CHAPTER 9

STUDENT PROFILES

'For every person wishing to teach,
there are thirty not wanting to be taught'.
W.C. Sellar

Introduction to Chapter.

The previous chapters have presented and analysed the responses provided by students to questions concerning speed. In particular, the Speed Response Model (SRM) developed in Chapter 8 is used in this chapter to examine individual student's concepts on speed and presents a meaningful way of profiling students' levels of responses.

Five students in each of Years 9 and 12 and six students in Year 6 were interviewed concerning the speed questions. In this chapter two students from each Year level are profiled using responses from the written test and the transcripts of the interview. The students who are included in these profiles were selected to illustrate diverse ideas about speed at each Year level. Each student was interviewed regarding some of the questions they had attempted in the speed test. Excerpts from these interviews are used to illustrate students ideas about speed and also to compare and contrast student responses, both written and verbal. Copies of the original transcripts of the written test for these students is provided in Appendix G.

As in previous chapters, the case studies in this chapter are written in four parts: Description of Speed, the Single Focus Questions of Part A, the Dual Focus Closed Comparison Questions of Part B, and the Dual Focus Open Comparison Questions of Part C.

The codes used are those established in earlier chapters. A starred code indicates that this level was reached only after probing or prompting from the interviewer. In the interviews, presented in the profiles, raised dots (··) are used to represent pauses in students speech, where each dot represents about one second of time.

YEAR 6 STUDENTS

STUDENT PROFILE 1: LUKE

Luke (Student 610) is a boy aged 11 years and 10 months. He has a lively personality and was very willing to do the interview. It will be noted from his responses that Luke has reading and writing difficulties. In the written test, Luke had five correct responses out of the thirteen questions.

Description of speed

In the test, Luke gave the following response for the meaning of speed:

SpeeD mens you go over the lemet of that Rodo or track. [sic]

At the beginning of the interview, before talking about the speed questions, Luke's verbal response to a description of speed was:

S: *When you go over the limit of the track or the road.*

Thus Luke's idea of speed is related to the real world of vehicles and roads. He does not associate speed with kilometres per hour, distance or time.

At the end of the interview after the speed questions had been discussed Luke was again asked for his description of speed.

S: *When ·· you get ·· when you put your foot on the accelerator.*
I: *Are you confusing speed and acceleration?*
S: *Yea* [an enlightened expression on face].
I: *Now tell me what speed is?*
S: *um its just the same as I told you last time.*
I: *What did you tell me last time?*
S: *When it goes over the track or the road limit.*

Luke's concept of speed was very stable, even after discussing the speed questions. His idea of speed was fixed firmly in the reality of vehicles and the speed they can go on the road or race track. This response was given even though in the interview on the speed questions he referred to both the distance and the time when comparing the speed of objects.

Luke's mode of functioning when giving a description of speed, was in the unistructural level of the ikonic mode U_i for both the test and interview (see Table 8.1 for characteristics of the different levels for descriptions of speed).

Part A Questions

Of the four Single Focus Questions in Part A, Luke gave correct responses to Questions V1 and V2 but not to Questions N1 and N3. He gave no explanation for any of the questions. His responses are presented in Table 9.1 where QN represents the Question Number and FR represents the Fullness of Response.

Table 9.1
Luke's responses to Part A questions

QN	Answer	Explanation	FR	SRM level
N1	4	-	IU	U_i
N3	~~10~~ 5 hours	-	IU	U_i
V1	2x	-	CU	M_i
V2	1/2 the time	-	CU	M_i

The answer to Question N1 has no apparent logical relationship to the numbers in the question. In Question N3, he originally had '10' but it was crossed out, his answer of '5 hours' could come from halving the ten. Both these questions were coded as U_i since there is no apparent rationale as to how Luke might have done these questions, but he did give a response.

For Question V1 and V2 Luke used direct variation and inverse variation, respectively, though he was unable to give an explanation. The SRM level was coded

as M_i for these questions since it is reasonable to infer what he might have done, but to him it could be common sense.

During the interview, however, he gave reasons for his answers. Interview, Question N1:

S: ··· *180 I timesed it by three.*

Luke gave the correct solution to this problem in the interview and used the correct operation giving a SRM level of U_1.

In Question N3 he could not provide a suitable response and the interviewer had to give some visual examples and help for Luke to understand what was meant by twenty kilometres per hour. In the middle of a discussion on how long it would take to go forty kilometres he said:

S: *Half an hour, because it takes half an hour for the umm it takes one hour for it to go twenty kilometres so it will take half an hour to go ten.*

Obviously, Luke had been processing the original question while the discussion was going on. He obtained the correct response with a suitable explanation using direct variation. This response was coded as R_1^* (the asterisk indicates that this level was only reached after prompting)

Luke's approach in the interview to Question V1 was the same as that given in the test.

Luke seemed to have an intuitive concept of how speed varied with distance and time, and it appeared that he could use direct and inverse variation to arrive at correct results. In the interviews it was clarified that he was able to use direct variation. However, the use of numerical information seemed to provide some difficulty, and he was not consistent with calculations. When attempting speed problems, Luke was able to go further than a strict interpretation of his description of speed.

Part B Questions

Of the six Questions given in Part B, Luke obtained correct responses to only Questions B1 and B8 in the test. His focus seemed to be mainly on the positions of

the trolleys without concern for the time. His responses are given below in Table 9.2 with a possible translation given in the square brackets below each response.

Table 9.2
Luke's responses to Part B questions

QN	Answer	Explanation	FR	SRM level
B1	Same	Because the sterc at the samplace and stop at the samplace. [Because they started at the same place and stopped at the same place.]	CP	M_i
B2	B	it stared a fenest arter it. [It started and finished after it.]	IP	M_i
B3	B	then startet a the samPlace and B is en fot. [they started at the same place and B is in front.]	IP	M_i
B5	Same	-	IU	U_i
B8	B	-	CU	U_i
B10	B	-	IU	U_i

In the explanations to the first three questions, Luke referred to the starting and finishing positions of the trolleys. His focus in all questions appears to be completely on the diagrams with no reference to the time of each trolley that was provided in the question. In the last three questions he did not provide an explanation. A possible reason for this is that the effort required to write explanations for the first three questions may have used a lot of his resources in thinking and writing and he could not keep it up and finish the test with his peers.

During the interview Luke sometimes took the variable time into account, as the following excerpts show.

Interview, Question B1:

S: *The same* [no hesitation].
I: *Why?*
S: *Because they went the same distance in the same amount of time.*

Luke used the names for the variables and referred to both of them. The SRM level would now be in the concrete symbolic mode M_1 compared to his test result in the ikonic mode M_i.

Interview, Question B2:

S: *They are the same again.*
I: *Why?*
S: *They went the same distance but one started before.*

Luke could see the same distance but omitted the time, giving a correct partial response (SRM U_1). He does indicate, however, an awareness that one trolley had started in front of the other.

Interview, Question B3:

S: [Trolley] *B.*
I: *Why?*
S: *Cause it took longer and went further.*
[Both time and distance considered]

S: *Which one do you want shorter or longer?*
I: *I want the one with the bigger speed?*
S: *That one [pointing to B].*
I: *Why?*
S: *Because it went further than that one. No! they would be the same.*
[Note how he initially focused on the distance]

I: *Why have you changed your mind?*
S: *The further that that went ·· it went two metres more and that [A] was two metres before and it was three seconds so I add just another two seconds on it here ···· .*
[Here he is focusing on the time as well as the distance]

He could see that if Trolley A went for another two seconds then it would travel to five units and then the time and distance would be the same as diagram B, thus the speed would be the same (M_1^*). Thus the probing made him focus on the different data that were available until he had a full grasp of the problem.

Interview, Question B5:

S: *··· um B because they took the same amount of time but it went further.*

He was referring to both time and distance again (SRM M_1). He now realises the importance of considering both variables and does so in the remainder of the questions in Part B.

Interview, Question B8:

S: ·· *Um ·· B because they went the same distance but B took the shorter time.*

This response is at M_1, since he has again focused on distance and time.

Interview, Question B10:

S: ·· *A.*
I: *Why?*
S: *Because it is going about a few metres more in one second more time.*

In Question B10, Luke has seen that if Trolley A was going the same speed as Trolley B, then it would have gone four units in four seconds but since it has reached five units, it must have greater speed (this strategy has been discussed previously). This type of thinking was typical of those students who had this question correct. Also, Luke focused on the distance and the time, compared two ratios and made a correct deduction. His SRM level would be M_1 on the response to this question and reaching into R_1.

In summary, his approach to comparing the speeds was not consistent. Sometimes he focused only on distance, as in the test. Eventually, in the interview, he considered both the distance and the time as significant for solving the problems. He attempted to give his explanations using the variable names in the interviews, and has consistently avoided the use of any numerical calculation. The quality of response was higher in the interview than that given in the test. This may be attributed to his verbal skills being of a higher order than his written skills and, more importantly, the probing made him reflect or gave him cues to direct his attention to his responses.

Part C Questions

For the Dual Focus Open Comparison Questions of Part C, Luke had only Question C1 correct. His responses are given in Table 9.3.

Table 9.3
Luke's responses to Part C questions

QN	Answer	Explanation	FR	SRM level
C1	2	it was harf wy [It was half way.]	CP	R_i
C2	B	-	II	U_i
C7	6	2 wi sa far but the same speep. [2 was as far but at the same speed.]	II	U_i

Luke recognised the direct variation of the variables in Question C1 but his explanation appears to indicate that he focused on the diagrams and noticed that Trolley B would only go half the distance of Trolley A, no reason was provided for deciding to do this. The response to Question C2 is a carry over from Part B questions where a choice had to be made between Trolley A and B. The response to Question C7 does not make sense though he has tried to incorporate the distance and maybe the time, referred to as speed.

During the interview on Question C1 Luke provided the correct response, his explanation was on the correct path but there seems to be confusion at the end, either of words or concepts. He knew the answer to be a distance of two units.

Interview, Question C1:

I: [An explanation of what is required was given] *If you halve the time, where does it get to?*
S: *Two* [no hesitation].
I: *Why?*
S: *It* [Trolley A] *took two seconds to get to there and it is the same speed so half that* [distance], *so it will go ··· twice as fast.*

He used the word speed correctly, since the speed of the two trolleys is the same. His use of the term fast was associated with time. It appears that Luke had lost the thread of his argument when he had to pause, and he ended up referring to the time instead of the distance that was required (one second was written down under Trolley A). This response was an optimal level for Luke as he tried to tie all the

variables together but he was unable to complete his argument. His SRM level was M_1 but he was reaching out to R_1.

Interview, Question C2:

S: *Umm that will go four.*
[he is correct]
I: *Why?*
S: *Because if that took one second to get there and that took two so you half it.*
I: *But you said the answer was four!*
S: *Oh! you double it.*
I: *You double it!* [said with surprise].
S: *mm* [agreement].
I: *So if I halve the time, what will that do to the speed.*
S: *.. if the time got halved and you double that* [the speed].
[He is now on to inverse thinking but only after prompting.]

Luke started this question by using the values of the variables and after some prompting tried to talk about the variables but he did not always use the variable name, for example, *"if the time got halved and you double that"*. He referred to the variable 'time', but for the variable speed it was only implied with a 'that'. He seemed to know intuitively that the answer was four units per second but while explaining, he saw the time was halved and so used direct variation terms to say that the answer was halved also. Again he had lost the logic of the argument but when he was reminded of his answer he used inverse variation. His SRM level is M_1^* since he could state the variables but only after prompting.

Luke could not start Question C7 in the interview, so the interviewer showed him how to work out the speed by finding how far the trolley would go in one second. With a little encouragement he did this by counting off the distance for every second to obtain the total distance. He had agreed that the speed must be two units per second, then:

S: *No! the speed went 6.*
I: *Why?*
S: *Because that one* [A] *is three and it* [B] *is twice the distance. Oh! it will be the same because it took more time.*

So here he was equating speed and distance in some way and then he saw that the time was also greater but deduced falsely that the 'speed' stayed the same. His response shows that he was trying to account for the variation in the variables. He

was operating at M_1^* since he cou d not explain how the two variables related to each other.

Summary

Luke gave clearer responses in the interview than in the test. This was due primarily to difficulties he had with writing. Since he had to justify every statement he made in the interview, he was being forced to think about the question. Even so, he sometimes confused the meaning of speed with the other variables. He could not convert his application of speed to a suitable definition of speed. When he referred to variables, he used the variable name more often than he did the value of the variable in his explanations. In the test he used one-variable type strategies and thus was at Level 2 in the Strategies Model, developed in Chapter 7. However, in the interview he often reverted to two-variable strategies under prompting and thus shifted to an earlier Level 3. Luke used direct variation without prompting, but for inverse variation he needed considerable help.

A summary of the different SRM levels for Luke's responses in both the written test and the interview is provided in Table 9.4.

Table 9.4
Summary of SRM levels for Luke

	Questions													
	des.	N1	N3	V:	V2	31	B2	B3	B5	B8	B10	C1	C2	C7
Test	U_i	$\underline{U_i}$	$\underline{U_i}$	M:	M_i	M_1	$\underline{M_i}$	$\underline{M_i}$	$\underline{U_i}$	U_i	$\underline{U_1}$	R_i	$\underline{U_i}$	$\underline{U_i}$
Interview	U_i	U_1	R_1^*	M:	-	M_1	U_1	M_1^*	M_1	M_1	M_1	M_1	M_1^*	M_1^*

An asterisk (*) indicates level after some prompting.
Underlined levels indicate that the student's answer was incorrect.

Figure 9.1 shows, in a diagrammatic way, the SRM profile of Luke's responses as taken from the SRM levels of Table 9.4.

