6. Method 2: APPLICATION OF THE MODEL

Cointegration analysis, carefully applied, allows the analysis of long-run economic relationships.
(Cuthberson and Hall 1992)

6.1 Introduction

This study was conducted in various stages. The first stage was to test and determine the long-run relationships between government defensive expenditure and some selected macroeconomic variables. The second stage was to impute for the value of leisure. The five methods of imputation in the literature were applied. In the third stage, the GNE measure was adjusted for environmental disturbances. Finally, the study attempted to integrate the changes in leisure and changes in the environment into one account.

Following the above sequence of analysis, this chapter is divided in six sections. Section 6.2 describes the tests used to determine and estimate the long-run relationships. The various methods attempted to impute for the value of leisure are outlined in Section 6.3. Section 6.4 discusses the procedures used to solve for the environmental productivity disturbances (ε_t). The procedures employed to integrate the changes in leisure and the environment in the national accounts are presented in Section 6.5. A summary is given in Section 6.6.

6.2 Estimating Long-run Relationships

Using the Adjusted GNE model (1) presented in Chapter 4, the long-run relationships among variables were explored. However, before the analysis of the econometric relationships between these time series data was possible, a standard testing procedure was followed to determine the nature of the variables to be regressed.

Since GNP is a long-run concept, it is reasonable to expect stable long-run relationships between the ratio of government defensive expenditures to GNE (g_{3t}^{w}) and variables like the ratio of consumption to GNE (c_t^{w}), the ratio of investment to GNE (i_t^{w}), the ratio of government expenditure on goods and services to GNE (g_{1t}^{w}), the ratio of government investment expenditure to GNE (g_{2t}^{w}), and the leisure-labour ratio (x_t^{w}). Hypothesised long-run relationships were formulated using the cointegration framework.

6.2.1 Testing for Non-stationarity

6.2.1.1 Unit root processes

There are important differences between stationary and non-stationary time series. Shocks to the stationary time series are necessarily temporary; over time, the effects of shocks will dissipate and the series will revert to its long-run mean level. As such, long-term forecasts of a stationary series will converge to the unconditional mean of the series.

On the other hand, a non-stationary series necessarily has permanent components. The mean and/or variance of the non-stationary series are time dependent. It is important to determine whether the variables are stationary or non-stationary. If the variables are non-stationary, then coefficient estimates based on ordinary least squares (OLS) regression will be biased and inconsistent (unless they are cointegrated). This means that if the data were not transformed according to the nature of the series, standard estimates and statistics calculated for the data could be considered spurious (Granger and Newbold 1974). A spurious regression has a high R^2, t-statistics that appear to be significant, but results that are without any economic meaning.

6.2.1.2 Unit root tests

To determine whether a time series is stationary or not, unit root tests were undertaken. There are several procedures for testing unit roots, including those developed by Fuller (1976), Dickey and Fuller (1979,1981), Said and Dickey (1984), Phillips (1987), and Park, Ouliaris and Choi (1988). The most popular unit root test in the applied literature is the Augmented Dickey-Fuller (ADF) test which is based on the autoregression of differences:

$$\Delta Y_t = a_0 + \gamma Y_{t-1} + \sum_{i=1}^{m} \beta_i \Delta Y_{t-i+1} + \varepsilon_t \tag{6.1}$$

The reason for the augmentation of the lagged differences is of course to ensure that the errors, ε_t in equation (6.1), are uncorrelated. In equation (6.1), the coefficient of interest is γ. If γ=0, Y_t has a unit root. The presence of a unit root is tested using the Dickey-Fuller statistics. The appropriate statistic to use depends on the deterministic components included in the regression equation (see Dickey and Fuller 1981). If the coefficients of a differenced equation sum to 1, at least one characteristic root is unity. Here, if $\gamma = 0$ then the system has a unit root. The null hypothesis of the unit root is given by $H_0{:}\gamma = 0$, and the alternative is $H_1{:}\gamma < 0$.

In this study the Phillips-Perron (PP) and the Park-Choi (PC) tests were also applied. Phillips and Perron (1988) proposed a modified version of the Dickey-Fuller test. The correction is designed to account for certain kinds of heteroskedasticity and serial correlation in ε_t. The following regression equations briefly explain the procedure:

$$Y_t = a_0^* + a_1^* Y_{t-1} + \mu_t \tag{6.2}$$

and

$$Y_t = \bar{a}_0 + \bar{a}_1 Y_{t-1} + \bar{a}_2 (t - T/2) + \mu_t \tag{6.3}$$

where

> T= number of observations and the disturbance term μ_t is such that $E_t\mu_t = 0$, but there are no requirements that the disturbance term is serially uncorrelated or homogeneous.

Unlike the Dickey-Fuller assumption of independence and homogeneity, the Phillips-Perron test allows the disturbances to be weakly dependent and heterogeneously distributed. Phillips and Perron characterise the distributions and derived tests statistics that can be used to test hypotheses about the coefficients a_i^* and $\bar{a}_i$ under the null hypothesis that the data are generated by

$$Y_t = Y_{t-1} + \mu_t \tag{6.4}$$

The tests are usually denoted $Z(\hat{\tau}), Z(\hat{\tau}_\mu), Z(\hat{\tau}_\tau), Z(\phi_1), Z(\phi_2$ and $Z(\phi_3)$. The ϕ_1, ϕ_2 and ϕ_3 statistics are constructed in exactly the same way as the ordinary F-tests. Likewise, the limiting distribution of $Z(.)$ is exactly the same as the Dickey-Fuller test, so the same critical values apply (Enders 1995).

The Park- Choi test aims to capture the presence of a stochastic trend by using time polynomials. Any integrated process involves a stochastic trend (s), the lag of the variable, and/or a deterministic trend (t). Since the stochastic trend (s) has the same properties as the time polynomial, that is, heteroschedastic and non-constant mean, it is possible to replace the stochastic part by time polynomials such that $s = \alpha_j t^j + \ldots + \alpha_p t^p$, where j should be greater than the degree of time trend included in the initial model. Therefore, if the stochastic trend is present in the integrated model, the time polynomials should be significant. The PC test involves two OLS regressions:

$$\text{(i)} \quad z_t = \alpha_0 + \alpha_1 t + e_t \tag{6.5}$$

(ii) $z_t = \alpha_0 + \alpha_1 t + \alpha_2 t^2 + \alpha_3 t^3 + \alpha_4 t^4 + \alpha_5 t^5 + e_t$ (6.6)

The test statistic for a typical random walk with a drift and trend is $J_2(1,5)$ = (RSS1-RSS2) / RSS2, where RSS1 and RSS2 are the respective residual sum of squares from the first and second regressions. The relevant critical value of five per cent significance level, for the statistics is 0.295 (for a random walk with a drift and a trend), and hence a calculated $J_2(1,5$ value greater than this critical value indicates the presence of a unit root.

6.2.2 Cointegration

6.2.2.1 The concept of cointegration

The concept of cointegration was introduced by Granger (1981) and was made popular by the work of Engle and Granger (1987). The idea of cointegration is based on the concept of equilibrium as discussed in economics. In the long-run, economic variables tend to move together, reaching a particular stationary point; when they move away from that point, market forces or government intervention push them back to equilibrium. Examples of this kind of co-movement may be the relationships between income and expenditure, money supply and prices, and exports and imports. According to Engle and Granger (1987), the formal definition of cointegration is best expressed by the following theorem.

> ***Theorem***: The components of vector Z_t are said to be cointegrated of order *d*, *b*, denoted by $Z_t \sim CI(d,b)$, if there exists a vector $\beta(\neq 0)$ such that $E_t = Z_t\beta \sim I(d-b)$, $b > 0$, and given that all components are integrated of order *d*. The vector β is called the cointegrating vector, and E_t is the disequilibrium error.

Since economic series are mostly I(1) series, take the case of d=1 and b=1. In such a situation, the equilibrium error will be I(0), and if E_t has a zero mean, it cannot move too far away from the origin and will cross the point frequently. This means that the components of Z_t are always converging towards the equilibrium, and thus the value of E_t will be as small as economic theory suggests. When E_t is not I(0), it will always move apart from zero and in that case the concept of equilibrium is violated (Engle and Granger 1987).

6.2.2.2 The Johansen's approach to cointegration

The methodology used in this study was developed by Johansen (1988) and Johansen and Juselius (1990). This procedure is well known, thus the explanation is mainly designed to set up the notation.

Suppose the vector of p-variables, $Z_t = (Z_{1t},....Z_{pt})'$, is generated by the k-order vector autoregressive process with Gaussian errors

$$Z_t = A_0 + A_1 Z_{t-1} + ... + A_k Z_{t-k} + D_t + e_t, \qquad t = 1,....,T \qquad (6.7)$$

where Z_t is a $p \times 1$ vector of stochastic variables, $e_1,.....,e_T$ are i.i.d. $N \sim (0, \Sigma)$ and D_t are centered seasonal dummies, and μ is a vector of constants. Since the object is to distinguish between stationarity by linear combinations and by differencing, the process may be written in error correction form as

$$\Delta Z_t = \Gamma_1 \Delta Z_{t-1} + ... + \Delta Z_{t-k+1} + \Pi Z'_{t-k} + \mu + \psi D_t + e_t \qquad t = 1,....,T \qquad (6.8)$$

The matrix Π contains the long-run information in the system and is analogous to the error correction representation of Engle and Granger (1987). Information about the number of cointegrating vectors is found in the rank of Π. Denote rank of Π as $r < p$. If there exists a representation of Π such that $\Pi = \alpha\beta'$, where α and β are both $p \times r$ matrices, the matrix is called a cointegrating matrix, and has the property $\beta' Z_t \sim I(0)$, where $I(d)$ indicates "integrated of order d". Thus the relationship $\beta' Z_t'$ can be interpreted as the stationary relations among potentially nonstationary variables, that is, as cointegrating relations. Johansen (1988) and Johansen and Juselius (1990) developed a maximum likelihood estimation procedure for $\mu, \Gamma_i, \alpha, \beta$ and Σ. They also provided tests for a number of cointegrating vector. For those wanting an intuitive explanation, notice that the Johansen procedure is nothing more than a multivariate generalisation of the Dickey Fuller test (Enders 1995). In addition, Johansen's method does not require that all variables are I(1). His method performs a cointegration analysis using a mixture of I(1) and I(0) variables.

