CHAPTER NINE

OVERVIEW:

AN HOLISTIC APPROACH TO UNDERSTANDING FRACTION CONCEPTS

> ***... the number of such sequential UMR learning cycles discovered within a single mode is determined by the size of the microscope used to analyse the individual components of skill acquisition.***
>
> **Watson *et al.* (1992a, p. 16)**

INTRODUCTION AND ORGANISATION OF THE CHAPTER

This chapter investigates the responses to all questions, irrespective of the theme into which individual questions may have been placed. As all the questions have now been analysed in some form in the previous four chapters, this chapter's primary focus is on obtaining an holistic interpretation. This is achieved by a combined quantitative analysis for the responses to all the questions. As a consequence, the responses from the previous chapter, which were based on analysis of only question, must also be compressed to only four categories, labelled 0, 1, 2 and 3, in order to facilitate the quantitative analyses. The modified table can be found in Appendix J.

RASCH ANALYSIS

ALL QUESTIONS

This section of the work focuses on the fit of the data involving the responses to all fraction questions, to the Rasch model. The Infit MNSQ value was 1.00 with a standard deviation of 0.19 and an Infit-t value of 0.04 with a standard deviation of 1.23. These results indicate that the model is appropriate to use with respect to the above data. Individual Infit Mean Square values can also be calculated for all questions, and are presented in graphical form in Fig. 9.1. Statistics that lie within the two vertical lines are considered acceptable.

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INFIT

MNSQ          .63       .71       .83      1.00      1.20      1.40
--------------+---------+---------+---------+---------+---------+-------
    item 1                                  |              .  *
    item 2a                         *       |              .
    item 2b                            *    |              .
    item 3a                                 |              .           *
    item 3b                  .              |      *       .
    item 6                   .              |      *       .
    item 5                   .              |    *         .
    item 6a                  .              *              .
    item 6b                  .      *       |              .
    item 7                   .          *   |              .
    item 8                   .              |  *           .
    item 9                   .              |  *           .
    item 10a                 .  *           |              .
    item 10b                 .  *           |              .
    item 10c            *    .              |              .
    item 10d                 .      *       |              .
    item 10e                 . *            |              .
    item 10f                 .      *       |              .
    item 11                  .    *         |              .
    item 12                  .          *   |              .
    item 13                  .              |  *           .
    item 14                  .              | *            .
    item 15                  .              | *            .
    item 16                  .             *|              .
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FIGURE 9.1

Map of Item Fit for all Fraction Questions

Of the 24 question parts, three Infit MNSQ values lay outside the vertical lines. Hence, there is a tendency for some students to answer these items correctly, but respond incorrectly to easier items, or vice versa. The three items with such a reverse response pattern are:

> Q1 which asked students to describe a fraction;
> Q3a which asked students to rank 1/3, 1/2 and 1/4; and,
> Q10c which required students to perform 3/4 - 2/3.

Variations with respect to these questions were discussed in earlier chapters. For example, some responses to Q1, indicated that a number of adult learners were unclear at what 'level' to pitch their answer. This is likely to result in high achievers describing fractions in simpler terms as one might use for a young child. In the case of Q10c, the application of the most popular incorrect strategy (considering fractions as made up of individual integers which could be used independently) resulted in an answer of 1. This represents an obvious error and it created a re-think of the question in the minds of some students. As a result, there were fewer students obtaining an incorrect result for the question than for similar questions involving addition. Finally, in the case of Q3a the Rasch procedure identified this question as having the most reversals. Many students applied a 'bigger denominator, smaller fraction' approach.

This strategy is successful when the number 1 is in the numerator (as it is in this question) but is unsuccessful in most other situations. It would appear that this is the feature that separates out this question as different.

Overall, the test item scale is consistent in measuring a latent trait. Reasons for the three items which show most reversals have been identified, although in the cases of both Q1 and Q10c the answering patterns are very close to those required by the model and are included in the analysis.

THRESHOLD VALUES

Figure 9.2 provides a graphical representation of the distribution of item difficulties and case (student) estimates. The figures on the left (Thresholds) represent the logit scale at which both items (indicated by question numbers) and cases (indicated by an X) are calibrated (exact values for both these measures can be found in Appendix K). On the right hand side of Fig. 9.2 are the response categories. As in the previous chapters, only the top three response categories are given. The lowest category (response category 0) serves as baseline data. Each X represents a particular student from the sample. This means that a student (X) has a 50% chance of being able to provide the response category of an item located at the same logit score.

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------------------------------------------------------------------------
Logit Scale  Student Estimates        Item Response Category Estimates
------------------------------------------------------------------------
                                  |    12.3 (2.54)
                           XXXX   |
                                  |
                                  |
                          XXXXX   |
                                  |     7.3
                            XXX   |
                                  |    2a.3
                             XX   |
                                  |     1.3
                             XX   |
                                  |    6b.3
                              X   |    3b.3
  1.0                       XXX   |     9.3
                                  |     5.3
                            XXX   |    2b.3   16.3
                           XXXX   |
                            XXX   |    16.2
                             XX   |
                                  |     8.3
                           XXXX   |    2b.2   15.3
                         XXXXXX   |     1.2
                         XXXXXX   |    2a.2    4.3    6a.3
                            XXX   |
                         XXXXXX   |
                             XX   |
                          XXXXX   |    10f.3
                                  |    10e.3
                            XXX   |    13.3
                           XXXX   |     5.2
                            XXX   |     9.2
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                                  |      3b.2    6b.2    15.2
                             XX:  |      12.2
                              X:  |     10e.2   10f.2
                              X:  |      14.3    15.1
  .0                          X:  |      3b.1     7.2    11.3
                                  |     10d.3    12.1
                           XXXX:  |      3a.3    13.2
                               :  |      2b.1     5.1    14.2
                             XX:  |      3a.2     4.2    16.1
                               :  |      11.2    13.1
                            XXX:  |
                                  |     10d.2
                                  |      6a.2     9.1
                               :  |      3a.1   10f.1
                                  |     10b.3    11.1
                                  |     10c.3
                             XX:  |      2a.1   10c.2
                               :  |       8.2   10b.2    14.1
                                  |     10c.1
                                  |     10a.3
                              X:  |      6b.1   10e.1
                                  |
                                  |     10d.1
                                  |       1.1
                                  |     10b.1
                                  |       8.1   10a.2
 -1.0                             |
                                  |      6a.1
                                  |
                                  |
                                  |
                                  |       4.1
                                  |       7.1
                                  |
                                  |     10a.1 (-1.38)
------------------------------------------------------------------------
  Each X represents    1 student
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FIGURE 9.2

Map of Thresholds for all Fraction Questions

Several groups (clusters) of responses were discernible from the results of item difficulties in Figure 9.2. There were three questions (Q10a, Q7, Q4) in which a majority of students attained the first response category. These were 10a.1 (1/2 + 1/4 = 2/6), 7.1 (2 bottles were greater than 3 bottles of punch) and 4.1 (cannot divide 9 pizzas between 15 people). In this grouping, students were required to relate fractions to a familiar situation (Q7 and Q4) or to add two common fractions (Q10a). However, the responses to these questions, at this level, indicated that the students did not really consider the relevance of fractions to the question at all. There was a clear gap between the responses to these questions and the remaining responses.

In addition to this group, there were five other main groups in the analyses. The following discussion looks briefly at each in order to describe common features.

The first group of responses ranged from 6a.1 to 10d.1 (Threshold values of -1.03 to -0.81). A majority of these responses focused on treating fractions as if they were dealing with whole numbers, e.g., 10b.1, 10d.1. In addition, responses in this category indicated a dependence on the fractions 1/2 and 1/4. For example, these fractions were selected repeatedly for all of the pizza sharing questions, irrespective of the numbers involved in the question. Students' responses appeared to be focused on the denominator.

The second group of responses ranged from 6b.1 to 10d.2 (Threshold values of -0.72 to -0.33). Responses in this grouping indicated a broadening of the range of fractions not evident in the previous section. For example, simple fractions, such as 1/3 and 1/5, were considered, e.g., Q9.1. However, these responses again appeared to be focused on the denominator. There were many responses which consisted of unusual answers to the questions in this section. There was one major exception to this, 10a.3. This equates to 1/2 + 1/4 = 3/4. This implies that a majority of students found this question to be particularly easy. The next section on Overall Difficulty and Step Difficulties discusses this issue further.

The third group of responses ranged from 11.2 to 10f.3 (Threshold values of -0.21 to 0.39) and consisted largely of responses in which fractions were treated as if students were completing a number pattern. Many unusual responses were noted in this grouping, and indicated that students realised that treating fractions as if dealing with whole numbers was not appropriate, but a viable and consistent alternative to completing patterns was not yet available to them. Some responses indicated that students guessed the size of a fraction, e.g., 3/4 because it looks like it, (15.2). Other responses, such as 3b.2, indicated that students focused on the denominator, almost irrespective of the numerator.

The fourth group of responses ranged from 2a.2 to 16.2, (Threshold values of 0.57 to 0.80) and were qualitatively different to the previous responses. For example, a majority of responses treated fractions as numbers and appeared to be able to focus on both the numerator and the denominator concurrently, irrespective of the often non-unitary values of the numerators, e.g., 2b.2. The responses to the 'pizza' questions (4.3, 6a.3, 8.3) indicated that students were able to select appropriate fractions. These responses indicated that the notion of equivalent fractions was just starting to occur.

