

5. Generalised CCD Multilateral Index Accounting for Spatial Autocorrelation

In Chapter 3, a stochastic approach is designed to estimate the parameters of the CCD multilateral index number formulae. In Chapter 4, it is found that the use of geographically mapped data in linear regression modelling creates spatial autocorrelation among the disturbances. Since the present study utilises prices across countries, which is found to be spatially autocorrelated in the study of Aten (1994), it is then necessary to incorporate spatial autocorrelation of disturbances in the context of CCD multilateral index number estimation.

Given the presence of spatial autocorrelation among the residuals of the CCD multilateral index model, a spatial disturbances model is constructed and a simultaneous estimation procedure is developed following the approach of Miron (1984). A more general form of the proximity matrix W is also constructed based on the three proximity measures mentioned in Section 4.2. The stochastic approach to CCD multilateral index number estimation as well as the simultaneous estimation procedure for the above model that accounts for spatial autocorrelation are illustrated using the 1985 ICP data. Empirical application and results are summarised and discussed in the later part of the Chapter.

In Section 5.1, the specified spatial disturbances model for the CCD multilateral index number is presented. This model may be viewed as an extension to the work of Prasada Rao and Selvanathan (1992a, 1994). It is aimed at providing efficient estimates for the PPP's and their standard errors. Section 5.2 outlines the steps in the simultaneous estimation of the parameters in the spatial model developed in the previous section. In Section 5.3, the specifics of the data used in this dissertation are explained in sufficient detail. Some illustrations of the proximity variables or matrices used in constructing the spatial disturbances model are also included in this section.

In Section 5.4, the outcomes of the simultaneous estimation procedure for the generalised CCD multilateral index model that allows for spatial autocorrelation (GCCD(SA)) are discussed. The result from testing for no spatial autocorrelation among the residuals of the CCD multilateral index model is also presented. Particular attention is focused on the estimated PPP's and their standard errors for the $M-1$ countries. Lastly, the results of the latter estimation procedure are evaluated to determine the sensitivity of the PPP estimates in terms of the proximity variables used

in the present study. The spatial autocorrelation parameter estimates obtained for each of the proximity measures are also compared in terms of their magnitudes.

5.1 Model Specification

Following the CCD multilateral index model already established in Section 3.2, given by

$$Dp_{ikj} = \Pi_j - \Pi_k + u_{ikj}, \quad (5.1)$$

where $i = 1, 2, 3, \dots, N$; $k = 1, 2, 3, \dots, M-1$; $j = k+1, \dots, M$; and $\Pi_1, \Pi_2, \Pi_3, \dots, \Pi_M$ are the parameters leading to estimates of the PPPs. The disturbance term u_{ikj} are assumed to have the following properties,

$$\begin{aligned} E(u_{ikj}) &= 0; \\ \text{Var}(u_{ikj}) &= \frac{\sigma^2}{\bar{w}_{ikj}}; \text{ and,} \\ E(u_{ikj}u_{i'k'j'}) &= 0, \text{ for } i \neq i', k \neq k' \text{ and } j \neq j' \end{aligned} \quad (5.2)$$

After adjusting for heteroscedasticity, equation (5.1) becomes

$$\sqrt{\bar{w}_{ikj}} Dp_{ikj} = \sqrt{\bar{w}_{ikj}} \Pi_j - \sqrt{\bar{w}_{ikj}} \Pi_k + u_{ikj}^*, \quad (5.3)$$

where $Dp_{ikj} = \ln(p_{ij} / p_{ik})$, the log-change in the price of the i th commodity of country j relative to country k and \bar{w}_{ikj} is the average budget share for i th commodity of countries k and j .

When the disturbance term exhibits spatial autocorrelation, the assumption that $\text{cov}(u_{ikj}^*, u_{ikj}^{*'}) = \sigma^2 I$ is no longer satisfied. Least square procedure could produce inefficient parameter estimates and the sampling properties of u_{ikj}^* follow from equations (4.12) and (4.13). Hence, the disturbance term can be specified as (4.9).

The above model can be written in matrix form as

$$Y = X\Pi + u^* \quad (5.4)$$

where

- Y is an $NM(M-1)/2$ column vector;
- X is an $NM(M-1)/2 \times M$ matrix of observation;
- $\Pi = [\Pi_1, \Pi_2, \dots, \Pi_M]'$ vector of unknown parameter;
- u^* is a vector of transformed disturbances;
- N is the total number of commodities; and
- M is the total number of countries.

When the disturbances suffer from spatial autocorrelation, following Cliff and Ord (1981), the disturbance term in (5.4) can be modelled as

$$u^* = \rho W^* u^* + v \quad (5.5)$$

where $E(v) = 0$ and $E(vv') = \sigma_v^2 I_{N^*}$ and $N^* = NM(M-1)/2$; and σ_v^2 is the variance of v_t , a t th element of the vector v , a vector of unobserved disturbances.

In order to construct the ρ and W^* matrices in equation (5.5), firstly rearrange or permute all the observations, such that the Y matrix in (5.4) takes on the form of a string of $M(M-1)/2$ column vectors, where each column vectors pertain to a certain commodity. Let $M^* = M(M-1)/2$, thus, the column vector Y is arranged in the following way

$$Y = \begin{bmatrix} \tilde{Y}_1 \\ \tilde{Y}_2 \\ \tilde{Y}_3 \\ \vdots \\ \tilde{Y}_{M^*} \end{bmatrix} \quad (5.6)$$

where a typical 'block' of Y for a particular i th commodity will be given by an $M^* \times I$ column vector

$$\tilde{Y}_i = \begin{bmatrix} \mathcal{D}p_{i12} \\ \mathcal{D}p_{i13} \\ \vdots \\ \mathcal{D}p_{i1M} \\ \text{-----} \\ \mathcal{D}p_{i23} \\ \mathcal{D}p_{i24} \\ \vdots \\ \mathcal{D}p_{i2M} \\ \text{-----} \\ \vdots \\ \mathcal{D}p_{ikj} \\ \vdots \\ \text{-----} \\ \mathcal{D}p_{iMM-1} \end{bmatrix}. \quad (5.7)$$

Assuming that ρ is different for each of the commodity, i.e., a different spatial autocorrelation parameter ρ_i is specified for each commodity i , then based on the arrangements of the observations given in matrix (5.6), the ρ matrix can be constructed as

$$\rho = \begin{bmatrix} \rho_1 \cdot I_{M^*} & \tilde{0} & \cdots & \tilde{0} \\ \tilde{0} & \rho_2 \cdot I_{M^*} & \cdots & \tilde{0} \\ \vdots & \vdots & \ddots & \vdots \\ \tilde{0} & \tilde{0} & \cdots & \rho_N \cdot I_{M^*} \end{bmatrix} \quad (5.8)$$

where $\tilde{0}$ is an $M^* \times M^*$ matrix of zeroes. Simplifying matrix (5.8) using kronecker product notation we obtain

$$\rho = \begin{bmatrix} \rho_1 & 0 & \cdots & 0 \\ 0 & \rho_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \rho_N \end{bmatrix} \otimes I_{M^*} \quad (5.9)$$

or

$$\rho = \hat{\rho} \otimes I_{M^*}$$

where $\hat{\rho}$ is an $N \times N$ diagonal matrix of spatial autocorrelation parameters of the N commodities. Based on matrices (5.9) and (5.8), it is clear that the ρ matrix is therefore an $N \times N$ block diagonal matrix whose element of the i th block are the i th commodity spatial autocorrelation parameter ρ_i . Obviously, the said matrix is symmetric.

With the matrix W^* , it would be different in structure from the previously mentioned proximity measures W defined in Subsection 4.2.1. This is due to the fact that spatial relationships among the price relatives between two countries for all the possible permutations of these countries, in each commodity need to be considered. In order to construct such W^* matrix the following rules are applied.

Rule 1. For a fixed i th commodity, ($i=1,2,3,\dots,N$), price relative of country j , relative to country k , (Dp_{ikj}), is then spatially related with the price relative of country l , relative to country k , (Dp_{ikl}), where $k=1,2,3,\dots,M-1$; $j, l = k+1, \dots, M$; and, $j \neq l$.

Rule 2. For a fixed i th commodity, ($i=1,2,3,\dots,N$), price relative of country j , relative to country k , (Dp_{ikj}), is then spatially related with the price relative of country l , relative to country m , (Dp_{ilm}), where $k, m = 1,2,3,\dots,M-1$; $j, l = k+1, \dots, M$; $j \neq l$; and, $m \neq k$.

Rule 3. For each of the i th commodity, ($i=1,2,3,\dots,N$), price relative of country j , relative to country k , (Dp_{ikj}), is then not spatially related with the price relative of country m , relative to country k , (Dp_{ikm}), for two different commodities i and i' , where $k = 1,2,3,\dots,M-1$; $j, m = k+1, \dots, M$.

For these three rules to apply, we also need to look at the disturbance term closely. Again, following the arrangement of our observations, particularly the column vector Y

in (5.6), the corresponding column vector for the disturbance term u^* in the model (5.4) is given by

$$u^* = \begin{bmatrix} \tilde{u}_1^* \\ \tilde{u}_2^* \\ \tilde{u}_3^* \\ \vdots \\ \tilde{u}_N^* \end{bmatrix}. \quad (5.10)$$

It is clear that the column vector u^* is simply a string of N , M^* column vectors of residuals. And for a particular commodity i ,

$$\tilde{u}^* = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ \vdots \\ u_{M-1} \end{bmatrix} \quad (5.11)$$

where u_k is a vector of $(M-k)$ unobserved random variables, and

$$u_k = \begin{bmatrix} u_{k,k+1} \\ u_{k,k+2} \\ u_{k,k+3} \\ \vdots \\ u_{k,M} \end{bmatrix}. \quad (5.12)$$

Clearly, $u_{k,k+1}$ is defined to be the disturbance term for country $k+1$ relative to country k , for the i th commodity. Hence, u_1 would be an $M-1$ column vector, u_2 is an $M-2$ column vector and so on, until u_{M-1} being a scalar matrix.

Therefore, the covariance structure for \tilde{u}^* can be expressed as

$$E(\tilde{u}^* \tilde{u}^{*'}) = E \begin{bmatrix} u_1 u_1' & u_1 u_2' & \cdots & u_1 u_{M-1}' \\ u_2 u_1' & u_2 u_2' & \cdots & u_2 u_{M-1}' \\ \vdots & \vdots & \ddots & \vdots \\ u_{M-1} u_1' & u_{M-1} u_2' & \cdots & u_{M-1} u_{M-1}' \end{bmatrix}. \quad (5.13)$$

Taking one particular matrix element of the above covariance matrix, say, $u_1 u_1'$, this matrix element could be expressed as

$$E(u_1 u_1') = E \begin{bmatrix} u_{12} u_{12} & u_{12} u_{13} & \cdots & u_{12} u_{1M} \\ u_{13} u_{12} & u_{13} u_{13} & \cdots & u_{13} u_{1M} \\ \vdots & \vdots & \ddots & \vdots \\ u_{1M} u_{12} & u_{1M} u_{13} & \cdots & u_{1M} u_{1M} \end{bmatrix}. \quad (5.14)$$

From matrix (5.14), it would suggest that $u_{12} u_{13}$ is the spatial autocorrelation between country 2 and country 3 relative to country 1 and consequently, $E(u_{12} u_{13})$ is the covariance between the price relatives of countries 2 and 3 relative to country 1 price.

