CHAPTER 7

ANALYSIS OF STRATEGIES USED

Be wise with speed;
A fool at forty is foul indeed.
Love of Fame, Sat. ii 1.281

This chapter continues to analyse the outcomes presented in Chapter 5 in an attempt to answer more of the research questions arising out of the Pilot Study in Chapter 3. It focuses on the strategies students employed in answering the speed questions.

The issues addressed: include a comparison of students' descriptions of speed with the strategies they used on the speed questions (Research Question 10); the effect of question structure on strategy (Research Question 11); the influence on strategy as the number of variables in a question differ (Research Question 12); and, mapping diagrams of strategies.

DESCRIPTION VERSUS STRATEGIES

This section considers the descriptions of speed students provided at the beginning of the speed test and makes a comparison with strategies students used in solving the speed questions. In solving these questions, some of the strategies selected by students were suitable and some were not, that is, some strategies were successful in providing the correct answer if carried through appropriately, while others were not successful. (For examples of descriptions of different categories see Chapter 5.)

CATEGORY A

To be able to produce a Category A response meant students had some idea of the formal definition of speed, that is, they mentioned that speed is the quotient of distance and time. Only five Year 12 students, all of whom had undertaken a physics course, gave a Category A response in their description of speed.

To assist in showing trends of the strategies used, and for succinctness, the codes in Table 7.1 are used to identify strategies employed by students who gave, spontaneously, a Category A description for speed.

Key for Table 7.1

Strategy	code
Formula	1
Proportion	2
Direct/inverse	3
Dist & time	4
Time only	7
Unknown	?
Incorrect	Underlined
Physics student	P

Table 7.1
Strategies employed by students
who gave Category A descriptions for speed

Student No.	Question (Year 12) N3	V1	V3	V5	B1	B4	B5	B8	B10	C3	C5	C7	C8	C9	C1	total Correct
107 P	2	1	3	1	4	1	1	1	1	1	1	1	1	1	1	15
104 P	3	3	2	3	4	1	1	1	1	1	1	1	1	1	1	15
101 P	1	3	3	3	4	2	2	2	1	3	3	1	1	1	1	15
102 P	1	2	3	1	4	1	1	1	1	1	1	1	1	?	?	12
114 P	1	3	3	3	4	7	7	7	?	3	3	1	1	1	1	10

All five of these Year 12 students had studied physics. Three of these students used the formula to answer most questions in the test and the other two students (101, 114) used the formula on the same five questions (namely, Questions N3, C7, C8, C9, C1). Question N3 was written in a typical textbook way which would be expected to trigger physics students into using the formula to solve it (some Category

B students used the formula on this question also). All of these students used the formula on Questions C7, C8 and C9. These three questions involved two variables changing at the same time, whereas the questions preceding them (C3, C5) had only one variable changing and so were solved easily by using direct and inverse variation strategies, respectively. The students, therefore, were 'forced' to change strategies to successfully solve two-variable type problems by using the formula. Question C1 was placed after Question C9 (as shown in the table) and students continued to use the formula even though the problem was solved easily using direct variation. It appears that once students used a particular type of strategy that is working well then they continued to use it. This occurs even though there maybe quicker and simpler strategies available that they have used before.

Most students did not use the formula in Questions V1 and V3 even though they had used it in Question N3. Question N3 was a numerical type question which made it easy to apply a formula and use substitution to solve the problem. Questions V1 and V2 are direct and inverse variation problems which were not numerical in nature. These students used either proportion or direct variation to solve them. Question V5 was an inverse variation type problem, three students used this method to solve it and the other two students reverted to the formula with quite detailed calculations to solve the problem as shown below.

Ans: $1/2x$.
Exp: $t_1 = 10$ *mins* $t_2 = 20$ *mins* $d_1 = x$ $d_2 = x$
$\Rightarrow v_1 = x/10$ $v_2 = x/20$ $\Rightarrow 1/2x = v_2$.

Student 102 Question V5

The formula is imbedded in the equation $v_1 = x/10$.

Ans: *1/2 as fast.*
Exp: *Mon* $s = d/t = d/10$ *Tues* $s = d/t = d/20$
Let $d = 20$ *speed is* $20/10 = 2$ *on Mon*
Speed $= 20/20 = 1$ *on Tues.*

Student 107 Question V5

These two responses show sophisticated uses of algebra and proportion with the formula as the supporting structure. These responses can be compared with the inverse strategy of Student 104 on Question V5:

Ans: *1/2 as fast.*
Exp: *If it takes 10 minutes to do distance one day and it takes 20 minutes to do this same distance the next day he must be going 1/2 as fast.*

This means, if time is doubled then speed is halved for the same distance.

All of these students answered Question B1 by mentioning that both trolleys went the same distance in the same time. Three students then reverted to the formula to find the speed of each trolley for the remainder of the questions. Student 101 continued to use the proportion strategy until she reached Question B10 where the ratios were more difficult to compare, and she then found the speed of each trolley using the formula.

Student 114 answered Questions B4 to B8 using time as the determining factor in deciding which trolley had greater speed. She seemed to infer, that greater time implies greater speed. This resulted in incorrect responses. In Question B10 she had the correct answer but gave no explanation. In the questions of Part C she reverted to using direct and inverse strategies correctly and with two-variable changing problems she used the formula. With Questions C9 and C1 she had difficulty substituting the correct information into the formula.

Thus, students who gave a definition of speed in their description of speed were able to access the formula and apply it to solve speed problems when other methods seemed too complicated. When easier strategies were available students used these if they had not already started using the formula on similar types of problems. When they did not use the formula, most of these students used strategies that were suitable to solve the problem, that is, they were able to access and use strategies 1, 2, 3 and 4.

CATEGORY B & C

A Category B response is characterized by students referring to both distance and time, but not to the formula or to the quotient of the two variables. Category C is characterized by a response that refers to only one of the variables of distance or time. Only two Year 9 students gave a Category C response and both referred to time as the variable. Both these categories are considered together since only two students in Year 9 gave a Category C response.

To assist in showing trends the codes in Table 7.2 are used to identify strategies employed by students who could spontaneously give a Category B or C description for speed.

Key for Table 7.2

Strategy	code	Strategy	code
formula	1	Calculation	5
proportion	2	Arithmetical operation	6
Direct/inverse	3	Time only	7
distance & time	4	Distance or front	8
		Unknown	?
Category B	B	Category C	C
Physics Student	P		
Incorrect	Underlined		

Table 7.2
Strategies employed by students who gave
Category B or C descriptions for speed

Student No.		Question (Year 12) N3,V1,V3,V5, B3,B4,B5,B8,B10, C3,C5,C7,C8,C9,C1														Total Correct
113	B P	1	1	1	1	4	1	1	1	1	1	1	1	1	1	15
111	B P	2	1	1	1	4	1	1	1	1	1	1	1	1	1	15
117	B	2	3	1	1	4	4	1	1	1	1	3	1	1	1	15
115	B	2	3	3	3	4	1	4	1	1	3	?	1	?	?	15
108	B P	3	2	2	3	1	1	4	4	1	3	3	1	3	1	14
110	B	5	3	2	3	4	?	4	4	4	3	3	5	3	?	14
116	B P	2	3	3	2	4	2	4	4	2	1	1	?	1	?	13
112	B	3	3	3	3	4	2	4	2	2	3	3	?	?	?	12
		Question (Year 9) N1,N3,V1,V3, B3,B3,B5,B8,B10, C1,C2,C7,C8,C9														
912	B	6	6	3	6	?	?	4	2	2	3	5	6	3	6	7
915	C	6	6	3	3	4	2	7	?	2	3	5	5	5	5	11
913	C	6	6	?	3	?	7	8	7	8	?	?	5	5	?	6

Of the eight students in Year 12 who gave a Category B response, six (two of whom did not study Physics) used the formula explicitly when solving the problems while the other two students, who did not study Physics, never used the formula.

Only Student 113 used the formula to solve Question N3 but then he used it in most other problems except for Question B1. (His responses are considered in more detail in the student profiles of Chapter 9.) Student 108 used the formula to explain Question B1 while all the other students referred to the trolleys as having travelled the same distance in the same time.

Students 108 and 115 appeared to use a mixture of the formula and other methods whereas other students tended to continue using the formula once they had started. A similar trend was found with students who gave a Category A description.

Student 112, not having recourse to the formula, did not respond successfully to Questions C7, C8, C9 where two variables were changed. It appears that he was able to return to direct variation on Question C1 to obtain the correct answer, for which he did not give any explanation. Student 110 used novel strategies to handle the more difficult problems without referring to the formula though there are some explanations which indicate that she was aware speed could be evaluated by dividing distance by time. (Her responses are also considered in greater detail in the student profiles of Chapter 9.) Two students (111 and 113) used the formula consistently and Student 117 used the formula on most questions.

It is of interest to note that of the eight Year 12 students in Category B six quoted the formula for speed in the problems but did not provide the formula when asked for a description of speed. It would seem that the word 'speed' triggers different reactions when asked for a description of speed compared to when asked to solve a problem about speed. The formula was used by some of these students only as the problems increased in difficulty and when they did not trust their less formal methods.

The one Year 9 student (912) who responded at Category B did not use the formula at all. Once, in Question C2, however, she divided distance by time but with no reference to a formula. Even though this student gave a high category description to speed, she did perform consistently with her description, that is, she used the information in the question concerning distance and time but without a formal knowledge of how distance and time were related to speed. When she used a mathematical operation (code = 6) she applied it incorrectly and the calculations appeared to be random operations on numbers. For example:

Question N1

Ans: *20 km.*
Exp: *Divide 60 by 3.*

The correct response was : 60 x 3 = 180 km.

Question N3

Ans: *200 h.*
Exp: *Times 20 km/h by 10 km.*

The correct response was:

20 km in one hour = 10 km in 0.5 hrs.

Of the correct responses, she used proportion or direct variation strategies more often, though she had difficulty in providing satisfactory explanations.

For the two students who had Category C descriptions (see Table 7.2) Student 913 answered six out of the fourteen questions correctly and used a variety of strategies with no particular one predominating. In the Part C questions, he divided the distance value by the time value at least twice but the responses suggested that there was no algorithmic procedure being followed.

Student 915 had eleven of the fourteen questions correct, however, only five of them were presented as a full response. He used distance and time separately and together in his responses to the Part B questions, but in the Part C questions he divided the values of distance by time by focusing on Trolley B. However, he continued to divide two values even when it was inappropriate. For example, he divided the speed value by the time value in Questions C8 and C9. It is possible, therfore, that he was just grabbing at an answer for Question C7 by applying an arithmetical operation.

In summary, for the majority of problems, the Year 12 students used the top four strategies that involved both the variables distance and time, as they had indicated in their descriptions of speed. The Year 9 students, however, knew of distance and/or time but did not employ appropriate strategies often to solve the problems correctly.

CATEGORY D

Students responding in this Category referred to speed as something that goes fast but did not limit the term 'fast' to high speed objects. A coded table of strategies for each student in this category is given in Table 7.3.

