



**AN INVESTIGATION INTO THE ASSESSMENT
OF STUDENTS' VAN HIELE LEVELS OF
UNDERSTANDING IN GEOMETRY**

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Certificate

I certify that the substance of this thesis has not already been submitted for any degree and is not being currently submitted for any other degree.

I certify that any help received in preparing this thesis, and all sources used, have been acknowledged in this thesis.

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(Signature)

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ABSTRACT

This study investigated the difficulties associated with the design of appropriate test items, and the advantages of different methods of assessing the understanding of students within the framework of the van Hiele theory. It is based on the diagnostic instrument developed by Mayberry (1981) for assessing van Hiele levels in geometry, and addresses the four design problems which Mayberry identified as associated with her study.

A detailed testing and interview program of 60 first-year primary-teacher trainees was undertaken. Initially, the responses of students to a written version of the Mayberry items were assessed using her method of evaluation. The QUEST program (which utilises the Rasch measurement theory) was applied to the results, leading to the identification of design problems in the original test, and the development of an amended version of the Mayberry test. Finally, Mayberry's method of assessment was compared with the method developed by Gutiérrez, Jaime and Fortuny (1991). The testing of four research themes provided the context for the investigation.

The highest van Hiele level of understanding for the majority of students (77%) was shown to be Level 2, even though 60% had studied a senior secondary geometry course in which the instruction was assumed to be at least Level 3 content. Apparent inconsistencies in the thinking of some of the students also emerged. Interviews, while supporting the levels displayed by the students in the test, did not clarify reasons for the inconsistencies. Application of the QUEST program confirmed the results of the test. Two issues related to the inconsistencies emerged. First, the difficulty thresholds of some items indicated their unsuitability to measure reasoning at the specified thought levels. Second, the setting of the subjective success criteria for the Mayberry method of assessment did not appear to be consistent.

Analysis of the inconsistencies led to the identification of four features in the Mayberry design which have the potential to lead to an incorrect assessment of a student's level of understanding in geometry. An amended version of the test, which retained the essence of the original Mayberry instrument, yet eliminated the difficulties caused by two of the identified design faults, was developed. The re-assessment of the students' responses using the amended version of Mayberry's coding system confirmed that the students' highest van Hiele level of understanding geometry was Level 2, as initially determined.

This re-assessment of the students' responses did not rectify all inconsistencies. Hence, the need to investigate another form of interpreting results was recognised. The responses were,

therefore, assessed again using the alternative method developed by Gutiérrez *et al.* This third assessment gave a more complete and realistic evaluation of the students' van Hiele levels of understanding geometry, and addressed the remaining inconsistencies which had emerged in the Mayberry analysis. The evaluation of two coding systems, namely, that of Mayberry, dependent on discreteness of the van Hiele levels, and that of Gutiérrez *et al.*, based on levels having a more continuous nature, provided a comparison of the two methods, and indicated the advantages of each.

Overall, in addressing Mayberry's four design problems, this investigation provided insight into the determination of students' van Hiele levels of understanding geometry. In particular, it allowed a deeper understanding of the van Hiele theory, and provided finer detail about what it means to work at a particular van Hiele level.