6.2.3 Test for exogeneity

The error correction formulation given by equation (6.8) is used as the basis for testing for weak exogeneity among variables. In this study, an investigation was carried out to determine which of the macroeconomic variables are exogenous. The idea of defining exogeneity for a given set of parameters of interest is central to the concept proposed by Engle, Hendry and Richard (1983). Generally, a variable X_t can be regarded as weakly exogenous for a set of parameters of interest, say θ, if the marginal process for X_t contains no useful information for the estimation of θ. This means that the inference for θ can be efficiently made conditionally on X_t alone, and its marginal process contains no relevant information. The concept can also be formulated in reverse. The alternative definition reads - X_t is weakly exogenous for the parameters of interest θ, if knowledge of θ is not required for inference on the marginal process of X_t (see Spanos 1986, pp 376 and 421-422).

To understand the exogeneity test in the context discussed above, consider equation (6.5). Following Juselius (1991), he vector $Z_t = (Y_t, X_t)'$ is partitioned, where Y_t denotes the endogenous variables and X_t denotes the potential set of weakly exogenous variables. Assuming further that k=2, equation (6.5) becomes

$$\begin{bmatrix} \Delta Y \\ \Delta X \end{bmatrix} = [\Gamma_1] \begin{bmatrix} \Delta Y \\ \Delta X \end{bmatrix}_{t-1} + \begin{bmatrix} \alpha_{11} & \alpha_{12} \\ \alpha_{21} & \alpha_{22} \end{bmatrix} \begin{bmatrix} \beta_{11} & \beta_{12} \\ \beta_{21} & \beta_{22} \end{bmatrix} Z_{t-2} + \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} + \theta D_t + \begin{bmatrix} \varepsilon_1 \\ \varepsilon_2 \end{bmatrix} \quad (6.6)$$

Using equation (6.6), it can be inferred that the condition for $\{X_t\}$ to be weakly exogenous is $\alpha_{2.} = \alpha_2 = (\alpha_{21} = \alpha_{22}) = 0$. This suggests that the equations for X_t do not contain information about the long-run parameters β. Likewise, this also means that the long-run parameters β can be estimated efficiently without the equation for ΔX_t. Thus, testing for exogeneity of a particular variable comes to testing whether the corresponding row of α is zero. The results of the exogeneity test are discussed in Chapter 7.

6.2.4 Impulse response analysis

Impulse response or dynamic multiplier analysis is a common tool for investigating the interrelationships among the variables in dynamic models. The tool is also valuable in cointegrated systems (Lütkepohl and Reimers 1992 p. 54). In such systems, it is assumed that although the individual variables are non-stationary, there are linear combinations of them which are stationary. These linear combinations are often interpreted as long-run equilibrium relations. It is assumed that deviations from equilibrium relations are stationary.

In applied work it is often of interest to know how one variable will respond to a shock in another variable; especially when a system involves a number of variables. Thus, one would like to investigate the relationship between two variables in a higher dimensional system. If there is a reaction by one variable to an impulse in another variable, it can be said that the latter is the cause of the former. This type of causality was established between g_{3t}^{w} and the other variables by tracing out the effect of an exogenous shock in one of the variables on all of the other variables. This kind of impulse response is often called multiplier analysis.

Impulse response analysis was usec to determine the effect of an exogenous shock in the ratio of government defensive e:penditure to GNE (g_{3t}^{w}) in a system containing the ratio of government expenditure o1 goods and services to GNE (g_{1t}^{w}), the ratio of government investment expendit re to GNE (g_{2t}^{w}), the ratio of consumption expenditure to GNE (c_t^{w}), the ratio of investment to GNE (i_t^{w}) and the leisure-labour ratio (x_t^{w}). To isolate the effect of g_{3t}^{w}, it is supposed that all six variables assume their shock mean prior to $t=0$, $g_1^{w} = \mu_1, g_2^{w} = \mu_2$ .... *etc* for $t< 0$ and g_{3t}^{w} increases by one unit in period $t=0$ that is $\varepsilon_{g_{3,0}} = 1$. With these assumptions, it is then possible to trace out what happens to the syst:m during periods t=1,2 .... if no further shocks occur, that is, when $\varepsilon_{g_{3,1}} = \varepsilon_{g_{3,2}} = \varepsilon_{g_{3,3}} = ... = 0$. Further, it is assumed that all six variables have a zero shock mean. A detailed discussion of this form of analysis can be found in Lütkepohl (1990) and L itkepohl and Reimers (1992).

After determining the response of se lected macroeconomic variables to changes in the ratio of government defensive expenditure to GNE (g_{3t}^{w}), the dynamic relationships within the system were depicted graphically. In instances where the variables have different scales, it is sometimes useful to consider changes in one standard deviation rather than unit shocks. In this stucy, the units of change were expressed in standard deviations. This is graphically illustrated by having units in the vertical axes equal to the standard deviations of the resid ıals corresponding to the variables whose effects are considered. Such rescaling may sometimes give a better picture of dynamic relationships because the average size of a change occurring in a system depends on their standard deviation (Lütkepohl .990).

An impulse response is said to be zero if one of the variables does not Granger-cause the other variables taken as a group. More precisely, an increase in government defensive expenditure (g_{3t}^{w}) has no e fect on other variables if the former variable does not Granger-cause the set of the remaining variables. As previously mentioned, in applied work it is often of foremost interest whether one variable has an impact on another variable. That is, one would like to know whether, for some

$g_{3t}^{w} \neq g_{1t}^{w}, \phi_{g_{3t}^{w} g_{1t}^{w}, i} = 0$ for i=1,2..... If $\phi_{g_{3t}^{w} g_{1t}^{w}, i}$ represents the actual reactions of variable g_{1t} to one standard deviation shock in g_{3t}^{w}, then the latter is called noncausal for the g_{1t}^{w}th variable if $\phi_{g_{3t}^{w} g_{1t}^{w}, i} = 0$ for i=1,2....

6.3 Imputing for the Value of Leisure

Attempts to value leisure on a national scale will now be discussed. They are also classified in Table 6.1. This classification is based on the concepts of leisure hours and leisure price. The various attempts (I to V) are presented consecutively.

Estimate I, which uses average wage and total quantity of leisure, is calculated by

(a) multiplying total leisure hours per year by the average wage rate per hour, ignoring unemployment and
(b) adding value of leisure computed in (a) to conventional GNE (Y^t) to get the adjusted GNE ($\hat{Y}^t$).

Estimates IIa and IIb are computed in the same manner but the leisure of the unemployed is included. The value of the leisure of the unemployed is taken as the prevailing average wage rate per hour for the period. In computing estimate IIa, it is assumed further that the growth rate of the productivity of leisure is in proportion to the growth of real wage. In contrast, for estimate IIb, it is assumed that the growth of the productivity of leisure was constant over time.

Table 6.1
Procedures for imputing for the Value of Leisure

Concept of leisure hours	Concept of leisure value	
	Average Wage	Marginal Wage
Total Quantity of Leisure	*(Estimate I)* without leisure of the unemployed *(Estimate IIa)* with leisure of the unemployed (productivity of leisure constant overtime) *(Estimate IIb)* with leisure of the unemployed (productivity of leisure growing at the same rate as real wage)	*(Estimate IVa)* using the actual marginal wage *(Estimate IVb)* using the computed marginal wage
Marginal Quantity of Leisure	*(Estimate III)* using actual marginal quantity of leisure	*(Estimate Va)* using the actual marginal wage *(Estimate Vb)* using the computed marginal wage

Estimate III employs the marginal quantity of leisure and the average wage as the unit value of leisure. These estimates is determined by

(a) computing the extra, or marginal, number of hours of leisure from year *n* to year *n+1* by subtracting total leisure hours of year *n* from year *n+1;*

(b) multiplying the marginal leisure hours in *n+1* by the average wage rate per hour in year *n+1,* and

(c) adding the total leisure value from (b) to the reported GNE to calculate the adjusted GNE value.

Estimates IVa and IVb are derived by

(a) multiplying the total quantity of leisure per year and the marginal wage per hour, and

(b) adding the leisure value derived in (a) to the unadjusted GNE.

On the other hand, estimates Va and Vb are computed by

(a) multiplying the marginal quantity of leisure and the marginal wage to calculate the leisure value, and

(b) the leisure values from (a) is then added to the conventional GNE.

For estimates IV and V, the marginal wage rate for a given year is calculated as the change in total wages divided by the change in work hours between years. These computations are labelled 'actual marginal wage'. Initial calculations reveal that the actual values for the marginal wage, varied between \$1 and \$2 032 per hour. The large difference in the size of the marginal wage computations of \$1 and \$2 032 per hour can partly be explained by the variation in increases in the number of persons employed. The number of persons employed fluctuates depending on whether the economy is experiencing a boom or recession. In periods of recessions, the tendency is for the number of persons employed to fall and in periods of economic boom for it to rise. Likewise in periods of rapid economic activity, total wage increases faster than number of persons employed. There are also some negative values is some years, because the number of working hours or the total wages, occasionally decrease between years. Because of the unlikely value of $2 032 per hour and the negative values, another way of computing for the marginal wage rate is explored. In an attempt to obtain a generalised procedure to assess marginal wage, a model that relates the marginal wage to selected variables is developed. The model is specified as:

$$MW = f\,(E,\ WR,\ WH,\ GR) \tag{6.7}$$

where

MW = marginal wage rate per week

E = total number of persons employed

WR = average wage rate per week

WH = average number of weekly working hours per person

Variable GR is the percentage growth rate of the economy per quarter and is calculated as $\left[(GNE_{t+1} - GNE_t)/GNE_t\right] \times 100$. This variable is included to represent technological changes, policy changes and trends in the economy.