Key for Table 7.3

Strategy	code	Strategy	code
formula	1	Calculation	5
proportion	2	Arithmetical operation	6
Direct/inverse	3	Time only	7
distance & time	4	Distance or front	8
		Unknown	?
Physics student	P	Incorrect	Underlined

Table 7.3

Strategies employed by students who gave Category D descriptions for speed

Student No.	Question (Year 12) N3,V1,V3,V5,				B1,B4,B5,B8,B10,					C3,C5,C7,C8,C9,C1						Total Correct
103	5	3	3	3	4	2	4	4	2	3	3	5	2	3	3	15
106 P	1	3	2	3	4	2	4	2	4	3	3	1	1	1	1	14
105	2	3	?	3	?	?	?	?	?	?	3	?	?	?	?	9
	Question (Year 9) N1,N3,V1,V3,				B1,B3,B5,B8,B10,					C1,C2,C7,C8,C9						Total Correct
905	6	2	3	3	4	7	8	7	4	3	3	5	5	?		11
918	6	2	?	5	4	4	4	2	5	5	5	3	3	?		11
908	6	?	3	3	4	?	8	4	?	3	?	?	6	6		11
917	6	2	3	3	4	7	7	4	4	3	3	5	5	?		10
909	6	2	3	3	4	2	4	?	7	3	?	?	?	?		8
902	6	2	3	6	7	7	8	8	8	5	?	?	?	?		8
911	6	6	3	3	?	?	?	8	?	?	?	?	?	6		6
910	6	6	3	3	4	8	4	4	4	?	?	?	6	?		4
	Question (Year 6) N1,N3,V1,V2,				B1,B2,B3,B5,B8,B10,					C1,C2,C7						Total Correct
606	6	2	3	3	4	4	2	4	4	?	3	3	6			12
609	6	2	3	?	4	7	2	4	4	4	3	3	?			11
616	6	2	3	3	8	8	7	4	7	7	3	?	?			8
615	6	?	3	?	4	8	8	8	4	8	3	3	5			6
611	6	6	?	?	7	7	8	8	7	8	3	3	?			4
619	6	6	6	3	7	8	7	7	7	7	3	7	3			4
612	6	6	8	3	?	8	8	8	8	8	3	3	3			4
601	6	?	3	?	7	8	8	8	7	7	?	?	?			3

Three Year 12 students responded in this Category for the description of speed and used a variety of approaches in the problems, with Student 106 (a physics student) using the formula in the last four questions. The other two students did not use the formula. Student 105 was a recent migrant to Australia and the other student (103) had not studied Maths II or Physics. The latter student has, however, completed all questions successfully, even those in Part C with two variables changing. For example, her response to Question C9, where it is stated that the distance is doubled and the speed is changed by one third and the time is required, was:

Ans: *6 secs.*
Exp: *It would go 1/3 the distance so it would go 1 cm in 1 sec => it would go 6 cm* [in 6 sec].

This student realised that if the speed was reduced by a third then the distance would be reduced by a third also (direct variation). Thus the speed was one centimetre per second (calculated by her and stated in the question). The distance was doubled to six centimetres so it would take six seconds to do this at one centimetre per second. She was able to use variation techniques in a quite sophisticated fashion that the Physics students could not, or would not, use when they had a formula to rely on.

Of the eight Year 9 students who used a Category D description of speed, two used the distance value divided by the time value on Questions C7 and C8. Two other students used a calculation based on the speed of Trolley B.

For example Student 918 on Question C1:

Ans: *2.*
Exp: *If the car travels at 2 units per second it must only go to number 2.*

These students showed an understanding of speed based on the units of speed even though they could not refer spontaneously to speed as related to distance or time.

Students 905, 917, 909, 908 used direct variation successfully on Questions V1, V3 and C1 but only Students 905 and 917 applied inverse variation in Question C2.

None of the eight Year 6 students in this Category attempted to divide the distance by the time values in any section of the test. Four of them referred to a trolley as 'faster' or 'in front' at least once in the Part B questions. Student 612 did this consistently.

The interesting point about the students in this category is that in their meaning for speed they did not say anything about distance or time but in the problems twelve of them referred to distance and time in at least one of the problems and fourteen of them used one of these variables in trying to solve the problems in Part B (code 7 & 8). Again it appears that the description of speed triggers different ideas from those used involving problems about speed. These students did not refer to speed as related to distance and/or time but were able to use these ideas in solving problems.

CATEGORY E

A Category E response was characterised by students referring to speed as an object which went very fast, that is, they did not associate slow objects with the term speed. Table 7.4 provides a summary of the strategies these students used in solving the problems.

None of the Year 12 students responded in this category. Their greater experience with speed did not limit their ideas of speed to the travelling of 'very fast' objects.

Even though students who responded in Category E had a different description of speed from those in Category D the types of strategies they employed reflect similar trends. Five Year 9 students applied the direct variation strategy in Question V1 and V3, but only three of these students used it in Question C1. Only Students 919 and 920 could use inverse variation in Question C2 appropriately, and they answered Question B10 correctly also. (Student 907 used a direct variation type of approach successfully based on the speed.)

Eleven Year 6 students responded in this category. Most of them used a simple calculation to answer Question N1, but they found Question N3 more difficult. Some of them guessed, with some of them guessing wrongly, others just multiplied two numbers.

For example

Ans: *2 hours.*
Exp: *10 x 2 = 20.*

Student 623

Key for Table 7.4

Strategy	Code	Strategy	Code
formula	1	Calculation	5
proportion	2	Arithmetical operation	6
Direct/inverse	3	Time only	7
distance & time	4	Distance or front	8
		Unknown	?
Incorrect	underlined		

Table 7.4

Coded strategies used by students who gave

Category E descriptions for speed

	Question (Year 9) N1	N3	V1	V3	B1	B3	B5	B8	B10	C1	C2	C7	C8	C9	Total Correct
919	6	3	3	3	4	2	4	4	4	3	3	5	5	?	13
920	6	6	3	3	?	5	4	4	5	3	3	?	?	6	10
907	6	2	?	3	7	4	4	4	4	4	3	5	3	5	10
914	6	2	3	3	8	7	4	?	?	3	3	6	?	?	8
916	6	2	3	6	4	4	8	8	8	4	?	?	?	?	8
901	6	6	3	3	8	8	8	8	8	8	3	?	8	3	5
904	6	?	3	3	4	8	4	4	8	?	?	?	?	?	5
903	6	6	3	6	8	4	4	8	4	3	3	3	3	?	4

	Question (Year 6) N1	N3	V1	V2	B1	B2	B3	B5	B8	B10	C1	C2	C7	Total Correct
607	6	2	3	?	7	4	2	4	4	?	3	?	?	9
622	6	?	3	3	4	8	8	8	8	8	3	3	3	8
617	6	2	3	3	4	4	2	2	2	4	?	?	?	7
604	6	?	3	3	7	8	7	7	8	8	3	3	3	6
620	6	?	3	3	8	8	8	8	8	8	8	?	?	6
608	6	?	?	?	8	?	8	?	8	8	?	6	8	6
625	6	?	3	?	8	8	8	8	4	?	3	6	?	5
618	6	6	6	3	7	8	8	8	7	7	3	8	6	5
613	6	?	?	3	8	8	8	?	8	8	8	?	?	5
623	6	6	?	?	8	8	7	8	6	6	8	8	6	3
605	6	6	6	6	?	8	7	7	7	7	8	6	?	2

Six students used direct variation in Question V1 and only four of these students used direct variation on Question C1. For inverse variation problems, only four students were successful on Question V2 but only one student was successful on Question C2. It appears that Year 6 students had trouble recognizing and using inverse variation in this problem.

In the Part B questions, eight Year 6 students focused on either the greater distance or the trolley in front at least once, with four students doing so consistently. This would fit in with their concept of speed being 'very fast' with the fast trolley being in the lead. Five students referred to time as the discriminating variable at least once.

When solving the problems in the test, seven students in this category referred to a trolley as either going fast or having greater speed in their responses, six were from Year 6 and one was in Year 9. The Year 6 students were more likely to use the idea of 'fast' as given in their meaning of speed to explaining how they did the problems.

In a similar way to the Category D students, some Category E students used distance and/or time as determiners in solving speed problems even though they could not refer to them in their descriptions of speed.

CATEGORY F AND G

Students' descriptions in Category F were characterised by having an idea for speed but it was vague and showed a misunderstanding of the proper concept of speed. Category G responses were those from students who had no idea what speed was or did not answer the question. Table 7.5 provides a summary of the strategies these students used in solving the speed problems.

The one Year 12 student who answered with a Category F response was a recent migrant to Australia and English was not her first language. From her responses it would appear she did not understand fully what the questions were asking. It is interesting to note that she made an attempt to use the formula on two problems but was unable to complete either problem successfully, for example on Question C7:

Ans: 6.
Exp: *speed is 6,* $s = d/t = 6/1 \quad d = 6.$

She stated the formula correctly and substituted in the correct distance for Trolley B but used the time for Trolley A. She may have tried a variation type of strategy for Questions C3 and C5 and was 'forced' to use the formula on Question C7 where two variables were changing. Her confidence in the use of the formula could not have been very high since she used it only once more in Question C9.

Key to Table 7.5

Strategy	code	Strategy	code
Formula		Calculation	5
Proportion	2	Arithmetical operation	6
Direct/inverse	3	Time only	7
Distance & time	4	Distance or front	8
		Unknown	?
Category F	F	Category G	G
Incorrect	Underlined		

Table 7.5

Coded strategies used by students

who gave Category F and G description for speed

Student No.	Question (Year 12) N3,V1,V3,V5, B1,B4,B5,B8,B10, C3,C5,C7,C8,C9,C1	Total Correct
109 F	3 ? ? 3 ? 7 ? 8 8 ? ? 1 ? 1 ?	2
	Question (Year 9) N1,N3,V1,V3, B1,B3,B5,B8,B10, C1,C2,C7,C8,C9	
906 G	6 6 3 3 4 2 2 2 5 3 3 5 6 6	12
	Questions (Year 6) N1,N3,V1,V2, B1,B2,B3,B5,B8,B10, C1,C2,C7	
602 F	6 2 3 ? ? 4 4 4 8 ? 3 3 ?	10
621 F	6 2 ? 3 8 8 4 4 4 8 3 ? ?	9
614 F	6 ? ? 3 8 8 2 8 8 8 3 ? ?	7
610 F	? ? ? 3 8 8 8 ? ? ? 3 ? 8	5
624 F	6 6 3 6 8 8 7 8 7 7 8 8 6	2
603 G	? 2 ? 3 4 4 2 4 4 4 3 ? 2	9

The Year 9 student wrote that he did not know what speed was, but he gave a response equivalent to Category D for velocity. He performed well, however, in the speed problems and in the interview he was successful in giving an acceptable description for speed. (The responses of this student are examined in the Student Profiles of Chapter 9.)

Although Year 6 students gave descriptions of speed that showed a lack of understanding of the concept, all of them indicated that they had some idea of speed at least once in the problems. Students 624 and 610 had the most difficulty in doing the problems successfully. The other students, including Student 603 (Category G) completed more than half the question successfully.

Students 602 and 621 were coded with strategy 4 (distance and time) in some of the Part B questions, but they sometimes used the term 'speed' synonymously with time, for example:

Ans: *Same.*
Exp: *They both go the same distance & the same speed.*
Student 602 Question B2

Even though these students could not provide a suitable description for the term speed, they had some ideas and strategies that they could use to attempt to solve problems concerning speed. It appears that most students have at least an 'intuitive' idea that speed is related to distance and/or time as they solve problems.

SUMMARY

In Chapter 5, in the discussion on students' description of speed, it was noted that in the interviews Students 612, 606, 915 and 906 provided a Category B level response in the interviews, that is these students improved their description of speed. From the above tables (7.4, 7.3, 7.2, 7.5, respectively) it can be seen that all these students have performed well in the speed problems compared to their peers. Although these students could not give a high category for a description of speed in the test, they could use the components, distance and time, to solve speed problems. In the interviews, after being questioned about the speed problems, they were able to specify what were the important components of speed. Students who could not give a better description of speed in the interviews, did not perform as well as their peers, see for example Students 903 (Table 7.4), 910 (Table 7.3) and 112 (Table 7.2).

