

**AN INVESTIGATION INTO STUDENTS'
UNDERSTANDING OF STATISTICS**

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Thesis submitted in fulfilment of the
requirements of the degree of
Doctor of Philosophy
of the
University of New England

February 1996

Certificate

I certify that the substance of this thesis has not already been submitted for any degree and is not being currently submitted for any other degree.

I certify that any help received in preparing this thesis, and all sources used, have been acknowledged in this thesis.

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(Signature)

ACKNOWLEDGEMENTS

I will be forever indebted to a number of people who provided assistance and support during the research undertaken for this study and the writing of the thesis.

In particular, my supervisor, Associate Professor John Pegg, who not only provided advice and support over a long period of time, was responsible for initially encouraging me to embark on the project. A greater gift than my thanks is to give John back his Saturday mornings which he has selfishly shared with me for the last twelve months.

Professor Don Fitzgerald provided valuable information concerning Luria Tests and assistance in processing of the data from these tests. Dr Ken Vine generously provided information concerning Rasch analysis and assistance in processing data with the QUEST system. I am greatly indebted to them both.

The principal, staff and students of the targeted school willingly cooperated during the two years of testing. I acknowledge the patience of the staff and the positive approach taken by the students, especially those who agreed to continue to be involved during the second year of the study. Without such cooperation this study would not have been possible.

My friends and relatives who have encouraged me to complete this thesis. I thank them all for the support. Especially my mother who, for as long as I can remember, has encouraged me to pursue life to its fullest and to achieve to my full potential. The example she set enrolling in her first ever university studies at the same time as I was enrolled for my doctorate has given me the incentive to complete what at times has felt like a never ending task.

Finally, to my long suffering son, who has never known a time in his seventeen-year life span when I have not be studying towards some award, my apologies for the times when I have had to work rather than share in his life and my assurances that I have finally finished.

TABLE OF CONTENTS

ACKNOWLEDGEMENTS		iii
LIST OF TABLES		vii
LIST OF FIGURES		xii
ABSTRACT		xiv
INTRODUCTION		1
CHAPTER		
1	STATISTICS	4
	What is Statistics ?	4
	Understanding in Probability in a Statistical Context	16
	Teaching Strategies Effect on Understanding in Statistics	25
	Conclusion	29
2	LITERATURE REVIEW	31
	Understanding in Statistics	31
	Levels of Understanding in Statistics	51
	The SOLO Taxonomy	63
	Conclusion	71
3	EXPERIMENTAL DESIGN - STUDY 1	76
	Pilot Study	76
	Main Study	87
	Conclusion	95
4	RESULTS - STUDY 1	96
	Data Collection	96
	Data Tabulation and Representation	114
	Data Reduction	132
	Interpretation and Inference	148
	Conclusion	167

5	OVERVIEW - STUDY 1	170
	Overall Student Performance	170
	Question Difficulty	174
	Level Thresholds	178
	Conclusion	184
6	EXPERIMENTAL DESIGN - STUDY 2	191
	Background	191
	Luria Study	193
	Longitudinal Study	197
	Interview Study	200
	Conclusion	202
7	RESULTS - STUDY 2	204
	Luria Study	204
	Longitudinal Study	209
	Interview Study	220
	Overview - Study 2	225
8	DISCUSSION OF RESULTS AND CONCLUSIONS	227
	Possible Limitations of the Study	227
	Summary of Results	228
	Post Study Developments	230
	Implications of the Study	233
	Conclusion	237
	REFERENCES	239
	APPENDICES	256
A	Written Test for Study 1	256
B	Statistics Components of Mathematics Syllabuses	261
C	Permission Letters for Study 1	269
D	Selection of Responses to Question 1 Part II	274

E	Selection of Responses to Question 2 Part II	277
F	Selection of Responses to Question 3 Part II	281
G	Selection of Responses to Question 4 Part II	286
H	Tables for Results - Study 1	291
I	Tables for Overview - Study 1	298
J	Figures for Overview - Study 1	300
K	Permission Letter for Study 2	304
L	Luria Test	305
M	Written Test for Study 2	323
N	Interview Schedule for Study 2	327
O	Tables for Results - Study 2	328
P	Figures for Results - Study 2	341