Specification of the W^* matrix

With spatial autocorrelation present among the disturbances of the model (5.4), such spatial relationship between two pair of countries could be capture and modelled using the elements of any of the proximity matrices discussed in Chapter 4. Given a proximity measure W , the spatial relationship being shown by the covariance matrix (5.14) can be specified using the elements of the W matrix and will be constructed following rule 1. Let

$$\omega^{11} = \begin{bmatrix} w_{22} & w_{23} & \cdots & w_{2M} \\ w_{32} & w_{33} & \cdots & w_{3M} \\ \vdots & \vdots & \ddots & \vdots \\ w_{M2} & w_{M3} & \cdots & w_{MM} \end{bmatrix}. \quad (5.15)$$

Careful examination of the matrix (5.15) reveals that ω^{11} is simply a submatrix of the original proximity matrix W in which the first row and first column elements are deleted. Evidently, it is a square matrix.

Supposing we consider another element of the covariance matrix (5.13), say $u_1 u_2'$. This can be expressed in matrix form as

$$E(u_1 u_2') = E \begin{bmatrix} u_{12}u_{23} & u_{12}u_{24} & \cdots & u_{12}u_{2M} \\ u_{13}u_{23} & u_{13}u_{24} & \cdots & u_{13}u_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ u_{1M}u_{23} & u_{1M}u_{24} & \cdots & u_{1M}u_{2M} \end{bmatrix}. \quad (5.16)$$

From matrix (5.16), it would suggest that $u_{12}u_{23}$ is the spatial autocorrelation between country 2 and country 3 relative to country 1 and 2 respectively. Consequently, $E(u_{12}u_{23})$ is the covariance between the price relatives of countries 2 relative to country 1 price and of country 3 relative to country 2. In this situation, applying rule 2, the spatial relationship for covariance matrix (5.16) could be modelled using

$$\omega^{12} = \begin{bmatrix} w_{2.} & w_{24} & \cdots & w_{2M} \\ w_{3.} & w_{34} & \cdots & w_{3M} \\ \vdots & \vdots & \ddots & \vdots \\ w_{M3} & w_{M4} & \cdots & w_{MM} \end{bmatrix}. \quad (5.17)$$

Again, careful examination of the matrix (5.17) reveals that ω^{12} is simply a submatrix of the original proximity matrix W in which the first row and first two columns are trimmed. Moreover, if the first column of ω^{11} is deleted, we would arrive also at the ω^{12} . Obviously, it is a rectangular matrix.

Continuing the process for the rest of the elements of (5.13) until the last scalar matrix $u_{M-1}u_{M-1}'$, we would obtain a more general form of a proximity matrix given by

$$\tilde{W} = \begin{bmatrix} \omega^{11} & \omega^{12} & \omega^{13} & \dots & \omega^{1(M-1)} \\ \omega^{21} & \omega^{22} & \omega^{23} & \dots & \omega^{2(M-1)} \\ \omega^{31} & \omega^{32} & \omega^{33} & \dots & \omega^{3(M-1)} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \omega^{(M-1)1} & \omega^{(M-1)2} & \omega^{(M-1)3} & \dots & \omega^{(M-1)(M-1)} \end{bmatrix} \quad (5.18)$$

where \tilde{W} is an $M^* \times M^*$ square matrix. In general, the submatrix ω^{ij} is a matrix from the original proximity matrix W where the first i row(s) and first j column(s) deleted. This would make ω^{ij} an $(M-i) \times (M-j)$ matrix and it is a square matrix if $i=j$. \tilde{W} may or may not be symmetric and symmetry will depend on the structure of the original W matrix. Hence, if W matrix is symmetric then \tilde{W} will also be symmetric.

Since \tilde{W} is defined for each particular i th commodity, then using rule 3, the big W^* matrix in equation (5.5), can be written as

$$W^* = \begin{bmatrix} \tilde{W} & \tilde{O} & \tilde{O} & \dots & \tilde{O} \\ \tilde{O} & \tilde{W} & \tilde{O} & \dots & \tilde{O} \\ \tilde{O} & \tilde{O} & \tilde{W} & \dots & \tilde{O} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \tilde{O} & \tilde{O} & \tilde{O} & \dots & \tilde{W} \end{bmatrix}. \quad (5.19)$$

Furthermore, the above W^* matrix should be standardized to conform the requirements in regression modelling of any spatial models.

Normalisation of the W^* matrix

In order to provide a natural interpretation for the spatial autocorrelation parameter ρ_i , the above W^* matrix must undergo a standardisation process, where each element of the matrix will be divided by their corresponding row total. This would create a new W^* matrix such that the sum of each row must equal to one. This new W^* matrix is the one to be used in specifying the disturbance model given in (5.5) as well as in the estimation

process. This normalisation has the effect of making the W^* always asymmetrical and usefully restrains the value of ρ_i within the feasible range of -1 to 1 when estimated. However, the magnitude of ρ_i would really depend on the structure of the original W matrix (Odland 1988). There have been some cases where the estimated value of spatial autocorrelation parameter would be outside the feasible region especially when generalised proximity variables are used in the estimation process (Upton and Fingleton 1985).

Characteristics of the disturbance term u^*

Following equation (4.11), equation (5.5) can be rewritten as

$$u^* = (I - \rho W^*)^{-1} v, \quad (5.20)$$

or

$$u^* = \Omega^{*-1} v, \quad (5.21)$$

where $\Omega^* = (I - \rho W^*)$.

Thus, the mean and variance-covariance matrix for u^* are expressed as

$$\begin{aligned} E(u^*) &= E[(I - \rho W^*)^{-1} v] \\ &= (I - \rho W^*)^{-1} E(v) \\ &= 0, \end{aligned} \quad (5.22)$$

and

$$\begin{aligned} E(u^* u^{*'}) &= E\left\{[(I - \rho W^*)^{-1} v][(I - \rho W^*)^{-1} v]'\right\} \\ &= E\left\{(I - \rho W^*)^{-1} v v' (I - \rho W^{*'})^{-1}\right\} \\ &= (I - \rho W^*)^{-1} E(v v') (I - \rho W^{*'})^{-1} \\ &= \sigma_v^2 [(I - \rho W^*)' (I - \rho W^*)]^{-1} \\ &= \sigma_v^2 [\Omega^{*'} \Omega^*]^{-1} \\ &= \sigma_v^2 \Phi^*, \end{aligned} \quad (5.23)$$

where

$$\begin{aligned}\Phi^{*-1} &= [(I - \rho W^*)'(I - \rho W^*)] \\ &= [\Omega^{*'} \Omega^*].\end{aligned}\tag{5.24}$$

5.2 Simultaneous Estimation Procedure

An initial step in the estimation of the generalised CCD multilateral index accounting for the spatial autocorrelation (GCCD(SA)) is to perform a GLS estimation procedure for the CCD multilateral index model and test the presence of spatial autocorrelation in the estimated residuals using the Moran's I statistic. Since it is assumed that spatial autocorrelation would be different for each commodity, the Moran's I statistic is estimated for each commodity. Moreover, since the present study deals with cross-country comparisons, computation of the said statistic is limited to a particular reference country. The W matrix to be used should be standardised such that all the elements of the matrix will have a value between 0 and 1, except for the contiguity matrix.

With the presence of spatial autocorrelation detected among the estimated residuals, a generalised CCD multilateral index model that allows for spatial autocorrelation is specified. An efficient estimation procedure is needed to estimate the parameters of the model. A simultaneous estimation procedure is developed to estimate the parameter vector $\Pi = [\Pi_1, \Pi_2, \dots, \Pi_M]'$, the variance σ_v^2 and the spatial autocorrelation parameter matrix ρ . This simultaneous estimation procedure is a combination of the stochastic approach to CCD multilateral index number estimation proposed by Prasada Rao and Selvanathan (1994) and the simultaneous GLS procedure of Miron (1984). Moreover, construction of the ρ matrix is done through individual estimation of the ρ_i element in the matrix equation (5.10).

The following six steps procedure is designed for the said estimation scheme:

Step 1. Perform an ordinary least square estimation procedure for the model (5.3) assuming a null matrix for ρ . This would imply that for the matrix (5.10), it is assumed that $\rho_i = 0$, for all i ($i=1,2,3,\dots,N$). Hence, we are assuming the value zero for the spatial autocorrelation parameters. It is important to note that, the least squares procedure here is identical with the stochastic procedure developed by Prasada Rao and Selvanathan,

that is discussed previously in Section 3.2. This step will give an initial output estimate for the parameter vector Π and the variance σ_v^2 .

Step 2. From the above initial estimates of the OLS procedure, an estimate for the disturbance vector u^* is calculated using the formula

$$\hat{u}^* = Y - X \Pi. \quad (5.27)$$

Step 3. Obtain the ρ matrix using the estimated disturbance vector \hat{u}^* . Following Miron (1984), an estimate of the ρ matrix is constructed by simply estimating the individual commodity spatial parameter ρ_i , which are element of the ρ matrix given in equation (5.10). An estimate of the i th commodity spatial parameter is obtained using the formula

$$\hat{\rho}_i = \frac{\hat{u}_i^{*'} \tilde{W} \hat{u}_i^*}{\hat{u}^{*'} \tilde{W}' \tilde{W} \hat{u}_i^*}, \quad i = 1, 2, 3, \dots, N, \quad (5.28)$$

where \hat{u}_i^* is an $M^* \times 1$ vector of the estimated residuals for the i th commodity. The \tilde{W} matrix is obtained using equation (5.20). It is important that \tilde{W} be standardised such that each row sums to unity. Let $\hat{\rho}_i = \bar{\rho}_i$, for all i .

Step 4. Using the estimated ρ matrix ($\hat{\rho}$) obtained in the previous step, we obtained the $\hat{\Omega}^*$ matrix, where

$$\hat{\Omega}^* = (I - \hat{\rho} W^*). \quad (5.29)$$

Step 5. Using the estimated $\hat{\Omega}^*$ matrix, transform the model (5.6) such that

$$\hat{\Omega}^* Y = \hat{\Omega}^* X \Pi + \hat{\Omega}^* u^*, \quad (5.30)$$

and perform a GLS estimation procedure for the above model following the stochastic approach of Prasada Rao and Selvanathan. The resulting GLS estimate for the parameter vector Π is

$$\hat{\Pi} = (X' \hat{\Phi}^{*-1} X)^{-1} X' \hat{\Phi}^{*-1} Y, \quad (5.31)$$

where $\hat{\Phi}^{*-1} = [\hat{\Omega}^* \hat{\Omega}^*]^{-1}$. An unbiased estimate for the variance of $\hat{\Pi}$ may be computed using equation (4.38), where the parameter σ_v^2 can be estimated using equation (4.38), Ψ^{-1} replaced by $\hat{\Phi}^{*-1}$ and $\hat{\beta}$ by $\hat{\Pi}$. It is also assumed that σ_v^2 is the same for all the i th commodity.

Step 6. Using the GLS estimate for $\hat{\Pi}$, repeat the estimation of the ρ matrix using the formula in Step 2 and 3, whereby individual diagonal elements of the $\tilde{\rho}_i$ matrix is computed and let the estimated matrix be $\hat{\tilde{\rho}}_{i1}$. Compare the obtained values in here with the $\bar{\rho}_{i0}$ values obtained in Step 3. If $\hat{\tilde{\rho}}_{i1}$ is approximately the same or identical with $\bar{\rho}_{i0}$, then the whole simultaneous procedure stops and the final estimates for all the parameters of the spatial CCD multilateral index model will be the last GLS results. Otherwise, proceed to step 4 and continue again the process.

The above six-steps procedure is an iterative process that only stops if the spatial autocorrelation parameter estimates for each individual commodity (ρ_i) is approximately the same with the previous estimates. Even though the above procedure starts with a simple OLS procedure, it becomes tedious as the process takes more complicated matrix estimations and these may be done only through the use of advanced computer software or statistical packages.

