the reasoning expected at that level allow for a more rapid acquisition of the level for other related concepts. The similarities within a level are dependent upon the constancy of the level definition, i.e., the expectations in understanding across different concepts for the same level are consistent.
That the van Hiele levels demonstrate all four of Steffe and Cobb’s (1988) criteria, indicates that movement from one level to the next higher level is not automatic, but a complex series of procedures. The five teaching/learning phases which are part of this procedure, have been described by van Hiele-Geldof (1957, in Fuys, Geddes & Tischler, 1984). If students are to grow through the five phases and achieve the next higher level, time is needed. Several researchers have demonstrated the considerable amount of time required for students to explore and reflect on ideas, to lead them through the five phases of this process, and, finally, bring them to a crisis in thinking which must be experienced to reach the next higher level. However, researchers have also shown that a higher level can be simulated through the application of certain strategies, such as level reduction and learning by rote, i.e., application of algorithms that are not understood, and the learning of rules by rote. Teachers need to be aware that a crisis in thinking can be avoided through use of such practices, and that such use can inhibit growth in students, preventing them from achieving a higher level of thinking. Researchers need also be aware that level reduction, and, hence, simulation of a higher level could result from prompting in interviews.

In conclusion, there appears to be general agreement regarding, and empirical evidence to support properties and features associated with the van Hiele levels. In particular, all properties and features have been shown to relevant to Steffe and Cobb’s (1988) four criteria for levels, and, hence, support the notion that the van Hiele levels do represent discrete stages of major knowledge reorganisation.

Research Designed to Use Knowledge of the Levels and Discuss their Properties
Research based on the van Hiele theory has examined both descriptive and predictive aspects of the theory. The research can be categorised into two broad groups. The first group consists of studies in which test items were developed which could be used to determine students’ van Hiele levels of thinking. In the second group of research studies, the researchers’ understanding of the levels and their properties has formed the basis for further investigation of the van Hiele theory. In some studies, specific instructional materials have been developed.

Research involving test materials
As mentioned in the previous chapter, the presentation of Wirszup’s (1976) paper in the United States aroused much interest in the van Hiele theory, and was the genesis of much research. This immediately gave rise to several related issues. In particular, the question arose as to whether a student’s van Hiele level could be determined. If this were so, what kind of instrument would be suitable for measuring a student’s predominant level of
reasoning? Several of the early projects developed a series of test items that were then used to produce empirical evidence, either in support of the existence of the van Hiele levels themselves, or to investigate a particular feature of the levels. These have since been used by many other researchers as a basis for their research.

Five instruments designed specifically to assess students’ van Hiele levels were developed by several investigators (e.g., Burger & Shaughnessy 1986; De Villiers & Njisane 1987; Fuys et al 1985; Mayberry 1981; Usiskin 1982). Mayberry, Fuys et al, and Burger and Shaughnessy developed items to be used in an interview situation, while Usiskin, and De Villiers and Njisane developed written test instruments. The different characteristics displayed by the various tests are detailed below.

Mayberry (1981) developed a set of sixty-two items to be used in interviews. Each item was designed to examine a student’s ability to reason at a specified level. A student was said to have mastered a level on a particular topic if the student reached a critical score set in the test for that level and topic. The first four van Hiele levels were tested over seven geometric topics. “These concepts were chosen because they occur in the elementary curriculum and, therefore, seemed essential to the geometric understanding of the potential teacher” (p.49). The fifth level, the discernment of mathematics, was considered to be topic-free. Experts in the fields of mathematics and mathematics education, among them Pierre van Hiele, were asked to validate the items by judging whether the items satisfied certain criteria (p.53). Each interview took approximately two hours. Mayberry found that a van Hiele level of thinking could be assigned to students for each topic tested, but that a student did not necessarily demonstrate the same level of thought processes across different topics (p.92).