Key:

+ response in test	■ response in interview
± interview and test the same	* prompt in interview
des = description of speed	A, B, C are for Parts A, B, C of test.

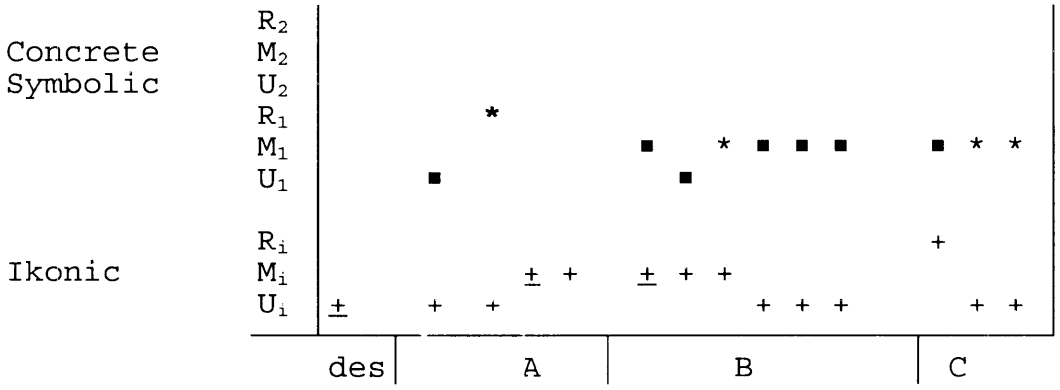

Figure 9.1 SRM profile of Luke.

This SRM profile shows clearly that Luke's responses were at a higher level in the interview than in the written test. The main reason for this was his poor writing and reading skills. His optimum level was M_1 and his functional level was normally below this in the ikonic mode in the written test. In addition his level of operation on problems in the interview was well above the level of his description of speed. Luke expressed himself verbally very well. When he chose two-variable strategies in the interview he lifted his performance to higher levels.

Luke's ideas of speed have not been consciously thought out, thus he could not state specifically the components that constitute speed. His written responses were all in the ikonic mode and four of his responses in the interview were also in this mode. However, as the interview progressed, his level of functioning stabilised at the multistructural level of the first cycle of the concrete symbolic mode. He was not able to link the different variables to reach the relational level in this cycle.

STUDENT PROFILE 2: KIRSTY

Kirsty (Student 622) is a Year 6 girl aged 10 years and 11 months and was born in Australia. She is a quiet but conscientious student. Her speed test result was nine correct responses out of the thirteen questions.

Description of Speed

In the speed test Kirsty's response for a description of speed was:

It's going as fast as you can.

This was an ikonic multistructural (M_i) response because the implication is that speed means something that goes very fast.

Three weeks later, at the beginning of the interview she was asked for a meaning of speed:

I: *What is speed?*
S: .....
I: *Is that a hard question? Have you ever heard of the word before?*
S: *I have, Mum and Dad talk about it and on TV.*
I: *Yes and what do you think it means when they talk about it?*
S: *How fast the car is going.*

She was a little unsettled and unsure of what was going on at the beginning of the interview and this continued until the beginning of the Part B questions. Her idea of speed was related to the idea of a car and her response did not indicate a shift in levels. Her answer could well mean 'very fast' as in her written response. She was again operating at the multistructural level in the ikonic mode, M_i. At the end of the interview, after talking through the speed questions, she was again asked about speed:

S: *How far in some time.*

She has a better understanding of the idea of speed after having had a discussion on the speed questions. (This did not always happen with students, see Luke above.) The level of response was now at the relational level of the first cycle in the concrete symbolic mode (R_1^*) since she could now refer to both distance (far) and time. The distance was done "in some time" which suggests an attempt to relate the two

variables. Thus she has been on a 'learning curve' during the interview on speed problems.

Part A Questions

In the Single Focus Questions in Part A of the Speed Test Kirsty had three of the four questions correct, two of them with full explanations. Her responses are presented in Table 9.5.

Table 9.5
Kirsty's responses to Part A questions

QN	Answer	Explanation	FR	SRM level
N1	4860 km	I times 60 & 180	IF	U_1
N3	30 mins	Because he is going 20 km/hr.	CP	U_1
V1	Twice as much as the first.	Because the second one is twice as long as the first one.	CF	R_1
V2	Half of the Smith's car	Because he went twice as fast as the Smith car.	CF	R_1

Kirsty was nervous at the beginning of the interview and this may have been true in the test, because as she progressed through the questions her responses improved. In the first question (Question N1), she just used the two numbers in the problem and multiplied them together (incorrectly) which indicated a unistructural level in the first cycle of the concrete symbolic mode, U_1.

Her response to Question N3, in the written test, does not explain how she arrived at her answer but she has used the speed of '20 km/h' in some way to arrive at that answer. In the interview on this question she could not even start answering the question, it was if her mind was a complete blank. After some discussion of the meaning of '20 km/hr' she provided the correct response of thirty minutes (level U_1^*).

For the variation questions V1 and V2 Kirsty used a proportion type process to arrive at the correct answer. She seemed to have the correct idea for both direct and

inverse variation placing her responses at R_1. In the interview, with Question V1, at first she misunderstood the question but after prompting she gave a correct response. She could not give an explanation of the processes used (level U_1^*).

Part B Questions

Of the six questions in Part B, Kirsty had three correct answers in the test, with Questions B3 and B8 incorrect and Question B5 correct, but with an inappropriate explanation. Her responses are provided in Table 9.6.

Table 9.6
Kirsty's responses to Part B questions

QN	Answer	Explanation	FR	SRM level
B1	Same	They stoped [sic] at the same time.	CF	M_1
B2	Same	They went the same distance.	CP	U_1
B3	B	It went further than A.	IP	M_i
B5	B	It had a different speed than A.	CI	U_1
B8	Same	Went the same distance.	IP	U_1
B10	A	It went further than B.	CP	M_i

Question B1 was interpreted as a full explanation since she referred to their stopping places and the time. Most of the other questions focused only on the distances and ignored the time. Thus she was using one-variable strategies with distance as the focus for determining the trolley with greater speed. The explanation in Question B5 was not clear enough to deduce how she was using the word speed. She could be meaning that it went further, or that they actually had different speeds. The explanations given to Questions B3 and B10 need to be interpreted carefully when assigning SRM levels. There are two ways that the word 'further' may be interpreted. First, if Kirsty was focusing on the diagrams and noted that one trolley was in front of the other, and had therefore travelled further, then her response is in the ikonic mode. Second, she may have noticed the distances that were stated on the diagrams and deduced which trolley had travelled a larger distance. Since she has not

explicitly mentioned the given distances the responses were coded as multistructural ikonic, M_i.

In the interview she gave the same answer and explanation for Question B1 and B3 as she did on the test. The interviewer tried to encourage her to take note of the differences in the time for Trolley A and B in Question B3 but Kirsty was unable to mention time in her response. However, for Question B5 her response was:

S: *B, because it went furtherer in the same amount of time.*

This was a correct and full response at M_1 and she did not refer to the speed as she had in the test. The previous discussion may have started to take effect by encouraging her to mention time.

Interview, Question B8:

S: *The same because they went the same distance just that A had moved up.*

She has totally neglected the time again. The interviewer then confronted her with her answer to Question B1 where she had said the trolleys were the same speed because they went the same distance in the same time but in this question (B8) they had different times.

Interview continues, Question B8:

I: *Does that make any difference?*
S: *Yes ·· that one [A] went slower.*
I: *Why?*
S: *·· because it took 5 seconds to go the same length as cart B.*

After prompting she could explain quite clearly why Trolley B had the greater speed.

For Question B10, Kirsty could not start the question by herself. She said that she was going to try and find the speed of each one, but she did not know how to proceed without help.

In most of her replies Kirsty referred to the variable names and she did not refer to the value of the variable. In Question B8 she only referred to the value of the variable after she had been prompted with other material by the interviewer.

Kirsty was functioning at SRM levels that spanned from M_i to U_1 and with prompting she could sometimes reach M_1 (Multistructural first cycle). Thus she was making a transition from the ikonic thinking mode to the concrete symbolic mode. Within the Strategies Model she alternated between Levels 2 and 3 in using two and one-variable strategies.

Part C Questions

In the Part C questions Kirsty had the first two questions correct. Table 9.7 provides her responses.

Table 9.7
Kirsty's responses to Part C questions

QN	Answer	Explanation	FR	SRM level
C1	2	Because it is half the speed of A so it will go half the distance.	CF	R_1
C2	4	If they went the same distance & the time is halfed [sic] the speed of B is 4.	CF	R_1
C7	6	Because on A its 3 and B went twice as far so it has to be 6.	IF	M_1

Kirsty used direct variation in Question C1 and inverse variation in Question C2. The explanation to Question C1 used the variable names, and in Question C2 the variable names and the final value were used. With Question C2, however, she referred to all the variables and mentioned the necessary constraint, that the distance be constant, for the inverse variation to work. In giving this response, she is going beyond R_1 in linking all the variables, including restraints but has omitted to say how the speed would change. This response has been coded R_1. For Question C7 Kirsty has confused speed and distance in attempting to link the variables. Since the link has been done incorrectly the response is coded as M_1.

In the interview Kirsty gave her answers to Part C with great confidence, before this all her responses were tentative with a questioning tone in her voice.

Interview, Question C1:

S: *It got to 2, the time is halved so you halve the distance.*

Since she has seen the direct variation but has not referred to the constraint that speed is constant, then the level is R_1.

Interview, Question C2:

S: *4.*
I: *How?*
S: *The time is halved and they go the same distance, the speed will have to double.*

This answer has taken into account all three variables and shows the relationship between them. Her response in the interview was similar to the test response, however, in the interview she has kept account of how the variables are changing including speed, which she did not do in the test. Thus she has reached U_2. For Question C7, she gave the same answer as in the test but provided a different explanation, which was:

S: Oh! the distance [of A] is halved of cart B so I just assumed the speed was 6.

Kirsty has again confused the terms for speed and distance. She had problems dealing with more than one variable being different in the question. She has just focused on the one variable that was most obvious in the picture.

Overall view

Table 9.8 provides a summary of the SRM levels that Kirsty responded with for the test and the interview.

Table 9.8
Summary of SRM levels for Kirsty

	Questions													
	des.	N1	N3	V:	V2	B1	B2	B3	B5	B8	B10	C1	C2	C7
Test	M_i	$\underline{U_1}$	U_1	R:	R_1	M_1	U_1	$\underline{M_i}$	$\underline{U_1}$	$\underline{U_1}$	M_i	R_1	R_1	$\underline{M_1}$
Interview	R_1*	-	U_1*	U:	* -	M_1	-	M_i	M_1	M_1*	P	R_1	U_2	$\underline{M_1}$

An asterisk (*) indicates level after some prompting.
Underlined levels indicate that the student's answer was incorrect in the test.

Figure 9.2 shows the SRM profile for Kirsty's responses to speed as taken from Table 9.8.

Key:

+ response in test	■ response in interview
± interview and test the same	* prompt in interview
des = description of speed	A, B, C are for Parts A, B, C of test.

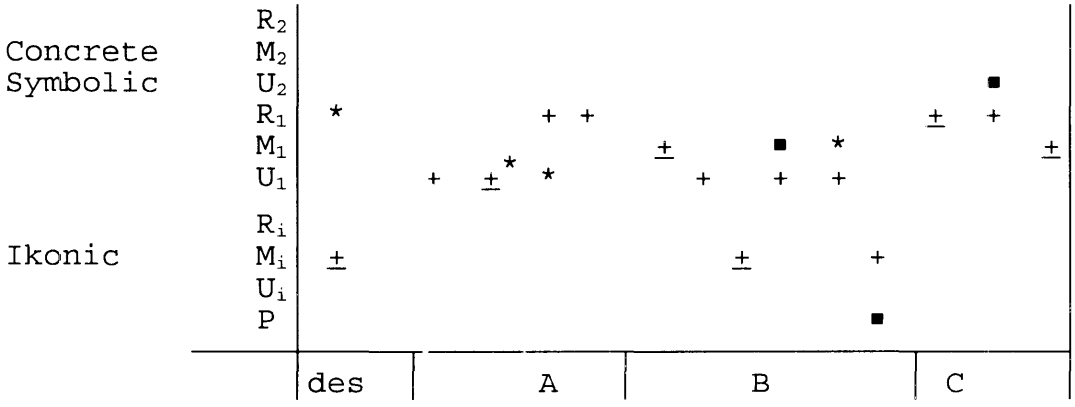

Figure 9.2 SRM Profile of Kirsty.

Kirsty has shown, that given the right prompts, she can move from lower levels to higher levels both in giving the description of speed and in answering the questions. Kirsty's initial nervousness hindered her using higher levels both in the test and in the interview. She functioned well at U_1 but could move to M_1 and sometimes to R_1 when she was performing at optimal level especially in the Part C questions. This could be attributed to two factors, first, her growing confidence as the test and interview proceeded, and second, the Part C questions were variation questions, which she was capable of doing when one variable was being changed.

The description Kirsty gave for speed at the end of the interview (R_1) reflected the level she used in the Part C questions and the direct variation questions in Part A. In the test, and at the beginning of the interview, her description of speed was well below the level that she reached in the problems. These types of answers reflect the 'two perspectives outcome' of Hewson (1981) where students can have two alternative views, one to be employed in everyday use and the other to be used at school. In Kirsty's case the everyday use predominates and only on reflection could she give a more scientifically acceptable version.