The fifth group ranged from 2b.3 to 7.3 (Threshold values of 0.93 to 1.38). These responses indicated that students could focus on both the numerator and the denominator simultaneously. A majority of responses in this grouping consisted of correct responses to questions and indicated that students had a clear overview of both fraction and context.

The final category consisted of the responses to only one question. This was 12.3, and was both qualitatively and quantitatively different to the previous responses. It indicates that the most difficult response on the quiz was to attain the third (or final) step to Question 12 (12.3) (Threshold value = 2.54). This was the step which required students to assimilate a diagram showing 5/12 and the incorrect written statement 1/4 + 1/6 = 1/10. This result indicates that very few students have the overview required to reconcile the two different approaches commonly used by students.

DIFFICULTY VALUES

Table 9.1 presents the Difficulty and Step Difficulty values using the Tau option for the 24 question parts analysed on the Fraction Quiz. The table has been arranged to indicate the overall easiest question to the most difficult question. The range of difficulty scores, in this case, is from -1.02 (easiest) to 0.90 (most difficult). Step Difficulties provide a measure of the difficulty to get from one response category to the next response category within a question. The higher the score, the more difficult the step. These numbers can only be considered within each question. For example, Q10a is the easiest question, but students found it relatively difficult to get started, not so difficult to achieve the next jump, and once achieved, found the final step was comparatively very easy.

TABLE 9.1

Overall Difficulty and Step Difficulties for all Fraction Questions

Question Number	Overall Difficulty	Step Difficulties 1	2	3
10a	-1.02	.49	.31	-.80
10b	-.67	.85	.62	-1.46
10c	-.56	2.10	.44	-2.54
10d	-.41	.27	.55	-.82
4	-.27	-.64	.27	.37
6a	-.26	-.17	-.30	.47
14	-.25	.33	.96	-1.29
3a	-.24	1.25	.61	-1.86
11	-.23	1.04	.22	-1.26
8	-.23	.45	-1.28	.84
10e	-.12	-.29	1.07	-.77
10f	.01	.35	.32	-.67
13	.02	1.48	-.95	-.54
7	.04	-1.04	-.06	1.11
6b	.18	-.48	-.04	.52
9	.30	-.01	-.22	.23
1	.31	-.99	.86	.13
15	.33	1.44	-1.20	-.24
5	.38	.37	-.39	.02
2a	.44	-.67	.41	.27
3b	.44	1.35	-1.63	.27
2b	.45	-.28	1.15	-.88
16	.46	-.44	2.17	-1.73
12	.90	.93	-2.54	1.61

The table confirms many of the major observations noted in the previous section. For example, the easiest question on the Fraction Quiz was Q10a (-1.02), and the most difficult question was Q12 (.90). In both cases there were considerable gaps between the Difficulty values associated with these questions and the remaining questions. In general, the questions appear to be ranked into five broad categories, based on approximate order of difficulty.

The first group of questions (Q10b to Q10d) (Difficulties of -.67 to -.41) all consisted of context-free operation questions, such as would be found in a traditional textbook. Although there was a considerable gap between Q10a and the other questions, Table 9.1 indicates that students had little difficulty in addressing these types of questions. The data suggest that students had difficulty starting (step 1) and continuing (step 2) these questions. Students found reaching response category 1 of Q10c, which dealt with subtraction, to be the most difficult. This is the response category associated with treating fractions as separate whole numbers. As previously stated, students who used this strategy would have obtained an answer of '1', which caused many students to re-think this strategy. Also, division of fractions was not included in this group, indicating that students find division of fractions considerably more difficult than questions involving the other three operations.

It is plausible (and this was supported by the interviews) that some students had rote learned many of the algorithms required to complete the above questions. This could explain, at least partially, why these types of responses were the easiest ones on the quiz. It is feasible that many students found these types of questions familiar, and applied standard procedures that they had rehearsed many times over. In some cases, this included applying incorrect procedures.

The second group of questions (Q4 to Q8) (Difficulties of -.27 to -.23) consisted of three of the four 'pizza' questions (Q4, Q6a and Q8) as well as Q14, Q3a and Q11. With respect to the 'pizza' questions, which required students to select fractions to divide pizzas, the table indicates that students found all of these questions to be of approximately equal difficulty, i.e., students did not find one question to be particularly more difficult than the other three. Students were able to deal with questions concerning nine pizzas and fifteen people (Q4), three pizzas and five people (Q6a), or the reverse situation (Q8) in similar ways. However, the data suggest that, of all the 'pizza' questions, students found Q8 to be the most difficult to start (Step Difficulty 1 was .45). This was the only 'pizza' question which had a greater number of objects (watermelons in this case) than people. This reversal may have confused some students.

Of the other three questions (Q14, Q3a, Q11) in this group, Table 9.1 suggests that students found it considerably more difficult to reach response category 1 for Q3a and Q11 (Step Difficulty 1 was 1.25 and 1.04, respectively). This was the step associated with moving from no response to drawing diagrams of fractions (Q3a) or to treating fractions as separate whole numbers when a fraction subtraction was required. In the case of Q11, it is possible that the context of the question, coupled with the needed to subtract two different fractions, may have confused some students, causing many of them to not answer the question. Irrespective of this, Table 9.1 also indicates that if students persevered, it became comparatively easy to reach the final response category for all three of these questions, i.e., Step Difficulty 3 of all three questions was well below -1.

The third group of questions (Q10e to Q7) (Step Difficulties of -.12 to .04) suggests that students find division of fractions (Q10e, Q10f) to be considerably more difficult to work with than any of the other three operations (Q10a, Q10b, Q10c, Q10d). For example, students found reaching response category 1 of Q10f to be difficult (Step Difficulty 1 was .35). This is associated with moving from no response to writing unusual answers, as if students were experimenting with numbers. It is possible that the fractions in Q10f (2/3 ÷ 5/9) were considerably less familiar to students than those fractions in Q10e (1/2 ÷ 1/4). This may have caused some students to simply experiment with numbers, whereas some students may have been able to provide more 'reasonable' answers to Q10e based on 'intuition'. Following these questions, both Q13 and Q7 marked the start of a majority of questions which either required students to think of fractions in ways that they may not have previously considered, or did not appear in a traditional textbook style presentation, e.g., add 2 to both the numerator and denominator of 1/5 (Q13). Table 9.1 indicates that students had considerable difficulty in starting Q13 (Step Difficulty 1 was 1.48) which is associated with moving from no response to stating that the fraction had not changed. However, once this response category was reached, students found it considerably easier to attempt the rest of the question (Step Difficulty 2 and 3 were -.95 and 3 -.54, respectively). This is associated with moving to response category 2, which involved experimenting with fractions as numbers (for example, adding two fractions), and then reaching response category 3, i.e., commenting on the way in which the fraction had changed. In the case of Q7, students found it comparatively easy to start (step 1 was -1.04), and continue (Step Difficulty 2 was -.06). However, students found reaching the final response category (Step Difficulty 3 was 1.11) considerably more difficult. This was the drink mixing question, which a majority of students attempted to solve by using ratios. It was not until the final response category that students used fractions and considered all aspects of the problem.

Q6b, an in-context question in which students were asked to divide two pizzas between five people, appeared to belong to a category all of its own. Table 9.1 suggests that students found this question to be considerably more difficult than other pizza questions. It is feasible that this question lies on or defines a boundary between the questions that the students found comparatively 'easy' or 'difficult'. As discussed in Chapter Five, students found this question difficult to complete, i.e., to arrive at 2/5 independently of the context.

The final questions were a mixture of context-free and in-context problems, although the in-context problems may have been less familiar and more non-routine (e.g., Q16) than many of the earlier in-context problems which the students had been asked to solve, e.g., students were asked to consider the possibility of two people able to save the same amount if one person saves 1/3 and the other 1/5 of their respective salaries (Q9). The table suggests that, although students found this question comparatively easy to start and continue, many found it difficult to complete (Step Difficulty 3 was .23). This is associated with moving from stating that it was possible, to being able to describe how it was possible, e.g., if 1/3 of the first person's salary was equal to 1/5 of the second person's salary. A majority of the questions in this group required students to have a clear understanding of the significance of both the numerator and denominator. For example, students were required to order three fractions of different numerators and denominators (Q3b) or to write the equivalent fraction for 14/16 (Q5), given that the denominator had to be 24. In the case of Q3b, students had difficulty in reaching the first response category for this question (Step Difficulty 1 was 1.35). This is associated with moving from no response to drawing diagrams of three fractions with different numerators and denominators. As Table 9.2 indicates, this task would appear to be too difficult for a majority of adult learners in this sample. In contrast, it was comparatively easy to reach the next response category (Step Difficulty 2 was -1.63) which is associated with experimenting with fractions as numbers. It is possible that some students recognised the difficulties associated with drawing the three fractions and chose to either not respond (response category 0), or to attempt to manipulate the fractions as if completing a pattern (response category 2).

The most difficult question on the quiz was Question 12. This question required students to have a complete and general overview of fractions as both diagrams (5/12) and symbols (such that 1/4 + 1/6 is not 1/10). As the table suggests, very few students were able to obtain this overview and compare the two approaches to reach the final response category (Step Difficulty 3 was 1.61).

It is unlikely that many responses in the last three categories (Overall Difficulties ranged from .18 to .90) could be rote learned, since a majority of the questions were unfamiliar, non-routine and unrehearsed by students.

In general, the table suggests that when questions based on similar difficulty scores are ranked and then placed into categories (based on similar difficulty scores), the different categories contain similar types of questions with respect to either being context-free or in-context. In general, this suggests that students found context-free questions easier to solve than their companion questions that were placed into a context.

