# CHAPTER 1

# LITERATURE REVIEW

The spirit of the time shall teach me speed.
Shakespeare King John IV

## Introduction to Chapter

Children in western society are made aware of the importance of speed very early in life: they are often exhorted to 'speed up' or to 'be faster' than their peers at some tasks. It has been noticed that in other cultures (in particular the South Pacific Islands) that this idea is not 'pushed on' to children by adults. This results in a society that has a different concept from 'having to do things today' or having to get the task done in a minimum of time, much to the exasperation of the Westerner who visits them.

For western society, speed is also one of the basic concepts necessary for a suitable understanding of applied mathematics, physics and related sciences. It not only occurs in these academic disciplines but it is an important feature of everyday living: such as in driving a car, or in children having a race. Most children develop a concept for speed from their own experience before they are formally introduced to them at school. When these ideas are introduced in mathematics and science classes, the notions are either refined, and growth takes place, or they tend to become more confused and alternative conceptions occur.

While dealing with the concept of speed in scientific literature the term velocity is often used. In physics there is a distinction between these two terms, however, in the general community 'speed' and 'velocity' are usually used synonymously. Because of this, the literature reviewed in this chapter is focused primarily on children's concept of speed without regard to the scientific meaning of velocity. The review considers the conceptual development of speed in two ways: firstly, as a qualitative concept for which children gain an intuitive notion; and, secondly, as a method of determining if children can see mathematical relationships between speed, distance and time.

As an introduction to this work, an overview is provided of Piaget's study of children's ideas on motion as a background to other research literature. Most recent studies in this area either aim to confirm Piaget's work, to refine it for particular instances, or to make some modifications where it seems to be appropriate. Such investigations are often referred to as 'Piaget-type experiments'. The overview of Piaget's study is followed by a review of some of the literature dealing with children's ideas about speed that is not related explicitly to the studies of Piaget.

## PIAGETIAN-TYPE INVESTIGATIONS ON SPEED

A number of studies have been carried out in child psychology and in child development research on the ways that young children perceive distance and length as well as motion (Piaget, Inhelder & Szeminska 1960; Lovell 1975; Witthuhn 1979; Shrieider 1974; Bartsch & Wellman 1988; Poduska & Phillips 1986). Several others have investigated how young children (up to seven years of age) perceive their surroundings and, in particular, the development of their ideas concerning distance, time and motion (for example, Herman, Norton & Roth 1983; Kelly, Kelly & Miller 1986; Miller & Baillargeon 1990, Witthuhn 1979). Most of these studies have, as a foundation, the work of Piaget (1970).

It is, therefore, important to review the work that has been carried out by Piaget and his colleagues concerning the child's understanding of motion and speed before considering other research on speed. Piaget's study on speed is discussed in great detail in his book *Child's Concept of Movement and Speed* (1970). His book was first published in 1946 in French and has had considerable impact on all work that has been done since on the concepts of speed and motion.

This Section describes some of Piaget's investigations that seem most relevant to an investigation into the concept of speed. In the Preface to his book Piaget said:

> Now the concepts of movement and speed especially touch upon the fields of mathematics and general science teaching, in which it would be

> of great value to know precisely the way in which these concepts develop.
>
> (p. ix)

In the first five chapters of his book, Piaget described his work on children regarding the notions of "Successive order or placing" and "Change of Location". Most of the interviews that were conducted with children were concerned with demonstrations with concrete models of objects that were moving subject to certain conditions. He found in these studies that the "Primary intuition of speed is ... that of overtaking, that is, change of order" (p. 62). Thus, Piaget found that children, aged five to twelve years, who are watching the motion of two objects see the object with the greater speed as the one that is catching up or passing the other object.

Some of the detail of his study on "Relative Movements" is discussed first to give the flavour of Piaget's methods and initial conclusions on children's perception of movement. Then consideration is given to the remainder of Piaget's investigations on speed.

**Relative Movements**

In the study of "Relative Movements" (pp. 102 - 120) the children were shown a snail shell on a piece of cardboard. The snail was said to move along the cardboard while the cardboard was also in motion. Sometimes the cardboard was moved in the same direction as the snail and other times in the opposite direction.

> The problem [for the child] is to locate again the point the snail has reached, after the materials have been removed, or else where he will arrive, given certain conditions which are stated verbally.
>
> (p. 102)

Sixty-seven children between the ages of five and fourteen years were interviewed and Piaget was able to distinguish four stages of intellectual development. In the first stage, the child usually paid attention to only one of the motions and generally it was that of the snail. In the second stage the child gave attention to the two motions but considered them as side-by-side, not end-to-end. The child failed to add or subtract the two motions. Piaget split the third stage into two parts. Initially, the child could solve the problem for paths in the same direction or opposite direction if the paths were equal, for both concrete and verbal methods. Piaget called this stage the beginning of the concrete operations and this was first evident in children from about eight years of age. In the second part of the third stage, the child was able to "gradually produce empirical solutions" (p. 104) for all questions. The children that could do this were nine-to-ten years of age. In the fourth and final stage the children were able to solve all questions from the very beginning. Most of these children were from ten-to-eleven years of age. Piaget identified this stage as the Hypothetico-deductive or formal level.

> The concrete operations ... bear upon reality itself, perceived or conceived, whilst formal operations proceed simply upon verbal supposition or on the hypothesis based on symbols (as in mathematics) which represent the former.
>
> (p. 116)

These four stages for a child's perception of relative movements were identified also in a similar way with his studies on speed. There is a progression from one stage to another that is related to the chronological ages of the children.

The remainder of Piaget's book is specifically related to the child's concepts of speed. His study concerned two aspects of speed: Qualitative Speed and Quantitative Speed. Each of these is now discussed with descriptions of the relevant investigations and results.

## QUALITATIVE SPEED

While the experiments on Qualitative Speed concerned comparing the speeds of two objects, the child did not have to calculate the speeds of the objects. Piaget considered Qualitative Speed under three headings, these are: Intuition of Speed, Speed in Synchronous Movements and Relative Speeds.

### The Intuition of Speed

Piaget developed three different tests to find what were a child's initial intuitions of speed:

Test I

In the first test, children had to judge the speed of two movements when only the starting and stopping points were visible. The rest of the motion was hidden in a tunnel. In this way children could not use the ideas of catching up or overtaking as methods of judging speed. The paths had unequal length and they were parallel to each other.

Piaget found three stages for this task:

**Stage 1**. The children in this stage failed to compare the speeds. If the children were shown the whole movement, they could see one object overtaking another and were able to determine correctly the one with greater speed. They had no intuitive feeling for distance or time associated with the task. When the whole motion was not visible, children focused only on the starting and stopping points.

**Stage 2**. After the children had seen the whole motion, they could visualise what was happening when only the starting and stopping points were shown. They deduced that the one that was ahead was the faster. "The earliest intuition of speed is that of overtaking" (p. 129).

**Stage 3.** The children were able to find an operational solution to the problem. They could tell which object had greater speed by the longer tunnel and objects having the same time. They seemed to know that there was a relationship between distance and time.

> Such a coordination of the relationships involved thus implies understanding of the equality of synchronous durations and this is why the operational conceptions of speed, unlike the intuitive conception, implies a spatio-temporal construction of the whole.
>
> (p. 132)

Thus, in this simple test, Piaget was able to identify children reaching across three levels from failing to determine what speed was, to those who considered overtaking, through to children who were able to consider distance and time.

Test II

In the second test, the children had to compare the speed of two objects that were open to view while travelling along two unequal paths. In this test the starting or stopping points were either the same or side by side.

Piaget again found that "the earliest intuition of speed is overtaking, conceived simply as an inversion of order" (p. 132). He also hypothesised that as growth takes place the starting and stopping points are considered, then distance and duration (time) gain the attention of the children. There was "a restructuring of the idea of speed, conceived from then on as the relation between these two kinds of intervals" (p. 133).

Piaget found two stages in this test:

**Stage 1.** In this first stage he found two reactions. In the first, reaction the children said that the cars travelled at the same speed, since no overtaking occurred. For the second reaction, the children thought that the car that travelled along the shorter road went more quickly because it had a shorter distance to travel or, alternatively, they finally recognised that the speed of the other car was greater, but did not know why. This second reaction "rests on ambiguity common to the child's logic and also to speech itself" (p. 137). In everyday speech, the words "finish quicker" can mean earlier where the speed is the same or lower.

**Stage 2.** Here, the children's understanding of the relationship between time and space "is limited to seeing that at equal speeds the times are proportional to the spaces" (p. 138). "They think the speeds are equal where different distances are traversed in the same time ... because no overtaking [occurred]" (p. 139).

In summary, Piaget stated:

> In the beginning the order of stopping points alone determines the speed even when the paths are entirely visible, while later the relationships of the spaces traversed and the durations formed in spite of the absence of overtaking.
>
> (p. 140)

Again there is growth as children focus on the length when the times are equal. There are alternate conceptions of speed that are rooted in speech itself, and also whether time or distance is the best characteristic to determine which object has gone the greater speed.

Test III

This test concerned Circular Movements. The child had to judge the speed of two moving objects travelling 'neck and neck' on two concentric circles that started and finished along the same radii. In this test there was no overtaking.

Piaget found four stages in Test III:

**Stage 1.** A spontaneous response was that the speeds were judged equal by the child. If the attention of the child was drawn to the fact of unequal lengths of the tracks they would give a response consistent with the inverse ratio of speed and distance. "Speed has temporal precedence and no longer spatial superiority" (p. 145), that is, the child focused on the time that the objects moved rather than the distances that they had moved.

Piaget (p. 145) commented that the use of the words "quicker" and "faster" as used in common speech caused some misunderstanding. Children at this stage often used the word 'quicker' to mean 'arrived sooner' or 'faster'.

"Shorter = finished first = travelled faster" (p. 146).

**Stage 2.** Piaget designated this stage as consisting of Intermediate Reactions.

> The attention of the child focused on three factors in turn: (1) The order (spatial or temporal) of starting and stopping points, ... (2) The disparity in length of their trajectories conceived in terms of precedence in time, ... (3) This same inequality conceived in terms of overtaking in space.
> (p. 148)

The children oscillated between these factors until the third factor was stabilized.

**Stage 3.** An operational solution to the problem was achieved. The child could see that "The relationships of distance and time immediately become factors on the operational whole" (p. 152). This occurred with children as early as six years of age.

In **Stage 4** the children were able to note the difference in speed due to difference in the distances travelled. However, some eleven-year-old children still became confused if the objects were connected by solid rods.

Piaget summed up this section on intuition of speed by stating:

> The facts indicate that the simplest intuition of speed, as well as of movement, is based on an intuition of order: at any age a moving object is held to be faster than another when it overtakes on a path parallel to its own, i.e., when it has originally been behind the other one according to the direction of travel, or side by side with it, and afterwards is ahead of it.
> (Piaget 1970, p. 121)

This test which considered equal angular speeds showed that some children could determine the linear speed of the objects correctly only if they took into account the distances that the objects travelled in equal time.

Reactions to Tests

Levin and Gardosh (1987) challenged the assumptions that Piaget and other investigators (Ehri & Muzio 1970; Levin & Simons 1986; Lovell, Kellet & Moorhouse 1962) have made about students' understanding of circular speed. In school, students are taught about linear motion well before they are taught about rotational motion. It is assumed that students also learn about them in this order, but students already have ideas about rotational speed at the same time they develop concepts of linear speed. Piaget assumed that children would only deal with linear speed until they had been formally taught about rotational speed. His results are explained with this assumption in mind. In fact, from a rotational frame of reference, the two objects going 'neck and neck' on concentric circles do have the same rotational speed. So the students placed in Stage 1 were quite correct when they deduced that the two objects were travelling at the same speed. Thus students use a frame of reference that is dictated by the context of the problem and it is improper to assume that young students are only limited to a linear frame of reference. Levin and Gardosh (1987) found that:

> By at least the age of 10, children have a core concept of speed. This concept is concerned with the relation of output to time and captures the fact that different dimensions can serve as appropriate output. In linear motions, the appropriate output is typically distance. In rotational motions, it is typically the number of rotations. In discrete motions it may be rate.
>
> (p. 307)

Levin, Siegler and Druyan (1990) looked at alternative conceptions that both children and adults have about motion and concluded that a common misconception was that single objects must have the same speed at all places on that object for a single-object/single-motion situation. This is true for linear motion but is incorrect for circular motion.

It was noted in Stage 1 above that Piaget commented on children's meanings of certain words. The word "faster' had to be used carefully since it could carry more than one meaning. Children do not easily distinguish between whether an object is faster because of its shorter time to travel or because of its actual speed. Similar problems occur in English, French and Japanese. In Thailand, however, there are two distinct words that can distinguish temporal precedence and speed, as well as, temporal length and spatial length. In English, for example, the word 'long' (and 'short') can be used in two ways: one can have a long (short) time and a long (short) distance. The work of Mori (1976) on the differences in Japanese and Thai pre-school children's understanding of the speed of objects suggests that "children's conception of speed and duration considerably depends on the differences of their mother tongue" (p. 112). Thus the use of the English word 'fast' can lead to misconceptions of speed because it can be used to refer to speed as well as to less time.

**Speed in Synchronous Movements**

Two different tests were used by Piaget to find how children viewed speed when the starting and stopping times were the same (synchronous movements) over equal and unequal distances. The child had to decide which object had the greater speed. It is of interest to note that Piaget used the word 'harder' with reference to the object that moved with the greater speed in these tests.

Test I

This test considered synchronous movements over unequal distances. Four tasks were given to the children to compare the speed of two cars that started and stopped simultaneously, with only one question being asked: "Which one went faster or harder?" (p. 157).

In the first task (T1) one car started at a point behind another car, they started moving at the same time, stopped at the same time and at the same place. Thus the first car catches up to the second car. The second task (T2) has the same initial conditions as T1 but the first car does not quite catch up to the second car. The third task (T3) has the same initial conditions as T1 and T2, but this time the first car definitely catches up with and overtakes the second car and stops at a noticeable distance in front of the second car. In the fourth task (T4) the cars set off in opposite directions towards each other, with unequal speeds, and stop beside each other to give unequal distances travelled.

Three stages were obtained.

**Stage 1**. The children focused on the visual aspects of the problem. "Overtaking alone produces correct judgements, and other arrangements are evaluated in terms of the placing of the finishing points" (p. 157). Thus the children were able to observe correctly that if one car overtakes another, task T3, then it was the faster, otherwise the children focused on the stopping positions of the two cars.

In summarising this stage, Piaget commented:

> It is as if the child judged speed by the finishing point alone, regardless of the distance travelled, and as if 'more quickly' meant 'finishing in front of' or 'before' in a spatial and temporal sense at one and the same time.
>
> The very perception of a movement immediately provides some impression of speed: a moving car will thus appear faster than a horse, whether overtaking actually occurs [or not].
>
> (p. 164)

**Stage 2**. This stage "is characterised by intermediate reactions between this initial intuition [stage 1] and logic".

The children in this stage "... gradually correct themselves in the course of the experiment" (p. 167) whereas children in Stage 1 do not (or cannot) correct themselves. There was a "progressive decentration of intuition fixed on the stopping points" (p. 170).

**Stage 3**. In stage 3 the "operations of correlation are introduced" (p. 157).

> Three correlated constructions are observed:-
> - temporal order is dissociated from succession in space
> - distances travelled are conceived as lengths occupying the interval between the given starting and stopping points
> - in the case of synchronous movements speed is defined in terms of these lengths covered in equal times
>
> ... Relations of speed are now expressed only in terms of lengths or distances and no longer of order.
>
> (p. 170)

Thus in stage 3 children of seven-to-eight-years of age could focus on the different distances that the cars had travelled as a criteria for determining speed if the times are the same.

Test II

In this test, partly synchronous times (stopping) and equal distances were considered. Thus the objects moved equal distances and stopped simultaneously but started at different times.

Piaget found that children go through three stages similar to that found in Test I.

**In Stage 1** the children evaluated speed intuitively in terms of the order of the starting and stopping points:

> ... either speeds were equal because they reach the same point simultaneously or else that the speed of the moving object which left first is greater, because it goes ahead of the other one.
> (p. 171)

In **Stage 2** the children started by attributing the greater speed to the one that was in front (space and time) then they considered the time factor.

When children were able to realise, that the car with the shorter time had the greater speed for the same distance, then Piaget placed them in **Stage 3.**

In his summary of the section on Synchronous Movements, Piaget made the following significant comments about children's initial conceptions of speed.

> The intuitive starting point of the conception of speed seems, all things considered, to be based on a sensori-motor schema belonging to the subjects own activity: managing to be first in line, or in a word, ahead. ... getting ahead is equivalent to overtaking. ... catching up is in fact almost overtaking. Even in a purely intuitive form, the child actually begins by judging speed solely from the stopping points: ... the speeds are equal regardless of distances covered and starting times.
> (p. 176)

Thus in the early stages, children consider the visual aspects of the motion as being significant. Consideration is now given to some other investigators work that are similar to Piaget's research.

Related investigations

Thus a child learns that if one object passes another in the same direction then it is said to be going faster. In the child's mind, however, the word 'faster' can mean something different from what is understood by an adult. "In the child's mind it is merely the word to denote the movement of one of the objects when there is a change in relative position" (Lovell 1975, p. 90). A child may usually consider that it has nothing to do with distance covered in a certain time. It is thought by the child that the object that gets to a particular point first goes faster. Lovell, Kellet and Moorhouse (1962) found that it was not until nine years of age that children could see that speed was related to distance and time.

These two studies of Piaget, on Synchronous Movements, were replicated by Raven (1972) who found close agreement with Piaget's results. Part of his results are given in Table 1.1. The majority of children of eight years of age and over were able to solve the unequal distances in equal times, this he called Task S1. For unequal times and equal distances, Task S2, it was only from ten years of age that most children could attain positive results.

Table 1.1
Percent of children attaining a positive score by task and age
on speed problems (N = 96)

| Task | age (years) | | | |
|---|---|---|---|---|
| | 8 | 9 | 10 | 11 |
| S1 | 80 | 93 | 87 | 93 |
| S2 | 7 | 26 | 67 | 87 |

(Part of Table 1 Raven 1972, p. 204)

Identical Piaget experiments were given to college-age students by Trowbridge and McDermott (1980) and they reported that these tasks presented no challenge to these students.