By comparison, Usiskin’s (1982) project set out “to test the ability of the van Hiele theory to describe and predict the performance of students in secondary school geometry” (p.8). To achieve this, a thirty-five-minute written test to identify the van Hiele levels of students was designed. The test consists of twenty-five multiple-choice items across a number of geometric concepts, with five items set to test each of the five levels. Criterion for each van Hiele level is at least three correct answers for each group of five questions. Many questions were first constructed, each question being designed to the very extensive list compiled by Usiskin (p.9), of behaviour characteristics for each van Hiele level. The questions were piloted with students in oral interviews (p.19). Using the responses from the students, the multiple-choice items were created. Usiskin found that with his test, “the vast majority of students can be assigned a van Hiele level”, and that “the levels assigned to students are a good descriptor of concurrent student performance in geometry” (p.89).
Another instrument for use in an interview was developed by the research team of Fuys, Geddes, and Tischler (1985) as part of their project. Their instrument, designed to be used in a one-to-one situation, consists of three modules of activities in which assessment tasks are integrated with instructional materials. This results from Fuys et al’s notion that if a student’s prior learning is suspect, then an assessment of the student’s ‘initial’ level of thinking may be an underestimation of ability to think in geometry, and, hence, an assessment of the student’s ‘potential’ level of thinking following instruction, may be more accurate (p.239). This study, since it is mainly concerned with student development through the levels, will be detailed in the next section.

The test developed by Burger and Shaughnessy (1986, pp.33-36) consisted of eight tasks requiring the drawing, identifying, defining, sorting, and informal and formal reasoning about geometric shapes (triangles and quadrilaterals). Burger and Shaughnessy kept the tasks open-ended, feeling that an opportunity to probe was necessary in establishing a student’s van Hiele level. The test was designed to identify understanding of the first four van Hiele levels. No attempt was made to use the tasks to investigate the fifth van Hiele level, “a level that requires the ability to compare different geometries” (p.34). Validation of the tasks was achieved through a developmental series consisting of preparation, pilot interviewing, analysis and revision. The developmental series was repeated several times. Interviews with the final version of the tasks took between 40 and 90 minutes. The researchers found that from the students’ responses to the tasks, predominant van Hiele levels of reasoning were able to be assigned to the students (p.42), behaviour among students at the same level was consistent, and the results helped characterise the levels operationally (p.47).

The written test instrument developed by Dc Villiers and Njisane (1987) as part of the Research Unit for Mathematics Education at the University of Stellenbosch (RUMEUS test) consisted of fifty-three questions, testing understanding in five geometric concepts over four levels. The nature of the questions ranged from simple yes/no items to those requiring construction of a formal proof. Several of the questions were of an open-ended nature, the responses being classified according to the level displayed in the answer. The test was used by Smith (1989) to make a theoretical and empirical comparison between two written test instruments that could be used to determine van Hiele levels of thinking. The second test was a slightly modified version of Usiskin’s test. Twenty-six multiple-choice items covering three concepts were used to test the first four van Hiele levels. The scoring scheme for both tests was similar to that devised by Mayberry. Results in which students who mastered Level \( n \), but not all the preceding levels, were considered unclassifiable. Both tests were found to be useful for the allocation of students’ van Hiele levels. More results were able to be
It is clear from the literature that there is a perceived need for reliable instruments that assess the van Hiele level of understanding of students in geometry, and that there are few instruments to choose from (Usiskin & Senk 1990, p.242). Whereas it is important to be able to determine a student’s van Hiele level of thinking with some degree of accuracy, the format of the test materials appears to need further investigation. Interviews, especially those in which the questions are of an open nature may give a more complete result than a written test, however, they are time-consuming to administer, and, hence, not suitable for the assessment of a large population. Written tests are more practical for such groups, yet the test developed at the University of Stellenbosch and Usiskin’s multiple-choice test appear to be the only instruments available upon which empirical data are readily available.

Further studies
Several researchers (e.g., Bobango 1987; De Block-Docq 1992; Fuys et al 1985) have used their understanding of the levels and their properties to investigate further the van Hiele theory. Some (Crowley 1987; De Block-Docq 1992; Fuys et al 1985; Nasser 1990) have investigated the effectiveness of instructiona material that can be used to lead students through the phases of learning, towards the next van Hiele level. Others (Bobango 1987; Chaiyasang 1988; Denis 1987; Mason 1989) have used test items already developed and validated, as part of their further investigations.