In contrast to Luke's responses, Kirsty performed at a higher level in the test but Luke provided responses in the interview that matched some of Kirsty's test responses. However, Kirsty was able to bring together all the variables and constraints in some questions, whereas Luke could not do this at all. Kirsty understood what variables were significant in solving the problems, though this did not occur at first. At the end of the interview, Kirsty could verbalise her understanding of speed by giving a suitable description of speed. Luke could not do this and he stayed with his basic definition of speed being related to roads and tracks.

YEAR 9 STUDENTS

STUDENT PROFILE 3: LEISA

Leisa is a girl in Year 9 (Student 910) aged 14 years and 3 months, and born in Australia. She is a quiet, but not academic, student. In the speed test she had four correct responses out of the fourteen questions.

Description of Speed

Leisa's description of speed as given on the written test was:

Speed means how fast you go and if you can go faster or slower.

Since she knew that speed was related to fast or slow objects she has responded at SRM first cycle relational R_1.

At the completion of the interview on the speed questions, Leisa was asked what she meant by speed:

I: *What do you think speed is?*
S: *I have forgotten now.*
I: *Have a guess.*
S: *If you go fast.*
I: *What do you mean by fast?*
S: *How much speed they put into it.*

Leisa associated speed with the word "fast" and confused it with the concept of acceleration. Even when prompted she could not get out of the speed-fast-speed cycle

and was thus responding at level M_i, which was lower than her test response. Her response 'How much speed they put into it' may incorporate the idea of fast or slow objects but it is unclear, nevertheless, she is definitely responding in the upper levels of the ikonic mode.

Part A Questions

Leisa had three of the four questions correct in the Single Focus Questions of Part A but only gave one full response. Her responses are presented in Table 9.9.

Table 9.9
Leisa's responses to Part A questions

QN	Answer	Explanation	FR	SRM level
N1	180 km	Times by 3.	CF	U_1
N3	2 hrs	divided	IP	U_1
V1	double	-	CU	M_i
V3	2 laps	-	CU	M_i

For Question N3 she had an answer of "2 hrs" when the expected answer was half an hour. Her explanation was "divided", so her answer could have been obtained by dividing twenty kilometres per hour by ten kilometres.

In the interview concerning Question N3, she was given several prompts. The following extract shows that she not only has a poor concept regarding speed but that she also has problems in recognising the reasonableness of her answers. This was related to her losing the focus of the question.

Interview, Question N3:

S: [she reads question] *Now do I work it out?*
I: *Yes try thinking out loud.*
S: *That is hard to do.* ···
I: *Do you know what you have to find?*
S: *Yes find the time it takes him to ride ten kilometres um* ·· *about 400.*
I: *400 what?*
S: *400 kilometres per hour.*
I: *Where did you get 400 kilometres per hour from?*

S: ····

I: *Think out loud.*
S: · *I just times'd it. Twenty by ten.*
I: *Twenty by ten?*
S: *Yes.*
I: *Why?*
S: *200.*
I: *200?*
S: *Yes.*
I: *Why?*
S: *I don't know.*

I: *Lets have a little think. What are you looking for?*
S: ··
I: *What does the question ask you to find?*
S: *How many hours will it take him oh, to ride.*
I: *So you think it will take him 200 hours?.*
S: *No,* [very quiet, unsure].
I: *Think out loud.*
S: *I am reading the question.* ··· *I don't know.*

I: *OK then, what does this mean?* [Point to 20 *kilometres* per hour] *the twenty kilometres per hour.*
S: *How fast he is going?*
I: *OK so what does the twenty kilometres per hour mean about how fast he goes?*
S: ··
I: *How far would he go in one hour?*
S: *Twenty kilometres.*
I: *How far would he go in two hours?*
S: *Forty.*
I: *How long would it take him to do ten kilometres?*
S: ···· *Ten hours.*
[The order of the sequence is reversed and she can not reverse her thinking.]
I: *Ten hours?*
S: *Im.*
I: *He does twenty kilometres in one hour. And you reckon he can do ten kilometres in ten hours?*
S: ···

I: *Lets do that again. How long does it take to go twenty kilometres?*
S: *1 hour.*
I: *If he did forty kilometres how long would it take him?*
S: *Two hours.*
I: *If he did sixty kilometres how long?*
S: *Three.*
I: *Good, if he goes ten kilometres how long will it take him?*
S: · *oh half half an hour.*

This illustrates a clear case of early closure on an answer without thinking of a rational procedure or how reasonable the answer is. She was satisfied to just perform an operation, which suggests why she gave different answers in the test to those in the

interview. On prompting it was obvious that she had no clear meaning for 'Km/hr'. Even after prompting was given, her understanding of the problem did not increase very much at all. She was still using arithmetical operations to close in on an answer.

In the test, Leisa had Questions V1 and V3 correct without providing explanations. However, in the interview she could not start Question V1. After a picture of the problem was drawn for her, she still could not respond correctly. So a familiar situation was presented to her of a journey taking ten minutes to drive to the local shopping centre.

I: *If you drove from here to* [local shopping centre named] *and it takes ten minutes how long would it take you to get from here to the shops and back again?*
S: *Twenty minutes.*
I: *Why?*
S: *Because if it takes ten minutes to get there it will take ten minutes to get back.*
I: *Good what have you done with the time?*
S: *.. I doubled it.*
I: *Good, you can see that alright? Could you see that here?* [pointing to the problem].
S: *Yes I could see it but I didn't know how to explain it.*

This time she could say, after prompting, that the time would be doubled, but it did not come easily to her. She appeared to be operating at **SRM** level of U_1^* (that is with prompting).

Thus, Leisa had a very limited knowledge of speed and could not do all the Single Focus Questions. She did not have a good grasp of how the variables changed with each other except when put in simple and familiar concrete situations.

Part B Questions

Of the five questions in this section, Leisa solved only the first one correctly. Table 9.10 provides her answers and explanations for these questions.

These responses show confusion over the term 'speed'. It could be used to indicate a short time or an object having gone a long distance (e.g., her explanation to Question B5). There is a focus on the positions of the two trolleys with respect to each other.

Table 9.10
Leisa's responses to Part B questions

QN	Answer	Explanation	FR	SRM level
B1	Same	Because they both did it in 2 seconds.	CF	M_1
B3	B	It went further than A stopped first.	IP	M_i
B5	Same	They both started together and had the same speed further up and B had longer speed.	IP	M_1
B8	A	it started further up and B had least speed.	IP	T_i
B10	A	A was just in front by a second.	CI	T_i

In Question B1 she could be referring to both variables if the 'it' refers to distance. This response may indicate a transition from unistructural to multistructural levels and has been coded M_1. The same may be true for the response to Question B5, but the confusion of the names for variables makes it difficult to judge. Both trolleys had the same time (her first 'speed') and Trolley B travelled the larger distance (her second 'speed'). Assuming this to be the case, then this response is coded as M_1 since both variables have been attended, but not linked, to each other.

She has tried to use variable names but has confused the terminology for them and has not expressed the relationships in a clear way. Only in Questions B1 and B10 is there any reference to the value of a variable.

The following are excerpts from the interview with Leisa.

Question B3:

S: *B .. because it went further than A.*
I *But the times are different.*
S: [No response].

Leisa was focusing on the trolley that went the greater distance and could not associate the time element even when prompted, so her response remained at M_i.

Question B5:

S: *Both had the greatest speed because they went the same speed.*

I: *How do you know?*
S: *Because it has three seconds on both.*
I: *But B went further.*
S: [No response].

In this question the times were the same so she thought that the speeds would be the same, but she could not incorporate the different distances into her explanation, even when prompted. Thus this response is at U_1 since she is only focusing on one variable. The interviewer then decided that if an everyday example was used she might perform at a higher level.

Every day example:

I: *If I went from here to the shops* [named] *in two minutes and you took five minutes is there a difference in speed?*
S: *Yes you will be going faster.*
[So she can do constant distance different times.]
I: *If I went from here to the shops in three seconds* [pointing at diagram Trolley A] *and you went to the shops and back in three seconds* [pointing at Trolley B] *who has the highest speed?*
S: *A because it is going further.*
[did she misunderstand the original question?]

Leisa knew that less time for same distance meant greater speed, but for different distances she seemed to have difficulty. The interviewer then asked Leisa two similar questions that were not in the test but had been designed in the pilot study.

Question B7 (distance the same $t_B=3$, $t_A = 5$):

S: *B because it is three and the other is five.*
I: *Because B took a longer time?*
[checking]
S: *No a shorter time.*

Leisa obtained the correct answer to this question and she knew that the trolley with the shorter time for the same distance would have the larger speed. This response is at U_1.

Question B9 ($d_A= 5$, $d_B= 4$, $t_A=5$, $t_B= 3$):

S: ··· *B because it took three seconds which is less than five seconds.*

In this question, Leisa completely ignored the distances and focused solely on the time as the determining factor to find the greater speed (U_1).

Question B10:

S: *A because it took a second more.*
I: *But you said before that the smaller time meant a greater speed.*
S: *But A had a greater distance to go.*

This question was a little more complex with the two trolleys starting at different places and she tried to incorporate distance and time into the response thus giving a level of SRM M_1^* after prompting.

In summary, Leisa could identify that shorter times for the same distance implied greater speed, but when the distance and time were different she was not consistent with her approach. She did not have a strategy to compare two variables that were different.

Part C Questions

In all the Dual Comparison Open Questions of the speed test, Leisa was unsuccessful. In fact with Question C2 and C7 she confused her answer with just giving 'B' as the answer which indicates that she is confusing this Part of the test with Part B questions where 'A, B or the same' were the required answers. Table 9.11 presents her test responses to the Part C questions.

Table 9.11
Leisa's responses to Part C questions

QN	Answer	Explanation	FR	SRM level
C1	3	It is near the three between.	IP	R_i
C2	B	B could of started later.	II	U_i
C7	B	Has greater speed.	II	U_i
C8	5	-	IU	U_i
C9	-	-	IU	P

For Question C1 she appears to know that the answer must be less than four units but could not see that a direct variation approach is required. The other responses show a lack of understanding of the questions and a complete failure of how the variables are related.

In the interview concerning Question C1 Leisa had no problem with the idea, that when time was halved then the distance was halved also.

S: *It will get to two.*
I: *How did you get the two?*
S: *I halved the four.*
I: *Very good.*
[she was needing positive reinforcement after Part B]

This response indicates that Leisa could use direct variation as a strategy to solve the problem successfully, though she could not fully explain why this was so. The interviewer thought it best not to continue to probe as it appeared that the student was tiring. It appears she could apply a direct variation approach with some encouragement (SRM M_1^*).

Interview, Question C2:

S: ·· *two* [speed].
I: *Why?*
S: *Because it will be the same as the other one.*
I: *This one* [A] *got there in two seconds and B got there in one second ··· Have they got the same speed?*
S: *It is four then.*
I: *How?*
S: *I had a guess.*
I: *It halved the time and doubles the speed, does that make sense?*
S: ··· [She shakes her head]
I: *You are right* [getting the four for speed] *but I want to know why you think it is right?*
S: ·· *I don't know.*

She tried the same principle (direct variation) in Question C2 that she had used in Question C1, but this time she was incorrect. When this was shown to her she quickly tried for another answer but then admitted that it was only a guess. Clearly, she is trying to close quickly on any answer to finish the question. She did not see the inverse relationship, thus her response was at U_i.

Question C7 was too difficult for Leisa to attempt and she could not even start the question.

Summary

From the above profile it can be seen that Leisa's concepts of speed were vague and that she used the word speed in place of time and/or distance. She was not consciously aware that speed was related in any specific way to distance and time, but she was aware that if the distance was halved then the time was halved also (at constant speed).

Table 9.12 shows a summary of her SRM levels for both the written test and the interview.

Table 9.12
Summary of SRM levels for Leisa

	Questions														
	des.	N1	N3	V1	V2	B1	B3	B5	B8	B10	C1	C2	C7	C8	C9
test	R_i	U_1	$\underline{U_1}$	M_i	M_i	M_1	$\underline{M_i}$	$\underline{M_1}$	$\underline{T_i}$	$\underline{T_i}$	$\underline{R_i}$	$\underline{U_i}$	$\underline{U_i}$	$\underline{U_i}$	$\underline{P}$
interview	M_i	-	U_1*	U_1*	-	-	M_i	U_1	(U_1)	M_1*	M_1*	U_i	P		

An asterisk (*) indicates level after some prompting.
Underlined levels indicate that the student's answer was incorrect.

From Table 9.12 is derived the diagrammatic SRM profile in Figure 9.3.

Key:

+ response in test	■ response in interview
± interview and test the same	* prompt in interview
des = description of speed	A, B, C are for Parts A, B, C of test.

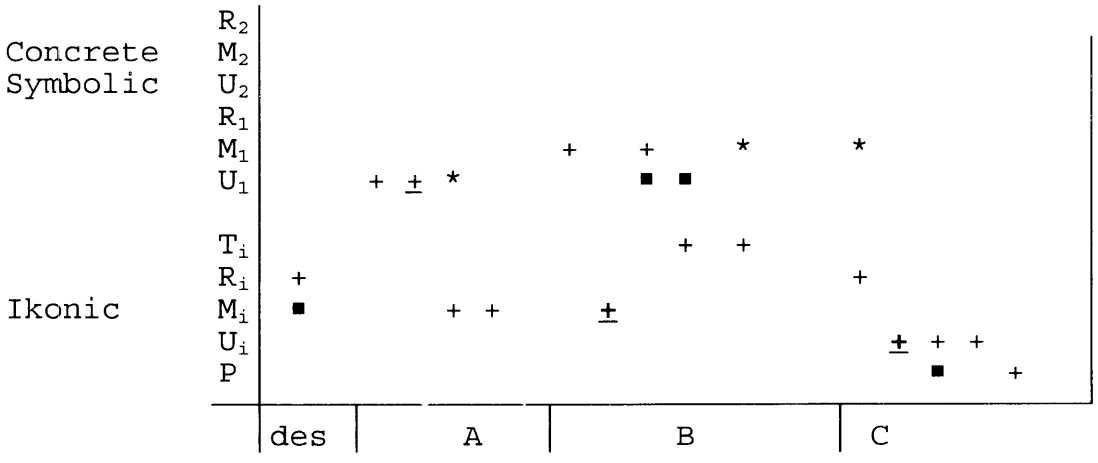

Figure 9.3 SRM profile for Leisa.

