CHAPTER THREE

THE DESIGN AND ANALYSIS OF THE PHASE ONE STUDY

> ***... it would be incorrect to assume that this implies the theories are correct - only by testing the ideas in practice can decisions of right and wrong be justified.***
>
> **Pegg (1984, p. 9)**

INTRODUCTION AND ORGANISATION OF THE CHAPTER

While the first chapter focused on the literature surrounding childrens' understandings of fractions, the second chapter highlighted the feasibility of interpreting adult learners' responses to fraction questions in terms of the SOLO Taxonomy. However, as the above quote suggests, this is only a theoretical possibility, and has not, as yet been tested in practice. Given this, it is valuable that a preliminary study be carried out prior to a more substantial exploration of these issues. The aim of this initial or phase one study is to provide some insight into mature-age students' responses to fraction concepts. The following questions serve to structure this analysis:

(i) What evidence is there to suggest that adult learners' responses to fraction questions form a hierarchy?

(ii) Is it possible to 'group' adult learners' responses into a small but cohesive collection of subgroupings which consist of similar responses?

(iii) How do adult learners' responses compare to children's responses to similar questions?

The aims of this chapter are to:

(i) explore the performances of adults on fraction questions; and,

(ii) compare the responses and determine if any similarities exist between the adult learners' responses and known children's responses from previous research.

For these reasons, a group of adult students and a subset of test items from the Kerslake (1986) study were selected. These issues are presented in the first part of the chapter which describes the study. The second part of the chapter presents the results of an experiment in which the subset of Kerslake items were administered to adult learners. The responses to the questions are presented under three themes which relate to: (i) models of fractions; (ii) fractions as numbers; and, (iii) equivalence of fractions. These are the same titles used in the Kerslake study. Summaries of the findings on a question by question basis are presented, and a discussion section including examples of typical responses from students is provided after each theme. The final section of the chapter compares, where practicable, the findings of the study involving adult learners to that of the children involved in the Kerslake study.

METHODOLOGY

A quiz based on a subset of the items reported in Kerslake (1986) was administered to adult learners in their first week of classes. A brief discussion of the student sample and the items selected now follows.

THE STUDENT SAMPLE

The sample selected for this study consisted of 103 adult learners or mature-age students attending a Technical and Further Education (TAFE) college. Unlike previous studies, which investigated childrens' understandings of fractions, this study focused on adults. This is a major difference for several reasons. First, students who attend TAFE do so voluntarily. There is no compulsion of adult learners to continue their education beyond the legal age for withdrawing from school (typically 15 years of age), although many choose do so for a variety of reasons. The TAFE students in this sample can be divided into two broad groups. Students in the first group, which usually consists of young school leavers, apply to enter a particular course at the college via a statewide scoring system (called the TE-score which is based on Year 12 results). Many of these students view TAFE as a means to an end (i.e., to facilitate their entry to university), rather than as an end in itself (i.e., to become a TAFE graduate). Students enter the second group by applying to the college directly via a 'mature-age entry' scheme. This group consists of older students who choose to return to study, usually after several years absence from school. In many cases these students have not completed high school.

The second major difference between TAFE students and school children, is that TAFE students come from a wide variety of backgrounds. It is not uncommon for students, with a broad range of ages, abilities, educations, cultures, gender differences and lengths of time since the student has last formally studied, to be enrolled in the same class. This has the potential to provide a wider scope and greater depth to the learning experience which can be both an asset and an impediment.

Irrespective of these differences, it is highly likely that most students have come across fractions at least once, if not on several occasions, throughout their schooling. Given this, it is feasible that adult learners conceptions of fractions may be more varied because of their previous out-of-school encounters than secondary students. The main aim of the preliminary study is to determine if such differences appear to influence students' understandings of fractions.

TEST DEVELOPMENT

A subset of the Kerslake (1986) items was selected which appeared to offer the greatest potential range of diversity of student responses. There were several reasons for selecting the sample from the Kerslake study. First, Kerslake's work has achieved widespread acceptance and follows on, and builds upon, the earlier research of Hart (1981). Second, it provides a wide variety of question types which appear suitable to an older audience. Finally, the items selected were broad enough to capture all major aspects of fractions.

A sample of seven items were selected to be representative of the three themes identified by Kerslake. These were: (i) models of fractions; (ii) fractions as numbers; and, (iii) equivalence of fractions. A summary of the different questions and how they relate to the three themes identified by Kerslake (1976) is presented in Table 3.1.

TABLE 3.1

Various aspects of fractions by question in the Phase One study

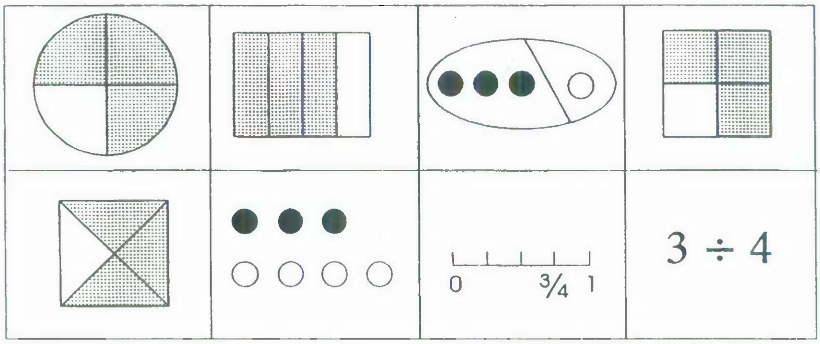
ASPECT OF FRACTIONS	QUESTION NUMBERS
Models of Fractions	1, 2, 3
Fractions as Numbers	4, 5
Equivalence of Fractions	6,7

The questions chosen formed the basis of a 'fraction quiz' and are provided below. A brief justification for the inclusion of each question follows the quiz.

FRACTION QUIZ

1. *How would you explain to someone, who didn't know, what a fraction was?*

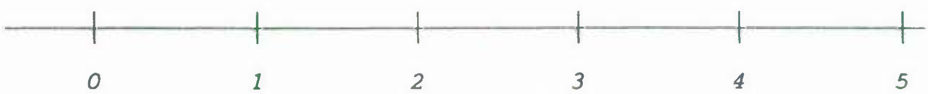
2. *Which of the following cards would help someone to understand what the fraction 3/4 is? Explain why.*



3. *You have three cakes. Could you share them equally between five people? Explain what you would do. (Use diagrams if necessary).*

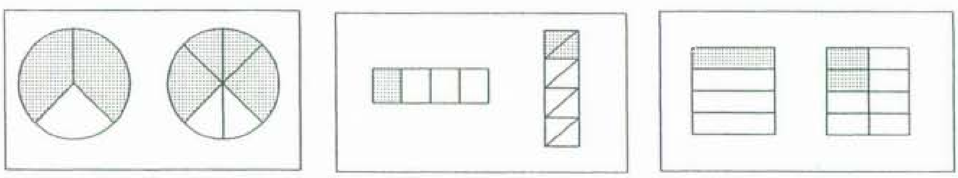
4. *How many numbers are there between 2 and 3? And between 0 and 1?*

5. *Where would the number 4 go on this number line? And the number 3/5? And the number 1 1/5?*



6. *Would you rather have 2/3 or 10/15 of a cake you particularly liked? Explain why?*

7. *Suppose you saw these diagrams in a textbook. What could you tell from them?*



Question 1 This question was selected in an effort to elicit responses based on adult perceptions of how fractions might be defined. This question asked students to describe fractions, and has seldom been addressed (as noted in the literature review in Chapter One). Despite this, some of the mathematical hierarchies (Chapter Two) suggested that the language that students use in their descriptions plays an important role in their understandings of a concept. As a consequence, it is believed that the responses to this question may be pivotal in establishing a mathematical hierarchy, if such a hierarchy of responses exists.

Question 2 This question was used to monitor the acceptance by students in the sample of certain models of fractions. The decision to accept (or reject) a certain model will depend on the individual's perception, preference and familiarity with that particular model. For example, it is difficult to pre-empt the range of answers possible to this question. Clearly, some other models of fractions relate to the part-whole construct as outlined in the previous chapter. However, other models of fractions are better suited to different aspects of fractions. This question was seeking to see which model, or models, adult learners in this sample associated with fractions.

Question 3 This question was chosen since it placed a fraction in a familiar everyday context, as opposed to a traditional textbook approach. Much of the literature was divided on the issue of 'context specific' versus 'context free' aspect of fractions (Chapter One) and the aim of this question was to investigate to what extent an individual's perception of fractions determine their approach to solving a problem presented in a real-world context. By placing a fraction in a familiar situation, it questioned the ability of adults to relate the use of fractions to their everyday knowledge.

Question 4 This question investigated the issue: are fractions numbers? Answers to this type of question may produce major clues as to the development of a mature understanding of fractions (Chapter One). It is possible that some adult learners, absent from formal mathematics for some time, may not recognise fractions as numbers since they do not, as a rule, make use of them in this context. This lapse in time may also limit their use of fractions to a comparatively small number, (such as one-half), or make their use imprecise, e.g., 'half way home'.