As discussed earlier, total wages and total working hours are the information needed to calculate marginal wages. The variables specified for equation (6.7) are variables that directly affect either total wages or total working hours. Equation (6.7) is estimated to derive a model for the marginal wage for men (*MWM*), the marginal wage for women (*MWW*) and the marginal wage for both (*MW*). The expected sign of variable *WH* (number of weekly working hours) is negative. Using the marginal productivity argument, the marginal wage rate should decrease as the number of work hours increases. A negative sign is also envisioned for variable *E* (number of employed persons). Firms demand labour up to a point where the marginal wage is equal to the marginal revenue product of labour. As the number of employed persons increases, marginal wage rate should decrease. The sign for the variable *GR* (growth rate) is expected to be positive. Likewise, a positive sign is also anticipated for *WR* (wage rate) because marginal wage should decrease with average wage.

As explained by Usher (1980), a true measure of economic growth would include increases or decreases in leisure since the base year. An improvement in the individual's welfare cannot be measured solely by the total quantity of leisure, but should also include the value of any changes in leisure time. Conceptually, the marginal wage measures the wage per additional hour of discretionary work, and so is an appropriate value of leisure. The individual's decision to undertake extra work or not is influenced by the additional wage he gets by working an extra hour. For these reasons, estimates IV and V are theoretically relevant, and Usher (1980) attempted to implement estimate V.

In this study, attempts are made to implement and compare all of the five procedures discussed. There are no previous studies undertaken in Australia using these valuations, probably because, as Usher says, it is difficult to measure the marginal wage. In this study, two ways of measuring the marginal wage are attempted. These are the actual arithmetic average and the smoothed regression-derived estimate.

6.4 Solving for the Environmental Variables

The third stage of analysis for this study was to solve for environmental disturbances variables (ε_t). But before the value of (ε_t) is determined, there is a need to first solve for the $g(G_{31t}, G_{32t})$ function. This function is greater than 1 if the government's environmental program is successful, and is less than 1 if the government's environmental function is not successful.

6.4.1 Solving for $g(G_{31t}, G_{32t})$

Since no studies have been done on government defensive expenditure in Australia, the nature of the function $g(G_{31t}, G_{32t})$ is not known. To solve for $g(G_{31t}, G_{32t})$, the Adjusted Model (1) presented in Chapter 4 is used. From equations (4.38a) and (4.38b), at equilibrium

$$\frac{\phi_2}{1-n_t} = \frac{\phi_1 g(G_{31t}, G_{32t}) \alpha n_t^{\alpha-1} k_t^{1-\alpha}}{C_t^w} \quad (6.8)$$

$$\therefore g(G_{31t}, G_{32t}) = \frac{\phi_2 C_t^w n_t}{\phi_1 \alpha n_t^{\alpha} k_t^{1-\alpha} (1-n_t)} \quad (6.9)$$

However, the value of ϕ_1 and ϕ_2 are unknown. Therefore, to solve for the $g(G_{31t}, G_{32t})$ function, equation (4.38d) was used. At equilibrium

$$\begin{aligned} C_t^w + I_t^w = g(G_{31t}, G_{32t})(n_t^{\alpha} k_t^{1-\alpha}) - [(1-\theta_1)G_{1t} + (1-\theta_2)G_{2t} \\ + (1-\theta_{31})G_{31t} + (1-\theta_{32})G_{32t}] \end{aligned} \quad (6.10)$$

$$\therefore g(G_{31t}, G_{32t}) = \frac{C_t^w + I_t^w + (1-\theta_1)G_{1t} + (1-\theta_2)G_{2t} + (1-\theta_{131})G_{31t} + (1-\theta_{32})G_{32t}}{n_t^{\alpha} k_t^{1-\alpha}}$$

The parameters were then classified into known and unknown variables. The unknown parameters were

θ_1 = degree of substitutability between C_t and G_{1t}

θ_2 = degree of substitutability between I_t and G_{2t}

θ_{31} = degree of substitutability between C_t and G_{1t}

θ_{32} = degree of substitutability between I_t and G_{2t}

α = production coefficient of labour

Since there are several unknown parameters, the simultaneous equation procedure was employed using SHAZAM. The same technique was used by Tobin (1984) and Barro (1989). The equations used to derive the unknown theta parameters are as follows:

$$C_t^w = C_t + \theta_1 G_{1t} + \theta_{31} G_{31t} \tag{6.11a}$$

$$I_t^w = I_t + \theta_2 G_{2t} + \theta_{32} G_{32t} \tag{6.11b}$$

$$\begin{aligned} GNE = {} & C_t^w + I_t^w + (1-\theta_1)G_{1t} + (1-\theta_2)G_{2t} \\ & + (1-\theta_{31})G_{31t} + (1-\theta_{32t})G_{32t} \end{aligned} \tag{6.11c}$$

$$G = G_{1t} + G_{2t} + G_{31t} + G_{32} \tag{6.11d}$$

Likewise, the parameter α was determined by regressing

$$\log GNE = \alpha \log n_t + (1-\alpha) \log k_t \tag{6.12}$$

After all the unknown parameters were determined, the value of $g(G_{31t}, G_{32t})$ was computed for each quarter.

The labour (n_t) values were taken from *ANA Gross Product, Employment and Hours Worked, ABS Catalogue No. 5222.0*. The capital (k_t) values refer to gross fixed capital expenditure and increases in inventories. Although there are controversies regarding the estimation of capital for production (see Walters and Dippelsman 1985), the conventional measure of capital was used in this study.

6.4.2 Solving for ε_t

Following the derivation presented in Chapter 4, at equilibrium

$$\frac{\phi_2}{1-n_t} = \frac{\phi_1 \varepsilon_t \alpha n_t^{\alpha-1} k_t^{1-\alpha}}{C_t^n} \tag{6.13}$$

$$\therefore \varepsilon_t = \frac{C_t^n \phi_2 n_t}{\phi_1 \varepsilon_t \alpha n_t^\alpha k_t^{1-\alpha} (1-n_t)} \tag{6.14}$$

Since no information is available on the probable values of ϕ_1 and ϕ_2 for Australia, equation (4.16d) was instead used to derive the value of the parameter ε_t. At equilibrium the relationship between consumption and income using the Neo-classical framework of income approach is

$$\varepsilon_t (n_t^\alpha k_t^{1-\alpha}) = C_t^n + I_t^n + (1-\theta_1)G_{1t} + (1-\theta_2)G_{2t} \tag{6.15}$$

$$\therefore \varepsilon_t = \frac{C_t^n + I_t^n + (1-\theta_1)G_{1t} + (1-\theta_2)G_{2t}}{n_t^\alpha k_t^{1-\alpha}}$$

The total output with government defensive expenditure (Y_t^w) is formulated as

$$Y_t^w = g(G_{31t}, G_{32t}) n_t^\alpha k_t^{1-\alpha} \tag{6.16}$$

while the output without governmen defensive expenditure function (Y_t^n) is

$$Y_t^n = \varepsilon_t n_t^\alpha k_t^{1-\alpha} \tag{6.17}$$

where in both equations $Y_t = n_t^\alpha k_t^{1-\alpha}$. Since the function $g(G_{31t}, G_{32t})$ was determined previously, the output level (Y_t) can now be solved using the equation

$$Y_t = \frac{Y_t^w}{g(G_{31t}, G_{32t})} \tag{6.18}$$

In a similar manner, the unknown variable effective consumption C_t^n without government defensive expenditure is determined using the equations:

$$C_t^n = C_t + \theta_1 G_{1t} \quad (6.19)$$

(without government defens ve expenditure)

$$C_t^w = C_t + \theta_1 G_{1t} + \theta_{31} G_{31t} \quad (6.20)$$

(with government defensive expenditure)

Together, equation (6.19) and (6.20) give

$$C_t^n = C_t^w - \theta_{31} G_{31t} \quad (6.21)$$

Likewise, I_t^n was derived using the same process. Since all the parameters needed to solve for the value of ε_t are now known, the next task is to solve for the value of ε_t for Australia. The results are detailed in Chapter 9.

6.5 Integration of the Changes

This section discusses the various ways of presenting changes in leisure and the environment in one account. In the succeeding analyses, it is assumed that the utility function is additive and that GNE can further be adjusted by adding the imputed value of leisure to the Unadjusted GNE or Adjusted GNE.

6.5.1 Unadjusted GNE (1)

It is of interest to policy makers and the general public to know what happens to the economy's output if the government does not spend on the environment. The derived value of the parameter ε_t was used to adjust the national accounts for environmental changes. This is to determine the effect of changes in the environment, represented by ε_t, on the national accounts. Output values with and without the environmental variable were compared. The growth rate for each measure was also computed. A detailed discussion of the results is given in Chapter 9.

6.5.2. Unadjusted GNE (2)

After the national accounts have been adjusted for changes in the environment (Y_t^n), the imputed value of leisure using Estimate 1 was added to it. The revised account was then estimated for 1962:3 to 1991:2. This account is referred to as Unajusted GNE (2) in the study. This method shows the impact of changes in leisure on the national accounts. This account integrates changes in leisure and the environment when the government spends nothing on the environment.

6.5.3. Adjusted GNE (1)

Adjusted GNE is equivalent to reported GNE. The term adjusted is used to describe a situation where some form of mechanism has been in place to correct environmental damages. In this study, the mechanism refers to government defensive expenditure. This method hopes to determine whether government programs for the environment are having some positive impact on the economy's productive capacity. The value of $g(G_{31t}, G_{32t})$ for each quarter was calculated and discussed in Chapter 9.

6.5.4. Adjusted GNE (2)

Again, the imputed value of leisure using Estimate 1 is added to Adjusted GNE (1). The purpose of this exercise is to determine what happens if the government continues to spend on environmental programs. Further, the inclusion of leisure will show whether the individual consumer's welfare has improved or not, using the rough estimate of Adjusted GNE (2).

