



ADULT LEARNERS' UNDERSTANDINGS OF FRACTION QUESTIONS

Kerryn A. Hayman

Bachelor of Science (La Trobe University)
Diploma in Education (La Trobe University)
Bachelor of Education (La Trobe University)

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DEDICATION

This thesis is dedicated to:

my parents ...

*who gave up so much to make sure that I had a good start in life,
and never lived to see the accomplishments;*

my husband ...

who never doubted that I could do it;

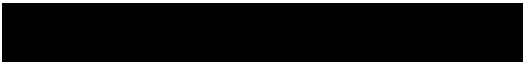
my baby son ...

who showed me what I am really doing this all for.

CERTIFICATE

I certify that the substance of this thesis has not already been submitted for any degree and is not being currently submitted for other degrees.

I certify that to the best of my knowledge any help received in preparing this thesis, and all sources used, have been acknowledged in this thesis.



Kerryn A. Hayman
31 March, 1998

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ABSTRACT

The main aim of this thesis was to investigate adult learners' understandings of fractions. A particular focus of this work was to determine if students responses to fraction questions could be grouped together on the basis of similarity of response; and, to explore the feasibility of such groupings into a notional hierarchy, such as the theoretical framework of the SOLO (Structure of the Observed Learning Outcome) Taxonomy of Biggs and Collis (1977, 1982).

Initial investigations into the literature surrounding fraction understanding revealed little evidence with respect to adult learners' conceptions of fractions. Given the abundance of courses available to adults, such research would appear vital if syllabi are to be designed to meet the needs of adult learners and, in particular, redress any misconceptions that adult learners may bring to such courses. For these reasons, the topic of fractions was chosen since it is feasible that many adult learners may have had previous experience with them, and fractions is one topic that may not be prevalent in many adults lives.

However, the literature review revealed a considerable amount of detail with respect to childrens' understandings of fractions. In particular, fraction understanding appears to require substantial development with respect to identifying wholes, subparts and the acknowledgment that the subparts are both equal and add up to produce the whole. The literature indicates that while these are necessary conditions required prior to treating fractions as numbers, there is little evidence to suggest that any of these conditions occur spontaneously, simultaneously or naturally.

An initial study was conducted in which seven fraction items (Kerslake, 1986) were administered to 103 adult learners in a TAFE college. Results indicated that adult learners' responses were comparable with the responses of the children from the United Kingdom. In addition, evidence was beginning to accumulate which indicated that adult learners' responses could be classified into a notional hierarchy.

Given the above, a series of research questions were constructed and a fractions quiz was designed which incorporated four themes with respect to fraction concepts. These themes were: Understanding Fractions, Comparison of Fractions, Operations on Fractions, and Description of fractions. Each theme, apart from the last one, investigated fractions questions that were placed into two different contexts. 'Context free' questions represented typical textbook style problems, and 'in-context' questions placed fractions into more familiar or non-routine situations. In all cases, the themes were subjected to both qualitative and quantitative analysis. Typical examples of

students' written and verbal responses are presented where appropriate. The qualitative and quantitative analyses suggest that the adult learners' responses may be interpreted into the structure of the SOLO Taxonomy. Considerable consistency can be seen when both local (responses across all themes) and global findings are combined. In general, a two-cycle UMR (unistructural, multistructural, relational) level interpretation within the concrete-symbolic mode appears to be the most viable explanation for the variety and consistency of adult learners' responses to fraction questions. In general, the first cycle is related to describing fractions in terms of concrete objects, while the second cycle treats fractions as numbers.

There are three main findings from this work. The first was that adult learners' responses to fraction questions can be interpreted within the theoretical framework of the SOLO Taxonomy. The second finding was that there was some similarity observed between the structure of mature-age learners' responses to fraction questions and those offered by younger children. Finally, the issue of placing a fraction question into a context (*in-context*), or presenting them in a traditional textbook style (*context-free*) is also discussed, although the evidence from this study was inconclusive.

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