Question 5 This question addressed the issue: how do fractions relate to the number line and how can they be represented this way? It is worth recalling that children, in particular, found locating fractions on the number line to be one of the more difficult aspects of fractions (Novillis, 1976). By focusing on the number-line representation, students were required to treat fractions as numbers.

Question 6 This question investigated the equivalence of fractions that were written in standard symbolic form. It involves the comparison of two equivalent fractions placed in an everyday context.

Question 7 This question also investigated the equivalence of fractions. However, in this case, diagrams were used to represent fractions. The diagrams chosen were similar to those featured typically in many junior secondary mathematics textbooks.

ANALYSIS OF RESPONSES

Students' responses to the seven questions, on a question by question basis, were grouped according to the similarity of the responses. Summary tables were then produced which indicated the different types of classifications obtained, and the number of responses in each category. Following each table, samples of responses, which are representative of each category, were provided to give further depth to the summary table and to allow for a more meaningful discussion. For research purposes, students' initials are shown in brackets, but have been altered to maintain privacy. Responses that did not appear to address the question or whose meaning was not able to be determined were coded 'Uncodable'. It is possible that an interview may have clarified the response, but this was not undertaken in the phase one study. A 'No response' coding indicates that the student did not attempt this question. Full details are provided in Appendix A. A summary of the findings, on a question by question basis, is presented for each question in this part of the chapter. A discussion section is provided for each theme.

MODELS OF FRACTIONS

Question 1, 2, and 3 addressed the concept of 'models of fractions'.

QUESTION 1

Question 1 was the first question which asked students to consider what fractions were.

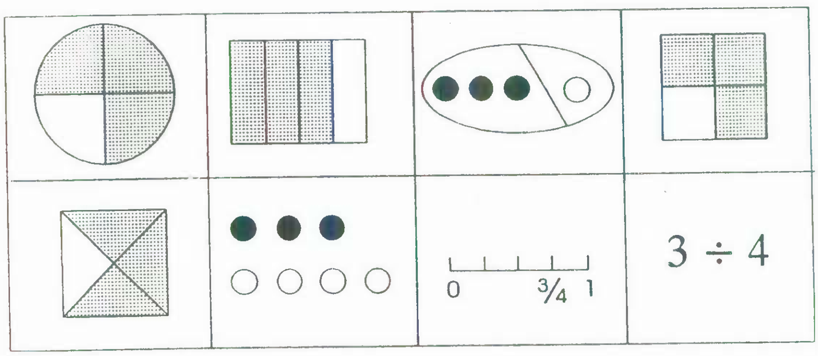
> ***How would you explain to someone, who didn't know, what a fraction was?***

In general, the adult learners in this sample exhibited a wide range of descriptions for fractions. However, these can be divided into three broad groups. The first group, approximately 10%, could not address this question at all. The second group, approximately 50% (n=43) could only describe fractions as 'part of a whole', and could add little else. The final group (n=28) described fractions as 'part of a number', and added no further information, i.e., only one in three adult students described fractions as numbers.

QUESTION 2

Question 2 asked students to choose between a small selection of cards which demonstrated different aspects of fractions.

> ***Which of the following cards would help someone to understand what the fraction 3/4 is? Explain why.***



The results to this question confirm the overall prevalence of the area model to other representations of fractions. Only three students accepted one model that was not an area model. Although it is difficult to determine the reasons for this, it is plausible that a majority of adult learners recalled seeing similar diagrams in textbooks as children or were more familiar with the area models than other aspects of fractions. However, given the diversity of adult learners' backgrounds, this explanation may be too simplistic.

QUESTION 3

Question 3 placed the division of three by five into a situation that should be familiar to many adults. It was seeking the connection between 3 ÷ 5 and 3/5.

> ***You have three cakes. Could you share them equally between five people? Explain what you would do. (Use diagrams if necessary).***

Many students used inadequate, inappropriate or misleading strategies to respond to this question. For example, some students failed to select any fraction at all. This made equal and fair division of the cakes impossible. A minor variation on this technique was the selection of a familiar, although inappropriate fraction, such as 1/2. This leads to inequitable distributions. However, some students could exercise a greater degree of control over their selection of fractions, and, once the cakes had been halved, could then select a more appropriate fraction, such as 1/5 or 1/10, and proceeded to divide the remaining cake by fifths or tenths. Some students drew diagrams, and appeared to depend on these diagrams as they worked towards a solution. The responses to this question suggest that the connection between 3 ÷ 5 and 3/5 was not immediately obvious to the majority of adult learners in the sample.

DISCUSSION

The responses to all the questions in this section revealed a notional hierarchy. However, Question 1 appeared to be the most discriminating question. Typical responses to this question provided clear boundaries which differentiated between different levels, and, hence suggest appropriate classifications and potential placement of such classifications of responses into a hierarchy. In addition, students' responses appeared to be aligned to one of two main approaches for dealing with fractions, and the data suggested that there was growth both within and between these two extremes. For example, some students could describe fractions only in terms of concrete objects, such as cakes or pies. However, the early responses to Question 3 demonstrated that this strategy was by no means foolproof, i.e., despite a dependence on concrete referents, approximately one third of the sample of adult learners were unsuccessful at dividing three by five in the disguise of a familiar everyday situation. These early stages of the hierarchy, suggest that when fractions were referred to, it is strictly in the context of cutting a cake or pie into (usually) a half. The next stages of the hierarchy would appear to depend on the skill of the student who would then determine if the halving strategy, once applied, could still be useful in gaining an appropriate answer, i.e., other more appropriate fractions, such as fifths or tenths are suggested. In this way, some students could, eventually, arrive at the correct answer. However, the solutions to the question appeared to be embedded in the context of the situation. The dependence on diagrams was also confirmed in the responses to Question 2, which indicated that a majority of adult learners preferred the 'area' models over other all other models of fractions. It is plausible that the area model, due to its prevalence in traditional textbooks, may have been the only model of fractions to which the adults had previously been exposed. However, it is also possible that this may be the only way some adults view fractions.

In contrast, other students' responses focused on fractions as possessing properties usually associated with numbers. For example, responses would refer to percentages and/or decimals in describing a fraction (Question 1), and, when a 1/2 was suggested, responses indicated that students attempted to use it in an example of how to add two fractions. The acceptance of all of the models including the division aspect (Question 2), further indicated that these types of responses were qualitatively different to the earlier ones, and hence occurred later in the hierarchy. Answers which did not rely on diagrams or did not appear to be confused by, or only focused on, the context of a question (such as Question 3) were ranked at the highest levels of the hierarchy.

However, caution must be exercised in interpreting adults' responses, since some adults clearly responded at a lower level, rather than at a higher one of which they were capable, i.e., at least one adult rejected the division model, even though they understood its significance. For example, in responding to Question 2, one student (SC) rejected the division model stating that: "The last example [3 ÷ 4] I feel would be too abstract to show someone who has never seen fractions. Even though it means the same as a fraction". Clearly, this student knew enough about fractions to be able to target their answer to suit a specific audience. This issue could represent a confounding variable when analysing written responses unless every effort is made to establish the context in which the question resides.

FRACTIONS AS NUMBERS

Questions 4 and 5 asked the question: are fractions numbers?

QUESTION 4

Question 4 required students to treat fractions as numbers and asked students to determine how many numbers there were between two consecutive whole numbers.

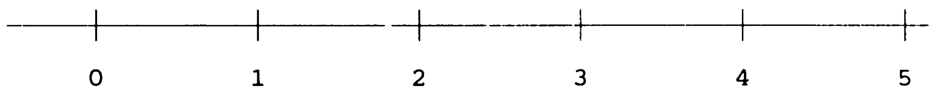
> ***How many numbers are there between 2 and 3? And between 0 and 1?***

The responses to this question indicated that many adult students relate the word 'number' to 'whole numbers'. This assumption would appear to limit significantly the possibilities of understanding numbers other than whole numbers, such as fractions. It is interesting to note that, even though some students knew there were other numbers beside whole numbers, some of them could still not come to terms with considering fractions as numbers, e.g., "well, if you want to call fractions numbers". Some students considered decimals to either one, two or three decimal places and subsequently arrived at (incorrect) answers of 10, 100 or 1000. Less than one-third of adult learners in the sample acknowledged that there were an infinite number of fractions between two integers.

QUESTION 5

Question 5 asked students to plot three numbers on a typical number line. One of the numbers was expressed as a whole number (4), one as a fraction less than one (3/5) and one as a fraction greater than one (1 1/5).

> ***Where would the number 4 go on this line? And the number 3/5? And the number 1 1/5?***



A majority of adult learners (approximately 75%) could successfully place a proper fraction and a mixed number on a number line. Despite this, there were still a sizeable number (approximately 25%) that did not complete this task successfully.

DISCUSSION

The results from the last two questions indicated that many adult learners do not treat, or did not classify, fractions as numbers. It is difficult to determine the reasons for this observation without a follow-up interview. However, it is plausible that there was some confusion surrounding the use of the term 'number', since some students clearly interpreted it to mean only whole numbers (Question 4). This observation would appear to be partially confirmed by the results to Question 5, which indicated that thirteen students in the sample could only plot a whole number (4) at the correct position on a number line. A majority of students could plot successfully both proper fractions and mixed numbers on a number line.