6.6 A Summary

This chapter presented the three stages of the study. Each stage uses a different method of analysis. The first stage is to determine the macroeconomic relationships between the ratio of government defensive expenditure to GNE (g_{3t}^{w}) and selected variables. To establish the nature of the variables, a unit root test is conducted. A cointegration test is also run to determine the existence of long-run relationships. Likewise, impulse response analysis is carried out to ascertain the impact of government defensive expenditure on identified variables.

Next, the value of leisure is calculated. All five of the methods of imputation discussed in the literature are attempted in the study. Before the third stage could be carried out, the values of the parameters (ε_t) and $g(G_{31t}, G_{32t})$ need to be determined using the models derived in Chapter 4. Finally, changes in the environment and leisure is integrated in one account.

7. Empirical Results 1: MACROECONOMIC RELATIONSHIPS

We found everywhere deep public concern for the environment....... The challenge is to ensure that these new values are more adequately reflected in the principles and operations of the political and economic structure.

(World Commission on the Environment 1990)

7.1 Introduction

Defensive expenditure was defined in Chapter 3 as the additional costs arising from the pursuit of income, production and consumption, related to economic activity. Expenditure that corrects the deterioration of living and environmental conditions or attempts to prevent deterioration is an additional monetary expense incurred to achieve positive production returns. Thus, the purpose of this chapter is to establish the relationship between the environmental component (government defensive expenditure) and macroeconomic variables like effective private consumption expenditure, effective private investment, government consumption expenditure on goods and services, government investment and the leisure-labour ratio.

The empirical work is presented as follows: Section 7.2 discusses tests for the order of integration. Section 7.3 examines the adequacy of the model. . Section 7.4 details the exogeneity test results. Section 7.5 identifies the rank of cointegration. Section 7.6 describes the short-run dynamics. Section 7.7 reports on the cointegrating relationships. Section 7.8 presents the impulse response analysis. Section 7.9 evaluates the models presented. Finally, Section 7.10 offers some conclusions.

7.2 Tests for the Order of Integration

It is important to determine the nature of the variables, especially when dealing with time series observations. If the variables are non-stationary, then the coefficient of estimates based on ordinary least squares (OLS) regressions will be biased and inconsistent. Thus, unit root tests were performed to determine whether the variables government consumption expenditure on goods and services G_{1t}, government investment expenditure G_{2t}, government defensive expenditure G_{3t}, effective investment I_t^w, effective consumption expenditure C_t^w and hours of leisure L_t^w are stationary.

The unit root tests were performed sequentially and the results are presented in two parts. The first part presents tests of stationarity of the levels of the time series while the second part reports on the results on first differences of the variables.

7.2.1. Unit root tests on the levels of the variables

The variables of government consumption expenditure on goods and services G_{1t}, government investment expenditure G_{2t}, government defensive expenditure G_{3t}, effective investment I_t^w and effective consumption expenditure C_t^w were transformed into ratios of GNE. The variable hours of leisure L_t^w was divided by hours of labour N_t^w to derive the variable x_t^w (leisure-labour ratio). Tests for unit roots were performed on all data using the Augmented Dickey-Fuller (ADF), Park-Choi (PC) and Phillips-Perron (PP) tests. The null hypothesis states that the variables under investigation have a unit root, while the alternative is that they do not. The results of the tests are presented in Table 7.1.

The first column of Table 7.1 reports on tests of stationarity of the levels of the time series using the Augmented Dickey-Fuller (ADF) with three lagged differences. The critical values of the test statistic (t_1) are tabulated in Fuller (1976) and discussed in

Dickey and Fuller (1979). The reported test statistics indicate that the null hypothesis cannot be rejected for any of the variables besides c_t^w. The computed value of -2.78 is less than the critical value of -2.57, thus the null hypothesis of non-stationarity is rejected. The variable c_t^w is stationary.

On the contrary, the PC unit root test shows that the null hypothesis of non-stationarity is not rejected. At 10 and 5 per cent significance levels for the PC, the null hypothesis of non-stationarity is not rejected. This implies that the variables are non-stationary (ie. with a unit root), may be at least I(1) processes, and need to be differenced at least once to become stationary.

Table 7.1

Results of the Augmented Dickey-Fuller (ADF), Park-Choi (PC) and the Phillips-Perron (PP) Unit Root Tests at Level

Variable	ADF	PC	PP
g_{3t}^w (government defensive expenditure/GNE)	-2.25	1.76	-2.23
g_{1t}^w (government final consumption expenditure/GNE)	-1.59	10.66	-1.60
g_{2t}^w (government investment expenditure/GNE)	-1.95	4.31	-2.28
i_t^w (private investment/GNE)	-1.59	11.11	-1.59
c_t^w (private consumption expenditure/GNE)	-2.78*	0.70	-2.79*
x_t^w (leisure/labour)	-1.81	13.06	-1.81

The critical value at the 10% significance level is -2.57 for the PP test (Phillips-Perron 1988) and ADF test, which corresponds to the unit root with constant without trend at level. A test value less than the critical value indicates stationarity. The critical value at the 5% level is 0.33 for the PC test (Park and Choi 1988), which corresponds to the unit root without trend. Calculated value greater than the critical value indicates the presence of a unit root.

A closer examination of the table reveals that all three tests indicate that the calculated values, except for the PP and ADF tests on effective consumption—GNE ratio c_t^w, are greater than the critical values. Truncation lags of one, three, five and seven are used in the PP unit root tests for each variable. The use of different truncation lags has been conventionally adopted to determine whether the results are uniform across different lag lengths. Note that the reported test-statistics for the PP unit root tests are those of $Z(t_{\tilde{\alpha}})$ with three truncation lags only (Perron 1988). Detailed results of the PP unit root tests are presented in **Appendix C**. These results indicate that, except for the ratio of effective consumption to GNE, c_t^w, the decision were consistent regardless of the length of the truncation lag. The null hypothesis of non-stationarity is accepted. With regards to c_t^w, as the truncation lag increases the null hypothesis of non-stationarity is accepted.

While unit root tests based on the ADF, PC and PP for the variable c_t^w are somewhat inconsistent with regards to its stationarity, results of the ADF and PP unit root tests were adhered to. This is because, two out of three tests gave the same conclusion.

7.2.2. Unit root tests on first differences

To determine the level of integration, unit root tests for first differences of the variables were undertaken. The results of these tests are summarised in Table 7.2.

A close look at Table 7.2 reveals that the null hypothesis of a unit root is rejected for all the time series using differenced data. These results are broadly consistent with the hypothesis that the individual time-series are individually I(1) except for c_t^w which is I(0). Because these data appear to be stationary in first differences, no further tests were performed.

Table 7.2

Results of the Augmented Dickey-Fuller (ADF), Park-Choi (PC) and the Phillips-Perron (PP) Unit Root Tests on Fist Differences

Variable	ADF	PC	PP
g_{3t}^{w} (government defensive expenditure/GNE)	-6.06*	0.04*	-10.22*
g_{1t}^{w} (government final consumption expenditure/GNE)	-5.60*	0.11*	-11.59*
g_{2t}^{w} (government investment expenditure/GNE)	-6.52*	0.15*	-12.67*
i_t^{w} (private investment/GNE)	-5.58*	0.11*	-11.50*
c_t^{w} (private consumption expenditure/GNE)	-6.00*	0.12*	-10.54*
x_t^{w} (leisure/labour)	-5.75*	0.13*	-10.30*

The critical value at the 10% significance level is -2.57 for the PP test (Phillips-Perron 1988) and the ADF test, which corresponds to the unit root with constant without trend at level. A test value less than the critical value indicates stationarity. The critical value at the 5% level is 0.33 for the PC test (Park and Choi 1988), which corresponds to the unit root without trend. A calculated value greater than the critical value indicates the presence of a unit root.

7.3. Testing the Adequacy of the Model

Let $Z_t = \left[g_{1t}^{w}, g_{2t}^{w}, g_{3t}^{w}, i_t^{w}, c_t^{w}, x_t^{w}\right]'$, where g_{1t}^{w} is the ratio of government consumption expenditure to GNE; g_{2t}^{w} is the ratio of government investment to GNE; g_{3t}^{w} is the ratio of government defensive expenditure to GNE; i_t^{w} is the ratio of effective consumption to GNE; c_t^{w} is the ratio of effective consumption to GNE; and x_t^{w} is the ratio of leisure to labour. The effective sample size is 116 quarterly observations from 1962:3 to 1991:2, as discussed in Chapter 5.

7.3.1 The univariate statistics

In choosing the specification of the VAR model as in equation (6.7) which is:

$$Z_t = A_0 + A_1 Z_{t-1} + \ldots. A_k Z_{t-k} + \mu + \psi D_t + e_t \qquad t = 1,\ldots\ldots.T \tag{7.1}$$

it is necessary to select the number of lags in the autoregressive specification. Constant and quarterly dummies are included although seasonal effects seem to be very small. After some experimentation, a lag length for the VAR of k = 5 has been settled upon. The diagnostic statistics for the residuals of the model are presented in Tables 7.3 and 7.4.

The Q statistic, based on statistical results obtained by Box and Pierce (B-P), has been applied. The null hypothesis is that the residuals are white noise. In all cases the Q statistic had a value smaller than the critical value of 21.03. Therefore, we fail to reject the null hypothesis and conclude that residuals are white noise.

Traditional econometric models assume a constant one-period forecast variance. To test this assumption, there is a need to test for the presence of ARCH (auto-regressive conditional heteroscedastic) in the residuals. This test is based simply on the autocorrelation of the squared OLS residuals. The existence of an ARCH effect can be interpreted as evidence of misspecification, either by omitted variables or through structural changes. Test results indicate that the disturbance are not conditionally heteroscedastic.

R^2 figures are given in column 4 of Table 7.3. The R^2 measures the 'goodness of fit'. Based on the results, it can be concluded that most of the variables were able to explain 40 per cent of their own variations.