The above outcomes relate to Research Question 10 concerning the relationship between descriptions and strategies. From these results it is clear that just because students can give a Category A or B description for speed does not infer that they use that description to solve problems on speed. Students using these categories were more likely to solve the problems successfully. In general these students can be placed in two groups: first, those who used the formula consistently from the start of the written test and second, those who used other strategies until the problems became more complex and they then reverted to the formula to solve the problem. All the students who studied physics eventually used the formula to solve the problems. Two Year 12 students who had not studied physics also employed the formula.

In contrast to this are those students who provide Category D and E responses for a description of speed and who often used distance and/or time in some way to solve the problems. That means they demonstrated an awareness that distance and time were important in solving problems even if they could not mention them when asked for a meaning for speed. It appears that when eliciting information about descriptions of speed, students can not 'see' the relationships that they would use in solving problems to clarify the components necessary for a suitable description. The reverse situation is also true for some students, that is, while attempting to solve problems on speed they appear to be unable to access the information that they have provided for in their description of speed. Thus, there appears to be two different conceptions of speed held by some students which depends on whether they are asked for a description of speed or to solve a problem concerning speed. They can not access one area to help with understanding in the other area. Most students, however, do have an intuitive sense that speed is related to distance and or time in some way when solving problems concerning speed.

STRATEGY AND QUESTION STRUCTURE

In this section the effect of question structure on strategy across the different types of questions is considered. This is discussed in three parts, namely, the effect of numerical questions and non-numerical questions, the influence of word questions versus questions with diagrams and, finally, the effect of open and closed questions.

NUMERICAL AND NON-NUMERICAL QUESTIONS

In this section a comparison is made between the strategies chosen by students when they do numerical and non-numerical type questions. The questions that are best suited for this type of analysis are the numerical questions, N1 and N3, and the non-numerical questions, V1 to V5. Questions N1 and N3 were designed to be straight-forward numerical questions that would involve a simple calculation with which most students would be familiar. Questions V1 to V5 were also designed as straight-forward questions but required the use of a variation-type approach, that is, students would not have to do a numerical calculation to solve the problem.

From an analysis of Table 5.7 it can be seen that for Question N1 most Year 6 and 9 students performed a simple calculation on the question as expected. Five Year 12 students, however, chose to use the formula to solve this question. The idea of obtaining distance by multiplying speed by time appeared to be an accepted method of solving the problem. However, for Question N3 a variety of strategies were used. To divide distance by speed to find time was not a natural approach for this problem. Only eight students in Years 9 and 12 attempted the question this way, no Year 6 students attempted to divide at all. On the other hand, twenty-four students from Years 6, 9 and 12 used a variation type of approach. They had seen the speed of '20 km/hr' as meaning twenty kilometres is travelled in one hour, which is a correct conclusion, therefore ten kilometres is covered in half an hour. This is the direct variation strategy. This type of strategy could have been used in Question N1, that is, '60 km/hr' means sixty kilometres is travelled in one hour therefore how many kilometres is covered in 3 hours? There is no evidence from the written responses that any student attempted it this way.

In Questions V1 to V5 most students used either direct or inverse variation appropriately, as expected. They were aware that if one variable changed then the other would also change in the same ratio. Any simple calculation approach was usually unsuccessful in solving the problem because numerical information was not provided that lent itself to simple calculations. Some Year 12 students used the formula to solve these questions.

Considering the overall strategies used in the calculation questions (labeled with an N) as compared to the variation questions (labeled with a V), the variation approach in the variation questions was used about three times more often than the variation approach in the calculation questions. Mathematical operations were used

over five times more often in the calculation questions than in the variation questions. (In this analysis only Questions N3 and V5 were considered for Year 12 students so as not to bias the figures in favour of the variation questions.)

Thus the structure of the questions did influence strategies students employed. Generally, numerical questions elicited mathematical operations whereas non-numerical questions tended to evoke direct and inverse variation strategies.

WORD QUESTIONS VERSUS DIAGRAMS

The Single Focus Questions in Part A of the test were presented as word questions whereas the questions in the Dual Focus Closed Questions of Part B were given in diagrammatic form. Apart from the initial instructions given in the Part B questions, no sentences were given for students to read and interpret. The questions of Part B were all of a similar format and required only one of three answers, namely, whether trolley A or B had the greater speed, or whether they had the same speed. This difference in the structure of the questions elicited quite different types of strategies from students.

Students could solve more Single Focus Questions correctly than they could solve questions with a double focus. Individual differences, however, were noted between some students. In general, students understood the written questions as well as the diagram type questions. Those that had difficulty reading the instructions were told verbally what was required in the Part B questions. The aim was to test students concepts of speed, not their reading ability. One will impact on the other, however, and this must be considered.

That the diagrams influenced the way students responded to the question is indicated by responses which focused on the diagram without referring to the written information in particular; for example, 'it looked like it', 'it was in front', 'covered more units', etc. Students who gave these responses were not consistent with such a response across questions. In one question they might focus on the 'trolley in front', but in the next question they may have taken into consideration the time and actual distances travelled. These same students did the questions of Part A with varying degrees of success and some accessed a variety of strategies. Thus their understanding of speed was not limited to visual cues only. They could also relate to contextual clues to solve word-type problems. One student in Year 6 (612) used the

word faster consistently to explain which trolley had the greater speed for the Part B questions. The context suggests he was referring to the trolley that was in front in each problem, that is, he was using a visual comparison of the diagrams. In Part A questions, though, he did have three correct responses to the word problems indicating that his concepts of speed were better in the written form and the diagram acted as a hindrance which lead him to look at the front trolley.

Some Year 12 students used the formula and proportion in both types of questions, that is, they had strategies that they knew would be successful so they were not influenced by the type of question as much as the younger students. The proportion strategy was also used by a few Year 6 and 9 students but only in the questions presented diagrammatically.

The diagrams focused most students attention towards the relevant variables, that is, time and distance. Clearly these variables were seen and indicated explicitly in the Dual Focus Questions, whereas in the Single Focus Questions of Part A students were more inclined to give mathematical-type responses related to variation and arithmetic operations.

In the responses to the Part B questions, the written explanation using both distance and time was used across all Year levels. The Year 6 students focused more on one variable in their explanations than did the Year 9 students with very few such responses from Year 12. A similar trend was found across years for visual-based strategies, namely, the faster trolley, speed, or one in front.

Thus, the very nature of the questions brought forth differing strategies from students with the written questions eliciting more mathematical types of approaches, and the diagrams prompting responses that required some comparison strategy such as proportion and the use of visual cues.

OPEN AND CLOSED QUESTIONS

One of the major differences between the Closed and Open Dual Focus Questions is that some of the closed questions could be attempted without any numerical processing and only three alternative answers were possible. In the open questions students had to process data in a mathematical way to obtain a suitable numerical response.

With increasing Year level, students gave better explanations of the strategies they used to arrive at their answer. There was a tendency for younger students to focus on one variable, either time or distance, whereas older students took both variables into account.

In the Dual Focus Open Questions, Year 6 students did not perform well and none of them answered correctly the question in which two variables changed (Question C7). Most students at all Year levels responded correctly to the direct variation question (C1) but younger students were not so successful in the inverse variation Question C2. None of the Year 6 students used a formula to solve the problems but some Year 9 students performed calculations that resembled, very closely, the use of the formula that Year 12 students utilized.

For students to do Part B questions successfully they used both diagrams to obtain all the information. In effect, they saw where the initial and final positions of the trolley were, read the distance indicated on the ruler provided, as well as read the times given above the diagrams. In Part C questions, some students (mostly Year 12 and two Year 9 students) obtained enough information from the second diagram to solve the problem without using the information on Trolley A. They took into account the speed displayed on Trolley B and the distance it travelled.

Thus, in Part B questions students tended to use written explanations that took into account either the time or the distance, or both, whereas in the Part C questions more students used the variation strategies and mathematical operations. A significant number of Year 6 and 9 students focused on only one variable in both parts of the test. Some Year 12 students used the formula in both parts of the test.

SUMMARY

This section has addressed Research Question 11 concerning the effects of the structure of questions on strategies selected by students. It has been shown that different types of questions elicited different types of responses and strategies. Questions written with specific values (numerical) elicited mathematical operations in the responses whereas those questions that indicated how variables changed (non-numerical) evoked direct and inverse variation type strategies. Questions presented as word-questions had responses that were mathematical in nature whilst the questions with diagrams elicited comparison strategies also as well as those relying on visual

cues. The closed questions had more responses that were of a written nature compared to the responses of the open questions which evoked more responses using variation type strategies and mathematical operations.

The Year 6 students were likely to focus on one aspect of the problem and so chose low level strategies that were not always successful. The Year 9 students had a mixture of strategies, some focused on one aspect of the problem while others focused on both variables and chose strategies which were a little more sophisticated than the Year 6 students. The Year 12 students did not use such a wide variety of strategies as they tended to choose those strategies that incorporated all the significant aspects of the data. Thus, as expected, with increasing age there is a growth in the types of strategies students have at their disposal and that they are likely to use in solving the problem successfully. However, of importance is the nature of these strategies. Whether the strategy is successful or not, is not the emphasis here, but rather the type of strategy that is selected by students attempting to solve the problems. Some of these strategies only require the use of one variable, while others require all variables. Some of the low level strategies do not require the use of variables at all but rely on the visual impact of the diagram. The nature of strategies used in relation to the variables changing in the question is the focus of the next section.

THE NUMBER OF VARIABLES CHANGING IN A QUESTION

One of the important issues to be considered concerned ways students would respond to questions as the number of variables that differ increased. In general, questions were given in order of the number of variables differing (see Table 3.7 for Part B questions and Table 3.8 for Part C questions). Questions in the Dual Focus Comparison Open and Closed tests had differing numbers of variables. These ranged from Question B1 with no variables differing to Question B10 with all variables differing, and Questions C1 to C5 with one variable differing to Question C7 to C9 with two variables differing.

In order to explore this issue three aspects are considered. First, consideration is given to the common Questions B1, B5, B8, and B10 which have 0, 1, 2, 3 variables displayed as different, respectively. Second, a detailed look is taken of strategy choices made by students as they proceed from Questions B5 and B8. Third, consideration is given to strategy choices for Questions C1 and C7.

QUESTIONS B1 TO B10

The percentage of students with Questions B1, B5, B8 and B10 correct and with a full explanation (CF) is displayed in Table 7.6 for each Year level.

Table 7.6
Percentage of students with a correct answer
and Correct Full (CF) explanation

	Question Number of variables that differ			
Yr Level	B1 0	B5 1	B8 2	B10 3
6	100	56	32	48
CF	24	32	20	8
9	100	75	50	65
CF	60	45	35	35
12	100	88	88	82
CF	88	82	82	76
total	100	71	53	63
CF	53	50	42	35

This table indicates that as the number of variables which differ increases, the success rate decreases. The reason for the increase in correct answers as students proceeded from Question B8 to B10 has already been discussed in Chapter 5 for the Year 6 and 9 students. This was due to incomplete strategies being employed by students which resulted, by chance, with the correct answer. Thus the Correct Full (CF) explanations gave a clearer trend of students' responses. In general, the number of CF responses decreased as the number of variables increased. Also, the percentage of students who obtained both correct answers and Correct Full responses increased with Year level. Such a trend was to be expected due to the greater experience of students as they become older. However, the rate of decrease of Correct Full (CF) responses is much greater for the Year 6 and 9 students as they progressed to those questions with more variables differing.

An overall summary of strategies used by students on these questions is provided in Table 7.7. This table also shows the strategies listed in order of frequency, from most frequent to least frequent for each Year level. Unclear strategies are listed at the end of each section.