CONCLUSION

In this section all data were analysed using the Quest program. This allowed for both the difficulty of the question, across themes, to be compared and, on the same scale, enabled student estimate scores to be calculated. Overall, the quantitative analysis suggests that it is possible to describe a notional hierarchy of the adult learners' in this sample responses to fraction questions. In addition, such a hierarchy would seem to be compatible with the results of the previous four chapters. It is now appropriate to interpret these areas with respect to the SOLO Taxonomy.

A SOLO INTERPRETATION

Results from this study indicate that a single UMR structure in the concrete symbolic mode does not explain adequately many of the phenomena observed above. It has therefore been proposed that at least two UMR cycles exist within the concrete symbolic mode to explain adult learners' responses to fraction questions. This is consistent with the approaches noted in the previous four chapters which analysed the students' responses by theme. All of these four chapters demonstrated that a majority of adult learners' responses to fraction questions could be classified into a two-cycle UMR approach within the concrete symbolic mode. The purpose of this section is to present an holistic approach to adult learners' understandings of fractions.

CONCRETE SYMBOLIC RESPONSES

THE FIRST UMR CYCLE

The main characteristic of the first cycle is that of ikonic *dependence*. Ikonic dependence suggests that students treated fractions as if dealing with tangible objects only, i.e., fractions are not treated independently of diagrams, but are related to tangible objects, such as cakes and pies. Fractions are used as labels for describing these diagrams, but do not hold any intrinsic meaning in the absence of an object, or at least a suitable substitute, such as a diagram. While many of the scripts contained student diagrams, it was not until the interview stage, when the dependence on diagrams by students whose responses were able to be coded in the first cycle, that the importance of these diagrams became apparent. It was during the interviews that many students claimed that they 'saw' diagrams or 'saw' specific fractions as parts of concrete objects, such as cakes or pies.

In addition, and possibly directly because of the ikonic dependence on diagrams, fractions were used in two main ways. When fractions were used successfully, they usually had only the numbers 2 or 4 in the denominator, (later broadening to 3 or 5) and had only small numbers, typically one, in the numerator. When students were given fractions that were more complex than these, and, in the absence of diagrams, they appeared to lapse into treating fractions as if dealing with whole numbers. In general, the responses indicated that students could not imagine or represent many fractions with differing numerators and denominators. Students' responses indicated that they could not, rather than did not, function with respect to fraction questions independently of diagrams or other icons. For this reason, the first cycle, as an intuitive or qualitative understanding of fractions, represents a critical precursor to being able to function with fractions as numbers.

Unistructural 1 responses focus on only one aspect. However, there were few responses to the fraction questions from any of the adult learners to provide detail for this level. The best example of the type of response expected at this level came from the responses to Q1 which described fractions in terms of tangible objects. Typical responses described fractions as 'parts of a whole' or an "easy way to break a whole thing into sizeable pieces".

The easiest fraction of all is one-half, and it is feasible that, since 1/4 is 1/2 of 1/2, these two fractions are the dominant ones at this level, and may even be the only fractions, in any practical sense, available at this level. For example, they were the most common fractions used, even when the problem clearly required more appropriate fractions to be selected, e.g., in Q4, Q6a, Q6b, Q7 and Q8. Some students did not select any other fractions other than 1/2 or 1/4, irrespective of the context or the fractions involved.

The 'bigger denominator, smaller fraction' rule, which predominated many adult learners thinking, would have its roots in this level. This is because it is possible to compare 1/2 to 1/4 successfully at this level. However, fractions that involved different denominators with numerators greater than one are unlikely to be ordered successfully by relying on this rule.

The lowest level of response to Q9, which asked students to compare the wages of two people, in which one saved 1/3 and the other person saved 1/5 would occur later at this level, i.e., "No, they cannot save the same amount". This is because, at this level, 1/3 could be compared to 1/5, but only within a very limited context. At this stage 1/3 and 1/5 are visualised, and decisions as to bigger/smaller would be based on comparing the relative, but fixed, size of each fraction compared to the whole. There is no attempt to quantify this response, or to clarify it.

Multistructural 1 responses still treat fractions as 'parts of wholes' but add restrictions, such as "not a whole" or "the bits add up to the whole" (Q1). Responses at this level may focus initially on 1/2 or 1/4, as in the previous level, to solve the pizza problems, but will spontaneously adopt another, usually more appropriate fraction, such as 1/5, towards the end of their solution, in order to address the question, i.e., in Q6a, typical multistructural 1 responses are able to deal with the remaining left over pieces. In general, remaining halves were treated as new wholes and the process of selecting a new and different fraction then occurred. This process may have been repeated more than once, depending on the choice of fraction. However, this type of approach means that it is not yet possible to solve questions, such as that posed in Q6b (2 pizzas and 5 people), since responses at this level indicate that students do not have an overview of fractions, and do not realise that there is a more efficient process of pre-selecting an appropriate fraction for the question, such as 1/5, from the outset of the problem.

When fractions are compared, it will again be on a visual basis. However, responses at this level indicated that students may attempt to describe or quantify their decisions as if using a diagram of the fraction. For example, some of the responses to Q9 indicated that students believed that it was possible for two people to save the same amount provided that one increased their amount or worked longer to catch up. Clearly, responses such as these did not consider the two people in the question to have different wages, but did acknowledge the possibility of saving the same amount, by describing how they would do this pictorially, i.e., by increasing the one amount, (however absurd this may have been in the context of the question), until both amounts were equal.

It is feasible that three fractions could be ranked at this level, provided that the fractions were comparatively simple, such as 1/2, 1/3, 1/4 (Q3a). Although it is possible to compare any two of these fractions in the previous level (U1), the added complexity of keeping track of all three fractions simultaneously, appears to add a new intricacy to the problem. In general, comparisons of fractions at this level depend upon how accurately the visual interpretation can represent the fractions, and how obvious any discrepancies are to the viewer.

As a consequence, this is the first level at which it is possible to work with some fractions that contain both numerators and denominators that are slightly different to each other. This is provided that both numerators and denominators are comparatively small numbers. Responses, regarding fractions of this nature, usually involved either vague statements, such as 'bigger' or 'smaller', or invented intermediary numbers, such as 1 or 1/2, with which to compare and contrast the given fractions. For example, responses to Q2a (compare 5/7 to 7/5) and Q2b (compare 2/3 and 3/5) indicated that individual fractions were compared to a 'whole' or a 'half' shape, and then the respective sizes were judged, before a final decision regarding the ordering of the original two fractions could be obtained. This technique would seem to be of limited use and restricted to 'user friendly' fractions. For example, it is not particularly useful in addressing the fractions in Q2b (2/3 and 3/5). This is confirmed by the overall difficulty values which indicated that students found the comparison of two fractions, such as 2/3 and 3/5 (Q2b), as well as ordering of three fractions, such as 2/3, 5/7 and 3/4, to be, relative to the questions in the test, difficult questions.

Attempts to answer questions which require students to perform operations on fractions were beyond this level. Typical responses, such as 1/2 + 1/4 = 2/6 (Q10a), 3/5 + 2/7 = 5/12 (Q10b) and 3/4 - 2/3 as 1/1 (Q10c), were classified at this level. All of these responses indicated that students treated the fractions, not as diagrams, but as if adding or subtracting separate whole numbers. This is despite the fact that there were many individual fractions that could be represented diagrammatically. Responses at this level knew what 1/2 and 1/4 (Q10a) looked like. However, there did not appear to be any attempt to seek a connection between the independent fraction diagrams (of 1/2 and 1/4) and the symbols associated with adding 1/2 and 1/4, i.e., the fraction diagrams were not used as useful tools to aid problem solving. Typical operation questions of this type (i.e., traditional textbook style), which included only written symbols and no familiar context to fall back on, precluded the notion of accessing diagrams to address this problem. As such, the absence of such diagrams and the complexity of the written symbols caused some students to treat fractions as if adding and subtracting separate whole numbers, i.e., the intrinsic usefulness of representing fractions as diagrams as an aid to solving the problem was not applied. There was no attempt to use common denominators at this level.

There is some evidence to suggest that the fractions 1/6 and 1/10 were also visualised at this level. However, any comparison that was made between these fractions, was achieved by comparing the relative sizes of diagrams.

Results of applying the 'bigger denominator, smaller fraction rule' (established in the previous level) may still be occasionally correct at this level and serve to reinforce its use.

Relational 1 responses indicated that this was the first level at which it was feasible to visualise a considerable number of fractions with different numerators and denominators. Responses from this study indicated that many fractions containing different numerators and denominators could be compared and ranked visually. However, the process is limited since it depends upon a direct comparison between two visual representations of the fractions and requires any differences to be 'obvious'. In some cases, students appeared to become quite expert at their diagrams and could differentiate comparatively similar fractions. However, for most of the fractions in the quiz, this direct comparison technique was too difficult for students. For example, the two fractions involved in Q2b (2/3 and 3/5), cover similar amounts of space on a diagram. As a consequence, it proved to be very difficult for some students to judge or differentiate clearly between these two fractions if they were only represented on a diagram. This process would seem to become even more complex, when three fractions, such as in Q3b (2/3, 3/4, 5/7), would need to be sketched to scale, compared and then ranked in order.

Responses indicated that many students at this level were looking for an alternative framework with which to manage their choice of fractions. Some responses indicated that students 'opted out' of the process and selected one fraction, such as 3/4 for Q15, because the diagram "looked like it". Another student (DK) also appeared to be operating at this level in responding to Q11 (2/5 - 3/8). The student drew a container (a punch bowl?) to represent a volume change from 2/5 to 3/8. Although the student appeared to have an overview of the diagram, the student could not perform the calculation successfully, but went on to describe the remaining volume in the container as 1/8. In both cases, this was because their diagrams looked like the fractions they chose to be their answers.