From these data, a notional hierarchy appeared to exist. For example, some responses did not relate fractions to numbers at all. These responses would form the early stages of a hierarchy. Growth through the hierarchy would consist of responses which gradually came to recognise fractions as possessors of number properties. For example, the response 'if you want to call fractions numbers' indicates some growth towards acknowledging this. There was also some minor evidence to suggest that students found fractions in mixed-number form slightly more difficult to handle than proper fractions. Only at the very highest levels did responses indicate that the number of fractions between two integers was infinite.

EQUIVALENT FRACTIONS

Question 6 and 7 dealt with the 'equivalent fractions' aspect of fractions.

QUESTION 6

Question 6 required students to compare two equivalent fractions, but placed the fractions into a familiar situation, such as comparing two equal portions of a cake.

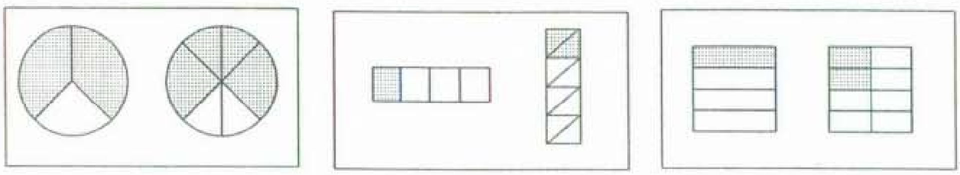
> ***Would you rather have 2/3 or 10/15 of a cake you particularly liked? Explain why?***

There may have been some confusion caused by placing fractions in a familiar everyday context, since many students responded as if dealing with a real cakes. This may have misled some students who appeared to make their decisions based on personal, rather than mathematical reasons, e.g., "I have a dainty mouth". In addition, some students indicated vivid imaginations with respect to the context of the problem; and this could often be to the detriment of the mathematics of the situation. For example, one student (DS) wrote: "I would prefer 2/3 of a cake because by the time you cut it (especially if a fresh sponge) (with cream and strawberries) into 15 pieces you would end up with a pile of crumbs". However, overall, a majority of adult learners' responses indicated that the adult learners in this sample did equate 2/3 to 10/15.

QUESTION 7

Question 7 asked students to select equivalent fractions from a variety of models of fractions.

> ***Suppose you saw these diagrams in a textbook. What could you tell from them?***



In general, the responses to this question indicated that a majority of adult learners related fractions to area diagrams. However, there was a considerable difference in the number of responses which stated that the diagrams represented fractions to the number of responses which stated that the diagrams represented equivalent fractions. It is plausible that many more students may have identified the fractions as equivalent, but omitted this detail in their answers, or did not feel obliged to put such an 'obvious' statement in their answers. It is of course also possible that they did not 'see' that the fractions were equivalent or related in any way.

DISCUSSION

The responses to the above two questions indicated that adult learners' responses could be classified into a notional hierarchy. In Question 7, responses which focused only on the fact that the diagrams were representations of fractions would be ranked lower than the observation that the diagrams represented equivalent fractions, i.e., diagrammatic equivalence. The responses to Question 6 also suggested a notional hierarchy. In this case, there were a number of responses which did not acknowledge that two fractions were equivalent. This type of response would be considered to be at a low level. Responses which clearly identified the fractions as equivalent would be ranked at a higher level. However, the effects of placing two equivalent fractions into a familiar context may have misled some adult learners, since they treated the situation as if working with real cakes, rather than focusing on the mathematics of the problem.

In general, the above two questions indicated that a majority of adults in the sample could recognise equivalent fractions in both (i) diagrammatic form (Question 7); and, (ii) symbolic form (Question 6). However, there was a greater variety of answers to Question 6 than Question 7. It is plausible that the placement of fractions into a familiar context may have confused some students.

CONCLUSION

Overall, the analysis of this research indicated that students' responses to the seven questions were of varying quality (see Appendix A for more details). Responses could be grouped, based on similarity of response, into a comparatively small, but coherent set of categories or classifications for each question. These groupings appeared to form a notional hierarchy.

In the early stages of the hierarchy, fractions were related to objects. This was first noted in the responses to Question 1, but continued to occur throughout the remaining themes. Cakes and pies where often drawn on students' answer sheets in both Questions 1 and 3, although the latter question may have prompted this approach. Irrespective of this, the responses to Question 3, when grouped, demonstrated a clear hierarchy. For example, some students drew three 'cakes' and proceeded to work with the diagrams as an aid to problem solving. Responses to this question suggested that some students not only needed the diagrams, but they needed to work on the actual diagrams as if they were real cakes, to find the answer. Further evidence to suggest that students related fractions to objects or diagrams was noted by the comparatively widespread acceptance of the area diagrams shown in Questions 2 and 7.

The middle levels of the hierarchy would appear to consist of an intermediary stage in which students appear to be easily confused or misled by fractions which are not presented in a typical area model form. For example, some students did not consider there to be any numbers between 2 and 3 (Question 4) or were confused when asked to plot two fractions on a number line (Question 5). Despite this, a majority of students could identify successfully equivalent fractions diagrammatically (Question 7) and symbolically (Question 6), although some students appeared to be misled by the placement of fractions into a context.

Only at the very highest levels of the notional hierarchy were fractions treated as numbers. This is partially confirmed by the answers to Questions 1 and 2, in which only a comparatively small number of responses indicated that fractions and two numbers which are divided were interchangeable. It is also worth noting that the number of responses increased in the 'No response' and 'Uncodable' sections for the questions which required more abstract answers, e.g., Question 4.

Finally, it is clear that the sample of adult learners enrolled in TAFE had difficulties with fraction concepts. In many cases, only one-third of the sample could answer the questions at the very highest levels. Typically, the students' responses exhibited a range of strategies, many of which highlighted weaknesses in their understandings. There may have been many reasons for this. For example, some students may have come from non-English speaking backgrounds, and may have had difficulties in understanding the question or providing an answer in language which they were still acquiring. However, the quiz was given in the usual mathematics time-slot and administered by the mathematics teacher; so it would seem fair to assume that most students would be operating within a mathematics framework with respect to the quiz.

Given all of the above, it is now relevant to compare the responses obtained in this study, with those of younger students' responses in the Kerslake study. There are two main purposes for doing this:

(i) to seek further evidence for the existence of a notional hierarchy, i.e., are the adult learners' responses similar to the children's responses?

(ii) to investigate any significant differences between adults' perceptions of fractions to those of younger children, i.e., are the adult learners' responses a 'continuation' of the children's responses or do they offer a different perspective?

COMPARISON WITH KERSLAKE'S FINDINGS

A comparison between the questions in the first phase study and the subset of questions directly related to the Kerslake work is now presented. Responses to both studies have been presented in summary tables, which are similar to those produced in Appendix A. A discussion section follows each table. Results have been converted to percentages in both cases and rounded to the nearest whole number. In some cases totals do not equal 100% due to rounding error. 'No response' and 'uncodable' classifications have been combined for this section of the analysis.

QUESTION 1

How would you explain to someone, who didn't know, what a fraction was?

Table 3.2 gives a comparison between the children in Kerslake's study and the TAFE adults.

TABLE 3.2

Comparison between Kerslake (1986) and adult learners' responses to Q1 on the Fraction Quiz for TAFE students

CHOICE OF MODEL	KERSLAKE (1986) PERCENTAGE OF CHILDREN (n = 23)	PERCENTAGE OF ADULTS (n = 103)
No response/uncodable	0	7
Couldn't say	17	2
Part of a whole	43	42
Drew a diagram or showed area or drew 1/2	n/a	8
One number over another	26	4
Part of a number	13	27
A percentage or decimal	n/a	6
Mathematical expression for division	n/a	5
Total	99	101

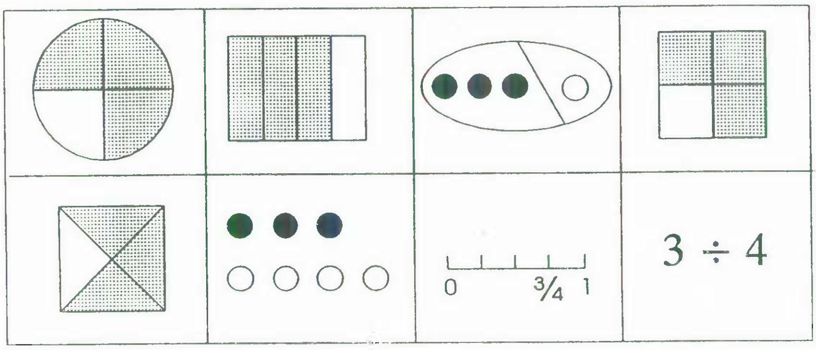
The table indicates that both the Kerslake and TAFE studies shared similar responses in all but three categories. Other classifications observed in this study were not reported in the Kerslake book. A comparatively smaller percentage of children compared to adult learners described fractions as 'part of a number', providing some justification for the order of the classifications. Similarly, fewer adult learners, compared to the study on children, responded with 'couldn't say'. Of interest is that a relatively large percentage (42-43 %) of both populations who referred to a fraction as a part of a whole. This implies that many of the adult learners in this study have not developed beyond this notional description of fractions.