Table 7.7
Types of strategies used in Question B1 to B10
as the number of variables differing increases

Question code	No. variables differing	Year 6 Strategy	Freq	Year 9 Strategy	Freq	Year 12 Strategy	Freq
B1		Written d & t	7	Written d & t	11	Written d & t	14
	0	Time	6	Faster	3	Formula	1
TDS	Variables	Distance	5	Time	2	Unclear	2
		Speed	4	Unclear	4		
		Unclear	3				
B5		Written d & t	7	Written d & t	10	Written d & t	8
		Distance	5	Distance	5	Formula	6
TdS	1	Faster	4	Proportion	1	Time only	1
	variable	Time	3	Time only	1	Unclear	2
		Proportion	1	Front	2		
		Unclear	3	Fast	1		
B8		Written d & t	7	Written d & t	7	Formula	7
		Time	7	Distance	6	Written d & t	4
tDs	2	Distance	6	Proportion	3	Proportion	3
	Variables	Faster	1	Time	2	Faster	1
		Proportion	1	Unclear	2	Time only	1
		Speed	1			Unclear	1
		Unclear	2				
B10		Time	6	Distance	5	Formula	10
		Distance	5	Written d & t	4	Proportion	2
tds	3	Faster	3	Calculation	3	Written d & t	2
	Variables	Written d & t	3	Front	3	Unclear	3
		Speed	2	Proportion	1		
		In front	1	Time only	1		
		Unclear	5	Unclear	3		

An analysis of this table reveals two trends, both have been alluded to previously. First, as the Year level increases there is a reduction in the number of

strategies students use. This is more noticeable if the unclear strategies, and those strategies which occurred only once, are not taken into account. Second, as the number of variables differing increased, more strategies were used. Older students were more aware that all variables in the problem needed to be accounted for, therefore they chose strategies which took this into account. The number of strategies available to do this are limited. Younger students were not so aware of the complexities and hence used other strategies that took into account only one variable or relied on visual aspects of the diagrams.

As the number of variables differing increased, it became more difficult for students to focus on all relevant variables. Thus across all Year levels, a wider range of strategies were used to solve the problems with more variables differing. It was as if there was information or processing overload and students focused on one variable rather than account for them all. Also, as the number of variables increased, the more advanced students in Year 6 and 9 were able to 'create' different strategies to cope with the situation.

The strategies that do account for all variables include: formula, proportion, and written accounts which refer to distance and time. All of these strategies were utilised by Year 12 students. Only the latter two strategies were adopted by some Year 9 students. Year 6 students who employed two-variable strategies only used the written account but its use decreased, especially when three variables differed, as in Question B10. The responses of Year 6 students, who used the written account, employed the same language as the Year 12 students. While it is possible that these Year 6 students had the same cognitive level as the Year 12 students on these problems, it is most likely that different processing of data was taking place. For example, Year 12 students took a conscious examination of the data and wrote a concise sentence which expressed the correct result. This idea is reinforced by the responses of some Year 12 students, who stated data and then summarised these into a general principle. As an example from the test, consider the response of Student 103 on Question B5:

Ans: *B.*
Exp: *A went only 3 cm in 3 sec and B went 5 cm in 3 sec so therefore it took the same time to go a greater distance.*

The interview with Student 12 also brought this out in Question B5:

S: Well B has gone, say it is kilometres, from zero to five kilometres it has done five kilometres in three seconds and A has only done from one to ·· zero to three so it has three kilometres in three seconds so B

has travelled more distance in the same time as A so it must be going faster.

The Year 6 students, on the other hand, mirrored data displayed on the diagram but did not generalise. Of those Year 6 students who did attempt a general principle, none of them gave a response which reflected the actual numerical data first then synthesised it into a general principle. As a result, Year 12 students were operating at a much more sophisticated level than the Year 6 students.

This idea of effect on strategy and operational level as the number of differing variables changes warrants more attention and is considered below.

QUESTIONS B5 AND B8

Question B5 has only the distance 'd' as the changing variable. The variables that change in Question B8 are the time 't' and the starting position 's'. From Table 7.7 it can be seen that the explanation used most often by students for both questions was the written account. This involved distance and time in some way. The exceptions were Year 12 students who, in Question B8, used the formula, which incorporates both distance and time, more often.

The Year 6 students who focused on just one variable, chose the changing variable more often, that is, in Question B5 more students paid attention to the distance 'd' than the time 't'. However, in Question B8 seven students concentrated on 't' and six students on 'd'. The Year 9 students who chose a one-variable strategy focused more often on 'd' in Question B8. All Year 12 students who provided an explanation chose two-variable strategies for these two questions. Attention is now given to individual student responses to compare strategies used as they progressed from Question B5 to Question B8.

Year 6

Table 7.8 provides a summary of strategies used by Year 6 students. This table is arranged into three groups based on the responses the students provided to Question B5. The first group incorporates those strategies that use two variables while those responses that relied on one variable for solution occur in the second group. The last group of responses includes those strategies that are visual in nature, for example, students focused on the trolley in front or referred to the one with greater speed.

Beneath the coded strategies of Question B5 are the corresponding strategies that each student provided for Question B8. Thus, the data allow a comparison to be made for each student's response as they proceeded from Question B5 with one variable different to Question B8 with two variables altered.

Key for Table 7.8

Strategy	code	Strategy	code
Proportion	p	Speed	s
Distance & time	b	Distance or front	d
Time only	t	Unclear	?
Faster	f	Incorrect	_

Table 7.8
An overview of strategies used by Year 6 students on Question B5 and B8

Question	Coded Strategies for students (n = 25)		
	Two-variable strategies based on Question B5	One-variable strategies based on Question B5	Visual based on Question B5
B5 TdS	p b b b b b b b	t t t d d d d d	f f f f s s ? ? ?
B8 tDs	p b b b b b t s	t t d t d d b ?	t t f f t d d d ?

The one student who used proportion to solve Question B5 used this method in Question B8 and did so successfully for both questions. Seven students referred to both distance and time in Question B5 and five of them continued to do so in Question B8. Of the other two students, one student used the term 'speed' inappropriately twice in Question B8 and the other student noticed a change in time. Hence, most students who used two-variable strategies in Question B5 continued to use them in Question B8.

Three students referred to time in Question B5 and two of them continued to refer to time in Question B8. The other student wrote "diffrent [sic] speeds but same lenght [sic]". His use of the term "speeds" might have referred to the different times, if so he has referred to both variables but he had an incorrect answer.

There were five students in Year 6 who focused on the distance 'd' in Question B5. Two of these students continued to focus on the distance in Question B8. One focused on the time 't', another considered both variables distance 'd' and time 't': "Because the [sic] both have the same distance but A has longer secsond [sic]". The fifth student responded to Question B8 with "A is three times faster than B" which was an incorrect and inappropriate response. Thus three students continued to use one-variable strategies and one student mentioned both variables but was not able to relate them appropriately.

The term "faster" was used by four students in Question B5, two of these students continued to use it in Question B8, but one also referred to the distance as well. The two other students used time as the determining variable in Question B8. Two students referred to speed in their explanation for Question B5. In Question B8 one referred to distance and the other used time.

Three students had unknown strategies for Question B5 of whom one could not give an explanation to Question B8. One of the others used the 'in front' strategy for Question B8 and the other focused on distance.

In summary, of the eight students who used two-variable strategies (written distance and time or proportion), six continued to use the same type of strategy. Thus these students did not change strategy as the number of differing variables increased. All the other students (except one) who had an explanation for Question B5 used one-variable strategies or the visual cue in both questions. Only one of these student went on to consider two variables in Question B8, but unsuccessfully. Most students who considered one variable were influenced in the question by the variables that were different, that is, they were more likely to refer to this variable.

Year 9

A summary of strategies employed by Year 9 students is provided in Table 7.9 which has the same structure and purpose as Table 7.8.

Only one Year 9 student chose the proportion strategy for Question B5 and he continued to use it in Question B8 successfully.

Key for Table 7.9

Strategy	code	Strategy	code
Proportion	p	Faster	f
Distance & time	b	Front	r
Time only	t	Unclear	?
Distance only	d	Incorrect	_

Table 7.9
An overview of strategies used by Year 9 students on Question B5 and B8

Question	Coded Strategies for students (n = 20)		
	Two-variable strategies based on Question B5	One-variable strategies based on Question B5	Visual based on Question B5
B5 TdS	p b b b b b b b b b b	d d d d d t	r r f
B8 tDs	p p p b b b d b b ? ?	d d d d t b	b t d

Ten students selected the written account to refer to both distance and time for Question B5. In Question B8 two of these students used the proportion strategy to compare the two ratios. Five students continued to use the written account of distance and time in Question B8 but two of them gave incorrect answers. One thought the times were the same and the other confused the term 'speed' with time. Another student focused on distance while two other students did not give an explanation to Question B8.

Five students focused on the changing variable 'd' in Question B5, and four of them continued to use distance to determine the greater speed in Question B8 while the other student used time in Question B8. One student used the time in Question B5 but in Question B8 referred to both distance and time successfully. The last three students focused on the visual aspects of Question B5 by using the front trolley or the term 'faster'. In Question B8 one of these students used both distance and time successfully while the other two fixed on time and distance alone respectively, but unsuccessfully.

In summary, most Year 9 students who chose two-variable strategies for Question B5 continued to do so in Question B8. Most students who used one-variable strategies and visual cues for Question B5 applied one-variable strategies in Question B8. Two students who chose either a visual or one-variable strategy for Question B5 chose a two-variable strategy for Question B8.

Year 12

A summary of strategies is given in Table 7.10 for Year 12 students on question B5 and B8. This table is similar to the previous tables for Years 6 and 9 except that there is no visual strategy presented.

The six students who used the formula in Question B5 also used it in Question B8, they were all successful in solving the problem.

Key for Table 7.10

Strategy	code	Strategy	code
Formula	F	Unclear	?
Proportion	p	Faster	f
Distance & time	b	Incorrect	_
Time only	t		

Table 7.10
An overview of strategies used by Year 12 students on Question B5 and B8

Question	Coded Strategies for students: two-variable strategies based on Question B5	One-variable strategies based on Question B5
B5 TdS	F F F F F F b b b b b b b b	t ? ?
B8 tDs	F F F F F F F p p p b b b b	t f ?

Eight students selected the written account using distance and time, but only four of them continued to do this with Question B8. Of the other students, one chose

the formula and three selected the proportion strategy. The proportion strategy enabled them to compare variables

Three students did not use a two-variable strategy, one used time for both questions, the other two students gave no explanation for Question B5 but one student had the correct answer for both questions. The other student in Question B8, suggested that the trolley that was faster had the greater speed.

In summary, Year 12 students were very consistent in the strategies they chose in that most of them selected two-variable strategies for both questions. The three that did not choose two variable strategies did so on both questions.

Conclusion

The above discussion explains some of the outcomes reported in Chapter 5 concerning strategies used to solve the speed problems. Most students who applied a different strategy across questions continued to use a strategy that was consistent with the number of variables that they were already using, that is, if they were using a two-variable strategy then they were more likely to choose a two-variable strategy for the next question.

Year 12 students were consistent in their approaches to Questions B5 and B8 and this was to be expected. For the Year 6 and 9 students, however, there were some changes as they progressed from Question B5 to B8. Five students changed from a two-variable approach, and in all cases were unsuccessful in solving the problem. Two Year 9 students changed to a two-variable approach successfully, while two Year 6 students changed to a two-variable approach unsuccessfully.

This suggests that Year 12 students were operating at a higher cognitive level than Year 9 students, while Year 9 students could successfully choose more appropriate strategies than Year 6 students. The number of variables that differ did affect the type of strategy students used. Some could not focus on the correct variables and so chose one, others were aware that more variables were changing and so tried to select strategies that would accommodate this.

From this analysis there are four major levels of functioning that can be identified:

Level 1. The responses suggest that the student is not aware of an appropriate variable to find the speed. There is a focus on visual stimuli such as the object that is

in front or the one with greater speed, where the term 'speed' is confused with other variables. Also, responses in this category show closure on a response with no consistency or no response at all.