Although responses at this level can now visualise a variety of fractions in terms of diagrams successfully, they do not go outside the context of the problem, but remain focused on the content of the question as if dealing with a real-world predicament. For example, responses indicated that some students still treated the pizza questions as if dealing with real pizzas, but rejected the notion of selecting inappropriate fractions, such as 1/2 or 1/4. Instead, they had an overview of the problem and would suggest that there had to be an appropriate number of pieces to be distributed among the people. More appropriate fractions, such as 1/5, were selected from the outset. Many responses classified at this level, took the context of the problem seriously and, typically, added the total number of pieces as if reassuring themselves that there would be enough for all the people, as in a real-life situation.

In general, by the time students have completed the first UMR cycle, responses indicate that they have a broad repertoire of fraction concepts which they can visualise as diagrams. In addition, many fractions can now be compared and ranked (1/3 > 1/5), and simple individual diagrams are related, e.g., 1/2 + 1/4 = 3/4 (visually). This suggests that by the end of the first cycle, equivalent fractions may exist as examples of different diagrams that cover the same amount of area as each other (diagrammatic equivalence). The example used as the basis for Q12 would be a version of a relational 1 response. Most students in this study knew enough about fractions to acknowledge that the written answer of 1/10 was wrong, although they may not have been able to offer an alternative written response. Separation of the numbers from the diagrams does not occur until the next level.

Typical responses to Q1 at this level, described fractions as 'parts of whole or numbers'. This transference between the first cycle and the second would appear to occur simultaneously as students recognise (i) the inadequacies and inefficiencies of continually visualising diagrams, and (ii) the coincidence of particular number patterns that are associated with diagrams that represent the same amount of space. The next step is to take up the use of fraction symbols, independently of diagrams.

THE SECOND UMR CYCLE

In this cycle, fractions are treated as numbers. Diagrams are no longer an integral part of calculation. As such, fractions have a unique role to play with respect to their use, calculation and interpretation. The levels in this cycle would therefore appear to develop a coherent and systematic system which enables the precise use of fractions as numbers to emerge.

Unistructural 2 responses focus on fractions as numbers, and typically described fractions (Q1) as 'parts of numbers'. Responses indicated that the need for a more general and systematic process to replace the use of diagrams will be answered by treating fractions as numbers. Hence, the first stage of this process is largely exploratory. Sometimes a strategy was used consistently across all types of problems, or different strategies were employed to address similar questions. For example, typical responses to Q5 (14/16 = ?/24) involved treating the numbers as if dealing with a number pattern. Responses at this level tended to be 'unreasonable' when related to the original question. In many cases this may have involved selecting an inappropriate operation (e.g., addition instead of multiplication). For example, some students attempted to add the two fractions in Q15 when they were asked to calculate the area of a piece of carpet in a room. Some responses obtained 22/15, which suggests that the area of carpet was larger than the room in which it was placed. However, responses at this level did not question the result, since they were focused on the arithmetic of the situation, and did not relate the (incorrect) answer back to the original question.

Students experimented with a variety of number patterns, until eventually some of the number patterns appear consistently useful or at least worth remembering. In the case of equivalent fractions, for example, the realisation that there is a consistent and reliable number pattern which will always generate equivalent fractions, irrespective of the choice of numerator or denominator, and, that this procedure will always enable any number of fractions to be compared, marks the start of the next (multistructural 2) level.

Multistructural 2 responses mark the first stage in which 'patterns' of fractions have been replaced by a systematic 'process', i.e., the use of equivalent fractions in number form. This transformation now enables comparison and ranking of fractions to be undertaken consistently, reliably and independently of diagrams. Responses indicated that students did not simply guess at answers or play with numbers until an answer was obtained.

Fractions at this level were again described as 'a part of a number'. However, responses indicated that students provided additional information, as if recognising that this simple statement was incomplete and required further details. For example, typical responses included: "not a whole number, can be written as a decimal, percentage etc". It is worth noting that all of the responses at this level referred to decimals or percentages as something different to fractions, but which could be a useful technique in solving problems that included fractions. Decimals and percentages were not explicitly referred to as being interchangeable with fractions at this level.

Responses at the multistructural 2 level indicated that students can write equivalent fractions using common denominators successfully. However, since there is no overview of where or when it is appropriate to do so, common denominators were utilised when it was inappropriate (in division questions, such as Q10e) or ineffective (in multiplication questions, such as Q10d) to do so.

Although, the early classification of the multistructural 2 level is marked by the ability to generate common denominators in symbolic form, there was growth within this level. For example, responses indicated that some students should leave this level with the ability to critically select common denominators, such as when to use lowest common denominators or when to use alternative strategies, such as percentages. In contrast, there was evidence to suggest that some students selected any common denominators, rather than streamlining their selection by utilising lowest common denominators when solving fraction questions that involved addition and subtraction. This may have lead to an increase in errors in the performance of operations on fractions.

Relational 2 responses typically exhibit control over all four operations on fractions. Students no longer guess at answers or need to depend on diagrams to visualise fractions. It is at this level that students concede that fractions and division of two numbers, are interchangeable, as are percentages or decimals with fractions (Q1). All of these aspects of fractions are now viewed as an integrated package of ideas.

RESPONSES BEYOND THE TARGET MODE

Findings from this work suggest that there were some responses which were more sophisticated than those shown above. Some of these responses, such as those to Q9, have been discussed in more detail in Chapter Six. All of these responses used algebra spontaneously, and have been placed outside the concrete symbolic mode. Although there was little evidence of other responses occurring in the formal mode, some responses to Q1, suggest that by the end of the second cycle some students were beginning to question the most fundamental assumptions usually associated with the concrete symbolic mode, e.g., the role of zero in either the numerator or the denominator or both. For example, one student (IS) described a fraction (Q1) as "a/b and b $\neq$ 0 is a division (a is divided by b). It's also a ratio". This kind of thinking marks the foundations of the type of thought required for access to the formal mode with respect to fractions.

Table 9.2 presents a summary of the adult learners' responses for an holistic interpretation of fractions, based on the findings of two UMR cycles within the concrete symbolic mode.

TABLE 9.2

Summary of adult learners' responses for all Fraction Questions

UMR	CONTEXT-FREE	IN-CONTEXT	Q1
<U1 (CS)	one part out of 4	Q4, Q6a, Q6b, Q8 - divide the pizzas by pieces Q7 - 2 bottles stronger	
U1 (CS)	can order simple fractions with 1 in numerator, e.g., Q3a biggest denominator, smallest fraction 'rule'	Q4, Q6a, Q6b, Q8 - 1/2 or 1/4 selected Q9 - not possible, assumes wages are the same, i.e., 1/5 > 1/3 always (earliest possible level) Q15 - wrote fractions on diagram	a part of a whole
M1 (CS)	Q5 - 22/24, 'add 8' Q10a - 1/2 + 1/4 = 2/6 Q10b - 3/5 + 2/7 = 5/12 Q10c - 3/4 - 2/3 = 1 can order simple fractions with >1 in numerator, e.g., Q2a, compares 5/7 and 7/5 to 1 or 1/2	Q4, Q6a, Q6b, Q8 - 1/2 or 1/4 and later 1/5 selected Q9 - is possible, assumes wages are same, i.e., person A saves longer Q12 drew diagrams of 1/4, 1/6 or 1/10, and compared the diagrams Q11 - treated fractions as if dealing with separate whole numbers, e.g., 3/8 - 2/5 = 1/3 Q15 - selected an incorrect operation, e.g., instead of multiplication, and did not manipulate the fractions correctly	a part of a whole and the bits add up to the whole
R1 (CS)	Q5 - 22/24 can compare more difficult fractions diagrammatically such as 2/3 and 3/5 (Q2b), but will prefer to avoid difficult diagrams and 'guess', e.g., 3/4 is the biggest because it seems that way (Q3b) Q10a - 1/2 + 1/4 = 3/4 (because it looks like it)	Q4, Q6a, Q6b, Q8 - 1/15, 1/10 or 1/5 selected from the outset; still context dependent Q9 - is possible, but cannot provide details Q12 - stated that the answer '1/10' was wrong and the diagram (5/12) was correct, but no further details were provided Q11 - 1/8 or 1/5 or 1/2 Q15 - 3/4 because it looks like it	a part of a whole or a number

UMR	CONTEXT-FREE	IN-CONTEXT	Q1
U2 (CS)	Q5 - 21/24 by patterns only Q10 attempted to use fraction 'rules' (e.g., common denominators) in absence of object - plays with numbers as if completing a pattern, e.g., selects inappropriate processes such as the 'cross division' rule for multiplication (Q10d) Q13 - fraction stays the same attempts to 'play with numbers' as if completing a pattern	Q4, Q6a, Q6b, Q8 - 1/5 or 1/10 selected from outset; context independent Q9 - is possible, states that the wages must be different, but unable to say how Q12 - focused on the written expression '1/6 + 1/4 =', but did not obtain the correct answer, and became confused by the diagram in the question Q11 - 2/5 - 3/8 = 24/15 = 8/5 NB. the answer is unreasonable Q15 - rules are applied incorrectly, e.g., 4/5 x 2/3 = 6/8 = 3/4 and the operation may also be incorrect, e.g., 4/5 + 2/3 = 13/15 Q16 - wrote 3/2 = 1½	a part of a number
M2 (CS)	equivalent fractions Q5 - 21/24 by using equivalence of fractions Used common denominators correctly, but made calculational errors, e.g., inappropriate use of common denominators for multiplication and division of fractions (10def) Q13 - fraction changes Q14 - equivalent fractions	Q4, Q6a, Q6b, Q8 - correct answer for pizza questions Q9 - is possible, if 1st wage earner earns > 2nd wage earner focused on common denominators (Q12, Q11, Q15) Q16 - attempted to manipulate 3/2 ÷ 5/3 unsuccessfully	a part of a number and can be changed to a percentage or decimal
R2 (CS)	all operations on fractions correct Q13 - fraction progression moves closer to 1	all operations on fractions correct Q9 - is possible, if A earns $500 and B earns $90	a part of a number and is interchangeable with fractions and decimals
U1 (F)		Q9 - is possible, if 1/5 of 1st wage earner = 1/3 of 2nd wage earner	a number a/b; $b \neq 0$