The 'drew a diagram' category was not formally noted by Kerslake. It is possible that this classification did not occur with the children in the study, or that due to the age of the children, it was absorbed in the 'part of a whole' classification. However, it is possible that this technique could be related to the maturity of the students or it could be an indication of the access of adults to a greater range of problem-solving strategies than available to children.

In general, it is clear that there are two distinct methods of dealing with fractions - and this was observed in both the Kerslake and TAFE studies, namely, (i) the notion that a fraction was 'part of a whole'; and, (ii) the more formal notions of fractions as having number properties. While it is reasonable to assume that adults have had more dealings with percentages, decimals and simple operations in their everyday lives than the younger students, only a comparatively small percentage (5%) of adult learners in this sample stated this.

QUESTION 2

> ***Which of the following cards would help someone to understand what the fraction 3/4 is? Explain why.***



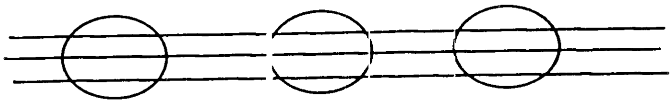
Although both studies indicate that students, irrespective of age, accept all the 'area' models, the Kerslake study revealed a greater spread of acceptance of other models. The adult learners, on the other hand, preferred the 'area' models to the exclusion of other models. One plausible explanation for this observation may be that the children had been exposed to a wider variety of models than the adults. However, in the 'rejection' case, no child rejected the area models, indicating that there appeared to be a strong affinity for the 'area' models demonstrated by both groups. In both cases only a comparatively small number of children and adults accepted the division model. This possibly confirms the difficulty associated with this interpretation of fractions.

QUESTION 3

> ***You have three cakes. Could you share them equally between five people? Explain what you would do. (Use diagrams if necessary).***

This question was asked as a follow-up question to a similar one involving 3 ÷ 4 and 3/4 (Kerslake, 1986, p. 31), and was incorporated into the discussion. A direct comparison between the Kerslake study and the TAFE students responses is therefore inappropriate and a summary table could not presented. However, the results indicated that ten out of fourteen students were able to answer this question correctly. Six students out of the ten could also see that 3/5 was the same as 3 ÷ 5. However, there were four children who divided the cakes into halves first and then further subdivided the remaining half. They did not arrive at 3/5.

It is important to note that many responses to the companion question in the Kerslake study, which asked students to divide three cakes by four people (Kerslake, 1986, p. 15), included diagrams. One example, in particular, stands out. One child (KP), (Kerslake, 1986, p. 15) who had difficulty with the task, drew diagrams and used a similar strategy to the following:



This particular method of sharing was not used by any of the TAFE students, and represents an important difference between the young children's responses in the Kerslake study and TAFE students' responses. The implication is that adult learners may have taken on board the importance of sharing and their focus of attention is on other features of fractions.

Overall, there were many similarities observed between the students in Kerslake's study and the TAFE adults on these two questions. For example, students in both samples drew circles and then proceeded to divide them into halves, sometimes dividing the remaining half again in order to obtain the correct answer. Allowing for the difference, while 3 ÷ 4 is a much easier question to answer than 3 ÷ 5, the following TAFE student's response is almost identical to the response (SB) in Kerslake (1986, p. 16).

Both studies indicated that many students who drew diagrams and divided the cakes into halves could obtain correct answers. However, in the 3/5 question, four children and 11 adults could not obtain the correct answer, e.g., some adults wrote '1/2 + 1/5' and no more. This suggests that they may have been unaware of relationships involving the fractions 1/2, 1/5 and 1/10, or could not add two fractions with different denominators.

QUESTION 4

> ***How many numbers are there between 2 and 3? And between 0 and 1?***

Table 3.3 provides a comparison between the children in Kerslake's study and the TAFE adults. Although relative percentages have again be used, only seven children were asked how many numbers were between 2 and 3 in the Kerslake study (1986, p. 18).

TABLE 3.3

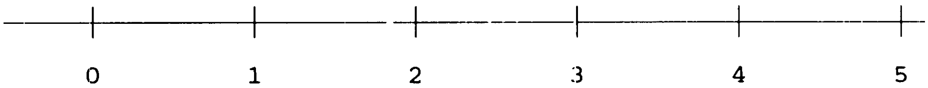
Comparison between Kerslake (1986) and adult learners' responses to Q4 on the Fraction Quiz for TAFE students

RESPONSE	KERSLAKE (1986) PERCENTAGE OF CHILDREN (n = 7)	PERCENTAGE OF ADULTS (n = 103)
No response/uncodable	n/a	19
0 or zero or none	14	17
No whole numbers, only fractions or decimals	n/a	18
One or one whole number	n/a	3
A small number 4 or 9 or 10	57	11
A large number or 1000 or 999	29	7
Infinite		25
Total	100	100

The table indicates that both the Kerslake and TAFE studies shared similar responses in three categories. The other classifications observed in this study were not reported in the Kerslake book. The table suggests that in both studies, some students related the word 'number' to whole number. Although the Kerslake study did not state explicitly the 'no whole numbers, only fractions or decimals' category, Kerslake (1986, p. 18) noted that one child who rejected the notion that there could be any numbers between 2 and 3, stated that there were 2 numbers (3 and 4) between 2 and 5. Additional information confirming this type of response was also obtained as not one child in the Kerslake study suggested that there were an infinite number of numbers between 0 and 1. However, it is also plausible that this type of response was outside the experiences of the children in the study. This is partially confirmed since only two out of seven children suggested that there were 'a large number'.

QUESTION 5

Where would the number 4 go on this number line? And the number 3/5? And the number 1 1/5?



Tables 3.4 and 3.5 compare the responses between the Kerslake study and the TAFE students on final two parts of Question 5, respectively.

TABLE 3.4

Comparison between Kerslake (1986) and adult learners' responses to Q5 on the Fraction Quiz for TAFE students

FRACTION 3/5

RESPONSE	KERSLAKE (1986) PERCENTAGE OF CHILDREN	PERCENTAGE OF ADULTS
Placed at 3/5	14	94
Placed at 3.5	36	-
3/5 of line length	50	6

TABLE 3.5

Analysis of adult learners' responses to Q5 (1 1/5) on the Fractions Quiz for TAFE students

FRACTION 1 1/5

RESPONSE	KERSLAKE (1886) PERCENTAGE OF CHILDREN	PERCENTAGE OF ADULTS
Placed at 1 1/5	64	99

Both the students in the Kerslake (1986, p. 33) study and the TAFE students had more difficulty plotting 3/5 than 1 1/5. It is possible that once students could locate a whole number, such as '1', it was a comparatively easy step to pinpoint a number, such as 1 1/5, since this number is just over 1. However, it would seem to be considerably more difficult to locate some proper fraction, such as 3/5, using this technique, since it is still not comfortably close enough to any established whole number. In addition, the positioning of 3/5 as representative of 3/5 of the length of the line indicated that some students partitioned the total length of the line into fifths and then plotted 3/5 of the entire length of the line.

QUESTION 6

> ***Would you rather have 2/3 or 10/15 of a cake you particularly liked? Explain why?***

Table 3.6 compares the students' responses of the Kerslake (1986) study to that of the responses of the TAFE students to Question 6.

TABLE 3.6

Comparison between Kerslake (1986) and adult learners' responses to Q6 on the Fraction Quiz for TAFE students

RESPONSE	KERSLAKE (1986) PERCENTAGE OF CHILDREN	PERCENTAGE OF ADULTS
Equal/equivalent	64	88
2/3 is larger	22	11
10/15 is larger	14	1

Although 64% of children and 88% of adults recognised that the two fractions were equivalent, there were still 36% of children and 12% of adults who did not equate the two fractions as being the same. Kerslake (1986, p. 34) offered the following explanation to the '10/15 is larger' phenomena: "GW's difficulty in reconciling what he knew about fractions and his feeling that 10/15 must be bigger because of the separate numbers being five times bigger". This indicates that, at some early stage, the student had abandoned his intuitive notions of 'sharing' and began to rely on number 'facts'. However, the student's response indicated that this 'jump' was probably premature, and hence, the arithmetic used was associated more closely with whole numbers than fractions. The fact that only one adult selected this response may indicate that most adults do recognise equivalent fractions in either the 'area' form or the numerical or symbolic form. Although some scripts indicated that this conclusion was reached only after the student cancelled 10/15 down to 2/3. Similarly, those students who selected '2/3 larger' "all referred to the fact that there was only one piece left if one took the 2/3" Kerslake (1986, p. 35). This implies that some students focused on the real-world setting of the problem, rather than the mathematics of the situation. A comparatively higher percentage of children selected 2/3 (rather than 10/15).

QUESTION 7

Suppose you saw these diagrams in a textbook. What could you tell from them?

