PERIODIC-PARABOLIC LOGISTIC EQUATION

WITH

SPATIAL AND TEMPORAL DEGENERACIES

By

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DECLARATION

I certify that the substance of this thesis has not already been submitted for any degree and is not currently being submitted for any other degree.

I certify that to the best of my knowledge, any help received in preparing this thesis, and all sources used, have been acknowledged in this thesis.

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Publications

Journal papers published

(1) Yihong Du, Rui Peng, Mingxin Wang, Effect of a protection zone in the diffusive Leslie predator-prey model, Journal of Differential Equations, 246 (2009), 3932-3956.

(2) Rui Peng, Junping Shi, Non-existence of non-constant positive steady states of two Holling type II predator-prey systems: strong interaction case, Journal of Differential Equations, 247(2009), 866-886.

(3) Rui Peng, Asymptotic profile of the positive steady state for an SIS epidemic reaction-diffusion model–Part I, Journal of Differential Equations, 247(2009), 1096-1119.

(4) Rui Peng, Dong Wei, Guoying Yang, Asymptotic behavior, uniqueness and stability of coexistence states of a non-cooperative reaction-diffusion model of nuclear reactors, Proceedings of the Royal Society of Edinburgh: Section A-Mathematics, 140(2010), 189-201.

Papers submitted/in preparation

(1) Yihong Du, Rui Peng, The periodic logistic equation with spatial and temporal degeneracies, submitted.

(2) Rui Peng, Dong Wei, The periodic-parabolic logistic equation on $\mathbb{R}^N$, submitted.

(3) Yihong Du, Rui Peng, Spatial-temporal patterns in the degenerate periodic logistic equation, in preparation.

Academic activities

(1) I attended the International Conference on Partial Differential Equations and Related Topics, Wuhan Institute of Physics and Mathematics, Chinese Academy of Sciences, Wuhan, China, October 25-28, 2008.

(2) I gave a 40 minutes invited plenary talk in the International Conference on Asymptotic Analysis and Infinite-dimensional Dynamical Systems, City University of Hongkong, Hongkong, June 19-22, 2009.

Talk title: Positive steady state for an SIS epidemic reaction-diffusion model.

(3) I gave a 45 minutes invited plenary talk in the International Workshop on Reaction-Diffusion Models and Mathematical Biology, Harbin Institute of Technology and Harbin Normal University, Harbin, China, June 24-27, 2009.

Talk title: Effect of a protection zone in the diffusive Leslie predator-prey model.

(4) I gave a 30 minutes contributed talk in the Pacific Rim Mathematical Association (PRMA) conference, University of New South Wales, Sydney, Australia, July 6-10, 2009.

Talk title: Non-existence of non-constant positive steady states of two Holling type II predator-prey systems: strong interaction case.

(5) I gave a 30 minutes contributed talk in the 53rd Annual Meeting of the Australian Mathematical Society, University of South Australia, Adelaide, Australia, September 28-October 1, 2009.

Talk title: Effect of a protection zone in the diffusive Leslie predator-prey model.

Abstract

A basic goal of theoretical ecology is to understand how the interactions of individual organizations with each other under a certain inhabiting environment determine the spatiotemporal structure of distribution of populations. The diffusive logistic equation, which describes the spatial and temporal distribution of the population density of a single species, is one of the fundamental reaction-diffusion equation models in population biology.

In this thesis, we are concerned with the periodic logistic equation with homogeneous Neumann boundary conditions. The theory for this basic case forms the foundation for further investigation of multispecies problems. The thesis consists of four chapters.

In chapter 1, we introduce some notations and recall the related theory of *Sobolev Spaces*. We also collect some fundamental theories for partial differential equations, and recall the definitions of sub/super solutions and the theory on principal eigenvalue of the periodic-parabolic problems. Finally, we recall the well-known Krein-Rutman theorem. These preliminaries will be frequently used in the subsequent chapters.

In chapter 2, we first introduce the biological background of the logistic equation. We then recall the existing research works on the logistic equation with spatial degeneracies and the periodic-parabolic logistic equation. As an ending of this chapter, we outline our investigation in this thesis.

In chapter 3, we study the periodic logistic equation:

$$\begin{cases} \partial_t u - \Delta u = au - b(x,t)u^p & \text{in } \Omega \times (0,\infty), \\ \partial_\nu u = 0 & \text{on } \partial\Omega \times (0,\infty), \\ u(x,0) = u_0(x) \geq, \not\equiv 0 & \text{in } \Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^N$ $(N \geq 2)$ is a bounded domain with smooth boundary $\partial\Omega$, a and $p > 1$ are constants. The function $b \in C^{\theta,\theta/2}(\overline{\Omega} \times \mathbb{R})$ $(0 < \theta < 1)$ is T-periodic in t and nonnegative on $\overline{\Omega} \times \mathbb{R}$.

This equation describes the population density $u(x,t)$ of a single species with initial density $u_0(x)$ and intrinsic growth rate a in a habitat Ω that has carrying capacity a/b. The Neumann boundary condition means that the species is enclosed in Ω with no population flux across its boundary $\partial\Omega$.

Our main interest here is to examine the case that $b(x,t)$ vanishes in a proper subset of $\Omega \times \mathbb{R}$. We will call such a case a degeneracy in the logistic equation. The region where b vanishes represents the extreme environmental situation that the species experiences no self-limitation for its growth there. A good understanding of such an extreme case is important in order to understand the scope of the possible behavior of the model as the environment varies heterogeneously.

We examine the effects of various natural spatial and temporal degeneracies of $b(x,t)$ on the long-time dynamical behavior of the positive solutions. Our analysis leads to a new eigenvalue problem for periodic-parabolic operators over a varying cylinder and certain parabolic initial and boundary blow-up problems not known before. Our investigation shows that the temporal degeneracy causes a fundamental change of the dynamical behavior of the equation only when spatial degeneracy also exists; but in sharp contrast, whether or not temporal degeneracy appears in the equation, the spatial degeneracy always induces fundamental changes of the behavior of the equation, though such changes differ significantly according to whether there is temporal degeneracy or not.

In chapter 4, we consider the perturbed periodic logistic equation:

$$\begin{cases} \partial_t u - \Delta u = au - [b(x,t) + \epsilon]u^p & \text{in } \Omega \times (0,T), \\ \partial_\nu u = 0 & \text{on } \partial\Omega \times (0,T), \\ u(x,0) = u(x,T) & \text{in } \Omega, \end{cases}$$

and determine the asymptotic behavior of the positive periodic solution as $\epsilon \to 0$. This reveals how the model evolves as the environment approaches the extreme degenerate case.

Under the same assumption on $b(x,t)$ as in chapter 3, our conclusions show that the temporal degeneracy generates patterns only when spatial degeneracy also exists. Moreover, in sharp contrast with the case where only spatial degeneracy exists, the perturbed periodic logistic equation is capable of generating some very different spatiotemporal patterns when both spatial and temporal degeneracies occur.

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