INTRODUCTION

Research in Mathematics Education has concentrated on attitudes, beliefs, Mathematics anxiety and test anxiety. Few studies have investigated all the emotions that children experience as they participate in Mathematics. The research described in the following pages attempted to identify the emotional states that a class of nine and ten year olds experienced as they participated in Mathematics lessons over a six month period.

The Research into Emotional States

The investigation begins by reviewing the relevant literature.

Chapter One commences with a discussion about the need to reform Mathematics Education. It continues with a detailed description of two of the reports that were fundamental to the changes made in Mathematics programs in the United Kingdom, the United States and Australia during the latter half of the twentieth century. The first section of Chapter One also points out that the changes made in Mathematics Education did not extend to the components of the affective domain.

In order to gain a thorough understanding of the Mathematics program that was used in the research, the second section of Chapter One describes in detail the Mathematics K-6 (New South Wales Department of Education, 1989) syllabus. This section of the chapter notes that the syllabus, like the reports upon which it was based, pays little attention to affective variables.

In an attempt to determine the significance of affect in the learning of Mathematics, the second chapter of the literature review discusses the affective domain. Particular reference is made to research into affect in Mathematics Education.

The review of the Mathematics literature shows that few investigations have studied the emotions. The review also shows that most of the research that has been carried out on the subject has concentrated on negative rather than positive emotions.

The history of the development of the theories of emotion are described in the literature review in Chapter Two in an attempt to explain the tendency of Mathematics research to concentrate on negative emotions. Also described in that chapter are George Mandler

(1984; 1989) and Andrew Ortony, Gary Clore and Andrew Collins' (1988) theories of emotion because they are found to be relevant to Mathematics Education.

From the literature review it is concluded that little is known about the role of affect in the learning of Mathematics and even less is known about the significance of the emotions in that process. This conclusion, and the scarcity of references to affective variables in the Mathematics K-6 (New South Wales Department of Education, 1989) syllabus, lead the researcher to formulate a major research question and two subsidiary questions:

Can the emotional states of children in a Mathematics environment be described?

Can indicators of emotional states be developed?

Can descriptors of emotional states be developed?

The Research Design

Chapter Three describes the design of the research into emotional states. As the study is qualitative in nature, the chapter begins by discussing the suitability of qualitative research to the study of affect.

Chapter Three contains the rationale behind the researcher's investigation into the emotions as well as background information about the subjects, the teacher and the components of the class's Mathematics program. The chapter also contains a detailed description of the planned data collection methods that were used in the research. The use of an independent observer, respondent validation and two methods of triangulation were strategies employed by the researcher to minimise data contamination and to strengthen the validity and reliability of the research.

The third section of Chapter Three contains a description of the main data analysis strategy used in the study: the constant comparative method. Chapter Three concludes with an evaluation of the research design. In this section potential threats to the validity and reliability of the research are noted as well as the limitations of the research.

Findings from the Research

The fourth chapter of the thesis describes the data that was collected during the research. The data showed that the children in the sample experienced many emotions as they participated in Mathematics lessons. The emotions ranged from an extreme positive emotional state to an extreme negative emotional state. Descriptors and indicators of the states were able to be devised.

As the data from the study was analysed themes began to emerge. It was found that one theme, non-verbal behaviour, needed to be explored further. Consequently, the researcher reviewed the literature on 'body language' and non-verbal behaviour and carried out a series of case studies. The design of the case studies and the procedures used in the them is also described in Chapter Four. The theme of 'body language' or non-verbal behaviour was an unexpected outcome of the study.

Chapter Five describes the five themes that evolved from the study of children's emotions. The themes were: testing, the subject of Mathematics, attitudes, beliefs and language. Four of the themes were found to contain sub-themes and many of the themes appeared to be interrelated. Chapter Five concludes with a description of a model of emotional states that was able to be constructed from the data. The model was composed of three levels of emotion that were hierarchically arranged. The levels were: the State of Elation, the State of Equilibrium and the State of Apprehension. Each level or state was found to have its own descriptors and specific non-verbal indicators of emotion. The themes that were found in the research were able to be related to the model of emotional states.

The final chapter of the thesis discusses the conclusions that were drawn from the research and the implications they have for future research as well as for educators and policy makers in Education.

Conclusions drawn from the Study of Emotional States and the Implications the Study has for Future Research

The investigation into emotional states found that the emotions children experienced during Mathematics lessons depended on three conditions or factors: the enjoyment level of the task on which the children were working, the nature of the task and its difficulty level. The intensity of children's emotions was found to be related to the unexpectedness of success or failure on a task. The emotions that children experienced appeared to influence their attitudes to, and beliefs about, Mathematics, as well as their beliefs about their Mathematical competence. These findings indicate that policy makers in Education should not only produce documents that emphasise the significance of attitudes in the learning of Mathematics but also beliefs.

Participation in tests, especially written tests, was mainly found to produce negative emotions in children while participation in Mathematics activities produced positive emotions. Physiological reactions often accompanied the negative emotions that the children experienced during written tests, and thoughts about marks dominated the children's minds during the tests. Memories of positive emotions and the tasks in which they were experienced were found to be overshadowed by memories of negative emotions and the tasks in which they were created. These findings suggest that teachers should not emphasise testing as a method of assessing student progress, but should give equal emphasis to a variety of assessment strategies. Many of the findings from the study of emotional states support findings from the literature, particularly those about Mathematics anxiety and test anxiety.

Mathematics activities such as the ones found in the Mathematics K-6 (New South Wales Department of Education, 1989) syllabus were found to produce enjoyment in children and to lead to the production of positive attitudes to Mathematics. However, the study also revealed that when children participated in Mathematics tasks that did not involve numberwork they did not realise they were doing Mathematics. The findings from the research suggest that teachers need to carefully plan their teaching and learning strategies so that children will frequently experience enjoyment and come to realise that Mathematics involves more than mere numberwork.

One problem that was found to permeate the study of emotional states and to raise concern about the validity of the research, was the use of language in describing the emotions. This problem may need to be addressed in future research.

Data from the research revealed several additional issues that were related to the study of emotions. They included: co-operation between students, motivation, gender differences and the content of Mathematics programs. Each of these issues were considered worthy of investigation in future research. Similarly, the researcher's model of emotional states raised a number of questions that will need to be addressed by future investigations. The model itself and the specific non-verbal indicators of emotion that are thought to be associated with each level in the model, are in need of verification by future research. In order to do this, studies need to be carried out in larger samples and in different educational settings.

CHAPTER ONE

THE LITERATURE REVIEW

Introduction

The first section of Chapter One discusses the reports and some of the research upon which the Mathematics K-6 (New South Wales Department of Education, 1989) syllabus was based, and notes that both the reports and the research have tended to ignore affective issues. Mathematics K-6 (New South Wales Department of Education, 1989) is the mandatory Mathematics syllabus for state primary schools in New South Wales, Australia.

The second section of Chapter One describes the philosophy, rationale and content of the Mathematics K-6 (New South Wales Department of Education, 1989) syllabus. This is considered necessary because the research described in the following pages was conducted in a Mathematics classroom in which the Mathematics K-6 (New South Wales Department of Education, 1989) syllabus was used.

THE REPORTS

The Need for Reform in Mathematics Education

During the late 1970s and early 1980s there was a growing concern in the Western world about the appropriateness of Mathematics programs that were being taught in schools. At the heart of the matter were the issues of relevance and usefulness. Doubt was cast on the ability of Mathematics programs to meet the Mathematical needs of students.

As a consequence of the concern about Mathematics Education, investigations into the need for reforms in Mathematics were undertaken. They resulted in the production of Mathematics Counts: Recommendations for School Mathematics in the 1980s (Cockcroft, 1982) in the United Kingdom, and the National Council of Teachers of Mathematics (NCTM) documents An Agenda for Action (NCTM, 1980) and Priorities in School Mathematics or PRISM project (NCTM, 1981) in the United States. Each of the studies proposed a number of recommendations for reform in Mathematics instruction. Through the adoption of the reforms it was hoped that students would leave school being better prepared Mathematically to cope with life in their rapidly changing technological societies.

The recommendations in the reports discussed eight main issues:
* the significance of problem solving
* language
* computers and calculators
* individual differences
* concrete materials
* real-life situations
* estimation and assessment .

However, affective issues such as students' attitudes and beliefs were given limited attention.

The eight main issues that were described in the reports will be discussed in the following pages because each one was addressed by the committee responsible for the formulation of the Mathematics K-6 (New South Wales Department of Education, 1989) syllabus and was subsequently included in the document. Reference will also be made to the affective variables that were mentioned in the reports.

Problem Solving, Estimation and Everyday Situations

An important feature of Mathematics Counts (Cockcroft, 1982) was its specification of a broad range of learning outcomes which distinguished facts and skills, conceptual structures, and general strategies and appreciation. The document stated that the ability to solve problems was at the heart of Mathematics.

> Mathematics is only 'useful' to the extent to which it can be applied to a particular situation and it is the ability to apply mathematics to a variety of situations to which we give the name 'problem solving'.
>
> (Cockcroft 1982, p. 73)

The first recommendation of An Agenda for Action (NCTM, 1980) went one step further by stating that problem solving was so important that the Mathematics curriculum should be organised around it. It added that problem solving required a wide repertoire of knowledge, not only of particular skills and concepts but also of the relationships between them and the fundamental principles that unified them (An Agenda for Action NCTM, 1980). The first recommendation also stated that problem solving involved applying Mathematics to the real world. Therefore, it was felt that practical and meaningful experiences rather than contrived textbook-type exercises needed to form the core of Mathematical work for children.

An Agenda for Action (NCTM, 1980) recognised the transitional aspect of applying problem solving skills and knowledge to new situations. Similarly, Mathematics Counts (Cockcroft, 1982) and PRISM (NCTM, 1981) claimed that children needed experience in using a broad range of strategies for solving problems and that they should be given the opportunity, through problem solving, to apply Mathematical strategies to everyday situations. Furthermore, Mathematics Counts (Cockcroft, 1982) stated that problem solving necessitated the use of estimation to a greater degree than had previously been considered necessary.

Recommendation Four in An Agenda for Action (NCTM, 1980) added to the discussion about the importance of structuring the Mathematics curriculum around problem solving by noting that higher-order skills could be developed through problem solving.

Mathematics Counts (Cockcroft, 1982) raised three additional but significant points about problem solving. First, it considered that children needed to develop persistence in solving problems. Second, it argued that high achievers would benefit from problem solving and practical investigations. Third, the Report (Cockcroft, 1982) suggested that discussion was an important means by which problem solving strategies could be developed and explored.

Language, Individual Learning, Concrete Materials and Evaluation

Mathematics Counts (Cockcroft, 1982) considered discussion to be especially necessary for the promotion of Mathematical success in low-achievers, girls and children from non-English speaking backgrounds. The Report (Cockcroft, 1982) stressed the importance of oral and written language as a means of communication. It acknowledged that children commenced school with different levels of language competence and that the development of language work in Mathematics was vital.

As language had been recognised as playing an essential part in the formulation and expression of Mathematical ideas Mathematics Counts (Cockcroft, 1982) stated that children needed to learn specific Mathematical terms and symbols. Unless children were able to distinguish between the Mathematical and the common meanings of these terms, it was argued that they would have difficulty understanding and completing problems and computations (Cockcroft, 1982).

Mathematics Counts (Cockcroft, 1982) also stated that there was a need for children whose first language was not English to converse in their own tongue whilst developing their ability to communicate Mathematically (Cockcroft, 1982). The Report (Cockcroft, 1982) argued that teachers should draw upon the various cultures of their students so that particular Mathematical concepts could be highlighted (Cockcroft, 1982).

In addition to language being considered a necessary tool for the development of Mathematical knowledge, Mathematics Counts (Cockcroft, 1982) stated that greater emphasis on language would enable educators to more effectively determine their students' levels of development and that this in turn would assist educators in planning appropriate learning experiences for their students. Language, problem solving and practical work were all considered to play an increasing role in the assessment of students' concepts and understandings in Mathematics.

The contribution of psychologists to Mathematics learning can be found in the emphasis Mathematics Counts (Cockcroft, 1982) placed on language. This emphasis evolved from the ideas on learning that were held by socioconstructivists. Socioconstructivists claimed that children constructed Mathematical knowledge by reorganising their social experiences in the course of their social interaction with each other and with the teacher (Schoenfeld, 1991). Language was considered to be at the heart of social interaction and therefore at the heart of learning.

A further example of the contribution of research theorists to the reports can be found in An Agenda for Action's (NCTM, 1980) claim that children learn at different rates and in different ways and therefore need to be catered for in ways that are appropriate to their particular stage of development. An Agenda for Action (NCTM, 1980) stated that this could be achieved by providing children with concrete materials, a variety of instructional techniques and adequate Mathematical resources. It pointed out that these strategies would necessitate a change in the role of the teacher from that of a *passive transmitter* of knowledge to an *active facilitator* of knowledge.

The use of concrete materials was discussed in some depth in the third part of Mathematics Counts (Cockcroft, 1982). It suggested that concrete materials could be used to assist concept development in number, two dimensional (2D) and three dimensional (3D) work and measurement. The Report (Cockcroft, 1982) encouraged the use of books of general interest to pupils and displays of pupils' work.

Mathematics Counts (Cockcroft, 1982) and An Agenda for Action (NCTM, 1980) stated that evaluation should play a significant role in structuring Mathematics programs within schools. For example, the Report (Cockcroft, 1982) argued that the evaluation of a Mathematics program should cover program effectiveness, student learning, teacher performance and the quality of the materials being used.

Both documents emphasised the role of assessment procedures in the evaluation of a school or classroom program, and stated that they should be appropriate to the goals of the Mathematics program and be used to improve the learning programs, teaching strategies and resources within the school (NCTM, 1980; Cockcroft, 1982).