The SRM profile indicates that Leisa had an optimal level of M_1 and a functional level spread across the ikonic mode and sometimes into U_1. On prompting, in the interviews, her responses moved from the ikonic mode to the concrete symbolic mode. Within the Strategies Model she is operating most often at Level 1, that is, she did not select appropriate variables.

The description of speed given by Leisa was in the ikonic mode. She used the same mode for the majority of her responses to the speed questions. The responses in the test that were in the concrete symbolic mode may have been more fortuitous than planned.

One reason for her low level of performance was that when she did attempt a question it was either on the visual level that she solved the problem or she could only focus on one variable and was thus using one-variable strategies. She could not relate two variables even when prompted.

STUDENT PROFILE 4: ANTHONY

Anthony is a Year 9 boy (Student 906) aged 14 years and 2 months and was born in Australia. He is a quiet student with academic ability. His speed test result was twelve correct responses from the fourteen questions.

Description of Speed

In the written test Anthony did not describe speed but he had an idea of velocity.

Speed - *Don't know*
Velocity - *How fast an object is going*

Such a response seemed to be a temporary lapse or lack of concentration. However, if his response to velocity is considered it would be coded as R_i since he has related velocity to fast objects but not limited it to very fast objects. At the completion of the interview on the questions in the speed test Anthony was asked:

I: *What is speed?*
S: *Um ·· how fast an object will be going, I guess.*
I: *So how could you tell if one object is going at a higher speed than another?*
S: *By measuring it.*

I: *What would you measure?*
S: *Um how much distance it covered um in how much time.*
I: *Oh! where did you get that from? Did you learn it somewhere?*
S: *No. Like you have ten kilometres per hour.*
I: *Righto.*
S: *So how much distance, yes, how much distance was travelled .. in how much time. I don't know if you can make much sense of that?*
I: *Yes that is good.*

Anthony's initial response for a description of speed, towards the end of the interview, was at the same level as his response to velocity in the test, that is, SRM R_i. With prompting he was encouraged to reflect upon how speed was measured and was able to give the information that speed was related to distance and time. These variables, however, were not connected mathematically thus the level of this response was R_1^*. This response was not a learned response. He appears to have deduced it from the units of speed. This does indicate that some responses in tests are not the highest level at which a student is capable of performing (optimum). It seems as if some students give a quick response so that they can proceed with the test. In the interview situation, the context was different for students because it did not matter what the answer was the interviewer always asked for an explanation or more information.

So Anthony's SRM level was prestructural in the test (or R_i if the description of velocity is accepted) and R_i in the interview; then, after prompting, his response shifted up to R_1^*.

Part A Questions

The responses that Anthony gave to the Part A questions in the test are provided in Table 9.13. He had all the answers correct but Question V3 had an inappropriate explanation.

In Questions N1 and V1 Anthony gave full and correct responses, that is, he gave explanations that clearly indicated how he had arrived at his answers. In both cases, however, he has only used a simple arithmetic operation which codes as a U_1 response. This arithmetic operation approach appears to be confirmed in Question N3. The answer was correct but the explanation he provided did not give the correct answer. Either it was just a mistake in writing or Anthony had intuitively found the correct answer then attempted to explain it. He knew that the answer was '½ hour'. He needed two numbers that would provide this ratio and they were ten and twenty but he wrote it down incorrectly. Such a process as knowing the answer then finding

an explanation that is not correct is common. This intuitive type processing of answers and then giving explanations was discussed in some detail in Chapter 6.

Table 9.13
Anthony's responses to Part A questions

QN	Answer	Explanation	FR	SRM level
N1	180 km	multiplied 60 by 3.	CF	U_1
N3	30 mins, ½ hour	divide 20 by 10.	CP	U_1
V1	Twice as much time	take the first journey time and multiply it by 2.	CF	U_1
V3	2 times	if she goes twice as fast, she is halving her speed, which means she does it in half the time. so she goes around twice.	CI	R_1

In Question V3 he had the correct answer but an inappropriate explanation. There was some confusion in this case over the use of the terms 'speed' and 'fast'. The question indicates that the speed is twice as big, which he had identified with 'fast', but then he stated that if the speed was halved then the time must be halved also. He has made an attempt to link the variables which indicates a move into R_1.

His misuse of the word speed was highlighted in the interview on Question N3:

S: ·· *It will take him ·· it will take him half an hour.*
I: *Why?*
S: *Because if he is travelling at twenty kilometres per hour ten is half of twenty and half an hour is thirty minutes.*
[An attempt is made to see if he can generalise this.]
I: *So you are saying that halve the -*
S: *Halve the speed he is going to halve the time.*
[He took the lead but on the wrong variables. He has equated speed and distance]
I: *But you have not halved the speed, he is still going at twenty kilometres per hour.*
S: ·· *um.*
I: *He has not halved his speed has he?*
S: *No. ·· You have a point there.*
I: *So why did you halve? You were talking of halving.*
S: ··

This line of questioning confused Anthony, and he eventually had to be helped back on to the right path. It was interesting to note that Anthony could provide the correct answer to the problem, but when asked to generalise what he was doing he could not do so. In the written test and the interview his SRM level was M_1 since he was trying to relate the variables but was unsuccessful.

During the interview on Question V1, however, Anthony gave a second cycle unistructural response U_2 using a direct variation strategy.

Interview, Question V1:

S: *Rightio ··· take twice as long.*
I: *Why?*
S: *Because they are travelling at the same speed but they go twice the distance, it takes twice the time.*

He took into account all the variables and he considered the constraint of speed staying the same for the variation to work. In the response to this question he was using the term 'speed' appropriately.

Part B Questions

For the Speed Test Anthony had all the Part B Questions correct with full explanations. His responses to these questions are provided in Table 9.14.

Table 9.14
Anthony's responses to Part B questions

QN	Answer	Explanation	FR	SRM level
B1	Same	Because they finish at the same spot at the same time.	CF	M_1
B3	Same	Because it took A 3 seconds to go 3 units & it took B 5 seconds to go 5 units.	CF	R_1
B5	B	Because it took A 3 seconds to go 3 units & B 3 seconds to go 5 units.	CF	R_1
B8	B	beause [sic] A took 5 seconds to g [sic] 3 units & it took B 3 seconds to go 3 units.	CF	R_1
B10	A	Beause [sic] A is going just over 1 unit per sec, but B is going only 1 unit per sec.	CF	M_2

The responses to Questions 33, B5 and B8 used the values of the variables to compare the distance and the time in explaining the answers, with no indication of how the values had been compared. In Questions B5 and B8, though, one of the variables does remain constant and so it was easy for a capable student to compare the values. For Question B10 he actually worked out the quotient of the distance to the time and effectively calculated the speed of both Trolley A and B which is a calculation strategy and thus coded as M_2.

During the interview on Question B5 Anthony gave the same response as in the test but after prompting he was able to start generalising the explanation.

Interview, Question B5:

S: *B* [no hesitation].
I: *Why?*
S: *Because it took three seconds for A to get to three but it took three seconds for B to get to five, it is further.*
I: *Because it went further?*
S: *In the same time.*

His initial response did not include this important concept of the times being the same, though it might have been implied by his mentioning the actual values for the time. With gentle probing he stated this correctly, and thus related distance and time using the names of the variables. If this was the same strategy used in the test then he was comparing distance and time values and not comparing the ratios. His initial response was at SRM level R_1 but on prompting he gave the principle that included the constraint, thus the level of response was U_2.

Interview, Question B10:

S: · *Oh! no!* [little laugh] *um* ···.
I: *Why did you say "Oh no" for?*
S: *Because it is harder.*
I: *Why?*
S: [another nervous laugh] *Um I think that ·· A · is ·· travelling · faster.*
I: *Why?*
S: *Well it took three seconds for B to travel three units whereas it took four seconds for A to travel five units.*
I: *Yes, so how does that make A travel faster?*
S: *Well · B goes one unit per second whereas A goes just over one unit per second.*

Anthony had to be prompted before giving the same answer in the interview that he had given in the test. While he gave the ratio of the values, it was only after he was asked how he had compared these two ratios that he was forced to do the calculation. It does show some consistency with the written test and he thus responded finally at SRM level M_2^*, but initially at R_1.

At this point the tape recorder was turned off for a few minutes as Anthony seemed to be under considerable pressure during the interview (he gave a great sigh of relief when the tape-recorder was turned off). This nervousness may account for the need to prompt him to obtain some of his responses.

Part C Questions

Anthony was successful in all the questions of Part C except for Question C8. Below are his responses to the test in Table 9.15.

Table 9.15
Anthony's responses to Part C questions

QN	Answer	Explanation	FR	SRM level
C1	2 units	Because speeds the same, but time is halfed [sic] so distance is halved 4 - 2.	CF	U_2
C2	4	distance is the same but time is halfed [sic] so B would have to be going 2 x as fast.	CF	U_2
C7	2	divide the units by the seconds.	CF	U_2
C8	6	if the sped [sic] is doubled it becomes 2 units per sec. time is increas [sic] by three times, it becoms [sic] 3 ses [sic] 2 units x 3 seconds = 6 units.	IF	U_2
C9	6 sec	if the speed goes dow [sic] by 1/3 it makes it 1 unit per sec, multiply 6 unit x by speed 1 = 6 seconds.	CP	M_2

In all the questions except Question C7, he used the variables that were indicated as changing in the question. With Question C7, however, he has referred only to Trolley B using distance divided by time to calculate the speed.

In Question C8 he made a mistake in thinking that the speed of Trolley A is only one unit per second and doubled it to obtain "2 units per sec". This might indicate that he is not able to control all the elements by confusing the speed for the time of one second. If he had not made this mistake he would have arrived at the correct answer since the rest of his explanation is full and clear.

The interview shows that he could obtain the correct answer.

Interview, Question C8:

I: *Notice that the speed has gone up two times and the time increased by three times.*
S: *Yea, so instead of three per second* [A] ·· *it* [B] *will travel at six per second. Is that right? So that means it is off the scale.*
I: *Why?*
S: *Why?*
I: *How far has it gone?*
S: *By three units.*
I: *So where do you think it will get to?*
S: ·· *Oh hang on* ·· [some muttering] ··· *If it takes three, one second to do three units, twice the speed one second will be six units at twice the speed* ·· *so that makes it* ··· *off the scale I think.*
I: *Yes, where would it be if that scale continued on.*
S: *Eighteen* [sigh].
I: *Where did you get the eighteen from?*
S: *Well three times six, because it takes one second to travel six units so three seconds like two seconds it travels twelve units and three seconds it travels eighteen units.*

The pressure that Anthony was feeling during this interview could have affected his concentration and required the prompting that was needed for the correct response. But his eventual reply showed that he had a working knowledge of the concept of speed and could do the necessary calculations on the variables, thus he was performing at the SRM level of U_2 and reaching up to M_2 but not quite making it.

Interview, Question C2:

S: *Because it has to be going twice as fast to get to the same distance in half the time.*

This response was again at SRM level U_2 as in the written test.

Overall, Anthony had a good grasp of the concept of speed. He could handle direct and inverse variations appropriately, and could see how speed was related to

distance and time, especially if on y one variable was changing. When two variables changed, he reverted to calculating the speed and comparing the answers.

Summary

Table 9.16 supplies a summary of Anthony's SRM response levels comparing the test response levels with the interview ones.

Table 9.16
Summary of SRM levels for Anthony

	Questions														
	des.	N1	N3	V1	V3	B1	B3	B5	B8	B10	C1	C2	C7	C8	C9
test	R_i	U_1	U_1	U_1	$\underline{R_1}$	M_1	R_1	R_1	R_1	M_2	U_2	U_2	U_2	$\underline{U_2}$	M_2
interview prompt	R_i* R_1*	-	R_1	U_2	-	-	-	R_1* U_2*	-	R_1* M_2*	-	U_2	-	U_2*	-

An asterisk (*) indicates level after some prompting.
Underlined levels indicate that the student's answer was incorrect.

Figure 9.4 shows the SRM profile for Anthony's responses to speed as taken from Table 9.16.

Key:

+ response in test	■ response in interview
± interview and test the same	* prompt in interview
des = description of speed	A, B, C are for Parts A, B, C of test.

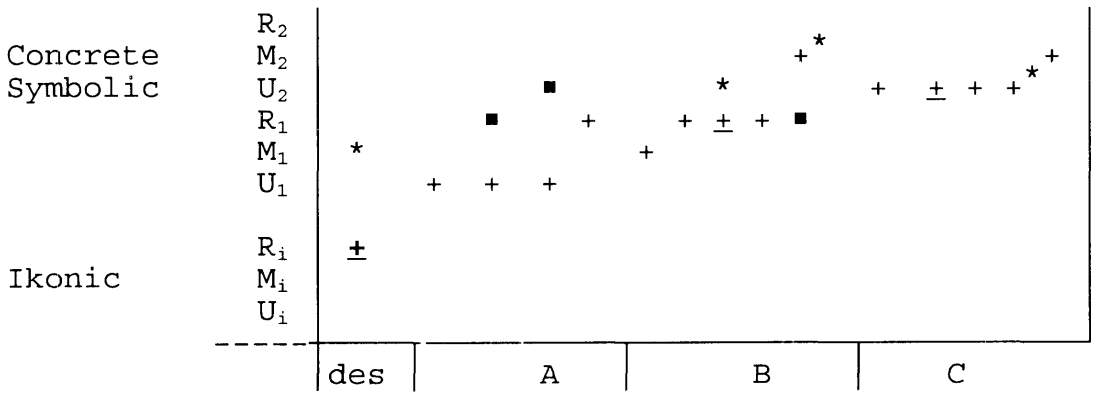

Figure 9.4 SRM profile for Anthony.