A study on the order of acquisition of the qualitative notions of speed by Perry and Obenauf (1987) considered experiments that were nearly the same as that of Piaget. They studied students in the 1st, 3rd, 5th grade at school and obtained results that were similar to those of Piaget. They, however, used a scoring system corresponding roughly to the stages that Piaget had found and used this in an ordering analysis to develop a hierarchy of the different tasks (notions) that were logically equivalent and/or logically independent.

They identified six categories of difficulty within which the ten tasks could be placed (p. 560). These can be stated, in broad terms as:

Category 1: The simplest task was similar to Piaget's Intuition of Speed Test II (discussed above) where the motion of the two objects was open to view for unequal distances where the stopping and starting points were side by side. The child was asked which object would finish first if the speeds were equal.

Category 2: There were two tasks in this category. One concerned Piaget's Intuition of Speed Test I where only the starting and stopping points were visible to the child, The second was Piaget's Circular Motions Test III in Intuition of Speed. Both these categories have spatial and temporal alignment and are entirely synchronous. The child could then focus on the difference in the distances of travel.

Category 3: The only task at this category was the Piaget's Synchronous Movements Task T1 where the starting points were not aligned but the stopping points were aligned.

Category 4: Piaget's Synchronous Movements Test I, Tasks T2 and T3, were both in this category. Here the first car either overtook, or did not quite catch up to, the second car. Both the starting and stopping points were aligned temporally.

Category 5: Two tasks were in this category: firstly, Piaget's Intuition of Speed Test II where the times were equal and the starting and stopping points coincided, but the distances travelled were different; and, secondly, Piaget's Synchronous Movements Test II where the stopping points were only synchronous, but the cars travelled unequal length but in equal time, the motion was wholly visible to the children.

Category 6: Stopping points only are synchronous and the distances are unequal.

In the discussion of the implications of their research Perry and Obenauf (1987) stated that:

> The ordering diagram [categories] suggests that the individual tasks presented, encompassing the intuition of speed and speed in synchronous movements, are interrelated and intertwined. The alignment or non alignment, either spatially or temporally, provide indicators of the difficulty of the tasks presented in this research.
>
> (p. 563)

Thus the difficulty of problems was related to the number of variables that differ. A possible seventh category would be when all variables are different, that is, when the stopping and starting points are different, and the times and path lengths are unequal. This would be complete unsynchronous motion that possibly leads into quantification of speed as a method of successfully completing such questions.

**Relative speeds**

This experiment consisted of having an observer (a doll) watch as eight cyclists passed by. The observer was in one of three states: stationary; moving in the same direction as the cyclists; or, moving in the opposite direction to the cyclists. The children were asked to predict how many cyclists would pass the observer. An additional question was given to younger children asking them to predict if a single cyclist would take more or less time to reach the doll if it stood still, moved away from or moved towards the cyclist.

Piaget found four stages:

**Stage 1**. The child could not answer the question concerning the single cyclist and gave random answers to the other questions.

**Stage 2**. (Articulated intuition) The children in this stage were from six to seven-and-half-years of age. The question of the single cyclist was solved but for the eight cyclists there was no relativity of speeds.

**Stage 3.** (Concrete operations) The children in this stage ranged in age from eight-to-eleven years. The children could not predict the result before hand, but, on seeing the demonstration, were able to explain the relations correctly.

**Stage 4**. (Formal operations) At this stage the children were ten-and-half-to-eleven-years of age. They were able to deduce the results prior to the experiment, and gave excellent explanations of the relativity of speed.

In summarising this section on Qualitative Speed, Piaget was able to identify various stages of development concerning children's methods for comparing the speed of two objects that moved simultaneously. The children did not have to do any calculations to actually find the speed of each object since they were could visually compare the speed using the relative motion of the two objects.

## QUANTIFICATION OF SPEED

The experiments on Quantitative speed involved children visually comparing simultaneous motions. To avoid children using concepts related to overtaking and catching up as a means of ascertaining the relative speeds of objects, the experiments in Quantitative Speed involved showing children motions that were in succession, that is, one after another. In this way, they could not do a visual comparison of the two speeds. The quantification of speed is considered under two headings: Unequal Distances in Unequal Times; and Conservation of Uniform Speeds.

### Unequal Distances and Unequal Times

This method involved the recording of the time taken for an object to move along a path. A second object then moved along a parallel path for a different distance and the time was again recorded. The subject was then "asked whether these two objects travelled at the same speed or if one went harder than the other" (p. 201). This experiment was conceptually different from the previous ones in that the child could not compare the parallel motion of the two objects and use the ideas of 'catching up' or overtaking.

**Substage 3a**. Piaget found that a child who was at the concrete level in Stage 3 in understanding speed, when tested on Partly Synchronous Times (Stopping) and Equal Distances in quantitative speed experiments,

> ... [could not] compare speeds of movement in succession, and this is so even when their times are equal (though no longer synchronous) over unequal distances, or their distances are equal in unequal times.
> (p. 202)

When the movements were shown together, the child could focus on the concrete situation (overtaking, catching up or the front one), but in successive motions the child was required to hold the information in his/her head. Piaget called this type of operation a 'formal hypothetico-deductive' operation.

**Substage 3b**. At this level the child was successful when the times were the same and the distances were different, or when the distances were the same but the times different. The child was unsuccessful when both the times and the distances were different.

Piaget (1970) described children as using the concrete operational mode in problems:

> ... as soon as unequal times and space are involved, even if the ratios remain fixed and maintain a constant speed, ... the problem becomes more complicated. The child is unsuccessful at the very outset of the new construction, because he thinks either of the time, or of the distances, in a kind of alternating intellectual centration, without being able to unite them in a single ratio.
>
> (p. 214)

**Substage 4a**. The child was successful in solving the problem "by trial and error though formal operations are implied" (p. 202). The correct conclusion was reached by comparing ratios.

**Substage 4b**. At this stage the child has systematic procedures for solving the problem apart from calculation mistakes.

At the conclusion of these tests Piaget stated that the issue was whether the child could handle proportional relations that were really a formal mode of thinking. Before this occurs it was

> ... necessary to reason from drawings symbolising in retrospect journeys made in succession, this special role of the stopping points tends to diminish and the problem simply becomes comparison of lengths in relation to time taken.
>
> (p. 218)

Piaget found that two types of errors often occurred. First, when the distances were unequal and times were the same, the children deduced that the objects had the same speed. Secondly, when the distances were equal and the times differed, then the one with larger time was the faster, that is, speed was proportional to time ($v \propto t$).

Thus time seems to be a determining factor, for some children, in deciding which object has the greater speed. If times are the same, the speed is the same, or the object with the larger time has the greater speed.

Trowbridge and McDermott (1980) investigated these Piaget type experiments with first-year college students enrolled in different types of physics courses offered by their institution. They found that most of the students were successful on the given speed tasks. The students were able to make predictions using intuitive reasoning based on the simple ratio of 2:1 in the Piagetian tasks. When the tasks were modified a little, so that the ratio was non-integral, many academically disadvantaged students could not perform as well as they had on the standard tasks. Students who had a background in calculus and physics were usually successful on the modified Piaget tasks. It was also found that:

> ... it is common among students untrained in physics to think of speed, or velocity, as a simple association between a distance travelled and a corresponding period of time (for example, 1½ seconds for 45 centimetres, 3/4 second for 22½ centimetres) instead of a quotient [as 30 cm/s].
>
> (p. 1022)

This was not quite the level of understanding that Piaget would attribute to adult-like reasoning. Piaget stated that "formal operations imply an understanding of abstract, logico-mathematical relationships of which velocity = distance/time is an example" and that most adults could operate in this formal mode (Piaget 1970).

**Conservation of uniform Speeds**

In this investigation, Piaget's aim was to see if children could predict the position of two given objects at later time intervals, given the time and distance that the objects had covered in the first interval of time. Thus the child's understanding of the relationships concerned with speed would be tested. Children were shown a lorry that moved a distance two centimetres in one day and then a bike that moved on a parallel path a distance one centimetre in one day. The children were then asked to predict the positions of the lorry and the bike on successive days given that the lorry and bike travelled the same speed as they did on the first day. Piaget identified four stages for children who did this test.

**Stage 1**. Children from five-to-six-years-and-nine-months of age did not understand conservation of speed. Hence, they could not predict the position of one object on the second day given that it had the same speed and time as on the first day.

**Stage 2**. By using trial and error, the child at this stage could conserve speed for one object at a time, but with two objects moving at different speeds the child kept them at a constant distance apart, that is. both were attributed with the same speed. These children were between six-and-seven years of age.

**Stage 3.** By using concrete operations these children were able to construct the paths of both objects at the same time. They could not think in a logical deductive way to anticipate where the objects would be, nor could they generalise about the relative motions. Their ages ranged from seven-to-eight years.

**Stage 4**. The questions were solved by children using formal deduction at ten-to-eleven-years of age.

In going from Stage 3 to Stage 4, children operating in the formal mode were aware of the concept of proportionality. Children operating in the concrete mode did a gradual construction of the journeys and did "not presuppose the schema of proportions" (p. 258). Formal solutions are based on "reflections of concrete constructions" (p. 258).

Boulanger (1976) suggested that a characteristic of thinking in the Piaget formal operational level (Stage 4) is proportional reasoning. Here, students must be able to compare two ratios and make deductions about them.

Raven (1972) replicated this experiment and found similar results. Table 1.2 summarises his findings showing the results for synchronous movements (as in Table 1.1) as well as for this task called S3 for conservation of uniform speeds.

It is apparent that the tasks became more difficult for the children so that in task S3 only the older children were likely to solve the problem.

Table 1.2
Percent of children attaining a positive score by task
and age on speed tasks (N = 96).

| Task | Age in Years 8 | 9 | 10 | 11 |
|---|---|---|---|---|
| S1 | 80 | 93 | 87 | 93 |
| S2 | 7 | 26 | 67 | 87 |
| S3 | 0 | 20 | 53 | 80 |

(Part of Table 1 Raven 1972, p. 204)

Siegel and Raven (1971) found that children up to nine-or-ten years of age needed concrete objects to illustrate the variables that were being changed. If only verbal data was used or successive viewing "the child of this age [was] unable to set up the problem in his thinking so that he [could] compare simultaneously the differing relationships" (p. 373).

**SUMMARY**

Piaget (1970) summarised his investigations on speed in the last chapter of his book. The introduction of this last chapter of Piaget's book is reproduced in Appendix A. Briefly, Piaget said that there were six "great operational systems" working together of which the first four depended on "qualitative logic". The first of these was 'placement' where children usually moved to order things in space. The second was 'displacement' in that children perceived that the objects moved their positions. The third he called 'co-displacement' where children could combine the ideas of placement and displacement and order things in time and speed or movement. The fourth was the operations of "relative displacement and co-displacement" allowing a correlation of movements and speeds. These four "operational systems" were qualitative in nature. The last two systems, numbers five and six, are quantitative.

The fifth operation is 'extensive', allowing mathematical relations to be used such as ratios and proportions. The last, numbered six, allows metrical operations, that is, children could construct basic units that could be used repeatedly to measure distance and time relationships to obtain speed.

To conclude this section on Piaget's work a few points stand out as worthy of further investigation.

Piaget was interested in the early development of the concepts of motion and speed in children. Most of these children were under the age of twelve years. While other researchers such as Trowbridge and McDermott (1980) have studied tertiary students, there has not been many investigations on older children and young adolescents, that is, between say twelve and eighteen years of age.

Most of the children that Piaget studied were academically able. These children used several methods (or strategies) to decide that one object is faster than another. It would be of interest to know whether the responses were consistent over a greater range of children's academic abilities and whether some of the strategies persisted into later years.

Piaget's studies were mostly in the concrete context with visual models and all by interview. For older children a more abstract mode of operation should be available so that a questionnaire type test instrument should reveal some worthwhile information in some quantity, with follow up interviews eliciting quality responses.

The investigations conducted by Piaget were concerned with how children judge one dimension independently of the others, that is, when velocity, time and distance are intermixed. They have not studied children's understanding of the interrelationships between these variables.

In summary the stages Piaget found that children went through may be generalised (approximately) as:

**Stage 1**. Children could not solve the problem even when help was given to them.

**Stage 2**. Children could resolve a simple or one variable changing problems after some help was given.

**Stage 3.** Children could solve the question in an appropriate way, often realising their own contradictions and growing through the problem.

**Stage 4**. Children had a formal (or mathematical) approach to solving the problem in a clear way.

In the remainder of this chapter consideration is given to other literature that deals with the concepts of speed.

## OTHER INVESTIGATIONS CONCERNING SPEED

This section deals with the literature concerning speed concepts of students in four parts: first young children and their ideas of speed; second, secondary school students' concepts of speed; third, adults and tertiary students' beliefs relating to speed and velocity; and finally, investigations that go across these groups. Some of the literature that falls within these groups has already been referred to in the previous section where they directly bear upon the work of Piaget.

### YOUNG CHILDREN

At a very early age, children begin to develop an idea of speed. It can be described by an example such as: the fastest person is the one who completes the task first even though the starting times are different and the tasks are of different complexity. The task might not have anything to do with distance but it does have to do with the "amount of output per time" (Levin & Gardosh 1987, p. 303). By age ten most children have a core concept of speed that is dependent on the context of the situation but may have different output dimensions other than distance (linear speed); for example, angle, as in angular motion. When children do have a concept of linear speed (as opposed to any other type of speed) they do not always work with the scientifically accepted concepts but use other more intuitive ideas.

> The earliest notion of velocity [speed] is based on an intuitive feeling for velocity [speed] independent of any duration, because order is easier to grasp than the concepts of interval or measurement.
>
> (Kolodiy 1977, p. 226)

A number of ideas that children might have about speed and speed comparison tasks have been identified in the literature and are introduced below. While these ideas are common they are not found in all children. Some children, in the course of their cognitive development, never have any of these concepts but on the other hand some children never grow past them. Four of these ideas warrant some explanation. They are: proximity, passing, distance and time, and overtaking.

**Four Conceptions**

One of the early concepts that a child receives about speed is the idea that the object that is closest to, or arrives at, a destination first must have been faster. This is often referred to as the proximity view (see Lovell 1975, Piaget 1970, Shire & Durkin 1984). In this case the children focus on the object that is closest to the finishing point (or the one in front) at the instant that they are required to judge which one is going fastest, all other factors seem to be ignored. Such situations can occur in at least four ways and lead to wrong conclusions: the slower object started before the faster object; the slower object has a shorter path to travel; the slower object started in front of the faster one; and finally, a combination of these three. Such factors are based on the child's senses of motion and are thus identified as related to the sensori-motor mode of operation.

Some children and adults have the conception that objects that pass each other must have the same speed even if just for an instant. It is the passing object, however, which has the greater speed. This idea of objects having the same speed as they pass each other is persistent in some people right into adulthood (Trowbridge & McDermott 1980). While they may agree that as it is catching up it has greater speed they often insist that when the objects are 'level' with each other then they must have the same speed.

At some stage of the children's development, the concept of speed being related to distance and time starts to emerge. Some children know this from about the age of six whereas others probably never come to fully understand it. Most adults associate speed as the distance covered in a specified time but do not always see it as the quotient of distance and time. Often speed is referred to as the distance over the time taken. The word 'over' is not a precise term and some people have difficulty in stating what it means (McDermott & Trowbridge 1980, p. 1022).

An object that can overtake another has the greater speed. This is the earliest and simplest intuition of speed and is based on order (Piaget 1970). This intuitive idea is independent of duration and occurs before the child has an understanding of the components of speed, that is, distance and time (Kolodiy 1977). This concept is to be contrasted with the previous notion of passing, where it is believed that objects have the same speed when passing.

In summary, these four conceptions are not independent of each other and must be considered when examining children's responses to questions on speed. Children may employ these conceptions as possible strategies in solving speed problems.

**Direct/inverse relationships**

Piaget (1970), Acredolo, Adams and Schmid (1984) and Boulanger (1976) suggested that the start of the concrete operational stage of reasoning is characterised by children starting to think correctly about direct relationships. Even very young children (five or six years of age) have an idea about direct relationships (Wilkening 1981). Speed is an appropriate concept for the study of this relationship, as it incorporates both the direct variation and the indirect (inverse) variation.

Direct variation is associated with speed in two ways: if time is constant then an increase in distance implies an increase in speed; and, if speed is constant then increasing the distance increases the time. Inverse variation is associated with speed when the distance is kept constant then increasing speed implies less duration of time.

These variations are often referred to as compensatory operations, that is, "recognising the direction of change in the value of one variable due to the change of value in the other variable" (Boulanger 1976, p. 3). Piaget (1970) and others (for example, Raven 1972; Siegel & Raven 1971) have "demonstrated that the natural evolution of understanding proceeds from compensatory relationships to the quantitative symbolic form and not the reverse" (Boulanger 1976, p. 6). Thus, in the development of the concept of speed it is expected that children will understand the direct variation notion before they would be able to appreciate the symbolic form of speed being distance per unit time

The study of Acredolo, Adams and Schmid (1984) was also concerned with the interrelationships between speed, time (duration) and distance. The "central issue [of their study] is whether children recognise the direct relationship between speed, distance and duration ... before they recognise the inverse relationship between speed and duration" (p. 2151).

They found that:

> ... the direct relationship between speed and distance is recognised by the majority of children in second grade [6 years 8 months] whereas the inverse relationship between speed and duration is recognised later and understood less universally throughout middle childhood.
> (Acredolo, Adams & Schmid 1984, p. 2157)

In addition they found that "relationships involving distance are more salient than those involving speed or duration. ... The data suggested that duration was more salient than speed" (p. 2152). An unexpected result was that the children gave "more attention to or [had] greater faith in the direct relationship between duration and distance than in the direct relationship between speed and distance" (p. 2158).

Also, when asking children to explain how they arrived at their answer "there is no guarantee that verbal explanations mirror the line of reasoning taken in arriving at a judgement" (Acredolo, Adams & Schmid 1984, p. 2158). A study (Shire and Dunkin 1984) on strategies that young children (three to eight years) use to determine velocity (speed) of objects found that preschool children gave irrelevant reasons as to why they gave their answers, whereas the older children used both proximity and speed as cues to decide which object would arrive at its destination first. It has been shown that direct teaching using demonstration and manipulation by students can help them to a better understanding of compensatory concepts in speed, force and work (Siegel & Raven 1971). The questions used by Siegel and Raven were of the format: if one variable is kept constant and the second one is either increased or decreased how will this effect the third variable? Will it be the same, larger, or smaller? Why do you think so?