UMR	CONTEXT-FREE	IN-CONTEXT	Q1
M1 (F)		Q9 - is possible, if 1/5 a = 1/3 b	
R1 (F)		Q9 - is possible, if second's salary is 5/3 times the first's salary	

CONCLUSION

The main finding from this chapter is that a hierarchy of adult learners' responses to fraction questions could be interpreted within the existing framework of the SOLO Taxonomy. In general, a majority of these responses could be classified into a two-cycle UMR approach within the concrete symbolic mode. A minor number of responses also fell outside these groupings but could be accommodated either just prior to the unistructural 1 level in the concrete symbolic mode, or as the first UMR cycle in the formal mode.

The first cycle in the concrete symbolic mode is characterised by the dependence on diagrams as representations of fractions. The second cycle relates fractions to numbers. It is not surprising that two fractions, 1/2 and 1/4, are the most predominant fractions used throughout the responses, and have been classified comparatively early in the hierarchy. In contrast, other fractions, such as 1/3 and 1/5, are more difficult, as shown by Rasch threshold and difficulty values, for the adult learners in this study to conceptualise. Finally, fractions which contained different numerators and denominators were the most difficult.

The evolution of skill which consistently and reliably generates equivalent fractions would appear to be achieved only after an exhaustive exploratory stage. Initially, fractions represent diagrams. Eventually, fractions are seen as numbers independently of diagrams, and are subsequently and systematically 'tested', as number patterns, to determine their suitability for future use with fractions. Finally, these number patterns are screened until a procedure is established which guarantees equivalent fractions. This process paves the way for typical operations (+, -, x, ÷) to be performed on fraction questions.

CHAPTER TEN

CONCLUSIONS AND CONSOLIDATIONS

Everyone agrees that quality is important, but no one is quite sure how to incorporate it.

Biggs and Collis (1980, p. 19)

INTRODUCTION AND ORGANISATION OF THE CHAPTER

Evidence gathered from the literature review chapters (Chapters One and Two), the preliminary study (Chapter Three) and the main study (Chapters Five to Nine) indicate that adult learners' responses to fraction questions in this study can be interpreted within the SOLO Taxonomy. In addition, adult learners in this sample responded to fraction questions in ways which were similar to those of younger learners presented in other studies, notably in the investigations of the CSMS team presented in Hart (1981) and the follow up study of Kerslake (1986), and Watson *et al.* (1991ab, 1992ab, 1993). Finally, the results of this study suggest that the effect of placing fraction problems into a context warrants further research.

The main aims of this final chapter are to: (i) summarise the findings of the previous chapters; (ii) reflect on the limitations and implications of the findings; and (iii) suggest further areas of research.

The chapter is divided into four main sections. The first focuses on the SOLO Taxonomy interpretation. This includes a comparison between the main findings of this investigation to a second study undertaken since this study was commenced. The second section of the chapter discusses the limitations of the study, followed by the third section which discusses the implications of the study to TAFE and adult learners in general. Finally, future research possibilities are presented.

SUMMARY FINDINGS

There were three main findings from this research. The first was that adult learners' responses to fraction questions can be interpreted within the theoretical framework of the SOLO Taxonomy. The second finding was that there is some similarity observed between the structure of mature-age learners' responses to fraction questions and those offered by younger children. Finally, the issue of placing fraction questions into a context (*in-context*), or presenting them in a traditional textbook style (*context-free*) is worthy of discussion, although the evidence from this study was inconclusive.

The findings presented in this study were established in three main ways. First, a preliminary investigation was undertaken by administering a sample of the Kerslake (1986) items to 103 TAFE students within the first few weeks of attendance at TAFE. The results of this phase one study formed the basis of a more substantial fraction quiz which was administered to two different groups (AD and TP) of TAFE students (n = 107). The questions in the main study were analysed, both qualitatively and quantitatively under four broad themes. These were: understanding fractions, comparing fractions, operations on fractions, and description of fractions. The findings within these themes were the focus of chapters Five to Eight. Second, the responses to all questions were analysed collectively (Chapter Nine). This process was designed to provide both thematic and holistic perspectives on adult learners' understandings of fractions. Overall, there was considerable overlap between all interpretations irrespective of whether the investigation was viewed at a more local level (the themes) or within an holistic interpretation. It is now relevant to provide an overview of the implications of these interpretations with respect to both the data and the model of cognitive development.

A SOLO INTERPRETATION

Combined findings from the previous chapters suggest that most of the adult learners' responses to fraction questions in this study can be interpreted using a two-cycle UMR structure within the concrete symbolic mode in the SOLO Taxonomy. Two cycles were chosen since there appeared to be two major approaches used by students to address fraction questions. However, within each approach, there was evidence to suggest that there was an internal hierarchy (the levels) which developed in complexity from focusing on only one aspect (unistructural) to considering all aspects of fractions as an integrated package (relational). In addition, there were some responses which fell outside this structure and could be interpreted within adjacent modes of thinking. For example, some of the responses to Q9 could be interpreted within the next (formal) mode of thinking.

In general, the first UMR cycle in the concrete symbolic mode depends on an intuitive or qualitative approach to fraction understanding. The main characteristics of this cycle are that fractions are dependent upon tangible objects, or at least, reasonable facsimiles that can be provided by diagrams. As such, fractions are a shorthand, useful for describing such diagrams, but do not have any significant meaning in the absence of such imagery support. This contrasts with the second cycle, in which fractions have meaning independent of any appropriate diagrams, and students are not easily misled in the absence of such support. Responses at the end of the second cycle indicated that adult learners possess the necessary requirements to perform successfully all four operations on fractions irrespective of the difficulty of the fractions involved. A summary of the levels in the two cycles is now presented.

A **unistructural 1** response indicates that learners focus on only one aspect of the problem and describe fractions in terms of parts of wholes. Fractions at this level contain 1 in the numerator, while the denominator is a comparatively small whole number, i.e., fractions are easy to visualise and comparatively common, e.g., 1/2 or 1/4. Fractions, in this category, can be ordered since the bigger the denominator, the smaller the fraction.

A **multistructural 1** response attempts to work with the common fractions established in the previous level. For example, responses to the pizza questions selected 1/2 or 1/4, but then selected a more appropriate fraction, such as 1/5, with which to divide any remaining pieces. Fractions, in absence of a diagram, are treated as if dealing with separate whole numbers, e.g., 1/2 + 1/4 = 2/6.

A **relational 1** response to a fraction question focuses on how the fraction appears as a diagram, e.g., 3/4 for the area of carpet in Q15. Responses at this stage indicated that learners were aware that fractions were not added/subtracted as if dealing with whole numbers, but these learners did not know about other techniques, such as equivalent fractions.

A **unistructural 2** response indicates an exploratory stage on behalf of the learners. Fractions were described as parts of numbers, and there was a wide variety of responses in which learners appeared to play with the numbers as if searching for, or attempting to complete, a pattern. Complex diagrams may confuse some learners at this stage and operations on fractions often yield inappropriate answers. Incorrect rules, such as the cross division rule, are seen at this level.

A **multistructural 2** response focuses on generating common denominators (accurately), although their use may not be appropriate, e.g., their use was observed in multiplication problems. Typical responses at this level list procedures to be followed, such as how to add two different fractions. Diagrams are generally ignored at this level.

A **relational 2** response relates all aspects of fractions, such as decimals and percentages. Learners indicated that they had an overview of both cycles and could deal with fractions being portrayed both diagrammatically and expressed as numbers, e.g., only the top responses to Q12 indicated that learners could explain both the diagram of 5/12 and the written statement of 1/6 + 1/4 = 1/10.

In general, the above structure implies that learners progress through an apparent dependence on, and then removal of, diagrams as they develop their understandings of fractions, and, clearly, learners who did take this journey view all aspects (diagrammatic and symbolic) of fractions as an integrated and inseparable package of ideas. However, it is too simplistic to presume that all learners reach the end of the two cycles via progressing through every level as suggested by the two-cycle mechanism. Given this, there is some indication that age, and experience, may play a more pivotal role in the development of fraction concepts than previously thought. This issue is now taken up in more detail by comparing the findings from this study to that of a comparable study by Watson, Campbell and Collis (in press).

COMPARISON WITH RECENT DEVELOPMENTS

Recent analysis of young childrens responses to fraction questions, by Watson *et al.* (in press) have affirmed a two-cycle UMR structure within the concrete symbolic mode. However, an additional UMR strand within the ikonic mode, and not noted in their earlier work, has also been postulated to offer a fuller explanation of fraction development in young children.

