PERSISTENT ERRORS IN INDICES: A COGNITIVE PERSPECTIVE

Colin Anderson

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To Narelle

and

Ross, Peter and Katrina

CERTIFICATE OF ORIGINALITY

I certify that the substance of this thesis has not already been submitted for any degree and is not currently being submitted for any other degree or qualification.

I certify that to the best of my knowledge any help received in preparing this thesis, and all sources used, have been acknowledged in this thesis.

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Colin J Anderson

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ABSTRACT

This thesis explores the understanding of indices shown by students in their senior years of secondary schooling. Quantitative and qualitative data are used to investigate how students perceive, and work with, an index as a raised number in arithmetic and in algebra. Earlier research focused largely on identifying errors students make. However, little has been done to explore the thinking being applied when those errors are made.

An extensive examination of errors, made by students preparing to enter the senior years of schooling, was undertaken initially through an analysis of responses to index questions in the N.S.W. School Certificate Moderator examination. This is the first time that an analysis of these data has been undertaken. Students were found to have considerable difficulty with such questions and many systematic errors were identified. The findings agreed, substantially, with those of other researchers. However, some errors reported by other researchers occurred infrequently.

A pilot study established that errors from the Moderator examination persist into the senior school and that errors emerging in a free-response format were similar to those in the multiple-choice items of the School Certificate. A free-response format could therefore be used in the main study. This format was preferable in that it is the common one used in senior work and allows for responses outside a prescribed set of answers.

The main study was carried out within themes relating both to content areas and to theoretical frameworks which may explain errors. This allowed a more purposeful and focused research of students' understandings than would an analysis which dealt with many, finely differentiated, errors. Senior students, sixteen and seventeen years of age, were tested using free-response format questions developed within each of five content related themes. The data, coded dichotomously, were analysed quantitatively by applying the Rasch model using QUEST software. Threshold values established the order of difficulty of items. Responses of individual students were examined and, on the basis of the persistent errors made, 25% of subjects were selected for interview. Routine strategies leading to persistent errors were identified for operations relating to: multiplication and division of expressions involving numerical bases; raising expressions to a power; simplifying algebraic fractions involving indices; and, taking the square root of expressions involving indices.

The data were further examined for evidence of levels of cognition. A classification of responses, based on the SOLO Taxonomy of Biggs and Collis (1982), was employed. Responses were coded polychotomously as: relational; high multistructural; and, lower multistructural or below. Item thresholds were used to compare step difficulties across items and category delta estimates used to compare step difficulties within items. It was established that students are responding consistently at the levels identified.

The form of understanding being applied was researched in terms of students' use of routine strategies as against accessing underlying concepts. This was done within the framework of 'relational' and 'instrumental' understanding as proposed by Skemp (1978). Students were found, almost invariably, to apply understanding which was instrumental in nature. The way in which the persistent errors emerge from these routine strategies was explored within the concepts of 'connections' (Shevarev 1946) or 'frames' (Davis 1984). Under these constructs, errors result when students adopt approaches which, while correct for certain other items of similar appearance, ignore important aspects of the question. Particular 'connections' or 'frames' were identified for the operations previously mentioned, namely: multiplication and division of expressions involving numerical bases; raising expressions to a power; simplifying algebraic fractions involving indices; and, taking the square root of expressions involving indices.