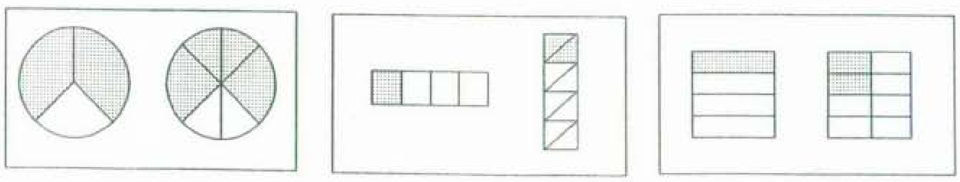


Table 3.7 shows the results of the students in the Kerslake study and the TAFE adults on Question 7.

TABLE 3.7

Comparison between Kerslake (1986) and adult learners' responses to Q7 on the Fraction Quiz for TAFE students

RESPONSE	KERSLAKE (1986) PERCENTAGE OF CHILDREN	PERCENTAGE OF ADULTS
Diagrams not the same	24	7
Diagrams represent fractions	not stated	7
Diagrams represent equivalent fractions	76	86

A majority of children in the Kerslake study (76%) and the TAFE adults (86%) recognised the above diagrams as equivalent fractions. Despite this, nearly one quarter (24%) of children and 7% of TAFE students did not realise that the diagrams were the same. This would imply that there is some stage at which 'sharing' is related to only the same number of pieces or the size of the piece, but not both.

It is also worth noting that a much higher proportion of both groups of students recognised fractions as equivalent in diagrammatic form rather than symbolic form, e.g., compare the results of Question 6 to Question 7.

CONCLUSION

Overall, the responses to the phase one study suggest that responses to fraction questions form a notional hierarchy, which is, in general, independent of age. Although only exploratory in nature, the phase one study indicated considerable similarity between categories of responses for both children (based on results of the Kerslake (1986) study) and 103 adult learners enrolled in TAFE. Although the proportions tended to be more widely spread with the children, and there were some questions in which a broader selection of answers appeared for one group, there was general agreement across all themes and most questions.

In particular, the responses to Question 1, indicated that a sizeable percentage of both groups (approximately 43% for both groups) could only describe a fraction as 'part of a whole'. This description continued to be used by many students on a variety of questions throughout the quiz. For example, some students could only divide cakes by drawing diagrams and then relying on these diagrams to solve problems. In many cases, it appeared as if this was the only method that was available - even when the strategy failed, e.g., students cut the cake into halves and then could not adapt the technique to distribute the rest of the cake equally. This type of example was observed by both adult and child learners. Similarly, both groups indicated a strong dependence on the area models as exemplars of fractions. These type of responses would be considered to occur in the early parts of a hierarchy.

The middle sections of the hierarchy appeared to consist of responses in which students were required to treat fractions as numbers, but were not able to do this successfully. Although a higher proportion of adult learners were able to handle this interpretation, it was clear that there were students in both groups, who had difficulty understanding the concept that fractions could be classed as numbers. For example, some students could not plot two fractions on a number line (Question 5) or did not consider that there could be any numbers between any two whole numbers (Question 4). Despite this, many students, in both studies, could interpret fractions correctly when area diagrams were provided as part of the question (e.g., Question 2 and Question 7).

It is evident that the ability to deal with fractions independently of diagrams, i.e., as numbers, occurs via a gradual process, and it is only in the final stages of the hierarchy that students are not confused or misled by visual distracters, such as those implicit in the pizza/cake questions. It is evident that responses at this level do not relate necessarily to the word 'number' to mean only 'whole numbers' and can accept the feasibility of there being an infinite number of numbers (fractions) between any two whole numbers. Again, this observation was confirmed partially by both studies, since only one-third of adult learners and no children responded with this type of answer to the theme 'fractions as numbers'.

In general, the Kerslake study and the phase one study have highlighted and differentiated the understanding of fractions into two broad classifications. The first involves the use of diagrams on which some students appear to depend to be able to solve problems. The second involved viewing fractions as possessors of number properties. However, since this study was only exploratory in nature, it is unclear if either approach aids or hinders further development in understanding fraction concepts. In addition, this study did not investigate the full range of applications of fractions, such as operations on fractions. The influence of placing fractions (including standard operations) into familiar situations is also worthy of further research. These issues are taken up further in the second phase of the study.

CHAPTER FOUR

TEST DEVELOPMENT AND DESIGN OF THE MAIN STUDY

... care must be taken not to categorize students too readily based on a few questions.

Watson *et al.* (1988, p. 278)

INTRODUCTION AND ORGANISATION OF THE CHAPTER

Findings based on the phase one study imply that it is possible to categorise adult learners' responses to fraction questions. Furthermore, it became evident that such classification has the potential to form a hierarchy of understanding of adult learning with respect to fractions. However, the preliminary study was largely exploratory, and, as the above quote suggests, further research is necessary before such conclusions can be verified. The research did, however, raise several issues worthy of further research. As a consequence, and to guide this work further, a number of specific research questions are provided which are addressed in the following chapters. These questions, to be answered by considering responses provided by adult learners enrolled in mathematics courses at a TAFE college, are:

(i) Can adult learners' responses to questions be grouped into classes of similar responses; and, be interpreted within the SOLO Taxonomy?

(ii) What understandings do adult learners have of fractions?

(iii) What strategies do adult learners employ to compare fractions?

(iv) What techniques/strategies do adult learners apply to questions involving operations on fractions?

(v) Do adult learners' performance on similar questions placed into either a context-free or in-context situation differ? If so, how?

(vi) Do adult learners' descriptions of fractions provide an indication of how they might perform on a range of fraction questions?

(vii) Is there any statistical evidence to support or refute the qualitative analysis?

The chapter is divided into three broad sections. First, an explanation is presented which gives insight into the development of the test items used in the main study. Second, the composition of the two different groups of TAFE students to which the fraction quiz was administered is presented. Finally, the qualitative and quantitative aspects of the data analysis plan are presented. This includes analysis of the written fractions quiz, and details of a structured interview technique, and use of the statistical package, designed to explore category-type data.

TEST DEVELOPMENT

Based on the results from the preliminary study and literature review, a new quiz was designed with a tighter focus. Several questions in the earlier study delivered ambiguous responses, or did not offer students the opportunity to tackle questions typically found in many textbooks. Additional questions were constructed in an endeavour to 'tease out' more fully the tendency, observed in the preliminary study and reported in the literature review, by some students to work with diagrams. Also of interest was the influence of the question context on performance. Adults have many life experiences involving fractions in some form and it is appropriate to explore whether there are differences when questions are placed in a familiar real-world context (referred to as in-context questions), as opposed to questions that resemble traditional textbook exercises and involve the manipulation of fractions (referred to as context-free questions).

The main study focused on four main concepts or themes with respect to fractions. The first three were: understanding fractions, comparison of fractions, and operations on fractions. As indicated above, these three aspects were considered within both in-context and context-free situations. The fourth theme - description of fractions - was also added. A summary of which questions applied to which aspects is indicated below in Table 4.1. Where some questions clearly include more than one aspect, the main aspect is the one classified.

Table 4.1 shows the various aspects of fractions and the questions to which they relate. It is worth noting that Question 1 could be interpreted equally as in either a context free situation or that of being placed in a context (*) and for this reason it has been included in both of these aspects.

TABLE 4.1

Aspects of fractions by question in the main study

ASPECT OF FRACTIONS	IN-CONTEXT QUESTIONS	CONTEXT-FREE QUESTIONS
UNDERSTANDING FRACTIONS	Q5, Q13, Q14 (equivalence)	Q4, Q6a, Q6b, Q8 (sharing)
COMPARISON OF FRACTIONS	Q7, Q9	Q2a, Q2b, Q3a, Q3b
OPERATIONS ON FRACTIONS	Q11, Q12, Q15, Q16	Q10a, Q10b, Q10c, Q10d, Q10e, Q10f
DESCRIPTION OF FRACTIONS	*1	*1

QUESTION DESCRIPTION

A copy of the complete quiz is provided in Appendix B. A brief explanation of each question follows.

QUESTION 1

Imagine you are writing a dictionary of Mathematical terms. Explain, giving as many details as possible, how you would describe what a fraction is.

This question replaced the original Question 1 on the preliminary study, i.e., *How would you explain to someone, who didn't know, what a fraction was?*

The aim of this question was to minimise the problem of adult learners responding to this question at a considerably lower level of sophistication than which they were capable. By proposing the 'dictionary', it was hoped that students would give more appropriate definitions which reflected more accurately their true understandings of the definition of a fraction. Similarly, adding the words "giving as many details as possible" was an effort to elicit as many details as possible from the student. This question gives students the opportunity to provide a detailed answer, which is not limited by the constraint "to someone who didn't know". The phrase caused some confusion as at least one student indicated that they understood enough about fractions to avoid showing "someone who didn't know" that a fraction could be interpreted as the division of two numbers. Instead, students should be able to select from a broad range of options which are no longer constrained by this condition. In addition, this question appears to offer the greatest potential to differentiate between intuitive responses and algorithmic responses noted in the first phase of the study. The question was asked first in order to quiz students regarding their descriptions of fractions before they had attempted other fraction questions.