The experiment consisted of interviewing from three to six children in each of the grades from pre-grade 1 to grade 4 (Watson et al., in press, p. 6). Each interview lasted for approximately 45 minutes. The responses from the interviews were taped and then collated based on similarity of response. This process was repeated independently by each of the three authors, until concordance was reached.

Their conclusions were largely based on the results of two experiments. One of these, referred to as the *pancake problem*, has the most relevance to this study. The question asked children to share a pancake fairly between three dolls, and resembles the 'pizza' questions asked in this thesis.

Pancake Problem Results

Watson *et al.* (in press) argued that there were two issues which needed to be resolved before responses could be said to be characteristic of concrete symbolic reasoning.

These were associated largely with the concept of conservation of number. In the case of this fraction question, this means that the children must be able to provide each doll with both the correct number of pieces, and take into account the relative size of each piece. For these reasons, responses which fell outside these concepts were classified in the ikonic mode. A typical response in the ikonic mode for each level is now presented.

A **unistructural** response (IK-U) was observed when one child split the pancake into two and gave two of the three dolls one piece each. "The concept present is that of sharing but it cannot be carried out to satisfy the constraints of the problem" (Watson *et al.*, in press, p. 7).

A **multistructural** response (IK-M) was observed when one child stated that three pieces were needed, but actually produced four pieces when asked to cut the pancake. "a more complex splitting was attempted but the dilemma of sharing could not be resolved" (Watson *et al.*, in press, p. 8).

A **relational** response (IK-R) was observed when one child split the pancake into approximately four equal pieces. The child gave two pieces to one doll and one piece to each of the other two dolls. This response indicates that students related the concept of fairness in so far as it meant that each doll received some portion of the pancake. The concept of "conservation of number of pieces or quantity required for fair sharing, however was not present" (Watson *et al.*, in press, p. 8).

A typical response in the concrete symbolic mode for each level is now presented.

A **unistructural 1** response (CS-U1) was observed when six students "exhibited a sharing based on conservation of number, not size, and if there were any leftover bits from the distribution they did not create a conflict for the student" (Watson *et al.*, in press, p. 8). For example, students divided the pancake into quarters and distributed one quarter of the pancake to the three dolls, leaving one quarter as a leftover.

A **multistructural 1** response (CS-M1) was "the realisation that having the same number of pieces is not enough to constitute fair sharing and that left overs are not possible when wholes are shared fairly" (Watson *et al.*, in press, p. 10). For example, additional cuts were made to the pieces of pancake until the child was satisfied that all the dolls had received a fair share, however, unequal this may have been in practice, i.e., solutions "are piecemeal and inelegant, and quantity may not actually be equal though the intention is there" (Watson *et al.*, in press, p. 11).

A **relational 1** response (CS-R1) indicates that both the number of pieces, and their relative sizes, were integrated to the extent that students attempted to subdivide leftover pieces equally. For example, three students divided the pancake into quarters, and then divided the remaining quarter into three equal parts. Each doll was then given a quarter and a third of the remaining quarter. Other variations of this technique all resulted in the dolls being given a fair share of pancake, although it was presented to the dolls in multiple pieces.

Watson *et al.* (in press) also noted that two students attempted to divide the pancake into thirds, but were not successful since they made the three cuts parallel to each other. It was not until the next level that the concept of one-third was visualised successfully.

A **unistructural 2** response was observed when one student spontaneously cut the pancake into thirds from the outset of the problem. "It appears that the more complex relationships needed to solve the problem in the first cycle of the concrete symbolic mode are replaced by a single, new, more sophisticated concept" (Watson *et al.*, in press, p. 13).

Overview

Clearly, there are similarities between the responses in the Watson *et al.* (in press) investigation and this study. In particular, a similar basic underlying structure can be identified at each of the levels that are common between the two studies. For example, a focus on single, simple and familiar fractions is evident at U1(CS), then a marked sequential processing in M (CS) responses and where problems arise they are dealt with (although no overview is evident), a linking of all relevant elements was possible with a consistent answer provided, could be seen in R1(CS), and in U2(CS) there was an added sophistication of previous responses.

However, there were differences. Most significantly was the focus by the students in the Watson *et al.* study on the notion of fair sharing. None of the adults in this study sought to address this issue or acted in any way that would suggest that equal fractions of an object or number could have a different size or quantity. Watson *et al.* alluded to this feature when they said: "'Fair' is thus shown to have a different meaning from an adult's in so far as quantity is not included within it" (Watson *et al.*, in press, p. 10). The investigation in this thesis has provided the empirical evidence to elaborate on this comment.

What is evident is that because of the many adult experiences in cutting up and distributing equal materials, and in using simple common fractions that they bring to the questions an extensive (albeit limited in some ways) experiential background. This has shown up in adults taking certain underlying aspects for granted. In the case of young children, however, it is this lack of ongoing, general experiences which seem to be the cause for them to focus on issues, such as fair sharing.

It is this additional focus which accounts for the slight discrepancies that can be seen in the allocation of some levels to the students' responses in Watson *et al.* and to the adults in the present thesis. For example, in Watson *et al.* a response coded R1(CS), **if** one is to ignore the notion of fairness in the responses, then there is a sequential processing evident that is similar to the coding of M1(CS) responses of the adults in this thesis.

This is a complicated issue that is not able to be resolved without the appropriate data available. What is needed is a study in which the same questions are administered to both young children and adult learners. The study should build upon the preliminary study in this thesis but should go much further. In such a study, careful probing and prompting of responses should be undertaken and the background experiences of the students should be identified and linked to the results. At the heart of such a study, is the nature of the different interpretations that can be given to responses categorised in the first cycle in the concrete symbolic mode.

It is clear from the preliminary study in the thesis that at a macroscopic level there are similar broad categories of responses between young and adult students. However, the comparison with the findings of Watson *et al.* indicates a tension at the microscopic level. This issue needs to be resolved. The importance of such future work to the coherence and viability of the SOLO model would seem to be critical.

THE TWO GROUPS OF STUDENTS

Unlike previous investigations, which dealt with young children, a central aim of this study was to determine how adult learners responded to fraction questions. Given this, one of the focuses of this study was the impact of age, or length of time since adult learners had seen fractions. This was achieved by administering the quiz to two different groups of adult learners. One group of students, the Associate Diploma group, consisted of a majority of school-leavers, who entered their course via a traditional application/acceptance scheme operated by the State of Queensland, which allocated places based on the attainment of a Tertiary Entrance score obtained by the learners over the last two years of their senior secondary schooling, i.e., traditional

Year 12 entry. The other group of students, referred to as Tertiary Preparation students, applied directly to the college on the basis of mature-age entry. A majority of these students had not completed Year 12, and were entering this course to provide them with a pathway into university that they may not have considered previously. The Tertiary Preparation course contained more mature-age students than the Associate Diploma course, and there was a greater time gap between when they left school and their enrolment in TAFE.

Analysis for this section of the study focused on recording the responses to fraction questions separately and performing a chi-square analysis on each of the summary tables presented for each question. The results indicated that, despite the differences stated above, both groups exhibited the same broad range of responses to each question. For approximately half the questions (13 out of 24), there were no significant differences observed when a chi-square analysis was performed. There were, however, some exceptions. A summary of these differences is presented in Table 10.1.

TABLE 10.1

Summary of Significant Differences between the AD and TP Groups

THEME	CONTEXT-FREE	IN-CONTEXT
I	-	Q14(χ^2=12.73, d.f.=5, $p<0.03$)
II	Q2a(χ^2=16.81, d.f.=6, $p<0.01$) Q3a(χ^2=27.29, d.f.=5, $p<0.00$) Q3b(χ^2=31.44, d.f.=6, $p<0.00$)	-
III	Q10a(χ^2=7.81, d.f.=3, $p<0.05$) Q10c(χ^2=9.84, d.f.=3, $p<0.02$) Q10e(χ^2=8.31, d.f.=3, $p<0.04$) Q10f(χ^2=8.31, d.f.=3, $p<0.04$)	Q12(χ^2=10.03, d.f.=4, $p<0.04$) Q11(χ^2=13.28, d.f.=4, $p<0.01$)
IV	Q1(χ^2=13.49, d.f.=7, $p<0.06$)	

As Table 10.1 indicates, there was some evidence to suggest that there was an association between the categories of responses and the group to which a student belongs. However, care must be exercised in this interpretation, since there is no baseline data to indicate which group was stronger or weaker at the beginning of the study. Allowing for this, it is still appropriate to compare the different approaches used by the two different groups on the above fraction questions. Since some of the questions are similar within each theme, it is appropriate to compare the responses of the two groups on a theme by theme basis.

Theme I: Understanding Fractions

There was only one question (Q14) for this theme, in which there was a significant difference between the two groups of students and their performance on this item. This was Q14 which asked students to describe the effect of doubling both the numerator and the denominator of 2/3. Hence, it is feasible that some of the more mature-age students may not have been able to describe this effect (there was an increase in non responses from the Tertiary Access group) since they were not as familiar with equivalent fractions as the younger learners. It is also feasible that some students may have felt that a change had occurred since the individual numbers had also changed (there was an increase in the 'fraction changes' category from the Associate Diploma group), i.e., the Associate Diploma students focused on a more literal meaning for the phrase 'describe the effect this will have on the 2/3', stating that a change had occurred because the individual numbers had altered.