Anthony had a wide functional SRM response level which spans the first cycle and the beginning of the second cycle of the concrete symbolic mode. The level of the responses depended on the complexity of the question. For the Part A questions, he was generally responding at U_1 and R_1, for Part B the responses were at R_1, while for the Part C questions he was operating mainly at U_2. On only two occasions did the prompting of the interviewer increase his level of response, that is, on the description of speed and for Question B5. Twice he reached an optimal level of M_2 in the written test for Question B10 and C9. Both these questions required this level of response to be solved satisfactorily and Anthony was able to do this.

The trend of the Anthony's profile was upwards from the questions in Part A to Part C (this compares with the erratic performance of Leisa). This upward trend also reflects the different minimum level that is required to solve the questions satisfactorily. His level of operation on the problems was above that of the level he used in his description of speed without prompting. When he was prompted for his description of speed he gave a M_1 response, but in most of the questions he was operating above this level to solve the problems.

The two students in Year 9 show more differences than similarities. Anthony used appropriate strategies to solve the problems in a consistent way whereas Leisa had little idea of the relationships between the variables and often reverted to ikonic type strategies to solve the more complex questions such as those in Part C. The profiles of these students in Figures 9.3 and 9.4 indicate clearly these differences.

YEAR 12 STUDENTS

STUDENT PROFILE 5: HEATHER

Heather is a Year 12 student (Student 110) aged 17 years and 4 months, and was born in Australia. She was one of the top students in the Year 12 cohort. Although she did Mathematics I she has not taken Mathematics II nor Physics so she has not been introduced to the concepts of speed as rigorously as some other students. Her results on the speed test was fourteen correct responses out of the fifteen questions.

Description of Speed

In the speed test, Heather gave a description of speed at the first cycle of the concrete symbolic mode at the relational (R_1) SRM Level.

The distance an object travels in a particular time.

In the interview she was still consistent with this description after the discussion of the speed problems.

Speed is the distance something takes in an amount of time.

There was no suggestion in her response that speed was the quotient of distance and time.

Part A Questions

Heather had all four questions in Part A correct with the first three being with a full explanation. Her responses are displayed in Table 9.17.

Table 9.17
Heather's responses to Part A questions

QN	Answer	Explanation	FR	SRM level
N3	½ hour i.e. 30 mins	10 km/hr - 20 km/hr.	CF	U_1
V1	Twice as long	x2 because is twice as far.	CF	R_1
V3	twice	$5 - 2 = 2.5$ $\quad 2.5\overline{)5}\ 2$ $\Rightarrow$ 2 times (twice)	CF	U_1
V5	2 x slower	$10 \times y = 20 \quad y = 2$	CP	U_1

In Question N3, it was not clear whether Heather knew the answer then searched for a method to explain it or not. But her use of ten kilometres per hour instead of just ten kilometres could show a lack of understanding of the problem. There is no evidence that she used the formula, *time = distance ÷ speed*, to obtain time in this question or any of the others. It appears as if Heather had looked for an operation that would fit her answer.

In Question V1, it was apparent that Heather was aware that if an object goes twice the distance it will take twice the time, so she used direct variation. She used a similar approach in the interview.

The response to Question V3 showed at least two operations taking place and it was not completely clear what she had done. A possible approach that corresponds to her answer is: To find the time to go around the block once at the new speed divide the old time by two obtaining *2.5* minutes. Now how many *2.5* minutes occur in *5* minutes? answer 2, so the girl will go around the block twice in the 5 minutes. It is not clear whether Heather knew that twice the speed then twice the distance in the same time.

Question V5 was a correct response but it was not clear how she decided on the explanation. She may have been searching for operations that supported her answer. In the interview on this question, there appeared to be a gap between her thinking and giving a clear explanation.

Interview, Question V5:

S: ····· *Half as slow because if it takes him ten minutes to walk but this time it took him twenty minutes which is twice ten, which is half.*
I: *Twice ten is a half?*
S: *Two times ten is twenty so you get a half.*
I: *That is you went two times ten and got a half.*
S: ··· *That has to be slower by two times.*

Heather was having trouble expressing herself, she obviously knew that if one doubles the time then the speed would be halved, but she could not generalise, which places her response at SRM M_1. She has used the values of the variables in all these questions.

Part B Questions

Heather gave correct and full explanations to all five questions in Part B of the speed test. Table 9.18 shows her responses to the Part B questions.

Each response refers to both the variables time and distance, and she has not resorted to a formula, calculation or ratio method in any of them. Each answer is reasoned by looking for the biggest distance travelled in the shortest time. Also,

Heather has not referred to the value of the variables which indicates that she was working solely in the second stream of the first cycle in the concrete symbolic mode.

Heather's answer to B4 shows an attempt to justify her answer, that is, reaching into the next cycle. She does not use ratios but common sense is applied to the problem as an attempt is made to compensate one variable change with another. The same type of thinking occurred in Question B10 which has been explored in Chapter 5.

Table 9.18
Heather's responses to Part B questions

QN	Answer	Explanation	FR	SRM level
B1	Same	both going same distance in the same time.	CF	M_1
B4	Same	As above but extra sec needed to go one centimetre further but if A had to go further would take same time.	CF	R_1
B5	B	because travelling further in the same time. is faster	CF	R_1
B8	B faster	because it takes a [A] longer to travel same distance.	CF	R_1
B10	A faster	Travels further in less time than if B travelled same distance.	CF	R_1

In the interviews, however, this thinking style was not always followed and other methods were used. When Heather answered Question B4 in the interview she focused initially on the time and said that Trolley A had the greater speed because the time was shorter. Then, as she was explaining it, she realised that the distance of Trolley B was greater, so she stopped, and, after some thought, realised that both Trolleys had the same speed:

S: *Because the distance travelled* [Trolley A] *is three metres and it takes three seconds so its a second a metre and this one* [Trolley B] *travels four metres and it takes four seconds and that is a second per metre so therefore they are the same.*

A different approach was used in this problem compared to the way she answered it in the test. In the test she reasoned it through by saying it was the same distance in the same time if Trolley A had an extra second of travel time. In the interview Heather was comparing the ratios of time to distance of each trolley. This is not a typical approach and was not used by students who have studied physics, they would use distance per unit time. Maybe this method was used when she realised that she was making mistakes with her original approach of comparing the times only. This final approach is at SRM level M_2, since she has compared the quotients of time to distance to compare speeds.

Interview, Question B8:

S: ··· *B goes faster.*
I: *Why?*
S: *Because it is travelling the same distance and it takes three seconds whereas A travels the same distance and takes five seconds.*
I: *Ok, But how does that make it faster?*
S: *Because it has taken less time to travel over the same distance.*

Heather's initial explanation was typical of other students' responses to this question, that is, she gave the distance and/or time values as an explanation. (She identified that the distances were the same.) After gentle prompting Heather generalised by using the variable names, instead of the values of the variables in the final part of her answer. She has included also the constraint that the distance is the same. Thus she is operating at R_1 and is reaching to U_2^*, after prompting.

In the interview of Question B10, Heather obtained the correct answer but her explanation just contained the values of the variables.

S: ··· *A is faster.*
I: *Why?*
S: *Because it takes four seconds for it to travel five metres and B takes three seconds to travel three metres.*

This response is coded R_1. She was then challenged as to how that could help her make a decision.

S: *I don't know I can just see it.*
I: *What can you see?*
S: *[she laughs]* ·· *um.*

She then tried to recall a formula for speed to justify her answer but was not sure about whether it was correct or not. The interviewer asked if she had used that formula to get her original answer, she said she had not. It was then explained to her that one of the purposes of the study was to try and identify how students can make decisions such as she had. She was willing to let the interviewer pursue with the questioning.

S: *OK I don't know* [how she thought about the question] *It just sort of come to me.*
I: *I want to pursue it.*
S: *How can I tell you when I don't know?*
I: *But you made a decision and it must have been based on something. Now when you read the question you were quiet then you told me the answer.*
S: *Yep.*
I: *And then you gave me an explanation, now was that explanation based on your original thinking or was it just to satisfy me.*
S: *Both, well that is what I did when I read the question I looked at the time and the distance and then I looked at the time and the distance with each other and then that was it.*
I: *And that was it?*
S: *yes mmmm.*
I: *OK let us look at the explanation. You said it went five in four seconds did you do any processing on that five and four?*
S: *No.*
I: *Then you said this one went three in three seconds did you do any processing on that?*
S: *No*
I: *Could you have said "Hey that went a bigger distance"?*
S: *A bigger distance in yea ··*
I: *But it took a longer time?*
S: *Yes but this one went a smaller distance and they took nearly the same times*
I: *Ok.*
S: *I don't know I can just see it.*
I: *Yes I know.*
S: *And I don't know how I came to the conclusion that I just do.*

This type of intuitive thinking was difficult to explore and the questioning may have confounded the original line of pursuit (even when one has a fairly willing subject). One of the problems was that if the student was thinking intuitively then an explanation needed to be given in the concrete symbolic mode to communicate and explain, which tends to run counter to an intuitive operation. This was confirmed when she wanted to use the formula to justify her answer.

From Heather's answer, towards the end of the above dialogue, it appeared that she saw the times were not very different and that Trolley A had travelled a bigger distance so it must have the greater speed. "It just looked right". Such thinking may have been common to other students also. This may help to explain why the number of correct responses for Question E10 was higher than expected in Years 6 and 9 (this issue was discussed in Chapter 5). This type of processing of data, without being able to give a method for comparing the ratios, is typical of the first cycle in the concrete symbolic mode. Students in the ikonic mode intuitively answer the question from the diagram, while students in the first cycle of the concrete symbolic mode, know the answer from the data. When the method of comparison is clear the response is in the second cycle.

Heather can work in the second cycle, as indicated in the interview on Question B4, but she does not do this all the time. It appears that such responses as that given to Question B4 are more fortuitous than planned.

Part C Questions

Heather obtained the correct answer in all the questions in Part C but the last one (Question C9) contained an inappropriate explanation. Four of these answers were given with full explanations. Her responses are presented in Table 9.19.

Table 9.19
Heather's responses to Part C questions

QN	Answer	Explanation	FR	SRM level
C3	8 cm	Twice as fast => twice as far.	CF	R_1
C5	1/2 sec	twice as fast so less time to reach same distance.	CF	U_2
C7	2	B go 2 cm in 1 sec 6 ÷ 3 = 2 [Originally she had 3 ÷ 6 = 2 but she changed it].	CF	M_2
C8	18 cm	3 x 2 x 3 = 18.	CF	M_2
C9	6	6 cm in 1 sec 18 cm in 3 sec.	CI	M_1
C1	2 cm	A went to 4 in 2 seconds so B will go to 2 in 1 sec.	CF	R_1

Heather used direct and inverse variation successfully on Questions C3, C5 and C1. The dialogue of the interview were also consistent with this.

In Question C7 she did not use variation but saw that Trolley B was going two centimetres every second. To justify this, she divided one number by another and on checking saw that they were the wrong way around and altered it accordingly. It did not appear that she consciously used the speed formula.

Heather used direct variation twice in a row in Question C8, by using the speed of Trolley A as the base and doubled the speed and trebled that answer.

Her answer to Question C9 was correct but the explanation did not relate to the question. It may be a carry over from the answer in Question C8. It appears that she did not see that if the trolley was going at one unit per second then it would take six seconds to travel to six units.

Interview, Question C9:

S: *·· mm ·· that is hard.*
I: *Why?*
S: *I don't know, can't do it.* [laugh] *Too many variables in it and the distance is being doubled. I guess I can take a number of steps to work it out. It would take one third of a second to travel one metre ·· three to three um ··. Did I get it right* [in the test].
I: *Yes.*
S: *I did!!*
I: *But no reason.*
S: *Mm ···· I suppose I ·· divided that by three or something and multiplied it by two or I don't know.*
I: *Why would you divide it by three?*
S: *Because this one is going at a speed of three and this one is going at a speed of one, it took three to get to three it would be one second divided by number times by three to get to there and multiply by two because ·····. Oh I would say six seconds.*
I: [covered up Trolley A] *Would six seconds to go from there to there be reasonable?* [pointing to Trolley B]
S: *Yes one second to travel each metre 1,2,3,4,5,6 seconds* [as she counted she pointed up the scale on the diagram].

Heather's initial response to this question was that it would be to "hard" to do it because too many variables were changing. However, she was prepared to think of a suitable strategy, that is, do one step at a time. Her next attempt was not very fruitful. After prompting and an awareness that she had done it correctly in the test, Heather was then prepared to suggest a method that might have given the correct answer. Following the variations given in the question she wanted to divide by three

and multiply by two. Her reasons for following such a process are not clear and she seems to have intuitively done some processing and arrived at an answer. When her answer was challenged she could justify it correctly. This response was coded as R_1^*.

Summary

Overall Heather seemed to be a very intuitive type thinker not relying on special learned techniques or formulae as some other Year 12 students did. In the questions of Part B, and C3 and C5 she referred to how the variables were changing in relation to each other, without having to use the values and do calculations. In the other questions, however, she referred to the values of the variables. This type of approach was discussed in Chapter 6.