The Mathematics Counts (Cockcroft, 1982) document stressed that assessment was essential to Mathematics programs and that it should be carried out in a variety of ways and for a variety of purposes. The Report (Cockcroft, 1982) added that assessment should be both diagnostic and supportive and that children should be given the opportunity to explain their thinking during the evaluation of their Mathematical understanding, skills and knowledge. The Report (Cockcroft, 1982) noted that there was a place for standardised tests in student assessment but that the over-reliance of teachers on paper and pencil tests should become a thing of the past.

The shift away from written tests as the most important method of assessing student progress was also mentioned in An Agenda for Action (NCTM, 1980). Within that document, it was stated that evaluation should not be limited to testing. Testing was recognised as being only one source of data and that for assessment purposes it needed to be used in combination with other methods of evaluating student progress. An Agenda for Action (NCTM, 1980) emphasised that test scores alone should not be considered synonymous with achievement or program quality. The document pointed out that the evaluation of problem solving would require new approaches to be taken in the assessment of students.

Technology

Mathematics Counts (Cockcroft, 1982) and An Agenda for Action (NCTM, 1980) emphasised the fact that children needed to be competent in their use of calculators and computers.

Calculators and computers were viewed as tools which had multiple roles in learning. Mathematics Counts (Cockcroft, 1982) stated that calculators and computers should: assist and improve Mathematics teaching, affect the content of what was taught, and relieve the stress applied to certain areas of that content. The Report (Cockcroft, 1982) also stated that the use of calculators and computers would: help reduce the feelings of failure in students, enable students to be free from lengthy computations, and allow students to participate more fully and enjoyably in Mathematics lessons and investigations.

By using computers and calculators Mathematics Counts (Cockcroft, 1982) claimed that children would come to develop positive attitudes towards Mathematics. It was also envisaged that the use of technological tools would enable students to develop confidence in their ability to perform Mathematical tasks.

The Agenda for Action (NCTM, 1980) and PRISM (NCTM, 1981) documents both highlighted the significance of computers in Mathematics instruction. The third recommendation of An Agenda for Action (NCTM, 1980) discussed the need for teachers and students to become computer literate. The recommendation also discussed the need for manufacturers to produce good quality computer software.

Competence in the use of computers and calculators was considered to be necessary to enable students to participate effectively in day-to-day experiences outside the classroom. However, the Mathematics Counts (Cockcroft, 1982) document stressed that both calculators and computers were to be viewed only as tools, and not as a replacement for Mathematical understanding or certain types of mental computations.

Additional Research Findings

In the United States in 1986, the National Council of Teachers of Mathematics (NCTM) established the Commission on Standards for School Mathematics. The creation of the Commission was an attempt to improve the quality of school Mathematics (Blane, 1992). The Commission's efforts resulted in the formulation of a document called the Curriculum and Evaluation Standards for School Mathematics ('The Standards'). This document set out the standards for Mathematics curricula in North American schools (K-12) together with a proposed means of evaluating the quality of both the curriculum and student achievement (Blane, 1992). The preface of the Standards stated that:

> The Standards is a document designed to establish a broad framework to guide reform in school mathematics in the next decade. In it a vision is given of what the mathematics curriculum should include in terms of content priority and emphasis. The challenge we issue to all interested in the quality of school mathematics is to work collaboratively to use these curriculum and evaluation standards as the basis for change so that the teaching and learning of mathematics in our schools is improved.
>
> (p. v)

The Standards were divided into three segments: K-4, 5-8, 9-12. Each segment contained topics that were very similar to the ones discussed in Mathematics Counts (Cockcroft, 1982) and An Agenda for Action (NCTM, 1980). Moreover, the Standards showed a strong resemblance to the content that was chosen for the Mathematics K-6 (New South Wales Department of Education, 1989) syllabus.

The Standards (NCTM, 1989) considered evaluation to be a vital component in improving instruction, learning, and Mathematics programs (Blane, 1992). The document acknowledged the importance of gathering valid information about student growth and achievement (Clarke, Clarke and Lovitt, 1990) and emphasised the need to use multiple sources of information in the assessment of students. The Standards (NCTM, 1989) also stressed that assessment instruments and assessment methods needed to be used appropriately (Grouws and Meier, 1992).

The major shortcoming of the Standards (Romberg, 1989) however, was its failure to describe how excellence in school Mathematics programs was to be achieved (Blane, 1992).

Even though the Standards (NCTM, 1989) was produced at the same time as the Mathematics K-6 (New South Wales Department of Education, 1989) syllabus and therefore did not directly influence its construction, knowledge of the document is important for several reasons (Kroll, 1989). First, because it was based on the same research and reports as the syllabus and was therefore also influenced by the constructivist perspective. Second, because it pointed out how students learnt Mathematics. Third, because it described what Mathematics was and why various Mathematical concepts should be developed in children (Kroll, 1989).

The overriding message of Mathematics Counts (Cockcroft, 1982), PRISM (NCTM, 1981) and An Agenda for Action (NCTM, 1980) was that competence in Mathematics needed to be considered to be more than mere proficiency in computational skills. That idea represented a major shift from the traditional view of what constituted Mathematical competence and what constituted Mathematics, and it had far-reaching implications for the teaching of Mathematics. However, the change in ideas did not include a corresponding change in the role that affect was thought to play in the learning of Mathematics.

Affective Variables

A shortcoming of Mathematics Counts (Cockcroft, 1982) and An Agenda for Action (NCTM, 1980) was the small amount of space that each document spent discussing students' affective qualities.

Mathematics Counts (Cockcroft, 1982) mentioned the need for the development of confidence, enjoyment and positive attitudes in students, in only thirteen of its eight hundred paragraphs. It also failed to give specific strategies for the development of those qualities. An Agenda for Action (NCTM, 1980) only once referred to the development of positive attitudes in students. Both documents completely neglected other affective issues such as student beliefs and appreciation. Therefore Leder and Forgasz's (1992) argument that little attention is given to affective factors beyond related curriculum statements, would appear to be true of the reports on which the Mathematics K-6 (New South Wales Department of Education, 1989) syllabus was based.

The following section discusses the Mathematics K-6 (New South Wales Department of Education, 1989) syllabus in more detail.

THE NEW SOUTH WALES MATHEMATICS K-6 SYLLABUS

The dissatisfaction with Mathematics programs that was witnessed in the United States and United Kingdom during the 1970s and 1980s was also evident in Australia. In New South Wales in 1982 the Board of Studies established a committee to develop a Mathematics syllabus that was based on current learning theory and research into the Mathematical needs of children. As a consequence of the committee's investigations, the Mathematics K-6 (New South Wales Department of Education, 1989) syllabus was created. The document became the mandatory Mathematics program for New South Wales (NSW) state primary schools.

Philosophy, Attitudes, Competence and Confidence

The Mathematics K-6 (NSW Department of Education, 1989) syllabus was based on many of the recommendations proposed in Mathematics Counts (Cockcroft, 1982) and An Agenda for Action (NCTM, 1980) and on the change in theory about how children learn. The Director-General's Foreword in the Mathematics syllabus stated that:

> ..... Mathematics K-6 responds to considerable recent research concerning how students learn Mathematics. It recognises that students learn at different rates and in different ways. It also acknowledges the importance of concrete materials and language use in mathematical learning. The nature of mathematical experiences undertaken must enable students to see the relevance of school Mathematics to aspects of their everyday lives ....
>
> (NSW Department of Education 1989, p. v)

The philosophy underlying the Mathematics K-6 (NSW Department of Education, 1989) syllabus was that Mathematics Education should provide all students with adequate access to Mathematics skills, knowledge and understanding so that they would be able to participate in society in a confident, competent and effective manner. To accomplish this, Mathematics instruction had to be seen by students to be relevant to them and to have an integral place in their lives

As the Mathematics K-6 (NSW Department of Education, 1989) syllabus aimed at having the findings from research incorporated into Mathematics programs in New South Wales schools, current learning theory was encapsulated in it. The constructivist approach to Mathematics became central to the theoretical framework upon which the syllabus was built (Masters and Doig, 1992).

The key position of problem solving in the Mathematics K-6 (NSW Department of Education, 1989) syllabus reflected the significance attributed to it by Mathematics Counts (Cockcroft, 1982) and An Agenda for Action (NCTM, 1980).

In the Mathematics K-6 (NSW Department of Education, 1989) syllabus, problem solving became more than a type of question or a question with words. It became a philosophy, a way of teaching Mathematics. Students were taught about, for, and through problem solving. The problem solving section emphasised the use of problem solving in real situations and in classroom incidents and included a number of strategies that could be used to solve problems. It also discussed the importance of the development of Mathematical language and the role of reflection in the problem solving process. Problem solving, language and reflection were all issues that had been raised in Mathematics Counts (Cockcroft, 1982) and An Agenda for Action (NCTM, 1980).

The Mathematics K-6 (NSW Department of Education, 1989) syllabus emphasised the need for students to develop positive attitudes to Mathematics. The syllabus also encouraged teachers and parents to develop positive attitudes towards Mathematics so that they would pass on to students their enthusiasm for the subject.

Attitudes were the only affective variable discussed in the Mathematics K-6 (NSW Department of Education, 1989) syllabus. However, a definition of the term was not provided. The four pages of the document in which attitudes were mentioned merely gave teachers and parents a list of the strategies they could use to develop positive attitudes in children.

Teaching and Learning Strategies and Assessment

As the Mathematics K-6 (NSW Department of Education, 1989) syllabus was based on changing theories about how children learn, and included much new content, it required a change in the types of teaching and learning strategies that were to be used in the classroom. This in turn required a change in the procedures used to assess student progress.

The Mathematics K-6 (NSW Department of Education, 1989) syllabus and support documents provided teachers with a variety of learning activities and teaching strategies. However, unlike its predecessor, Mathematics K-6 (NSW Department of Education, 1989) did not make any of the teaching strategies or learning activities prescriptive. This enabled teachers to cater for their students' varying stages of development and to take into account the context in which the Mathematics was being taught.

The Mathematics K-6 (NSW Department of Education, 1989) syllabus considered assessment to be a significant part of the learning process, both in its role within the classroom and school, and in its appropriateness to the topics covered in Mathematics lessons. Consequently, a number of assessment procedures were included in the document. The methods of assessment advocated by the syllabus were: paper and pencil tests, observations, listening to students, structured interviews, student-teacher discussions, student explanations and demonstrations, samples of work, and practical investigations.

The various assessment techniques listed in the Mathematics K-6 (NSW Department of Education, 1989) syllabus allowed teachers - as the Mathematics Counts (Cockcroft, 1982) document suggested - to evaluate the goals of Mathematics programs more effectively and to form a complete picture of students' Mathematical understanding.

Masters and Doig (1992) claimed that curriculum reform needed to be accompanied by assessment methods that supported and reinforced the desired reforms. Furthermore, Masters and Doig (1992) stated that assessment methods needed to be consistent with current understandings of student learning and how it occurred, and needed to provide feedback capable of promoting that learning. Clarke et. al. (1990) also argued that assessment needed to contribute constructively to student learning. To accomplish that task Clarke et al. (1990) advocated the use of teacher observations, student journals, self-assessment questionnaires and work folios. Even though the research by Clarke et al. (1990) and Masters and Doig (1992) was published after the development of the Mathematics K-6 (NSW Department of Education, 1989) syllabus, it can be seen to support the assessment methods recommended in the document.

According to Clarke (1992) assessment is a high profile classroom activity which impacts greatly on student self-esteem and on student classroom behaviours. Moreover, Clarke

(1992) has stated that there is a link between classroom assessment practices and the ideas students have about their own Mathematical competence and that of their peers.

McDonough and Clarke (1994) argued that contemporary assessment practices offered new insight into the interrelationship of previously separated affective and cognitive responses. According to Clarke (1994) the centrality of affective response in the understanding and promotion of learning (through explicit assessment practices) was now being acknowledged. Clarke (1994) also claimed that the implications of the reconception of the affective-cognitive link were likely to prove most significant in the assessment practices of the Mathematics Education community.

It has long been noted that testing can become an end in itself rather than a means to an end (Grouws and Meier, 1992) and that an over-reliance on written tests can restrict curricular content. Paper and pencil testing has also been found to devalue critical thinking and to stress simple rather than higher-order thinking skills (Grouws and Meier, 1992). However, despite these criticisms, testing is considered to have a legitimate place in student assessment. Testing has been included as an assessment method in the Mathematics K-6 (NSW Department of Education, 1989) syllabus but its importance in the evaluation of student learning has diminished. The preceding Mathematics syllabus, the Curriculum for Primary School Mathematics (NSW Department of Education, 1967) implied that testing was to be used as the main form of assessment. The introduction of Mathematics K-6 (NSW Department of Education, 1989) saw a shift in the traditional role assigned to testing. The 1989 syllabus considered testing to be only one of many forms of evaluating student progress.

Masters and Doig (1992) claimed that there was a tendency to view Mathematics learning as a process of memorising isolated facts and algorithms and recalling and applying them on command. Masters and Doig (1992) stated that, if methods of assessment were to be consistent with knowledge about how students learnt Mathematics, then new approaches to assessment and new assessment tools were required. The approach to assessment taken by the Mathematics K-6 (NSW Department of Education, 1989) syllabus appears to satisfy this requirement.

Planning, Programming and Content

The Mathematics K-6 (NSW Department of Education, 1989) syllabus was composed of three strands: Number, Space and Measurement. The inclusion of the Space strand made the syllabus significantly different from its predecessors. Having three distinct strands that were equally important reinforced the idea that being proficient at Mathematics was more than merely having the ability to competently use the four computational skills of addition, subtraction, multiplication and division.

Mathematics K-6 (NSW Department of Education, 1989) encouraged teachers to present students with topics from each strand of the syllabus simultaneously. By doing this it was hoped that students would come to understand the subject as a whole and see how its parts were interrelated.