LIST OF TABLES

4.1.1	Response Level by Academic Year - Question 1 Part I	102
4.1.2	Response Level by Mathematical Ability - Question 1 Part I	102
4.1.3	Response Level by Gender - Question 1 Part I	103
4.1.4	Response Level by Academic Year - Question 1 Part II	105
4.1.5	Response Level by Mathematical Ability - Question 1 Part II	106
4.1.6	Response Level by Gender - Question 1 Part II	106
4.1.7	Level Comparison for Parts I and II - Question 1	108
4.1.8	Group Comparison within Students - Question 1	109
4.2.1	Response Level and Path by Academic Year - Question 2 Part I	120
4.2.2	Response Level by Mathematical Ability - Question 2 Part I	121
4.2.3	Response Path by Mathematical Ability - Question 2 Part I	121
4.2.4	Response Level by Gender - Question 2 Part I	122
4.2.5	Response Path by Gender - Question 2 Part I	122
4.2.6	Response Level and Path by Academic Year - Question 2 Part II	124
4.2.7	Level Comparison for Parts I and II - Question 2	126
4.2.8	Path Comparison for Parts I and II - Question 2	126
4.2.9	Group Comparison within Students - Question 2	128
4.2.10	Path Comparison within Students - Question 2	129
4.3.1	Response Level and Path by Academic Year - Question 3 Part I	140
4.3.2	Response Level and Path by Academic Year - Question 3 Part II	142
4.4.1	Response Level and Path by Academic Year - Question 4 Part I	158
4.4.2	Response Level and Path by Academic Year - Question 4 Part II	161
5.1	Estimates of Understanding - Analysis of Variance	172
5.2	Estimates of Understanding - Main Effect Means	173
5.3	Question Difficulty	175
6.1	Luria Study Sample	194
6.2	Luria Test Bank	195
6.3	Longitudinal Study Sample	198
7.1.1	Factor Analysis of Luria Test Scores	205

7.1.2	Regression of Rasch Score on Successive and Simultaneous Factors	206
7.1.3	Regression of Path Score on Successive and Simultaneous Factors	206
7.1.4	Regression of Path Score on Successive Factor	207
7.1.5	Regression of Path Score on Simultaneous Factor	207
7.1.6	Paths by Successive Processing	208
7.1.7	Paths by Simultaneous Processing	208
7.2.1	Levels by Mathematical Ability	211
7.2.2	Paths by Mathematical Ability	212
7.2.3	Levels by Gender	212
7.2.4	Paths by Gender	213
7.2.5	Levels by Question Part	214
7.2.6	Paths by Question Part	214
7.2.7	Estimates of Understanding - Analysis of Variance	215
7.2.8	Estimates of Understanding - Main Effect Means	216
7.2.9	Estimate Differences - Academic Years	218
7.2.10	Estimate Differences - Mathematical Ability	218
7.2.11	Estimate Differences - Gender	218
7.2.12	Estimate Differences - Analysis of Variance	219
7.3.1	Response Level Changes - Written Test versus Interview	221
7.3.2	Response Level Changes due to Probing	222
7.3.3	Response Level Changes due to Prompting	223
H.1.1	Level Comparison within Students - Question 1	291
H.2.1	Response Level by Mathematical Ability - Question 2 Part II	291
H.2.2	Response Path by Mathematical Ability - Question 2 Part II	291
H.2.3	Response Level by Gender - Question 2 Part II	291
H.2.4	Response Path by Gender - Question 2 Part II	292
H.2.5	Level Comparison within Students - Question 2	292
H.3.1	Response Level by Mathematical Ability - Question 3 Part I	292
H.3.2	Response Path by Mathematical Ability - Question 3 Part I	292
H.3.3	Response Level by Gender - Question 3 Part I	292
H.3.4	Response Path by Gender - Question 3 Part I	293
H.3.5	Response Level by Mathematical Ability - Question 3 Part II	293
H.3.6	Response Path by Mathematical Ability - Question 3 Part II	293
H.3.7	Response Level by Gender - Question 3 Part II	293
H.3.8	Response Path by Gender - Question 3 Part II	293
H.3.9	Level Comparison within Students - Question 3	293