The Geddes team (Fuys et al 1985) noted that, according to the van Hieles, thinking levels are determined in part, by prior learning experiences, and hence, ‘initial’ assessment may not accurately assess students’ ability to think in geometry. As part of their project, the team developed three assessment/instruction modules to provide a context for interviews “which could shed light on the student’s level of ‘thinking, cognitive processes, and learning difficulties that may adversely affect thinking in geometry’” (p.16). The objective was to determine whether students revealed a more realistic ‘potential’ working level as they progressed through instructional activities during the interviews. The activities were correlated with specific van Hiele level descriptors and were based in part, on van Hiele-Geldof’s material in her doctoral research (p.2). Other tasks were similar to those used by Burger and Shaughnessy (1986). The geometry topics treated were properties of quadrilaterals, angles and angle sums for triangles and quadrilaterals, and area of rectangles, parallelograms and triangles (Fuys 1985, p.455). Each student was interviewed for between six and eight 45-minute sessions. The results indicated that, for many students, the level as
assessed by their responses during and after instruction was significantly higher than their ‘entry’ or ‘initial’ level, suggesting that their ‘potential’ level, i.e., their ability to think in geometry would not have been accurately assessed in a test that did not involve instruction (Fuys et al. 1985, p.238).

Crowley (1987), in contrast, focused on the teaching phases when she designed a set of instructional activities to develop students’ ability to think. She developed a useful series of exploratory activities, resulting from her conviction that children should be presented with a wide variety of geometric experiences if they are to grow through the van Hieles levels. The series of activities were designed to fit into the five developmental phases. Crowley emphasised the importance of the activities being placed in context. To develop a student’s thinking, an activity needs, not only to be appropriate to the student’s van Hiele level, but also, to be placed in context to the student’s learning phase within the level. If the concept that is the subject of exploration in an activity is too difficult for the student to follow, there is the risk that level reduction will occur.

Nasser (1990) also designed her own activities for her investigation into the van Hiele levels of thinking in congruency by English and Brazilian secondary school students. The work was part of an investigation to determine whether the learning and understanding of ‘congruence’ by Brazilian secondary school students could be improved when the instruction is based on the van Hiele theory (p.297). The tasks were open-ended in nature, allowing for responses to be evaluated for the van Hiele level demonstrated therein. As well as developing level descriptors for congruency and the appropriate activities, Nasser described sets of expected responses, classified according to van Hiele levels. Despite the different approach to the activities by the English and Brazilian students (resulting from the different teaching methods), their responses were found to fit the level descriptors. Since the students responding at a certain level also gave correct answers to questions at lower levels, the study supported the hierarchical nature of the levels (p.302).

In contrast to the researchers above, who designed their own instructional materials, De Block-Docq (1992, 1994), in her study, replicated with a similar group of students, the set of seventy lessons Dina van Hiele-Geldof (1957, in Fuys, Geddes & Tischler, 1984) found necessary to move 12-year-old students from Level 1 to Level 3. De Block-Docq’s aim was to compare the outcomes resulting from the two situations. However, while the van Hiele theory formed a common basis to the two situations and the content was the same, the teaching situation experienced by each group of students and the results of the two studies were found to differ. Van Hiele-Geldof used the theme of tiling as a tool with which to
develop the importance of deduction, and to verify the already existing theory of levels through their hierarchical attainment. In contrast, De Block-Docq encouraged her students to use deduction spontaneously in resolving the problems presented to them in the context of a natural situation. In her lessons, she favoured the spontaneous elaboration of conjectures, and the perspective of looking towards infinite grouping of objects. De Block-Docq found that an aim of attainment of levels, if allowed to dominate instruction, could limit the maturation of mathematical thinking.

While the above studies have all focused on specifically designed teaching materials, other studies have incorporated test materials already developed and validated into their further investigations into the van Hiele theory. The very many (over one hundred) investigators receiving permission to use Usiskin’s multiple-choice test in a research study (Usiskin & Senk 1990, p.242) include the two researchers, Bobango (1987) and Chaiyasang 1988. In contrast, Denis (1987) evaluated van Hiele levels using Mayberry’s test items, while a fourth researcher, Mason (1989), used both Usiskin’s and Mayberry’s tests to determine van Hiele levels.

Bobango (1987) designed a set of phase-based lessons for use on a computer, to investigate whether students’ van Hiele levels of geometric thought could be raised as a result of such instruction, in preparation for a formal proof course (p.11). The effectiveness of the teaching was measured by assessing both the experimental and the control students’ van Hiele levels before and after the courses of instruction. The lesson instructions were given over a period of twenty school days by a teacher whose major role was the guiding of the discussion segments of the lessons. The exercises, designed by the researcher as part of the lessons, used microcomputers and Geometric Supposer programs as tools. A control group worked with another computer program for the same twenty days, but there was no planned class discussion associated with the lessons (pp.53-59). Some students were interviewed to confirm van Hiele levels determined through Usiskin’s test (p.62). The researcher found that the instruction did raise the van Hiele levels for a significant number of students in the experimental group compared to those in the control group, particularly, the students whose pre-test showed their understanding to be at van Hiele Levels 1 or 2.

