

**APPLYING THE PROCEDURAL ANALOGY THEORY  
IN MATHEMATICS TEACHING**

*by*

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## DECLARATION

I certify that the substance of this thesis has not already been submitted for any degree and is not being submitted for any degree.

I certify that any help received in preparing this thesis, and all sources used, have been acknowledged.



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## ABSTRACT

This study involved research into cognitive processes and mathematics education. It investigated the Procedural Analogy Theory (Ohlsson and Hall, 1990), as a basis for designing instruction in mathematics, and for explaining the value of concrete materials in teaching arithmetic skills.

The use of concrete materials in the teaching of school mathematics has long been supported by mathematics educators and by teacher educators. Such materials are common in schools, but generally used by few teachers, their use tends to be largely with young children and then in idiosyncratic manners. The research literature has for many years reported contradictory findings about the effectiveness of concrete materials. The procedural analogy theory attempts to provide guidelines for the planning and comparison of pedagogies before their implementation in classrooms, particularly through the use of a formula to calculate an isomorphism index which seeks to predict the effectiveness of a given pedagogy.

In addition to being concerned with planning for group instruction, this research is concerned with individuals as learners. Luria's research, resulting in his identification of simultaneous and successive patterns of brain functioning, appears to be an appropriate way in which to examine individual differences in the context of the application of the procedural analogy theory in classrooms, because they both have their foundations in cognitive processing, and because simultaneous processing with its spatial emphasis seems particularly appropriate to a theory concerned with the use of two and three dimensional objects in teaching. Interactions between implementation of the procedural analogy theory and dimensions of cognitive processing were also studied.

Data were gathered using mathematics achievement tests, and tests of cognitive processing styles, from 112 nine and ten year old students who were arranged into three groups for this research, through systematically assigning names from the alphabetical class rolls. Qualitative data, gathered through videotaping lessons and semi-structured interviews, were used to verify that the teaching approaches were implemented appropriately, to gather fine-grained data on teacher talk and action, and on student talk and action, and to gain insights into student understandings.

Results give support to the value of the procedural analogy as a tool for both planning instruction and predicting learning outcomes, and demonstrate the importance of high level simultaneous processing for learning mathematics through concrete materials. These findings are discussed in the contexts of constructivism, good teaching and computer mediated learning in mathematics. And suggestions are made for both classroom practice and further research.

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