As the above computational scheme consider estimation of individual spatial autocorrelation parameter, it would imply that when either N or M gets larger, difficulties in the processing of the ρ matrix as well as the $\hat{\Omega}^*$ matrix may occur. In the case of the present study, it considered $N=8$ commodities and originally $M=56$ countries. However, due to time constraints and computer facility limitations, the total number of countries considered on the empirical application for the generalised CCD multilateral index accounting for spatial autocorrelation is reduced to $M=20$ countries only. With $M=20$ countries, still $M^*=190$ and this would satisfy the important assumptions of Cliff and Ord (1973, 1981) with regards to spatial autocorrelation. More detailed discussion on how the 20 countries are selected is given in the next section.

It could also be noticed that the procedure is designed following the GLS approach of Miron. It is expected that the results using GLS approach will be approximately the same with that of using the ML approach, as evidently shown in the theoretical results of the two methods in Section 4.2. Although further research is much needed to verify

such claim. In addition, the iterative schemes developed for the present study is just an initial step in the estimation of the multilateral index numbers formula being used for cross-country comparisons exercises. Moreover, as of this moment, no explicit form of the parameter estimator for the generalised CCD multilateral index accounting for spatial autocorrelation is obtained. Further research with regards to this matter is also necessary.

Estimates of the PPP and its standard error may also be obtained following the computational results given in Section 3.4. Inferences regarding the parameter estimates of the model as well as evaluation of its goodness-of-fit may be conducted using the results in Section 4.3.

5.3 The Data

In this section, the main features of the data employed in this dissertation are explained. The nature of the ICP data is as well as the specifics of such data are explained. Then, the spatial proximity variables used for the measurement of spatial autocorrelation is briefly examined. The reasons behind choosing these types of proximity matrices are also explained. The procedures used for the estimation of the data are summarised in sufficient detail. In addition, the reader is also made aware of the study's constraints that are due to the sources and limitations of the data used particularly on the spatial proximity data.

5.3.1 ICP Data

In 1994, phase V results of the ICP were published. There were 64 countries included in the phase V publication of the ICP with 53 detailed expenditure categories on GDP of which 39 items referred to private consumption expenditure. However, only 56 countries are covered and tabulated in the final processing of the 1985 world results. The expenditure and price data used in the present study are drawn from the summary tables 5 and 10 respectively in the United Nations (1994) report on World Comparisons of Real Gross Domestic Product and Purchasing Power for 1985.

In the 1985 ICP, price and expenditure data are collected on a large number of items with specific characteristics, which are aggregated into over 250 detailed expenditure categories of the GDP. For purpose of analysis, the study will focus only on the eight basic categories in private consumption summarised in Table 5.1. Only the private

consumption expenditure component of the GDP is considered because it is deemed that the standard economic theory underlying much of the work of Diewert, Caves and Christensen may not apply adequately to other categories of the GDP.

Table 5.1 Description of Categories

Category	Description	Abbreviation
1	Food, beverages, and tobacco	Food
2	Clothing and footwear	Clothing
3	Gross rents, fuel and power	Rent
4	Household equipment and operation	Furniture
5	Medical care	Medical
6	Transport and communication	Transport
7	Recreation and education	Education
8	Miscellaneous goods and services	Others

Source: Adapted from Freeman 1992, p34.

This study requires observed data on both the price and quantity of the i th commodity consumed for the j th country ($j=1,2,3,\dots,56$), denoted respectively by p_{ij} and q_{ij} , so that estimates of PPP's may be obtained by various methods. Such information is contained in the UN (1994) publication. Table 10 of this publication provides the expenditure data used in the form of per capita nominal value of final expenditure on GDP at national prices expressed in national currencies. The expenditure for the major category, say 'Food', is a simple aggregation of its subcategories. For example, the final expenditure on GDP for United Kingdom, in pounds, for the category 'Food' is 724 pounds. This is made up of an amount of 520 pounds for food, 96 pounds for beverages and an amount of 108 pounds for tobacco. The aggregated expenditure data for each country can be found in Table A.1 of the Appendix 1.

The other input needed in the estimation of the CCD multilateral index models is a price data (p_{ij}). As the commodity categories are highly aggregated, the price data that are used in this study are in the form of a price index. It is meaningless to make any cross-country comparisons using prices in terms of raw national currencies. To give an example of this, consider the sub-category beverages that is a part of the category 'Food'. The beverages category is split into five detailed items ranging from mineral water to beer. These beverages prices, which will be in national currencies, cannot be compared

between countries and so must be transformed into a price index. These price indexes for the detailed categories must then be aggregated to form a price index for beverages in international dollars. That is, since commodity groups represent aggregates of many narrower groups, the price for that group or category is necessarily a weighted average of each of the prices of each of the commodities contained therein. Purchasing power parities with respect to the international dollar for the various sub-aggregates of GDP can be regarded, in some sense, as a price for that commodity.

The ICP uses the Geary-Khamis method to obtain such price information with USA as the base or reference country. For example, consider the case of India and the category 'Food'. One US dollar's worth of food, beverages and tobacco costs 5.817 rupees in India. This price, p_{ij} , referred to here are the PPP_{ij} obtained from Table 5 of the UN report (1994). The aggregated price data for each of the 56 countries considered in the present study can be found in Table A.2 of the Appendix 1.

Generally, the level of accuracy and reliability of ICP results is believed to be high, that is why for most cross-country demand analyses this information is widely used. Theil and Suhm (1981) work with the 1975 ICP in their system-wide approach to a global demand system. Dancer (1990) made use of the 1980 ICP data in cross-demand analysis, to be able to obtain improved values of the purchasing power parities. Most recent works of Prasada Rao and Selvanathan (1992, 1994), as well as Freeman (1992) are based on the 1980 Phase IV ICP data. All these studies used the price and expenditure data in forms similar to that used in the present study.

5.3.2 The Spatial Proximity Variable

Concerning the development of the spatial disturbances model given in equation (5.7), some necessary proximity matrices need to be constructed to be able to obtain a big W^* matrix defined in equation (5.21), the elements of which are said to be submatrices of proximity measures W . These spatial proximity measures W are also used in calculating the spatial autocorrelation statistic's I mentioned in Section 4.2, a statistic used in testing for the presence of spatial autocorrelation in the residuals of the CCD multilateral index model.

Three spatial proximity measures are considered in the present study. These three measures are found to be useful in establishing the nature of geographic relationship existing among prices of different commodities across-countries (see Aten 1994). The concepts behind the selection and construction of these three measures have been

thoroughly discussed in Section 4.2 in Chapter 4. The three measures considered are the contiguity measure, the great circle distance between capital cities, and the volume of trade between countries measured by the exports and imports.

The contiguity matrix

A simple contiguity matrix W , of the 56 countries covered by the 1985 ICP publication is created. Each element of the W matrix, w_{kj} equals one if country k and country j share a boundary, and zero otherwise. For example, $w_{21}=1$, since country 2 (France) shares a common boundary with country 1 (Germany) while $w_{31}=0$ since country 3 (Italy) and Germany are not neighbours. The 56×56 contiguity matrix constructed here is given in Appendix 2. Most studies on spatial autocorrelation refer this matrix as matrix of binary weights. It is advantageous to use this kind of spatial proximity measure when one is investigating spatial autocorrelation in irregularly spaced locations such as countries of the world. Moreover, this is very easy to construct as one needs only a world map to identify pair of countries with common boundary. However, it could be noted that problem seems to occur when isolated islands or countries isolated by water are included in the study. Aten (1994) suggested that for this case, certain variations such as distances to the nearest neighbour, proportion of coastline that is close to another country, or islands near to a certain continent, each would establish the closeness of any pair of countries are subject for consideration in constructing any contiguity matrix.

Past studies that made use of the contiguity matrix in analysing spatial autocorrelation are that of Cliff and Ord (1973), Hordijk (1974), Ord (1975), Sen and Soot (1977), and Bradsma and Ketellapper (1979b). Miron (1984) and Aten (1994) made use of the same form of proximity measures in the development of economic models that allow spatial autocorrelation in the error term.

The Distance Matrix

The second measure of spatial proximity used in the present study is the distance matrix. Measured in kilometres, it is defined as the shortest great circle distance between each country's capital city. So, for the distance matrix W , w_{kj} equals zero if $k=j$ and a value greater than zero for all the other elements of the matrix. In this measure, it is not the actual distance in kilometres which are used in the analysis but the inverse of the said distance value, that is, $w_{kj}=1/d_{kj}$, where d_{kj} is the great circle distance between capital city of country k and j . The reason for this specification is that the greater the distance the larger the value and as compared to the contiguity matrix where a value of 1 denotes

nearest neighbour, the direction of proximity should be similar to that of contiguity measures for comparisons. The rationale for using the distance matrix, although may be regarded as a crude one, is that for the islands and countries in the sample which are not clustered (eg. Japan, Philippines, Madagascar, Australia, etc.) would have a non-zero weight attached.

For this proximity measure, the author utilised distance matrix used in Aten (1994)'s. Aten established that the relative prices for the goods and services in the GDP aggregates are very similar among physically close countries, hence, the use of distance matrix is considered important in the analysis of spatial autocorrelation among the disturbances of any economic models which uses spatial data. However, due to the confidentiality of the said distance measure, Aten reserved the right for this data, hence, the present study cannot publish nor reproduce the whole 56×56 distance matrix in the Appendix. For those researchers who like to have an access to this matrix, is therefore referred to the above author. However, a small portion of the distance matrix used in the present study is given in Table 5.2 below for illustration.

Table 5.2 A Matrix(5×5) of Distance Proximity Measure

Country	Belgium	Netherlands	Australia	India	Bangladesh
Belgium	0	0.005208	0.000060	0.000155	0.000129
Netherlands	0.005208	0	0.000060	0.000156	0.000130
Australia	0.000060	0.000060	0	0.000097	0.000111
India	0.000155	0.000156	0.000097	0	0.000703
Bangladesh	0.000129	0.000130	0.000111	0.000703	0

Source: Aten 1994.

Since Belgium and Netherlands share common boundary as revealed in the contiguity matrix, it is expected that the corresponding element for such pair of countries in the distance matrix would be relatively close to 1. In contrast for Australia and Belgium, since this pair of countries is geographically far from each other, it would have a corresponding proximity value that is almost close to zero. In the above table it is approximately 0.00006. Moreover, the above illustration shows that the distance matrix considers in the study would be symmetric.

The Trade Matrix

The third proximity measures considered in the dissertation is the trade matrix. It reflects the trade flows between pair of countries and is calculated using the exports converted to US\$ between these two countries. The 56×56 trade matrix used in the present study is the same trade matrix constructed by Aten (1994) in her study. Again, just like the distance matrix, it cannot be reproduced or published in this paper. Aten made use of the 1992 International Monetary Fund's 'Direction of Trade Statistics' publication to create the trade flows for all the pair of countries considered in the 1985 ICP publication. Again, a portion of the said matrix is illustrated below, where each element is expressed in US dollar.

Table 5.3 A Matrix(5×5) of Trade Proximity Measure

Country	Belgium	Netherlands	Australia	India	Bangladesh
Belgium	0	9,595,346	160,467	185,445	41,002
Netherlands	7,633,262	0	263,049	130,683	0
Australia	180,060	288,78	0	101,586	13,661
India	708,575	263,034	563,067	0	19,282
Bangladesh	0	0	22,173	106,230	0

Source: Aten 1994.