In a similar manner, Usiskin’s test was used to evaluate the effectiveness of the radically new geometry syllabus which had been introduced into Thailand in 1978. Chaiyasang (1988) used the first twenty multiple-choice questions of Usiskin’s test to measure the van Hiele levels of geometric thinking of students. He tested 3047 students from grades 6 to 9. The results regarding the effectiveness of the new course were disappointing, showing that more than
50% of the students for each of grades 6 to 9 were at Level 1, and that there was little progress in levels of geometric thinking as students moved through the grades. However, Chaiyasang’s study showed that application of Usiskin’s test resulted in allocation of van Hiele levels for over 90% of the students, their results giving evidence that they passed through the van Hiele levels in order (pp.127-128).

While the above studies used Usiskin’s test to evaluate the effectiveness of courses, Denis (1987) used Mayberry’s interview items to investigate the relationship between Piagetian stage of cognitive development and van Hiele level of geometric thought among a selected group of Puerto Rican adolescents. Denis (p.47) chose Mayberry’s test in preference to Usiskin’s for two reasons, because it was of an interview format, recommended by van Hiele, and, because the items were designed for specific concepts, allowing for students to be at different van Hiele levels for different concepts. Following division of the students into two groups, those at the Piagetian concrete-operational stage and those at the Piagetian formal-operational stage, a random selection of forty students was interviewed to determine their van Hiele levels of thinking (p.45). For this process, Mayberry’s questions pertaining to the four concepts with the highest rate of reproducibility, i.e., circle, congruency, right triangle, and square, were used, thus reducing the time required for each interview (p.48). Denis found that the highest van Hiele level of the students at the formal-operational stage was significantly higher than the highest van Hiele level of the students at the concrete-operational stage (p.88).

The interview-based items designed by Mayberry were used also by Mason (1989) to validate the van Hiele levels determined pre- and post-operationally by means of Usiskin’s multiple-choice test. Mason designed a twenty-hour course in geometry, using the computer program LOGO, when investigating the geometric understanding and misconceptions of gifted students in grades 4 to 8. Most of the van Hiele levels of understanding of the students assessed using Usiskin’s test, were confirmed through a 30-to-45 minute interview based on Mayberry’s items. The tests and interviews covered understanding of the seven topics used by Mayberry in the preparation of her items. The difference in the van Hiele levels determined by the pre- and post-tests was found to be significant (p.3). Mason stated that the post-test levels tended to support Dina van Hiele’s statement that twenty hours of instruction may facilitate movement from one level to the next. Other findings resulting from Mason’s study included misconceptions resulting from the influence of textbook-oriented or prototype figures, difficulty in focusing on angles, and a tendency to focus on the non-critical attributes of a figure. The results confirmed Mayberry’s rejection of the hypothesis that a student exhibits the same level of understanding across all concepts in geometry. Mason assessed the
gifted students as frequently indicating ability to reason at van Hiele Levels 3 and 4, while not demonstrating mastery of lower levels. Mason suggested that this apparent divergence from the van Hiele theory may be due to their lack of exposure to formal geometry instruction (pp.12-14).

The evaluation methods developed for their tests by Mayberry and Usiskin are based on the premise that the van Hiele levels are discontinuous. This has resulted in each question being designed to test for a specific van Hiele level, and hence, each response being assessed for its meeting the specified level. Other researchers have used questions of an open-ended nature, assessing each response for the van Hiele level displayed (Burger & Shaughnessy, 1986; Nasser, 1990). This has allowed for development of the notion that the van Hiele levels are of a more continuous nature, and that some students' responses reflect more than one level of thinking. This notion forms the basis for the alternative method of evaluating students' responses designed by Gutiérrez, Jaime and Fortuny (1991). Each response is quantitatively assessed for the level and type of answer displayed, resulting in a more encompassing evaluation of the degree of mastery of each van Hiele level for the student.