Table 7.3:

Results of Residual Analysis

Variable	B-P Q(1)	ARCH(2)	R^2
g_{3t}^{w}	11.36	0.81	0.40
g_{1t}^{w}	20.45	3.14	0.51
g_{2t}^{w}	16.42	2.68	0.38
i_t^{w}	12.97	2.56	0.50
c_t^{w}	17.07	2.97	0.49
x_t^{w}	11.83	16.66	0.44

*Critical value at 95% level $\chi^2(5)$ = 11.07 , $\chi^2(12)$=21.03.
(1)The Box-Pierce Q Statistics sum 17 autocorrelations, and degrees of freedom are given by the number of autocorrelations summed minus the order of the AR(5) process, namely 12.
(2)ARCH (*f*) is a test statistic for autoregressive heteroscedasticity appropriately distributed as $\chi^2(f)$, where *f*=4 in this case.

The literature on testing for normality is vast. Three of the most commonly used measures of deviations from normality are the skewness (SKEW), kurtosis (KURT) and the J-B Normality tests. Results of these tests are presented in Table 7.4.

Consider the usual linear model $y = X\beta + e$ where $E[\mathbf{e}] = 0$ and $E[\mathbf{ee}'] = \sigma^2 I$. If in addition, ($e$) is normally distributed, then the third and fourth moments for an element in (e) are given by

$$\mu_3 = E[e_t^3] = 0 \tag{7.2}$$

and

$$\mu_4 = E[e_t^4] = 3\sigma^4 \tag{7.3}$$

Table 7.4

Tests for Normal Errors

Variable	Skewness[1]	Kurtosis[2]	JB-Norm[3]
g_{3t}^{w}	1.24	8.42	10.81
g_{1t}^{w}	0.35	3.52	3.42
g_{2t}^{w}	-0.22	2.87	0.98
i_t^{w}	0.33	3.54	3.41
c_t^{w}	-0.94	4.12	4.28
x_t^{w}	0.81	8.11	16.66

* Critical value at 95% level $\chi^2(2)=5.99$
[1]Skew is the third moment around the mean.
[2]Kurtosis is the fourth moment around the mean.
[3]JB is the Jarque-Bera test statistic for normality distributed as $\chi^2(2)$.

A large number of tests for normality are based on how far estimates of the third and fourth moments, $\tilde{u}_3$ and $\tilde{u}_4$, deviate from 0 and $\tilde{\sigma}^4$, respectively, where $\tilde{\sigma}^2$ is an estimate of $E[e_t^2] = \sigma^2$. In this regard it is conventional to consider scaled versions of μ_3 and μ_4 that are known as measures of skewness and kurtosis respectively. The skewness measure is given by

$$b_1 = \frac{\mu_3}{\sigma^3} \tag{7.4}$$

The measure of kurtosis is

$$b_2 = \frac{\mu_4}{\sigma^4} \tag{7.5}$$

The skewness of a distribution refers to its degrees of symmetry (or lack of it), whereas the kurtosis of a distribution is indicated by the 'peakness' of the distribution and the thickness of its tail. The skewness (b_1) value ranges from -3 to +3. If the data are perfectly symmetric $b_1 = 0$ because the mean is equal to the median. If b_1 is positive, the mean is larger than the median, by implication, the data exhibit a pattern with a right tail. Similarly, a b_1 which is negative suggests that the data is skewed to the left. Since all of the skew values are within the -3 and +3 range, it implies that all of the identified variables have a non-skewed distributed error term.

The kurtosis values are relatively small which indicates that the frequency of observations close to the mean are high and the frequency of observations far from the mean are low. These results support the contention that the error terms are normally distributed in at least 4 of the variables in Table 7.4.

The Jarque-Bera Normality test is a joint test of whether or not the estimates of b_1 and/or $(b_2 - 3)$ are significantly different from 0. Since the error vector **e** is unobservable, estimates of b_1 and/or $(b_2 - 3)$ are based on least square residuals. Under the null hypothesis that the errors are normally distributed, the J-B normality test statistic has an asymptotic $\chi^2_{(2)}$ distribution and is given by

$$\begin{aligned} \lambda &= T\left(\frac{(b_1)^2}{6} + \frac{(\tilde{b}_2 - 3)^2}{24}\right) \\ &= T\left(\frac{\tilde{\mu}_3^2}{6\tilde{\sigma}^6} + \frac{(\tilde{\mu}_4 - 3\tilde{\sigma}^4)^2}{24\tilde{\sigma}^8}\right) \end{aligned} \tag{7.6}$$

Table 7.4 shows high values for g_t^w (ratio of government defensive expenditure to leisure) and x_t^w (ratio of leisure to labour) with respect to the J-B normality test. These results imply that errors of the two variables are not normally distributed. A few sharp movements in g_{3t}^w and x_t^w variables appear to be the cause of this problem (refer to **Appendix D**). Several k values have been tried, however, little improvement has been achieved with these different values.

The normality tests reject the null hypothesis that the errors g_{3t}^{w} and x_t^{w} are normally distributed. This result seems to be due to the presence of kurtosis, or fatter tails. Gonzalo (1994) has shown that the Johansen maximum likelihood estimator is relatively robust to the presence of fat tails. Thus, some kurtosis, in two of the variables should not affect the overall results of this study.

7.3.2 Multivariate statistics and information criteria

The multivariate residual analysis in this study is based on the output of RATS. Most of the statistics are standard output from many statistical package but some of the statistics require slight modifications to take account of the cointegration restriction. Detailed discussion of these tests is presented in CATS in RATS manual (Hansen and Juselius 1994).

To assess the performance of the whole system, the following tests are evaluated:

Test for 'Goodness of Fit'

INFORMATION CRITERIA:	SC	=	-59.54
	HQ	=	-61.79

The information criteria of Schwarts (SC) and Hannan-Quinn (HQ) statistics show the goodness of fit of the function. The Schwartz Criterion and those of Hannan-Quinn are used because they are most commonly used in empirical work. The two criteria involve a function of residual sum squared corrected for the number of parameters. Optional lag length is determined by minimising the function

$$SC = (RSS + K \log T\delta^{2}) / T \tag{7.7}$$

where K is the number of regressors and T is the number of observations. The SC always chooses the lag length which is not bigger than that chosen using the Akaike Information Criteria (AIC), as SC puts heavier penalty on the number of parameters.

Likewise the equation

$$HQ = n \log s(s^2) + 2k \log(\log n) \quad (7.8)$$

is also minimised. The parameter s^2 represents the estimated variance of the residuals, k is the total number of the estimated parameter and n is the number of estimations used in estimating the ARMA model.

These two criteria were used to determine the final lag length involved in cointegration. The purpose of these tests is to avoid the bias associated with an arbitrary choice of lag length. After several experimentation the VAR of k=5 was found to be the lag length that minimises the values of SC and HQ.

The possibility of residual auto correlation was checked using three different LM-type tests. Results were as follows:

Test for Autocorrelation

L-B(27)	CHISQ(570)	=	598.07	p-val=0.20
LM(1),	CHISQ(36)	=	45.66	p-val=0.13
LM(4),	CHISQ(36)	=	30.51	p-val=0.73

The first test is a multivariate Ljung-Box Test based on the estimated auto-and cross correlation of the first $\left[T/4\right]$ lags (Hosking 1980). The Ljung-Box Q-statistic is calculated as

$$Q = \frac{T(T+2)\sum_{k=1}^{s} r_k^2}{T-k} \quad (7.9)$$

where

T = number of observations

s = degrees of freedom

r_k^2 = squared sample autocorrelation at lag k

If the sample value of Q calculated from equation (7.9) exceeds the critical value of χ^2, then at least one value of r_k is statistically different from zero at the specified significance level.

The second and third tests are LM-type tests for first and fourth order autocorrelations respectively. Since calculated p-values are larger than 5 percent in all three tests, the possibility of autocorrelation is rejected.

7.3.3 Eigenvalues of the companion matrix

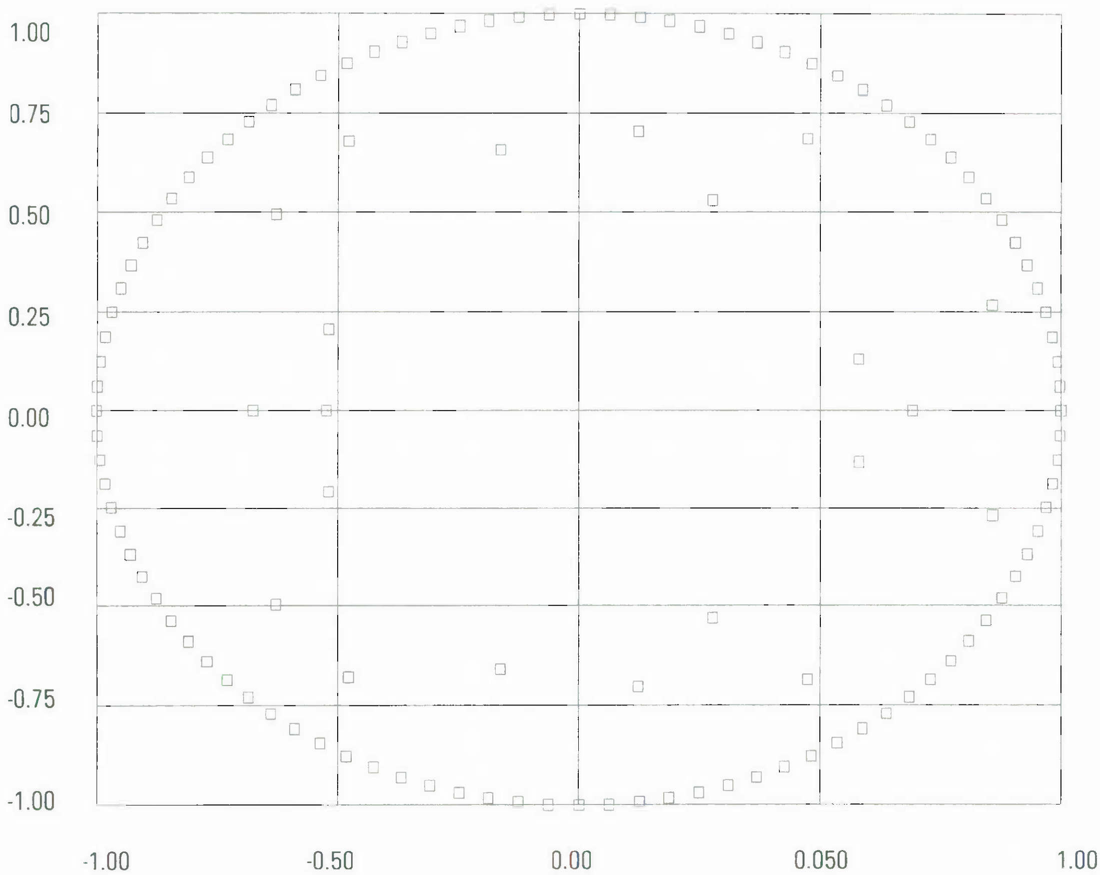
By investigating the roots of the companion matrix, one can get information about the $p \times k$ roots describing the dynamic properties of the process. The eigenvalues of the companion matrix is given by in which A_i is defined by equation (6.8) and I_p is the p-dimensional identity matrix (Hansen and Juselius 1994).