H.3.10	Level Comparison for Parts I and II - Question 3	294
H.3.11	Path Comparison for Parts I and II - Question 3	294
H.3.12	Group Comparison within Students - Question 3	294
H.3.13	Path Comparison within Students - Question 3	294
H.4.1	Response Level by Mathematical Ability - Question 4 Part I	294
H.4.2	Response Path by Mathematical Ability - Question 4 Part I	295
H.4.3	Response Level by Gender - Question 4 Part I	295
H.4.4	Response Path by Gender - Question 4 Part I	295
H.4.5	Response Level by Mathematical Ability - Question 4 Part II	295
H.4.6	Response Path by Mathematical Ability - Question 4 Part II	295
H.4.7	Response Level by Gender - Question 4 Part II	296
H.4.8	Response Path by Gender - Question 4 Part II	296
H.4.9	Level Comparison within Students - Question 4	296
H.4.10	Level Comparison for Parts I and II - Question 4	296
H.4.11	Path Comparison for Parts I and II - Question 4	296
H.4.12	Group Comparison within Students - Question 4	297
H.4.13	Path Comparison within Students - Question 4	297
I.1	Comparison of Estimate Means for Academic Years - Matrix of Pairwise Mean Differences	298
I.2	Comparison of Estimate Means for Academic Years - Scheffe Test - Matrix of Pairwise Comparison Probabilities	298
I.3	Comparison of Estimate Means for Mathematical Ability - Matrix of Pairwise Mean Differences	298
I.4	Comparison of Estimate Means for Mathematical Ability - Scheffe Test - Matrix of Pairwise Comparison Probabilities	298
I.5	Comparison of Estimate Means for Gender - Matrix of Pairwise Mean Differences	298
I.6	Comparison of Estimate Means for Gender - Scheffe Test - Matrix of Pairwise Comparison Probabilities	299
I.7	Item Estimate Tau Values	299
I.8	Item Estimate Threshold Values	299
O.1.1	Successive Processing Style	328
O.1.2	Simultaneous Processing Style	328
O.2	Tau Value Comparisons	329
O.3	Threshold Value Comparisons	329
O.4.1	Response Level by Academic Year - Question 1 Part I	330
O.4.2	Response Level by Academic Year - Question 1 Part II	330

O.4.3	Response Level and Path by Academic Year - Question 2 Part I	330
O.4.4	Response Level and Path by Academic Year - Question 2 Part II	331
O.4.5	Response Level and Path by Academic Year - Question 3 Part I	331
O.4.6	Response Level and Path by Academic Year - Question 3 Part II	331
O.4.7	Response Level and Path by Academic Year - Question 4 Part I	332
O.4.8	Response Level and Path by Academic Year - Question 4 Part II	332
O.5.1	Response Level by Mathematical Ability - Question 1 Part I	332
O.5.2	Response Level by Mathematical Ability - Question 1 Part II	333
O.5.3	Response Level and Path by Mathematical Ability - Q. 2 Part I	333
O.5.4	Response Level and Path by Mathematical Ability - Q. 2 Part II	333
O.5.5	Response Level and Path by Mathematical Ability - Q.3 Part I	334
O.5.6	Response Level and Path by Mathematical Ability - Q.3 Part II	334
O.5.7	Response Level and Path by Mathematical Ability - Q.4 Part I	334
O.5.8	Response Level and Path by Mathematical Ability - Q.4 Part II	335
O.6.1	Response Level by Gender - Question 1 Part I	335
O.6.2	Response Level by Gender - Question 1 Part II	335
O.6.3	Response Level and Path Gender - Question 2 Part I	336
O.6.4	Response Level and Path Gender - Question 2 Part II	336
O.6.5	Response Level and Path by Gender - Question 3 Part I	336
O.6.6	Response Level and Path by Gender - Question 3 Part II	337
O.6.7	Response Level and Path by Gender - Question 4 Part I	337
O.6.8	Response Level and Path by Gender - Question 4 Part II	337
O.7.1	Comparison of Parts I and II - Question 1	338
O.7.2	Comparison of Parts I and II - Question 2	338

0.7.3	Comparison of Farts I and II - Question 3	338
0.7.4	Comparison of Farts I and II - Question 4	339
0.8.1	Comparison of Estimate Means for Academic Years - Matrix of Pairwise Mean Differences	339
0.8.2	Comparison of Estimate Means for Academic Years - Scheffe Test - Matrix of Pairwise Comparison Probabilities	339
0.8.3	Comparison of Estimate Means for Mathematical Ability - Matrix of Pairwise Mean Differences	339
0.8.4	Comparison of Estimate Means for Mathematical Ability - Scheffe Test - Matrix of Pairwise Comparison Probabilities	340
0.8.5	Comparison of Estimate Means for Gender - Matrix of Pairwise Mean Differences	340
0.8.6	Comparison of Estimate Means for Gender - Scheffe Test - Matrix of Pairwise Comparison Probabilities	340