Table 9.20 compares the SRM levels of the interview with the test.

Table 9.20
Summary of SRM levels for Heather

	Questions															
	des.	N3	V1	V3	V5	B1	B4	B5	B8	B10	C3	C5	C7	C8	C9	C1
test	R_1	U_1	R_1	U_1	U..	M_1	R_1	R_1	R_1	R_1	R_1	U_2	M_2	M_2	$\underline{M_1}$	R_1
interview	R_1	-	R_1	-	M..	-	M_2	-	U_2^*	R_1	-	U_2	-	-	R_1^*	-

An asterisk (*) indicates level after some prompting.
Underlined levels indicate that the student's answer was incorrect.

Figure 9.5 shows the SRM profile of Heather's responses taken from Table 9.20.

Key:

+ response in test	■ response in interview
± interview and test the same	* prompt in interview
des = description of speed	A, B, C are for Parts A, B, C of test.

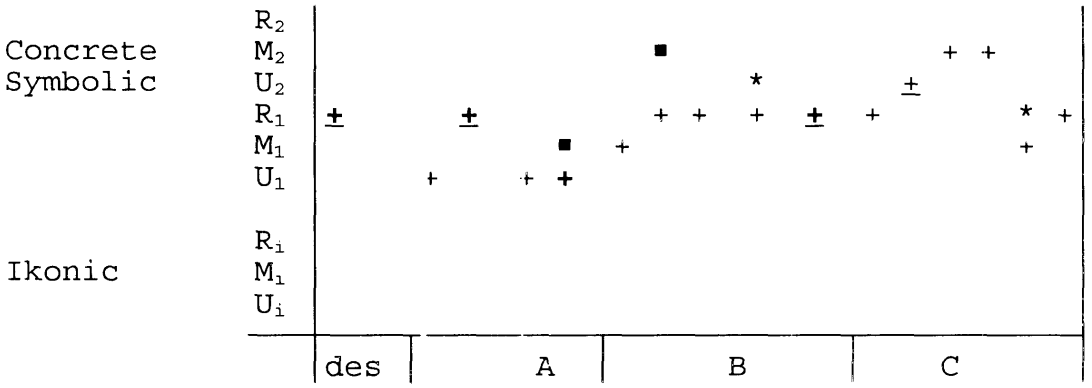

Figure 9.5 SRM Profile of Heather.

From the profile, Heather has a functional level in the first cycle of the concrete symbolic mode around the M_1 and R_1 levels. She is optimal at the second cycle M_2. She was consistent with her description of speed which was about the same level as that used for the problems on speed. She had very successful strategies for solving the problems that did not involve using the formula. Heather considered all the variables and used their relationships successfully, though she was unable to always explain how she accomplished this, typical of first cycle responses. It is interesting to note that three times Heather used strategies compatible with the SRM M_2 level but could not revert to these strategies on other questions even when prompted to explain how she obtained her answer. Such high level responses were not part of her functional level. There is a slight upward trend in the level of responses as progress was made from Part A questions to Part C questions. Such trends have been noted before and are due to the greater demands of the question to be solved.

STUDENT PROFILE 6: RICHARD

The last student to be considered in this chapter is Richard who is in Year 12 (Student (113) the dux of the school. He is 16 years 11 months of age and born in Australia. He had studied Mathematics I, Mathematics II and Physics. He had all the questions correct in the test.

Description of Speed.

In the test Richard's response for a description of speed was:

> *Speed is the distance travelled by an object in a given amount of time. e.g. 20 m/s, 20 km/hr, 100 feet per sec etc. Speed is a non-vector quantity.*

This is a SRM R_1 level response as he has referred to the distance and time but has not referred to it as a quotient or rate of change.

Richard's response in the interview was:

S: *It's um ··· far out ·· um ··· it's the time taken for an object to move over a given period····*

The discussion was then side tracked by Richard talking about vectors. The investigator then asked Richard what he meant by the term 'over' in his statement concerning speed. The rationale for this was to find if he was using it in a mathematical sense or just a period of time in which the object moved. Eventually, after some discussion, he said:

S: *Oh right yes ·· denominator and numerator of a sum like metres per second is the speed unit so therefore it would be distance over time.*

The response in the interview was at a higher level than in the test since the student was applying the term 'over' to the quotient of distance and time. Hence, at this point, he was operating at U_2 and making the transition to M_2. It took some discussion before Richard could clarify what he meant by 'over'. He was very verbose with his replies trying to give multiple answers to cover every possibility.

Part A Questions

Richard had all four Single Focus Questions of Part A correct with full responses. Table 9.21 provides his responses to the Part A questions.

Table 9.21
Richard's responses to Part A questions

QN	Answer	Explanation	FR	SRM level
N3	1/2 hr	d = vt rearrange formula to obtain t and subs [substitute].	CF	R_2
V1	2 times more	d = vt while letting the first journey be the value rearrange and subs[titute].	CF	R_2
V3	twice	d = vt let d = value of d subs & rearrange formula to find normal journey = d/5 & running time = d/2.5 => faster.	CF	R_2
V5	2 times slower	d = vt => t = d/v as t α 1/v => as t gets larger v gets smaller as d is constant.	CF	R_2

The most obvious difference between Richard's responses and those of the previous students was that he used the formula for speed ($v = d/t$) and did so from the very first question. The first two responses involved substitution into the formula, whereas the responses to Questions V3 and V5 showed some deeper mathematical insights in the way variations occurred but they were still formula orientated. In Question V5 he has generalised the inverse variation while keeping in mind the constraint that the distance was constant, thus he has also incorporated a U_2 level response as well.

Part B Questions

In the Dual Focus Closed Comparison Questions (Part B) Richard again gave correct and full responses to the questions. Table 9.22 presents Richard's responses to these questions.

Except for Question B1, Richard has used the formula to calculate the speed of each trolley, then he has compared the answers. He has not used the methods of the

previous students in this chapter by comparing distances and times. This type of approach was typical of at least half the students who had studied Physics.

Table 9.22
Richard's responses to Part B questions

QN	Answer	Explanation	FR	SRM level
B1	Same	Same distance and time.	CF	M_1
B4	Same	From the equations v = d/t both speeds are 1 cm/s.	CF	R_2
B5	B is faster	From the equation v = d/t A = 1 cm/s B = 5/3 cm/s => B faster.	CF	R_2
B8	B is faster	From the equation v = d/t A = 3/5 cm/s B = 1 cm/s.	CF	R_2
B10	A is faster	from the equ v =d/t A = 5/4 cm/s B = 1 cm/s.	CF	R_2

In the interview of Question B5, Richard went into great detail explaining what he had done in the test. Part of this interview is reproduced here to illustrate the detail that Richard was prepared to give.

S: *First I see the time for the top one so I put the time down for the first one. I put three seconds for the second problem* [He was writing these down] *I will put this a bit further down the page. This is three seconds also. Therefore I can see the ruler indicating maybe metres, centimetres I don't know. But I would say metres sticking with convention. So we would say three metres for the first problem* [he put the three metres under the three seconds] *and the next problem is the five metres that is five minus zero so I can see that is final minus initial so that is five metres so I would go to an equation that has those two problems there relating to the speed because we are trying to find out if one is greater or one is the same so I look at my equations. I think of an equation that has both of those and the speed so I have got speed I have got time and I have got distance so I know an equation that says speed equals distance over time .. so I, just for the first problem here at the top of the page I plug in the equation so speed is equal to three metres over three seconds which is equal to three metres divided by three seconds equals one metre per second that is speed one* [he writes S_1]. *Speed two* [He writes S_2] *for the second problem is equal to the same distance over time which is equal to three metres over three seconds which is equal to* ....

I: *You mean five metres not three*

S: *Oh yes five metres and two thirds metres per second therefore as speed two is greater than speed one you would say that the second problem is going at the fastest speed, so therefore B is going faster.*

By giving all that detail Richard lost the initial thrust of the question, obviously he did not write all those details when doing the written test, some of it would be routine, but he has gone to the 'ntl degree' to please the interviewer with detail. The SRM level would still be R_2. It is also of interest to note that Richard was the only student interviewed that actually used the paper and pen that was provided in all the interviews. Richard was then challenged to do the question without using the formula.

S: *I don't usually try it like that.*

So he was given an example of how it could be done using the idea that greater distance in the same time implies greater speed, then:

I: *It is quicker like that.*
S: *Yes. If you wanted a mathematical response to it though.*
I: *So you are calling yours a mathematical response?*
S: *Yes.*
I: *Is that the way you normally go?*
S: *Yes. The way you just did it you did it in your head you didn't have any mathematical response to that, you didn't have any proof for that or anything like that.*

It was interesting to see how Richard defined a mathematical response. If the formula was not used and the response was not written down, it was not mathematics. He was then encouraged to do Question B9 without using the formula.

S: *Yea! Um I can see the second one* [Trolley B] *has gone three seconds and it has gone four metres and the first one* [Trolley A] *has only gone five metres in five seconds so I would say that B is the fastest one.*
I: *Why?*
S: *As it's in the lesser time gone much more distance. It has gone five seconds for only one metre more so I would have to say that B would be greatest.*

Twice in his response Richard had referred to the time before the distance. For a formula orientated person where speed is known as distance over time, then using distance followed by time would be a more routine procedure. Richard then

calculated the answer by using the formula and 'amazingly' he got the same answer. He had used a mixture of variable names and values in his response (U_2^*).

For Question B10 the interviewer suggested to Richard that he could choose any method, Richard chose to use the formula but this time refined it by using final distance minus initial distance all divided by the time, thus he was again operating at Level R_2.

Part C Questions

Once again Richard had all the questions correct with a full response in the Dual Focus Open Comparison Questions of Part C. In each question, Richard used the formula on Trolley B to calculate the missing variable. Except for Question C3 he seemed to ignore Trolley A and in all questions did not refer to the information that was given on how the variables had been changed. Table 9.23 provides Richard's responses to the first three questions in Part C. The answers to the other questions were all of the same type of response and are not repeated here.

Table 9.23
Richard's responses to Part C questions

QN	Answer	Explanation	FR	SRM level
C3	d = ~~4 cm~~ 8 cm	from the equation d = vt v= 2 m/s t = 2 secs => d = 4 cm v = 4 m/s t = 2 sec => d = 8 cm.	CF	R_2
C5	t = ½ sec	d = vt - working out from equation v = 6 cm/s d = 3 cm => t =d/v = 3/6 = ½.	CF	R_2
C7	v = 2 cm/s	d =vt t = 3 sec d = 6cm => v = d/t = 2 cm/s.	CF	R_2

In the interview Richard was asked to solve Question C2, this question was not included in the Year 12 written test (although it was in the Year 6 and 9 written test). In this question the time was halved, the distance remained the same, the student was required to find the speed.

S: *If the time is halved and the distance remains the same then the speed is doubled.*

I: *The speed is doubled!* [surprise! he has not used the formula] *Why?*

S: *Because it has taken one second less than the first one or halved and by the equation d over t, half the time at the bottom means double the time at the top.*

He used "the equation d over t" as a ratio, not to obtain the answer but to justify his answer. Initially, he gave a SRM response of U_2 which was sufficient for this question but when prompted for more detail he went to Level R_2 to use the formula. To check if that was just a 'one off' response Richard was then asked to solve Question C6 where the two times remained the same but the distance was halved, what would happen to the speed?

S: *See it has taken the same amount of time so we take that out of the solution but we see that the distance between them is different with a constant time. So we would say that it has taken ... It's faster for the first one it is obvious because it has taken ... it is double, double with the um distance and the same variables with it. And so I am saying that the speed is halved.*

This was not the easiest way of arriving at the solution but he did get there in the end. It was not obvious here whether he used a formula or not. Using a relevant calculation, as these comments suggest, is a SRM level of R_1. In the questions with two variables changing Richard always used the formula on Trolley B as in the written test.

Summary

Richard's responses are in sharp contrast to Heather's responses (given above) especially in the strategies that were used to solve the problems. Heather had been more intuitive about examining the way the variables related to each other, whereas Richard used the formula even in the most simple of problems. As a physics student he had access to the formula whilst Heather did not have access to the formula though she was aware that one did exist. Richard's perception of what mathematics was, is an interesting insight as to how some capable students might perceive mathematics. If it is not writing down formulae and manipulating them it is not 'real' mathematics. This might well reflect the current teaching styles and perceptions of some school teachers.

Table 9.24 provides a summary of the various levels with which Richard responded to the different questions for both the test and the interviews.

Table 9.24
Summary of SRM levels for Richard

	Questions															
	des.	N1	N3	V1	V2	B1	B4	B5	B8	B10	C3	C5	C7	C8	C9	C1
test	R_1	R_2	R_2	R_2	R_2	M_1	R_2	R_2	R_2	R_2	R_2	R_2	R_2	R_2	R_2	R_2
Interview prompt	U_2* M_2*	-	-	-	-	-	-	R_2	(U_2*)	R_2	(U_2) R_2*	(R_1)	R_2	-	-	-

An asterisk (*) indicates level after some prompting.
Underlined levels indicate that the student's answer was incorrect in the test.

Figure 9.6 shows the SRM profile for Richard taken from Table 9.24.

Key:

+ response in test	■ response in interview
± interview and test the same	* prompt in interview
des = description of speed	A, B, C are for Parts A, B, C of test.