Level 2. The responses indicate that only one relevant variable is used as the discriminator for solving the question and thus only one-variable strategies are selected.

Level 3. At this level, responses take into account the relevant variables and include a two-variable strategy but there is no attempt to state a general principle that links these variables to speed.

Level 4. A response in this category takes into account all of the data and an appropriate strategy is selected to use that data. Students can state a general rule that links the variables together, either by using the formula or a written account.

There is a correlation with age across these levels where the type of strategy chosen is a reflection of the level. For a student to respond at their highest level depends, in a large part, on the demands of the question. Table 7.11 summarises the frequency of each level for each Year group for the common questions in Part B of the test. Year 6 students responded across the levels from Level 1 through to Level 3. However, as the number of variables that differed in the question increased less students reached Level 3. Just over half the Year 9 students operated at Level 3 for most questions. The Year 12 students used Level 4 and Level 3 responses with over half of them using Level 4 on Question B10.

Table 7.11

Frequency of levels by Year level

Level	Year 6 (n = 25)				Year 9 (n = 20)				Year 12 (n = 17)			
	B1	B5	B8	B10	B1	B5	B8	B10	B1	B5	B8	B10
1	7	9	6	11	7	3	2	6	2	2	2	2
2	11	8	10	11	2	6	9	6	0	1	1	1
3	7	8	9	3	11	11	9	8	14	8	7	4
4	-	-	-	-	-	-	-	-	1	6	7	10

There is clear evidence that the level of operation is related to age but there is not a sharp cut off from one level to another as Piaget's work implied. Students in

Year 12, who could operate at Level 4, could also 'reach' down to and operate at lower levels if the question did not demand higher level processing. This was true for lower Year levels as well, but it appears impossible for some students who consistently used Level 1 strategies to apply strategies associated with higher levels. This was verified in interviews when some younger students, even when prompted quite explicitly to reach for higher responses, did not respond at the next level. (For an example, see Leisa's responses in the interviews in the Student Profiles in Chapter 9.)

QUESTIONS C1 AND C7

A similar analysis could be provided for other questions to show that levels occur as students move from one type of question to another. As a brief analysis, consideration is given to Questions C1 and C7 which also had a different number of variables changing. Question C1 had only one variable that changed and Question C7 had two variables that changed.

All seventeen Year 6 students who used direct variation in Question C1 had it correct. Eight of these students did not attempt to give an explanation for Question C7. Three tried multiplying some numbers, four continued to use a form of direct variation, one attempted proportion and one just guessed, they were all unsuccessful at solving the question.

Eleven Year 9 students attempted the direct variation approach on Question C1, and all were successful. When they tried Question C7 only one of them attempted to use direct variation, but four found an alternative strategy by focusing on the second trolley and noted that it had travelled a distance of six metres in three seconds so it must have a speed of two metres per second. Four other Year 9 students used a similar strategy to this on Question C1. They noticed that the second trolley was travelling at two metres per second and therefore would cover two metres in one second. They ignored the changes stated in the question. However, when they attempted Question C7 they were all unsuccessful, and only one tried the same strategy.

Year 12 students either used the formula or direct variation in Question C1. In Question C7 twelve students used the formula, two chose to focus on the second

trolley and calculate the speed from the distance and time in a similar way to the successful Year 9 students.

In summary, there were a few students in all Year levels who could not find a suitable strategy for Question C1. Year 6 students employed direct variation in Question C1 but could not unravel the more complex Question C7. Some Year 9 students used the direct variation strategy in Question C1 and some of these found an alternative strategy to solve Question C7. Year 12 students chose suitable strategies for both questions with the formula being the strategy used most frequently.

This shift in cognitive processing with age again reflects the levels suggested previously. Year 6 students worked mainly at the lower levels, Year 9 students at the middle level and Year 12 students at the higher levels. Those students who could not access a suitable strategy operated at Level 1.

Students within a particular Year level were not confined to a particular cognitive level. Students could operate across levels. For example, there were some Year 9 students operating at Level 1 in some questions, while there were some Year 12 students that operated at Level 2. A student who could operate at a higher level had access to the lower levels of processing if the question did not demand a higher level for successful completion of the problem.

CONCLUSION

In this section the effect of number of variables on strategy has been considered in some detail in addressing Research Question 12. In general, it was found that as the number of variables that differ increases, students used a wider range of strategies to solve the problems, although many of these strategies would not permit success. Some students (especially in Years 6 and 9) found it difficult to continue with two variable strategies as the number of variables that differed increased and reverted to referring to only one variable in their strategies. From this analysis a series of levels were identified that categorised the type of processing that students were using while attempting to solve speed problems. These levels were related, in a general way, to the Year level of students. Questions B5 and B8 were used to analyse these trends and they were further verified by considering Questions C1 and C7.

USING MAPPING DIAGRAMS

The different strategies employed by students to solve the speed problems can be divided into groups, based on strategy type. These are two-variable strategies, one-variable strategies and those that are visually based.

To illustrate these strategies the mapping diagrams (referred to in Chapter 4) of Collis and Watson (1991) are employed to elucidate the response structure of student answers.

In this thesis the symbols used are:

- x represents inappropriate or incorrect data
- * data given with potential to cue a response
- Φ concepts processes and/or strategies expected within the "Universe of discourse"
- o abstract concepts, processes and/or strategies within "Universe of discourse" but additional to those expected as part of understanding of the question.
- ■ responses - both intermediate and final

(Note: these symbols are not the same as that used by Collis and Watson.)

A simple example was provided in Chapter 4 (Figure 4.1) to show how the symbols are used. In the following discussion **Question B10** is used to illustrate different types of strategies employed by students. In this question, all the given variables are different, that is, the starting positions, the finishing positions, the distance travelled, and the times of travel. The resulting speeds are different also. This question has been discussed in previous chapters where students provided the final correct responses with incomplete strategies, also some interesting strategies were used by students to finish the question.

To complete the problem successfully, it is necessary to consider the time of travel, the distance that each trolley had travelled and then compare, in a suitable way, the ratio of distance to time for each trolley. Thus the data necessary for students to focus on are, the starting and finishing places, the actual distances travelled for each trolley, and the time for each trolley. These are intermediate responses. Using a concept of distance travelled per unit time (i.e., speed) students could then set up the required ratios for each trolley, another intermediate response.

A suitable comparison of these two results needs to be processed to reach the final response.

TWO-VARIABLE STRATEGIES

Successful two-variable strategies used by students were the formula, proportion and written accounts that utilised distance and time. All of these strategies if used appropriately gave successful solutions.

The formula strategy

A high level two-variable strategy used by Year 12 students was the formula strategy. An example of a student's response using this strategy is:

Ans: *A is faster.*
Exp: *Speed of A = d/t = 5/4 cm/s*
" " " " = 1 cm/s.

In this example, the student used the formula on both trolleys to evaluate the speed, then chose the answer with the greater magnitude as the trolley with the greater speed. Figure 7.1 shows the mapping structure for this response. The response utilises all the data and, the term speed has triggered the use of the formula for these students.

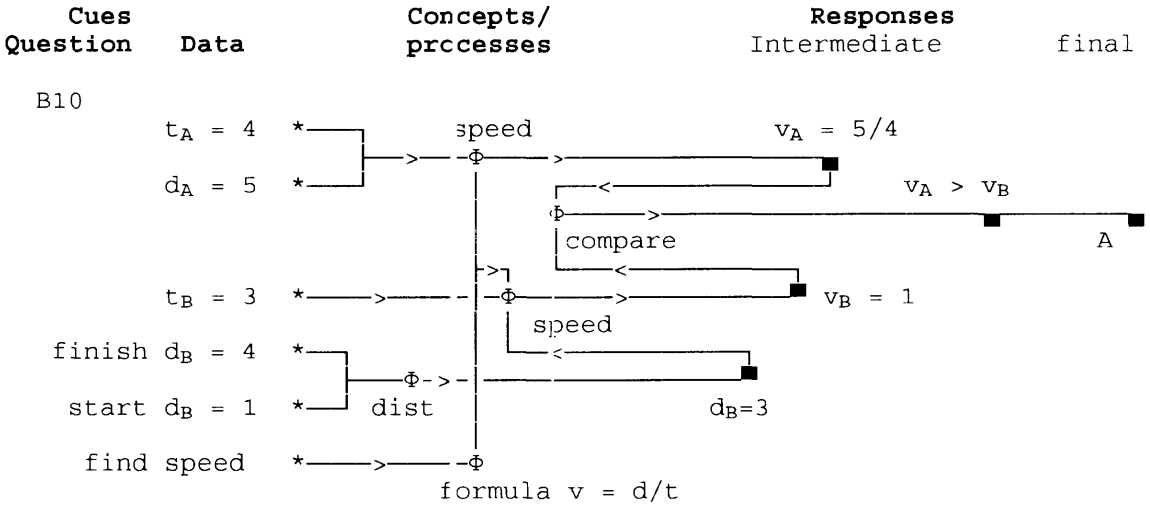


Figure 7.1 Map using a formula strategy

Proportion strategy

In this type of response the individual does not call upon the formula to calculate speed but found the ratio of distance to time for each trolley and compared the ratios. An example is:

Ans: *A.*
Exp: *A did 5 units in 4 secs and B did 3 units in 3 secs therefore A.*

Figure 7.2 illustrates the mapping diagram for the proportion strategy of Question B10. The dotted line indicates that the method of comparison is not clear for the above response.

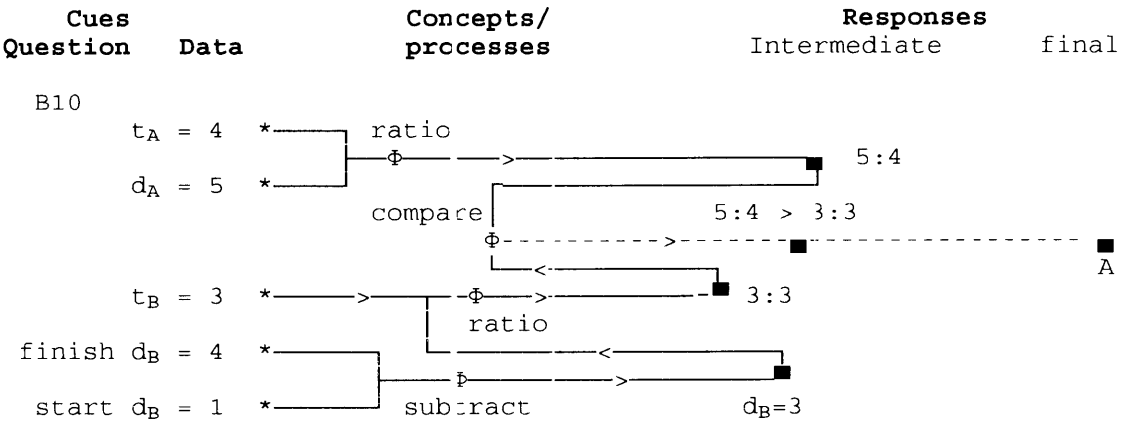


Figure 7.2 Map using a proportion strategy.

Most student responses using this strategy did not indicate how they had made this comparison. The interviews clarified this for some students who referred to "equivalent" fractions or who found the quotient of each ratio.

There is a similarity in this mapping structure to that of the formula strategy, the main differences lie in the processes involved. This mapping does not invoke the formula and thus instead of calculating the speed of each trolley from the formula the student has found the ratios of distance to time for each trolley.

Written distance and time

With this strategy, students refer to distance and time in written form. For example one student wrote:

Ans: *A*
Exp: *Trolley A went a bigger distance in its time than Trolley B would have travelled in the same time* [as A].