> The use of compensatory operations are dependant upon the ability to coordinate two or more variables that reciprocally oppose each other. At the beginning of concrete operations, partial correspondences can be made by the child if the factors vary in one direction and logical manipulation can be independently used when the variables reinforce each other.
> (Siegel & Raven 1971, p. 373)

The studies of Inhelder and Piaget (1958) found that seven and eight year old children could not coordinate factors that simultaneously have opposing directions nor could they acquire the idea of the possible equivalence of opposing variables. Also they found that, in the middle stage of concrete operations, the child can be assisted in solving problems with compensatory operations. It has been "suggested that the acquisition of this compensatory mechanism is a necessary condition to the development of proportional thinking" (Siegel & Raven 1971, p. 377). Children of age seven to nine years were "unable to independently focus on and separate the variables operating in the interactions" (Siegel & Raven 1971, p. 378).

Thus young children have elementary ideas about the concept of speed. They are aware of direct variations though they find it difficult to focus on all the variables in the problems. They are more aware of the idea of speed between two objects than either the distance or the time associated with these objects.

## SECONDARY STUDENTS

The literature dealing with secondary students concepts of speed is often not on simple concepts of speed but on studies dealing with learned notions of speed in the classroom. The study of Gorodetsky, Hoz and Vinner (1986) illustrated the type of investigations that take place at this level. In Israel, where their research was based, students as early as Year 8 are explicitly taught the formula for speed and how to solve problems related to it. In their study, 563 students from Years 9 to 11 were asked to solve questions similar to this example:

> Two cars start at the same time from city A. They go to city B, and return to city A without delay. The distance between the cities is 500 km. Car 1 goes to city B at the speed of 30 km/h and back at 50 km/h, and car 2 goes both ways at the speed of 40 km/h. Which car returns first to the city?
>
> (p. 565)

Questions such as this are not similar to the simple concrete situations used by Piaget, but they require the use of the formula $v = s/t$ and the algebraic manipulation of the formula. In the question above, the formula needs to be used twice on each car, for the problem to be solved successfully. They found that some students used the formal approach (that is using the formula), while others used more intuitive methods that often resulted in incorrect answers.

In the literature there appears to be a scarcity of research dealing with secondary students understanding of simple concepts of speed. This could suggest that researchers assume these students understand the concept of speed and look for more sophisticated notions to study, such as the meaning of velocity in the physics sense. For example, a New Zealand study by Jones (1983), using the instance-about-events technique developed by Osborne and Gilbert (1980), found that all students interviewed (Years 8 to 11) knew that an object that went a greater distance than another object, in the same time, went 'faster'. However, there appeared to be some students who thought that the object going faster was the one that was "catching up". He also found that from 80% of Year 8 to 41% of Year 11 students believed that when one object overtakes another then they are travelling at the same speed for a small interval of time.

Jones also categorised students' meanings of velocity into four groups, those who: did not know the meaning of velocity; stated that velocity and speed were synonymous; realised that velocity was different from speed; and, knew that velocity was speed in a certain direction. Only one student out of the thirty (all eleven to sixteen years of age) was able to give the correct response. In the discussion of these results Jones commented that "the students generally didn't understand what velocity was, even those students who had studied it in the formal setting" (p. 101).

One of the general characteristics of students' naive knowledge of motion is that their "concepts are poorly differentiated. For example, students (middle-school through to college) use the term speed, velocity and acceleration interchangeably" (Champagne, Gunstone & Klopfer 1983, p. 176).

It appears that not all secondary students develop the concept of speed at the same rate. Some of them still have elementary concepts of speed associated with younger children. There are, however, large numbers of students who do grow in their understanding and can associate speed with distance and time. How such knowledge is acquired is unclear, that is, is it taught or developed through experience out of school. What is clear is that most studies at this level are concerned only with students' understanding of what they have been explicitly taught.

## ADULTS AND TERTIARY STUDENTS

The studies that have been conducted with tertiary students and adults concepts of speed have focused on higher order concepts relating to further education in physics or mathematics. Sometimes, in the process of doing these investigations, the simple concepts of speed have also been addressed.

Adults are capable of perceiving velocity "directly and accurately" (Rosebaum 1975, p. 402) that is, they can view an object and extrapolate its motion as it passes out of view by a "direct extension of the movement that was seen" (p. 402). Thus adults usually have a stable idea of how speed can be used for simple everyday applications based on their experience.

In an investigation beyond Piaget-type experiments, Trowbridge and McDermott (1980) included two experiments to compare the speed of two objects: the first experiment consisted of two objects which passed each other twice and, the second experiment had two objects not passing (but with differing speed). They found that some college-age students believed that when two objects reach the same position then they must have the same speed. This occurred even when they knew that one object was going at constant speed and the second object was slowing down having started from a higher initial speed than the first. Thus some college-students still have the ideas that young children have concerning the speed of objects. It seems as if some people do not grow out of these ideas.

Some of these students also believed that if the two objects did not pass each other then they could not have the same speed. This occurred even when they had described the motion of one object as decreasing in speed to zero and the second object as increasing in speed from zero. Students seem to rely on the "perceptually obvious phenomenon of passing". Also they

> ... frequently do not relate their intuition of how fast an object is going to the ratio of the distance travelled to the elapsed time or to the idea of velocity at an instant. ... [This belief] seemed to remain intact in some students even after several weeks of instruction.
>
> (Trowbridge & McDermott 1980, p. 1027)

A hierarchical list for the development of simple concepts (space and time) and also for velocity was given by Trowbridge (1979) (these are reproduced in Appendix B). He found four levels in the hierarchy for velocity. Level one corresponds to the preoperational stage of Piaget, where children can not discriminate between speed and finishing points or arrival times. The second level is comparable to Piaget's concrete stage. Here the child can solve the problems if the motions are simultaneous by using the concept of overtaking or when the times and/or the distances are equal, but the child was not successful when the motion was in succession. Levels three and four include and extend Piaget's description of formal reasoning where the child could solve speed problems using ratio and could discriminate between average and instantaneous velocities.

Poduska and Phillips (1986) found a large percentage of college-age subjects who did not pass these Piagetian-type tasks. The only task that was given to students that was classically Piagetian was the circular, one-to-many speeds. In this task 41% of college students passed, where a pass was given only if the student could give an appropriate explanation for a correct answer. The other tasks that were given were more involved than the simple tasks of Piaget though they did involve the conservation of quantities.

In summary, it appears that not all adults have well developed ideas concerning speed. Many of them still hold onto alternative conceptions about speed and its relationships with distance and time. Many of their ideas are related to their previous experience, for example, travelling in cars.

## RESEARCH ACROSS AGE GROUPS

A number of studies have been conducted across different age groups in an effort to categorise the stages that individuals pass through as they develop the concepts associated with speed. Some of these studies are now considered.

### The Rule-assessment Method

A study that involved people from five years of age through to adulthood is that of Siegler and Richards (1979). They questioned Piaget's developmental Stage theory on three points:

Firstly,

> The account is inconsistently rendered; for example, in some passages 5-year-olds are said to confuse time with spatial points whereas in others they are said to confuse time with total distance.
>
> (p. 289)

Secondly, they questioned the methodology. Piaget claimed "that all three concepts [that is, spatial points, time and total distance] are mastered simultaneously", but he "never appears to have examined the same three children on all three tasks" (p. 289).

Finally, they noted that "Piaget's account of the intermediate period (Stage II) is vague with only a sketchy picture of the transition process emerging" (p. 289).

Siegler and Richards (1979) set out to replicate some of Piaget's work while at the same time attempting to eliminate the problems mentioned above. They used the "rule-assessment" method as one that would objectively assess children's knowledge of time, speed and distance.

> This approach involves formulating models of rules that children might use to perform various tasks and then formulating different types of problems that allow discrimination among the rules. Its virtues are that it is both revealing and reliable.
>
> (p. 289)

They designed three rules according to the first three stages that Piaget had developed. "Children using Rule I would judge the distance, time and speed solely on the relative stopping points of the trains" (p. 289). The train that was ahead would be judged "to have travelled farther, faster and for a longer time" (p. 289). If they stopped at the same place then they would be judged to have gone the same distance, time and speed. In Rule II, the children would judge as they did in Rule I except that for trains stopping at the same position they would "choose the train that started from further back". For Rule III, the children would "solve all problems correctly regardless of the positions of the stopping points" (p. 290).

They developed six basic problem types based on concrete experiments of trains travelling along parallel tracks. Each basic problem was presented to the subjects in four different ways, in random order, such that each concept (time, distance, and speed) was tested. For a subject to be considered as using a particular Rule, at least 20 of the 24 responses needed to be predicted by that rule.

They found that children, who used Rule II for trains that stopped at the same point, did not consider the starting position but rather they "consistently chose the train that stopped sooner" (p. 293). In their results of the speed concept, 38 of the 48 students and adults could be considered as using a rule. Seven of them used Rule I, "in which they judged speed on the basis of the stopping points" (p. 293). Six of these students were five-to-six years of age, and the other student was about eight years of age. Six students used Rule II, "in which they chose the first train to stop when the stopping points were equal and relied on end points when they were not equal" (p. 293). Four of these student were five-to-six years of age and the other two students were eight-to-nine years of age. Twenty-five subjects used Rule III, "in which they were consistently correct" (p. 294). Only four of these students were from eight-to-nine years of age, ten were eleven-to-twelve years of age and eleven were adults. Those students who were not classified as using a Rule appeared to be trying to use Rule III. Thus there appears to be a definite stage development taking place that is chronologically related.

Most of the results agreed with Piaget's findings,

> The only major departure from Piaget's findings regarding the initial and final knowledge states concerned with the age at which Rule III was attained; time and to some extent distance seemed to be mastered at much later ages than Piaget claimed.
>
> (p. 296)

Only adults and some eleven and twelve year old students were able to master time within the context of these questions. The major source of error on the speed concept was that students focused on the distance to determine speed, followed by those who focused on the end-points.

**Reactions to the Work of Siegler and Richards**

This work of Siegler and Richards was followed up by Acredolo and Schmid (1981) who replicated the experiment but with the added variation of trains travelling with either equal speeds or over equal distances or within the same time, as well as a mixture of all three variables. Many of their observations were similar to that of Siegler and Richards but with two notable exceptions. "The absence of an end-point end-time strategy among younger children for all three concepts and, for the duration concept, the absence of a distance strategy among older children" (Acredolo & Schmid 1981, p. 493).

Some of the results of Siegler and Richards (1979) were challenged by Wilkening (1981). He suggested that the reason that Siegler and Richards obtained the results of end-point end-time strategy was due to memory demands on children, that is, it was easier for children to fix on things that were visually present than to retrieve information from memory that was given or seen previously. For example, in young children the fixation on stopping places for comparing speed occurred because this information "was visually present at the moment of judgement; the information about other variables would have to be retrieved from memory" (p. 244). This was strong evidence for the sensori-motor stage where children relied on what was happening at a specific instant and were not reasoning from other information. Wilkening found that "young children do know that velocity, time, and distance are distinct entities and that [they] interact in some way" (p. 240). He stated that the research on children of five years of age indicated that they knew " - at least tacitly - that distance is directly related to time and velocity, and that time is directly related to distance but inversely related to velocity" (p. 240).

The process that enabled these children to make surprisingly 'sophisticated judgements' was revealed by observing their eye movements. "The children followed imaginary movements ... with their eyes for the time" required. When the time was up, "they pointed to that position on the scale where the imaginary movement had reached" (p. 241). Eye movements varied with the speeds which the children believed different imaginary objects would have moved. This leads to a conclusion that sensori-motor type responses, such as eye movements, may play a role in describing how children attempt certain speed questions. However, this strategy is particularly difficult to identify.

**Distorting Images**

A different approach to studying speed concepts in children was investigated by Acker (1983) who distorted the images of two trains. He showed some televised shots of two trains moving along parallel tracks to third and seventh graders and college students. The focal length of the lens, however, had been altered to make the trains appear different sizes and this gave the appearance of one train going faster than the other even though both trains went the same distance in the same time. Using Piaget's theory it was postulated that concrete operational people would be led to believe that the trains were travelling at different speeds and that the formal operational people would notice the relationship between the time and the distance and judge that they were going at the same velocity. It was found that: 56% of the third graders were in the concrete operational level; 79% of the seventh graders were in the formal mode; and, 98% of college students were in the formal mode. This did show that a transition from concrete to formal operational thinking occurs with age. But "a standard interpretation of Piaget's hierarchy would suggest that nearly all, rather than slightly over half, of the third graders should be using concrete operations" (Acker 1983, p. 346). An explanation for this is that "the task characteristics can either facilitate or inhibit the appearance and use of different modes of reasoning" (Acker 1983, p. 346). This means that children can arrive at the right result while using the wrong reasoning.

**Qualitative Studies**

Poduska and Phillips (1986) commented that:

> Piaget's model of intellectual development suggests that thinking about speed involves more than the physicist's definition of speed as a ratio of distance/time. Piaget proposed unique mental structures for thinking about speed. These speed structures are composed of a number of mental operations that are grouped together or coordinated to allow individuals to interpret movement in their environment. Speed structures are not data or facts, but rather mental operations used to transform, manipulate interrelate, and in general, process data relating to speed.
> (p. 842)

These operational structures about speed, first identified by Piaget, include elementary ideas such as placement and displacement which, when combined, lead to co-displacement and relative motion. In addition to these are the quantitative structures which allow measurements to be made using operations involving ratio and proportion through to the formula. A few studies have considered the use of the formula $s = vt$ (where s = distance, v = speed, t = time), in particular Gorodetsky, Hoz and Vinner (1986). They stated that a number of distractors or difficulties can hinder the student from applying the formula correctly. They noted eight characteristics:

> (1) the number of moving objects ... (2) whether the moving objects change speed and/or direction ... (3) the relation between the directions in which the objects move ... (4) the relation between the distances travelled by each object ... (5) the relation between the starting times and durations of the motion of each object ... (6) the relation between the speeds of the objects ... (7) the type and number of the unknown variables, and (8) whether it is necessary to introduce auxiliary variables or not.
> (p. 565)

Each of these characteristics needs to be considered when developing questions concerning speed and examining responses of students.

The ideas of the fastest object being due to proximity or 'the first one there' is based on a sensori-motor reaction on the part of young children. " ... Movement and speed give rise to a long elaboration of responses: at first sensori-motor, then intuitive, and, finally, operational" (Piaget 1970, p. 279). As the realisation grows that distance and time have something to do with speed, they have an intuitive idea of which one goes the faster when comparing the motion of two objects. Finally, the discovery of the scientifically defined relationships between speed, distance and time leads children into operational thinking.

Trowbridge and McDermott (1980) also observed that for a student to learn the correct concept there had to be an adequate connection between their previous concepts with which they were familiar and the new concepts. This was especially true for the academically disadvantaged student. They also found that students use of the technical vocabulary did not always mean that they understood the concepts involved, but "as students begin to disentangle one concept from another, the process is reflected in a more precise use of appropriate terms" (p. 1028).

Another study with a quantitative focus was conducted by Poduska and Phillips (1986). It was "based on the hypothesis that correct thinking about speed involves more than a simple distance-to-time ratio" (p. 842). It was hypothesised that students used a "coordinated group of mental operations called mental structures" (p. 842). The order of difficulty of the tasks used to test these mental structures was found to be (from least difficult) distance, asymmetric speeds (for example, uniform acceleration down an incline), one-to-many speeds (circular - with objects at different radii), symmetric speeds (different weights over a smooth pulley) and then time (p. 845). It was inferred that "the ability to pass the tasks in a certain sequence is based on intellectual development" since there was no "uniform program for teaching all students about the content of all the tasks" (p. 845). It is interesting to note that the mental structure of time was identified as the most difficult.

**Summary**

In 1928 Albert Einstein put the following question to Piaget "In what order do children acquire the concepts of time and speed?" (Siegler & Richards 1979, p. 288). Nearly twenty years later Piaget answered the question in his book "The child's Conception of Movement and Speed" (1946) (English translation in Piaget 1970). Piaget found that movement and speed are acquired before the concept of time. This was further verified by the investigations of Siegler and Richards (1979):

> Children do not understand the concept of time before they understand the concept of speed; in fact the reverse is the case, with speed being mastered well in advance of time. It has also become evident that neither time, speed, nor distance is an intuitive notion but rather that all three concepts undergo lengthy and regular developmental sequences.
> (Siegler & Richards 1979, p. 279)

Piaget's model was supported by the investigations of Poduska and Phillips (1986), they stated that "students develop at different rates but follow the same sequential steps" (p. 846). They also supported the theory that the "sequential order is inherent in the structure of knowledge rather than in some other factors" (p. 846).

Thus, there is evidence that the development of the concept of speed, especially when concrete experiments are used, is related to the age of the subjects. However, the age of an individual does not determine the sophistication of the concept employed by that individual when solving speed problems. Piaget's model of the development of the concept of speed is generally accepted by other investigators as a suitable framework, although there are still many gaps in the knowledge about children's understanding of speed. In particular these knowledge gaps appear to be related to children's use of strategies, the types of responses provided by older students and the way these responses fit into more recent frameworks in cognitive development. To further investigate these knowledge gaps is an appropriate area of study that will be addressed.

## CONCLUSION

This chapter has considered the research conducted by Piaget concerning motion in some detail. Six different studies were examined and each revealed stages that children progress through as they become older. The description of Piaget's work was followed by a review of relevant research that has been done on the concepts of speed for children ranging from early primary to tertiary levels of education.

The most detailed research on the concepts of speed have been conducted with young children. For example, Piaget's work has been very thorough in detailing the stages that children go through in building concepts of motion up to about the age of twelve years. Only a few studies have addressed secondary students and adults ideas of speed in any detail. It is appropriate, therefore, that further work be conducted in this area to find what ideas secondary school students have concerning speed and the strategies that they employ to solve problems about speed.

Some questions that arise out of this review which seem worthy of further investigation include: Do the findings of Piaget about the ideas that young children have on speed, still occur in older children? Do they still grow through different stages? Is distance a more salient factor for children than time in solving speed questions? Can a more detailed model for levels of development be produced? How do the strategies children employ in solving speed problems relate to their understanding of speed?

Before describing the research of this thesis, the next chapter reviews some of the important work that has been done on Concept Development and Levels of Thinking as it pertains to neo-Piagetian and post-Piagetian theories.