The content of the Mathematics K-6 (NSW Department of Education, 1989) syllabus was set out in stages rather than grade levels and its sequencing was that of a spiralling curriculum. By formatting the document in that way, the Mathematics K-6 (NSW Department of Education, 1989) syllabus allowed teachers to choose the content that was most appropriate to the needs and development of their students and to concentrate fully on developing children's Mathematical concepts and understanding.

The Teacher's Role

In the Mathematics K-6 (NSW Department of Education, 1989) syllabus the teacher was no longer viewed as being separate from the learning process. The teacher was considered to play an active and crucial role in the development of student Mathematical understanding. She/he was thought to be a facilitator of, and a guide to, children's learning. This role change can be attributed to the learning theories of the constructivists (Koehler and Grouws, 1992).

Resources and Concrete Materials

The influence of Mathematics Counts (Cockcroft, 1982), An Agenda for Action (NCTM, 1980) and the work of Bruner and Piaget can be witnessed in the Mathematics K-6 (NSW Department of Education, 1989) syllabus's suggestion that concrete materials and a variety of resources should be used in Mathematics activities.

Language

Mathematics Counts (Cockcroft, 1982), An Agenda for Action (NCTM, 1980) and the findings from research in Psychology all emphasised the significance of language in the development of Mathematical understanding in children. In its content, teaching and learning experiences, and methods of assessment, the Mathematics K-6 (NSW

Department of Education, 1989) syllabus stressed that children should be given every opportunity to use language. Mathematics K-6 (NSW Department of Education, 1989) can therefore be seen to reflect the socioconstructivists' view that language plays a significant role in the learning of shared Mathematical knowledge and in the acculturation of students.

The emphasis which the syllabus placed on oral and written language in Mathematics underscored the holistic approach the syllabus took towards Mathematics Education and the influence that the reports had in the syllabus's development.

SUMMARY

It is obvious that the Mathematics K-6 (NSW Department of Education, 1989) syllabus has been heavily influenced by learning theories from the field of Psychology and by the main recommendations made in Mathematics Counts (Cockcroft, 1982) and An Agenda for Action (NCTM, 1980). From both the research and the recommendations arising from these reports, the NSW Board of Studies appears to have constructed a Mathematics syllabus that is theoretically sound and suited to the needs of the children for whom it has been developed. The syllabus also appears to have taken into account the needs those children will have when they leave school and enter society. However, a shortcoming of the document is its general neglect of students' affective qualities. This includes a lack of recognition and definition of the elements of the affective domain, and a lack of discussion about the implications those elements have for the learning of Mathematics.

In an attempt to determine the value of Mathematics programs in catering for students' affective needs, Chapter Two will review the literature on the affective domain and the Mathematics Education literature on affect.

CHAPTER TWO

A REVIEW OF THE LITERATURE OF THE AFFECTIVE DOMAIN

Introduction

Chapter One reviewed the Mathematics K-6 (NSW Department of Education, 1989) syllabus and the reports upon which it was based. The review showed that the documents focused on students' cognitive needs but largely ignored their affective needs.

In an attempt to determine the significance of affect in the learning of Mathematics, Chapter Two reviews the literature on the affective domain and the Mathematics Education literature on affect. The chapter commences by defining relevant terms, discussing the elements that make up the domain and noting the absence of a theoretical framework for it.

Chapter Two continues by specifically discussing the Mathematics literature on affect. This section describes studies that have investigated attitudes, beliefs, the Mathematics context and the emotions of stress and panic. The related issues of anxiety, Mathematics anxiety and test anxiety are also discussed. From the review it is found that studies of negative emotions dominate the literature.

The third part of Chapter Two begins by pointing out that it was the absence of Mathematics research into positive emotions and the small number of references to affective factors in the reports upon which the Mathematics K-6 (NSW Department of Education, 1989) syllabus was based and the syllabus itself, that prompted the researcher to plan an investigation into emotional states. Within this section the major research question and two subsidiary questions are presented.

In an attempt to understand the reason for the dominance of studies of negative emotions in the Mathematics literature, the third section of Chapter Two provides an overview of the development of theories of emotion from their beginning in the discipline of Psychology to their present day status in the discipline of Mathematics.

Chapter Two concludes by discussing the ideas of cognitive theorists George Mandler (1984;1989) and Andrew Ortony, Gary Clore and Andrew Collins (1988), as their theories of emotion are considered to be relevant to Mathematics Education.

DEFINING AFFECT

What is meant by the term *affective domain*? What are the components of the domain? Can the components be measured? If so, how, and how accurate are the results likely to be? What, if any, is the relationship of the affective domain to the cognitive domain? If there is a relationship, is it important in the learning process? Can educators simultaneously cater for growth in the cognitive and the affective domains?

Questions such as these are only a few of the many that may be posed when one begins to investigate the affective domain.

Defining Terms

Term definition is the major problem faced by researchers studying the affective domain.

As Hart (1989) and Simon (1982) have indicated, describing the affective domain is not easy (McLeod, 1992). A review of the literature of the domain shows that there are almost as many definitions of the terms affective domain and affect as there are writers on the subject. In all instances the main emphasis of the definitions used - when definitions have been used at all - will be determined by the discipline from which the writer comes. The earliest writings on the affective domain originated in the discipline of Psychology but literature on the subject today can be found in Education as well as Psychology.

The term 'affect' has often been used as a general and inclusive label to refer to both mood and emotions (Forgas, 1991). Petty, Gleicher and Baker (1991) have used the term *affect* as a super-ordinate construct which encompasses emotions and transient moods and feelings (Forgas, 1991). Weiner (1992) has used the term *affect* synonymously with feeling, evaluation and emotion.

Frijda (1986) pointed out that the lack of accurate and universally accepted definitions for affect, attitude, emotion, mood and related terms remains an on-going problem in Psychology.

As Mandler (1989) declared:

> When defining 'affect' the dictionary is not much help. It defines the noun affect as, among other things, a "disposition of body or mind; affection, love; the emotion that lies behind action; pleasantness or unpleasantness of, or complex of ideas involved in, an emotional state". (quoted from Chambers Twentieth Century Dictionary 1977)
>
> (p. 238)

McLeod (1992) has defined the affective domain as a wide range of beliefs, feelings and moods that are generally considered to go beyond the domain of cognition; while Beane (1985) has described the affective domain as the area of human nature and conduct that deals with emotions, feelings, values, attitudes, predispositions and morals (Himsl and Lambert, 1993). In their taxonomy of educational objectives, Krathwohl, Bloom and Bertram (1964) stated:

> The objectives of the affective domain should be those which emphasise a feeling tone, an emotion or a degree of acceptance or rejection.
>
> (p. 7)

After reading the above definitions it becomes obvious that any investigation into the affective domain must begin with clearly defined terms.

A Definition of the Affective Domain

For the purpose of the investigation into children's emotional states in a Mathematics environment, the affective domain is considered to be, as McLeod (1992) and Beane (1985) have suggested, the area of human nature that deals with feelings, attitudes, beliefs and moods, and extends beyond the domain of cognition. This definition implies that the elements of the domain are those which are considered to have some form of relationship with the emotions and may therefore be thought to include additional constructs such as self-esteem, self-confidence and morals.

Elements of the Affective Domain

The problem of definition is not restricted to the terms *affective domain* and *affect*. It also extends to the constructs that make up the domain. The exact meaning of commonly used words such as *feelings*, *emotions*, *attitudes*, *beliefs* and *moods* often needs to be deduced from the specific context being described.

McDonough and Clarke (1994) demonstrated the difficulty associated with the terminology used in the study of the affective domain when they wrote:

> Psychology and mathematical education, sometimes have different meanings for terms (Hart, 1989) and even within Mathematics education studies which use the same terminology may not be studying the same phenomenon (McLeod, 1994).
> (p. 392)

An additional problem to that of defining the terms used in the study of the affective domain is the absence of a universally accepted and definitive list of components that make up the domain.

McLeod (1992) has argued that there is a strong affiliation between the affective domain and the cognitive domain. Consequently, he felt that researchers needed to recognise the fact that some topics which were usually identified with the cognitive domain should be considered to have strong affective ties. For McLeod (1992) subjects such as autonomy, aesthetics, intuition, metacognition, the social context and technology came under this heading. However, most writers consider attitudes, beliefs, feelings and emotions to be the main components of the affective domain.

Conceptual Framework

A further difficulty associated with investigations into the affective domain is the absence of a conceptual framework to which researchers can relate their findings. Though a taxonomy for the domain has been written (see Krathwohl, Bloom and Bertram, 1964) it has never proven to be, as the authors themselves have pointed out, as widely accepted as the taxonomy for the cognitive domain.

AFFECT AND MATHEMATICS

The following section investigates affect as it is considered within the discipline of Mathematics. It begins with a general discussion about the place of affect in Mathematics Education then continues by describing research into attitudes, beliefs, the Mathematics context, the emotions, anxiety, Mathematics anxiety and test anxiety.

Why Study Affect?

The relevance of the affective domain to students' learning of Mathematics is increasingly being recognised by Mathematics educators (Leder and Forgasz, 1992).

Reyes (1984) stated that there were two main reasons why studies of the affective domain needed to be undertaken in Mathematics. First, to assist educators to maximise students' Mathematical learning. Second, to develop in students positive attitudes towards the subject. Reyes's last point has been included in the aims of the New South Wales Mathematics K-6 (NSW Department of Education, 1989) syllabus. The first aim listed in the syllabus is: "To create in students favourable attitudes towards and stimulate interest in mathematics." (p. 8)

Throughout the twentieth century Mathematics curriculum developers have spent most of their time attending to the content of syllabuses and paying little attention to student affect (McDonough and Clarke, 1994). Research on affect in Mathematics Education has remained on the edge of the field of study even though affective issues are of great importance to teachers and students (McLeod, 1992). Similarly, the planning of instructional and assessment practices has traditionally neglected affective factors such as student motivation, confidence, satisfaction and self-esteem (McDonough and Clarke 1994). However, in the Mathematics community today, there is an increasing awareness of the importance of studying both the cognitive and affective aspects of student learning.

According to Reyes (1984) many educators claim that the study of affect would provide them with greater knowledge about the following factors: how to help students learn more Mathematics, how to develop in students positive attitudes to Mathematics and positive Mathematics self-concepts, how to foster the most appropriate Mathematical learning environment for students, how to encourage students to take more Mathematics courses in their later years of schooling and how to overcome the under representation of some minority groups (such as women, low socio-economic groups and coloured students) in Mathematics courses in which a high level of Mathematical knowledge is required.

According to McLeod (1992), in Mathematics Education, there are a number of topics that should be thought of as being associated with the affective domain. Confidence, self-concept, self-efficacy, Mathematics anxiety, causal attributions, effort and ability attributions, learned helplessness, and motivation have all been viewed by McLeod as coming under the heading of the domain (McLeod, 1992).

Like McLeod (1992), McDonough and Clarke (1994) have discussed the multi-dimensionality of the affective domain. The authors have investigated the domain in relation to the way it has been portrayed in major documents in Mathematics Education. According to McDonough and Clarke (1994) appreciation and enjoyment of Mathematics (Australian Education Council, 1991; Cockcroft, 1982) and the ideas of self-esteem and success (Victorian Ministry of Education, 1988) are facets of the affective domain that are considered to be important in the learning of Mathematics.

McDonough and Clarke (1994) have also noted that a great many variables have been placed under the heading of the affective domain. Moreover, they stated that the extensiveness of the list may have contributed to the difficulty of defining the terms of the domain.

Attitudes

As the majority of studies of affect have focused largely on cognitive behaviour many components of the affective domain have been defined in cognitive terms (Krathwohl, Bloom and Betram, 1964).

The word attitude has generally been defined by psychologists as a predisposition to respond in a favourable or unfavourable way to a given object, person, activity or idea (Hart, 1989). This definition has three components: an affective or emotional component, a behavioural component and a component containing beliefs about the object (Rajecki, 1982). Postle, Spunde and Hodgkinson's (1977) definition of attitudes reflects this idea. The authors considered attitudes to be learned patterns of organisation that were not easy to change, that were uni-dimensional in nature (one may be favourably or unfavourably disposed towards an object) and were predispositions to act with respect to certain objects. Attitudes were also thought to contain an emotional component.

Leder's conception of attitudes is similar to those given above. Leder (1993) has stated that attitudes have a cognitive component (they are learnt), a behavioural component (attitudes predispose towards action) and an evaluative component (the actions may be favourable or unfavourable).

Some psychologists, however, have defined attitudes in a different way, including only beliefs about the object (for example, Wyer 1974). Such a definition eliminates the emotional and behavioural components of the term and makes it synonymous with beliefs (Hart, 1989).

According to Hart (1989) in the context of Mathematics Education, a positive or negative attitude towards Mathematics could be inferred from one's emotional reaction to the subject, one's behaviour in approaching or avoiding Mathematics and one's beliefs about what Mathematics is and how it may be used.

Mathematics educators have often used the term attitude in a less clearly defined way than psychologists. Attitude in Mathematics Education is still most commonly used to mean liking, disliking and related preferences (McDonough and Clarke, 1994).

Hart (1989) has stated that research into attitudes indicates that the term may stand for any one of a number of perceptions about Mathematics, oneself, one's mother, father or teacher. Attitudes in these instances do not have a strong emotional component (Hart, 1989). However, the term attitude has also been used by some Mathematics educators to mean anxiety, which typically does contain a strong emotional component. Many social psychologists would not include anxiety among attitudes.

Riedesel and Burns (1977) stated that attitudes to Mathematics may be developed as early as grade three but that grades four to eight appeared to be the most crucial years for the development of attitudes. Similarly, Callahan (1971) found that eleven years was the most important age for the establishment of lasting attitudes to Mathematics.