Other questions in this theme, focused on dividing pizzas (Q4, Q6a, Q6b, Q8) or adding 2 to both the numerator and denominator (Q13), or writing the equivalent fraction of 14/16 with a denominator of 24. In the case of the pizza questions, no significant differences were observed between the two groups of students, i.e., both groups provided similar techniques with which to divide the pizzas. It seems plausible that both groups may have had similar real-world experiences with pizzas, and, hence, age and the number of years gap prior to schooling may not have affected significantly the techniques used to divide pizza. It is also plausible that neither group of students had been asked Q13-type questions previously, and, hence, neither group was advantaged (or disadvantaged) on the basis of age or years since formal study. In the case of Q5, it is feasible that students, irrespective of which group of students they were in, could manipulate the fractions to write the equivalent fraction of 14/16 in terms of a denominator of 24, however, this question did not ask students to describe the effect of doing this.

Theme II: Comparison of Fractions

The only questions for this theme, in which there was a significant difference between the two groups of students, were all context-free (Q2a, Q3a, Q3b). On all of these items (Q2a, Q3a, Q3b), the Associate Diploma group performed better as a group than the Tertiary Access students. In particular, the Associate Diploma students selected more 'mathematical' techniques, such as common denominators or percentages or decimals more often than the Tertiary Access students. In addition, there was some indication that the Tertiary Preparation students used rules, such as the 'bigger denominator, smaller fraction' more prevalently than the Associate

Diploma students. In some cases, this could lead to success, e.g., fractions, such as 1/3, 1/2, 1/4 (Q3a) can be ranked correctly, and no significant difference was recorded. In other cases, such as 2/3, 5/7, 3/4, (Q3b) this technique will produce incorrect answers, and, hence, there was a significant difference noted for this item. Again, it is feasible that more Associate Diploma students had comparatively more recent experiences with these types of questions, and knew enough about fractions to adapt their techniques for different questions, whereas the mature-age students were still attempting to recall which procedures where applicable to which questions.

The only exception to the above, was Q2b which asked students to compare 2/3 and 3/5. However, there was an increase in non responses from both groups of students indicating that students from both groups had difficultly comparing these two fractions. In the case of the in-context questions (Q7 and Q9), responses from both groups of students suggested that ratio was a more appropriate technique to use for Q7. Although there was no significant difference observed in Q9, there did appear to be more Associate Diploma students who focused on the manipulation of fractions, e.g., they calculated common denominators; whereas there were more Tertiary Access students who focused on the reality of the situation, i.e., that two people could be on different salaries. Again, this may have been a reflection on the different backgrounds of the two groups of students, with the Associate Diploma students being more focused on the fractions in the question, and the Tertiary Access students relating more to the different contributions of the workers.

Theme III: Operations on Fractions

This theme produced the most questions which indicated significant differences between the two groups of students and their performances on fraction items. In almost every case, the Associate Diploma students performed better on the operations on fraction questions than the Tertiary Access students. Even when a technique was clearly inappropriate, such as common denominators for multiplication, there were more students in the Associate Diploma group who employed this technique than the Tertiary Access group. In contrast, there were more Tertiary Access students who either did not attempt the questions, or who appeared to play with the numbers as if trying to remember appropriate techniques, irrespective of the operation or the context.

Although the responses to Q10d did not indicate that there was a significant difference between the two groups of students, there is some suggestion that more Associate Diploma students selected common denominators (although inappropriate) to solve this problem. In contrast, more Tertiary Preparations students worked with patterns as they attempted to address this question.

The main exceptions to this, were Q10b, in which an answer of 1/1 may have prompted students (in both groups) to re-evaluate their answers; Q15 which required the knowledge of the area of a rectangle; and Q16 which may have been unfamiliar to both groups of students.

Theme IV: Description of Fractions

The chi-square analysis to Q1 is technically not significant ($p<0.06$). However, it is close to the acceptable limit ($p<0.05$) and has been included in the analysis of this section. The main reason for this is that there were more Tertiary Preparation students who described fractions in terms of objects, such as wholes or cakes and pies, than Associate Diploma students. In contrast, there were more Associate Diploma students who described fractions as percentages or decimals. Once again, it is feasible that there were more Tertiary Preparation students who related fractions to cakes and pies, since this has been a majority of their fraction experiences in the real-world.

Overall, a trend was observed which indicated that students in the Associate Diploma group responded with more number-based answers to the questions. This implies that older students or students who have been absent from school for longer periods of time may have more difficulty interpreting and responding to fraction questions.

CONTEXT-FREE VERSUS IN-CONTEXT QUESTIONS

The third main finding from this study concerns the effects of placing fractions into either a specific, practical context-free situation as might be expected in a traditional textbook, compared to placing fractions into a context. Again, results from both the qualitative and quantitative analyses formed the basis of this section of the work.

Overall, the results suggest that at the lowest levels of fraction understanding, learners found the in-context questions easier to deal with than those that are context-free. However, with respect to operations on fractions, the reverse situation would appear to be true, i.e., learners found operations on fractions to be easier to deal with if they are presented in traditional textbook style than if they are placed into a context. These findings are summarised in Table 10.2.

TABLE 10.2

Summary of Difficulty of Context-free or In-context problems

THEME	CONTEXT-FREE	IN-CONTEXT
I	difficult	easier
II	difficult	easier
III	easier	difficult
IV	not applicable	not applicable

Although Table 10.2, at least superficially, suggests conflicting results, there is a plausible explanation for these observations. For example, it is feasible that some questions enable students to provide lower level responses which may lead to correct answers. In contrast, other questions, such as those that place operations on fractions into a context set up barriers to lower level responses, and, hence, prevent lower level answers from being correct. It is worth recalling that most of the in-context questions for the first two themes involved placing fractions into familiar situations, such as dividing pizzas or comparing salaries. Lower level responses to these types of questions consisted of diagrams which learners used as if dealing with real pizzas, or fixed quantities.

In the case of some of the in-context operations on fractions, learners may have required additional information usually not associated with solving operations on fraction questions. For example, Q15 required students to calculate the area of a rectangle without being given whole number dimensions. In addition, some questions may not have been familiar to many students. For example, Q12, which asked learners to reconcile a diagram of 5/12 to the written statement of 1/6 + 1/4 = 5/12.

The ability to succeed with operations on fractions questions has been placed within the relational level of the second cycle of concrete symbolic mode. This implies that, irrespective of the context, learners will only be able to answer these types of questions correctly if they are able to operate at this level. The results from this study indicate that many of the students may not be able to operate at this level.

SUMMARY

In general, there were three main findings from this study. First, adult learners' responses to fraction questions can be interpreted into a two-cycle UMR structure within the concrete symbolic mode. Both the responses from adult learners and the responses from younger children in the Watson *et al.* (in press) investigation exhibit considerable similarity.

Second, there was some indication that mature-age learners, or learners who have had a considerable time lapse in their schooling, may have more difficulty addressing fraction questions.

Third, the results suggest that adult learners find fractions easy to deal with when placed into a context, provided that the context is familiar, or does not involve operations on fractions.

LIMITATIONS OF THE STUDY

Notwithstanding the above, there were two main limitations to this study and these are now discussed.

First, it is reasonable that the two groups involved in the study may not have been a true representation of adult learners in TAFE throughout the country, nor of adult learners in general. Hence, questions of generalability of the specific findings of how this sample achieved on various items may not be taken as indicative of some benchmark appropriate to other similar institutions. Nevertheless, the students in the study covered a broad range of abilities and backgrounds, and their responses were sufficiently varied to provide a sound basis for coding.

Second, the research precluded the testing of some aspects of fraction concepts, e.g., there were no questions which involved mixed numbers and the four operations. Further, questions targeting ratio, percentages or decimals, or conversions between each of these were also not included, nor were negative or irrational fractions. There were a number of reasons for this. Time was a critical variable as many students had a heavy workload (had part-time jobs or extensive family commitments). As a result there was a clear limit on the time available for testing, and, despite its desirability, only a few students were available for interview.

Also, the issue of the students well being, in terms of self-image, was an important consideration. Given that the test was carried out before formal instruction had settled down, it was decided that the questions should be designed (as much as possible) not to intimidate the students and be of a form that might, in a gentle way stimulate their mathematical thinking. As such, the questions were designed to encourage responses and students were requested to supply as many details as possible.

It would seem, from the comments supplied by the students, that the test did achieve these aims. Students did provide fulsome details, none objected to undertaking the test, and the interviews did play an important role in clarifying written comments and providing deeper insights into the cognitive processing employed. This was especially true for the responses in the first cycle in the concrete symbolic mode.

FUNCTIONING IN THE FIRST CYCLE

Although this work suggests that the focus of the first UMR cycle in the concrete symbolic mode is characterised by the dependence on diagrams as if dealing with real objects, it is possible to interpret the results of this study with respect to earlier research on fractions, e.g., Collis and Romberg (1989) and Watson *et al.* (1992b). These studies advocated *ikonic support* (Watson *et al.*, 1992a, p. 7) and suggested that successful utilisation of the ikonic mode can:

> ... either provide an alternative method of solution to that of conventional mathematics ... or it can provide richness and flexibility when used in conjunction with concrete symbolic or formal modes, and can help to break unproductive sets.

Such an interpretation is, of course, always possible, and there were some responses noted in this investigation which appeared to coincide with the above description. For example, there were some responses which included diagrams, although the solution did not appear to depend on these diagrams. These responses indicated that some students included diagrams as a teacher would, to add detail when explaining fractions to a child. This observation was partially confirmed by an increase in the presence of diagrams in a question (Q6b) which prompted students to "draw a diagram". During an interview situation, responses, such as those described could present alternative answers to the fraction questions, more in keeping with the second UMR cycle, i.e., symbolic and independent of diagrams. This is consistent with the notion that ikonic support is used to supplement responses in the concrete symbolic mode by utilising diagrams. In these cases, the diagrams did not replace their concrete symbolic functioning.