Figure 7.3 provides the mapping diagram for the written response that uses distance and time.

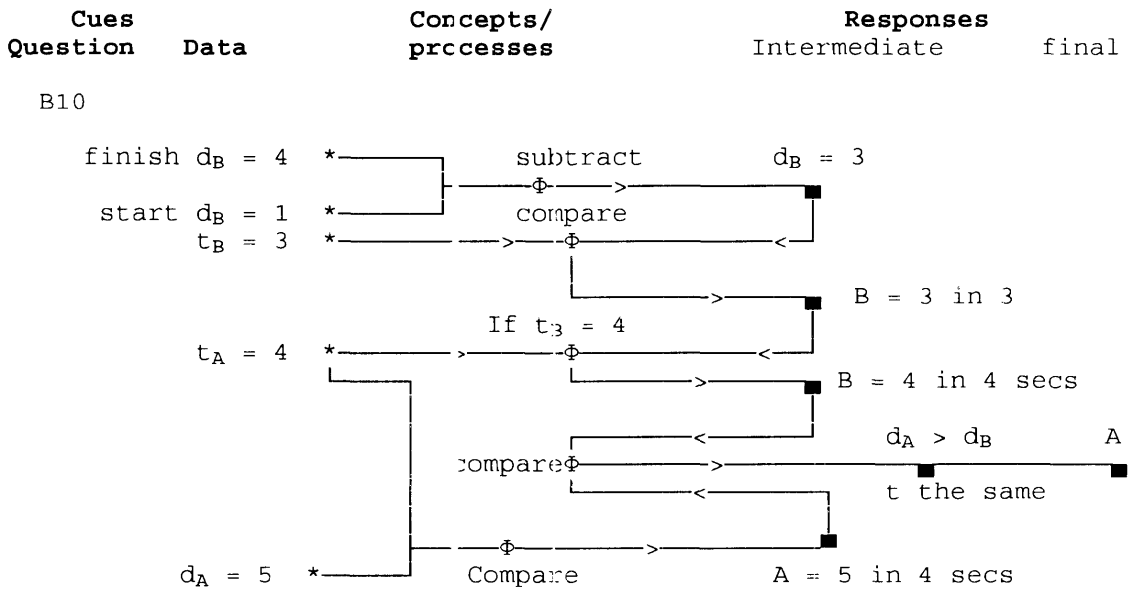


Figure 7.3 Map using both the distance and time as a written account.

This response indicates that the student is aware the larger the distance and the shorter time then the greater the speed. In applying this idea to this question, it is apparent that this student saw that Trolley B covered a distance of four units in four seconds but Trolley A traveled a distance of five units in the same time so it had the larger speed. Whether the student calculated the actual speed of Trolley B to be one metre per second or not, is not clear. In the above response, the student has not shown the calculation of the data to make the written summary. The interviews indicated that he processed the data to arrive at the intermediate results. The structure is a little different from the previous two strategies but multiple processing of data is occurring.

Calculation

Three Year 9 students, who did not have access to the formula, employed a calculation that resembled using the formula. On the previous questions they had

used other strategies since the comparison of ratios was a simple matter. In Question B10, with all the variables different, there was no easy way to compare the resulting ratios. The following is a typical example of this strategy from a Year 9 student.

Ans: *A.*
Exp: *A = 5 ÷ 4 secs = 1.25 units per sec.*
B = 3 ÷ 3 secs = 1 unit per sec so A went faster.

The mapping of this strategy is provided in Figure 7.4.

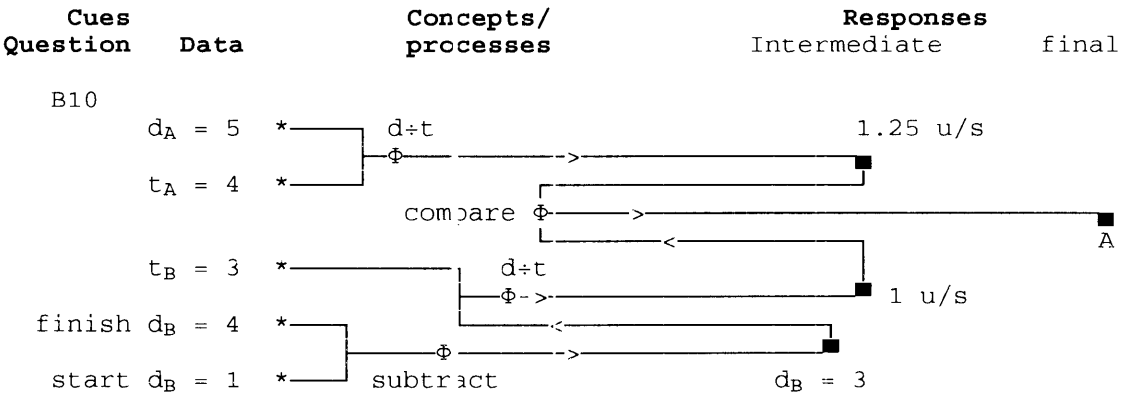


Figure 7.4 Mapping of the calculation strategy of Year 9 students.

The structure for the calculation and proportion strategy are similar. The essential difference in the two strategies is the method used to compare the two ratios. In the proportion method, students were not explicit in saying how they did the comparison. In the calculation approach, the responses showed clearly how the comparison was made, that is, students calculated the value of each ratio by using the quotient of the two numbers.

ONE-VARIABLE STRATEGIES

The one-variable strategies include time-only and distance-only strategies. These strategies were not always successful at producing correct answers. If the answer was correct it was based on insufficient data. If a student continued to use this strategy in other problems, incorrect answers would result.

Time-only strategy

A typical example of a time-only strategy is:

Ans: *B.*
Exp: *B took less time so is faster.*

Figure 7.5 shows a mapping diagram where the one variable time is considered. Only part of the data (time) are used and processing takes place on that data without regard to other relevant information such as distance.

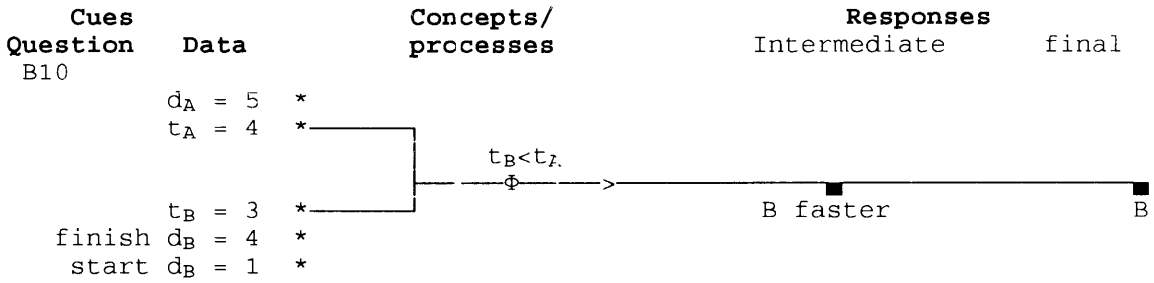


Figure 7.5 Map of time only strategy.

This map shows that only one major processing of data takes place and the other process to decide which is 'faster' is a carry-over from the comparing process.

Distance-only strategy

This strategy is similar to the time-only strategy in that distance is the determining variable and time is ignored. An example of this is:

Ans: *A.*
Exp: *A went a bigger distance than B.*

Figure 7.6 shows a mapping of this response.

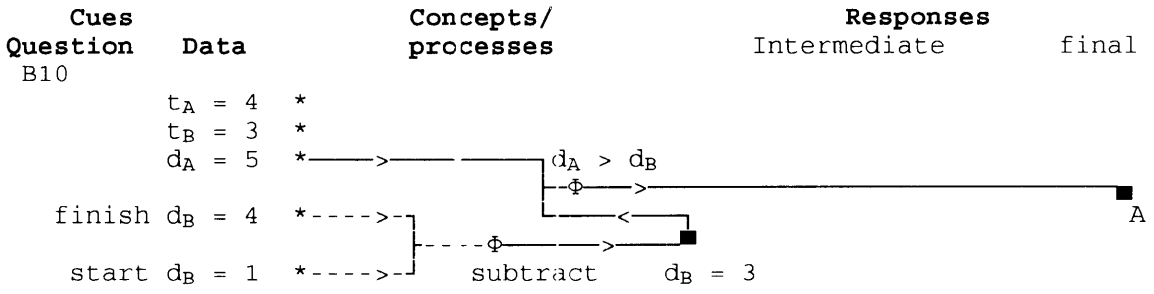


Figure 7.6 Map of time only strategy.

The major process used to answer the question was the comparison of distances to obtain the trolley with the greater speed.

VISUAL STRATEGIES

In these strategies, students focused on the diagram and not on the distance and the time supplied with the diagram. The strategies considered here are the in-front and faster strategies.

The 'In-front' strategy

In this strategy the student chose the trolley that was in front as the one with greater speed. The strategy based on the observation that one of the trolleys is in front of the other would appear to have strong visual support.

An example of such a response is:

Ans: *A.*
Exp: *cause [sic] it is in front.*

The mapping of this response is given in Figure 7.7.

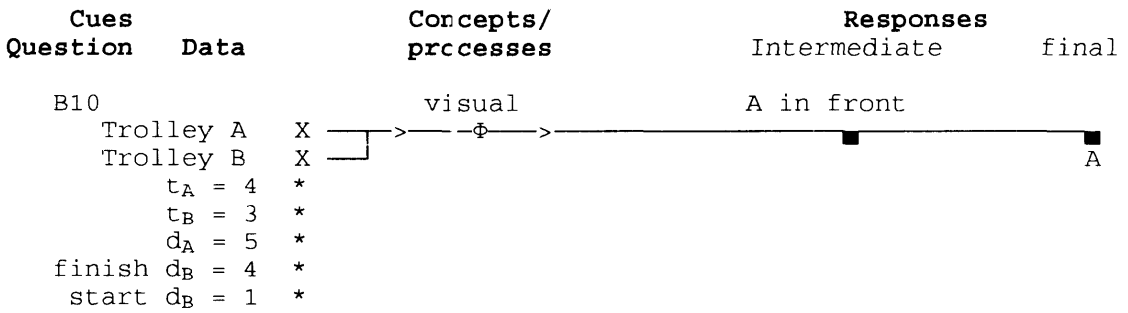


Figure 7.7 Map of in-front strategy.

In this case, all relevant data needed to solve the problem correctly are ignored. Only one process has taken place and this is at the visual level.

Some responses, however, do include data points as well as the diagram. The next example illustrates this linking of diagram with time.

Ans: *A.*
Exp: *A was just in front by one second.*

The mapping of this response is shown in Figure 7.8. The multiple processing and linking of information with the visual cues puts this response at a level above the previous response.

The student has noted that the times differ by one second and he focused on the diagram to see how 'far' one of the trolleys is in front. The student used visual support to help solve the problem with some data being accessed.