A Definition of the term Attitude

For the purpose of the study, the researcher will define the term *attitude* as meaning the liking or disliking of an object, person or event, and the manner in which one thinks about and reacts to, those constructs. Attitudes are thought to be related to the frequent experiencing of particular positive and/or negative emotions.

Research Trends, Methodology and Findings on Attitudes

The small amount of research on affect in Mathematics that has been carried out during the second half of the twentieth century has focused on attitudes towards Mathematics. The research has included both student and teacher attitudes, but particular interest has been taken in students' responses to the subject of Mathematics as it has been taught in schools.

According to McLeod (1994) research on affect in Mathematics initially focused on general attitudes to the subject then broadened to include the study of beliefs and more intense emotional reactions. McLeod (1994) stated that the change in the emphasis of research over the decades has been accompanied by a change in the methodology and theoretical foundations of the research, and by a change in ideas about teaching and learning.

According to Postle, Spunde and Hodgkinson (1977) the importance of attitudes in the learning process lies in the fact that they determine how one will respond to certain learning situations.

For many Mathematics educators the development of positive attitudes in students has become one of their main goals. Some educators advocate positive attitude development because they consider it to be an important educational outcome in its own right (Reyes, 1984). Others, however, argue that fostering positive attitudes in students will lead to an increase in their level of understanding of the subject and consequently, to an increase in their achievement. Advocates of this idea posit a causal relationship between attitudes towards Mathematics and Mathematics achievement, with a change in attitude supposedly bringing about a corresponding change in achievement. However, some see this relationship as being the reverse, with achievement being the causal factor in the development of attitudes (Hart, 1989). McLeod (1992) has pointed out that research into attitudes seems to indicate that attitudes and achievement are independent constructs that interact with each other in complicated and unpredictable ways.

Most of the research into attitudes in Mathematics Education has investigated the relationship between Mathematics achievement in students and their attitude towards the subject (McLeod, 1994). Work in this area began during the 1960s. It was usually limited to the administration of questionnaires to large groups of students in high school or college. The questionnaires measured student liking or disliking of Mathematics. Some items about anxiety, however, were included (Hart, 1989). Statistical analyses were used to interpret the results. A major concern with the investigations that were carried out during the 1960's has been the questionable quality (reliability) of the instruments that were used in the studies (McLeod, 1994).

The end of the 1960s saw the development of pencil and paper scales that provided a multi-dimensional view of attitudes towards Mathematics. The scales were used to measure specific components of attitudes and included clear descriptions of the construct to be measured (Hart, 1989).

In the late 1970s the first research into gender-related differences in Mathematics was undertaken. Fennema and Sherman (1976) developed an instrument specifically for that purpose. (For a summary of her findings and of the instruments used to measure attitudes, see Fennema and Sherman 1976, Fennema, 1989.)

McLeod (1994) pointed out that evaluation studies have been a main source of data on attitudes and that the studies have provided general information about attitudes and Mathematics. One of the main findings of the studies shows a tendency for attitude scores towards Mathematics to decline as students move from primary to secondary school (McLeod, 1994).

Marshall's (1989) investigation into affect and problem solving in grade six children yielded a majority of negative responses that the author classified as attitudes about the situation in which the students found themselves. Negative affect was found to be demonstrated most frequently in statements reflecting either dislike of the task ("I hate this") or in self-judgments about ability ("I'm no good at this").

Marshall (1989) stated that attitudinal responses resulted from the activation of previously stored affective memories (emotional memories). The attitudinal responses were dispassionate ('cold' rather than 'hot') and did not usually activate observable physiological reactions. However they were still able to impact upon students strongly enough to prevent them from willingly engaging in problem solving and to block their attempts to search their memories for appropriate techniques to solve problems. Marshall (1989) stated that attitudinal responses were in direct contrast to emotional responses.

Sharples' (1969) study, which compared nine, ten and eleven year olds' attitudes to various school subjects, found that in all cases Mathematics occupied a low position on her rating scale (Bell, Costello and Kuchemann, 1983).

Kyles and Sumner's (1977) investigation into Mathematics attainment found that Mathematics was considered to be useful by both primary and secondary students but that the subject was not particularly liked by primary school students nor found to be interesting by secondary students. Anxiety was seen to be the most relevant factor in the primary student's dislike of Mathematics, with about half of them indicating that they were affected by it (Bell et al., 1983).

The Assessment and Performance Unit (APU) Primary Survey (1980) found that children sometimes enjoyed Mathematics. Student responses were dependent upon the topic being treated in class at the time of the survey.

According to Reyes (1984) studies of attitudes and preferences towards Mathematics have only shown consistent positive correlations between the constructs of achievement in Mathematics and confidence in learning Mathematics.

A survey of TAFE students' attitude towards Mathematics, which was carried out by FitzSimons (1994), showed that students' attitudes towards the subject (liking/disliking, enjoy/do not enjoy, prefer/prefer not to) were overall more positive than negative. The results also showed that there was a preference in students for tasks that could be understood, completed and which they considered to be relevant to them. In the study, the majority of students stated that they enjoyed a challenge. Negative attitudes towards Mathematics were overwhelmingly seen to be the result of task complexity and test anxiety. Similarly, teachers' insensitivity to students' needs and feelings (especially in the case of work being rushed), repetitiveness and the feeling of inadequacy were additional reasons given for the development of negative attitudes towards Mathematics (FitzSimons, 1994).

Beliefs

Colby (1973) defined the term *belief* as a judgment of the credibility of conceptualisation. Credibility of conceptualisation related to whether one accepted, rejected or suspended judgment about a set of concepts. Beliefs were also thought to include the relationships between those concepts (Hart, 1989).

Like attitudes, beliefs are considered to be stable in nature (McLeod 1992; McDonough and Clarke, 1994). Some psychologists consider attitudes and beliefs to be different constructs while others view them as being the same. Leder (1993) claimed that a consensus was emerging within the Mathematical community that there could be a significant overlap between attitudes, beliefs and emotions.

According to McDonough and Clarke (1994) research into beliefs lies on the border between affect and cognition because beliefs and belief systems can be considered to encompass both cognitive and affective functions. However, most research into beliefs has been undertaken from a cognitive perspective (McLeod, 1992). The affective component of beliefs has largely been ignored.

Interest in beliefs and belief systems has been an offshoot of cognitive psychology (Hart, 1989). Much of the work in this area began in the 1960s and originated from psychological investigations into artificial intelligence.

An important investigation into student beliefs in Mathematics Education was carried out by Schoenfeld in the 1980s. Schoenfeld's (1983) study of the management and control systems that problem solvers used as they addressed Mathematical problems suggested that attitudes towards Mathematics and confidence about Mathematics were aspects of a student belief system and that the ideas students held about the nature of Mathematics had an important effect on how they managed their cognitive resources (McLeod, 1989).

Schoenfeld (1985) referred to beliefs as the individual's Mathematical world view, that is, the perspective with which an individual approached Mathematics and Mathematical tasks. Schoenfeld (1985) found that the purely cognitive components of his framework did a poor job of predicting the problem solving processes of students (Hart, 1989).

As well as investigating students' attitudes to Mathematics, the FitzSimons (1994) study explored students' beliefs about the nature of Mathematics. Results from the research showed that fifty-six percent of the students considered the subject of Mathematics to be about numbers, rules and formulae and that more students regarded Mathematics as a negative rather than a positive experience.

Borasi (1990) stated that students' conceptions of the nature of Mathematics and their expectations of school Mathematics constituted a powerful force operating behind the scenes in Mathematics classes. Borasi (1990) claimed that students' beliefs were usually stereotyped, deep-seated and unconscious and were therefore difficult for teachers to access and modify.

Only recently have Mathematics educators called for research to be carried out into beliefs. Silver (1985) included beliefs in his list of the ten most underrepresented themes in need of being studied in problem solving (Hart, 1989).

During the last twenty years research on beliefs has become an important strand joining a number of studies of both students and teachers. However, little emphasis has been given to providing an overall structure for the study of beliefs (McLeod, 1992). No single framework has been used by researchers. Moreover, studies have organised data in different ways, and within each context being investigated, individual researchers have put forward their own explanations for the influence of beliefs in Mathematics (McLeod, 1992).

McLeod's (1992) research into beliefs has highlighted their multi-dimensional nature. McLeod (1992) has divided the subject into two main areas: the beliefs of students and the beliefs of teachers. McLeod (1992) subdivided student beliefs into: beliefs about Mathematics, beliefs about self, beliefs about Mathematics teaching, and beliefs about the context in which Mathematics Education occurred. Of these subdivisions student beliefs about Mathematics has received the most attention (McLeod, 1994).

McLeod (1994) has pointed out that confidence about learning Mathematics, self-concept, self-efficacy, causal attributions, and motivation variables could also be considered to be kinds of beliefs. Each construct has been researched in relation to affect and Mathematics Education.

Lester, Garofalo and Kroll (1989) stated that beliefs often interacted with, and at times shaped, attitudes and emotions. Furthermore, Lester et al. (1989) claimed that beliefs influenced the decisions made during problem solving. According to the authors, research on metacognition has contributed to our knowledge of how beliefs influence performance in problem solving.

McLeod (1989) has suggested that in order to gain a better understanding of the relationship between affect and belief systems in problem solving, the two constructs need to be investigated from a more anthropological approach than most researchers in Mathematics Education have taken, or seem to be willing to take.

Research into beliefs, such as that carried out by Schoenfeld (1989), has indicated that students' problem solving performances are frequently undermined by their beliefs about Mathematics (McLeod, 1994) and that their beliefs about Mathematics are greatly influenced by the cultural setting of the classroom (McLeod, 1992).

The Social Context

The analysis of the social context has been of interest to psychologists and anthropologists as well as educators. In the Mathematics Education literature, studies have shown the significance of context in the learning of Mathematics and in the development of attitudes towards Mathematics.

The Thompson and Thompson (1989) investigation into affect and problem solving in elementary school Mathematics classrooms found that the type of learning situation, together with the teacher's manner, could influence the number of feelings (emotions) that children experienced. Thompson and Thompson (1989) claimed that those conditions contributed to the development of attitudes towards problem solving.

Bishop's (1988) analysis of Mathematical culture placed considerable emphasis on beliefs and attitudes. Bishop (1988) has argued for a curriculum that emphasises inducting students into the culture of Mathematics (McLeod, 1992). Research on the importance of student culture in the learning process (See Parsons, Adler and Kaczala 1982; Stevenson, Lee and Stigler, 1986; Stigler and Perry 1988) has indicated that the family has a significant role in the formulation of students' beliefs towards Mathematics (McLeod, 1992).

Clarke (1992) stated that the means by which competence is demonstrated and recognised, the nature of the messages by which the recognition is communicated and the attitudes and practices which result from it, are determined by factors embedded in the social context of the Mathematics classroom. Furthermore, Clarke (1992) considered classroom assessment to be the fundamental aspect of the process.

In Mathematics Education, the study of emotions, and to a lesser extent that of attitudes and beliefs, must be carried out with reference to the particular environment in which it occurs (Cobb et al. 1989). Once emotions, attitudes and beliefs are detached from a specific environment they have little meaning. This idea has been verified by Cobb, Yackel and Wood (1989) in their study and analysis of emotional acts in grade seven Mathematics classrooms.

Emotions

Unlike attitudes and beliefs which are considered to be relatively stable in nature, emotions are considered to be unstable, intense, 'hot' and transient. Emotions have a definite cause and a clear cognitive content (Forgas, 1991). Psychologists assume that emotions evolved with specific signalling functions about particular environmental events (Frijda, 1986).

Some cognitive theorists claim that emotions arise when a discrepancy exists between an individual's appraisal of an event and her/his expectations of it (Frijda 1986; Lazarus 1984; Mandler 1984).

Emotional expression is thought to be one of the most powerful types of affective information (Buck 1984, 1988; Izard 1977). There is also considerable cross-cultural agreement about emotional behaviour, particularly in relation to the meaning of facial expressions (Ekman 1973; Ekman, Friesen, O'Sullivan, Chan, Diacoyanni-Tarlatzis, Heider, Krause, LeCompte, Pitcairn, Ricci-Bitt, Scherer, Tomita and Tzavarus, 1987).

In our culture, children are encouraged to keep their feelings to themselves. As they grow older and become more acculturalised and socialised, observations of emotional responses in them become more difficult (Krathwohl, Bloom and Bertram, 1964). This necessitates a greater reliance on verbal descriptions of emotional experience.

The limitations of language are seen to be one of the major problems associated with the study of the emotions. Most of the evidence of emotional experience comes from the language used in self-report instruments. While people's reports of their emotions are usually treated as valid it must be remembered that they are not scientific observations. Emotional behaviour is private and unconfirmable (Lang, 1994) and reports of emotional behaviour may be synthesised (Ortony et al., 1988).

Just as the terms *attitudes* and *beliefs* have a variety of meanings, so does the term *emotions*. Moreover, it is not unusual to find the words *emotion*, *feeling*, and *mood* being used interchangeably in the literature. Dunlop (1984) has stated that the language of emotion is often vague and imprecise.

Mandler (1984) has pointed out that there is no adequate definition for the term *emotion*, despite the extensive history of the study of emotion in both Philosophy and Psychology (Hart, 1989).

According to Corsini (1984) the Encyclopedia of Psychology defined emotions as high-energy states of mind that give rise to feelings. The strength of the feelings is thought to vary and a direction is usually attached to the states so that they can be categorised as either positive or negative (Hart, 1989).

Other definitions of emotion exist. For example, Arnold (1960) defined emotion as the tendency towards anything considered to be good (beneficial) or away from anything considered to be bad (harmful). According to Arnold (1960) the pattern of attraction or aversion was accompanied by a pattern of physiological changes organised toward approach or withdrawal. The patterns differed for different emotions. Arnold (1960) made a distinction between emotion and feelings. For him, feelings were positive or negative reactions to some experience. In the dynamics of emotion Arnold viewed perceptual and cognitive processes as very important.