From the above table, each row element shows the exports from a country into different countries while each column would refer to imports into the given country. This explains the asymmetric nature of the trade matrix, as mentioned earlier in Subsection 4.2.1. The volume of exports from Belgium to Netherlands is relatively large at 9,595,346 US dollars. On the other hand, the trade flows from Netherlands to Belgium is 7,633,262 US dollars. It could be viewed also from the above table that the trade flows between Australia and Belgium is relatively smaller than the trade flows between Netherlands and Belgium. This result indicates that there are greater interactions between countries which are closely located to each other. Hence, the direction of the trade proximity measures would be similar to those associated with the contiguity and distance matrices. This is also exemplified in the case of the Bangladesh to Australia and Bangladesh to India, as Bangladesh is much nearer to India than the Australian continent.

The trade matrix can be viewed as a generalised proximity measure (Upton and Fingleton 1985) and when it is used in the computation of the spatial autocorrelation test

statistic, the statistic's distribution would be more closely approximated by a normal distribution. Moreover, Aten (1994) established that the Moran's I statistic increases in magnitude as well as in terms of its statistical significance when the trade matrix is used as a relative measure of spatial proximity. Also, Aten found out that when there is less trade flows between countries, prices are less similar, hence spatial autocorrelation can greatly be captured using the trade matrix.

As discussed previously in Subsection 4.2.1, both the trade and distance matrices need to be normalised such that row sums in the matrices sum to unity, before it can be used in the computation of the Moran's I statistic. Normalisation lends a natural interpretation to the Moran's I coefficient (see Ord 1975 p.121, Cliff and Ord 1973 p.90).

5.3.3 Benchmark Year and Country Coverage

Numerical application in the present case is limited to the 56 countries published in the 1985 ICP Worlds Report. Only eight personal consumption expenditure aggregates given in Table 5.1 are considered for the modelling procedures. The elements of the 56×56 contiguity, distance and trade matrices are taken as variables in the estimation of the Moran's I statistics. This choice ensures that the computations are of manageable size.

In the simultaneous estimation procedure for the CCD multilateral index model with spatial autocorrelation parameter in the disturbance term, only 20 countries selected by purpose is considered for analysis. The list of the 20 countries considered in the estimation is reported in the empirical results. Hence, the original 56×56 contiguity, distance and trade matrices are reduced to 20×20 proximity matrices only. This is due to the fact that the available computer package SHAZAM could only accommodate spatial disturbances model with 20 countries.

Selection of the 20 countries included in the analysis of the GCCD(SA) is based on the contiguity matrix as well as on economic status. The process of selecting the sample is done purposively such that the countries selected are somehow contiguous to each other and the sample is fully represented by both developed and developing countries. Countries are also taken from all parts of the world and all the major continents are represented. United States should also be a part of the sample so that the PPP results can be converted in terms of US\$ for comparison purposes.

5.4 Empirical Results

This section presents an application of the theoretical results of Chapter 3 to 5 using the price and quantity data from the Phase V of the ICP of the United Nation Statistical Office (1985). The list of countries includes the fifty six countries that participated in the Phase V exercise. The commodity list here is restricted to eight highly aggregated commodity groups of the private consumption expenditure, namely, Food, Clothing, Rent, Furniture, Medical, Transport, Education, and Others.

For the empirical estimation of CCD indexes, $N=8$ and $M=56$, thus a total of $NM(M-1)/2=12,320$ observations is taken for the GLS estimation of the CCD multilateral index model. To be able to set-up all the observations for the independent (Y) as well as the explanatory (X) variables of the CCD model given by equation (3.7), a FORTRAN program is developed and listing of the program is given in Section A.3.1 of Appendix 3. The output of the program arranges observations for X and Y for all cross-country permutations taken for each commodities. All the observations are created following the transformation given in equation (3.6).

After all the necessary transformed data are set-up, GLS estimation procedure using the SHAZAM software is employed. The SHAZAM program for the GLS estimation of the CCD multilateral model can be found in Section A.4.1 of Appendix 4. This initial step demonstrates the use of the stochastic approach in the derivation and computation of CCD multilateral indices which are transitive, base-invariant and possess useful least-squares properties. Later in this section, the results of the GLS estimation for 20 countries is presented and compared to the results of the simultaneous estimation of the generalised CCD model which allows for spatial autocorrelation.

At this point, we return to the main focus of the present study which is to determine the presence of spatial autocorrelation among the residuals of the CCD multilateral index model. To account for this, the residuals for each of the 12,320 observations are estimated using the previous GLS estimates. These estimated residuals are then considered in the computation of the Moran's I test statistic.

5.4.1 The Estimated Moran's I Statistic

The Moran's I test statistic, designed to test the presence of spatial autocorrelation among the estimated residuals of the CCD model is estimated. The I statistic is obtained for each commodity aggregate using the formula given by equation (4.16). It is

necessary to select a particular reference country such that the computation of the test statistic would be simpler and straightforward. Simpler in the sense that not all of the estimated residuals will be used in the computation, but only a set of residuals pertaining to all possible distinct permutations of country j relative to a reference country k in a particular commodity i . Choosing a particular reference country is straightforward since any of the 56 countries may be chosen. This would be the case since the general results on testing for spatial autocorrelation is invariant to the choice of the reference country. Invariant in the sense that the Moran's I statistic captures every possible spatial relationship of countries in the study as truly depicted in the computational formula. In the present study, Germany is chosen as the reference country. Being the first country on the list, it would be easy for the author to get the estimated residuals based on the GLS estimates for the CCD model.

Following the Moran's I formula in equation (4.16) and the CCD model specification given by (3.6), the residual vector \hat{u} which is used as input into Moran's I computation is based on the set of observations for a particular i th commodity with country 1 (Germany) as the base country. For example, the Moran's I statistic for commodity Food ($i=1$), with country 1 as a base can be obtained using the residuals coming from observations Dp_{11j} (i.e. $\log p_{1j} - \log p_{11}$), $j=1,2,3,\dots,56$. However, in model (3.6) we are adjusting for heteroscedasticity, that is, the residuals are actually based on $\sqrt{\bar{w}_{ikj}} Dp_{ikj}$, which are the weighted residuals. But generally, this weight will have no effect on the overall outcome, only a slight variation on the magnitudes of the estimated Moran's I statistic. In this case, the obtained residual can be expressed as

$$\hat{u} = \begin{bmatrix} \hat{u}_{11} \\ \hat{u}_{12} \\ \hat{u}_{13} \\ \vdots \\ \hat{u}_{156} \end{bmatrix}. \quad (5.32)$$

It is worth noting that we need to use all the values of the vector \hat{u} , including the first one, \hat{u}_{11} , which is obviously zero in value since the first country is the base. The above procedure is also applied for the rest of the commodities. It could also be noted from equation (4.2) that the Moran's I statistic is based on the deviations from the mean and not the raw values, hence, algebraically, it can be shown that the computed Moran's I

coefficient would be identical irrespective of which country is chosen as base particularly with model (3.4) (Cliff and Ord 1981). Moreover, the Moran's I statistic is calculated using all the three proximity measures, contiguity, distance and trade. The EXCEL software had been very helpful in computing all these results. Normality approximation could be used since the calculation of the I statistic is based on 56 observations.

The Moran's I statistic for all the eight aggregate commodities estimated using the three proximity measures are presented in Table 5.4. The column headed I , refers to the computed Moran's I value while SA refers to the nature (positive, negative, or none) of spatial autocorrelation present. The $E(I)$ and $Var(I)$ are the corresponding moments of I under the assumption of randomization, and Z-value is the standard normal deviate computed using (4.20).

From Table 5.4 it is clear that, positive spatial autocorrelation exists among the price relatives residuals of the CCD model for all the commodity headings. The magnitudes of the estimated I values have been relatively high for Food, Rent, Furniture and Education. This suggests that similarities in the price relatives for Food, Rent, Furniture and Education are likely due to the geographic proximity measures (contiguity and distance). This would mean that neighbouring countries are likely to have similar prices for the above four commodities. For example, price ratios on Food for a country like Australia tend to be similar with that of New Zealand. In addition, Food and Furniture price similarities are also likely to be due on trade interaction between countries irrespective of their geographic distances.

Furthermore, it is evident from columns 4, 7 and 10 of Table 5.4 that the spatial autocorrelation is significant among the residuals for all the commodities except for Clothing and Transport. Spatial autocorrelation for the Transport is reflected well using the distance measure, however, with the contiguity and trade, it was found not significant. This would mean that similarities on price relatives for the Transport commodity is unlikely for neighbouring countries and between countries with less trade activities. In the case of Clothing, it is not significant using the trade measure. Moreover, the Moran values for such commodity, using contiguity matrix are relatively higher suggesting that boundaries rather than trade interaction are more likely to capture price patterns for Clothing.

Table 5.4 Moran's I statistic for different commodity aggregates based on the three proximity measures, (Base country: Germany)

Commodity	Contiguity			Distance			Trade		
	I	SA	Z-value	I	SA	Z-value	I	SA	Z-value
1. Food	0.611 (0.0149)	+	5.17*	0.753 (0.00087)	+	26.22*	0.524 (0.00654)	+	6.73*
2. Clothing	0.399 (0.0148)	+	3.44*	0.218 (0.00086)	+	8.12*	0.055 (0.00647)	+	0.93 ^{ns}
3. Rent	0.558 (0.0148)	+	4.75*	0.619 (0.00086)	+	21.80*	0.406 (0.00647)	+	5.30*
4. Furniture	0.674 (0.0149)	+	5.69*	0.665 (0.00087)	+	23.28*	0.512 (0.00647)	+	6.61*
5. Medical	0.501 (0.0142)	+	4.37*	0.247 (0.00083)	+	9.28*	0.132 (0.00652)	+	1.88*
6. Transport	0.053 (0.0147)	+	0.60 ^{ns}	0.165 (0.00085)	+	6.33*	-0.019 (0.00625)	+	0.01 ^{ns}
7. Education	0.519 (0.0148)	+	4.43*	0.604 (0.00086)	+	21.25*	0.362 (0.00649)	+	4.74*
8. Others	0.389 (0.0147)	+	3.37*	0.232 (0.00086)	+	8.61*	0.193 (0.00645)	+	2.65*
$E(I)$	-0.02			-0.02			-0.02		

Note: (a) I refers to the Moran's I coefficient estimates

(b) SA indicates the nature of Spatial Autocorrelation: '+' for positive and '-' for negative

(c) $E(I)$ is the mean of I , calculated using equation (4.3)

(d) Figures in brackets are the approximate variances of I , i.e. $\text{Var}(I)$, calculated using equation (4.4)

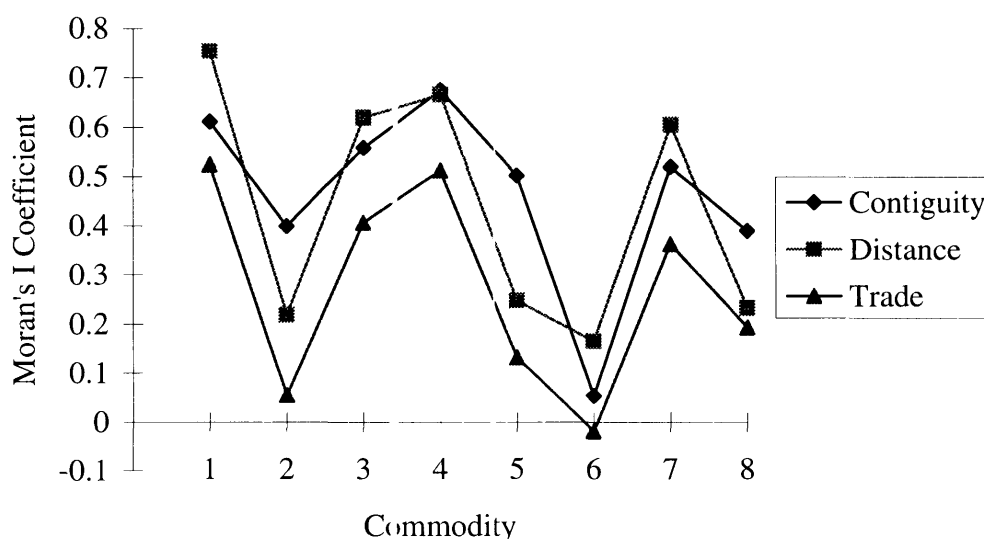
(e) * indicates significant at 5% significant level

It could be summarised that significantly positive spatial autocorrelated residuals are evident among the eight commodities considered in the present study, although for Transport this evidence is not much clear. The estimated Moran's I statistic for Food, Clothing, Furniture and Education have been consistently high for all the proximity specifications. This would suggest that price ratios for these four commodities are likely similar for geographically close countries, and between countries with high trade interaction. It is worth noting that under the assumption of randomization, the expected

value of the test statistic are the same, i.e., the expected value of I is always -0.02 for all the commodities. The estimated approximate standard errors for I are approximately the same or very close to each other for all the commodities in each proximity specification. However, such standard error estimates are relatively lower for the distance measure. The very small estimated approximate standard errors for them provide quite an encouraging measure of the efficiency of the estimation and testing procedure.