$$A = \begin{bmatrix} A_1 & A_2 & \cdots & A_{k-1} & A_k \\ I_p & 0 & \cdots & 0 & 0 \\ 0 & I_p & \cdots & 0 & 0 \\ \vdots & & \ddots & & \vdots \\ 0 & 0 & \cdots & I_p & 0 \end{bmatrix} \tag{7.10}$$

The estimated eigenvalues of A are the reciprocal values of the roots of the characteristic polynomial; hence, the eigenvalues should be inside the unit disc or equal to unity under the assumption of the cointegrated VAR-models given by equation (6.8). Eigenvalues outside the unit disc correspond to explosive processes, but looking at Figure 7.1, it can be seen that the described situation does not occur for this data. The number of stochastic trends in the model corresponds to the number of roots close to unity in the companion matrix. Figure 7.1 shows that the largest root is the only one on the unit circle. The result also supports the choice of k=5, since other lag lengths have eigenvalues outside the unit disk. The roots are presented graphically in Figure 7.1.

Figure 7.1

A Scatter Plot of the Eigenvalues of the Companion Matrix



7.4 Test for Exogeneity

A weak exogeneity test was conducted using an ECM with k=5 and 1 cointegrating vector. The hypothesis is that certain rows of α are zero. Since $\alpha_{i.} = 0$ implies that the cointegrating relations $\beta_j X_t$ do not enter equation *i*, this is in fact the test of weak exogeneity of Z_{it} when the parameters of interest are the long-run parameters in β. The exact form of the test statistic is discussed in detail in Juselius (1991).

From Table 7.5, it can be concluded that the ratio of effective consumption expenditure to GNE (c_t^w) and the ratio of leisure to labour (x_t^w) are weakly exogenous, but the ratio of government investment expenditure to GNE (g_{2t}^w), the ratio of government consumption expenditure on goods and services to GNE (g_{1t}^w) and the ratio of effective investment to GNE (i_t^w) are *not* weakly exogenous. These weak exogeneity test results provide further insight into the Adjusted GNE model. Since previous testing revealed that c_t^w, and x_t^w are excluded from the cointegrating relationships with g_{3t}^w, it is expected that these variables will not respond to any disequilibria. These results clearly support this expectation.

Table 7.5

Tests for Weak Exogeneity

Variable	c_t^w	i_t^w	g_{1t}^w	g_{2t}^w	g_{3t}^w	x_t^w
LR Test (χ^2 (r))	2.99	13.33	5.69	6.41	4.23	1.89
P-value	0.22*	0.00	0.03	0.01	0.04	0.39*

(*) Critical value at 5% level χ^2(1)= 3.84 and the weak exogeneity of the ratio of effective consumption expenditure to GNE (c_t^w) and the ratio of leisure to labour (x_t^w) is accepted.

7.5 Testing the Cointegrating Rank

The number of distinct cointegrating vectors are obtained by checking the significance of the characteristic root Π. The error correction model is specified as

$$\Delta Z_t = \Gamma_1 \Delta Z_{t-1} + \ldots + \Delta Z_{t-k+1} + \mu + \psi D_1 + e_t \qquad t = 1, \ldots, T \qquad (7.11)$$

$$Z_t = (i_t^w, g_{1t}^w, g_{2t}^w, g_{3t}^w) \tag{7.14}$$

where

i_t^w = ratio of effective investment to GNE

g_{1t}^w = ratio of government expenditure on goods and services to GNE

g_{2t}^w = ratio of government investment expenditure to GNE

g_{3t}^w = ratio of government defensive expenditure to GNE

The four characteristic roots of the estimated Π matrix are given in the first column of Table 7.6.

Table 7.6
Cointegrating Rank Test

Eigen Value	H_0	Trace	Trace (0.90)	λ_{max}	λ (0.90)
0.0264	$r \leq 3$	12.97	13.34	7.97	12.10
0.0653	$r \leq 2$	20.47	26.79	15.50	18.70
0.1348	$r \leq 1$	46.54	43.96	19.07	24.71
0.2113	$r = 0$	72.89	65.06	28.35	30.77

In Table 7.6, the likelihood ratio test statistics for the rank of Π are presented along with the 90 per cent quintiles of appropriate limiting distributions. Two versions of the test procedures are reported in Table 7.6. The first is based on the trace test and the second on the maximal eigenvalue. The test for the cointegrating rank begins with $H_0{:}r = 0$ and moves sequentially up until a non-rejection is found. With the maximal eigenvalue test, $H_0{:}r \leq 1$ cannot be rejected which suggests that the choice of $r = 1$. Likewise, the trace test leads to a choice of $r = 1$. The remaining tests and discussion are based on this choice.

7.6 Short-run Dynamics

Table 7.7 presents the estimates of the short-run effects of $\hat{\Gamma}_2$ and the corresponding 't-values'. In the short-run, the ratio of government defensive expenditure to GNE (g_{3t}^{w}) is positively influenced by the ratio government expenditure on goods and services to GNE (g_{1t}^{w}) but negatively influenced by the ratios of effective investment to GNE (i_t^{w}) and government investment expenditure to GNE (g_{2t}^{w}).

Table 7.7

Short-run Dynamics of Endogenous Variables $\hat{\Gamma}_2$

	i_t^{w}	g_{1t}^{w}	g_{2t}^{w}	g_{3t}^{w}
i_t^{w}	-6.01 (-1.54)	5.83 (1.47)	-0.08 (-1.51)	0.99 (1.70)
g_{1t}^{w}	-5.88 (-1.53)	5.70 (1.46)	-0.07 (-1.05)	0.96 (1.67)
g_{2t}^{w}	-7.99 (-1.23)	7.55 (1.18)	0.33* (2.85)	0.60 (0.64)
g_{3t}^{w}	-0.60 (-0.67)	0.61 (0.68)	-0.20 (-1.22)	0.04 (0.28)

Likewise, g_{3t}^{w} is a factor that positively influence i_t^{w}, g_{1t}^{w}, and g_{2t}^{w} in the short-run. Because these estimates are based on the unrestricted error correction model (ECM), many of the parameters are insignificant. However, as the number of lags increases, the 't-values' also increase. The results for $\hat{\Gamma}_1, \hat{\Gamma}_3$ and $\hat{\Gamma}_4$ are presented in **Appendix E.**

7.7 Cointegrating Relationships

Since the system is cointegrated, the matrix Π can be written as the product of two vectors α and β. The vector β is known as the cointegrating vector, which is 4×1 in this case. The column of β determines the vectors contained in the cointegrating space.

The maximum likelihood estimates for the unrestricted β are:

$$\hat{\beta}' = \begin{bmatrix} \beta_i & \beta_{g1} & \beta_{g2} & \beta_{g3} \end{bmatrix} \tag{7.14}$$

$$\hat{\beta}' = \begin{bmatrix} 666.49 & -592.81 & -11\ .74 & -69.52 \end{bmatrix} \tag{7.15}$$

The value of the matrix given by equation (7.15) was then normalised by the variable i_t^w (ratio of effective investment expenditure to GNE) and resulted to

$$\hat{\beta}' = \begin{bmatrix} 1.00 & -0.89 & -0.17 & -0.11 \end{bmatrix} \tag{7.16}$$

and the speed adjustment parameters are

		t-values
α_i =	0.44	2.44*
α_{g1} =	0.45	2.52*
α_{g2} =	1.55	5.28*
α_{g3} =	-0.07	-1.76

The null hypothesis tested here is that government defensive expenditure g_{3t}^{w} is cointegrated with the macroeconomic variable under consideration. The test was specified as

$$\beta' Z_t = \left[\beta_i \quad \beta_{g1} \quad \beta_{g2} \quad 0\right] \tag{7.17}$$

The p-calculated is 0.04 which is larger than 0.05. Hence, the null hypothesis that the cointegrating vector is $\hat{\beta}' = \left[\beta_i \quad \beta_g \quad \beta_{g2} \quad 0\right]$ is rejected. This means that there is cointegration among g_{3t}^{w}, g_{1t}^{w}, g_{21t}^{w} and i_t^{w}.

In the long-run, government investment appears to be a complement of government defensive expenditure. In other words, a rise in government defensive expenditure has a positive impact on government investment. In contrast, it was expected that government defensive expenditure g_{3t}^{w} and government investment expenditure g_{2t}^{w} have a substitute relationship. An increase in g_{3t}^{w} will result in a reduction in the amount of money available to g_{2t}^{w} assuming a fixed government budget. The complementary relationship between g_{3t}^{w} and g_{2t}^{w} can partly be explained by the fact that as government defensive expenditure increases, the foreseen productivity of some government investments might also improve. This will likely encourage the government to increase its investment.

Likewise, the result with regards to g_{1t}^{w} is quite a surprise. This result indicates that g_{1t}^{w} and g_{3t}^{w} are complementary or are moving in the same direction in the long-run. Looking at the nature of defensive expenditure, one would note that a bulk of this expenditure is classified as final consumption expenditure.