Averill (1982) claimed that emotions were often interpreted as passions rather than as actions (Weiner, 1992).

Weiner (1992) stated that there were five distinguishing characteristics of emotions. According to Weiner (1992) emotions had a positive or negative experiential quality, the positive or negative characteristics of emotions varied in magnitude, the emotion might be accompanied by certain facial expressions and body postures, the emotions were signals for certain types of behaviour, and emotions often followed particular thoughts. Weiner (1992) also noted that the antecedent thoughts, facial expressions and behavioural consequences were only the overt manifestations of emotion. The emotion itself was an amalgamation of all the components. It was an entire, organised system (Weiner, 1992).

A Definition of Emotions

Arising from the above discussion and for the purpose of the research described in subsequent chapters, emotions are considered to be the underlying sensations that are experienced as a consequence of the cognitive interpretation of a particular environmental stimulus. Emotions are interpreted as being either positive or negative, unstable and transient. The transiency and instability of emotions distinguishes them from the more stable and permanent constructs of attitudes and beliefs. While emotions are abstract in nature they become manifest in speech and/or physical actions.

Mathematics Research into the Emotions

The following section describes the Mathematics research into stress and panic as they are the two emotions that are most frequently investigated in the Mathematics Education literature. Research into anxiety has also been included in this part of the chapter because it is a construct that is often considered to be an emotion.

Mathematics is recognised as a study based on the intellect, with the emotions rarely being engaged (Buxton, 1981). It is possibly for this reason that the study of emotions during Mathematical problem solving lessons has only recently been given any attention.

Lo, Wheatley and Smith (1994) stated that emotional acts often occurred in Mathematics lessons when there was an attempt to resolve a problematic situation. At the individual level, the emotional acts included the expression of confusion, frustration and impatience. At the group level, students took sides or accused each other of being insincere.

According to Marshall (1989), emotional responses that occur during problem solving are the result of a reaction to an emotion that arises during the completion of the task. Marshall (1989) claimed that emotional responses were intense feelings that were accompanied by physiological changes such as nausea, increased heartbeat or shaking hands.

Sowder's (1989) study of affect in solving story problems showed that emotional responses such as sweating, gasps or bodily tension were not always obvious.

Like so many other aspects of the affective domain, the study of emotion, emotional states and emotional reactions during Mathematical problem solving lessons do not have a theoretical framework to which researchers can refer. McLeod (1992) has been one researcher who has been very conscious of the absence of such a framework. He has argued that Mandler's (1984) discrepancy theory could be used as a theoretical base from which to study the role of the emotions in the learning of Mathematics. Using Mandler's (1984) theory and the idea of emotional memory, which Mandler has also endorsed, McLeod (1992) claimed that it should be possible to explain how students develop permanent responses (attitudes) to Mathematics.

Mandler (1984) stated that his discrepancy theory could be used to explain students' poor attitudes to Mathematics. For Mandler, negative attitudes to Mathematics were thought to be the consequence of students' unhappy early experiences in the subject (Mandler, 1989). If this idea is correct, then it can be assumed that the emotional experiences students have in Mathematics lessons during their initial years at school act as the building blocks for the development of attitudes to the subject. Furthermore, it could be argued that students' emotional experiences might also form the basis of the beliefs they hold about themselves as learners of Mathematics and their beliefs about Mathematics.

A similar assumption could be made using Spielberger's (1972) ideas about anxiety. In terms of his state-trait conception, the emotional states that students experience during problem solving activities could, through repetition, become a trait with the result that the student would always be anxious when confronted with a problem solving task. Buxton (1981) has stated that this type of long-term negative reaction to Mathematics problem solving is common (McLeod, 1989).

The FitzSimons (1994) study investigated emotions as well as attitudes and beliefs. FitzSimons (1994) found that negative responses outnumbered positive ones. The terms that students most commonly used to describe their emotions were: frustrated, confused, bored and angry. The research found that negative responses were related to a lack of Mathematical understanding in students and to their inability to successfully complete set problems.

In the Buxton (1981) study of Mathematics anxiety, words such as panic, anxiety, embarrassment, irritation, frustration and fear were frequently used by the subjects when they had to describe the emotions they experienced during Mathematics lessons.

From their investigation into affect and problem solving Thompson and Thompson (1989) concluded that the absence of negative emotions in students should not be interpreted as meaning that a desirable affective state had been achieved. Thompson and Thompson (1989) explained that expert problem solvers experienced negative emotions and considered them to be an important component in the problem solving process. The authors maintained that the significant issue was not that negative emotions occurred but rather, how students coped with them when they did occur. In their study Thompson and Thompson (1989) noted that overt signs of enthusiasm for Mathematics were rarely observed.

Stress

Sarason (1984) differentiated between stress and anxiety. According to Sarason (1984), from a cognitive perspective, stress could be viewed as a call for action. Sarason (1984) considered stress to be instigated by appraisals of properties of situations and personal dispositions. Anxiety, on the other hand, was considered to be a problem of intrusive, interfering thoughts that diminished attention to, and the efficient execution of, a task (Sarason, 1984).

Mandler (1989) has attempted to explain the different effects that stress can have on individuals. Mandler (1989) stated that when emotional reactions occur, they are usually not conducive to the full utilisation of an individual's cognitive abilities: Thoughts become simplified or concentrated on peripheral events such as physiological reactions. When this happens the effect of stress on the individual is debilitative because she/he focuses on the aspects of the situation that she/he considers to be important, but which may not necessarily be the ones which will lead to the solution of the problem.

According to Mandler (1989) whether stress is debilitative or not depends on the other thoughts that are activated by the emotional experience and therefore become available for dealing with the situation. Mandler (1989) claimed that stress could be thought of as being helpful when it screened out irrelevant thoughts but a handicap when it caused the individual to focus her/his attention on the irrelevancies. In order to handle stress and use it constructively in a problem solving situation, Mandler (1989) stated that students needed to be equipped with a variety of problem solving strategies.

Stress, created by various classroom activities, has been found to be a contributing factor to Mathematics anxiety. Activities that are thought to induce stress include: timed tests, frequent quizzes, excessive competitiveness and being asked to work on the blackboard (Tobias, 1980).

Panic

Buxton's (1981) qualitative study of panic, a form of Mathematics anxiety, resulted in his construction of a model which showed how panic could block thinking. Buxton's model postulated that individuals switched their thoughts back and forward from one of three mental states which Buxton called delta one, delta two and delta three. According to Buxton (1981) a type of mental paralysis occurs as a result of the repeated switching between states. Buxton called the paralysis 'panic' and argued that it could prevent a task from being completed. In Buxton's theory, the emotions are considered to be capable of limiting an individual's performance to the degree of stopping all productive thought about a problem's solution (Reyes, 1984).

Buxton (1981) found that in Mathematics lessons students' mental anguish was caused by the content of the lessons and the elements of time (as in mental Arithmetic), pressure (in trying to obtain high grades) and evaluation.

The Buxton (1981) investigation emphasised the importance of students being internally motivated in Mathematics lessons. Buxton (1981) stated that students would become internally motivated if they experienced frequent success and if the number of failures they experienced were few. Buxton (1981) argued that one of the most important aims for teachers should be to have their students experience more success than failure. According to Buxton (1981) if this was achieved then students would not experience panic and/or anxiety.

Anxiety

Anxiety is the most studied of all the emotions (Hart, 1989). However, it may be examined in relation to either attitudes or emotions. Anxiety is viewed by most psychologists as a 'hot' reaction. However, those who view anxiety as an attitude would consider it to be a 'cool', less intense emotional reaction (Hart, 1989).

Many definitions of anxiety exist in the psychological literature. According to Sarason (1984) anxiety has often been defined as a complex state that included cognitive, emotional, behavioural, and bodily reactions. Benner (1985) defined anxiety as a subjective feeling of tension, apprehension, and worry, set off by a particular combination of cognitive, emotional, physiological, and behavioural cues.

Spielberger (1972) discussed two types of anxiety: state anxiety and trait anxiety. According to Spielberger (1972) state anxiety, or the *A-state* is the unpleasant emotional state or condition which is characterised by activation or arousal of the autonomic nervous system. State anxiety is time- and situation-specific and is aroused when an individual perceives a situation as potentially harmful or threatening. Trait anxiety is neither situation- nor time-specific. It is the relatively stable personality trait of being prone to anxiety (Reyes, 1984). Paper and pencil instruments have been used successfully to distinguish between state and trait anxiety (Hart, 1989).

Attempts have been made to create a model that explains anxiety in Mathematics learning. The models put forward by Byrd (1982) and Buxton (1981) have proven to be the most useful in doing this.

Using semi-structured, individual interviews, Byrd (1982) carried out an in-depth study of anxiety in college students. From her research, Byrd (1982) proposed a model of anxiety that showed the sequence of responses with which an individual reacted to an anxiety-arousing situation. Byrd (1982) found that Mathematics anxiety manifested itself in many ways and tended to be more a state than a trait anxiety.

In the model proposed by Byrd, the anxiety reaction itself consists mainly of physiological and behavioural signs that are associated with the reaction to the stressor (the cause). The physiological reactions include increased heartbeat, sweating palms, dilated pupils or other autonomic nervous system reactions. The behavioural responses, which are more able to be controlled by the student, include such things as the voice trembling, biting the fingernails and fidgety behaviour (Reyes, 1984).

Byrd's (1982) model may be summarised in the following way:

STRESSOR → PERCEPTION → A-STATE → COGNITIVE → COPING
OF THREAT REACTION REAPPRAISAL

Figure 1: Anxiety as a Process (from Byrd 1982)

In the Byrd (1982) model, cognitive reappraisal comes after the A-state reaction has begun. It consists of a method of coping with the stressor. According to Byrd (1982), coping strategies could take many forms. They could include actions to combat the threat, inaction, or defence mechanisms such as repression or rationalisation. Some forms of coping, called facilitative, could improve an individual's performance. More frequently, however, the methods of coping employed by individuals decreased performance or had a negative effect on the individual. Anxiety such as this was called debilitative.

Mathematics Anxiety

In the literature there are numerous definitions of the term anxiety. Many of them are general in nature and only vaguely define the construct. Furthermore, some definitions consider anxiety to be a transient state or emotion while others consider it to be a permanent state. The confusion surrounding the definition and delineation of the term anxiety also extends to the definitions of Mathematics anxiety (Hart, 1989) and test anxiety.

Mathematics anxiety has sometimes been viewed as a negative feeling or dislike of Mathematics, a fear of Mathematics or an attitude to the subject. According to Reyes (1984) the most commonly used definition of Mathematics anxiety is that of Richardson and Suinn (1972).

> Mathematics anxiety involves feelings of tension and anxiety that interfere with the manipulation of numbers and the solving of mathematical problems in a wide variety of ordinary life and academic situations.
>
> (p. 551)

The definition by Richardson and Suinn (1972) assumes that Mathematics anxiety is an A-state aroused during Mathematics situations. However, not all researchers agree with this idea.

Williams (1988) defined Mathematics anxiety as an emotional and cognitive dread of Mathematics. Lewis (1970) defined Mathematics anxiety as being a state of emotion underpinned by qualities of fear and dread. According to Lewis (1970) the emotion was unpleasant, directed towards the future and out of all proportion to the threat. Mathematics anxiety was thought to contain the special characteristics of feelings of uncertainty and helplessness when faced with danger.

Tobias (1976) stated that Mathematics anxiety was an emotional and cognitive problem associated with an 'I can't' syndrome. Tobias (1976) claimed that Mathematics anxiety manifested itself in fright of numerical concepts and notation. Buckley and Ribordy (1982) defined Mathematics anxiety as an irrational dread of Mathematics that interfered with number manipulation and Mathematical problem solving within everyday life and academic situations.

Hembree's (1990) investigation into anxiety found that many subconstructs which had been put under the rubric of anxiety were related to specific situations. The two most prominent of these were Mathematics anxiety and test anxiety. From his investigation, Hembree (1990) concluded that both test and Mathematics anxiety seemed to be learned conditions that were more behavioural in nature than cognitive. Hembree (1990) stated that Mathematics anxiety appeared to be comprised of a general fear of contact with Mathematics. That fear included such things as being in Mathematics classes, completing homework and doing tests.

Since 1977 a great deal has been written about Mathematics anxiety. While some research studies have involved experimental investigations most of the literature on the subject has consisted of discussions on how Mathematics curricula and different teaching methods contribute to Mathematics anxiety. The literature has also included descriptions of interventions designed to reduce Mathematics anxiety (McLeod 1992; 1994). These interventions have mainly involved individuals in a desensitisation process and/or in a change of beliefs about Mathematics (McLeod, 1992). Many of the discussions about the interventions however, have been theoretical in nature and have lacked statistically proven evidence (Reyes, 1989).

Hackett (1985) argued that causes of Mathematics anxiety range from factors such as the socioeconomic status and parental background of the students through to the influence of teachers and school systems. Oberlin (1982) stated that the following teaching techniques could cause Mathematics anxiety: assigning the same work to everyone, covering textbooks problem by problem, giving written work everyday (including weekends and holidays), insisting on one way to correctly complete a problem and assigning Mathematics problems as punishment for misbehaviour.

In her study of Mathematics anxiety in the classroom Sherard (1981) suggested eight strategies that teachers could use to reduce Mathematics anxiety in their students. According to Sherard (1981) teachers should:

* avoid stereotyping Mathematics as a male domain
* make students aware of the everyday usefulness of Mathematics, especially for future plans and careers
* help students develop self-confidence in their ability to do Mathematics
* concentrate on problem solving, spatial skills, language and the symbolism of Mathematics
* be aware of the negative effects that testing could have on attitudes towards Mathematics.

In addition to the above strategies Sherard (1981) stated that teachers should create a relaxed and supportive classroom atmosphere and show a sensitivity to students' feelings.