**CHAPTER 2**

# COGNITIVE DEVELOPMENT FRAMEWORK

Soon learnt, soon forgotten.
Proverb

## Introduction to chapter

The previous chapter reviewed the literature that pertains to children's understanding of the concept of speed. Much of the work in that chapter was based upon or around the experiments and ideas of Piaget. This work indicated that young children went through different stages of cognitive development that was related to their chronological age. Other studies indicate also that there are hierarchies associated with students ideas.

This chapter takes up this theme by considering neo-Piagetian theories which have grown out of Piaget's initial studies, and a post-Piagetian taxonomy that has its roots within the neo-Piagetian tradition. These structures provide vehicles for understanding the intellectual growth of students. Within the neo-Piagetian theories, consideration is given to investigations that have attempted to categorise levels of intellectual growth and ways students might think about knowledge. These are provided as examples of hierarchies, not as an exhaustive analysis of them. The post-Piagetian model explored is the SOLO Taxonomy of Biggs and Collis (1982, 1991).

## PIAGET AND NEO-PIAGETIAN THEORIES

In this section a brief description is given of Piaget's theory of cognitive growth before describing some neo-Piagetian studies and associated hierarchies that have been developed.

### Piaget's Theory

Consideration was given in the first section of Chapter 1 to Piaget's work as it relates to motion, speed and the structure of intellectual development. This structure, however, is not limited to the concepts of motion and speed. Piaget's other work on intellectual development (for example, Inhelder & Piaget 1958) reveal an overall structure in the general cognitive growth of children. The stages follow a similar structure to that of children's understanding of the concept for speed as outlined in Chapter 1.

In summary (see for example, Beard 1969; Lovell 1975; Driver 1983), Piaget found four chronological stages in cognitive growth.

#### Sensori-motor

The sensori-motor stage covers the period from birth to about two years. At birth, infants have no knowledge of the existence of the world or of themselves. As infants act in their environment their patterns of behaviour are modified and their sensori-motor systems become coordinated. While children grow physically during these two years, great intellectual progress is made as they interact with objects and relate experiences to these objects. Children's understanding of the world, however, is limited to properties of these objects and events that arise directly from other experiences related to them. Thus children develop and organise mental and physical activities as they grow.

#### Pre-operations

The pre-operational stage ranges from about two-to-seven-years of age. During this stage, children develop language while continuing to refine sensori-motor skills at the same time. Children view everything in relation to themselves (they are egocentric). The ideas of space and time are limited to their own 'world' and they have no conception of anything beyond it.

Concrete operations

The concrete operational stage begins around seven years of age and continues to about twelve years of age. As children interact with others their language becomes more refined and they are made aware of the world of others. The operations of children are performed directly on objects that themselves are either concrete or symbolic. This stage is characterised by: reversibility of thought; classifying objects into classes; and, trial and error methods in solving problems. Mental actions start to take the place of physical actions and internalisation of ideas begins. At the end of this stage children can make accurate observations from experiments.

Formal operations

The formal operational stage occurs in children who are older than twelve years. Students no longer use concrete operations to represent mental abstractions. This stage is characterised by: an ability to manipulate propositions; and, the consideration of combinations of variables. Students can set up mathematical models, especially those concerning ratio and proportion.

There has been concern expressed by some educationists (for example, Lovell 1975; Biggs & Collis 1991; Campbell, Watson & Collis 1992) about the rigorous ways in which Piaget's levels of thinking are applied to people. Chiappetta (1975), for example, claimed that up to 85 percent of people over sixteen years of age did not use the formal mode of thinking as predicted by Piaget but were functioning in the concrete operational stage. If this reflects the real situation, then some courses in upper high school and college are inappropriate for many of the students they serve since most of these courses are geared for students operating in the formal operational stage.

Since the work of Piaget and his colleagues has been published, many researchers have attempted to either confirm, adapt or challenge the results. From these studies has arisen theoretical perspectives which are collectively referred to as neo-Piagetian theories. These are similar to, but have important fundamental differences from, that of Piaget.

**Neo-Piagetian theories**

The neo-Piagetian theories incorporate important aspects of Piaget's theories as well as accommodate findings of more recent studies and research in cognitive developmental psychology. Like Piaget, the neo-Piagetians are interested in "cognitive competences of the developing person" but are more systematic in looking for "relations between competence and mechanism" and "context with cognitive growth" (Demetriou 1988, p. viii). These relations have immediate practical implications to the work of neo-Piagetians as compared to the theoretical thrust of Piaget's work. Neo-Piagetians bring the metaphors of mainstream psychology to identify ideas and concepts, and powerful statistical tools that help control variables that Piaget was not able to do. Another important feature of neo-Piagetians is the collaborative effort that is being made by many of them to try and understand each others' theories and to learn from them. Piaget, on the other hand, appeared to be in a one horse race "only against himself" (Sternberg 1988, p. 1).

One typical neo-Piagetian theory has been proposed by Case (1991). Case stated that "much of children's development stems from a change in their intellectual structures" (p. 344). There are three components to these control structures:

> a representation of the essential features of some particular class of problems, a representation of the goals that this class most frequently occasions, and a representation of a sequence of operations that will bridge the gap between the problem's initial and terminal state.
>
> (p. 344)

His theory implies that children, from different cultures, show different patterns of development. This occurs because of the types of problems they meet most often within their culture and how their culture provides models to help solve those problems. Also, the theory accommodates other factors such as soci-economic and emotional states that influence children's motives and methods for achieving their goals. The actions of children are limited by "a set of changes that are system wide and have a strong biological component" (p. 344). As these limits shift, so the control structures of the children progress through four recursive cycles for each type of topic on which the children are working. These cycles are referred to as: sensori-motor stage, interrelational stage, dimensional stage and vectorial stage. These have similar characteristics to Piaget's stages, described earlier.

Within each of these four cycles are three substages that children progress through.

> At the first substage, children assemble a new class of operations, by coordinating two well-established executive structures that are already in their repertoire. As their working memory grows and as they practice these new operations, they enter the second substage in which they become capable of executing two such operations in sequence. Finally, with further growth in working memory, and with further practice, they enter the third substage in which they become capable of executing two or more operations of the new sort in parallel, and integrating the products of these into a coherent system. Once consolidated, these integrated systems then function as the basic units from which the structures of the next stage are assembled.
>
> (p. 345)

Thus Case's theory has similar broad structures to that of Piaget's theory but there is some fine tuning occurring within the stages to accommodate the various levels of cognitive processing children use as they move from one stage to another.

The fine tuning that Case used within his stages is typical of the fine tuning that has occurred within other neo-Piagetian frameworks. The work of Fischer (1980), Biggs and Collis (1982) and Halford (1982) have each

> ... modeled children's mental functioning at a more microscopic level than Piaget had done, focusing on such constructs as 'skills', 'control structures', or 'symbolic systems'. However, each of these investigators also postulated general structural levels (i.e., levels of skill, or control, or symbolic functioning) that are hierarchically nested and whose attainment advances children's thought to a new level.
>
> (Case & Edelstein 1993, p. 6)

In the remainder of this section some of the theories that have attempted to suggest a hierarchy for intellectual development are presented with special emphasis given to those hierarchies that are related to mathematics, the physical sciences and, in particular, motion.

## HIERARCHIES IN MATHEMATICS AND SCIENCE

### Quality of Response

One method of attempting to determine students' cognitive levels is to consider the quality of their responses. An example of a scale of Quality of Response was developed by Renner (1979) for topics in mathematics using proportional reasoning. These were:

1. No response.
2. A mathematical manipulation without a class inclusion or confused explanations or factors which were invented by the student.
3. A grasp of the relationship between two quantities.
4. Recognition of the correct relationship but an inability to use this in the rest of the problem.
5. An attempt is made to apply the relationship to the rest of the problem.
6. Proper use of the relationship and an attempt to use sound logic.
7. Proper use of the relationship with a discussion of implications and other relevant variables.

In Renner's (1979, p. 279) study, four questions (incidents) were used that covered: proportional reasoning (two incidents); combinatorial logic; and, separation and control of variables. For each incident, a quality of response scale was developed similar to the one provided above. The level of response, given by students for each incident, was placed into a regression equation that had been developed to give a predicted 'Piagetian score' for each student. The average 'Piagetian score' over the four incidents was related to the stages developed by Piaget. Thus each student could be 'placed' somewhere within the Piagetian stages. If a student's score was between two stages then the student was considered to be in a transitional phase between these stages.

### Complexity of Questions

A particular aspect made clear in Renner's study was that the type of question determines the highest quality of response. While intellectual development can be deduced by observing the level of reasoning that is carried out by the individual, the level at which an individual is likely to operate is dependent upon the type of questions that are asked. Questions often set a certain level of reasoning for understanding. As an example, consider the following question:

> INSTRUCTION: Tell whether the following statements are ALWAYS true. If the statement is not always true, give an example in which it is false. In either case, explain your reasoning.
>
> THE QUESTION: On the freeway, if two cars can reach the same speed, then they MUST be side by side.
>
> (Trowbridge & McDermott 1980, p. 1024)

Trowbridge and McDermott in analysing this question commented:

> In order to answer a question of this type the students need to have developed both a certain conceptual discrimination and a special kind of reasoning ability. In particular, the meaning of the phrase 'reach the same speed' must be distinguished from the idea of 'reach the same place'. Secondly, the logical import of the 'if ... then ... ' construction has to be understood. The student must recognize the need for a counter example in order to negate the given statement and realize that the existence of a single counter example falsifies a 'must' statement. Such hypothetical-codeductive reasoning is an aspect of formal thought essential to success in the sciences.
>
> (1980, p. 1024)

So, by taking into account the complexity of the question, such that it does not detract from the knowledge that is being studied, it appears that the sequential order of intellectual development is inherent in the structure of the knowledge and not in some other factors like the school curriculum (Piaget 1970; Poduska & Phillips 1986). Hart (1981) pointed out that the sequential order of a hierarchy can be obtained in at least three ways, namely:

> (i) a learning sequence or sequences of understanding, which is essentially in the learner,
> (ii) a teaching sequence which the teacher uses,
> (iii) a logic sequence which is in the topic.
>
> (p. 205)

Thus care needs to be taken when examining hierarchies to see from whence they come and on what they are based. A worthwhile hierarchy that is useful to help organise the teaching topic needs to take into account the elements that are essential to the learner as well as the essential aspects of the topic. Teachers can then adapt their teaching sequence to take advantage of the level reached by the learner and the elements that compose the different levels within the topic.

**Levels of Thinking**

Lybeck (1979) and his colleagues found that students showed three categories of thinking. In the first, the explanations were simple and absolute. "The students base their explanations on one subsystem of the system. The viewpoint does not require them to think in terms of relationships between subsystems" (p. 122). In the second category the students took a relativistic point of view. "A feature of this category is that two subsystems are taken into account" (p. 122) but they may have some difficulties with the overall concept. In the third and final category the formal definition was known but some difficulties were encountered, especially when applied to some aspect of the everyday world that was outside of 'class-room physics'. Not only was the content a cause of conceptual difficulty but the context also provided a cause from which problems may arise (Lybeck 1979).

Applied Mathematics and Physics are subjects in which students have great difficulty in understanding the concepts involved whether it be due to content or context. Barnes and Barnes (1978) used Piaget-type questions before and after such courses, and they found that "no developmental progress is made" when students take introductory college physics courses for one semester.

One reason suggested for this lack of development was that the physicist and the student were thinking and communicating at two different levels. Fuller, Karplus and Lawson (1977) suggested that physicists use certain reasoning patterns more often than others. These patterns concern focusing, propositional logic, and proportional reasoning. They proposed that by using the work of Piaget it might be possible to understand why such reasoning occurred, and see how growth of the concept came about. As a result help could be provided to students of physics. This problem is not only true for physicists but occurs in other areas of knowledge where the teacher is thinking at a different level from that of the student and thus communication is on a different level.

Reif (1986) pointed out that

> Experts use some important knowledge so frequently that they take it for granted and do not teach it explicitly. Indeed, one can trace many student difficulties to a lack of knowledge or skills that were never explicitly taught.
>
> (p. 49)

This indicates strongly that the expert and the student are operating in entirely different ways of thinking.

> ... students' conceptual knowledge lacks coherence. For example, students usually interpret a scientific concept by retrieving miscellaneous associated knowledge fragments, many of which are incorrect; they rarely invoke a definition of a concept, usually cannot interpret it adequately even if they do; and they have no effective ways of resolving inconsistencies that they encounter.
>
> (Reif 1986, p. 50)

One way of categorising these different levels of thinking, between the expert and the novice, is to use Piaget's stages. This may be done by considering the novice as operating in the concrete mode and the expert operating in the formal mode. Fuller, Karplus and Lawson (1977) have described characteristics of the concrete and formal operational thinker.

Some characteristics of concrete reasoning include the ability to satisfactorily consider: class inclusion, conservation, serial ordering, and reversibility. A person with this level of reasoning is able to: understand concepts and simple hypotheses that make direct references to familiar actions and objects, and have simple associations; follow step by step instructions; and, to relate his/her own viewpoint to that of another in a simple situation.

The limitations of concrete reasoners are that they: search for, and can identify, the variable that influences a phenomenon but the approach is unsystematical; make observations, and draw inferences from those observations without considering all possibilities; respond to difficult problems by applying a related, but not necessarily correct, algorithm; and, process relevant information, but are not aware spontaneously of their own reasoning.

Formal reasoners, on the other hand, have the following characteristics: combinatorial reasoning, i.e., they consider all possible relations of experimental or theoretical conditions even though some may not be realised in nature; control of variables, i.e., all variables are considered and kept constant except the one being investigated; concrete reasoning about constructs, i.e., they apply concrete reasoning to concepts and abstract properties; functional relationships, i.e., interpret relationships into a mathematical form; probabilistic correlations, i.e., take into account the randomness that occurs in nature (Fuller, Karplus & Lawson 1977, p. 26).

Fuller, Karplus and Lawson (1977) commented also on the stages that were reached by students.

> Piaget's original notion was that all persons use formal reasoning reliably by their late teens. Yet recent studies strongly suggest that, although almost everyone becomes able to use concrete reasoning, many people do not come to formal reasoning reliably.
>
> (p. 26)

Thus the physicist's

> Fixation on the formal aspects of physics instead of its concrete experiences has made physics unnecessarily difficult and dry. We have moved the sense of exploration and discovery from the study of physics for the majority of students
>
> (p. 28)

Such statements as these may help to explain why students in Australia (and elsewhere) are often found to have a lack of interest in the physical sciences in Years 10 and 11 at school.

**Students' Concepts**

It is well documented that for success to occur in helping students to develop proper scientific concepts, and thus experience cognitive growth, they must first have their previous concepts and ideas challenged in some way (Acredolo & Schmid 1980; Gunstone & White 1981; Hewson 1981; Pines & Leith 1981; Gunstone, Champagne & Klopfer 1981; Tasker 1981; Rennie 1981). Some other studies that have also focused on students concepts of motion are: Gaskell 1973; Lebout-Barrell 1976; Sjoberg 1981; Gilbert, Watts and Osbourne 1982; Clement 1982; Watts and Gilbert 1983; McDermott 1984; Halloun 1985; Osbourne 1985; Minstrell and Stimpson 1986a, 1986b; Brown and Clement 1987; Clement, Brown and Zietsem 1989; Chee 1989; Steinberg 1990.

An example of different concepts students can bring to class is provided by Hewson (1981). He identified five different types of conceptions that students can have when interacting with science concepts. These are:

- **the undisturbed children's science outcome** in which the alternatives brought to class persist unchanged by science teaching;

- **the two perspectives outcome** where both views exist, one to be used at school, the other in everyday occurrences;
- **the reinforced outcome** the alternative view is strengthened by science teaching which is now misapplied to support it;
- **the mixed outcome** both views coexist in ways which are not integrated and may be self-contradictory;
- **the unified scientific outcome** where a correct understanding of the concept within in its scientific setting is understood.

When dealing with students, it is important to be aware of these conceptions so as to understand students' responses and to help in the analysis of those responses.

The different ways students use concepts is often an indication of the level at which they are operating. Therefore, it is important to know what is meant by a concept. Many authors give meanings and definitions of a concept. One of the more fitting definitions is that given by Pines and Leith (1981):

> When a symbol, such as a word, is used to designate a regularity among objects, events, or other concepts, so that this regularity can be communicated and thought about, we have the major ingredients of a concept. Two or more concepts that are related form a proposition. A concept is a locus of meaning - a sort of summary of all the propositional relationships in which that concept participates.
>
> (p. 15)

Care needs to be taken, nevertheless, when assuming that if students use the correct symbols that they also have the correct concepts.

> What a child hears and learns depends upon what he or she already knows. ... Children hold tenaciously to their misconceptions [alternative frameworks] because these are not freak pieces of knowledge, but instead are parts of well established frameworks that have been slowly constructed through experience.
>
> (Pines & Leith 1981, pp. 17, 18)

Thus in the study of motion

> Students do not come to the study of mechanics with a blank slate. They come with prior experience and with a good practical understanding of how objects move.
>
> (Green, McCloskey & Caramazza 1980, p. 4)

The students' interpretations and explanations of these motions, however, often get confused when an unfamiliar motion needs to be predicted. Such ideas can create problems in the formal learning situation:

> ... students of all ages have acquired concepts and primitive theories about the world. From a modern scientific perspective, their conceptions may appear to be misconceptions, and their primitive theories are often naive and fragmentary. Moreover, these theories are highly resistant to change and present major obstacles to new learning. The realization that learning requires the restructuring of preexisting knowledge has important implications:
> - Science curricula cannot merely specify what knowledge should be taught, but must also identify and deal explicitly with students' prior notions.
> - The task of restructuring a student's preexisting knowledge is often more difficult than that of simply imparting new knowledge - and thus requires more sophisticated teaching methods.
>
> (Reif 1986, p. 49)

**Controlling Variables**

One aspect, important to determining the level of a person's intellectual development, is how they control changing variables in simple everyday situations where more than one variable is changing at a time. Research on human information processing has demonstrated that there are limits to the number of pieces of information a person can keep in mind and 'work on' at one time (Miller 1956; Baddeley & Hitch 1974; Norman 1976). This capacity limitation appears to take a toll in complex reasoning tasks where the difficulty in solving a problem is often directly related to memory load (Simon & Kotovsky 1963, Denny 1969; Case 1991). This aspect of control of variables within a problem is an important feature that needs more attention.