The overall sensitivity of the estimated Moran's I statistic with respect to the specification of the proximity measure, it is well illustrated in Figure 5.1. This figure shows the comparison of the estimated Moran's I for all the commodities based on the three proximity measures. There is evidence of constancy in the movements of the estimated Moran's I coefficients. This would suggest robustness of the Moran's I statistic in measuring the degree of spatial relationship present among the residuals of the CCD multilateral index model. As mentioned earlier, the Moran's I statistic is a powerful test for spatial autocorrelation especially for large samples.

Figure 5.1 Comparison of Moran's I statistic computed using different proximity measures, (Base country : Germany)



The above findings confirm the claim that spatial autocorrelation is present among the residuals of the CCD multilateral index model. Such findings indicate possible inefficiency of the estimates obtained from the simple GLS estimation underlying the CCD index computation. This clearly suggests that modifications should be made with regards to the tested model and a more generalised form of the CCD model that

accounts for spatial autocorrelation is deemed necessary. This is applied in the next subsection.

5.4.2 The Iterative Process

With the presence of significant spatial autocorrelation detected among the estimated residuals, a generalised CCD multilateral index model that accounts for spatial autocorrelation (GCCD(SA)) is specified and estimated. The estimation of the model is accomplished following the iterative scheme layout in Section 5.2. A SHAZAM program is developed for this iterative procedure. The SHAZAM program is listed in Section A.4.2 of Appendix 4.

Computationally, the iterative scheme performed well as the estimates of the parameters ρ_i in matrix ρ converged after only a few iterations. In fact, the speed of convergence is amazingly fast. With the contiguity and distance measures used for the W^* matrix, convergence of the procedure is attained after 4 iterations while with the trade measure it converged in 5 iterations. This simply suggests that the iterative scheme works well in the case of large samples (in the present case $n=190$). This may be due to the fact that GLS and ML estimators for ρ tend to be the same for large sample sizes. This result is consistent with the findings of Ord (1975) and Miron (1984). In fact, Ord established the viability of an iterative procedure similar to the present scheme for only 26 observations. In his application, the iterative scheme performed well as the successive values of ρ converged to a final value after 5 iterations. Moreover the value of the likelihood function changed very little after the second iteration. This is similar to the present results as the changes for the log-likelihood values in last two iterations are at minimal. In Miron's application, the procedure converged in 4 iterations even with a sample of 10 observations, which is clearly fast. More importantly, Ord (1975), Cliff and Ord (1981), and Hepple (1976) observed the iterative method to work well, and convergence to a local minimum is always guaranteed. It is possible that several local minima may exist for small samples in the case of spatial data. However, these authors did not discuss any relevance of the number of explanatory variables in the convergence of the spatial autocorrelation parameter which may have been a factor too. In the present case, we are dealing with 20 explanatory variables for the GCCD(SA) model.

5.4.3 The Estimated Spatial Autocorrelation Coefficient

The estimates of the spatial autocorrelation parameter for each commodity after the final iteration are shown in Table 5.5. It shows the estimates of ρ based on the three

proximity measures using different W^* matrices. Looking at the estimated ρ_i values, unexpectedly, there have been values which lie outside the feasible region of ρ . This is possible since it was mentioned earlier that ρ_i values can be outside the possible limits. The estimation for the spatial autocorrelation parameter have been not completely satisfactory in terms of its magnitudes when the distance and trade proximity measures are used. This seems to suggest that the magnitudes of ρ_i are highly dependent on the structure of the proximity matrix W^* . This has been already noted in the studies of Ord (1975) and Miron (1984), however there is no straightforward interpretation of such results. It is the sign that is more important, whether it is positive or negative. This would necessarily imply that even though spatial autocorrelation is detected for such proximity matrices, still there is a need to further improve the measurement of the proximity measures such that it would give a much more realistic estimate of the spatial autocorrelation parameter in the regression analysis.

Table 5.5 Parameter estimates for ρ in each commodity aggregates based on the three GCCD(SA) specifications

Commodity	Contiguity	Distance	Trade
1. Food	0.800	0.855	1.366
2. Clothing	0.188	0.703	0.576
3. Rent	0.736	0.960	1.237
4. Furniture	0.775	0.982	1.101
5. Medical	0.540	0.381	0.086
6. Transport	0.270	-0.132	-0.247
7. Education	0.742	0.938	1.347
8. Others	0.808	1.232	1.214

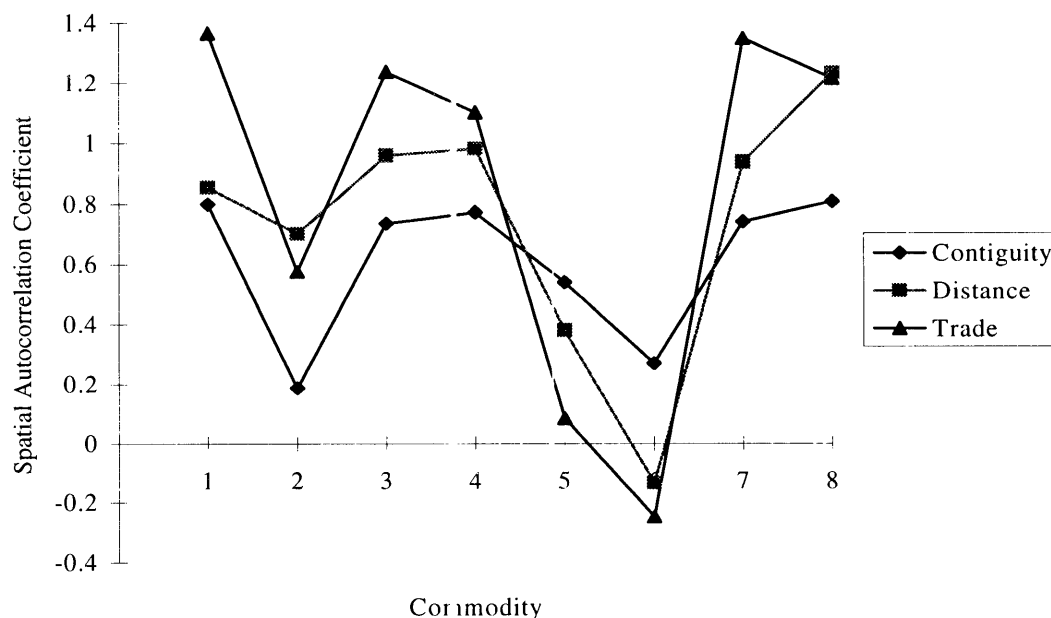
A large magnitude in absolute value for ρ_i estimate would mean a very strong spatial relationship present among the relative prices across countries. Just like the Moran's I coefficient, a positive value for ρ_i indicates similarity between neighbouring countries and a negative ρ_i implies dissimilarity in the price ratios across neighbouring countries. An estimate of the standard error of the estimated ρ_i would then be needed to complete the interpretation of the above results. As mentioned earlier, this could be approximated using the asymptotic-covariance matrix given by equation (4.37), however, due to time constraint and the complexity of the said approximation, the issue of significance of the estimated spatial autocorrelation coefficient has not been pursued.

Looking at the specific values in Table 5.5, the price relatives for the goods and services in all the commodities are positively correlated among physically close countries with the exception of Transport. This could be due to the fact that the Transport commodity is not tradeable. Under the column headings contiguity and distance, similarities in price ratios are well established for the commodities Food, Rent, Furniture, Education, and Others. The ρ_i estimates due to distance are higher suggesting that distance rather than boundaries are more likely to capture price patterns for these five commodities. This is also true when the trade matrix is used as the spatial proximity variable. That is, ρ_i increases in magnitude suggesting that although countries may be physically distant, if they engaged heavily in trade their relative prices for Food, Rent, Furniture and Education are likely to be similar, probably due to the concept of trade equilibrium in the national markets. Conversely, when there is less trade, prices for such commodities are less similar. Clothing and Transport are apparently less correlated with contiguity. Transport commodity appears to become negatively correlated with distance and trade, while the non-tradeable good like Medical care is less correlated with the two proximity measure. The above findings are consistent with results reported in Aten (1994).

To shed more light on the meaning of the individual estimated ρ_i values for the i th commodity, we take for example the commodity Food. Price relatives for Food across two contiguous countries like France and Germany are positively correlated (similar) as $\hat{\rho}_1 = 0.8$, thus we would expect that the price ratio for another pair of country like France and UK to be similar as they share common boundary too. Moreover, with $\hat{\rho}_1 = 1.366$ due to trade, countries which have greater trade interaction like India and Belgium as shown in the Table 5.3 are more likely to have similar prices for Food, regardless of its distance. This may also hold for neighbouring country like Belgium and Netherlands which have significantly high trade flows. Medical care and Transport are the only two commodities which exhibit similar price patterns for neighbouring countries regardless of the trade flows.

In terms of the constancy on the estimates of the ρ_i across different W^* specifications, almost all the proximity measures reveals the same pattern or structure as regards to the magnitudes of the spatial autocorrelation present on the price relatives. This is very much illustrated in Figure 5.2 .

Figure 5.2 Comparison of the estimated spatial autocorrelation coefficients for the GCCD(SA) models



5.4.4 The Estimated Purchasing Power Parities

The purchasing power parities and the estimated standard errors for each country obtained by the various methods are presented in Table 5.6 and 5.7. Table 5.6 presents the results for the generalised CCD model accounting for spatial autocorrelation with USA as the base. Columns 2, 4, and 6 of the table show the PPP estimates based on the three types of proximity measures namely the contiguity, distance and trade. These estimates are calculated using the formula given in Section 3.4. Columns 3, 5, and 7 present their approximate standard errors calculated using equation (3.14). Given the computational problem encountered in using SHAZAM, discussed earlier, estimation of the GCCD(SA) model is conducted only for 20 countries.

From Table 5.6, it is clear that the magnitudes of the estimated PPP's for each country for the GCCD(SA) model do not deviate substantially from each other. The PPP's calculated through contiguity-based model seem to be relatively higher, only marginally, for some countries as compared to the results for the distance-based and trade-based models. Moreover, the PPPs from the different specification are expected to lead unbiased estimates of the PPP's, thus, these specifications are supposed to have an effect

mainly on efficiency. This is what has been achieved more clearly with the trade matrix specification.