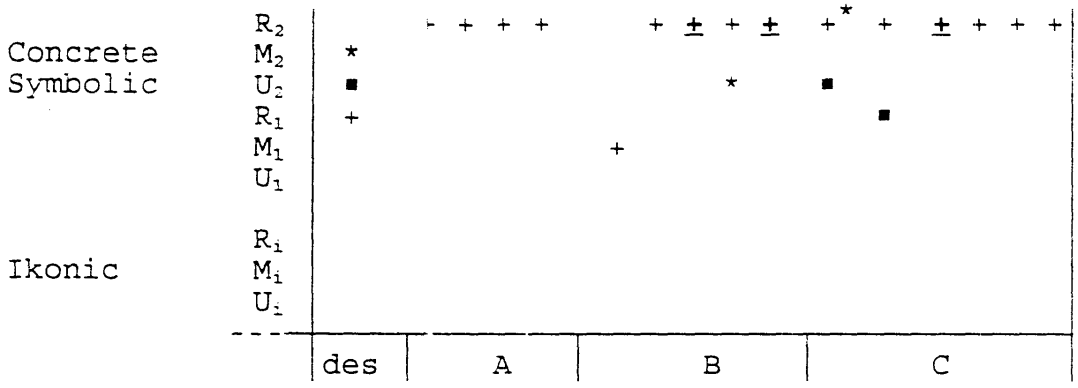

Figure 9.6 SRM Profile of Richard.

In the written test Richard was operating within the concrete symbolic mode in the second cycle, mostly at the relational level R_2, that is, he was using the formula explicitly to solve most of the problems. During the interview he was encouraged to use other strategies beside the formula on some questions but it was not the way he

liked to do the problems. If justif cation was necessary he used the formula and thus operated at level R_2.

Richard's optimal and functional level were the same at R_2. However, for a description of speed he did not spontaneously use mathematical relationships to define speed nor did he use the formula. Only on prompting was he encouraged to relate the two variables, but still the formula was not referred to.

CONCLUSION

From the results of Heather, Richard and Anthony it is apparent that functioning at the R_1 and/or second cycle can consistently provide the correct solution with appropriate explanations. These two cycles embody two alternate strategies of doing the problems. The relational level of the second cycle is more readily available to those who have been introduced to the 'mysteries' of the formula in a formal way. Given motivation and application, students who have not been shown the formula and are working at R_1 or U_2 could develop the formula for themselves. Such students would then be working in the formal mode of the SOLO Taxonomy. Once the formula had been established these students might under go a 'level reduction' as they routinise the procedures in the second cycle of the concrete symbolic mode.

In addressing Research Question 16 it is evident that the Speed Response Model (SRM) of the SOLO Taxonomy does provide a means of describing the response levels of students. Responses can be categorised to a level within a cycle. Although the intent of the SOLO Taxonomy is that the response is being categorised and not the student, the responses on a particular topic do provide an overall profile of students' performance. This profile of students' responses does give an indication of the functional level both in the written test and the interview for the student concerned. With prompting in the interview the optimal level that the student was capable of operating at can also be identified.

Research Question 17: "Can student's responses be understood and explained by the SOLO model?"

When students' responses can be placed within a certain level of the SRM then this gives understanding as to the type of concept students have about speed for that question. When a number of responses for different questions are categorised, and a

profile of a student's level of responses is produced, as in the student SRM profile diagram, then a better understanding is gained of their performance and conceptions concerning speed. For example, if most of a student's responses fall within the ikonic mode, then they are unaware of the variables associated with speed and are relying mostly on visual cues. Thus students levels of operation for giving responses on speed problems can be understood using the terminology of the SRM.

With the SRM as a basis, it may be postulated that students would grow through the levels within the cycles, that is, when students' initial concepts of speed are vague, problems are solved intuitively based on guesses or visual information. They then proceed to identify certain aspects of the diagram or problem as important in explaining their answers. With more experience they identify the variables associated with speed and determine which variables are used to find greater speed. This enables them to combine the variables and some students will eventually consider all possible variables and their conditions that are important to the problem. From the interviews it is known that not all students are capable of thinking through problems and taking into account all of the variables and the conditions under which the variables operate, even when encouraged to do so. The more able students use sophisticated type mathematics and/or calculations to help them solve problems. Usually, after students have been introduced to the concept of speed formally, through the formula, they then operate at the relational level of the second cycle either all the time or when the difficulty of the problem demands it, otherwise many of them use first-cycle operations or simple mathematical strategies that do not involve finding the speed of the trolleys.

One very important outcome from this chapter is the introduction of student SRM profiles that provides an overview of a student's cognitive response level of operation. Such profiles are worthy of more attention in further work and may provide a vehicle for examining student performance from their responses for other selected topics.

Another function of the student SRM profiles is to furnish a basis for classifying questions according to the level of expected response, that is, the minimum response level necessary to satisfactorily solve the problem. For this to be accomplished a range of profiles would need to be examined to see the trend of minimum levels required to solve a problem.

CHAPTER 10

CONCLUSION

'Reeling and Writhing of course, to begin with,' the Mock Turtle replied,
'and then the different branches of arithmetic - Ambition, Distraction,
Ugl fication, and Derision.'
Lewis Carroll. "Alice's Adventures in Wonderland" Ch 9

The three main themes of this thesis were first introduced at the end of Chapter 2. These themes are: 1. To ascertain students ideas concerning speed and the methods they employ to solve problems on speed; 2. To examine the strategy choices of students to determine any underlying structure; 3. To provide a theoretical framework to interpret the findings.

This chapter considers initially possible limitations imposed by the design of the study. Then consideration is given to the outcomes of the study in relation to the three themes. The final sections of this chapter discuss the implications of the study for further research and for the teaching/learning process.

LIMITATIONS OF THE STUDY

The outcomes described in the preceding chapters must be viewed in the light of possible limitations imposed by the written test and the features of the experimental design.

The Written Test

Three separate tests, one for each Year level, were constructed using the speed questions prepared in Chapter 3. Each test had a similar structure but they had a different mix of questions suited to the levels of the students for which they were designed. The length of the test had to be such that students in each Year level would

complete it in less than forty minutes which was the length of a typical class period. Any longer time than this would have caused disruption to at least three classes. Another factor that was also taken into account when considering the length of the test was the time that students would be able to stay on task without losing the motivation to complete the test. For these reasons not all question were given to all students so that the test would be of a suitable length.

For each section of the test there were common questions to all tests. These questions provided some comparability across groups. Though the length of the test and the number of questions on the test were a limitation in this study, the questions asked and the time given on those tasks were adequate in producing responses that could be analysed successfully.

The Interviews

It would have been ideal to interview all sixty-two students who responded to the written test. Again the time factor was a limiting component, not only from time constraints of the investigator but for the students, teachers and schools. For this reason six students from each Year level, with a range of abilities based on the outcomes of the written test, were selected. In hindsight it was just as well only eighteen students were selected for interview (two interviews had to be cancelled) since the problems encountered in organising these interview appointments would have been unmanageable if all students had been involved. The interviews that were conducted were rich in information and helped to identify some of the obscure strategies that students used. The interviews also clarified that the test responses were good indicators of students' initial methods and ideas of solving the speed problems, though prompting and probing in the interview did encourage some students to give better responses. The emphasis in this study was not on collecting data for quantitative analysis but in seeking quality responses that would yield students ideas on speed. The outcomes of the preceding chapters verify that such qualitative data were collected and were readily analysed to enable all research questions to be answered.

The Students

The students used in this study did not represent the population of students in Australia or even in Brisbane. The students who attend the schools used in this study come from a low socioeconomic community and thus most of them do not have the

advantages that come from living in a more affluent environment, both financially and educationally. Another consideration that affects the populations of the schools is the presence of three independent schools close by. The more affluent and/or the more educationally committed parents often see independent schools as alternative to a state school education. In spite of these restrictions, a wide range of responses was obtained from students involved in the study and there were some students who revealed academic talent. The study did not require a representative sample of Australian students to obtain suitable outcomes but rather the purpose of the study was achieved by receiving responses from students with a range of abilities that would span the weakest response to the most sophisticated response. This method of sampling referred to as 'purposive sampling' is one of the characteristics of a qualitative research project. Thus the sample of students chosen was not a limitation to the present study at all; in fact, these students gave quite a diverse range of responses at all Year levels.

SUMMARY OF OUTCOMES

The outcomes of the study are reported under the three themes of this thesis. First, as they apply to students concepts of speed; second, to strategies used in solving problems on speed; and, finally in relation to the post-Piagetian framework that was developed.

STUDENTS CONCEPTS OF SPEED

The level of students' responses is related to the nature of the question that is posed. Two types of questions are: a recall from memory of some concept; or, one that provided cues and data that could then be used to solve a problem. In different contexts other researchers (for example, Biggs & Collis 1980, 1982) have noted similar responses.

When students were asked to give their best meaning for speed, most students, who had not been introduced formally to the definition of speed, provided answers that were related to the everyday notions of speed associated with 'fast' or 'very fast'. Students who had, however, been introduced to the scientific/mathematical definition of speed gave responses that ranged from the concept of speed being something that goes 'fast' right through to the formula. Most of these students did mention that speed was related to distance and time but not always in a mathematical way.

A question that required a solution to a problem concerning speed allowed students to focus on the components of speed provided in the question, that is, distance and/or time. Some students did this even though their description of speed did not mention these two components. Thus, the nature of the question, in general, can elicit different levels of response.

The interviews highlighted the fact that those students who did not score well in the test provided consistently low responses in giving a description of speed for the written test and interview. This occurred whether they were asked for the description at the beginning of the interview or at the end of the interview, after discussion had taken place concerning the speed problems. On the other hand, those students who scored well on the test, but did not provide a high level response for a description of speed, increased the sophistication of their response, after discussion of the problems, by mentioning that distance and time were associated with speed. Thus, a student's optimum description of speed is related to the way they attempt to solve speed problems. They become aware of the essential elements that are required to solve the speed problems and are then able to articulate this into a better description of speed.

Thus, students' concepts of speed were often found to occur at two different levels depending on the nature of question to which they were responding. Often the responses provided for a description of speed by students, without probing or prompting, are at a lower level of cognition than the operational use of speed to solve problems. The case studies in Chapter 9 confirmed these findings.

STRATEGIES USED IN SOLVING PROBLEMS ON SPEED

A range of strategies were identified that students employed to solve speed problems. The strategies chosen by students appear to depend on the complexity of the question asked. In general four levels of strategy were identified, though they were not always needed for all questions. These levels were identified in the Strategies Model of Chapter 7.

The first level is identified by responses of students who use visual cues to explain how they had arrived at their answer, they ignored the relevant written data supplied. When one-variable strategies were selected this indicated the second level of strategy. At this level the student chose one variable as the determining factor in solving the question, ignoring all other relevant data. At the third level, students used

the two variables required to solve the problem but they did not mention any constraints that were necessary, or could not show the general rule that related these variables. This level was referred to as the two-variable strategy. Such strategies as direct variation and proportion were classed as two-variable strategies. Responses at the highest level strategy used all the relevant variables relating them together in some way. The most obvious specific strategy at this level is the use of the formula which incorporates all the variables that are associated with speed. Strategies that also referred to the variables that changed but used the condition under which the changes could occur were also at this level.

Students who chose the higher levels of strategy (Levels 3 and 4) were more likely to perform better on the speed questions and to successfully come to an answer. Students who chose the lower levels of strategies often provided incorrect responses and found it difficult to make progress to other levels during the written test or the interview.

These levels are similar but not identical to the three categories of thinking proposed by Lybeck (1979) (see Chapter 2). The most significant difference is that the four strategy levels pick up on the variables that the student focuses on whereas Lybeck's categories deal with the type of system or systems that are used. Lybeck's categories do not identify the students' responses that focus on the visual aspect of the problem. Also, the third category could be extended to students who could solve problems using the formula without difficulty.

Intuitive thinking was also identified as a common strategy to solve problems. Strategies of this type would often occur within levels one and two of the strategies outlined above. Collis and Romberg (1991) and Collis, Watson and Campbell (1992) indicate that students use intuitive (ikonic mode) and mathematical procedure (concrete symbolic) type approaches to solve problems. Furthermore, some students shift from one mode to the other as they move through the different tasks. Not only was this confirmed in the present study but it was noted that students often answered the question intuitively then attempted to explain their answer within the concrete symbolic mode. When a student's explanation did not match the correct answer, the student may have used intuitive thinking to provide the answer. On the other hand, some students may have initially employed an intuitive approach to arrive at the correct answer and then provided a correct concrete symbolic explanation. Such strategy shifts could not be detected in the written test since the same answer and explanation could occur for a student working solely within the concrete symbolic

mode. In the interviews, however, there were a number of instances where students stated the correct answer, and, when asked for an explanation, provided a concrete symbolic one. When they were challenged whether this method was used initially to provide the answer they admitted that it was not. They could "just see it".

Another shift in modes of thinking occurs with students who start using concrete symbolic logic and find it too difficult so they revert to an intuitive approach to answer the question. Their explanations are often meaningless. This selection of mode of thinking or shift in mode of students while doing problems was also noted by Collis and Romberg (1991) and Collis, Watson and Campbell (1992) (see Chapter 2). However, some students continue to use the concrete symbolic mode regardless of more intuitive type methods being available, for example, see the student Profile of Richard in Chapter 9.

POST-PIAGETIAN FRAMEWORK

The SOLO Taxonomy was chosen as a suitable post-Piagetian framework in which to gain an understanding of students' responses. The structured mapping diagrams were used as a tool to illustrate the different levels. When the outcomes were analysed within the framework of the SOLO Taxonomy, two findings stand out.

First, assigning SOLO levels, within the modes, to students' responses was not just a matter of identifying whether the response referred to one or two relevant pieces of data but it also included the number of processes that were used on that data. This was especially true when students had to solve problems. The number of processes that are employed in a response are indicative of the type of strategy used. Thus the strategy selected by students in their responses was a reflection of the level of the response. In contrast to this, when students were asked to give a description of a concept from their own repertoire of knowledge then the number of relevant pieces of knowledge that were selected and/or related determined the level of the response.