**Factors that effect determination of levels**

The techniques of obtaining data do elicit different types of knowledge. Those that are conceptually framed "tend to elicit propositional knowledge, whereas those that are framed by a situation or phenomenon elicit knowledge-in-action" (Driver & Erickson 1983, p. 46).

Students who are known to possess formal thought ability described by Piaget often prefer to use the concrete operational methods of solving certain types of problems (Dunlop & Fazio 1977 . This tends to frustrate any attempt to determine the level at which students are capable of operating, because both the 'concrete' and 'formal' student provide similar answers. Only by asking the right type of question, that triggers formal type of responses, can the optimal level of some students be revealed.

Research that identifies where students have had difficulties with scientific concepts has been summarised by Labudde, Reif and Quinn (1988, p. 81). The difficulties that they found were in four broad categories: a knowledge base that is fragmented, incoherent and prone to misconceptions; unsystematic or inefficient search and retrieval processes; an inability to apply knowledge appropriately after it has been retrieved; and finally, a failure to distinguish between concepts and reasoning modes used in science versus those used in everyday life.

In concept interpretation, Reif (1987, p. 312) identified three knowledge types which they called formal, compiled, and joint formal and compiled. The formal type of knowledge is "highly explicit and coherent, deliberately designed to make concept interpretation reliably unambiguous, precise, consistent and general". The compiled knowledge consists of a repertoire of knowledge "about various important special instances, or type of instances of the concept". This knowledge is stored in the memory and is usually retrievable by automatic key indicators in the problem. Its advantage is that it is 'fast and effortless'. Joint formal and compiled knowledge makes concept interpretation "reliably effective and efficient". It allows a quick solution of familiar situations but it permits unfamiliar situations to be dealt with in a reliable fashion by using formal thinking.

Reif made the following observation

> Novice students tend to store lots of special knowledge and then do relatively little processing after retrieval of such knowledge. On the other hand, experts tend to store smaller amounts of knowledge which are more general, but which, once retrieved, requires more reasoning to be applied in a particular case.
>
> (Reif 1987, p. 317)

Thus when analysing the responses of students, care needs to be taken in not confusing the ideas of 'experts' with that of 'novices', since both can offer the same response. Only on extended explanation will the difference be apparent.

In summary, the neo-Piagetian studies offer hierarchies that have common features. They consider responses that range from the simple to the complex, from poor conceptual backgrounds to those of the expert. There are as well an array of features that can confuse attempts to categorise levels of cognitive development, such as the type of question presented, the quality of response students choose to use, prior learning and teaching, and the number of variables used in the problem.

**Strategies**

Another fruitful area of cognitive development studies is that of the choice of strategies that students use in solving problems (see Siegler 1989(a), 1989(b), 1990, 1991; Siegler, R. S. & Jenkins, E. A. 1989; McGilly & Siegler 1989). Another way of deciding whether a response given by a student was at one level or another was to code the sophistication of the strategy that was employed by the student in solving the problem. Some experimenters have developed models that postulate that certain strategies are used by children at a particular age or stage of development. Siegler's work on children's strategies in doing simple adding has called these models into serious question. He found that students, who have the same age, use a variety of strategies in solving the same problem. Even the same student working on similar questions uses different strategies when asked these questions at different times.

> A view of development that depicts children as actively choosing among multiple competing strategies may turn out to be more generally accurate than the traditional view in which a child of a given age is depicted as invariably using a particular approach.
>
> (p. 94)

There is evidence that adults also seek alternative strategies when answering questions (Reder 1987). The reason people choose different strategies is that

> ... strategies differ in their accuracy, in the amounts of time needed to execute them, in their processing demands, and in the range of problems to which they apply. Strategy choices involve tradeoffs among these properties.
>
> (Siegler 1991, p. 90)

Each of these factors is important when considering quality of student responses in attempting to ascertain levels of cognitive development. The tradeoffs that students use among these properties determines the response that eventuates.

"Piagetian and neo-Piagetian approaches have found unities primarily in structural similarities in reasoning across tasks" (Siegler 1991, p. 100). Siegler's work has found other unities as well.

> Other cross task unities: unities in children's use of multiple strategies on varied tasks; unities in the adaptive quality of their strategy choices from quite young ages through adulthood; unities in individual difference patterns across domains; and most important, unities in the processes that produce strategy choices and strategy discoveries in different domains.
> (p. 100)

Thus, in analysing students' results it is important to consider the strategies that are employed and not to group intellectual levels based on any one particular strategy. The variety of strategies that a student can call upon when solving similar questions is indicative of the optimal level at which they might function.

**Conclusion**

Case (1991) highlighted dilemmas that confront neo-Piagetian theories.

> While neo-Piagetian theory provided a set of formalisms for analysing children's intellectual *processes* across domains, it offered virtually no guidance as to how to analyse the knowledge or other products to which these processes might lead. ... A second and ultimately more formidable problem was that the sort of results neo-Piagetian theory had led us to *expect* in various domains of development were not always what we discovered. In their place we often found phenomena that at first appeared completely different, yet on closer inspection were not so completely different as to warrant a rejection of the neo-Piagetian framework entirely.
> (p. xiv)

Thus the framework offered by Piaget is resilient enough to be used as a base for other research and the neo-Piagetian frameworks have been able to refine Piaget's work in an effort to explain some issues. Sometimes this has been achieved successfully and other times unsuccessfully but in the latter case it is not enough to reject these new frameworks.

The few studies that have been mentioned above that attempt to categorise students' concepts, all refer to similar characteristics in dealing with cognitive development. These range from the simple and concrete levels of thinking to the more formal and abstract levels of reasoning. These levels are usually based on at least two ideas: (i) the use, or lack of use, of relevant variables; and, (ii) the operations that are used on those variables. Such operations also involve different strategies that are applied to the same problem by different students. Does this then imply that there may be some broad framework from which to interpret the selection or use of different strategies? By working within a set theoretical perspective that considers, in a detailed way, students' responses it might be possible to answer this question.

The SOLO Taxonomy of Biggs and Collis (1982, 1986, 1991) is one system that has similar features to the above structures as it is within the broad Piagetian framework but it addresses also many of the concerns raised previously. (The acronym SOLO stands for Structure of the Observed Learning Outcomes.) It attempts to combine and systematise these structures across curriculum areas in a way that is internally consistent. The next section of this chapter considers the SOLO Taxonomy in some detail.

## THE SOLO TAXONOMY: A POST-PIAGETIAN FRAMEWORK

The SOLO Taxonomy (Biggs & Collis 1982, 1986, 1991) provides a useful framework to interpret student levels of understanding in various subject areas. The taxonomy has grown as a reaction to many of the problems identified in the Piagetian and neo-Piagetian literature. It has a similar broad structure to that of Piaget's theory, but since its initial conceptions in 1982 the descriptions of the SOLO Taxonomy have become more refined and sophisticated. The emphasis of the SOLO Taxonomy (as it will be amplified later) is classifying the responses that students give, rather than classifying the student at a particular level, as in Piaget's model. This classification has two dimensions: first, in terms of five modes of thinking and second, in terms of levels of understanding within each mode.

In the following sections consideration is first given to a brief description of the taxonomy, followed by a discussion of its relevance as a suitable framework to this study.

## THE MODES

The SOLO Taxonomy is classified into five stages or modes of functioning: sensori-motor, ikonic, concrete symbolic, formal, post-formal. Each mode is progressively available to individuals as they become older, such that the preceding modes are always available for the person to use.

### Sensori-motor

The sensori-motor mode is available from birth. It shares the same features as described by Piaget in his first stage. As such, this mode is related to the movement and co-ordination that is involved in reacting to the physical environment. In the earlier years this involves "skilled motor activity such as grasping, crawling, walking, running and jumping" (Collis & Biggs 1991, p. 188). This mode is available in the later years of development as more complex and practised actions take place, from playing tennis to gymnastics. The sensori-motor mode incorporates actions that feel right without having to describe the details of the "how" and "why". Such knowledge is "well described as *tacit* knowledge because it consists in knowing how to carry out an act without necessarily being able to describe or explain it" (Collis & Romberg 1991, p. 89).

### Ikonic

The ikonic mode is available from about 18 months and is similar to Piaget's second stage, pre-operations. This mode is characterised by the imaging and imagining of an action as an object; it is helped by language, and revealed through oral communication. Words and images stand for objects and events. It is in this mode of thinking in which Bruner (1964) said that the individual was able to form internal pictures, images or 'ikons' to aid in thought processes. Within the ikonic mode of functioning, children develop verbal communication skills that reach adult level in terms of its structure. These skills are then ready to be utilised for propositional reasoning. There is also a heavy reliance on imaging, often affect-laden, which lays a sound basis for much of the future development in both cognitive and non-cognitive areas.

Not only is the ikonic mode of thinking used by young children but it is utilised by adults in quite highly sophisticated forms such as that employed by mathematicians and scientists to find new ways of displaying knowledge (Biggs & Collis 1991,

p. 11). Such thinking is not to be "confused with geometrical, graphical or pictorial illustrations which form part of the concrete symbolic system of representing the aspects of the environment" (Collis & Romberg 1989, p. 10). It is common for a person to be able to find a solution for a problem using this mode, that is, by using images and visualisation, then justifying this solution using a higher mode.

> Ikonic thinking leads to a form of knowledge commonly known as 'intuitive' which is highly valued by creative thinkers in both the sciences and the arts.
>
> (Collis & Romberg 1991, p. 89)

The ikonic mode "is the basic mode for intuitive thinking" (Collis & Biggs 1991, p. 7). Within the context of elementary mathematics the ikonic mode has the following major characteristics (Collis & Romberg 1991, p. 113):

1. Intuitive - provides qualitative insights into a problem structure.
2. Forming of images - allows for visualisation of situations, concrete structures which leads to individuals making qualitative diagrammatic representations of elements of the problem.
3. Communication - "tends to conform to the oral structure of everyday language. Rather than being propositional, it is descriptive and closely tied to expressing feelings and intuitive and aesthetic associations".
4. Affective mode - includes "interest, perseverance, confidence, valuing" which effect an "individuals willingness to pursue mathematics study".
5. Social variables - implies the "capacity to work with others on mathematically related tasks".

Collis and Romberg added:

> It is clear that the ikonic mode can be used alone or in conjunction with other modes. It is also clear that it has been much neglected and deserves considerable attention in its own right in the mathematics classroom. Ability to use this mode is shown by a willingness to approach a problem with a relaxed and interested attitude similar to that of the preschool child's approach (Collis 1988). There should not be a feeling that mathematics belongs to the concrete symbolic mode and that a problem must be immediately translated into this form.
>
> (1991, p. 114)

That 'real' mathematics can likewise take place in the ikonic mode is a salient feature of the SOLO Taxonomy.

**Concrete symbolic**

The concrete symbolic mode is available to most students at about six years of age. In this mode, concepts and operations are experienced through the medium of symbols which are tied to the real world, for example, written language, mathematics symbols, maps, musical notation. "These symbolic systems have a logic and an order both within the system itself and between the symbol system and the world" (Collis & Biggs 1991, p. 11). One of the major tasks of compulsory schooling is teaching students the mastery of these systems.

Within the context of elementary mathematics the characteristics of the concrete symbolic mode are:

> 1. This is the mode in which *symbol systems* such as elementary mathematics are developed ... There is a logic and order in this system that allow for both independent and interdependent manipulation without breaking the essential direct link with the empirical world...
>
> 2. Mathematical *concepts* are the elements of thought for the concrete-symbolic mode. They are of a higher order of abstraction than the signifiers of the ikonic mode but are still directly related to the empirical world.
>
> 3. *Communication* is via propositional statements using the rules and concepts of the system.
>
> (Collis & Romberg 1991, p. 114)

A large part of school mathematics is concerned with developing this mode, but often more than one mode is used in problem solving; that is, both the ikonic and the concrete symbolic modes. Thus, in examining student responses one would possibly expect to find a mixture of ikonic and concrete symbolic responses not only across responses, from students of the same age, but also within the responses of the same student.

**Formal**

The formal mode becomes accessible for many students after sixteen years of age. In this mode, abstract thinking is important where the elements of knowledge do not necessarily refer to 'concrete' reality. The student can relate, not only to what is seen to be real but also, to what may be possible. Teachers in the upper secondary school and early university level are concerned with having students think in this mode in specific content areas.

**Post formal**

This mode is the most difficult to describe. It is believed to be rarely attained, and, if so, an individual would be in excess of twenty years of age. To do so students "must have an overview of their discipline such that they can challenge its basic tenets and conduct research to advance understanding in the area" (Collis & Biggs 1991, p. 9). Thus, this mode is dealing with those who are doing research in a discipline in restructuring knowledge at its very roots.

Overall, these five modes (or stages) define broad boundaries that are typically reached; at any given age prior modes are still available. On any given topic or question, therefore, individuals may respond in any of the modes that are available to them. This makes it difficult for an investigator to determine which is the highest mode at which an individual can operate. The emphasis is not on the individuals' level of performance, but rather on the level of response that is given for that topic at that time. Children do not abandon the use of the ikonic mode upon entering the concrete symbolic mode, but the former mode provides a necessary support for the latter (Campbell, Watson & Collis 1992). Similarly, it would seem that it is possible to identify ikonic and concrete symbolic support in formal responses.

## THE CYCLES

It will be noticed that the above modes are similar to Piaget's broad stages of cognitive development. However, of great significance is that Biggs and Collis have identified a series of levels which are not only associated with each mode but have the same characteristics within each mode, that is, they have identified recurring cycles which help classify responses as they become more complex as modes develop (Collis 1985). This learning cycle is referred to by a series of levels: Prestructural, Unistructural, Multistructural, Relational and Extended abstract.

**In the Concrete Symbolic Mode**

The examples and descriptions given below are provided with respect to the concrete symbolic mode, though the general principles apply across all modes.

Prestructural

Prestructural responses occur when students have appeared to use very little working memory. There is a lack of logical or sequential thinking. The cues in the question and the responses are likely to be confused and may be irrelevant to each other leading to three types of confused responses: 1. "Denial"- the response is likely to be "I dunno" or simply a guess. 2. "Transduction" - more like a "guesstimate" which is based on one cue that is most likely perceptual or emotional. 3. "Tautology" - a restatement of the question.

"The prestructural response is marked by very high closure and very low consistency" (Collis & Biggs 1979, p. 19). Quick closure is more likely to result in an inconsistent response to the original cues and lead to irrelevant answers.

Unistructural

Unistructural responses represent the use of only one relevant aspect of the mode; students "seize upon the first relevant dimension that comes to mind - but it is at least relevant" (Collis & Biggs 1979, p. 19). More working memory is required as students have to focus on the question and choose one cue that is sufficient for them to close on a response. As in the prestructural response, students closes quickly on this one cue from the many that may be available.

Multistructural

Multistructural responses represent several disjoint aspects, often in sequence; the closure to the answer is delayed while more information is received, but since the different aspects of this information are not necessarily inter-related, inconsistencies result. This type of response takes greater working memory since not only must the question be kept in mind but a number of possible aspects of the answer are considered as well.

Relational

Relational responses involve several aspects related into an integrated whole; there is "a definite answer (closure) and while this may be an excellent answer for that context, it will not do for other contexts - that is an over-generalisation may be made" (Collis & Biggs 1979, p. 19). At this level students not only have to hold the cue and the relevant data in working memory but they must also interrelate and process these data.

Extended abstract

Extended abstract responses take the whole process into a new mode of functioning. In this mode

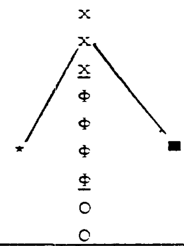
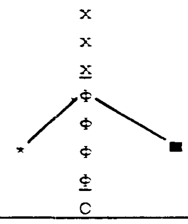
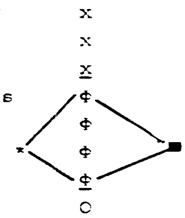
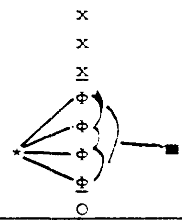
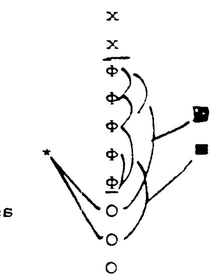
> . . . the student here needs not only to encode the given information, but to comprehend its relevance to over-riding abstract principles, from which he can deduce a hypothesis, and apply it to a situation that is not given: all of this obviously requires a larger memory capacity than the other SOLO levels.
>
> (Collis & Biggs 1979, p. 17)

The relevant features of the extended abstract level of the concrete symbolic mode are that the student can consider an abstract principle; make deductions from that principle; use analogies that are compatible with that principle; and "the outcome may be indeterminate, that is, there is an absence of closure (events might have been different under different circumstances)" (Collis & Biggs 1979, p. 18). Thus the outcome is open and alternative possibilities exist, depending on the particular context.

To determine at which level a response is likely to occur, within a mode, is found by considering the following four main variables: Capacity or memory space (M-space), this is related to the amount of information that can be held and operated with in the working memory; Relating operations or organising dimensions, concerns how information, such as cues and responses, are inter-related; Consistency and closure includes how complete and consistent the processing of information is in the response; and Structure, a diagrammatic representation of the nature of the responses showing the data that is used or not used for a particular response (Collis & Biggs 1979, Biggs & Collis 1980, 1982)

Conclusion

Figure 2.1 provides an overview of the SOLO model and response structure. This figure is adapted from Biggs and Collis (1982) and the symbols in the legend have been altered from that given originally.

| SOLO description | Working Memory capacity required | Logical operations involved | consistency and closure | Response Structure Cue Response |
|---|---|---|---|---|
| Prestructural | Minimal: cue and response confused | Denial, tautology, transduction, bound to specifics | No felt need for consistency Closes without even seeing the problem |  |
| Uni-structural | Low: cue + one relevant datum | Can "generalize" only in terms of one aspect | No felt need for consistency, thus closes too quickly: jumps to conclusions |  |
| Multi-structural | Median: cue + isolated relevant data | Can "generalize' only in terms of a few limited and independent aspects | Although has a feeling for consistency can be inconsistent because closes to soon on basis of isolated fixations on data, and so can come to different conclusions with same data. |  |
| Relational | High: Cue + relevant data + interrelations | Induction. Can generalize within given or experienced context using related aspects | No inconsistencies within the given system, but since closure is unique inconsistencies may occur when going outside the system |  |
| Extended Abstract | Maximal: cue + relevant data + interrelations + hypotheses | Deduction and induction. Can generalize to situations not experienced | Inconsistencies resolved. No felt need to give closed conclusions - conclusions held open or qualified to allow logically possible alternatives ($R_1$, $R_2$ or $R_3$) |  |

LEGEND

- x represents inappropriate or incorrect data
- ⋆ data given with potential to cue a response
- Φ concepts processes and/or strategies expected within the "Universe of discourse"
- o abstract concepts, processes and/or strategies within "Universe of discourse" but additional to those expected as part of understanding of the question.
- ■ responses - both intermediate and final

Figure 2.1 SOLO model and response structure (Adapted from Biggs & Collis 1982).