Table 5.6 Purchasing Power Parities and Standard Errors based on the GCCD(SA) formulation , (Base currency: US dollar)

	Contiguity		Distance		Trade	
Country	PPP	s.e.	PPP	s.e.	PPP	s.e.
1. Germany	2.5610	0.1229	2.5329	0.1165	2.5269	0.1000
2. France	7.4511	0.3577	7.3776	0.3383	7.3655	0.2891
3. Netherlands	2.5291	0.1214	2.4995	0.1141	2.4924	0.0971
4. Belgium	46.1938	2.1942	45.7856	2.0814	45.6863	1.7804
5. Luxembourg	44.2981	2.1263	43.9868	1.9948	43.6644	1.7116
6. UK	0.5767	0.0276	0.5701	0.0258	0.5663	0.0190
7. Denmark	10.3948	0.4917	10.3241	0.4648	10.2230	0.3897
8. Norway	9.5903	0.4565	9.4897	0.4270	9.3822	0.3587
9. Sweden	8.6945	0.4173	8.6448	0.3895	8.5720	0.3227
10. Australia	1.2369	0.0594	1.2549	0.0534	1.2222	0.0463
11. Canada	1.2501	0.0596	1.2430	0.0548	1.2385	0.0454
12. India	4.3478	0.2061	4.4237	0.1930	4.2950	0.1556
13. Sri Lanka	6.6440	0.3256	6.7084	0.2787	6.3293	0.2449
14. Pakistan	4.0425	0.2020	4.0822	0.1779	3.9151	0.1481
15. Philippines	6.3940	0.3056	6.3587	0.2649	6.2921	0.2520
16. Kenya	4.1734	0.1920	4.4174	0.1674	4.2940	0.1019
17. Tanzania	12.1507	0.4954	13.2780	0.4894	12.6702	0.3940
18. Zambia	0.9134	0.0227	0.9928	0.0257	0.9345	0.0228
19. Zimbabwe	0.4855	0.0155	0.4958	0.0142	0.4906	0.0132

To verify the plausibility of these estimates, these estimates are compared to the estimated PPP's using the binary TT index and the CCD model. This is illustrated in Table 5.7. It could be viewed from Table 5.7 that the PPP's estimated from the GCCD(SA) models seem plausible. It could be seen that there has been not much difference in the magnitudes of the PPP estimates for all the models. Moreover, the PPP's in columns 6, 8, and 10 are relatively higher as compared to the CCD estimates in column 4. A comparison of the estimated approximate standard errors in column 3 with 5, 7, 9 and 11 shows that the standard errors associated with the binary TT index are high. The difference in the estimated standard errors between the binary and the

Table 5.7 A comparison of Purchasing Power Parities computed using different stochastic formulations
(Base currency : US dollar)

Country	TT		CCD		GCCD(SA)-Contiguity		GCCD(SA) - Distance		GCCD(SA) - Trade	
	PPP	s.e.	PPP	s.e.	PPP	s.e.	PPP	s.e.	PPP	s.e.
1. Germany	2.5559	1.5667	2.5412	0.1171	2.5610	0.1229	2.5329	0.1165	2.5269	0.1000
2. France	7.4292	4.5538	7.3720	0.3397	7.4511	0.3577	7.3776	0.3383	7.3655	0.2891
3. Netherlands	2.4927	1.5279	2.5012	0.1153	2.5291	0.1214	2.4995	0.1141	2.4924	0.0971
4. Belgium	45.6951	28.0091	45.7336	2.1074	46.1938	2.1942	45.7856	2.0814	45.6863	1.7804
5. Luxembourg	44.5941	27.5345	45.7345	2.0155	44.2981	2.1205	45.9808	1.9948	45.6044	1.7110
6. United Kingdom	0.5730	0.3512	0.5685	0.0262	0.5767	0.0276	0.5701	0.0258	0.5663	0.0190
7. Denmark	10.2873	6.3057	10.2644	0.4730	10.3948	0.4917	10.3241	0.4648	10.2230	0.3897
8. Norway	9.2412	5.6645	9.3880	0.4326	9.5903	0.4565	9.4897	0.4270	9.3822	0.3587
9. Sweden	8.1521	4.9969	8.5333	0.3932	8.6945	0.4173	8.6448	0.3895	8.5720	0.3227
10. Australia	1.2468	0.7642	1.2218	0.0563	1.2369	0.0594	1.2549	0.0534	1.2222	0.0463
11. Canada	1.2376	0.7586	1.2358	0.0569	1.2501	0.0596	1.2430	0.0548	1.2385	0.0454
12. India	4.3432	2.6622	4.3004	0.1982	4.3478	0.2061	4.4237	0.1930	4.2950	0.1556
13. Sri Lanka	6.5663	4.0249	6.5195	0.3004	6.6440	0.3256	6.7084	0.2787	6.3293	0.2449
14. Pakistan	3.8575	2.3645	3.9479	0.1819	4.0425	0.2020	4.0822	0.1779	3.9151	0.1481
15. Philippines	6.4342	3.9439	6.2734	0.2891	6.3940	0.3056	6.3587	0.2649	6.2921	0.2520
16. Kenya	4.4022	2.6984	4.3175	0.1989	4.1734	0.1920	4.4174	0.1674	4.2940	0.1019
17. Tanzania	12.8877	7.8996	13.2331	0.6098	12.1507	0.4954	13.2780	0.4894	12.6702	0.3940
18. Zambia	0.0918	0.0563	0.9588	0.0442	0.9134	0.0227	0.9928	0.0257	0.9345	0.0228
19. Zimbabwe	0.4933	0.3024	0.4919	0.0227	0.4855	0.0155	0.4958	0.0142	0.4906	0.0132

multilateral indices is mainly due to the large degrees of freedom associated with the transitive multilateral indices resulting from CCD and GCCD(SA) models.

The very small standard errors for the estimated PPP's for the GCCD(SA) models are quite encouraging. A comparison of the estimated approximate standard errors in columns 7, 9, and 11 of Table 5.7 show very little difference. However standard errors associated with the trade matrix W^* are uniformly lower, suggesting the possibility that trade captures the correlation among price relatives better.

To supplement the significance of the PPP's obtained using GCCD(SA) model, it is desirable to assess the suitability of alternative specification of proximity measures using some available measures of goodness-of-fit. Table 5.8 presents some of the available statistical measures. It could be gleaned from this table that there is a significant gain in the efficiency when the CCD model is specified and estimated to account for spatial autocorrelation. It appears that the use of the simple model of CCD is inappropriate for data where spatial autocorrelation is present, as the estimated standard error for the said regression model is fairly high as compared with the GCCD(SA) models. It may also be mentioned that the nonspatial CCD model explains only about 94.8 percent of the variation in Y , whilst the GCCD(SA) model based on the three proximity measures explains 97 percent of the variation. This would suggest that the GCCD(SA) model has a good fit to the spatial data. These results are based on the r^{*2} coefficient computed using equation (4.39). This quantity measures the proportion of the variation in Y that is explained by the GCCD(SA) model. It is also identical to the ordinary r^2 (coefficient of determination) of any linear regression model if the CCD model is used. So just like the r^2 , the closer that r^{*2} is to 1, the better is the fit of the model. Furthermore, as shown by the formula in (4.39), it would imply also that the estimated regression standard error will comparatively be smaller when compared to the total variability of Y .

Table 5.8 Some goodness-of-fit measures for alternative spatial autocorrelation specifications

Measure	CCD	Contiguity	Distance	Trade
sigma	0.1457	0.1126	0.1186	0.1109
RSS	31.87	19.06	21.12	18.48
Log-likelihood	780.51	1171.1	1093.19	1194.56
r^{*2}	0.948	0.97	0.97	0.97

However, the choice between which proximity measures in terms of goodness-of-fit is not clear in terms of r^2 . Based on the estimated value of the log-likelihood function, it is the trade-based model that is preferable with the highest log-likelihood value of 1194.56. Moreover the estimated residual sum of squares (RSS) is lower for trade. This would imply that the trade interaction between countries could really capture the spatial relationship among the price relatives. In this particular context, the generalised CCD multilateral index model that accounts for spatial autocorrelation seems more compelling than the CCD multilateral index formulae in the estimation of the purchasing power parities.

5.5 Concluding Remarks

In this chapter we have shown how spatial autocorrelation can be considered in the context of CCD multilateral index number estimation to produce an improved class of index numbers for multilateral or spatial comparisons. For the 1985 ICP data, the existence of the spatial autocorrelation among the estimated residuals of the CCD multilateral index model was established using the Moran's I statistic. A more generalised form of the CCD model was specified leading to the GCCD(SA) spatial model. An iterative procedure was developed and has proven to be useful in the estimation of the spatial model. The problem of biasedness and inefficiency in the standard error estimation has been remedied. It was found by the application of an empirical data set, that the GCCD(SA) model are able to provide plausible estimates of the purchasing power parities.

Three alternative proximity measures, namely the contiguity, distance, and trade have been successful in capturing the spatial correlation in the price ratios of the different commodities across-countries. All these measures have shown their usefulness in the GCCD(SA) estimation. Numerical values of PPPs appear to be robust to the choice of the W^* matrix but trade matrix seems to perform better. However, at this point in time, significance of the spatial autocorrelation parameter estimates could not be assessed, and this could be a topic for further investigation.

6. Summary and Conclusions

Much of the work to date in the area of international comparison of prices has been focused on finding an improved classes of multilateral indices for spatial comparisons with which purchasing power parities (PPPs) can be estimated. Most of these have been based on two of most widely used methods namely the Elteto-Koves-Szulc (EKS) and the Geary-Khamis (GK) procedures. All of the ICP work to date is essentially based on the PPPs derived using the GK method of aggregation. Despite the use of the GK procedure by a number of international organisations in most of their international comparison exercises, the method has often been criticized as a heuristic procedure with less economic theoretic foundation. This has led to the use of some alternative EKS-based models in deriving multilateral indices, an example of which is due to Caves, Christensen, and Diewert (1982) who proposed a transitive and base invariant CCD index.

With the introduction of stochastic approach to index-number theory by Clements and Izan (1987), and to index number construction by Selvanathan (1989), Prasada Rao and Selvanathan (1992a) derived a multilateral log-change index number formula based on the CCD index which yields indices with attractive properties. This has led to an index number model which provides a viable alternative aggregation procedure for multilateral comparisons as well as estimates of the PPPs.

However, problems seem to arise in the use of price data across countries in the estimation of the CCD multilateral index formula, as price levels may exhibit geographic patterns. This phenomenon has led to the problem of spatial autocorrelation, which has an effect on the disturbances of the CCD model and the least squares estimation would then produce inefficient estimates of the multilateral indices and the PPPs associated with them. Moreover, in Aten (1994)'s exploration of the existence of spatial autocorrelation in the relative prices of goods and services across countries, her study established the presence of spatial autocorrelation among the residuals in the cross-country demand analysis using the 1985 ICP benchmark data.

In recognition of the existence of this problem, the present study explores the possibilities of allowing for spatial autocorrelation in the disturbance structure specification of the stochastic formulation of the CCD multilateral index proposed by Prasada Rao and Selvanathan (1992a), thus, leading to more efficient estimates of the PPP's and their standard errors.

In this dissertation, a review of relevant studies on index numbers for multilateral comparisons was first presented in Chapter 2. This conceptualised the origin of the multilateral indices. The concept of PPP was introduced and the properties of multilateral indices were presented. The binary Theil-Tornqvist (TT) was also examined, with a focus on the stochastic approach to derive TT indices. It was asserted that despite the elegant properties of the TT indices, they have no role in any multilateral comparison exercises due to the lack of transitivity.