Morris (1981) has also suggested a number of procedures that could be used to reduce Mathematics anxiety. The procedures are:

* creating a positive and supportive classroom atmosphere
* pointing out to students that everyone experiences difficulty

in Mathematics at some time or other
* stressing the understanding of the thought processes
* exposing students to unusual Mathematics situations
* having students fully understand given material before moving on to new work
* eliminating timed tests and individual blackboard work from class activities.

However, Morris (1981) stated that if tests had to be given then positive reinforcement should be included in the feedback.

In helping overcome Mathematics anxiety in students Wood (1982) recommended that teachers utilise three approaches from applied Psychology. Wood (1982) argued that teachers needed to: improve verbal recognition of mistakes, discourage gloating when someone committed an error, and help students understand the consequences of their mistakes.

Richardson and Suinn (1972) claimed that the Mathematics Anxiety Rating Scale should be used to determine the type of treatment necessary for students suffering from Mathematics anxiety.

Recent research into Mathematics anxiety has included studies involving perspectives with cognitive orientations, studies based on constructivist ideas, studies into the effect of Mathematics anxiety on the performance of elementary school teachers and studies which have been directed towards emotional intensity (McLeod, 1992). Some studies into Mathematics anxiety and instructional patterns (Clute, 1984) have suggested that anxious students do less well in discovery lessons than in expository lessons. Research studies have also indicated that learning style can influence the amount of Mathematics anxiety experienced by students (Hodges, 1983).

Most research into Mathematics anxiety has investigated its relationship to achievement. Investigations into Mathematics anxiety and student achievement have involved students of both sexes and have been carried out at all levels of schooling. Consistent results have been attained from these studies. They have shown that a negative relationship exists between anxiety and achievement and that females generally report higher levels of Mathematics anxiety than males. High achievement has also been found to relate to low anxiety. However, no clear cause-effect relationship between Mathematics anxiety and achievement in Mathematics has yet been discovered (Reyes, 1984).

Mathematics anxiety has also been found to cause Mathematics avoidance (Morris 1981; Clute 1984). Mandler (1989) claimed that in an attempt to overcome the problem of Mathematics avoidance, ideas from research into motivation should be applied to Mathematics. Studies of motivation have indicated that people seek out or repeat situations which give rise to positive emotional states and avoid situations which result in negative emotions. Mandler (1989) stated that by applying this principle to Mathematics, an explanation could be found to the phenomena of students' aversion to the subject.

According to the literature, it is important for educators to remember that Mathematics anxiety can handicap learning and interfere with career plans and educational goals, especially for females. Mathematics anxiety is a phenomena that is experienced by people of all ages (Gourgey, 1982) and since many variables can influence it, no single solution is considered capable of eliminating it.

Test Anxiety

The difficulties of definition and classification that are associated with the elements of the affective domain again surface when one reviews the literature on the construct of test anxiety. McLeod (1992) has pointed out that little differentiation is made in the literature between Mathematics anxiety and test anxiety.

In Psychology, test anxiety has been a widely studied personality variable because it has provided a measure of the personal salience of situations in which people are evaluated (Sarason, 1984).

Despite the vast number of studies in Psychology that have looked at the general construct of test anxiety, few studies in Education have specifically investigated Mathematics test anxiety.

For Reyes (1984) test anxiety is anxiety that is aroused by evaluative situations. Wine (1971) described the test anxious person as one who, in situations perceived as evaluative, focused attention on the self instead of the task at hand. The inattention to the task tended to reduce the level of performance on it (Reyes, 1984).

Test anxiety is thought to be composed of two components: worry and emotionality. Worry is the cognitive concern one has about one's performance, the consequences of failure and the comparison of one's ability to that of others (Kass and Fish, 1991). Emotionality refers to a person's awareness of bodily arousal (ANS) and tension in evaluative situations (Sarason, 1984). Deffenbacher (1980) has shown that although worry and emotionality are correlated, worry is related to performance decrements in the presence of an evaluative stressor, and emotionality is not. Morris and Liebert (1970) found that the cognitive aspect of test anxiety interfered with performance (Reyes, 1984).

Much applied research has focused on identifying and reducing evaluative threat in the test situation so that the degree of worry in students would be reduced and their optimal performance achieved (Kass and Fish, 1991). Even though it is not always possible to reduce evaluative threat, some methods have been found to be successful. The methods include: eliminating time pressure, fostering positive interactions with the examiner before and during testing, ensuring success experiences during testing, and altering the pre-performance information given to students (Kass and Fish, 1991).

Dew, Galassi and Galassi (1984), reporting on a study involving physiological measures and Mathematics anxiety, maintained that while there was an association between Mathematics anxiety and test anxiety, they were different constructs. Moreover, the authors claimed that Mathematics test anxiety was not identical to the more general construct of test anxiety (McCormick and Scott, 1993).

Some research studies, however, have found a connection between Mathematics anxiety and test anxiety. Wood (1988) found Mathematics anxiety to be strongly related to test anxiety. Wood (1988) argued that it was the individual's fear of Mathematics rather than her/his participation in the subject that caused her/him to be anxious. According to Reyes (1984), Mathematics anxiety is more likely to be related to test anxiety than to trait anxiety.

From the review of the affective domain it becomes obvious that we still know very little about the role of affect in the learning of Mathematics. The review of the Mathematics Education literature highlights the paucity of studies of positive emotions. In an attempt to understand why positive emotions have not been researched, the following section of the literature review investigates the evolution of the various theories of emotion.

THE DEVELOPMENT OF THEORIES OF EMOTION

The review of the Mathematics Education literature on affect revealed that many studies have investigated the negative emotions of panic and stress and the related issue of anxiety. However, the review also revealed that few studies in Mathematics have focused on positive emotions or on the emotions in general. The absence of such studies and the paucity of references to affective factors in the Mathematics K-6 (NSW Department of Education) syllabus and the reports upon which the syllabus was based, prompted the researcher to formulate a major research question:

Can the emotional states of children in a Mathematics environment be described?

Two additional research questions were created to assist the researcher in her study of emotional states:

Can descriptors of emotional states be developed?
Can indicators of emotional states be developed?

In order to comprehend the reason for the dominance of negative emotions in the literature, it is necessary to understand the theoretical background from which the emotions have been studied. To do this one must return to the inaugural years of the discipline of Psychology, for it was during those years that theories on emotion first emerged.

The following section provides an overview of the development of the theories on emotion.

History of the Development of the Theories of Emotion

According to Lang (1994) almost every text on emotion written this century has begun with the James-Lange theory (Lang, 1994) because modern concerns with emotion have dated from the publication of the papers by James and Lange.

James's contribution to the study of emotion has been significant, for it was his ideas that prompted the change in the understanding of mental processes from the content approach to the process approach. James's ideas have also greatly influenced the way modern psychologists have conceptualised emotion.

In the last decade of the nineteenth century, the newly formed science of Psychology was concerned with the study of mental life and essentially, with the conscious experience of human beings (Lang, 1994). The preoccupation with the analysis of consciousness dominated all areas of study in the nineteenth century. At that time emotions were considered to be unanalysable, simple entities from which all mental phenomena (complex ideas, feelings and thoughts) were constructed (Mandler, 1984). Wilhelm Wundt, one of the founders of the science of Psychology, was the proponent of this idea. Wundt (1896) argued that feelings (Gefuhle) were primary and unanalysable and in a sense were similar to primitive visual or auditory sensations. Wundt allowed that different Gefuhle might be combined or expanded and that physiological changes might exert a secondary influence. He conceded that the resulting broad 'Affect' was a complex mental event. Wundt maintained, however, that initially experienced feelings were the beginning and basic ingredients of emotion (Lang, 1994).

In the final years of the nineteenth century Wundt's ideas were challenged by the those of William James. Like Wundt, James based his theory on his introspective ideas rather than on scientific experimentation. James's philosophical theorising concerned what he called the 'coarser' emotions: anger, joy, fear, love, hate, grief, shame, pride and combinations of those emotions. James distinguished coarse emotions from the subtler emotions such as intellectual and aesthetic feelings (Allport, 1961).

James generally agreed with the theory of emotion of the time. However, he also claimed that it should be possible to understand the process that lead to the development of feelings (Lang, 1994). James, the forerunner of the organic tradition, was a functionalist but he was convinced that conscious feeling was important. As James argued that feelings required an explanation, he proposed that emotions were actually secondary phenomena prompted by the perception of somatic and visceral changes that had been elicited by external stimulation (Lang, 1994). At the same time, however, James felt that the idea of a disembodied human feeling was an impossibility (Allport, 1961).

> If we fancy some strong emotion and then try to abstract from our consciousness of it all the feelings of its bodily symptoms, we find we have nothing left behind, no mind-stuff out of which the emotion can be constituted, and that a cold and neutral state of intellectual perception is all that remains.
>
> (p. 246)

James placed heavy emphasis on the significance of external events in giving rise to bodily visceral changes without any awareness of the meaning of the environmental events occurring in the organism and without any interpretation of them occurring. Both James and Lange repeatedly stated that particular perceptions produced bodily effects, which in turn were perceived and experienced as emotions. However, neither mentioned how the individual became aware of the meaning of environmental events, how the events were subsequently interpreted, or how the perceptions of external events produced the bodily effects (Mandler, 1984).

William James and Carl Lange's theories shared the idea that emotion did not begin with the conscious experience of an affect. Both men proposed that in emotion, bodily and behavioural responses were prior events, that is, a physical reaction occurred then the thought about the feeling (Lang, 1994). In James's view, emotions began with stimuli that evoked instincts - reflex type reactions that were built into organisms (Lang, 1994).

> Instinctive reactions and emotional expression shade imperceptibly into each other. Every object that excites an instinct excites an emotion as well.
>
> (James 1890, p. 442)

James made conscious emotion the central focus of his theory. By doing this, he shifted the function of conscious experience from that of an initiator of action to a passive receiver of physiological change. Lange, on the other hand, maintained the idea that emotion *was* its physiology and that conscious experience was of little importance. Furthermore, James described a varied list of somatic and visceral responses that were prompts to conscious emotional experience, while Lange's hypothesis only specified that emotion was a cardiovascular event (Lang, 1994).

In James' theory, bodily changes meant *all* bodily functions as they occurred. This included responses such as running and crying as well as visceral changes. James stated that one's feeling of those changes was the emotion (Mandler, 1984).

> My theory is .... that the bodily changes follow directly the PERCEPTION of the exciting fact, and that our feeling of the same changes as they occur IS the emotion.
>
> (Quoted in Allport 1961, p. 242)

Lange's theory, on the other hand, restricted bodily changes to mean only those that occurred inside the organism. He held that: "various disturbances are due to disturbances in the vascular innervations." (Mandler 1984, p. 21)

As Lang (1994) has pointed out, over the years, the ideas of the two men have been moulded together so that the James-Lange theory has been seen to both be in support of, and in opposition to, physiological reductionism as the precursor of the behavioural analysis of emotion. The James-Lange theory has also been viewed as the starting point for cognitive theories of affect. Mandler (1984) stated that the James-Lange theory could be called the theory of emotional consciousness.

Both James and Lange were aware of the difficulties and subtleties surrounding the use of language to specify and investigate emotion. According to Lang (1994) it was mainly for this reason that James and Lange have had their names linked and are seen to be proponents of the same theory. While it is convenient for psychologists to place the names of the two men together, it needs to be remembered that each one had his own conception of emotion and that each man made his own contribution to the theory of emotion now called the James-Lange theory.

The idea that emotions may not be the fundamental, unanalysable entities they had always been considered to be developed from James' work. It is the distinction between emotion as a secondary derivative process (the process approach), and the nineteenth century view that feelings were unanalysable and simple (the content approach) that has become one of the main themes running through the history of the psychology of emotion (Mandler, 1984).

The James-Lange theory, may appear to be irrelevant and superficial today. However, its significance lies in the fact that from it evolved the two main traditions found in the study of emotion: the organic and the mental tradition. The distinction between the two traditions lies in the emphasis each one places on central as against peripheral theories (Mandler, 1984).

The organic tradition is concerned with central nervous-system mechanisms (content) while the mental or intellectual tradition concerns itself with peripheral reactions (processes), and particularly with autonomic nervous system (ANS) responses (Mandler, 1984). Both traditions have a long history of development.

The organic tradition which was based on the ideas of eighteenth century sensualists and, later, on James' ideas, states that psychological events rather than thoughts are the precursors of emotion (Mandler, 1984). Followers of the tradition claim that organic reactions influence mental emotional consequences.

The mental tradition developed from the work of Johann Friedrich Herbart. Herbart argued that emotion was a mental disorder based on discrepancies or conflicts among perceptions or ideas.

In the mental tradition organic events are seen as consequences of psychic events. Historically the tradition started with the unanalysable feelings idea but its main thrust was its insistence on the priority of psychological processes in the causal chain of emotions. Unlike the organic tradition which considers processes to be the result of organic visceral events, the mental tradition views organic visceral events as being the result of processes such as mental events, habits, conditioning mechanisms, sensation and feeling.

Within the mentalist camp there are two distinct approaches to the theory of emotion. Some mentalist supporters advocate a theory of fundamental emotions while others uphold a conflict idea of emotion. The most persistent tradition in the mentalist camp is that of the fundamentalists.

The fundamentalist theory, which dates back to the time of Charles Darwin, postulates that there are one or more fundamental, innate processes from which emotional expression and experience emerge. Fundamental emotions are usually ascribed to innate (neural) patterns that arise from the evolutionary history of the organism. These patterned packages are considered to be basic emotions and are thought to number between five and ten. Given these fundamental emotions, other emotional states are then considered to be the products of combination of fundamental states together with cognitive, perceptual and visceral attenuating and accenting influences (Mandler, 1984).

Recent supporters of the mentalist tradition include Tomkins (1962, 1963), Izard (1977), Plutchick (1962) and Ekman (1984). Each of these researchers had her/his own list of fundamental emotions and key differentiating responses, and each had her/his own view of the physiology-affect relationship (Lang, 1994).