"Perhaps the most outstanding feature of the above model is the marriage between the cyclical nature of learning and the hierarchical nature of cognitive development" (Collis 1985, p. 10). For example, the prestructural level of the target mode could be in the previous mode while the extended abstract level of the target mode is reaching into the next mode.

Not all responses fit appropriately into these levels, transitional responses occur when a student is "feeling" for the next level. A typical transitional response has more information than the lower level requires but the student appears to get lost in the argument without closing on an answer. Collis and Biggs (1979) warned that "a well-formulated transitional response at a lower level can be mistaken for an extended abstract response, which likewise doesn't reach closure but for a very different reason" (p. 20).

To illustrate the increasing complexity of responses as one proceeds through the levels, Biggs and Collis (1980, pp. 21-22) provided an example of students solving a simple equation. It was not the solution itself that shows the level but the method of justification. The students were asked to solve the equation $y + 4 = 9$. Typical responses were:

*Prestructural.* *We have not learned those yet.*

or

*five, that is my favourite number.*

In such responses "the response or justification is irrelevant in the context".

*Unistructural.*

Child: *Five.*
Experimenter: *Why?*
Child: *Because five and four make nine.*
Exper: *Could you do it by saying y = 9 - 4?*
Child: *No. that's a different sum altogether.*

The student has used a counting procedure and sees no relation between the addition and subtraction operations.

*Multistructural.*

Child: *Five.*
Exper: *Why?*
Child: *Because four from nine equals five.*

But the child could not explain why $9 - 4 = 5$ is equivalent to $5 + 4 = 9$

Child: *That is what we are taught.*

The child recognises the steps but there is no integration of operations.

*Relational.*

Child: *Five.*
Exper: *Why?*
Child: *Because* $9 - 4 = 5$.

The child could recognize that $9 - 4 = 5$ was equivalent to $5 + 4 = 9$ and saw them as the same thing, that is, the child was able to see reversibility of operations but was "unable to examine the abstract logical basis for the process".

*Extended Abstract.*

Child: *Five.*
Exper: *Why?*
Child: *Because* $9 - 4 = 5$.
Exper: *Why does that work?* $y + 4 = 9$ *does not look much like* $y = 9 - 4$.
Child: *Well, we have got to keep things balanced all the way and so you could think like this:* $y + 4 = 9$, *therefore* $y + 4 - 4 = 9 - 4$, *subtracting four from both sides.*

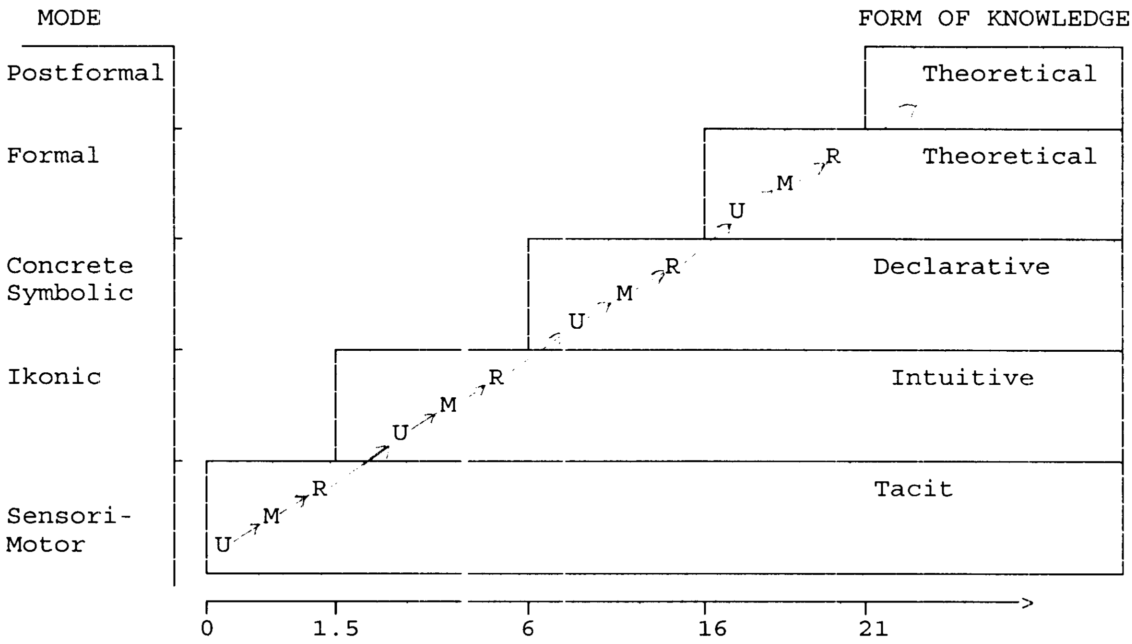
The child responding at this level has a good understanding of the abstract notion of the inverse operation. The child is prepared to consider relevant ideas that are not specifically given, such as 'balance', as a general principle and to withhold final commitment to a definite result until a formal logical rationale can be provided.

**Cycles and Modes**

The cycles encompass two aspects, namely, cycles that occur across modes and, cycles that occur within modes.

Cycles across modes

The levels of responses that have been described above, within the concrete symbolic mode, occur within the other modes of functioning. As progress is made from one mode to another, the unistructural, multistructural and relational levels are encountered. The extended abstract level is equivalent to the unistructural level of the next mode. With the focus of the responses in each mode being related to the characteristics of that mode. An overview of the modes and cycles of the SOLO Taxonomy is provided in Figure 2.2. (An earlier model of the SOLO Taxonomy is given in Appendix C which illustrates the development that is continuing to occur.)



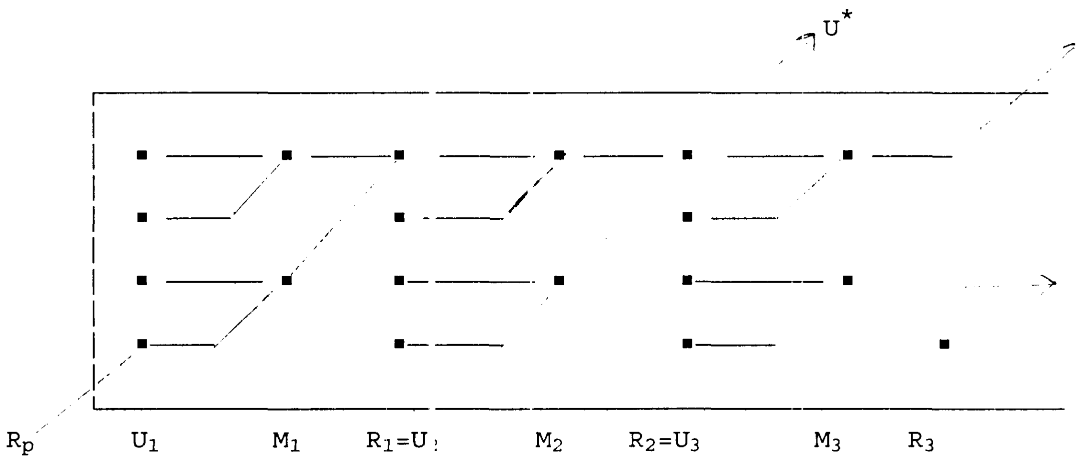

AGE (Years not to scale)

Figure 2.2 Modes, Learning cycles and forms of knowledge (From Biggs & Collis 1991, p. 66).

Cycles within Modes

More recent studies concerning the SOLO Taxonomy suggests that there may be recurring cycles within each mode that repeat before the next mode is reached (see, for example Pegg 1992; Levins & Pegg 1993; Campbell, Watson & Collis 1992). For example, in the concrete symbolic mode, the cycle of unistructural, multistructural and relational repeat at different levels of complexity. Whether there is one cycle in the mode or many is dependent on the knowledge that is being examined. Campbell, Watson and Collis (1992), in their study concerning the development of the concept of volume measurements by students, suggested "... that a second such learning cycle takes place in high school, with the second building on the achievements of the first" (p. 291). They have developed an intra-modal development model (Figure 2.3) that suggests that the unistructural level of one cycle is the same as or equivalent to the relational level of the previous cycle (p. 296). Also, their work suggests that the development of the ikonic mode continues in parallel with corresponding development of the concrete symbolic mode.

The model in Figure 2.3 also indicates that a number of unistructural responses combine to give a multistructural response within each cycle. Then different multistructural responses related together yield a relational response.




KEY
$U_*$ Unistructural level of target mode
$U^*$ Unistructural level of the following mode
M Multistructural level of target mode
R Relational level of target mode
$R_p$ Relational level of preceding mode
1, 2, 3 the number of cycle

Figure 2.3 A model for intra modal development as given in Campbell, Watson and Collis (1992).

Studies by Levins (1992), and Levins and Pegg (1993a, 1993b) likewise indicate that in the areas of science and mathematics at least two cycles within the concrete symbolic mode can be identified in student responses. Of interest, was the question that arose out of their work: "Are responses categorised as $R_1$ and $U_2$, within a given mode, equivalent or are there some qualitative differences in the answers which reflect different understandings?" (p. 11).

## DISCUSSION

The preceding discussion has identified the basic parameters of the SOLO Taxonomy. However, there are some recent issues that add to a deeper understanding of the Taxonomy. This discussion focuses on several of these.

The early work on the SOLO Taxonomy (e.g., Biggs 1980) indicated a close relationship between the levels of Piaget and the levels within the concrete symbolic mode of SOLO. Table 2.1 provides an overview of this relationship.

Table 2.1
Comparison of initial SOLO Taxonomy and Piaget's stages.

| Early SOLO levels in the concrete symbolic mode (1980) | Terms used by Collis & Biggs (1976) | Piaget's Stages |
|---|---|---|
| prestructural | preoperational | Stage I |
| unistructural | early concrete | Stage IIA |
| multistructural | middle concrete | Stage IIB |
| relational | concrete | Stage IIIA |
| extended abstract | formal | Stage IIIB |

(Adapted from Biggs 1980, p. 106)

The relationship is not as simple as it appears in the table since each model approaches student understanding in a different way. Furthermore, the SOLO model is an attempt to refine Piaget's theory and bring out some of the fine structure that is not included in Piaget's work. This table does, however, give a guide for a comparison of the levels.

SOLO is best described as arising out of the neo-Piagetian theories since it is based on the ideas of Piaget but these ideas are extended further and thus can be thought of as post-Piagetian. The developmental modes are similar in structure to those of Piaget, but with differing nomenclature and are considered from a different perspective. Within each mode there is also structure that identifies growth. Another important aspect is that,

> [In SOLO], in direct contrast to Piagetian theory, the emergence of one mode does not replace its predecessor. The modes in fact accrue, the later developing modes existing alongside the earlier modes.
> (Collis & Romberg 1991. p. 87)

As well as this link with Piaget's idea, the SOLO Taxonomy is associated with other cognitive theories in education. This is valuable because these have already influenced thinking and practice in education (see Collis & Biggs 1991, pp. 1, 5). The SOLO Taxonomy is not an isolated theory in the claims that it is making, since similar ideas have been proposed by many educational psychologists (for example, Fischer 1980; Halford 1982; Fischer & Pipp 1984; Mounoud 1985) which regard "cognitive development as a series of hierarchical skill structures that can be grouped into sets of levels" which "incorporate skills of gradually increasing complexity" (Collis & Romberg 1991, p. 86).

Most taxonomies provide a way of looking at levels of conceptual thinking that are being used by students. This is a rather difficult approach, as an outside observer can not hope to fully understand the thinking processes of another person and fully classify the thinking level of that person. An additional complicating factor is that people can perform at very high levels of conceptual thinking in some areas of knowledge but very poorly in others. This is affected by their backgrounds, motivation and interests (Collis & Biggs 1979, p. 3).

The SOLO Taxonomy provides a different approach to analysing students' concepts and this is done by observing the learning outcomes and responses that students are prepared to give to the observer. The focus is not on the person, but rather on the responses that the person gives. It is a means of measuring quality of responses from a theoretical perspective. By looking at the responses that students give, one can then say that, for a particular time, the response given occurs at a particular mode and/or level.

Collis and Biggs (1979, p. 13) suggested that the modes do not refer to the average capability of cognitive functioning, but to the optimum capability of thought at a given period. The student may operate at or below this level. Better responses have a more complex structure though both responses may be technically correct. The SOLO response is related to an immediate and particular result.

One of the challenges of analysing students' responses is in identifying the appropriate level. Some responses are clearly identified with a particular level, but with the wide range of responses that students provide it is often difficult to place some of these responses exactly in one level. In new areas of knowledge a large number of responses may be needed to get 'a feel' for the different levels, then an investigation is needed to determine whether these levels reflect a suitable taxonomy such as that offered by the SOLO model.

As students move through a mode, their responses becomes more complex, incorporating more features of the question, until they can move to the next mode. Moving from the prestructural to the extended abstract level in both the ikonic and the concrete symbolic modes is still being researched in a wide range of subject areas. In general, however, it is thought that the extended abstract level of a lower mode is similar to the unistructural level of the next mode.

In considering the concrete symbolic mode, the unistructural response is observed in the response of students who provide one statement or idea in an answer to a question. The statement is relevant but only focuses on one point and is not usually sufficient as a full answer. Nevertheless, the student is usually quite happy with this response as being an answer to the question. The transition from the unistructural to the multistructural level requires practice and the ability to store these 'bits' of knowledge so that the student is able to integrate this knowledge. A typical response at the multistructural level is to give a list of statements or ideas that are relevant to the question.

To make the transition from the multistructural to the relational level requires a greater intellectual demand since here the student must not only have all the knowledge elements but also understand how they relate to one another. Collis and Biggs (1991) suggested that in the concrete symbolic mode this level is not required in all subjects by all students. Such a decision depends on the demands of society and the interests and motivation of the student. In making the transition from the relational to the extended abstract level in the concrete symbolic mode does, however, depend on the special interests of students in particular subject areas at the senior secondary or early tertiary levels. In order to reach this level of thinking, it may be necessary to pass through a number of cycles of such transitions until a total grasp of the concept is apparent in the responses.

When students are faced with mathematically related problems they need to make a decision whether to solve it intuitively, in the ikonic mode, or use standard procedures of mathematical language, the concrete symbolic mode. A schema for students possible course of action on cognition tasks, devised by Collis and Romberg (1991, p. 103), is reproduced in Figure 2.4.

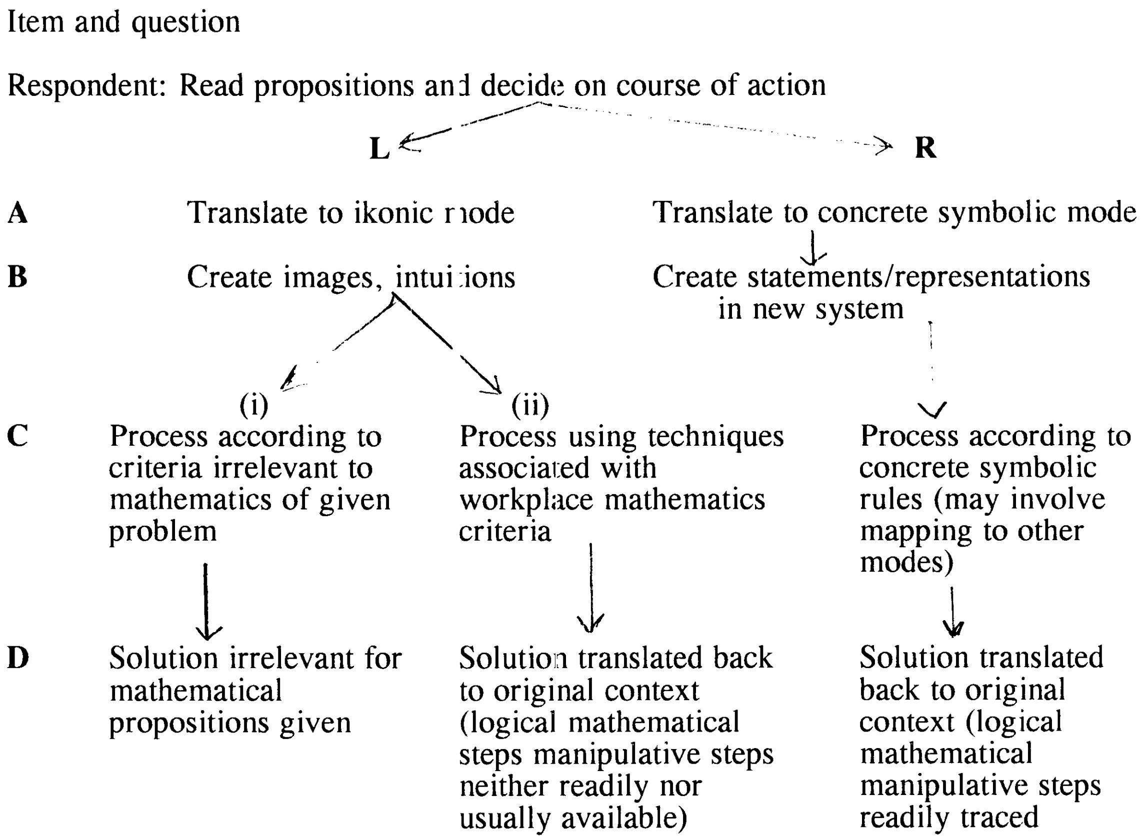



Figure 2.4 Possible course of action for cognition task.

