

**A DEVELOPMENTAL ANALYSIS OF STRATEGIES
EMPLOYED IN SOLVING SPEED PROBLEMS:
A POST-PIAGETIAN APPROACH**

by

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CERTIFICATE

I certify that the substance of this thesis has not already been submitted for any degree and is not being currently submitted for any other degree.

I certify that to the best of my knowledge any help received in preparing this thesis, and all sources used, have been acknowledged in this thesis.

Signature _____



ABSTRACT

This thesis is a qualitative study of student concepts of speed. Previously, Piaget's (1946) work on children's understanding of speed laid a solid foundation for such a study, and has provided a basis for several other investigators (e.g., Trowbridge (1979), with tertiary students). Students in this study, whose ages ranged from twelve-to-eighteen years, were questioned about their ideas concerning speed. In understanding students' concepts of speed, three major factors were considered: strategies they employed to solve a variety of problems on speed; their descriptions of speed; and, categorising and describing these responses within a suitable theoretical framework.

The strategies employed by students to solve speed problems were categorised into four levels: first, focusing on the visual aspects of the problem; second, attention was given to one variable that was provided; third, two variables were used but there was no attempt to consider constraints; fourth, all relevant variables were accounted for and the strategy was nearly always successful, aside from mechanical errors. In addition a number of issues are discussed that relate to students choice of strategy. In particular: the way students referred to variables; the methods employed in solving direct and inverse variation type problems; and, the role of intuitive thinking.

To enable the responses to be better understood, a post-Piagetian framework, the SOLO Taxonomy of Biggs and Collis (1982), was employed. Students' descriptions of speed were able to be categorised by extending this taxonomy. With the aid of mapping diagrams, students' responses to speed questions were placed within two modes of functioning, namely the ikonic and the concrete symbolic mode. The ikonic mode exhibited one cycle, while, within the concrete symbolic mode two cycles were identified. In general, ikonic mode responses relied on the diagram for cues, while the concrete symbolic responses relied on the given data and different strategies were employed to arrive at an answer. In addition, first cycle responses within the concrete symbolic mode, usually employed intuitive type strategies whereas second cycle responses had overviews of all the data and used strategies that incorporated the relevant variables. Within each of these cycles responses progressing through the different levels of the SOLO Taxonomy were identified. This revised model provides clarity when exploring the details of students' understanding of speed.

Profiles of students' responses, using this revised model, are presented and a diagrammatic representation of these profiles indicate an appropriate way to document the levels of responses provided by students.

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INTRODUCTION

This study was concerned with students' ideas and concepts of speed and the methods employed by students in solving questions about speed. Piaget (1970) laid a solid foundation concerning young children's beliefs regarding motion and speed. Other studies, such as Levin & Gardosh (1987), Ehri & Muzio (1970), Levin & Simons, (1986), Lovell, Kellet & Moorhouse (1962), have built on Piaget's work to confirm, refute or challenge his results. Piaget's results have been robust enough to withstand the test of time and still continue to form a valuable framework. Most studies have dealt with young students ranging from five-to-twelve years of age. Some studies have considered tertiary students, ideas about speed (e.g., Trowbridge (1979)) but the main emphasis was on higher order concepts associated with speed and velocity. Studies that have concerned secondary school students' ideas about speed were related to what students had been explicitly taught about speed and velocity, for example, the investigation reported by Gorodetsky, Hoz and Vinner (1986).

One of the main purposes of this study was to explore students' ideas of speed and strategies employed to solve speed problems. These students ranged in age from twelve-to-eighteen years and thus covered the final years of primary school right through to the end of secondary school. Another purpose of this study was to analyse students' responses and place them within a suitable cognitive framework to thus gain further insights into levels of cognitive development. A framework that was appropriate for this study is a post-Piagetian model developed by Biggs and Collis (1982) that focuses on Structure of Observed Learning Outcomes - referred to as the SOLO Taxonomy. In this model the move is away from categorising students to categorising students' responses.

A further feature of this study was to tease out some of the details within the SOLO Taxonomy. This reveals some of the finer structure embedded within the SOLO Taxonomy that helps to clarify students' responses from the iconic mode into the concrete symbolic mode of cognitive development.

At the beginning of this thesis, in Chapter 1, an overview of the research already carried out on students' concepts of speed is reported, with detail given to the work of Piaget.

In Chapter 2 some cognitive theories are considered and, in particular, emphasis is given to the work of Biggs and Collis on the SOLO Taxonomy, this taxonomy then forms the framework for interpreting the results in Chapters 8 and 9. Chapter 3 describes the questions and initial trials of these questions on students. A small pilot study was undertaken using the test instruments to examine students' concepts of speed. The research questions for the main study are also formulated in this chapter.

The Research design and context are outlined in Chapter 4, followed by the outcomes of the main study in Chapter 5. In this chapter the idea of quality of response is introduced covering two dimensions: fullness of response and strategy for solving problems. These dimensions are used to analyse responses in the written test and amplified with students' comments from the interviews.

Chapter 6 analyses the results, further addressing some of the issues that arose in Chapter 5 across different parts of the speed test. Such issues as the relationship between descriptions and problems, complexity of problems, intuitive thinking and students reference to variables are examined.

The strategies employed by students are examined both across questions and within questions in Chapter 7. Levels of cognitive processing are presented that increase with the age of students and are related to the type of strategy employed by them.

The SOLO Taxonomy is applied to students' responses in Chapter 8 and a model is developed that indicates at least three cycles. The first cycle is identified within the iconic mode and the other two within the concrete symbolic mode. Each of these cycles exhibit the levels of structure associated with the SOLO Taxonomy, that is, unistructural through to relational levels.

Six student profiles are presented and discussed in Chapter 9 using the Speed Response Model (SRM) developed in Chapter 8. The levels in this Model are used to identify the level of functioning of students across a range of problems. A graphical representation is used to illustrate the students' SRM profile for both the written test and the interview. This profile simplified the process of obtaining an estimate of the students optimal and functional SRM level of operation.

The final chapter summarises the results of the study and indicates the limitations of the study and the implications for further research and to the teaching/learning process.