QUESTION 2

Compare and contrast what is meant by:

a. $\frac{5}{7}$ ***and*** $\frac{7}{5}$***?***

b. $\frac{2}{3}$ ***and*** $\frac{3}{5}$***?***

QUESTION 3

Place in order from smallest to biggest:

a. $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{4}$

b. $\frac{2}{3}$, $\frac{5}{7}$, $\frac{3}{4}$

Although Question 2 and 3 were also designed to differentiate between the two approaches noted in the preliminary study, the main purpose in including them was to determine what difference, if any, item complexity made to students' responses. For example, if there was a substantial difference between the methods observed, then asking a less complex question (Q2a and Q3a) and more complex question (Q2b and Q3b) on the same aspect of how students compare fractions should highlight such differences. Students, who use an 'intuitive' or 'diagrammatic' schema, may need to adopt the more 'symbolic' form for the latter questions, otherwise they may not be able to answer it appropriately. The assumption behind this statement is that the more complex a fraction, the more difficult it is to perceive it 'pictorially'. Even if students could compare two fractions pictorially, (such as in Q2), it would be more difficult, to keep track of three 'diagrams' (such as in Q3b), without resorting to some technique for keeping a tally or score on each. This may require the student to adopt equivalent fractions or to convert the fractions to decimals or percentages. This question was designed to seek out students who would attempt to apply the 'bigger the denominator, the smaller the fraction' rule which appeared in some students' responses in the preliminary study.

QUESTION 4

You have 9 pizzas to be shared between 15 people. Describe how you would do this.

QUESTION 6

a. ***You have 3 cakes to be shared between 5 people. Describe how you would do this.***

b. ***You have 2 cakes to be shared between 5 people. Describe how you would do this. (You may use diagrams if you wish).***

QUESTION 8

You have 5 watermelons to be shared between 3 people. Describe how you would do this.

Question 4, 6 and 8 were all different versions of Q3 in the preliminary study, which was: *"You have three cakes. Could you share them equally between five people? Explain what you would do. (Use diagrams if necessary)".* They are referred to loosely as the 'pizza' questions.

There were several reasons for asking three versions of the one question. One of the major implications from the first study, was that more detail was required from the 'pizza' questions asked. For example, Question 4, concerning 9 pizzas and 15 people, was asked to determine if some students would literally sketch 9 pizzas and subdivide them into fifteenths. It was hoped that students would be motivated by the inefficiency of this approach and, therefore, adopt a more efficient method, such as equating this question to an easier one, focusing on the mathematics of the situation and not the context, or (eventually) cancelling' down to 3/5.

A simpler version of this question (Question 6a) was also asked and provided students with the opportunity to focus on the mathematics of the situation (3/5) without the added complexity of cancelling.

Question 6b was asked for two reasons. The first reason was to identify students who used the 'halving' strategy in the pizza questions. If students attempted to apply this to Question 3b, they would have insufficient pizza (or cake, in this case) in the first round of sharing. The second reason was to identify if prompting (in the question) influenced the solution process.

Question 8 was the logical opposite of Question 6b. This question was asked to see how students would cope with the reverse situation, namely, if there were more pizzas (or watermelons, in this case), than people.

QUESTION 5

Complete the following:

$$\frac{14}{16} = \frac{\_}{24}$$

Question 5 asked students to compare two fractions written in a typical text-book style, i.e., in a 'context-free' setting. However, the problem has been made more difficult than usual since the first fraction is not given in simplest form and there is no whole number that can be used to multiply both the numerator and denominator. As such, it is likely that this question will provide information on the two different approaches noted in the preliminary study. If students used a rule, such as 'whatever you do to the top you do to the bottom' without any understanding of the origins of this rule, then the answer would probably be incorrect. Also, it was anticipated that some mature-age students may resort to 'completing a pattern' or use the 'add on at all costs' method as identified by Hart (1981) with her work on ratio.

QUESTION 7

You have two recipes to choose from to make a drink of punch for a party. One recipe calls for 3 bottles of sherry and 6 bottles of soda water. The other calls for 2 bottles of sherry and 5 bottles of soda water. Which is the stronger drink. Why?

QUESTION 9

Two people, who have different occupations, save a certain part of their salaries each week. The first person saves 1/5 of their salary. The second person saves 1/3. Is it possible for each to save the same amount? Give details.

Question 7 and 9 were aimed at targeting the practical implications of equivalent fractions, i.e., these questions were placed in a familiar context. Of interest, was which strategies students might adopt and possible reasons for the choice. For example, while some students may use equivalent fractions to solve Question 7, using either decimals or percentages is also feasible. It is possible that if some students' command over fraction notation was insubstantial, then other methods, such as decimals and percentages, might be used. Of course, this problem can also be solved by using ratios without direct recourse to fractions.

In the case of Question 9, the problem involves comparing two fractions, '1/3' and '1/5', in a way that many students may not have thought about previously. It was felt that this question was potentially a key question in the test.

QUESTION 10

Complete the following:

a. $\frac{1}{2} + \frac{1}{4}$ *b.* $\frac{3}{5} + \frac{2}{7}$

c. $\frac{3}{4} - \frac{2}{3}$ *d.* $\frac{5}{9} \times \frac{3}{5}$

e. $\frac{1}{2} \div \frac{1}{4}$ *f.* $\frac{2}{3} \div \frac{5}{9}$

Question 10 provided an opportunity for adult learners to perform fraction tasks, such as those that would be encountered in traditional textbooks, i.e., the four operations (+, -, x, ÷) on fractions were placed within a 'context-free' environment. The preliminary study did not focus on this aspect of fractions, and there is little information with respect to operations on fractions and adult learners. However, the literature reported in this study highlighted that many children viewed fraction arithmetic as a series of rules to be applied in Mathematics examinations with little understanding of the reasons behind them.

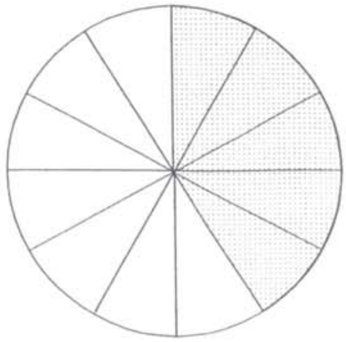
QUESTION 11

At a recent function, the punch bowl was 2/5 full. At the end of the function, the bowl was 3/8 full. How much punch was consumed during the evening?

Question 11 was designed to investigate strategies that involved the practical implications of equivalent fractions. There is value in comparing the results of Question 11 to its comparable part in Question 10 (Q10c).

QUESTION 12

A student was asked to add 1/6 to 1/4. She drew:



and then concluded that: $\frac{1}{6} + \frac{1}{4} = \frac{1}{10}$

Discuss her conclusion.

Question 12 first appeared as an example in Dickson (1984, p. 304), and was used to demonstrate that at least one student was unaware of any conflict in their answer to this question. The purpose of presenting this question to students, was to place TAFE students in a situation in which they were required to seek an explanation for the above observation. It is also possible that some students may not be aware of the apparent contradiction in the given solution.

QUESTION 13

If I add 2 to both the top number (numerator) and bottom number (denominator) of 1/5, describe, in detail, what will happen. [I'm looking for more than just the answer].

QUESTION 14

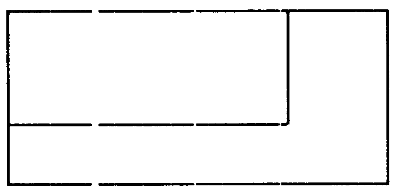
If I have the fraction 2/3 and I double the numerator (top number) and the denominator (bottom number), describe the effect this will have on the 2/3. Why?

Both these questions address special features of fractions. For example, Question 13 was designed to be open ended and it was anticipated that some students might generalise their answers beyond the immediate response of adding 2 to both the numerator and denominator. Question 14 asked students to consider the 'real' meaning of equivalence when dealing with fractions as numbers.

In both cases, the ability to interpret the questions with respect to treating fractions as numbers, and not as diagrams, would appear to be more appropriate. Of interest is the contrast between the possible answers to Questions 13 and 14.

QUESTION 15

A carpet piece is placed in the corner of a room as shown. The carpet is found to go along 4/5 on one wall, and 2/3 of the other wall. What fraction of the floor does the carpet cover?



Question 15 combines the concept of area with fraction multiplication. Although the context should be familiar to most adults, it is possible that some students may not have considered the dimensions of a figure as a fraction of a given length, although unspecified numerically. It is feasible that some students may focus on the reality of the problem and place actual lengths on the diagram in order to solve it. It is also possible that some students may work with the fractions irrespective of the context.

QUESTION 16
What does $\frac{3/4}{2/3}$ mean?

To attempt Question 16 required students to use fractions as numbers. The question was asked to see (i) what strategies were employed, and (ii) how did the complexity of the item influence the procedures used or quality of answers given.