Conflict theories have not been widely accepted within the mentalist tradition. They have appeared and reappeared throughout the history of the development of theories on emotion. Behaviourism and psychoanalysis both belong under the heading of conflict theories in the mentalist tradition (Mandler, 1984).

Conflict theory, whose major exponent was French psychologist Frederic Paulhan, is based on the idea that when an organism's behaviour is interrupted by some emergency situation, current behaviour is suspended and responses appear that are directed towards the resolution of the emergency. Conflict theories concern themselves with specific mechanisms whereby current behaviour is interrupted and 'emotional' responses are substituted (Mandler, 1984).

Recent supporters of the conflict theory of emotion include Dewey (1894), Angier (1927), Hebb (1949), Meyer (1955), Mandler (1964) and Miller, Galanter and Aibram (1960).

Developments since James and Lange

There have been several critiques of James's work (See Wundt, 1891; Titchener, 1896; Cannon, 1914). While most of them have had little effect on the development of emotional theory, Cannon's (1927) criticisms of James's ideas have been the most damaging.

Cannon's critique showed up what he considered to be the five major flaws in James's argument. His criticisms set the tone for the succeeding fifty years of psychological evaluation of the James-Lange theory. The Cannon (1927) critique also produced an extensive research tradition in the psychophysiology of emotion (Mandler, 1984).

Cannon's (1927) criticisms concerned themselves with the idea of visceral feedback for emotional behaviour. Cannon argued that the visceral changes in emotion were part of the body's general mobilisation for action ('fight or flight'), a process that did not vary significantly over affects (Lang, 1994). Cannon used experimental and logical analysis as well as theory to attack the ideas of James.

Following Cannon's (1927) criticism of the James-Lange theory, the next significant development in the evolution of theories of emotion came through the experiments and ideas put forward by Stanley Schachter in the 1960s.

Schachter (1959) proposed that visceral action was necessary in emotional experience because it set the stage for the experience. However, he also claimed that a cognitive evaluation was important. Schachter (1959) argued that emotion was the product of the two. Moreover, he stated that general autonomic arousal, rather than a specific pattern or an emotion-specific arousal was the visceral accompaniment to the emotional experience (Mandler, 1984).

Experiments carried out by Schachter and Singer (1962) lead the authors to conclude that cognition arising from the immediate situation and being influenced by interpretations of past experiences, provided the framework within which one understood and labelled her/his emotions (Tomkins and Izard, 1965).

As in James's analysis, Schachter gave little consideration to the process that initiated physiological reactions (Lang, 1994). Schachter's major contribution to the development of theories of emotion lay in the fact that his theory disengaged emotion from specific patterns of visceral behaviour and conjoined the visceral and cognitive components. As a consequence of this finding, the line from James and Lange was changed to a cognitive one (Mandler, 1984).

Mandler (1984) claimed that after the Schachter experiments, the organic and mentalist traditions had merged into a more general view in which conflicting interruptive (mental and environmental) events were considered to play an important role in emotion and in generating visceral responses. Conversely, Mandler (1984) stated that a visceral response alone was not a sufficient condition for emotional phenomena and that additional mental or cognitive events were also required. Mandler (1984) argued that the same blurring had occurred to the central-peripheral distinction. According to Mandler (1984) both central and peripheral events could be thought to contribute to the production of emotional experience and emotional behaviour.

Since the time of James and Lange our knowledge of the physical basis of emotion has increased in depth and complexity (Lang, 1994). Lang (1994) stated that the interaction of the literature from the study of behaviour, neurophysiology and neurochemistry and cognition should be used to further our understanding of emotion. Furthermore, Lang (1994) claimed that each area of study has already contributed to the theories of emotion that are currently in circulation. According to Lang (1994) the study of behaviour has provided information about conditioning and motivation while neurophysiology has provided greater knowledge about chemical analysis and brain imaging, and cognitive theory has added to our understanding of artificial intelligence, attribution, emotional memory and emotional expression.

In recent years psychologists have begun to consider emotion, emotional expression and emotional memory from the point of view of information processing (Lang, 1994). Lang (1985) stated that the memory of an emotional episode could be viewed as an information network that included units representing emotional stimuli, somatic or visceral responses and related semantic (interpretive) knowledge. Lang (1994) considered the memory to be activated by input that matched some of its representations, and that because of the implicit connectivity, the other representations in the structure were also automatically involved. As the circuit was associative, Lang (1994) suggested that any of the units could initiate or subsequently contribute to the process. The idea of emotional memory was first hinted at by James. He wrote:

> In both instinct and emotion the mere memory or imagination of the object may suffice to liberate the excitement.
> (James quoted in Allport 1961, 240)

McDonald (1989) stated that emotional memories were action dispositions and that they concerned highly motivated behaviour. According to McDonald (1989) expectations that were built up over time could sometimes unconsciously determine attitudes and feelings.

It has not been the aim of this section to provide an in-depth discussion of the historical development of the theories of emotion and the subjects of study that have evolved from them. An outline of the theories was considered necessary so that one could understand the theoretical background from which research into the emotions has evolved.

The preoccupation of the early theorists with visceral responses could account for the dominance of studies of negative emotions in the literature, for negative emotions are the ones that are most often found to produce strong visceral reactions.

Cognitive Theorists and the Contribution of George Mandler and Andrew Ortony, Gary Clore and Andrew Collins

To investigate the emotions in the context of Education, and specifically in relation to the Mathematics classroom, it is necessary to return to the third area of study suggested by Lang viz., the study of cognition.

The cognitive perspective, with particular reference to the work of Mandler (1984;1989) and Ortony, Clore and Collins (1988), will now be discussed, as cognitive theory provides a feasible explanation for the emotions that students experience in a Mathematics environment and because it is from a cognitive perspective that the study of emotional states will be interpreted.

Cognitive Theorists

In the latter part of the twentieth century, cognitive theorists claimed that the emotional quality of a stimulus was determined by a prior mental analysis. Many theorists viewed the role that Schachter assigned to evaluative judgment - attribution of an ongoing arousal - as being too narrow and coming too late (Lang, 1994).

Lazarus (1982), for example, proposed that the whole phenomenal field was subject to evaluative scrutiny and interpretation as part of the affective process. Elaborating a term created by Arnold (1960), he argued that all emotion began with an appraisal of the significance of the stimulus. Furthermore, appraisal was thought to occur in stages. Initially it was made to determine if an input was emotionally evocative, then a reappraisal was made to assess any late information and also to make a response selection (Lang, 1994). Many clinical psychologists adopted this idea. One such psychologist was Beck.

Beck and Emery (1985) described anxiety and panic as being the result of the misattribution of physiological arousal. Beck (1985) argued that the causal chain of emotion was a cognitive/physiological loop feeding off itself and perpetuating emotional responses to certain stimuli (Lang, 1994). The cognitive/physiological loop can best be described as follows:

> Neither cognitive nor physiological factors are antecedents [to emotion] ..... [They] are viewed as feeding back upon and modifying both continuing emotional experiences and future emotional responses.
> (Candland, Fell, Keen, Lesher, Plutchik and Tarpy 1977, quoted in Mandler 1984, p. 48)

The Theory of George Mandler

George Mandler (1975, 1984) accepted Schachter's attribution doctrine and was greatly influenced by it. However, one of Mandler's main objectives was to explain the primary stimulus issue.

Adopting the ideas held by advocates of the conflict theory of the mentalist tradition, Mandler suggested that activation in emotion resulted from an interruption or disruption in a goal-directed behavioural sequence (Lang, 1994). This concept had been suggested earlier by Lewin (1935, 1940) and also by Hebb (1949). Hebb (1949) described emotion as the consequence of discontinuity in a phase sequence (Lang, 1994).

Mandler's (1984) theory proposed that most affective factors arose out of emotional responses to an interruption of plans or planned behaviour. An interruption occurred when a structure failed to handle available evidence or action requirements. Interruptions were considered to be disruptive because they blocked the activity that was most appropriate to the situation.

According to Mandler (1984), plans arose from the activation of a schema. The schema produced an action sequence and if the anticipated sequence of actions could not be completed as planned, a blockage or discrepancy occurred that was followed by a physiological response. The physiological response was usually experienced as an increase in heartbeat or in muscle tension (McLeod, 1992). Mandler called the visceral reactions or gut feelings the 'hot' aspect of emotion, and stated that the physiological reaction of the organism determined the intensity of the emotion and therefore, its specific attributes.

Mandler (1984) argued that arousal served as the mechanism for redirecting the individual's attention and that as the arousal occurred, the individual attempted to evaluate its meaning. The cognitive evaluation of the interruption, or 'cold' aspect of emotion (which had been expressed in a similar way by James), provided the individual with a meaning to the arousal (McLeod, 1992).

The theory proposed by Mandler (1984) can be summarised as follows: interruption leads to arousal and the arousal may or may not produce an emotional state.

McLeod (1992) noted that there were several significant points in Mandler's theory. First, that meaning came out of the cognitive interpretation of the arousal and that the meaning ascribed to it was influenced by the individual's knowledge, beliefs and culture. Second, that the arousal which lead to the emotion was usually of limited duration. Third, that repeated interruptions in the same context were likely to result in emotions that became less intense. Mandler (1989) also emphasised two points in his theory. First, that there were an innumerable number of different emotional states because no situational evaluation was the same from one occasion to the next. Second, that purely cognitive evaluations could generate judgments of helplessness that eventuated in anxiety.

Mandler (1984) claimed that his theory bridged the gap between the conflict and organic theories. Moreover, he stated that it extended the importance of conflict and downplayed the role of visceral events in the production of emotional experience and behaviour.

The Theory of Andrew Ortony, Gary Clore and Andrew Collins

Ortony, Clore and Collins (1988) found Mandler's (1984) theory attractive because of its specificity with respect to the appraisal aspects of emotion, and because of its explicit recognition of the importance of plans, goals and knowledge representations (Ortony et al., 1988). However, the authors claimed that the theory's lack of information about specific emotions, especially the positive ones, and its absence of a systematic account of the relations among different emotions, were major weaknesses in it.

Ortony et al. (1988) therefore proposed a theory that attempted to overcome what they considered to be some of the deficiencies in Mandler's theory. Their approach tried to explain how people's perception of the world caused them to experience emotion.

The goal of Ortony et al. (1988) was to construct a cognitive theory concerning the origins of the emotions. They particularly wanted to specify the global structure interrelating different emotions and the characteristics of individual emotions (Ortony et al., 1988).

The main thrust of Ortony et al.'s (1988) theory was that the emotions were best represented as a set of substantially independent groups based on the nature of their cognitive origins, and that the particular emotion a person experienced was determined by the way in which she/he construed the world or the changes in it. Thus, Ortony et al. (1988) attempted to specify both the eliciting conditions for different emotions and the variables that influenced their intensity.

The authors defined emotions as valenced reactions to events, agents or objects, with their specific nature being determined by the way in which the eliciting situation was construed (Ortony et al., 1988). Ortony et al. (1988) defined events as being people's construals of things that happened when they were considered independently of any beliefs they might have about actual or possible causes. Objects were objects viewed qua objects while agents were things considered in light of their actual or presumed instrumentality or agency in causing or contributing to events. Agents could be people, non-human animate beings, inanimate objects or abstractions, such as institutions and situations, provided they were construed as causally efficacious in the particular context.

Ortony, Clore and Collins' (1988) theory started with the assumption that emotions arose as a result of the way in which the situations that initiated them were construed by the experiencer. The authors recognised that there were certain significant individual and cultural differences in the experiencing of emotions.

Ortony et al. (1988) argued that emotions issued from the cognitive interpretation imposed upon external reality rather than directly from reality itself. They claimed there was an essential and deep cognitive basis for emotions but that some emotions involved less cognitive processing and structuring than others (Ortony et al., 1988).

According to Ortony et al. (1988) most theories of emotion tended to be vague about the role that cognition played in emotion. The authors constructed a theory that attempted to overcome the vagueness found in other theories. Ortony, Clore and Collins (1988) stressed, that when they claimed emotions always involved some degree of cognition, they did not necessarily mean that the contribution of cognition was always conscious. The authors considered emotions to be determined by the structure, content and organisation of knowledge representations and by the processes that operated them. The representations and processes were only sometimes considered to be available to consciousness.

Like James (1884) and Lange (1922), Ortony et al. (1988) discussed the difficulty of using language in theorising about emotion. The authors argued that language often contained ambiguities, synonyms, lexical gaps and linguistic traps which limited its accuracy in describing the phenomena of emotion. Therefore Ortony, Clore and Collins (1988) stated that as a source of evidence in the description of emotions, language needed to be used with considerable care.

By trying to separate a theory of emotion from the language of emotion Ortony et al. (1988) claimed they had the potential to develop a theory which was relatively culture-free. Furthermore, the authors stated that they had been able to evolve a theory based on culturally universal principles. The principle of the theory was that the particular classes of emotion that existed in a culture were dependent on the ways in which the members of the culture viewed their world. In our culture, Ortony et al. (1988) saw the main divisions as being those of events objects and the actions of an agent, thus they based their theory on those constructs (Ortony et al., 1988).

Ortony, Clore and Collins (1988) acknowledged that it was important to identify characteristic behaviours associated with individual emotions but they also stated that the behaviours did not usually constitute the emotion. Instead, the behaviours were seen to represent a response (in conjunction with the particular initiating event) to an emotional state (Ortony et al., 1988). Other theorists, however, (for example Frijda 1987) have stated that action tendencies should be construed as part of the total emotional experience.

Ortony et al. (1988) argued that the problem with concentrating on behaviour when considering the emotions was that the same behaviour could result from very different emotions (or no emotions at all) and that very different behaviours could result from the same emotion.