The negative long-run relationship between g_{1t}^{w} and the ratio of effective private investment to GNE (i_t^{w}) is rather interesting. It implies that as government defensive expenditure decreases, the level of effective investment decreases. A possible explanation of this result is that government defensive expenditure is seen as a means of improving environmental quality, and any decrease is considered a deterioration of environmental quality. For instance, in Australia the overall assimilative capacity of the environmental resource is generally thought to be greater than what is common overseas because of the geographic size of the continent. But the magnitudes of these environmental residuals coming from the different sectors of the economy are large and pose a substantial environmental problem. Thus, a decrease in government defensive expenditure will be seen as a sign to increase private defensive expenditure so as to maintain the existing productive capacity. A rise in private firms' expenditure on the environment would mean less money available for conventional investment. This finding is consistent with results obtained by Aschauer (1988): that a rise in public investment (such as government defensive expenditure in the model), although minimal, has a negative impact on private investment.

The test on exogeneity indicated that ratios effective consumption to GNE (c_t^w), and leisure to labour (x_t^w) are weakly exogenous. These two variables were then assumed to be exogenous when the cointegration test was run. Thus, the results are not conclusive on the probable long-run relationship between the ratio of government defensive expenditure to GNE (g_{3t}^w) and the variables c_t^w and x_t^w.

7.8 Impulse Response Analysis

While the previous analyses have shown long-run relationships among variables, it is nontheless useful to find out what would happen to investment, consumption, leisure, government investment and government consumption if there was an unexpected shock or fluctuation in government defensive expenditure. This is done using Impulse Response Analysis (IRA) which can capture the relationships between government defensive expenditure and other variables. The results of the IRA for each variable are presented in Figures 7.2a to 7.2f. The ratio of government investment expenditure to GNE (g_{2t}^w) is adversely affected by an increase in the ratio of government defensive to GNE (g_{2t}^{w}) is adversely affected by an increase in the ratio of government defensive expenditure to GNE (g_{3t}^{w}), while the rest of the variables are initially positively influenced. The adjustments of the macroeconomic variables included in this study to the government defensive expenditure shock are slow. Only after approximately 28 quarters the stabilisation process begin, and it is not until the 32nd quarter that stability is once again achieved.

With regard to the ratio of effective investment to GNE (i_t^{w}), Figure 7.2a shows that it is positively affected by g_{3t}^{w} and that the effect is permanent. An effect of a one-time impulse on a variable is called permanent if it does not return to zero and settles at a different equilibrium value. Likewise, in Figure 7.2b the effects of impulses on g_{1t}^{w} are illustrated. They are also initially positive and the effect is also permanent. It is important to note that the result for g_{1t}^{w} is almost identical to that of i_t^{w}.

The initial negative relationship between g_{3t}^{w} and g_{2t}^{w} is partly explained by the substitute relationship between the two in the short-run. As the economy grows there is an increasing need for the government to provide more infrastructure in order to cater for the needs of the growing economy. However, the government also needs to spend on correcting environmental damages brought about by increased levels of production. Note that the reaction of g_{2t}^{w} contrasts sharply with the cointegration relations presented earlier. This sheds doubt on the meaning of the restrictions imposed on the cointegration vector. Impulse responses from a system estimated without constraints were also computed. The results are identical to Figure 7.2b and are therefore not repeated here.

Like the other variables, the effect of g_{3t}^{w} on itself is also permanent. In fact, none of the variables is transitory. A one-time impulse on a variable is called transitory if the variable returns to its previous equilibrium value of zero after some period. Unlike