SUMMARY

In general, the above items were selected in order to capitalise on the findings noted in the literature review and the preliminary study. The questions were seeking to clarify issues raised or to elicit further evidence concerning fraction understanding. The items were designed to be incorporated into four broad research themes, namely, understanding fractions, comparison of fractions, operations on fractions, and description of fractions. In addition, questions were also devised to provide students with the opportunity to attempt questions placed in each of the first three themes in two main ways, namely, context-free (such as in typical textbooks) or in-context (such as placing fractions into a familiar context). The questions, when classified and selected accordingly, provide an effective and efficient structure which provides the necessary depth and variety to enable a thorough analysis to take place with respect to the research issues noted at the beginning of the chapter.

THE STUDENT SAMPLE

COURSE BACKGROUND INFORMATION

Previous studies into students' understandings of fractions have primarily dealt with school-age children and they tended to present their results based on age. However, when dealing with adult learners age is an inappropriate criteria, and possibly time at school or time since formal mathematics study seem more useful aspects that could offer a broader basis to explore diversity. Conclusions from the last two chapters indicated that adult learners and child learners share both similarities and differences, when it comes to responses to fraction questions. The provision of two different post secondary groups of students at a TAFE college provided a convenient platform on which to administer the fraction quiz to investigate how adult learners approach fraction tasks. Students in the first group (Associate Diploma students) tend to be younger, having recently completed Year 12 by the traditional means. Students in the second group (Tertiary Preparation students) tended to be more typical of students who have not completed schooling and have usually been absent from formal study for several years.

STUDENT BACKGROUND INFORMATION

To provide a clearer picture of the sample used in the study, each student was asked to complete an information sheet (see Appendix C). This sheet provided background on gender, age (approximately), highest level of Mathematics previously completed and, the length of time (approximately) since they had last studied Mathematics. The information obtained from this survey is summarised below. In particular, the gender, age, highest level of Mathematics previously studied, and length of time since formal study are recorded. Throughout the analysis, the Associate Diploma students (n=55) are identified as AD students and the Tertiary Preparation students (n=52) as TP students.

GENDER OF STUDENTS BY COURSE

Table 4.2 summarises the gender balance in the two groups.

TABLE 4.2

Summary of gender distribution between the AD and TP groups

GENDER	AD students (n=55)	TP students (n=52)
Female	24 (44%)	28 (54%)
Male	31 (56%)	24 (46%)

The table indicates that the gender distribution is approximately evenly represented within each group. There were slightly more women in the TP course and this was expected since the overall philosophy of that course is to provide a pathway into university for those students who would not normally have been able to access university study.

AGE OF STUDENTS BY COURSE

Table 4.3 provides a descriptive comparison between the students' ages in the AD and TP courses.

TABLE 4.3

Summary of age distribution for the AD and TP groups

AGE	AD students (n=55)	TP students (n=52)
<21	36 (65%)	7 (13%)
21-30	9 (16%)	22 (42%)
31-40	7 (13%)	16 (31%)
41-50	3 (5%)	6 (12%)
51-60	0	1 (2%)

The information in this table confirms that school leavers are attending the AD programme in relatively large numbers, and that the TP programme is attracting older students. This was to be expected, since many tertiary institutions offer places based on a ranking system which tends to favour students who have completed traditional school pathways. As a result, there are more places in the Associate Diploma offered to Year 12 school leavers than mature-age students. In addition, the Tertiary Preparation course targets mature-age students, particularly those who have not completed a traditional Year 12 course.

LENGTH OF TIME SINCE COMPLETION OF MATHEMATICS BY COURSE

Table 4.4 is a comparison between students in the AD and the TP courses based on the number of years since they have encountered formal study of Mathematics.

TABLE 4.4

Summary of years since formal study in Mathematics for the AD and TP groups

YEARS SINCE LAST STUDIED	AD students (n=55)	TP students (n=52)
0	32 (58%)	1 (2%)
less than 3	8 (15%)	5 (10%)
3 to 11 years	4 (7%)	16 (31%)
12 years or more	6 (11%)	17 (33%)
Did not state	5 (9%)	13 (25%)

This table indicates that there are more students who have been absent from formal education for longer periods in the Tertiary Preparation programme than in the

Associate Diploma course. For example, 73% of AD students compared to only 12% of TP students have been out of school for less than three years. Again, this was to be expected, since the Associate Diploma is marketed as a viable option to university study by TAFE, and subsequently by many guidance officers in the local high schools. School leavers from the area are continuing, rather than re-entering, education. In contrast, many mature-age learners are preferring to re-familiarise and re-orient themselves for further study, by first enrolling in the Tertiary Preparation programme rather than directly into the Associate Diploma or other tertiary-level courses.

HIGHEST LEVEL ATTAINED BY STUDENTS BY COURSE

Table 4.5 is a comparison between the AD and the TP students based on the highest level of Mathematics previously studied prior to enrolling in TAFE in the year of enrolment.

TABLE 4.5

Summary of Mathematics background for the AD and TP groups

PREVIOUS HIGHEST LEVEL OF MATHEMATICS STUDIED	AD students (n=55)	TP students (n=52)
University	2 (4%)	1 (2%)
Year 12 Maths1/2	5 (9%)	3 (6%)
Year 12 Maths 1	17 (31%)	5 (10%)
Year 12 MIS	18 (33%)	10 (19%)
Year 11	0 (0%)	4 (8%)
Year 10	7 (13%)	23 (44%)
Year 9	4 (7%)	4 (8%)
Year 8	2 (4%)	1 (2%)
Did not state	0 (0%)	1 (2%)

The table indicates that a majority of students (77%) undertaking the Associate Diploma, had completed Year 12 or higher Mathematics, although approximately one-third (33%) of these had completed only a low level Mathematics course, e.g., Mathematics in Society (MIS). It was not possible to determine how successful the students were at completing these courses. In addition, the social justice stance of the Tertiary Preparation program can be seen clearly. Only 37% of TP students had completed Year 12 or higher successfully, with a majority of the students having completed Year 10 or higher successfully.

SUMMARY

In general, both Associate Diploma and Tertiary Preparation groups exhibited a number of similarities. For example, gender did not appear to play a significant role in choice of course, since there were approximately the same proportion of males and females in both courses. In addition, both groups had indicated a broad range of ages, mathematics backgrounds and numbers of years since formal schooling.

However, the major differences between the Associate Diploma cohort and the Tertiary Preparation students was the distribution of age and educational background. For example, there were considerably higher proportions of mature-age students in the Tertiary Preparation course compared to the Associate Diploma course, indicating that there was a substantial number of school leavers who were attracted to the Associate Diploma course. In addition, the level of mathematics background and the number of years since formal schooling, may indicate that many school leavers view the Associate Diploma as an entry ticket into university and not as an end in itself. It is worth noting that this last assertion is a primary aim of the tertiary preparation course.

DATA ANALYSIS PLAN

The data analysis plan can broadly be described under the descriptions qualitative and quantitative. The qualitative analysis includes grouping and coding responses to the written quiz, and interviewing a sample of students to provide further evidence or clarification with respect to their written responses. The interview structure was based loosely on the Newman (1977) technique. To supplement the qualitative analysis, and to validate the findings from it, a quantitative analysis using the Rasch method of partial credit was undertaken on the data. A brief description of each process follows.

QUALITATIVE ANALYSIS

The criteria for analysing responses was instigated via a two-phase approach. First, students attempted the Fraction Quiz in a typical classroom situation at the commencement of year, without access to calculators. Students had up to two hours within which to complete the test, although many submitted the test before this time had elapsed. Second, a small number of students volunteered to be interviewed within a few weeks of their completion of the quiz.

PROCEDURE FOR CLASSIFICATION OF RESPONSES

Responses, on a question by question basis, were classified based on grouping similar types together, i.e., key characteristics of similar responses were collated. This procedure was refined over the next few months so that initial classifications could be re-organised or redefined. Eventually, each classification could be identified by a 'title', i.e., the similarity of the responses could be grouped and given a label that specifically described the 'essence' of that category. In many cases, students used similar words or phrases, and the most common phrase was chosen to describe the category. This 'matching language' technique also provided a useful means with which to differentiate between the boundaries of disparate approaches. When this was not possible, responses were not classified in the same category. In some cases, these responses formed the basis of new classifications. Finally, responses which did not appear to have anything in common with any of the major classifications, or with any other responses, were coded as 'responses that require an interview for clarification'.

Second, the above procedure was repeated over the next eighteen months, on at least three separate occasions. Finally, the researcher was satisfied that the criteria for classifying a response into a particular category was justified, i.e., the classifications were 'stable' with respect to interpretation. In addition, a test of coder reliability was undertaken on all questions in the study which were coded by an experienced worker in the field. The technique employed involved the co-worker initially discussing the categories and an initial trial coding of 10% of randomly selected responses on each question. This process allowed for any anomalies or misunderstandings to be clarified. It also resulted in the refinement of some categories in some questions. The final test for coder reliability involved a random sample of 20% of all responses to all questions. The reliability was very high being in the range of 0.97 to 1.