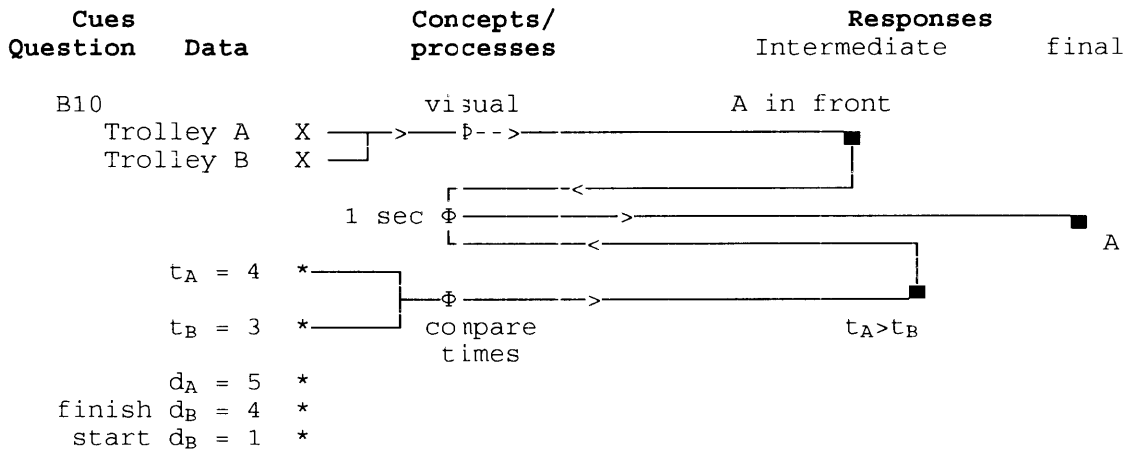


Figure 7.8 Map of in-front strategy with data.

Faster strategy

A student who explained his answer by suggesting that one trolley was faster than the other is included under this strategy. A typical example is:

Ans: *A.*
Exp: *It is faster* [than B].

This response does not provide detail as to what the student has done to solve the problem. Whether this student focused on the times or the distances is not clear. This is reflected in the mapping diagram of Figure 7.9.

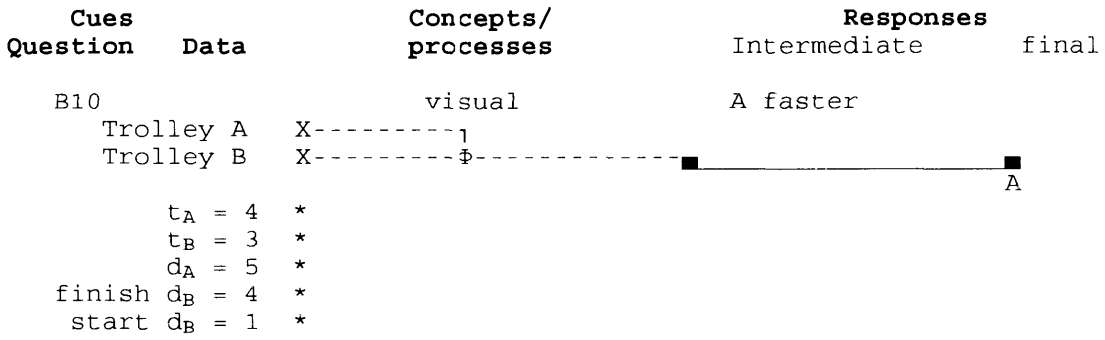


Figure 7.9 Map of faster strategy.

DISCUSSION

The mappings of the two-variable strategies at the higher level of cognitive processing show a linking together of the data using a concept or process to reach intermediate results. These intermediate results are then linked to provide the final result. The mappings of the one-variable strategy show the linking of some of the data via a process or concept to give a result. The other data were ignored and an answer was drawn from the intermediate result to provide the final result. The mappings of the visual strategies ignore the numerical data and rely on the visual impact of the diagram where there is some processing occurring to give the final result. The mapping diagram Figure 7.8 indicates a response that is a transition from visual to one-variable strategies in that both strategies are occurring.

The above mappings and discussion indicate that students do operate in different ways, some with strong visual support (for example, the trolley in front) and others with little visual support (for example, the formula). The same student can operate at both levels on the same question and also on different questions using the visual cues as support to using mathematical representations to aid thinking and vice versa.

Some students who had access to the formula saw some of these questions as rather simplistic and could do them without having to use large amounts of working memory. They had undergone, what Van Hiele (1986) called, a "level-reduction", that is, they were able to work at a level that used the formula, but for non-complex problems could use strategies like direct variation and proportion to solve the problems successfully. When problems became more difficult, they then accessed the

formula to attain a solution. This was noticed in the responses of some students who were thinking through the initial problems without the formula in the Part B questions but reverted to the formula when the problem became more complex.

CONCLUSION

The outcomes of this chapter have covered a range of issues related to strategies. In particular, the following four findings are of interest.

First, students descriptions of speed were not always related directly to the type of strategy that they employed to solve problems. Some students who could not identify speed as related to distance and time when asked to provide a description of speed, solved problems employing strategies that incorporated either one or both these variables to solve questions on speed. On the other hand, some students who knew the formula for speed did not automatically rely on the formula to solve speed problems. When the problem became more complex, however, students who knew the formula would then use it.

Second, different types of questions elicited different types of responses and strategies that were related to the Year levels of the students. Word-presented questions evoked more mathematical responses, while questions with diagrams elicited strategies that relied on visual cues from the younger students.

Third, there is strong support for different levels of strategies when solving speed problems. The strategy groupings students use, ranged from those with a strong visual support involving images to search for an answer, to those that require little visual support, such as, the use of a formula. Four levels of operation were identified. In summary these are: Level 1 is characterised by responses that show no understanding of the problem and rely on visual aspects of the problem only. Level 2 is characterised by responses that rely on one-variable type strategies. When two-variable strategies are used, then the response is at Level 3. However, the response does not tie all the variables into a general principle. When the response brings in the general rule and indicates any constraints that might apply then this indicates a Level 4 response.

Finally, Mapping diagrams are a useful tool to illustrate the different strategies used and provide a means of seeing commonalities and differences between the strategies and levels of operation.

The next chapter explores these mapping diagrams in the context of the SOLO Taxonomy as a suitable post-Piagetian framework. Working within a structure, such as the SOLO Taxonomy, provides reliability in interpreting the outcomes associated with the concept of speed.

CHAPTER 8

INTERPRETING THE OUTCOMES WITHIN A POST-PIAGETIAN FRAMEWORK

A wise Tory and a wise Whig, I believe will agree. Their principles are the same, though their modes of thinking are different.
Boswell's Life Vol iv. p. 117. 1781.

The previous chapters provided an overview of student responses to questions asked in the Speed Tests. In particular, Chapters 3 and 5 provided students' descriptions of speed that were considered within categories. Chapter 7 analysed the strategies employed by students over a range of questions and four levels of operation were identified as being related to the type of strategy students used. While there are benefits in seeing groupings of responses, for descriptions and strategies, it would allow for greater flexibility if these responses could be interpreted within a recognised theory or framework. A possible framework is Piaget's theory of cognitive development, which was introduced in Chapter 2, but its applications are rigid in the stages, it does not allow fine tuning and it is applied to the individual. Neo-Piagetian theories appear to offer more flexibility and the later developments of post-Piagetian (sometimes referred to as post-neo-Piagetian) frameworks give even more versatility in applying levels to responses and not to individuals. The SOLO Taxonomy, described in Chapter 2, is an appropriate post-Piagetian framework within which to describe the outcomes presented in the previous chapters.

It is the purpose of this chapter to examine the responses of students regarding the speed questions within the framework of the SOLO Taxonomy. Consideration is given initially to descriptions of speed provided by students (Research Question 13) and then to responses made by them to the questions concerning speed in the written

test (Research Question 14). Finally, an elaboration of the SOLO Taxonomy is discussed (Research Question 15).

SOLO AND THE DESCRIPTION OF SPEED

The categories given in Chapter 5 to student responses for a description of speed, may be considered also within the SOLO Taxonomy. The two main groupings of descriptions refer to two modes of responses, that is, the ikonic and concrete symbolic mode. In the ikonic mode students base their description on perceptual pictures of speed that they have in their mind. They then convert this picture into words to describe what they perceive as speed. Two cycles were identified within the concrete symbolic mode. The first cycle is different from the ikonic mode in that students have identified the components of speed and they can state that speed is related to distance and/or time in some way. This cycle has some ikonic support since students often relate this motion to real situations in formulating their responses. The second cycle starts when students can relate the components of speed in a mathematical way and do not provide, in general, any ikonic support.

A summary of the break down of responses in terms of levels and modes is provided in Table 8.1.

Table 8.1
The SOLO Taxonomy and descriptions of speed

	IKONIC MODE	**CONCRETE SYMBOLIC MODE**	
LEVELS		**First Cycle**	**Second Cycle**
Relational	Motion of fast or slow objects.	Relate time and distance but not mathematically.	Formula $v = d/t$
Multistructural	Have idea of motion but only very fast objects	Time **and** distance referred to but not connected.	Distance per unit time
Unistructural	Idea of simple movement	Time **or** distance referred to.	Mathematical idea relating distance and time

Responses coded in the ikonic mode have as their basis ideas that have been learnt from experience. Some students, starting at about Year 9, analysed speed for themselves and were aware that distance and time were involved and have thus made a transition into the concrete symbolic mode. By Year 12 most students had identified the key elements and were able to verbalise a connection between speed, distance and time whether or not they have been taught the definition for speed. Those who have done Mathematics II (Mechanics) and Physics do not all give the definition 'automatically' but most students related speed to the variables distance and time in some way. Responses that attempt to give mathematical relationships are coded in the second cycle. The more sophisticated the definition of speed that is given, the higher the level in this cycle until a formula is provided explicitly.

IKONIC MODE

Responses that appear to be at an intuitive level where students rely on their own experiences and simple everyday words to express themselves are coded as ikonic.

Unistructural level responses usually focus on one aspect associated with speed. In this case, speed is often confused with speed limits, speeding up (acceleration) and moving from one place to another.

At the multistructural level, students associate speed with the motion of very **fast** objects, such as a car. In this case, the idea of speed is related to speeding cars, that is, cars exceeding the legal speed limit. Vehicles that do not exceed the speed limit are not 'speeding' therefore may not be considered to have speed. The word 'speeding' is used in everyday language for cars travelling at excessive speed. Drivers caught 'speeding' are given a 'speeding ticket'. Thus there are usually two components, either stated or implied, in students' responses: the term fast, and an association with high speed.

A higher level response is when speed is associated with an object that is moving, whether it is at high or low speed. Such a response is coded as relational since students now have a complete idea of speed as the motion of some object. At this level, students do not analyse the components that determine speed. Instead they see it in global terms, clearly related to their descriptions of personal experience.

CONCRETE SYMBOLIC MODE

Responses that incorporate the variables associated with speed are coded in the concrete symbolic mode. Within this mode the responses can be categorised within two cycles, with levels within each cycle.

First Cycle

The responses in the first cycle of the concrete symbolic mode reveal a higher level of understanding regarding the concept of speed. Students relate speed to distance and/or time. At the unistructural level, they associate speed with only one variable. The object with the higher speed is the one that goes the greater distance, so distance is the significant variable. Other students associate speed with the object that 'gets there the quickest', that is, in the shortest time, in this case time becomes the critical variable.

The multistructural level is characterised by responses that refer to both distance and time. The term 'fast' may even be used in an attempt to describe a meaning to speed. There is no indication in the response to a relationship between these variables or terms.

When an attempt is made to relate the two variables in some way this indicates a relational level response. There is a reference to speed being "a distance in a certain time". There is no reference, however, to there being a mathematical relationship between these variables.

Second cycle

The second cycle is characterised by responses that express mathematical relationships between distance and time. Unistructural level responses indicate a relationship between the two variables in a structured way with the suggestion of a mathematical relationship. For example, the response of a student who chooses to say that speed "is distance over time" knows there is a mathematical relationship but does not explain what the term 'over' means, is coded at the unistructural level.

A multistructural response, however, indicates that speed is distance per unit time. Thus the student is aware of the correct mathematical relationship between the variables.