Research Questions and Themes

The issues in the literature have shown that there is strong support for the existence of the van Hiele levels, and, in general, for the properties and features of the van Hiele theory. However, there is still some debate whether the levels are discrete or continuous, and how many levels exist. Research has demonstrated that the behavioural characteristics of the levels can be defined, and that the level(s) at which a student is thinking can be evaluated according to these definitions. Several researchers have developed test instruments that can be used to determine the students' van Hiele levels of understanding geometry. Some of the tests have been designed for interview situations, others are written tests. While interviews provide opportunity to ensure questions are fully understood by students, and, hence, the reliability of the results is high, they are time-consuming and not suitable for application to large groups of students. This issue, the conflict between keeping a test brief and focusing on the reliability of the test was raised in the literature.

Of the instruments reviewed, the Mayberry (1981) study stands out as being important. She defined the levels operationally, and applied the levels across a series of topics. The seven topics were chosen because they occur in the elementary curriculum and, therefore, seemed essential to the geometric understanding of the potential teacher (p.49). The test items were designed to cover each level for every topic, and to match the operational definitions of the levels. The items were validated by experts in the fields of mathematics and mathematics
among them Pierre van Hiele, who were asked to confirm whether the items satisfied certain criteria (pp.52-57). Mayberry designed her items to be used in interviews, each interview taking approximately two hours. This limited the number of students who could be evaluated for their understanding of geometry. Her marking scheme was devised to match the design of her test, and, hence, the response to each item was evaluated solely for its expression of knowledge of the level for which the item was designed to test.

In her article (1983) based on her doctoral dissertation (1981) in which she summarised the main points of the research, Mayberry acknowledged certain design problems associated with her investigation.

This study was limited by several factors. First, the choice of representative questions for each thought level was difficult and needs further refinement. Second, the sample of students was homogeneous, with too many response patterns being on the lower two levels to demonstrate the five van Hiele levels adequately. Third, the determination of the success criterion for a given topic and level was rather subjective. Finally, the small sample size necessitated by the interview technique certainly limited the generalizability of the results.

(Mayberry, 1983, p. 68)

These factors identified by Mayberry are particularly significant and need to be addressed. A study to research the four main issues would provide valuable information in knowing more about the van Hiele theory. However, the context of Mayberry’s test was that the levels are discontinuous. Several researchers (Burger & Shaughnessy 1986; Fuys, Geddes & Tischler 1985; Gutiérrez, Jaime & Fortuny 1991; Pegg & Davey 1989; Usiskin 1982) have indicated that this is not the case, and that the levels are discontinuous. Gutiérrez, Jaime and Fortuny (1991) went further and sought to identify evidence of different levels within a single response. Hence, it would be of interest to investigate how the Gutiérrez, Jaime and Fortuny (1991) assessment method, coming from a different perspective of the nature of levels, might impact on the evaluation of the Mayberry items. Consequently, it seemed valuable both in a practical and theoretical sense, to design a study to address the following general issues:

1. to refine, and analyse more deeply, the questions used by Mayberry;
2. to investigate the understanding of geometric topics of students drawn from a greater diversity of geometric backgrounds;
3. to evaluate Mayberry’s assessment system in detail, looking at possible alternatives; and
4. to change the format to a written paper, allowing testing of a larger sample.

The above four issues led to the formulation of four research themes:
1. How does a sample of Australian students training to be primary teachers perform on a written test version of the Mayberry questions?

2. Can a quantitative analysis of the results using a partial credit model, offer insights into the nature of the Mayberry test?

3. Do the Mayberry items measure the van Hiele levels for which they have been designed? Are the success criteria established by Mayberry, valid?

4. What implications can be drawn when an alternative assessment system, such as the method used by Gutiérrez, Jaime and Fortuny, which does not have the discreteness of levels as a significant feature to influence the findings, is applied?

In developing the research design, several considerations were identified from the review of literature and by the demands implicit in the research themes. These can be summarised as:

1. The Mayberry test needs to be modified to enable a deeper insight into the representative nature of the questions, and to explore possible alternative interpretations of what level is tested by each Mayberry question.

2. The sample should be selected to contain students of non-homogeneous geometric experience, e.g., ranging from students who have recently completed the NSW Higher School Certificate or equivalent with Level 2 or higher geometry, to students who have little or no secondary geometric experience.

3. The success criterion for each concept and level should be analysed in association with the results and the occurrence of the response pattern errors, and the Gutiérrez et al scaling investigated as an alternative perspective.

4. A written test based on Mayberry’s interview questions would be set allowing for a larger sample size and enabling test-time to be reduced. Back-up interviews would be conducted to validate the results of the written test.

The following chapter considers these design aspects of the study in detail.