Figure 7.2a

Response of i_t^w to One-standard Deviation Shock in g_{3t}^w

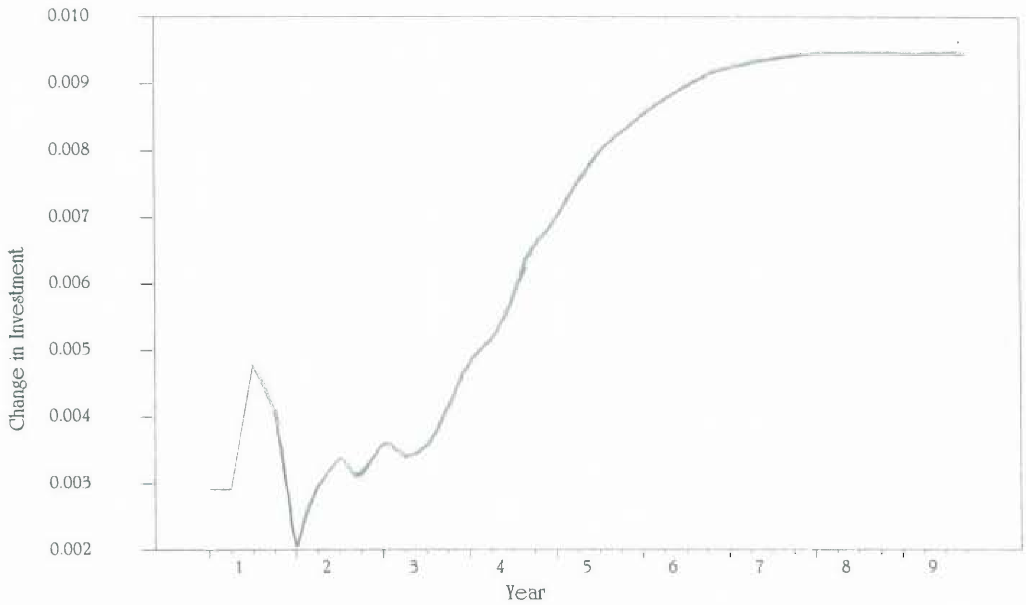


Figure 7.2b

Response of g_{1t}^w to One-standard Deviation Shock in g_{3t}^w

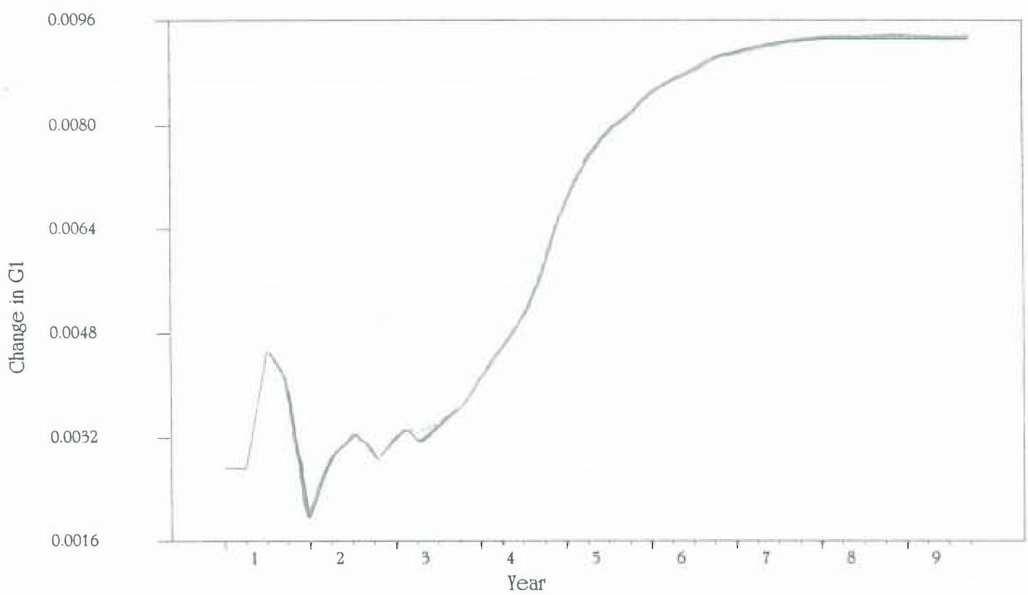


Figure 7.2c

Response of g^{w}_{2t} to One standard Deviation Shock in g^{w}_{3t}

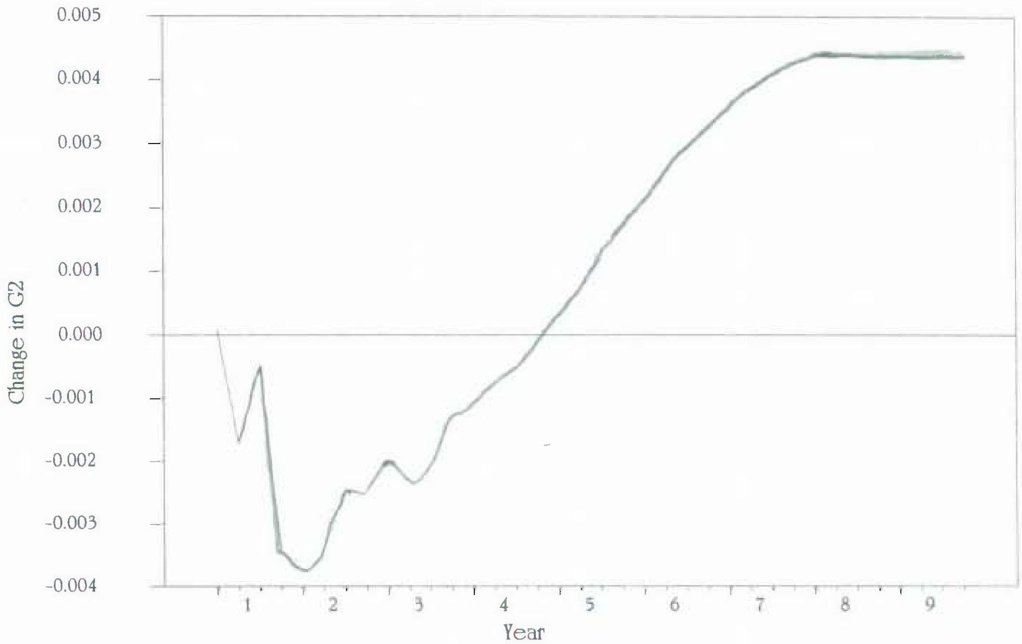


Figure 7.2d

Response of g^{w}_{3t} to One standard Deviation Shock in g^{w}_{3t}

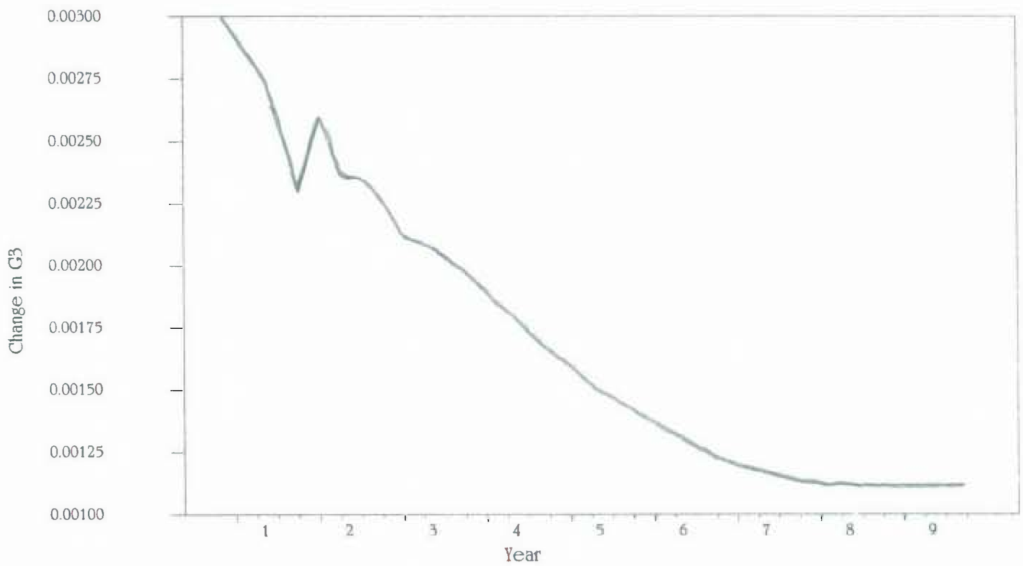


Figure 7.2e

Response of x_t^w to One-standard Deviation Shock in g_{3t}^w

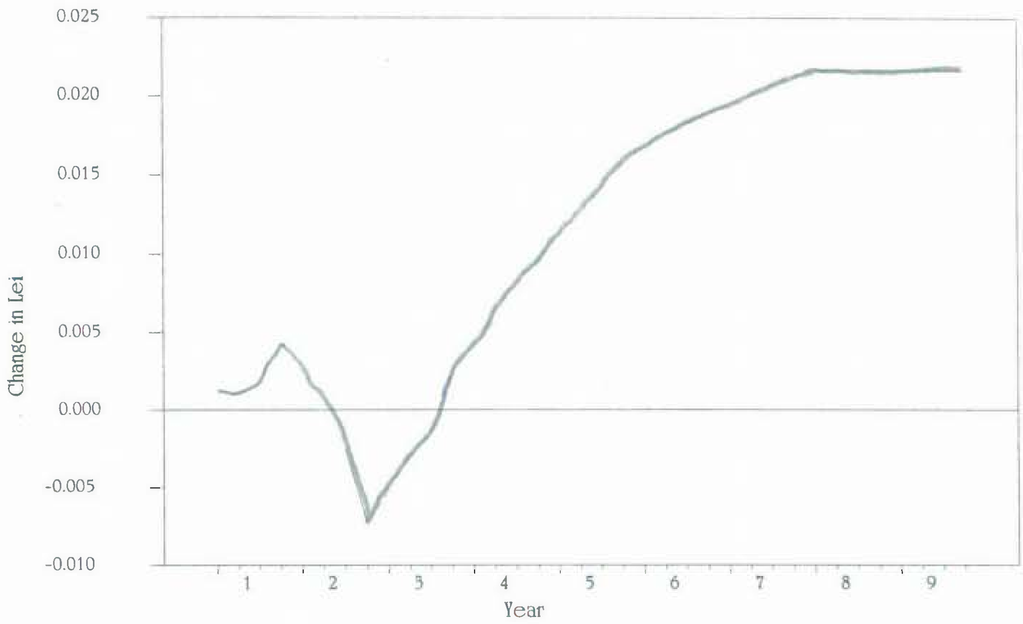
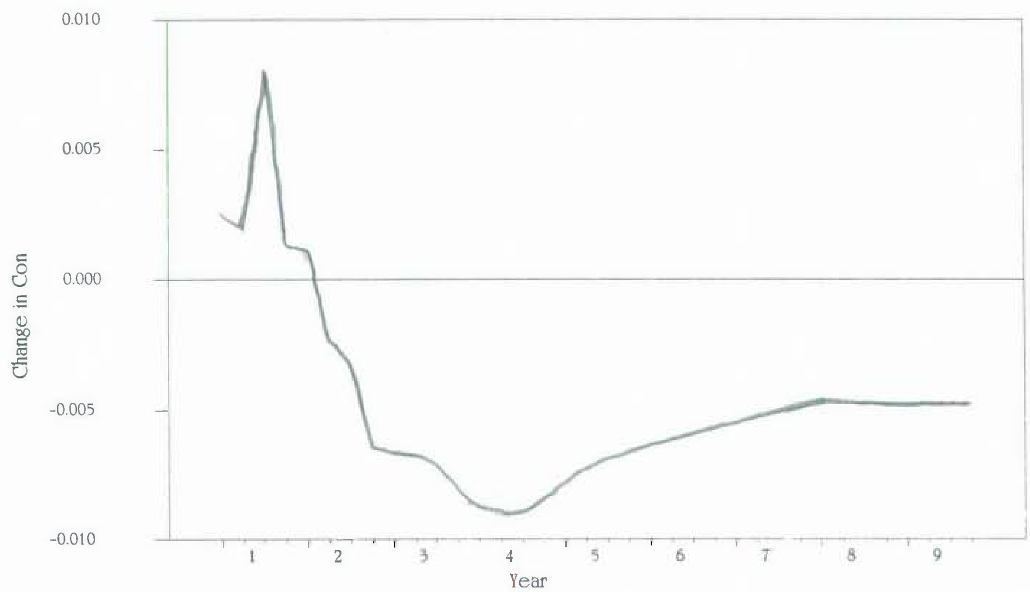


Figure 7.2f

Response of c_t^w to One-standard Deviation Shock in g_{3t}^w



the other variables, the impact of g_{t}^{n} on itself is decreasing after the 7th quarter and only became stable after the 30th quarter. This happens to g_{3t}^{w} after the 51st quarter.

The initial impact of a one standard deviation increase in g_{3t}^{w} to c_{t}^{w} is positive but after the 7th quarter it becomes negative. In contrast the impact on x_{t}^{w} on the 7th quarter is negative but it becomes positive again after the 13th quarter.

7.9 Evaluation of the Derived Model

Since the Cobb-Douglas production function was assumed for this study, the reduced form equations were easily derived. The framework developed to derive the reduced form equations can readily be used to other forms of production function. For instance, using a quadratic production function, the reduced form equations become

$$x_{t}^{n} = \frac{\theta_{1}\theta_{2}\Pi_{10}\varepsilon_{t}(a_{1} + b_{1}n_{t} + b_{3}k_{t})}{n_{t}}\left[1 - (1-\theta_{1})g_{1t} - (1-\theta_{2})g_{2t}\right] \quad (7.18)$$

$$c_{t}^{n} = \frac{y_{t+1}\lambda_{t}}{e_{t}\beta_{t}\varepsilon_{t+1}(a_{2} + b_{2}k_{t+1} + b_{3}n_{t+1})}\left[1 - (1-\theta_{1})g_{1t} - (1-\theta_{2})g_{2t}\right] \quad (7.19)$$

$$i_{t}^{n} = \left[1 - \frac{y_{t+1}\lambda_{t}}{e_{t}\beta_{t}\varepsilon_{t+1}(a_{2} + b_{2}k_{t+1} + b_{3}n_{t+1})}\right]\left[1 - (1-\theta_{1})g_{1t} - (1-\theta_{2})g_{2t}\right] \quad (7.20)$$

The resulting equations differ from hose derived using the Cobb-Douglas production function. The variables c_{t}^{n}, x_{t}^{n} and i_{t}^{n} are not only influenced by the ratios of govenment expenditures on goods and services to GNE (g_{1t}^{n}), government investment expenditures to GNE (g_{2t}^{n}) and govenment defensive expenditures to GNE (g_{3t}^{n}) but also by capital and labour. It is important to note however that the variables of government expenditures are still included in the derived reduced equations. The Cobb-Douglas production function was chosen because it is mathematically less complicated to manipulate than the other production functions.

The model will be applied to Australian data from 1962:3 to 1991:2. The results and the evaluation of the applicability of the model will be discussed in chapter 9.

7.10 A Summary of Results

Using the adjusted GNE model, the long-run relationships between the various aggregate economic variables are investigated. The cointegration method suggested by Johansen (1988) and Johansen and Juselius (1990) is used. The findings of the cointegration test are as follows: there is one cointegrating vector in a six-variable VAR model; cointegration exists among the ratios effective consumption to GNE (i_t^w), government expenditure on goods and services to GNE (g_{1t}^w), government investment to GNE (g_{2t}^w) and government defensive expenditure to GNE (g_{3t}^w).

The exogeneity test results show that the ratio of effective consumption expenditure to GNE (c_t^w) and the leisure-labour ratio (x_t^w) are weakly exogenous. Thus, the results are not very conclusive as to whether government defensive expenditure affects the ratios of effective private consumption expenditures to GNE and the leisure to labour. Therefore, a rise in the ratio of government defensive expenditure may or may not have an impact on these two variables. The results however show that government defensive expenditure does have long-run linear relationships with government spending on consumption, government investment and effective investment respectively. Therefore, a rise in g_{3t}^w has a negative impact on i_t^w in the long-run and a positive impact on both g_{1t}^w and g_{2t}^w.

With regards to impulse analysis, only g_{2t}^{w} was inversely affected by expected increases in government defensive expenditure initially. For all the identified variables, the adjustment process is slow: it took approximately 28th quarters for most variables to stabilise.