Students can choose to proceed down column L or column R. Column L is the intuitive approach in the ikonic mode whereas column R represents a concrete symbolic approach to the problem At row B, column L branches to C(i) to give "an irrelevant conclusion or, in the case of C(ii), to an intuitive relevant solution. ... It is rare that a student will stay purely on one track (L(ii) or R) if the problem is novel. In this case there is likely to be movement both ways at either rows B or C or both" (Collis and Romberg 1991, p. 103).

Some research results indicate that whether a student will use the concrete symbolic mode or the ikonic mode strategies in doing a problem is often determined by the nature of the problem. Collis, Watson and Campbell (1992) found that for students of nine and ten years of age:

> ... students tended to change modes significantly more often when answering CS-type problems [concrete symbolic] than when answering IK-type problems [ikonic] ... When students are working on a CS approach, and get stuck, they feel free to use their imagination to make progress whereas when they use their imagination/intuition to solve the problem they are typically satisfied with the result, be it correct or incorrect, and see no point in supporting it with mathematics.
>
> (p. 13)

This suggests that students are more likely to utilize ikonic strategies if they find some degree of difficulty in solving concrete symbolic problems and are unable to use concrete symbolic strategies effectively. In the case of problems directed at the ikonic mode, however, students are unlikely to use concrete symbolic strategies to assist in their problem solving.

In the conclusion of their paper Collis, Watson and Campbell (1992) stated that the concrete symbolic mode is that which is taught at school and the ikonic mode "appeals to common sense everyday life and visualisation, and the use of intuitive reasoning" (p. 14). Students who use an ikonic approach "see little point in backing up their solution by an appeal to mathematics" (p. 14). Students are able to move between modes but most often they move from concrete symbolic to ikonic.

From the above, it would seem that the SOLO Taxonomy is a valuable tool to use in analysing students' understanding, in particular, it addresses the following four issues: it has strong links with both Piaget and neo-Piagetian formulations; it helps explore other theoretical perspectives such as that provided by Bruner (Biggs & Collis 1991, p. 71); it overcomes many identified criticisms of Piaget's work (i.e., décalage, by focusing on the response rather than the student); and finally, it offers two measures to describe students' responses, namely the modes, which describe ways of thinking, and levels, for the degree of complexity that is exhibited within the modes.

The application of the Taxonomy to a given situation offers a broad external validity to the process. However, its use is not without penalty. In order to use the Taxonomy then it is important that the elements of coding and analysis are made very clear. One method to achieve this is to use the SOLO response structures (e.g., mappings, Collis & Watson 1991) to clearly indicate the processes, cues and data that are used by the student. These mappings provide a concise overview of the structure of the student's response showing possible logical pathways in cognition. The 'pattern' of the structure gives an indication of the level of the response. There also needs to be a logical sequence or path to the development, but these are required to be consistent with previous work in this area. The key issue here is that applying a known categorisation system to a new area is fraught with difficulties.

## CONCLUSION

Chapters one and two have introduced the literature relating to studies concerned with students' ideas on speed, and have surveyed the relevant theoretical cognitive frameworks that provide a basis for understanding student responses. The major contribution to both these areas is found in the work of Piaget. He has provided a detailed analysis of children's ideas concerning speed and furnished a theory for students intellectual development. Arising from Piaget's model comes a post-Piagetian theory, the SOLO Taxonomy, which promises to offer a useful tool for examining students responses in some detail.

From chapters one and two, three themes provide general direction for the present study:

1. To ascertain students ideas concerning speed and the methods they employ to solve problems on speed.
2. To examine the strategy choices' of students to determine any underlying structure.
3. To provide a theoretical framework to interpret findings.

The next chapter describes the initial qualitative investigation which gave direction for the main study in addressing the above themes. It reports on the speed questions that were developed and the Pilot study that was conducted to test these questions.

# CHAPTER 3

# DEVELOPING TEST QUESTIONS AND PILOT STUDY

All things began in order, so shall they end, and so shall they begin again; according to the ordainer of order and mystical mathematics of the city of heaven.
Sir Thomas Browne The Garden of Cyrus Ch. 5.

## Introduction to Chapter

Before proceeding with the main investigation, several initial investigations were undertaken to develop suitable questions and appropriate research strategies that would give direction to building a stable research design. The research themes, listed at the end of the previous chapter, provide a framework and direction for this work.

In particular, the following issues require investigation prior to the start of the main study.

1. To explore the range of responses students offer when asked for a description of the meaning of speed.
2. To develop a series of straight-forward questions concerning speed that could be viewed as familiar to students in the secondary school.
3. To develop a comprehensive series of questions about speed that would test students' understanding of variables.
4. To investigate implications of using subsets of the items on different students.
5. To see if the items were suitable to allow students to provide a variety of responses.
6. To check basic assumptions regarding suitability and difficulty of questions in various question bands.

This chapter consists of three sections. First, the initial outcomes are reported of students' responses for a description of speed. Second, the speed questions are developed, analysed and trialled on a group of Year 10 students. Finally, a small pilot study is described using the speed questions with students from Years 8 and 11.

## DESCRIPTIONS OF SPEED

An initial study was carried out to investigate how students would respond, in their own words, to the meanings of the terms speed and velocity. The senior students (Years 11 and 12) were also asked to indicate the subjects that they were studying in school as it was expected that this may influence their responses.

A sample of one hundred students from Years 8 to 12, twelve science teachers and three other adults were asked to give their best attempt at providing a description for the meaning of the words speed and velocity. A clear explanation (verbal and written) was given to the subjects describing the purpose of the study. In addition the students were told that the results were not going to be used for any school-based assessment. Each response for a description of speed was coded. Four students were interviewed concerning their responses to give some indication of the reliability of the questionnaire.

### Results

Most students provided descriptions that reflected some understanding of the concept of speed. Only five responses could be classed as inappropriate or 'smart' answers.

The responses of students concerning their meaning of speed were able to be categorised successfully. It became clear that students' responses could be arranged into two broad groups, and that, within these groups, different categories of understanding were observed. Table 3.1 reports these categories with examples of typical student responses.

Table 3.1

Categories of students' responses for a description of speed

**GROUP 1** Speed is related to distance and/or time in some way

| Category | Characteristic | Examples of students' responses |
|---|---|---|
| A | Speed is the quotient of distance and time. | - Speed is distance over time.<br>- How much distance is covered per sec. It is a scalar quantity. |
| B | Speed involved distance and time. but not necessarily as a quotient. | - Measurement of kilometres you move in an hour.<br>- Rate at which an object moves, it involved time and distance.<br>Certain distance done in certain time, how fast an object moves, depends on acc. time and distance. |
| C | Distance or time was mentioned in some meaningful way. | - How fast you travel over a certain distance.<br>- How fast something is moving, usually compared to hours.<br>- Is the time where you start moving to the time you stop moving and how long it took you. |

**GROUP 2** Speed is not related to distance and/or time.

| | | |
|---|---|---|
| D | A concept of speed as the movement of an object. | - How fast something is moving.<br>- Is how quick you will be.<br>- How fast it takes to reach an area.<br>- A measurement of how fast an object or thing is moving.<br>- When an object is moving. |
| E | Speed is only associated with objects that move very fast. | - Goes very fast. |
| F | Incorrect concept of speed but associated with it. | - The speed from 0 to whatever. |
| G | Nothing to do with the scientific concept of speed. | - Something you sniff.<br>- I dunno. |

Table 3.2 provides the frequency of the different categories of response for different groups of subjects. In attempting to interpret this information it is valuable to note that: Some of the Year 12 students were studying physics and mathematics; none of the Year 11 students were currently studying physics or mathematics; and, according to their teachers, the Year 9 class was a low academic class and the Year 8 class was a high academic class.

Table 3.2
Summary of frequency of student responses
for different categories in speed

| | A | B | C | D | E | F | G | total |
|---|---|---|---|---|---|---|---|---|
| adult | 7 | 2 | 2 | 4 | 0 | 0 | 0 | 15 |
| Year 12 | 8 | 3 | 0 | 2 | 0 | 0 | 0 | 13 |
| Year 11 | 0 | 1 | 2 | 10 | 2 | 0 | 0 | 15 |
| Year 10 | 1 | 2 | 3 | 15 | 0 | 1 | 0 | 22 |
| Year 9 | 0 | 1 | 1 | 16 | 2 | 1 | 2 | 23 |
| Year 8 | 0 | 2 | 2 | 20 | 2 | 1 | 0 | 27 |
| total | 16 | 11 | 10 | 67 | 6 | 3 | 2 | 115 |

The responses for velocity were not categorised but it was noted that most students did not know what velocity meant, with a large proportion saying that they had never heard of it. The exceptions were those who had completed a full semester of mechanics in Year 11 Mathematics II or Physics.

**Discussion**

The above results show a range of responses over the different age groups. The categories seem to be secure and are used for further discussion in this chapter.

Some brief informal interviews with a few students indicated that they gave similar verbal responses for a description of speed as they did in the test. If they were prompted for more information by being presented with diagrams, they could expand on their initial responses and provide more detail for a meaning of speed. For example, one student wrote that speed "was how fast something went". Initially, in the interview, he gave a similar response but, when he was provided with examples of two objects going at different speeds, he could identify which one had the greater speed. When asked by what means he had arrived at this conclusion, he said it was by comparing the distances the two objects went and "how fast [time] it took". Thus, this student was able to use the fact that speed was related to distance and time but could not spontaneously talk about the features. These ideas are explored further in the main study.

The above statement, which used 'fast' as a synonym for time, was not an isolated case. The word fast was used in two ways by students, namely, as speed and also as the shortest time. For some students these two meanings could be the same, whereas for others it would cause some confusion. This confusion is due to the way these words are used in common everyday language.

**Implications**

One of the major implications that comes from the investigation, described above, is that students need to be asked more about speed than just what they think it is. When students are given simple questions concerning speed they might also provide responses that indicate their understanding of the relationship between distance, time and speed. It is the purpose of the next section to address this implication and describe questions that were developed to further investigate students' understanding of speed.

Since the descriptions offered by students were able to be categorised into similar groupings, issue number one has been addressed. It is of interest to know whether these categories can be identified for different students and their degree of robustness, that is, how stable are the categories for individual students under probing and prompting.

## THE QUESTIONS AND THEIR ANALYSIS

To find how students understood the relationships between the variables distance, time and speed it was necessary to formulate a set of questions which would reveal the concepts and ideas already held by students.

To achieve this a set of questions was developed which needed to be of varying difficulty, both conceptually and procedurally, in order to obtain a range of responses from students over different Year levels. As a result, three different types of questions were devised. They can be summarised as questions:

1. which tested for straight-forward ideas familiar to the students. These were of two types: (a) those of a purely numerical nature; and (b) those that would investigate how students perceived the variation of one variable with another;
2. which included visual instances of motion where the students had to compare the motion in one instance with the motion in another instance;
3. which were based on instances that would seek simple relationships between the variables as well as more complicated ones.

This is not an exhaustive list of the types of questions that could be used. For example, typical word questions, as found in physics textbooks, that specifically seek students' understanding of the kinematic formulae in different and complex ways are not included. It was not the purpose of the questions in this study to test physics, although students were not hindered from using the kinematic formulae if they had access to them.

Each of the three types of questions are now discussed in detail.

## SINGLE FOCUS QUESTIONS

In developing questions in this section it was intended to have simple questions, in familiar contexts, as the first set of questions on the test (Part A). A number of questions dealing with cars were used, with their speeds and distances over different times being considered. The initial questions were tested on two primary school children, three adults and six Year 10 students to check for inconsistency, ambiguity and clarity. An effort was also made to use a vocabulary that was familiar to students, not the technical jargon of teachers or physicists. After informal discussion with these people a final set of questions was developed that covered all possibilities. (All the questions developed are included in Appendix D.)

These simple speed questions in familiar contexts were of two types:

(a) Numerical (labeled N): These are of a numerical nature involving a simple calculation based upon familiar situations. The aim was to see if students could use accepted concepts of speed.

(b) Variation (labeled V): These questions are concerned with direct and inverse variation of the variables associated with speed. It was hoped to elicit students' perceptions on how they thought the variables changed, i.e., if one variable changed, what would happen to a second variable, given that the third was kept constant?

**Numerical Questions (N)**

For the numerical questions, three 'real life' questions were developed, that were expected to be familiar to the students. Each question required that one of the variables be found, i.e., the speed, the distance travelled, or the time taken for vehicles to travel. For example, the first numerical question, N1, is:

*If a car has a speed of 60 km/hr, how far will it go in three hours?*

In this question the distance was required, the speed and time were provided.

Table 3.3 indicates the expected operations that students would carry out on each question to obtain the correct response. The coding for each question for the variables used is provided also (see the Key).

Table 3.3
Analysis of Numerical Questions (N)

| Operation | Question | Code |
|---|---|---|
| multiplication | N1 | VT**D** |
| division | N2 | **D**TV |
| | N3 | VD**T** |

The Key
Question Reference Code

| | |
|---|---|
| upper case | constant variable |
| Lower Case | changing variable |
| **Bold** | variable to be found |

Question N1 for example would be coded as 'N1 VT**D**'. This means that N1 is the question reference (Numerical Question 1), Speed (V) and Time (T) are given and are constant, and it is required to find the Distance **D** (it is in **bold** type).

### Variation Questions (V)

In the second type of question, six questions were developed to cover the six possible ways of testing variations of the three variables. For example, if speed (V) is held constant then the time (t) can be found given a change in distance (d), (using the key above this would be Vd**t**) or the distance could be found given a change in time (V**d**t).

As an example, the first question in Part A, Question V1, was coded Vd**t**.

*A car goes on two journeys at the same speed. The second journey was twice as far as the first. How much more time would the second journey take?*

Table 3.4 provides a summary of an analysis of the questions which are referenced by the code V. The key is the same as that provided for Table 3.3.

Table 3.4
Analysis of Variation Speed Questions (V)

| Type of variation | Question | Code |
|---|---|---|
| Direct | V1<br>V4<br>V3<br>V6 | Vd**t**<br>Vd**t**<br>v**d**T<br>v**d**T |
| Inverse | V2<br>V5 | vD**t**<br>**v**Dt |

A series of different tests was administered to a group of Year 10 students towards the end of the school year. The details of the method and results of this trial using the Single Focus Questions (Part A) are reported in Appendix E.

**Discussion**

The different tests were an appropriate length as far as time was concerned. From the responses, it was apparent that the questions were straight-forward for Year 10 students. For most of the questions, students were consistent in the methods that they used. These results confirmed the assumptions regarding difficulty of the questions and also that the various questions included in the different tests appeared to show no particular problems.

**DUAL FOCUS QUESTIONS**

In preparing questions for this section it was decided to use a set of diagrams so that each question was similar in layout and easy to follow. This would allow for the information to be displayed in a consistent manner on the diagram, and the student would not be required to read large sections of written information.

Two differing types of questions were devised: the open comparison and the closed comparison.

**Closed Comparison**

In these questions, included in Part B, different instances are given of two trolleys moving from left to right. The student had to select the trolley that had the greater speed, or state whether they had the same speed. The questions were ordered in a hierarchy from none of the variables being different to all the variables differing between the two instances.

For speed, the variables considered were distance (d), time (t), speed (v) and starting position (s). The variables $d$, $t$ and $v$ were included because they are related by the relationship $d = t \times v$. The variable $s$ was included because Piaget (1976) had found that some students do not take into account where the objects started and seem to believe that the fastest one is the one that is in front at the end of the journey. A tree diagram (Figure 3.1) was constructed to show all the possibilities of combining these variables. The numbers next to the final codes refer to the final question number that is used to identify these questions in this thesis.

Not all possibilities on the tree diagram are physically attainable, these are indicated by the brackets on the tree diagram. For example, if the speed of the two instances is different, then the distance and/or time **must** be different as well. Hence a question with speed different and time and distance the same is impossible. From the tree diagram ten questions of different instances were developed. These questions are referred to as Dual Focus Closed Comparison.

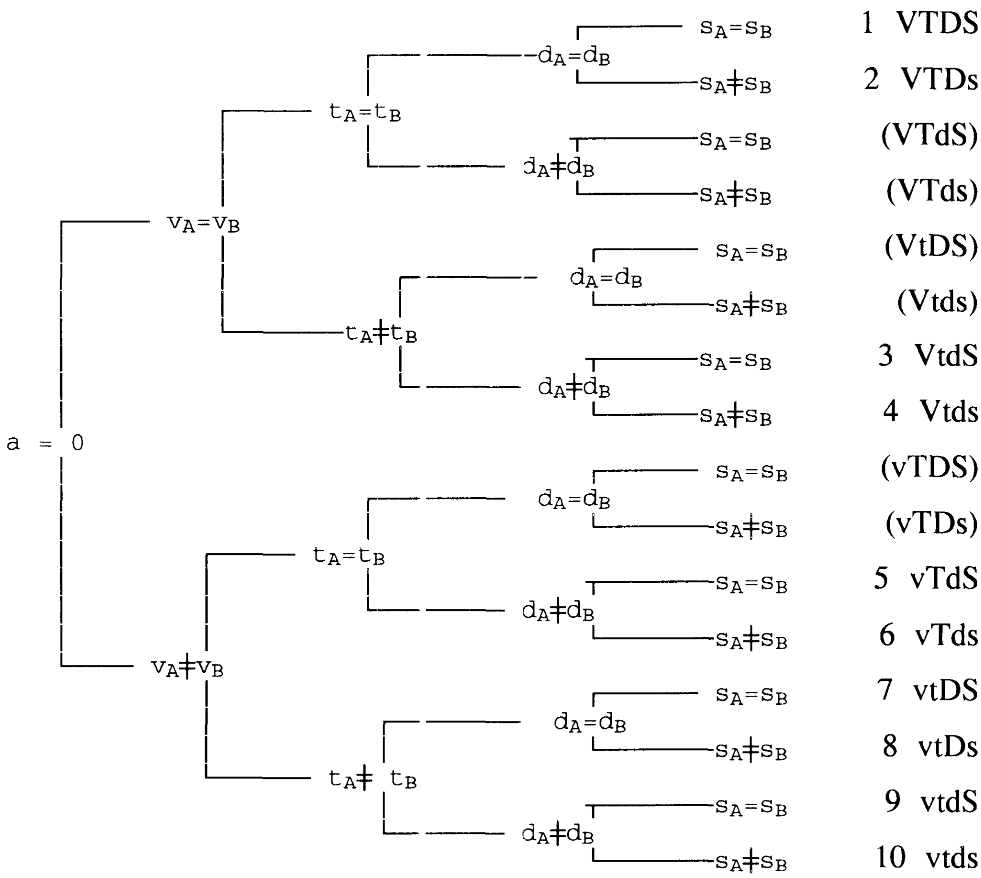



Figure 3.1 Tree diagram for Speed: Dual Focus Closed Comparison questions (coded B)

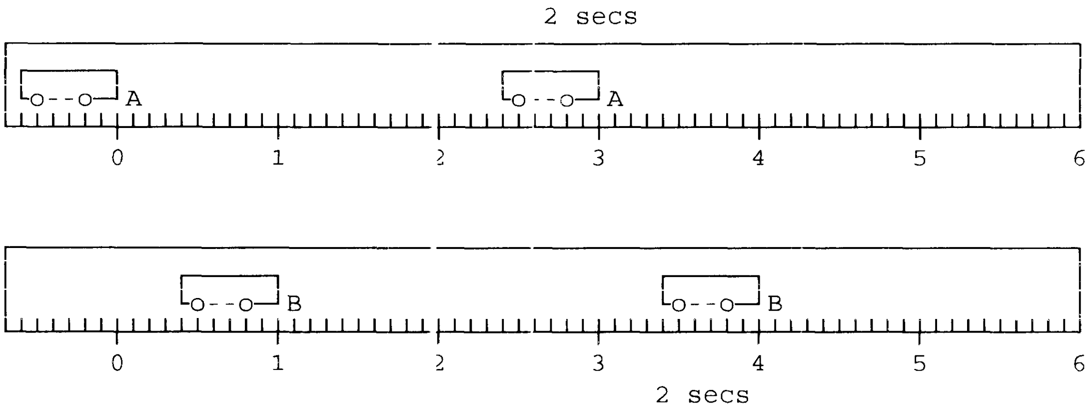
The ten questions that were developed are included in Appendix D. The questions that were used in the main study are included in the insert at the back of this thesis so that the reader can refer easily to any question.