Second, two modes of functioning were identified. One cycle within the ikonic mode and two cycles within the concrete symbolic mode were found and used to explore students' understanding of speed. This model, developed in Chapter 8, is referred to as the Speed Response Model (SRM) of the SOLO Taxonomy. In the

ikonic mode students rely on visual cues to solve problems, such as, identifying the object which is in front as having the greater speed. The first cycle of the concrete symbolic mode may have ikonic support and is characterised by responses focusing on the relevant data and selecting strategies to use that data to solve the problem. Such strategies usually involved some sort of mathematical calculation or a comparison of the variables. In the higher levels of this cycle, students were frequently successful in solving the problem. The second cycle has no apparent, or overt ikonic support, and is at the higher end of the concrete symbolic mode. In this cycle, students used the formula or calculations, similar to the formula, to solve speed problems. They were nearly always successful apart from mechanical errors.

Within each mode, the levels of response have been identified as unistructural, multistructural and relational. Within the first cycle of the concrete symbolic mode students can refer to the variables with their values or their names. For students to reach the relational level, however, it is easier to refer to the variable names when giving an overview of the general principles and taking into account constraints. In the second cycle students employ well established mathematical procedures and often use symbols to represent the variables as they use the formula.

These levels were characterised in the Speed Response Model (SRM) and applied to individual student responses. This allowed an overview of student performance to be profiled that gave an indication of the level of functional and optimal performance of the student in relation to the concept of speed.

Thus the SOLO Taxonomy is a suitable framework to use and is found to have the flexibility to be expanded and modified without losing its essential elements. Once the modified framework is in place it provides a very effective structure to categorise students' responses and allows an overview of student performance levels over a variety of tasks to be discerned.

IMPLICATIONS FOR THE SOLO TAXONOMY

The original SOLO Taxonomy proposed by Biggs and Collis (1982) has undergone some developmental changes in recent years. The work of Campbell, Watson & Collis (1992), Pegg (1992), Levins (1992) and Levins & Pegg (1993a, 1993b) have suggested that cycles of levels also occur within the modes in other topic areas. This project has confirmed this for the concept of speed, and identified two

cycles within the concrete symbo ic mode. For a qualitative research project this confirmation increases the validity of the study.

Campbell, Watson & Collis (1992), also indicated in the intra modal model that the relational level of one cycle was identical to the unistructural level of the next cycle. This conclusion was not supported in this project. The three levels of each cycle were found to be uniquely determined and the relational level of one cycle was not identical to the unistructural level of the next cycle. In fact, the cycles were found to have their own characteristics that made them different to each other so that no overlap was detected, though there are similarities in structure as revealed by the mapping diagrams.

The implication for this on the SOLO Taxonomy is that there is now more detailed structure available to help determine the levels of students' responses. The number of cycles within the concrete symbolic mode could well depend on the topic chosen.

Another implication is that when the SOLO Taxonomy is applied to students' responses on solving problems, the type of strategy employed and the number of processes within that strategy need to be considered to determine the level of the response.

The simple structures that have been identified in the mapping diagrams, for each level within the cycles and the modes, provides a meaningful way of identifying those levels. The mapping diagrams also highlight which processes take place and how they are interrelated.

IMPLICATIONS FOR FURTHER RESEARCH

This study has refined some of the detail within the SOLO Taxonomy with one cycle in the ikonic mode and two cycles in the concrete symbolic mode employing the concept of speed. Four issues stand out as worthy of further investigation.

First, it would be of interest to know how robust this model is across a wider population of students, that is, will the same range of responses occur in students from a school that does not draw its students from a socioeconomically disadvantaged area?

Second, would the Speed Response Model (SRM) be robust in other topics of a similar nature? For example, the concepts of acceleration and force which have simple scientific definitions with mathematical formulae associated with them, but also have everyday common usage, would provide a fruitful field of investigation.

Third, does the model predict a cognitive development model for individual student growth in the concept of speed? An answer to this question would require a longitudinal study involving students with a range of abilities to be interviewed over a number of years.

Fourth, to try and tease out more directly the nature of intuitive processing both in young children and in adults who have not been 'taught' more formal procedures.

IMPLICATIONS FOR TEACHING/LEARNING

This study has considered a well known concept that is used both in the general community and in a formal mathematical sense. It is therefore not surprising that students come to mathematics and science classrooms with alternative conceptions of speed. From this study a number of issues have already been raised that impact upon the teaching/learning situation. In particular, three issues stand out.

STRATEGY V'S DESCRIPTION

Chapter 7 presented a comparison between students' description of speed and solutions to problems on speed. Some students who did not state that speed was related to distance and time could often use distance and/or time to solve problems on speed. On the other hand, some students who gave descriptions of speed that included distance and time did not always use these ideas to solve problems concerning speed. This has implications for the teaching process. When students understanding of speed is based on a description this does not always mean that they can then use that concept to solve problems. These students must be guided across these two different cognitive domains. The reverse is also true, when students are taught how to solve problems on a certain topic, they may not be able to analyse what they have done and give a succinct description of the process. Most students need guidance in teasing out the essential elements (or variables) that are associated with solving problems. If this is true with a common concept such as speed, the

difficulties must be compounded when they are presented with concepts about which they have little previous experience in the sciences such as energy, momentum and vectors. The difficulties become even more acute with the abstract concepts that are often presented in mathematics.

INTUITIVE THINKING

The role of intuitive thinking in solving speed questions was discussed in Chapter 6. Some students gave an answer then searched for a method or strategy that would explain how they arrived at that answer. In the classroom, teachers need to be aware of the inbuilt intuitive skills that students possess for solving problems. Young students often 'see' the answer then seek for a way to explain that answer, only after the teacher asks for one. They will not write anything down until they are sure of an answer. In simple problems, students at junior high school will often have correct answers though they might not be able to explain how they solved the problem. One of the aims of secondary education is to develop skills in the concrete symbolic mode. Teachers often use the concrete symbolic mode to solve problems and give explanations to students, while students are not realising the importance of this mode of thinking. Often these students are still working intuitively in mathematics though they can use the symbols and structure of mathematics to a limited extent. Initially, they start to use the concrete symbolic mode to justify their intuitive answers when the teacher demands it of them, but it is not natural for them to do this. The teacher needs to be acquainted with these two very different levels of processing and lead students from one level to another as they become ready. It is only when working on more complex problems, in the middle high school and later, that students will see the need of using the concrete symbolic mode as an aid to solve problems. At this time special guidance is required from the teacher to help the student develop these skills and to proceed in solving the problem.

A typical method, that some Year 10 and 11 students use to solve numerical problems in mathematics and physics, is to try various calculations on their calculators until they have the 'correct' answer. Then they write the answer down. Often they have forgotten the processes that they used to obtain that answer since they have not recorded what they have done. It is the role of teachers to help students formulate a plan for solving the problem and to write this down in an appropriate way (using the concrete symbolic mode) then use a calculator to implement the calculation.

With the emphasis in mathematics, especially in Queensland, on good communication skills as a necessary component in assessment, it is imperative that students learn to communicate effectively the strategies they use in the concrete symbolic mode of mathematics. This does not deny students the use of the ikonic mode (or intuitive thinking) as a powerful tool to tackle problems, but it does ask students to then back this thinking up using relevant concrete symbolic logic.

USING SOLO

Teachers who are acquainted with the SOLO Taxonomy, and, in particular, are conversant with the cycles within the concrete symbolic mode have a useful tool for categorising the responses of students. They can adapt the learning sequence to match the level of the student in that topic. This acquaintance is not just limited to the cycles found for the concept of speed but may be extended also to other areas of knowledge. In general when students' responses are highly visual then the teacher may look at the details within the response to see if the student is ready to be lead to the relevant data that are necessary to make a correct response. If the student's response can be categorised as high in the first cycle the student can be guided to see the relevance of the data to reach correct conclusions. A response that does have some relevant data can be used to help students access other relevant data and thus move them into the multistructural level. It would not be wise for a teacher to try and lead a student who can focus on only one aspect of a problem (i.e., the unistructural level), directly into an understanding of the relationships of the variables (i.e., the relational level) without making sure that they have acquired an understanding of several elements, that is, be able to respond at the multistructural level first. It is only at this level that students are aware of all the variables important to the situation. Once students are aware of the relationships between variables they can then be introduced to the formula as a logical method of stating the relationship. In this fashion, the teacher can help students to move in logical small steps from one level to another that will enhance the teaching/learning experience.

The author has found it useful to discuss the SOLO Taxonomy with senior students. In this way they can see where they are within the cycles and consciously reach for higher levels. Thus, students have some control of their own cognitive development and can appreciate what the teacher is doing when new concepts are

introduced. Students can also plot their own SOLO or SRM profiles and monitor their own progress.

CONCLUSION

The conclusion of Chapter 1 (the Literature Review) posed five questions that arose out of Piaget-type investigations. First, do the ideas that young children have concerning speed still occur in older children? The present study indicates that in general the ideas that young children have concerning speed are still present in older children to some degree. The stages that Piaget found are not limited to the ages of the children involved in his study. Some students up to Year 12 (18 years of age) still use visual cues to solve problems. Thus, the stages that Piaget proposed are not limited to any particular age group but are apparent in later years as well. This gives strength to the assertion, of the neo-Piagetian theorists, that the earlier modes are still available for individuals to use even when they are capable of using higher modes. Piaget found that in the earlier stages, children tend to focus on one aspect of the motion to compare speeds. This study has demonstrated that even when students are not shown the motion but just a representation of the motion in a diagram that there is a tendency for them to focus on one variable to the exclusion of other variables. This research has provided more detail concerning the relationships and processes that students use when employing variables to solve speed problems. For example, the SRM provides levels based on the way students link variables in the first cycle and the sophistication of mathematical procedures in the second cycle of the concrete symbolic mode.

Second, do children grow through different stages? It was not the purpose of this thesis to conduct a longitudinal investigation into the stages through which individual students grow. The focus was on the range of responses that students, from various Year levels, would provide to problems on speed. Now that these levels of responses have been identified, it may be postulated that this provides a framework for possible growth through the levels. The outcomes suggest, however, that there is no fixed chronological age for which each level is expected to occur. There is a trend, however, that older students, with more mathematical and/or physics experience, tend to use mathematical procedures (i.e., in the second cycle concrete symbolic mode) in preference to visual or intuitive ideas (i.e., ikonic or first cycle

concrete symbolic procedures). Similarly, younger children will favour ikonic mode processes over mathematical processes.

Third, some investigators have queried: Is time or distance the more salient factor for students to use in comparing speeds? This study indicates that for students who do choose only one variable in their explanations, it is the one that is different that usually receives attention, regardless of whether it is distance or time. Students today, compared with those of Piaget's time, do appear to have a better idea that shorter time implies greater speed. Students who focus on the visual aspects of the problem (ikonic mode), however, are more likely to use distance, instead of time, as this has the visual impact in the diagrams. They often confuse distance and time with the terms 'fast', 'faster' or 'in front'.

Fourth, would further investigation into this area provide a more detailed model for levels of development? The whole thrust of the last few chapters has been to demonstrate that a more detailed model as provided in the SRM does indeed fulfil this requirement. Although this have been discussed already in this chapter, the key points are that the nine levels developed in the SRM encapsulate the four stages of Piaget and the first three levels identified by Trowbridge; the fourth level went beyond Piaget's fourth stage to examine discrimination of such tasks as instantaneous and average velocities. This level was not examined in the present study. While it is not possible to have a one-to-one correspondence between these different numbers of levels, the SRM does provide a valid framework in which to explore these levels. Further, both the findings of Piaget and Trowbridge can be identified within the more encompassing and finely detailed SRM.

In general, each stage described by Piaget can be identified within the SRM. For example: Stage 1 goes with the U_i and M_i; Stage 2 relates R_i and U_1; Stage 3 links within M_1, R_1 and U_2; and, Stage 4 fits into U_2 and R_2. The four levels identified by Trowbridge (1979) can be linked with the SRM. For example: Level 1 corresponds with U_i and M_i; Level 2 is related to R_i, U_1 and M_1; Level 3 ranges from R_1 to R_2; and, Level 4 reaches into the formal mode which was not identified in the SRM.

The last question posed was; what was the relationship between strategies employed by students and their understanding of speed? Simply, students, who could use high level strategies, could also identify the variables associated with speed and manipulate them using either a calculation or the formula. Thus they showed a better

understanding of the concepts of speed than those students who chose lower level strategies.

Overall the Speed Response Model (SRM) developed in Chapter 8 and applied to the student profiles in Chapter 9 provides a more detailed framework to explore student responses than do the previous models. Not only has it extended the work reported in the literature but it has helped place the work of Piaget and Trowbridge within a broader context. The descriptions given to the levels within the modes provides more clarity to understanding students concepts of speed.

In summary, this thesis explored student's understanding of speed and provided a link between the age-groups of two important previous studies in this area. In attempting to 'fill the gap', a more detailed integrated model of development, based upon the SOLO Taxonomy, was offered. The framework developed, based upon a series of questions designed to cover a broad range of speed situations, enabled a clearer perspective of the link between responses and strategies to be obtained.

As with so much research, the more questions that can be answered the more there appears still to be found. The framework proposed is an important start to a detailed analysis of the concept of speed and other related topics. Nevertheless, many adults from the general community will state that "speed is something that goes very fast" and confuse speed with "Exceleration" [sic]. Obviously, there are still further issues that can be addressed by research in this area.

Piaget was often questioned as to why he continued studying the development of the mind, his answer is still pertinent: "Our only excuse is the still surprisingly unrefined state of the knowledge we have of children's intellectual growth" (Piaget 1979, p. ix).