Finally, in preparation for the statistical analysis to be undertaken, some classifications containing comparatively small numbers of students were combined, so that in general, only 4 to 8 categories were listed. The 'labels' established above for the various classifications form the basis of a summary table, which is presented for each question. In addition, and to indicate the diversity of responses, the number of students who responded with each type of answer is included in the summary table. Typical responses for each classification are presented following each table. For the convenience of the researcher, both the Associate Diploma group and the Tertiary Preparation group were analysed independently, and although the results have been presented on the same table, are identified as AD or TP students. Any significant differences between the two groups are addressed in the analysis. To aid the reader, where a notional hierarchy appeared to be evident, responses were presented in the order of 'no response' gradually increasing in complexity with 'best response' indicated in the last position in the table. When the responses where coded, if the students' responses could be interpreted in more than one classification, students were given the benefit of the doubt and allocated to the 'highest' possible ranking that their response would allow. This technique was to maintain a consistent approach across the analysis. Typical responses for each summary table are presented, and where appropriate, examples from the students' scripts and interviews are included.

THE INTERVIEW METHOD

Eight students were interviewed. The interviews ranged from a total of 40 minutes to 1½ hours and were audio taped and then transcribed. Students' responses from the interview sessions are indicated in the analysis where appropriate.

Many authors (Booth, 1984; Clements, 1980; Ginsburg & Opper, 1979; Harrison, Brindley & Bye, 1989) have discussed and developed interviewing techniques to elicit student responses that may generate different cognitive levels as they reply to questioning. For example, Booth (1984) (based on Newman (1977)) gave an eleven step procedure for encouraging student responses in this way. An adaptation on this technique was used in this study and is outlined below:

	Process	Questions Asked (Example)
1.	Reading	The question was read out loud to the student
2.	Comprehension Interpretation	The student is asked if they understand the question, e.g., What does the question mean?
3.	Strategy selection Skills selection	How did you do the question?
4.	Process	Explain to me as you go through the question, what you did
5.	Memory	(Check for recollection of intermediate steps)
6.	Encoding	The acceptance or rejection of the original answer is reached
7.	Consolidation	Are you happy with that answer?
8.	Verification	Could you have reached that answer another way? How?
9.	Conflict	Suppose I play devil's advocate and change ...
10.	Similarity	Can you see any questions similar to this one? Can you think of somewhere or something like this question? What would happen if we kept going?
11.	Generalisation	What would eventually happen if we repeated this process again and again?

On occasions, it was not possible, nor desirable, to stick rigidly to this outline, due to different incidents that arose during the interview sessions. At times, some students focused on an aspect of the problem that required modification to the procedure outlined above. However, in general, the above format was adhered to throughout the interview.

SUMMARY

Overall, the written responses to each question were classified by grouping similar types together. A 'matching language' technique was used in which similar words or phrases became the key ingredient of that category. The most common phrase was chosen to describe the category.

Subsequent coding was repeated over an eighteen month period and involved an experienced co-worker in the field on two separate occasions. The reliability was found to be between 0.97 and 1.

In addition, a small sample of students were interviewed based loosely on the Newman (1977) technique. This technique endeavoured to generate a variety of cognitive responses from students, and assisted in classifying students' responses.

Finally, where a notional hierarchy appeared to be evident, responses were ranked in order from 'no response' gradually increasing in complexity to 'best response'. Summary tables are presented throughout the analysis. Typical responses, which include examples of students' scripts and interviews are included.

QUANTITATIVE ANALYSIS

BACKGROUND TO THE RASCH MODEL

The Rasch model (Rasch, 1960, 1977) facilitates the analysis of ordinal data observed from scores obtained from problem solving tasks. Scored responses to individual steps involved in solving a complex question are determined irrespective of the sample (Masters, 1982, p. 149). As a consequence, individual responses to items can be awarded partial credit rather than a simple 'pass' or 'fail' scenario, i.e., students' responses to questions can be quantified. Complete answers gain full credit, and partial answers are awarded partial credit. The amount of 'correctness' is proportional to the amount of credit awarded. In addition, the Rasch model acknowledged that it was not essential that a hierarchy be firmly established prior to analysis. Clearly, these two parameters provide a sufficient basis on which to interpret the data from the present study.

APPLICATION TO THE CURRENT STUDY

Due to a constraint associated with the application of the Quest package (a computer program designed to perform Rasch analyses) (Adams & Khoo, 1993), it was necessary that when responses were analysed there must be the same number of categories in each question. To achieve this, categories were compressed into four main classifications for the responses to each question. These were then awarded the values 0, 1, 2, or 3 depending on the apparent complexity of the answer.

Prior to interpretation of statistics with respect to item complexity and placement of categories into a hierarchy, it is necessary to establish how well the data 'fits' the

Rasch model. In the Quest package, this is achieved by producing a series of statistics known as 'item estimates'. The two main parameters which judge this goodness of fit are the Infit Mean Square (Infit MNSQ) values, which should be close to 1, and the Infit-t scores, which should be close to 0 to indicate perfect fit. The Quest package presents this information in the form of a map of Infit MNSQ values. These are presented throughout the following chapters.

Once the goodness of fit of the data to the model has been established, there are two main statistics relevant to the analysis of this study. These are:

(i) Threshold values

The threshold values provide estimates of item step difficulty in a form that allows for comparisons across items and cases (students). Clearly, there are fewer students who are able to attain the more difficult steps to the more difficult questions, and, conversely, there should be more students who have the opportunity of reaching the early steps on the easiest questions.

The Quest package presents the information in the form of a single graphic which indicates the number of students on the left hand side of the page (as X's) and the coded corresponding response on the right hand side of the page, e.g., there could be 7 students (this would appear as XXXXXXX) who reached step 3 on Question 5. This would appear as 5.3 on the right hand side. These two extremes are able to be compared by reference to a single logit scale. The students performance represented by a case estimates score implies a 50% probability of success on being successful with items with the same logit scale.

(ii) Difficulty calculated by using the Tau option

The Quest package uses tau values in two ways. First, they are able to indicate the overall complexity of a question (referred to as Overall Difficulty of an item). In general, the more negative the tau values are, the easier the item. The more positive the tau values are, the more difficult the item is. This enables items to be ranked from easiest to complex. For example, item 4 with an overall difficulty of -.20 was found to be an easier question than item 5 with an overall difficulty of .46.

Second, tau values provide a measure of the comparative difficulties involved in reaching each response category from a previous response category of a complex problem (referred to as Step Difficulty values). It is not essential to know how a response category was reached in the first place, only the comparative difficulty of attaining the next response category, having reached the current one successfully. For example, the difficulty of moving from response category 0 to response category 1 would be defined as Step Difficulty 1. The difficulty associated with moving from response category 1 to response category 2 would be defined as Step Difficulty 2, and so on. Similar scores on two or more adjacent steps indicate that the difficulty of moving from one response category to the next is comparable. However, a sizeable change in degree, or a change of sign from positive to negative, from one step to an adjacent step indicates considerable difficulty in going from a lower response category to the next.

Step Difficulty values can only be compared across individual items, and not across multiple items. For example, a step 1 difficulty of 0.17 on one item is not comparable to a step 1 difficulty value of 0.17 on another item. It is only appropriate to compare the various steps (1, 2, 3) associated within each item.

CHI-SQUARE

In addition, chi-square (χ^2) calculations were undertaken with each question to determine if there were any significant differences between the two groups of students. The chi-square test determines if any differences between the two groups of students' responses are statistically different ($p<0.05$).

SUMMARY

The Quest package would appear to be a useful tool to analyse students' responses to fraction questions. This is provided that the two main parameters, which are used to judge the goodness of fit. The Infit MNSQ values should approximate 1, and the Infit-t values which should approximate 0. Once this has been achieved then the two main statistics which are relevant to this study are the threshold and difficulty (using the Tau option) values. Threshold values are useful to allow comparisons across items and cases (students), e.g., fewer students should attain the more difficult steps to the more difficult questions. Tau values determine (i) the overall difficulty of a question, and (ii) a measure of the step difficulty associated with moving from one response category to another response category in a complex problem.

Finally, chi-square (χ^2) was calculated for each question to determine if there were any significant differences between the two groups of students.

CONCLUSION

Following the administration of the fractions quiz to 107 TAFE students, the responses to the quiz were analysed on the basis of similarity of response on a question-by-question basis. Subsequent interviews were then carried out and additional statistical information was also incorporated. The results of this data analysis form the focus of the next few chapters. The results from the main study have primarily been structured around four main themes, namely; understanding fractions, comparison of fractions, operations on fractions and the final theme which related to students' descriptions of fractions. This is summarised in the following table.

TABLE 4.6

Representation of Data Analysis plan by chapter and research theme

Chapter Number	Research Theme
Five	Understanding Fractions
Six	Comparison of Fractions
Seven	Operations on Fractions
Eight	Description of Fractions
Nine	Overview of Fractions

Throughout the analyses, summary tables and typical responses to questions are presented. Where appropriate, additional material gained from the interviews is presented to highlight the types of responses classified into each category. Findings from the Rasch analysis are presented and an interpretation within the guidelines of the SOLO Taxonomy postulated. In addition, Chapter Nine provides an overview of the strategies adults use in addressing fractions questions.