LIST OF FIGURES

2.1	Task Analysis Guideline	53
2.2	Task Analysis - Major Sections	54
2.3	Stochastics Discipline Control Circuit	55
2.4	Planning and Execution Stage	57
2.5	Data Tabulation and Representation Flowchart	59
2.6	SOLO Response Structures	65
2.7	Modes and Learning Cycles	66
2.8	Possible Course of Action for Cognition Task	67
3.1	Trialled Data Collection Question	79
3.2	Trialled Data Tabulation and Representation Question - Graph	80
3.3	Trialled Data Tabulation and Representation Question - Table	81
3.4	Trialled Data Reduction Question - Average	81
3.5	Trialled Data Reduction Question - Comparison of Measures of Central Tendency	82
3.6	Trialled Data Reduction Question - Best Measure of Central Tendency	83
4.1	Data Collection - Part I Question	97
4.2	Data Collection - Part II Question	104
4.3	Data Tabulation and Representation - Part I Question	115
4.4	Data Tabulation and Representation - Part II Question	123
4.5	Data Reduction - Part I Question	133
4.6	Data Reduction - Part II Question	141
4.7	Interpretation and Inference - Part I Question	149
4.8	Interpretation and Inference - Part II Question	159
5.1	Estimates of Understanding	171
5.2	Difficulty Moving Between Levels - Question 1	175
5.3	Difficulty Moving Between Levels - Question 2	176
5.4	Difficulty Moving Between Levels - Question 3	177
5.5	Difficulty Moving Between Levels - Question 4	177
5.6	Thresholds to Attain Levels - Question 1	179
5.7	Thresholds to Attain Levels - Question 2	180
5.8	Thresholds to Attain Levels - Question 3	180
5.9	Thresholds to Attain Levels - Question 4	181
5.10	Threshold Value: Comparison for Levels within Questions	183

J.1	Fit Statistics - Question Estimates	300
J.2	Fit Statistics - Student Estimates	300
J.3	Infit Mean Square Map - Parameter Fit	300
J.4	Estimates of Understanding by Academic Year	301
J.5	Estimates of Understanding by Mathematical Ability	302
J.6	Estimates of Understanding by Gender	303
P.1	Fit Statistics - Question Estimates	341
P.2	Fit Statistics - Student Estimates	341
P.3	Infit Mean Square Map - Parameter Fit	341
P.4	Estimates of Understanding	342
P.5	Estimates of Understanding by Academic Year	343
P.6	Estimates of Understanding by Mathematical Ability	344
P.7	Estimates of Understanding by Gender	345

ABSTRACT

This thesis is an investigation, both qualitative and quantitative, of students' understanding of statistics. Early investigations into the study of levels of understanding were directed at course content, but more recently the search has focused on student responses. This present study describes and analyses levels of statistical understanding identified in secondary students' responses.

Students were given open-format questions concerning four basic areas of statistics, namely, data collection, data tabulation and representation, data reduction, and interpretation and inference. Their responses were grouped according to the degree of statistical sophistication exhibited and these groupings were further refined to develop an hierarchy of levels of understanding in each of the four areas. The framework of the SOLO Taxonomy (Biggs and Collis, 1991) was used to describe the hierarchy. The responses fell into two modes, the iconic and the concrete-symbolic and features of each level within these modes were described.

The influence of a number of factors on these levels was investigated, quantitatively. The codings were found to increase in level with increasing academic year. Mathematical ability was also found to be an influencing factor on the level but gender had far less influence. By presenting data in raw form and as graphs, it was determined that in some instances the form of presentation of the data influences the level of understanding. During coding, responses were found to diverge into two distinct paths and a students' tendency to process simultaneously (Luria, 1966) was found to have some influence on this tendency. QUEST software was used to apply the Rasch model to the data to estimate overall statistical understanding for each student. From this analysis, tau and threshold values were used to provide quantitative evidence that the coding of the responses fell into two distinct cycles in the concrete-symbolic mode.

The longitudinal aspect was also investigated by retesting the same students twelve months later. The identified hierarchy proved sufficient to code the responses and analysis of the results substantiated most results observed in the previous year. The increase in understanding was not significant over the twelve months. This supported the earlier findings of little significant difference in understanding between successive academic years. Interviewing of a small sample of students showed that prompting usually provided more information in the response but often at the same level. Probing allowed some students to increase their level but if pressured some students reverted to lower levels for support, thus exhibiting multi-modal functioning.

Level descriptions and the cycles identified in this study are consistent with recent findings by other researchers in the field of statistical understanding.