The overall general framework within which, work on the stochastic approach to multilateral indices has extensively been undertaken was the main focus of discussion in Chapter 3. Furthermore, Chapter 3 also demonstrates as to how the PPP and its standard error can be estimated using the GLS estimation results of the CCD multilateral index model.

The concept of spatial autocorrelation and its role associated with the CCD multilateral index model was presented in Chapter 4. This chapter essentially focused on the conceptual framework associated with spatial autocorrelation, its measurement and then, the problem of specification, estimation and hypothesis testing. Several proximity measures which are used in the literature to account for spatial phenomena were discussed. Conceptually, it was argued that spatial autocorrelation plays an important role in the estimation of CCD indices.

One of the basic thrusts of the dissertation is to be able to test the presence of spatial autocorrelation among the disturbances of the CCD multilateral index model. The Moran's I statistic that was discussed in Chapter 4 had played a significant role in this context. With the 1985 ICP price and quantity data on 56 countries, and the three proximity measures namely, contiguity, distance and trade, the existence of the spatial autocorrelation among the estimated residuals of the CCD model was established. It was learned that positive spatial autocorrelation exists among the price relatives of the CCD model using the ICP data for all eight private consumption commodity headings using 54 countries in the benchmark year 1985. The estimated Moran's I statistics were consistently high, and statistically significant for Food, Rent, Furniture and Education in all the proximity measures. This supports the conclusion that price relatives for these commodities are likely to be similar for geographically close countries, and for countries with high trade interaction.

In response to these results, a more generalised form of the CCD model was specified following the conceptual framework underlying in Chapter 4. A different structure of

the disturbance term was created to allow for spatial autocorrelation. A more generalised form of the proximity matrix W was also constructed, leading to W^* , to capture the spatial relationship present among the price relatives residuals across countries. The W^* specification utilised the previously mentioned proximity measures. The model specified was general enough to allow a different spatial autocorrelation parameter ρ for each commodity group. The new model derived from the CCD model allowing for spatial autocorrelation results in GCCD(SA) multilateral index number formula.

An iterative procedure outlined in Chapter 5, was used to estimate the GCCD(SA) model. This iterative scheme has performed well, in fact, the speed of convergence for the spatial autocorrelation coefficients was very fast. As regards to the parameter estimates for ρ , it was found that the price relatives for the goods and services in all the commodity headings are positively correlated among physically close countries with the exception of non-tradeable goods like Transport. It was also revealed that countries which are distant from each other were more likely to have similar prices if their trade interaction was greater. The main conclusion from these results is that the three alternative proximity measures, namely contiguity, distance, and trade have been successful in capturing the nature of spatial correlation in the price relatives of the different commodities across countries.

A major aspect from the study is the improvement in efficiency as well as the plausibility of the PPP estimates derived using the GCCD(SA) model specifications. The results indicated that the magnitude of the estimated PPPs for each country for all the index models, TT, CCD and GCCD(SA) do not deviate substantially from each other. This enables us to conclude that the GCCD(SA) model provides plausible estimates of the purchasing power parities. Improvement in the efficiency of the standard error estimates was clearly achieved by the GCCD(SA) specification, as evidently shown by the very small standard error estimates for the PPPs. Moreover, standard error estimates associated with the trade matrix specification are uniformly lower, suggesting the possibility that trade captures the correlation among the price relatives better. Some available goodness-of-fit measures also support these conclusions.

Therefore, the principal achievement of the present study is that with the presence of spatial autocorrelation among the price relatives of commodities across countries, further improvement of the Prasada Rao and Selvanathan (1992a)'s CCD model can be affected by explicitly incorporating spatial autocorrelation in the disturbance structure,

leading to unbiased and efficient estimates of the multilateral indices and the PPP's associated with these indices.

Due to some computational problems encountered during the study, the numerical illustration is based only on a selection of 20 countries. It would be very useful to examine these computational problems closely and to devise algorithms or programs to handle larger sets that could take all the countries into the computation. This requires handling of much larger scale matrices, particularly the W^* specification, hence, a continuous search for more flexible computer package for this type of estimation is necessary.

Finally, this study brings a new facet in the multilateral comparison exercises. The concept of spatial autocorrelation should be taken into account in any econometric formulation of multilateral indices. But more importantly, this study brings to the attention of the econometricians the need to adequately address the problem of spatial autocorrelation in any standard cross-country regression analysis.

Appendix 1. Basic Data Used

Table A.1 Expenditure Data on Per Capita Basis in National Currencies for Various Sub-Aggregates of Private Consumption Expenditure for 56 ICP Countries, 1985

Country	Food	Clothing	Rent	Furniture	Medical	Transport	Education	Others
Germany	3256	1496	3723	1569	2792	2662	2979	2113
France	11258	3394	9855	4499	7620	7512	7652	8062
Italy	2237157	832899	1360762	753171	974399	1109265	1404702	1476563
Netherlands	3223	1130	3301	1219	2098	1801	3032	2458
Belgium	65857	22717	59653	30770	35317	38820	52420	47330
Luxembourg	70266	22432	71156	31816	27252	60223	49890	51672
UK	724	269	767	255	347	621	626	819
Ireland	1250	173	365	167	334	366	456	203
Denmark	15757	3921	16459	4612	7247	11257	14269	12532
Greece	120657	27824	40470	25856	21861	42680	28274	47486
Spain	136918	35600	78040	34326	37117	67855	51538	92365
Portugal	102880	25646	23256	21725	15827	34281	21569	24025
Austria	24142	11294	20119	7223	12001	18038	15111	18055
Finland	9757	1928	6748	2511	4110	6395	7203	7413
Norway	14377	4264	10051	4461	7530	9979	11218	7966
Sweden	12944	3952	14095	3418	8128	8350	14200	8519
Australia	1936	480	2042	656	974	1293	1534	816
New Zealand	1534	564	1269	756	840	1740	1357	1343
Japan	342763	97438	287828	85149	167955	144183	241261	275814
Canada	1892	667	2445	930	575	1680	2192	1471
USA	1501	699	2151	621	1629	1698	1712	1728
Turkey	182310	60954	51907	52017	14827	19914	17558	17161
Hong Kong	4687	2976	4670	1852	1916	2751	6054	7566
Korea	476608	71033	118110	49811	52578	102216	152819	105604
Thailand	5353	2189	983	787	749	1825	1177	1363
India	1335	264	249	73	71	178	138	99
Iran	84355	18561	48823	12077	11920	12585	12436	10897
Sri Lanka	4073	510	425	337	165	1188	506	243

Table A.1 continued

Country	Food	Clothing	Rent	Furniture	Medical	Transport	Education	Others
Pakistan	1641	243	623	166	37	505	390	245
Philippines	4787	335	1647	354	194	339	358	568
Botswana	246	67	73	87	73	67	231	37
Egypt	300	57	45	22	19	19	47	35
Ethiopia	109	12	28	16	6	17	8	5
Kenya	1573	246	403	301	101	268	437	126
Malawi	85	20	20	26	9	22	26	7
Mauritius	3465	604	2252	731	603	1244	1859	586
Nigeria	358	33	28	20	20	20	42	18
Sierra Leone	635	42	155	26	20	126	33	11
Swaziland	339	82	46	156	57	114	118	112
Tanzania	3012	452	355	143	149	90	153	102
Zambia	281	64	72	31	52	36	113	23
Zimbabwe	286	69	75	33	24	38	75	20
Benin	42266	14787	12459	8664	5082	14798	4944	3044
Cameroon	81423	20867	48396	14693	34375	35296	32087	29051
Congo	106977	14560	24529	10953	16182	38235	23461	22868
Ivory Coast	98926	18864	9737	19058	17352	19620	12271	4207
Madagascar	84065	7880	15499	6412	2531	4980	8015	4688
Mali	31358	3158	4112	2116	1228	5189	3096	682
Morocco	1826	457	367	343	202	345	418	331
Rwanda	7786	2215	3157	2689	630	1881	1818	626
Senegal	83523	17522	18022	7989	4656	8709	12109	6537
Tunisia	283	65	85	45	45	48	82	26
Poland	71946	15661	11190	15725	9863	14814	19561	15721
Hungary	18250	5035	5154	4697	2977	5171	8161	8265
Yugoslavia	119415	32016	30436	28362	20680	38177	36850	33482
Bangladesh	2649	313	732	134	82	133	88	78

Source: ICP 1994, *Comparisons of Real Gross Domestic Product and Purchasing Power, 1985*, Summary Table 10.

Table A.2 Purchasing Power Parities for Various Sub-Aggregates of Private Consumption Expenditure for 56 ICP Countries at International Prices in National Currency per US Dollar, 1985

Country	Food	Clothing	Recreation	Furniture	Medical	Transport	Education	Others
Germany	2.361	2.602	2.478	2.589	1.848	3.454	2.672	2.893
France	7.088	8.094	6.464	8.674	4.496	11.103	8.05	8.462
Italy	1396	1569	83	1631	1008	1975	1304	1473
Netherlands	2.487	2.49	2.182	2.74	1.585	3.469	2.765	2.874
Belgium	46.62	58.79	37.44	51.51	23.82	66.96	51.04	52.98
Luxembourg	42.44	55.8	35.72	52.59	26.7	53.54	59.42	48.16
UK	0.6131	0.5488	0.4075	0.6864	0.3215	0.9584	0.5855	0.6863
Ireland	0.9093	0.7007	0.4075	0.8612	0.5432	1.3669	0.6397	0.8486
Denmark	11.419	9.757	7.903	10.545	7.537	15.472	9.81	11.832
Greece	82.92	104.44	65.68	100.06	51.94	90.79	69.08	98.01
Spain	108.1	146.77	45.77	116.55	70.42	155.28	104.86	101.2
Portugal	103.41	115.93	22.11	99.75	43.28	134.52	40.12	85.47
Austria	18.23	19.78	14.02	17.51	10.37	26.92	19.13	18.45
Finland	8.106	7.523	4.117	7.234	3.686	9.724	6.241	7.415
Norway	11.958	10.217	6.288	8.9	4.991	13.931	9.273	11.855
Sweden	11.244	10.812	6.649	7.822	4.425	10.412	8.269	9.317
Australia	1.113	1.31	1.1	1.259	1.088	1.543	1.156	1.495
New Zealand	1.317	1.612	0.974	1.708	0.974	2.079	0.953	1.606
Japan	293.8	238	157.5	246.4	106.8	279.6	229.3	248.3
Canada	1.371	1.312	1.276	1.265	0.921	1.241	1.289	1.193
USA	1	1	1	1	1	1	1	1
Turkey	185.2	258.5	185.5	205	109.7	168.7	100.7	177.4
Hong Kong	4.718	4.623	6.415	4.454	2.718	5.575	4.435	3.559
Korea	629.6	453	419.5	419.9	263.8	574.9	474.1	308.9
Thailand	9.483	13.243	5.778	9.328	3.479	10.142	5.927	6.446
India	5.817	6.081	2.413	5.932	1.873	5.919	3.326	4.569
Iran	90.59	72.54	51.72	95.5	30.11	55.09	85.52	40.14
Sri Lanka	9.918	5.826	6.67	7.09	1.78	9.113	3.731	5.169