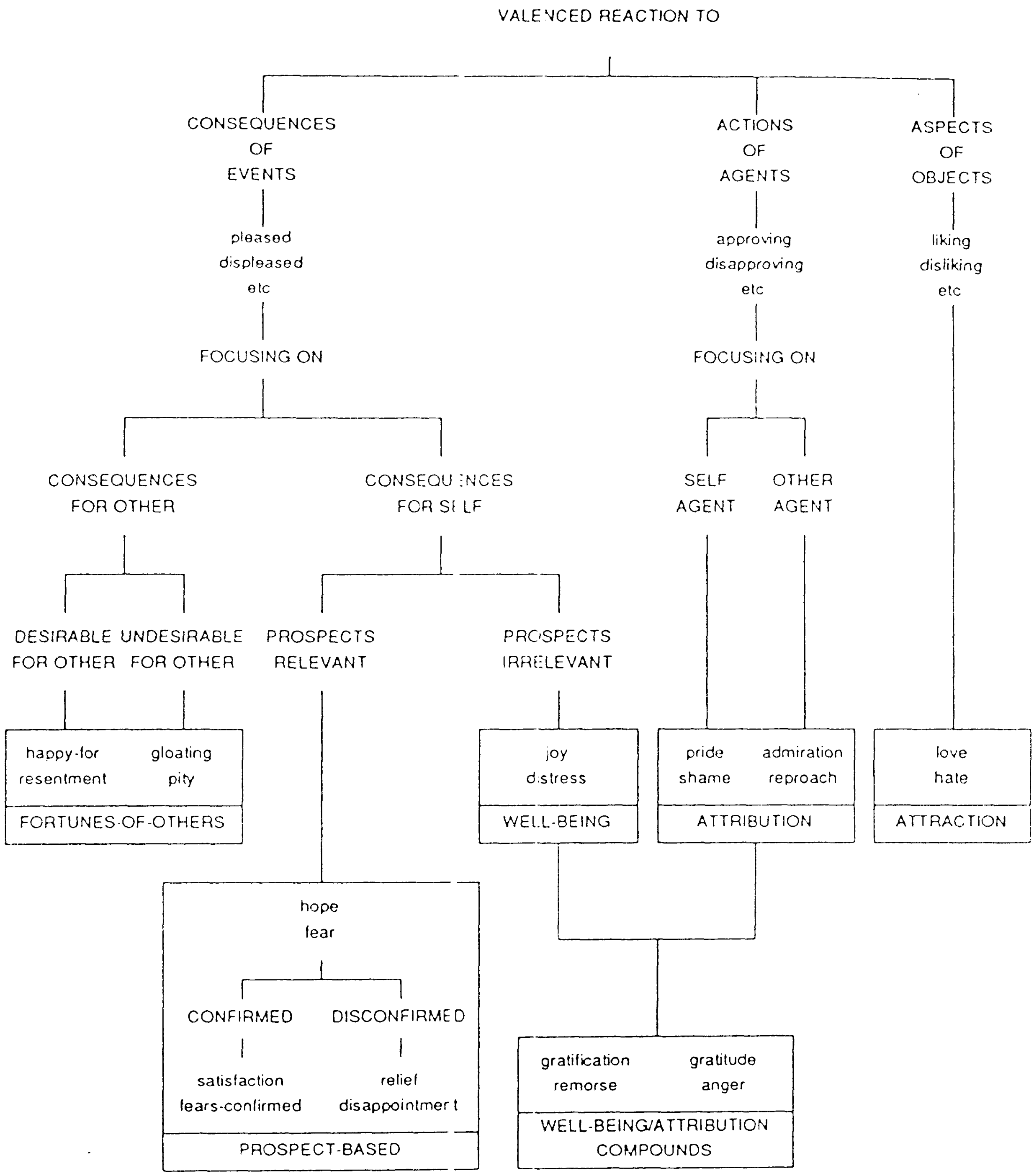
Ortony, Clore and Collins (1988) were sympathetic with the thought that different emotions involved different associated action tendencies. They concluded that action tendencies might be typical for some emotions and even normal, but they could not be constitutive of all emotions (Ortony et al., 1988). Ortony et al. (1988) argued that an analysis of emotion needed to go beyond differentiating positive from negative emotions and more towards giving a systematic account of the qualitative differences between individual emotions such as fear, envy, anger, pride, relief and admiration.

The remaining part of this section discusses the structure of the theory in more detail.

According to Ortony et al. (1988), their theory can be shown diagrammatically as a hierarchical structure indicating the relationships between the consequences of events, the actions of agents and the aspects of objects to others and the self (See Figure 1).

The theory commences with an introduction to the distinctions and associated emotions relating to reactions, events and their consequences. This branch includes emotional types such as pleased and displeased, joy and distress, hope and fear, relief and disappointment and pity and resentment. All of the emotions on the first branch are the reactions the experiencing person might have to the implications of events. The first branch is the most complex part of the diagram (Ortony et al., 1988).

The second main set of emotions are those emanating from the actions of agents. Basically these emotions are differentiated forms of the affective reactions of approving and disapproving of an agent's action (Ortony et al., 1988).



Global structure of emotion types.

Figure 2

The final branch shows a structureless group of emotions that result from reactions to objects. The emotions are called 'Attraction' emotions by the authors and are all variations of the affective reactions of liking and disliking. Thus they represent the undifferentiated affective and aesthetic reactions to objects for which love and hate are examples (Ortony et al., 1988).

Ortony et al. (1988) claimed that object-based emotions, or the emotions that are the momentary reactions of liking and disliking, are among the most salient experiences we have. At the same time, those emotions appear to be more immediate, more spontaneous and less affected by accessible cognitive processes than most of the other emotions.

A major feature of the Ortony et al. (1988) theory is that some of the factors that affect the intensity of emotions are specific to particular groups of emotions. The authors embraced a notion of emotion types rather than the idea of a set of basic emotions because they felt the term was too vague. Ortony et al. (1988) argued that the concept of basic emotions developed from the idea that some emotions were more salient than others, either through intensity or frequency.

Ortony, Clore and Collins (1988) did, however, treat some emotions as more fundamental than others and they had a compounding hypothesis for certain emotional states. The authors claimed that particular emotions could be considered basic because of the very specific meaning given to them. Their meaning was a result of their less complex specifications and eliciting conditions compared to other emotions. In the view of Ortony et al. (1988), emotions could *really* only be considered to be basic in the sense that they were either positive or negative reactions to something.

The underlying principle inherent in the structure of the Ortony et al. (1988) theory is that of successive differentiation. The authors stated that the complexity of an emotion was determined by the degree to which it was a more differentiated form of a simpler affective reaction. This meant that the distinctiveness of an emotional state stemmed in part from the particular form of the affective reaction that it was. Thus, the authors developed a hierarchical structure in which the top level contains two basic kinds of affective reactions, positive and negative. Ortony et al. (1988) argued that valenced reactions were the essential ingredients of emotions because all emotions contained some sort of positive or negative reaction to something. When additional factors were brought into consideration, increasingly differentiated emotional states were thought to result.

The most important of the factors was the nature of the thing (event, agent or object) to which the valenced reaction was a reaction (Ortony et al., 1988).

Ortony et al. (1988) explained that some states such as surprise or abandonment should be thought of as cognitive states rather than emotional states because they were not usually valenced.

In the Ortony et al. (1988) theory global variables and local variables were thought to affect the intensity of different emotions. Global variables were considered to be factors such as a sense of reality, proximity, unexpectedness and arousal. Local variables included likelihood, effort, realisation, desirability-for-other, deservingness, praiseworthiness, the strength of the cognitive unit, expectation/deviation, familiarity and appealingness (Ortony et al., 1988).

According to Ortony et al. (1988), two additional variables that could effect the intensity of emotions were the degree to which the object was appealing or unappealing, and the familiarity of the object. Appealingness originated in attitudes that were dispositional likes and dislikes. These dispositions were associated with representations of, or categories of objects. Thus momentary liking (or disliking) often resulted from how one categorised an object, what one's disposition was towards objects in that category, and from the characteristics of the individual object itself.

Ortony, Clore and Collins (1988) claimed that an important cognitive function of the feeling of an emotion was its ability to index memory. They stated that if an organism experienced a feeling similar to ones experienced in the past, that organism had the potential to respond on the basis of its memory of the success or failure of past responses. Emotional experiences could therefore be viewed as a method of indexing memory so that the organism could respond to a situation in the maximally appropriate manner.

When one experiences emotion one has to cope with the emotion-inducing situation and the experience of the emotion itself (Ortony et al., 1988). Ortony et al. (1988) have suggested that one of the functions of emotion is to focus attention on the emotion-inducing event. Consequently, situations that give rise to intense emotions would be expected to command more attention than those giving rise to less intense emotions or to no emotion at all. Ortony et al. (1988) have argued that a situation that evoked intense emotions might be brought into conscious awareness by its intensity, and that this would enable the organism to take action if it was considered necessary. Therefore, when a situation arose that was unexpected, Ortony et al. (1988) claimed that it could produce a great deal of cognitive disorganisation.

Ortony et al. (1988) stated that failure to cope was likely to intensify the emotion being experienced. According to the authors, failure to cope could be caused by two things: insufficient time and/or the individual's inability to handle the task or situation.

In the Ortony et al. (1988) theory, coping became an important aspect of the emotional experience. Coping was considered to be closely related to the degree of control one perceived oneself to have: The more control, the more one would be able to cope. Ortony et al. (1988) stated that the extent to which one coped, or thought one could cope, influenced not only the intensity of the emotions that arose, but in some cases, the new emotions that were created by the situation and the additional demands on the coping mechanisms.

In summary, the Ortony et al. (1988) theory suggests that the overall structure of the emotional system can be represented as groups or families of emotion types that share the same eliciting conditions, and that these types are differentiated forms of more general affective reactions to events, agents and objects. The main feature of the theory is the idea that the function of emotion is to represent in a conscious and insistent way (through distinctive feelings and cognitions), the personally significant aspects of construed situations.

The Relevance of the Theories of Mandler and Ortony et al. to Mathematics Education

Mandler (1989) saw his theory as making a contribution to the understanding of emotion in Mathematical problem solving situations. According to Mandler (1989) Mathematics is a subject known for its frequent production of errors and unexpected successes and for its evaluation of students. Mandler (1989) claimed that it was these conditions that made Mathematics lessons suitable for demonstrating his conflict theory.

Mandler (1989) stated that emotions were situation specific. Therefore, any discrepancy that occurred in the course of problem solving represented a potential affective episode. Whether or not an affective episode occurred depended on how the individual evaluated the situation (Mandler, 1989).

According to Mandler (1989), an error could be viewed as a discrepancy. An error was said to occur when the learner did something or thought something that was different from her/his original intention. An error could also be interpreted as being an unexpected action that the learner performed during the learning process (Mandler, 1989). Mandler (1989) stated that most errors were coupled with a negative evaluation of the current situation and therefore produced negative affect (unhappiness, disgust or despair) in varying degrees of intensity. Negative evaluation was also thought to interfere with ongoing cognitive processes because of both the pre-occupation of the mind with insignificant matters and the search for a solution. Mandler (1989) argued that if the learner was continuously producing errors, then the intensity of the negative affect would increase and this would lead to general confusion or the task being abandoned. If the task was not abandoned then a solution to it might be obtained from panic-initiated random guesses.

Mandler (1989) claimed that if students were made to realise that the making of errors in Mathematics was a natural part of the learning process then error production would be less interfering and less intense because there would not be a build up of experiential schemas.

Mandler (1989) pointed out that the intensity of the reaction that a student experienced during problem solving was related to the degree of organisation of the student's mental activity. In Mathematics a great deal of student time is spent practising solvable routine-type problems. When blockages occur that interrupt these types of problem solving sessions, intense emotions are likely to result (McLeod, 1989).

Mandler (1989) stated that in the case of a discrepancy being a success, the smooth progression of a planned course in problem solving would produce little in the way of arousal. However, if the learner was unsure of her/his ability an unexpected success in problem solving would produce a discrepancy. Joy, delight and satisfaction were likely to be the resulting emotions (Mandler, 1989). In problem solving situations both positive (happiness) and negative (unhappiness) reactions can occur. Either kind of emotion can result from the same type of interruption. Mandler (1989) therefore considered that the vital issue was the way in which students interpreted an interruption (McLeod, 1989).

Mandler (1989) considered his theory to be relevant to problem solving in Mathematics. However, Ortony et al. (1988) have not applied their theory to any aspect of Education. Both models appear to have applicability to children in a Mathematics environment.

Such theories can be used akin to a conceptual framework and have the potential to add validity to the investigation into emotional states.

SUMMARY

In order to determine the merit of considering affective variables in the learning of Mathematics, Chapter Two reviewed the literature on the affective domain and the Mathematics Education literature on affect. The review of the Psychological literature found that much has been written about attitudes and beliefs but less has been written about the emotions. The review of the Mathematics Education literature on affect revealed a similar pattern. The bulk of the research that has been undertaken in Mathematics Education has investigated measurable products such as attitudes and beliefs, and has neglected processes such as the emotions.

The second section of Chapter Two discussed findings from the research into stress and panic and the related issues of anxiety, Mathematics anxiety and test anxiety. In this section it was pointed out that most Mathematics research into the emotions has focused on negative emotions. The absence of studies of positive emotions and/or the full range of emotions, and the paucity of references to affective variables in the Mathematics K-6 (NSW Department of Education, 1989) syllabus and the reports upon which the syllabus was based, prompted the researcher to formulate a major research question and two subsidiary questions about emotional states.

In an attempt to understand the reason for the dominance of research into negative emotions the final section of Chapter Two traced the development of the theories of emotion. The section also discussed the cognitive theories of George Mandler (1984; 1989) and Andrew Ortony, Gary Clore and Andrew Collins (1988) because their theories appeared to offer an explanation for the emotions that students experienced in a Mathematics environment.

The following chapter will describe the design of the study into emotional states.