In contrast, there were a number of responses which did not possess any of the above three attributes associated with either the ikonic mode or ikonic support, i.e., did not provide:

(i) an alternative method of solution (using the ikonic mode only)
(ii) richness and flexibility (ikonic support)
(iii) help to break unproductive sets (ikonic support)

as described by Watson *et al.* (1992b, p. 7). Quantitative analysis confirmed the categorisation of these responses at a low level. However, the main evidence for the content of these responses came during interview situations in which students could not provide alternatives to diagrams. For all of these types of responses, students did not:

(i) respond with alternative pathways. In the absence of such diagrams, students did not have any alternative avenues to access, such as those indicative of the second cycle in the concrete symbolic mode.

(ii) provide richness and flexibility in conjunction with other modes. Responses which depended on diagrams did not support reasoning associated with the second UMR cycle in the concrete symbolic mode. In many cases, they were in direct conflict.

(iii) break unproductive sets. This term would appear to imply that ikonic support is utilised after some attempt has been made to deal with the problem in the concrete symbolic mode. There is some evidence from this study to suggest that the reverse situation occurs. For example, even when the diagrams the students had drawn did not appear to aid the students answers, many students still persevered with unproductive and inappropriate diagrams.

Although ikonic support "can lead the student woefully astray" (Watson *et al.*, 1992b, p.11) when applied incorrectly, there is no further indication in the literature acknowledging the dependence on diagrams, as if dealing with tangible objects, which was noted throughout this investigation. Given this, the phenomenon in which learners not only draw diagrams, but work with the diagrams as if dealing with tangible objects, has been referred to as *ikonic dependence*. As the term implies, learners cannot, in general, address a problem in absence of such diagrams or reasonable facsimiles.

It is feasible that this phenomenon has only been observed with adult learners. This suggests that some adult learners have adapted these techniques throughout their lives, and it is plausible that what is diagnosed as ikonic support in younger learners many manifest itself as ikonic dependence in adult learners. It is as if the adult students have grown beyond ikonic support and actually depend on the diagrams as their only method of solution. If this is the situation, then teachers of adult learners must become aware of the implications. This issue is discussed in the next section.

IMPLICATIONS

There are several implications from the findings of the study into adult learners' responses to fraction questions. These are now presented.

IMPLICATIONS FOR TEACHING

The findings from this study suggest two broad approaches to teaching fractions. The first approach would appear to be applicable to learners who have had little previous instruction in fractions. The second approach would involve re-teaching this topic to students who have been exposed to fractions previously, e.g., adult learners who return to formal education after several years absence from study. In the latter case it needs to be remembered that such students may have developed 'rule of thumb' techniques for dealing with fraction problems that routinely occur in their everyday life.

LEARNERS NEW TO FRACTIONS

The findings of this study and those of Watson *et al.* (1992b, in press) suggest that teachers need to become more aware of the intuitive approaches used by learners when dealing with fraction questions. For example, it is usually acceptable to have pieces of cake remain after a distribution in the everyday sense. Learners, therefore, should be provided with situations involving the more mathematical sense of fair division, i.e., situations where there is no remainder.

This study as well as the Watson *et al.* (1992b, in press) investigations have raised the issue of the difficulty associated with fractions other than 1/2 and 1/4. In general, learners find fractions which are less familiar to be considerably more difficult to work with. For example, the findings from Watson *et al.* (in press) demonstrated that young learners found the fraction 1/3 difficult to conceptualise. This implies that additional resources/time is needed than might be traditionally expected for students to achieve functional proficiency. Knowing and applying fractions, such as 1/2 and 1/4, does not imply effective transfer to other similar fraction types.

LEARNERS NOT NEW TO FRACTIONS

This study suggests that many adult learners are not proficient at fraction understanding. First, there are those learners who show little initial understanding of fractions. Learners in this category need to be provided with diagrams and objects to help facilitate their move through the first UMR cycle in the concrete symbolic mode.

Such preliminary work should provide a sound foundation for future fraction development. Second, for those students who are more capable (and have a good basic background in the first cycle), there is a need for catalysts which encourage these people to move from an object focus to one of number. This can be done by placing students in problem situations where solution using diagrams is problematic.

IMPLICATIONS FOR TAFE

Many adult learners in this sample entered TAFE with a variety of rote-learned rules which are poorly understood with respect to fractions. In many cases, students perceptions and subsequent misapplications go undetected, or uncorrected, in mainstream teaching techniques traditionally used in TAFE. However, a more detailed examination into adult learners' conceptions with respect to fractions, as occurred in this study, would appear to have many advantages for TAFE. First, a diagnostic approach to adult learners understandings of fractions would enable some students to progress at a faster rate, and hence complete this part of their mathematics courses ahead of time. Second, students who have major difficulties can be detected early and offered assistance.

Unfortunately, many adult learners and staff, are often not aware of any misconceptions until during, or even after, a final assessment. This is not the aim of education, since many of the students' mathematical problems may have been able to have been dealt with at an earlier stage, i.e., an early intervention approach may decrease the number of 'invented algorithms' that have become common in some students' responses. This may involve the development and implementation of new curricula to address the issues raised above.

Finally, there is one further constraint on educating adults which should be considered when plausible solutions are suggested for adult learners' misconceptions of fraction concepts. It is the phenomenon loosely termed 'rote learning'. Clearly, some adult learners' responses in this work can be, at least partially, explained by rote learning. For example, many students during interviews commented on the bigger denominator, smaller fraction rule. In some cases, this rule did lead to correct solutions, e.g., 1/4, 1/3, 1/2, which appeared to perpetuate its widespread acceptance even when its use was inappropriate. However, the notion of employing rote learning to explain every anomaly is too simplistic and superficial, and there clearly is value in students being able to recall information and being able to apply remembered algorithms quickly and efficiently. Nevertheless, learning by rote has become an art form for some students, and for many students it provides short-term successes, since it often yields instant results. The dilemma is that if it is left to persist or encouraged then the answer to every question becomes a unique answer, requiring individual attention, rather than an example of an overall principle within a specific case.

FUTURE DIRECTIONS

There are four main directions for further work that have developed out of the analysis. First, there would seem to be a need to clarify the role of rote learning in mathematics. In some instances, it may be a useful device. Clearly, some students see it this way. Alternatively, many mathematics educators would see it as offering short-term gains but long-term drawbacks.

Second, it is important to explore the differences in responses between the younger students in the Watson *et al.* (in press) study and the adult learners in the present study. Since fractions are an important element for further studies in many mathematics topics, such as algebra, there should be a consistent and coherent set of levels that describe learning irrespective of age. Such research would appear to have strong practical as well as theoretical ramifications.

Third, there was some indication that age, or the amount of time a student is absent from formal schooling, may play a role in determining the responses to fraction questions. These differences were more pronounced in the comparison of fractions theme and the operation on fractions theme. In particular, the Associate Diploma students utilised common denominators more readily than the Tertiary Preparation students. In contrast, there were more Tertiary Preparation students who did not respond to fraction questions. This was particularly apparent when asked to perform operations on fractions that were context-free. Although inconclusive, these findings suggest that more research is required to investigate the connection between the length of time since adult learners have encountered fractions, and the related background experiences on their performances on fraction questions.

Finally, the issues surrounding the placement of fractions into a context need to be addressed. Evidence from this study suggests that fractions that are placed into a familiar context, may activate lower level responses, e.g., first UMR cycle responses. However, questions which require students to treat fractions as numbers, such as the addition of two fractions, can only be addressed adequately at the relational 2 level. Placing questions of this type into a context complicates the fraction question to such a degree that many adult learners found the context to be considerably more difficult than a traditional textbook approach. This suggests that while initial fraction concepts are formed from real-life situations (as Streefland (1991) suggested), functional fraction knowledge is embedded in context-free situations, such as traditional textbook questions. Despite this, many adult learners in this study did not possess knowledge beyond the rudimentary basics needed to get through life or school situations. The effect of placing fractions into a context, particularly non-routine or unfamiliar, needs to be clarified.

CONCLUSION

This study has focused on adult learners' understandings of fractions. Fractions is but one topic in mathematics and mathematics is but one subject of the total curricula offered to the students in this study. The findings represent only a small window into the quality of adult learners' responses, and as the SOLO Taxonomy suggests: "SOLO levels are equivalent to test results; they describe a particular performance at a particular time. They are not meant as labels or to tag students ... It carries with it a warning not to overgeneralise" (Biggs & Collis, 1982, p.23). As a consequence, the SOLO Taxonomy is an appropriate vehicle with which to analyse adult learners' understandings of fractions. The framework is able to offer 'hope' to both learners and teachers since it has the potential to act as a diagnostic and prescriptive tool for intellectual growth.

The focus on the topic of fractions was valuable for several reasons. All students in the study had met fractions (in some form) in their daily life experiences, and all would have encountered them (again, in some form) at school. Hence, this work provides some insight into how mathematics is remembered, retained and applied by a group of students who were not usually seen as the most successful at school. The study also offers a perspective on how adults use experiences in other aspects of their lives to address mathematical ideas that are non-trivial.

Clearly, much more research is needed with this type of student sample. Irrespective of whether the adult learners have recently completed Year 12, or are re-entering formal education after several years have lapsed, it is important in a technological society that people are given the opportunity to continue education. Research, such as provided in this study, helps provide a basis on which older students can be better assisted to meet their potential.