Two examples are now given of the type of questions that were developed (the questions are coded with a B).

<u>Example No. 1</u>. Question 32 illustrates trolleys that have started from different positions. The student is given the time taken (two seconds for each trolley) and the

starting and finishing points, giving the distance travelled as three units for each trolley. Since the time taken and the distance travelled are the same, the speeds are also the same. This question is of type VTDs which means speed, time and distance are kept constant and the starting position is different.

Question B2 VTDs

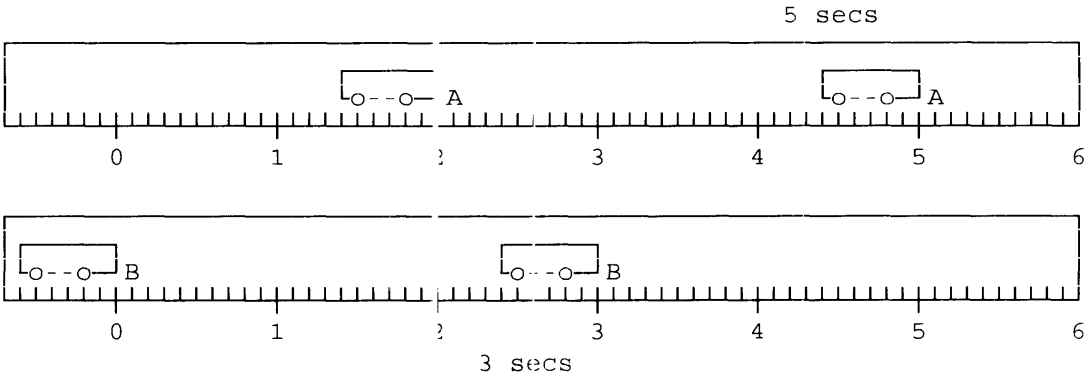



Answer: A B same speed

---

Example No. 2. Question B8 represents two trolleys that start and finish at different positions but have travelled the same distance of three units, However, the time taken for each trolley is different (five seconds and three seconds). Hence, the speeds are different with B having the greater speed. Since three variables are different, only the distance D staying the same, this question was coded vtDs. This question demanded more memory capacity of the student than Question B2 as it required more processing of information to obtain a correct response. If students were aware of the relationship between distance and time then they would have to compare the two ratios 3:5 and 3:3 and decide which one was larger. (There was a purposeful avoidance in asking which one had the faster speed since students have shown that 'fast' is often associated with time as well as speed).

Question B8 vtDs




Answer: A B same speed

---

Table 3.5 provides an analysis of the Dual Focus Closed Comparison questions in an attempt to categorise the questions. Each cell shows the Question number and the question code, for example B1 and TDS, respectively. The table also indicates the questions in which the two instances have the same speed and different speed.

Table 3.5
An analysis of the Dual Focus Closed Comparison speed questions.

| | No. of variables that differ | | | |
|---|---|---|---|---|
| | 0 | 1 | 2 | 3 |
| | Ques. code | Ques. code | Ques. code | Ques. code |
| same speed | B1 TDS | B2 TDs | B3 tdS | B4 tds |
| different speed | no cases | B5 TdS<br>B7 tDS | B6 Tds<br>B8 tDs<br>B9 tdS | B10 tds |

Key:

t = time; s = start position; d = distance; v = speed
(lower case := Variable is different in the two instances
Upper case := Variable is held constant for the two instances)

**Open Comparison**

In these questions, included as Part C, different instances were given again of two trolleys, but in this case the second trolley had one of the variables missing and the student had to find the value of this variable. The variables that differed usually did so by simple multiples of each other. Again, there is a hierarchical order of the questions from only one variable being different to all variables being different. This type of question focused students' attention on how the variables relate when they are changed.

By only displaying one variable changing and keeping the others constant in both instances, six different questions were developed (coded with a C). These questions with the coded variables are displayed in Table 3.6. The full set of questions is given in Part C of Appendix D.

Table 3.6
Questions in Part C with one variable changing.

| Question | code |
|---|---|
| C1 | V**d**t |
| C2 | **v**Dt |
| C3 | v**d**T |
| C4 | V**d**t |
| C5 | vD**t** |
| C6 | **v**dT |

KEY

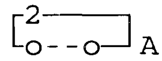
Upper case => variables same
Lower case => variables differ
bold => variable to be found
in the second instance.

An example of this type of question is the following:

Instructions (at beginning of questions in Part C):

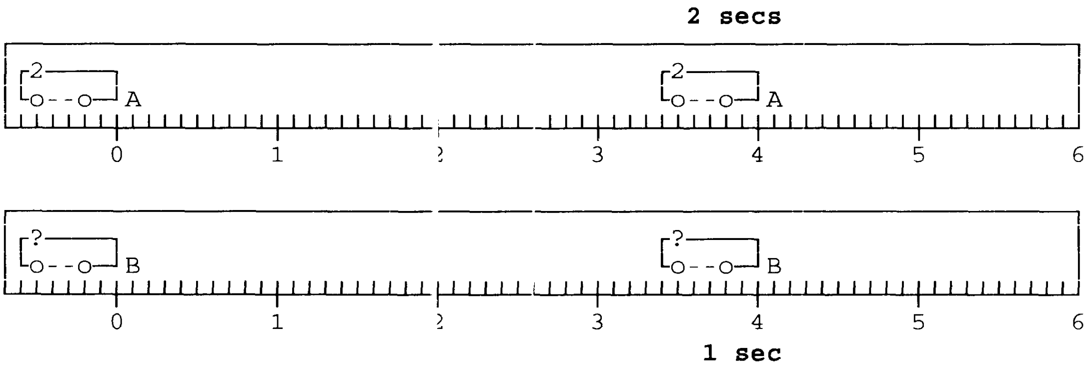
In the following questions the first diagram gives all the information but the second diagram has things changed (in darker print) and one bit missing.

By comparing diagrams or otherwise find the missing information required. (Note: the speed of the trolley is on the 'speedo' on the trolley)




This trolley has a speed of 2 units/sec.

Question C2




The time is halved. Find the speed?

---

In this question students are required to find the speed of Trolley B given the initial conditions of Trolley A. The new time for Trolley B was provided on the diagram. Also the question stated that the time had been halved.

In the problems where two variables are displayed as changing on the diagram, three further questions were developed, these are summarised in Table 3.7.

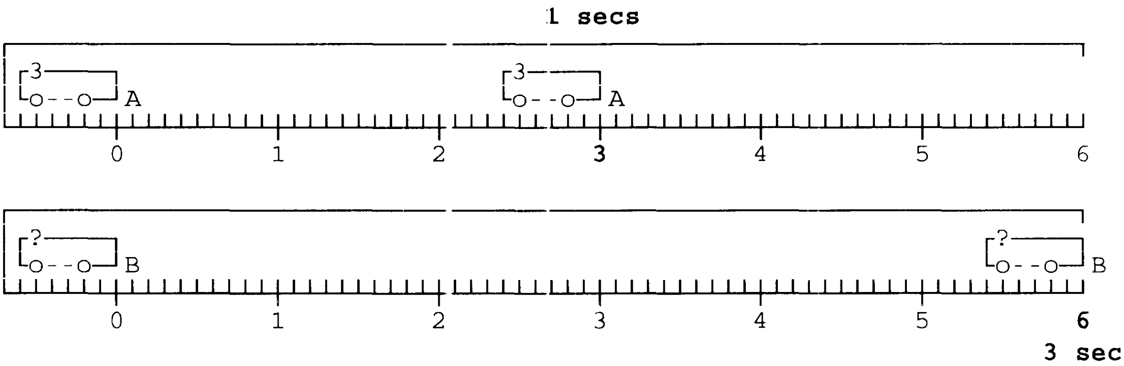
Table 3.7
Questions in Part C with two variables changing

| Question | code |
|---|---|
| C7. | vdt |
| C8. | dvt |
| C9. | tdv |

The key to the codes are the same as that given in Table 3.6 above.

An example of these questions is Question C7:

Question C7




The time has increased by 3 times and the distance has doubled.
Find the speed?

In this question, students are required to find the speed of Trolley B given the initial conditions for Trolley A. In this situation they are provided with new time and distance for Trolley B.

Table 3.8 summarises the nature of these Dual Focus Open Comparison questions with the number of variables that are displayed on the diagrams as different. The type of variation that students could use to solve the question successfully is also indicated.

Table 3.8
Analysis of the open comparison speed questions

| | **Type of variation** | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | **One Variable changing** | | | | | | **Two Variables changing** | | |
| | **Direct** | | | | **Inverse** | | **Complex** | | |
| Question No. | C1 | C4 | C3 | C6 | C2 | C5 | C8 | C7 | C9 |
| Code | Vdt | Vdt | vdT | vdT | vDt | vDt | dvt | vdt | tdv |

Key for Table 3.8

| |
|---|
| lower case := Variable is changed |
| Upper case := Variable is held constant |
| Bold := variable to be found |

These questions, Dual Focus Open and Closed Comparison, were also administered to a small group of Year 10 students. The outcomes of their responses are reported in Appendix E. Since there were more questions to be asked than could be reasonably completed in a class lesson, a series of different Speed Tests were developed with eleven questions in each one.

**Discussion**

The method of using a series of speed tests allowed all Dual Focus Questions to be tested on more than one student.

Most students were able to obtain the correct response since:

1. the questions were not seen to be particularly difficult;
2. the students had already covered recently the section on speed in Year 10 science classes, and some were able to recall the relationships between the variables.

It is interesting to note that there were a variety of responses given for the different items. Some students, who had the correct response, gave explanations that suggested that they obtained those responses from invalid reasoning. Sometimes the reason might have worked for a particular question but was not acceptable for the general situation.

The strategy that focused on the trolley that was in front as having the greater speed did occur a few times. Students frequently used the terms 'fast' or 'fastest' and this might have referred to a number of things: the trolley with the shortest time; or the one in front; or the one with the greater speed.

In the Closed Comparison questions (Coded C) it appears that most students used the direct variation or the formula to obtain a correct response. Those who obtained the wrong response often ignored one of the variables.

**Implications**

From this limited initial trial of the questions on Year 10 students, some items of interest started to emerge. In particular are the following:

- some students could provide the correct response, possibly with wrong reasoning, it would be of interest to identify these responses in the main study, and, during interviews, probe the reasoning that occurred;
- some students changed the methods they employed to solve similar questions. This peculiarity needs to be explored further in the main study to identify potential trends.
- how do the number of variables that differ within a question influence students' performance?
- how do students perform on the questions which involve different types of variation?

## PILOT STUDY ON YEAR 8 AND YEAR 11 STUDENTS.

Before the main study was commenced a small pilot study was undertaken with a group of Year 8 and 11 students A suitable test was constructed using the different types of questions that had been developed. The method and results of this pilot study are reported in Appendix F.

## DISCUSSION

The Descriptions given by the Year 11 students showed that they had been introduced to speed in some form in the past, most probably in Year 10 science and mathematics classes. Only some students, however, were able to recall the definition for speed.

It was clear that the Year 8 students had not been formally introduced to speed concepts by referring to common everyday ideas with which they were familiar, and using such expressions as "how fast something goes". They performed satisfactorily on only a few questions (N1, N3 and B1). This was to be anticipated, and justifies the previous assumptions concerning the difficulty and levels of the questions. From the Year 8 responses it became clear that at least three students used 'time' as a factor to determine which Trolley had the greatest speed. They seemed to have the idea that "the shorter the time the greater the speed" but they ignored other factors. Two students focused on the trolley that was in front as having the greater speed. Several students did not give any explanation on some questions, though they provided some correct answers. The reason for this was not always clear, some possible explanations are: sometimes they did not know the answer and guessed (if they had guessed, they were told to say so); they knew how to obtain the answer but could not explain it; they were not motivated and answered the question by whim; or they felt as if time was running out.

One student in Year 8 performed extremely well with nearly all items explained correctly. His responses showed that he had the correct ideas regarding the concept of speed. The next ranking student in the class had about one half of the questions correct.

The Year 11 students performed satisfactorily on all questions. Their responses showed a different trend compared to the Year 8 students, with at least eleven of the twenty one students using some form of the formula (such as $v = d/t$) to work out the speeds and comparing the results. Some students used proportion. One student started his response by using ratios and eventually switched to equations in the Dual Focus Open Comparison questions of Part C. Another student who focused solely on distances as the criteria for greater speed, ignored the time completely.

A direct comparison of the Year 8 and 11 results needs to be done cautiously as each test had different questions at different levels of difficulty. It is clear, however, that the Year 11 students performed better on the questions and with more confidence than the Year 8 students.

**Implications**

Since some of the items appeared to be difficult for some students it was decided to explain to the students (especially those in the lower classes) that some of the questions might be difficult. They would need to be encouraged to do their best with those questions that they thought they could do. This was done to remove some of the stress which some students seemed to experience in test type conditions and help them to progress through most of the questions.

The results showed that Year 11 students performed better than Year 8 students which was expected. A range of responses were received from both groups, and there was no easily discernable indication that rote learning of ideas was apparent. This implies that not only were the questions suitable but that the format did not trigger some specific procedural approach.

Since the students were able to do the Single Focus Questions (Part A) with confidence, pilot research issue number two was addressed, that is, the students were familiar with these types of ideas.

Question N1, which only required multiplication, was answered confidently by the Year 8 students, but with Question N3 they seemed to have more difficulty. Half the students had Question N3 correct, but those who had it incorrect did so for a variety of reasons. Some students multiplied instead of divided, others used direct variation instead of inverse. However, a sufficient number of students were able to approach the problems in an appropriate way, suggesting that these questions were not totally unfamiliar to students at this level.

The Year 8 students did not perform well on the variation questions (coded with V). This type of word question needed some skill in comprehension to understand what was being asked, this may partly account for the poor performance. Only five students gave explanations to Question V2.

These results address pilot research issues 3 and 5, in that the questions do seem to be comprehensive in testing students' understanding of variables associated with speed, and that a variety of responses were provided.

**SUMMARY**

In the main study, consideration should be given to managing the time that is required by students to answer the questions. One possible way would be to give the different parts of the questions out at different times, and ask the students to start the new set of questions immediately. This approach would have problems in organisation and student management during the testing period. It would be better to give all question types together and encourage the slower students to attempt the different types and so obtain a more complete coverage of questions attempted.

There were a variety of responses given for the different questions and these tests did give an indication of the type of answers and responses to expect in the main study. It is expected that interviews would help to clarify the way students view these concepts which provides information relating to pilot research issue 5.

The Single Focus Questions appeared to be familiar to all the Year 11 students and most of the Year 8 students, this satisfied pilot research issue 2. The questions allowed students to use different approaches to the questions and to demonstrate their understanding of the variables associated with speed.

## THE RESEARCH QUESTIONS

The following research questions were devised as suitable to fulfil the research themes listed at the end of Chapter 2, and were considered to be consistent with the directions suggested by the pilot study.

The research questions are grouped under headings that reflect the structure of the chapters that follow.

**Descriptions** (Chapter 5)

1. What are the main category groupings of students' descriptions of speed? Are these findings consistent with trends identified in the pilot study?
2. During the interviews what additional categories emerge when students are probed or prompted?
3. Are there broad bands of student descriptions consistent with year groupings? If so, what are their characteristics?

**Speed Questions** (Chapter 5)

4. What are the main identifiable features in students' performance on the speed test? How is this influenced by Year level and question type?
5. When students solve speed problems, what is the nature and consequences of the strategies employed to solve these questions?
6. How do the interviews concerning speed problems enhance understanding of strategies used by students?

**Analysis** (Chapter 6)

7. How do students refer to variables in their explanations? What influences the way these variables are used?
8. What differences in responses are associated with direct and inverse variation questions?
9. What role does intuitive thinking have in solving problems? If so, can these be identified? What methods do students use to explain their answers?

**Strategies** (Chapter 7)

10. How do students' descriptions of speed relate to strategies used in solving problems?
11. What effect does question structure have on strategies?
12. How is the strategy employed by students affected by the number of variables that differ?

**SOLO Taxonomy** (Chapter 8)

13. Does the SOLO Taxonomy provide a useful framework to explain the results and analysis?
14. Do the results help to identify different aspects of the SOLO Taxonomy?
15. Can a 'SOLO model' be provided for students' responses to speed?

**Student Profiles** (Chapter 9)

16. Can the SOLO model developed in Chapter 8 provide an overview of a students' performance on speed questions?
17. Can students' responses be understood and explained by the SOLO model?

These research questions provide a specific focus for the study. The next chapter describes the context and research design that form a basis of the investigation.