Table A.2 continued

Country	Food	Clothing	Rent	Furniture	Medical	Transport	Education	Others
Pakistan	5.046	3.31	4.027	6.165	0.696	5.532	5.077	2.367
Philippines	7.376	6.391	7.443	7.285	3.721	16.996	2.882	4.042
Botswana	0.7643	0.31	0.2558	0.6199	0.4501	1.0825	0.3815	0.7827
Egypt	0.4144	0.2413	0.0738	0.2371	0.1755	0.2496	0.1291	0.3656
Ethiopia	1.1671	0.308	0.4245	1.1026	0.5776	0.7585	0.3619	0.9932
Kenya	6.767	2.895	1.932	6.046	3.97	6.152	2.612	7.254
Malawi	0.4559	0.274	0.3126	0.6739	0.3003	0.9511	0.2736	0.6034
Mauritius	4.5908	2.4509	0.8421	4.3174	3.249	3.714	1.6481	5.0319
Nigeria	1.3253	0.626	0.2522	0.9786	0.5573	0.6862	0.3403	1.04
Sierra Leone	3.1715	1.0658	0.5692	3.4093	1.1603	5.0095	0.864	3.1995
Swaziland	0.8943	0.3295	0.2761	0.6821	0.4604	0.7144	0.3015	0.7939
Tanzania	24.95	12.34	3.24	28.88	4.39	17.47	3.93	26.05
Zambia	1.8502	0.6523	0.3883	1.4911	0.5775	0.9858	0.5313	1.8737
Zimbabwe	0.6608	0.4049	0.1839	0.69	0.6326	0.7813	0.3676	0.7245
Benin	137.51	70.03	33.07	128.93	67.97	187.67	49.26	148.63
Cameroon	197.7	92.3	78.8	219.3	106.4	204	82.3	170.6
Congo	277.6	133.3	91.4	259.8	122.3	223.6	62.6	243
Ivory Coast	178.5	156.1	82.1	187.5	84.9	283.1	96.5	259
Madagascar	383.9	134.7	113.4	358.7	124.8	242	86.7	326
Mali	256.5	98.9	110.2	204.8	60.2	244.6	63.6	203.3
Morocco	3.25	1.428	0.992	3.336	2.023	2.991	1.286	2.951
Rwanda	50.07	27.8	25.25	45.18	17.3	66.12	18.26	61.95
Senegal	205.5	88.8	64.3	173.4	112.1	145.8	83.7	159.5
Tunisia	0.3254	0.2425	0.1629	0.2573	0.1878	0.3601	0.1535	0.3941
Poland	111.28	106.59	19.69	104.25	29.95	147.58	58.22	75.08
Hungary	21.01	27.47	6.01	24.78	3.8	32.39	17.17	20.01
Yugoslavia	125.9	149.7	51.4	136.7	30.7	192.3	94.8	93.9
Bangladesh	8.925	7.181	4.796	8.718	3.96	4.212	2.994	3.735

Source: ICP 1994, *Comparisons of Real Gross Domestic Product and Purchasing Power, 1985*, Summary Table 5.

Appendix 3. FORTRAN Programmes for generating data observations

A.3.1 Programme for the CCD Model (56 Countries)

```

program index
double precision p,e,w,y,xx,dd
dimension p(56,56),e(56,8),w(56,8)
dimension y(15000),xx(15000,56)
open(unit=21,file='expend.dta',status='old')
open(unit=22,file='price.dta',status='old')
open(unit=23,file='xx',status='old')
open(unit=25,file='ws',status='old')
open(unit=26,file='y',status='old')
g=56
f=g-1
do 10 j=1,g
read(22,*)(p(j,i),i=1,8)
write(5,*)(p(j,i),i=1,8)
10 continue
do 20 j=1,g
read(21,*)(e(j,i),i=1,8)
esum=0.0
do 21 i=1,8
esum=esum+e(j,i)
21 continue
do 22 i=1,8
w(j,i)=e(j,i)/esum
22 continue
write(25,66)(w(j,i),i=1,8)
20 continue
c
c
c generate the Dpikj and X matrice ;
c
c
kkk=1
do 30 i=1,8
do 31 k=1,f
do 32 j=k+1,g
dd=sqrt(((w(j,i)+w(k,i))/2))
y(kkk)=(log(p(j,i))-log(p(k,i)))*(1d)
do 33 l=1,g
if (l-k) 76,78,79

```

A.3.1 continued

```

76  xx(kkk,l)=0.0
    go to 33
78  xx(kkk,l)=dd
    go to 33
79  if (l-j) 87,86,89
87  xx(kkk,l)=0.0
    go to 33
86  xx(kkk,l)=-dd
    go to 33
89  xx(kkk,l)=0.0
    go to 33
33  continue
    kkk=kkk+1
32  continue
31  continue
30  continue
    write(26,77)(y(j),j=1,(kkk-1))
    do 35 l=1,(kkk-1)
        write(23,99)(xx(l,j),j=1,g)
35  continue
c   ws-matrix
66  format(1x,8f8.4,1x)
c   y-vector
77  format(10f8.4)
c   xx-matrix
99  format(10f8.4)
    stop

```

A.3.2 Programme for the TT and CCD Model (20 Countries)

```

program index
double precision p,e,w,y,xx,dd,tt,pi,ccd
dimension p(20,20),e(20,8),w(20,8),tt(20,20)
dimension y(1600),xx(1600,20),pi(20,20),ccd(20,20)
open(unit=21,file='exp.dta',status='old')
open(unit=22,file='pri.dta',status='old')
open(unit=23,file='xxf',status='old')
open(unit=24,file='ttcc',status='old')
open(unit=25,file='wsf',status='old')
open(unit=26,file='yf',status='old')
g=20
f=g-1
do 10 j=1,g
read(22,*)(p(j,i),i=1,8)

```

A.3.2 continued

```

        write(5,*)(p(j,i),i=1,8)
10  continue
    do 20 j=1,g
        read(21,*)(e(j,i),i=1,8)
        esum=0.0
        do 21 i=1,8
            esum=esum+e(j,i)
21  continue
        do 22 i=1,8
            w(j,i)=e(j,i)/esum
22  continue
        write(25,66)(w(j,i),i=1,8)
20  continue
c
c  generate TT and CCD indices
c
    do 24 k=1,g
        do 24 j=1,g
            pi(k,j)=0.0
            do 25 i=1,8
                pi(k,j)=pi(k,j)+((w(k,i)+w(j,i))/2 *(log(p(j,i))-log(p(k,i))))
25  continue
            tt(k,j)=exp(pi(k,j))
24  continue
        do 50 k=1,g
            do 50 j=1,g
26  ccd(k,j)=1.0
                do 26 k=1,g
                    do 28 j=1,g
                        do 27 l=1,g
                            c1=1.0/g
27  ccd(k,j)=ccd(k,j)*((tt(k,l)*tt(l,j))**(c1))
28  continue
                write(24,88) (tt(k,j),j=1,g)
                write(24,88) (ccd(k,j),j=1,g)
26  continue
c
c  generate the Dpikj ( Y ) vector and X matrices
c
c
    kkk=1
    do 30 i=1,8
        do 31 k=1,f
            do 32 j=k+1,g
                dd=sqrt(((w(j,i)+w(k,i))/2))
                y(kkk)=(log(p(j,i))-log(p(k,i)))*( 1d)
                do 33 l=1,g

```


A.3.2 continued

```

      if (l-k) 76,78,79
76  xx(kkk,l)=0.0
      go to 33
78  xx(kkk,l)=dd
      go to 33
79  if (l-j) 87,86,89
87  xx(kkk,l)=0.0
      go to 33
86  xx(kkk,l)=-dd
      go to 33
89  xx(kkk,l)=0.0
      go to 33
33  continue
      kkk=kkk+1
32  continue
31  continue
30  continue
      write(26,77)(y(j),j=1,(kkk-1))
      do 35 l=1,(kkk-1)
      write(23,99)(xx(l,j),j=1,g)
35  continue
c   ws-matrix
66  format(1x,8f8.4,1x)
c   y-vector
77  format(10f8.4)
c   xx-matrix
99  format(10f8.4)
c   ttcc-matrix
88  format(10f8.4,1x)
      stop

```

Appendix 4. SHAZAM Programmes for calculating PPPs

A.4.1 Programme for GLS Estimation of the CCD Model (56 countries)

```

*****
* PPP Estimation using Stochastic Approach
*****
format(10f8.4)
file 33 xx
file 44 y
par 30000
sample 1 12320
read(44) y / byvar
read(33) x1-x56 / format
copy x1-x55 xx1
matrix xx1=-xx1
*
* generate CCD index ( stochastic approach)
*
ols y xx1 / noconstant coef=pi1 resid=ehat1
*
* generate PPP estimates
*
print pi1
genr epi1=exp(pi1)
print epi1
sample 1 55
file 22 ppp1
write(22) epi1
*
* generate the residuals
*
sample 1 12320
file 23 ehat.dta
write(23) ehat1
stop

```

A.4.2 Programme for Simultaneous Estimation of the GCCD(SA) Model (20 countries)

```

* First step *
*****

* GLS Estimation of the CCD Model
*****

format(10f8.4)
sample 1 1520
read(yf) y / byvar
read(xxf) x1-x20 / format
copy x1-x19 xx1
matrix xx1=-xx1
*
* generate CCD index ( stochastic approach)
*
ols y xx1 / noconstant coef=pi1 resic=ehat1
*
* estimate the PPP
*
genr epil=exp(pi1)
print epil
*
* generate the residuals
*
sample 1 1520
file 23 ehat10.dta
write(23) ehat1
stop

* Second Step *
*****

* Estimation of the Spatial Autocorrelation Parameter
*****

set nodoecho nowarn
dim w 190 190 y1 190 xxx1 190 20 num 1 1 denum 1 1
read(cmat20.dta) w / rows=190 cols:=190
sample 1 1520
read(ehat10.dta) u / byvar
genl k1=1
genl k2=190
?do %=1,8
copy u u1 / frow=k1;k2 trow=1;190
matrix num=(u1'w)*u1
matrix denum=(u1'(w'w))*u1
genl r%=num/denum
print r%
genl k1=k2+1
genl k2=k2+190

```

A.4.2 continued

```

endo
*
* compute the y and x transformed matrix
*
format(10f8.4)
sample 1 1520
read(yf) y / byvar
read(xxf) x1-x20 /format
copy x1-x19 xx1
matrix xxx=-xx1
*
* adjust for spatial autocorrelation
*
matrix in=iden(190)
genl n1=1
genl n2=190
?do %=1,8
matrix v=in-(r%@w)
copy y y1 /frow=n1;n2 trow=1;190
copy xxx xxx1/ frow=n1;n2 fcol=1; 9 trow=1;190 tcol=1;19
matrix yy%=v*y1
matrix xs%=v*xxx1
genl n1=n2+1
genl n2=n2+190
endo
matrix ystar=(yy1'lyy2'lyy3'lyy4'lyy5'lyy6'lyy7'lyy8')
matrix xstar=(xs1'xs2'xs3'xs4'xs5'xs6'xs7'xs8')
sample 1 1520
file 48 ystar.dta
write(48) ystar / format
file 49 xstar.dta
write(49) xstar / format
stop
*
* Third Step*
*****
* Estimation of the GCCD(SA) Model
*****
format(10f8.4)
sample 1 1520
read(ystar.dta) y / byvar
read(xstar.dta) x1-x20 / format
copy x1-x19 xx1
*
* generate CCD index ( stochastic approach)
*
ols y xx1 / noconstant coef=pi1 resid=e1

```

A.4.2 continued

```
*  
* this are the estimates  
*  
genr epi1=exp(pi1)  
print epi1  
*  
* this are the residuals  
*  
sample 1 1520  
file 23 ehat10.dta  
write(23) ehat1  
stop
```

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