A relational response provides the actual formula in symbolic form. Usually, these responses were accompanied with the multistructural response that speed is distance per unit time. Responses such as this indicate that the student had an overall view of speed. Such a concept could be stated in words and also as a mathematical relationship that related the words to the symbols and was in a form that could be used to find the speed of an object

Responses at these last two levels are usually optimal responses, that is, students can give them when they are asked for more information about speed. The functional response is usually lower, as indicated by Year 12 students who had studied Physics and Mathematics II, giving first cycle multistructural or relational responses in the test. On probing (i.e., asking for any detail without being provided with additional information) in the interview for a better description of speed, they could all give a second cycle response.

Summary

For responses to students' description of speed, two modes of functioning were identified: the ikonic and the concrete symbolic. Within the concrete symbolic mode are two cycles each with unistructural, multistructural and relational levels. The first cycle has some ikonic support but the second cycle is focused on the more mathematical aspects of speed. Also, it should be noted that the relational level of the first cycle is not identical to the unistructural level of the next cycle. There was an increased sophistication, in which the new unistructural level was based on a simple element which encompassed the previous relational level but in a more succinct form.

SOLO AND THE DUAL FOCUS CLOSED QUESTIONS

In this section, attention is given to the problems in the speed test and to the application of the SOLO Taxonomy to these responses in an effort to understand their structure.

In Chapter 7 four levels of strategies were found for the way students solved speed problems. These levels were related to the number of variables that were used to solve the problem. Level 4 was associated with responses that took into account all the relevant variables and showed an overview of the whole problem in using the appropriate formula or stated the general principle that linked all the variables.

The remaining levels do not show the sophistication of level 4. A Level 3 response used a two-variable strategy but there was no attempt to account for all the variables or give an overview of the problem. When responses used a one-variable type strategy to solve the problem this characterised Level 2. Level 1 strategies did not take into account the relevant variables but relied on visual aspects of the problem.

In the following discussion, the strategies used by students are looked at in more detail. In classifying responses, the variables suggested by Biggs and Collis (1982) need to be kept in mind. These variables are: capacity or memory space (M-space), relating operations or organizing dimensions, consistency and closure, and structure. These features have been discussed previously in Chapter 2.

One important aspect of the model, given in Figure 2.1, is the response structure in the final column. By mapping the structure of responses it is possible to obtain an overview of a student's response. These mapping diagrams were used successfully in Chapter 7 to illustrate the levels that were developed based on strategies. It is proposed to use the diagrams again to tease out more of the details associated with the SOLO Taxonomy as applied to speed problems.

As in Chapter 7, the symbols used are:

- x represents inappropriate or incorrect data, that is, the student selects information from the question that does not lead to a correct solution to the problem.
- * data given with potential to cue a response;
- Φ concepts processes and/or strategies expected within the "Universe of discourse";
- o abstract concepts, processes and/or strategies within "Universe of discourse" but additional to those expected as part of understanding of the question;
- ■ responses - both intermediate and final.

Questions B3 and B5, in the Dual Focus Closed Comparison Questions of Part B, are used to illustrate the different levels and structures of responses. Question B3 was attempted by students from Years 6 and 9, while Question B5 was attempted by all students from Years 6 to 12. The major features of these questions is that in Question B3 both the distances and the times for both trolleys are different but the speeds are the same. In Question B5 the times are the same, but the distance d is different. Thus, in this case, the speed is different also.

Three cycles were identified in the responses that were given. The first cycle was clearly ikonic; students had relied on the visual aspects of the diagram. The other two cycles were in the concrete symbolic mode of functioning. The following discussion identifies these three cycles and the levels within them as they relate to Questions B3 and B5.

IKONIC MODE

When most of the focus of responses were related to the diagram rather than the associated data, this indicated that students were using the ikonic mode to give an answer and an explanation. Within this mode, different levels of responses could be identified that relate to the different SOLO levels. Using the characteristic variables that help to determine the levels, typical responses that could occur at each level are considered.

Unistructural

An example of a unistructural response on Question B5 is:

Ans: *B*.
Exp: *guess*.

Responses like this show a lack of logical or sequential reasoning but the student has decided on an answer. Such responses are coded as unistructural.

Figure 8.1 illustrates a mapping of this type of response where no use has been made of the relevant data and no reference has been made to the diagrams. The process is a guess but a final response has been given which includes, at the very least, that one trolley has been considered. The dotted lines indicate the uncertainty associated in reconstructing the mapping of the response.

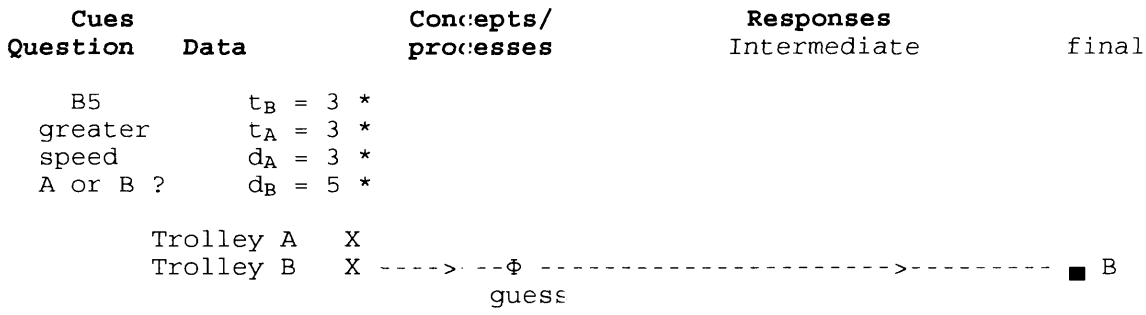


Figure 8.1 Map of unistructural ikonic mode response. A guess strategy.

Multistructural

A multistructural response attempts an explanation for the answer which implies both trolleys have been considered. The example, provided below, uses the 'front' trolley as the determining factor.

Ans: *B.*
Exp: *because trolley B is in front of A.*

The student has seen both trolleys and has identified one of them as being in front of the other, therefore this student has attributed the greater speed to the trolley in front. The mapping diagram, Figure 8.2, shows the two cues that have been used to process that data, there has been no reference to the other cues. If this type of strategy was continued on other problems then inconsistent responses would occur.

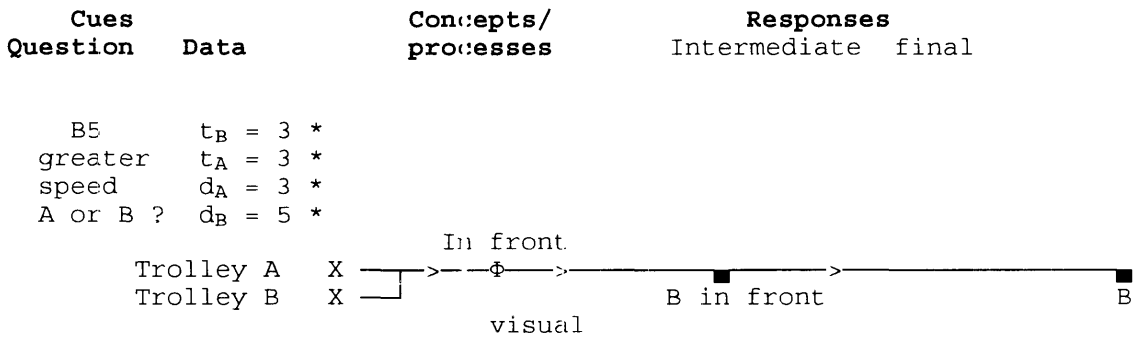


Figure 8.2 Map of multistructural ikonic mode response, using the 'in front' strategy (Correct response but incomplete strategy).

Relational

A response that is similar to the multistructural response above but which includes some integrating aspect, such as. the spaces between the two trolleys on the diagram is coded at the relational level. For example:

Ans: *B.*
Exp: *It was faster by two spaces.*

The mapping of this response is shown in Figure 8.3

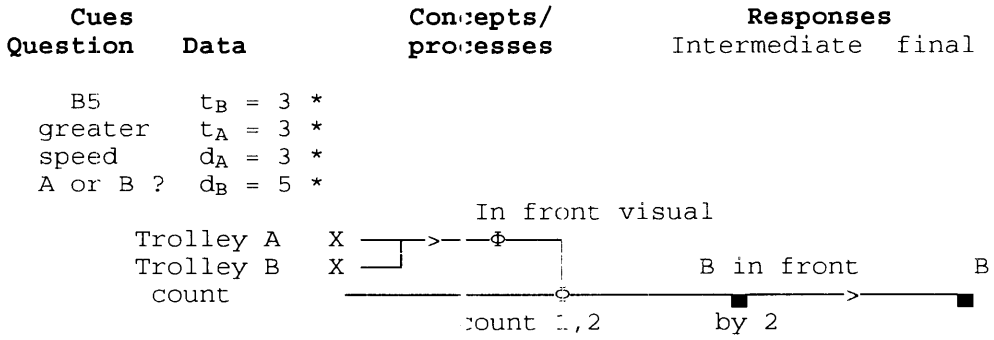


Figure 8.3 Map of relational ikonic mode response, using the 'in front' strategy with counting (Correct response but incomplete strategy).

This response shows the link that is occurring between the two trolleys, and a visual examination of the relative positions of the two trolleys. The student has considered both trolleys, and these two aspects have been related to help address the problem.

Transitional response

A transitional response between modes would indicate some use of the data points but not give any indication as to how the answer was obtained. For example

Ans: B.
Exp: it got to 5 units.

This response shows no relevant processing of the data but it does refer to one data point. Such a response is based on the diagrams which give a visual cue. So both the diagram and the data point are related to give a response. The mapping diagram in Figure 8.4 indicates this tentative idea by the dashed lines from the visual aspects.

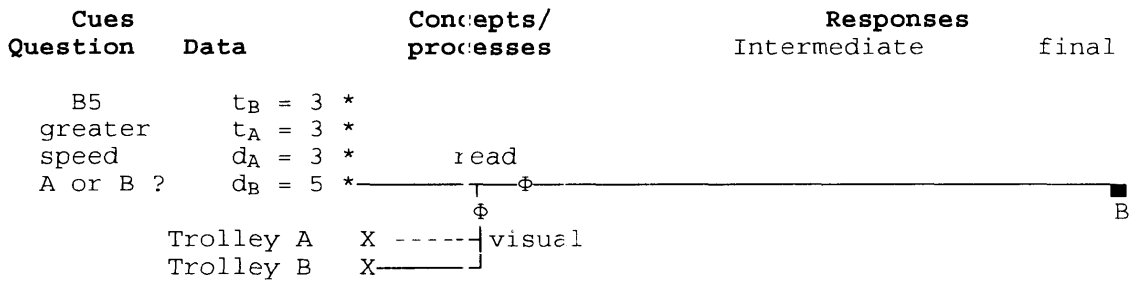


Figure 8.4 Map of a transitional response from the ikonic mode (Correct response but incomplete strategy).

CONCRETE SYMBOLIC MODE

First cycle

As the responses move into the concrete symbolic mode, there is less dependence on the visual aspects of the diagram. In the first cycle, the focus is not so reliant on the diagrams, as in the ikonic mode, but there is consideration being given to some of the numerical data that is supplied with relevant processing of that data.

Responses in this cycle of the concrete symbolic mode are characterized by the way the data are processed rather than by the number of data points that students decide to use. A response is coded in the first cycle if relevant variables are used either by value or name.

Unistructural

Ans: *B.*
Exp: *B got to 5 units and A only got to 3.*

In this response two data points are used and only one process takes place. The two distances are compared and the one with the greater distance is the one that the student considers to have the greater speed. This is typical of a unistructural approach where **one operation** is chosen for some of the data and other relevant data are ignored (see Collis & Watson 1991, p. 74). In this problem the correct answer is arrived at using this method because the times are both the same. The above response, however, has not indicated that the times have been taken into

consideration. The mapping of this level is given in Figure 8.5 where the process being used compares the two distances.

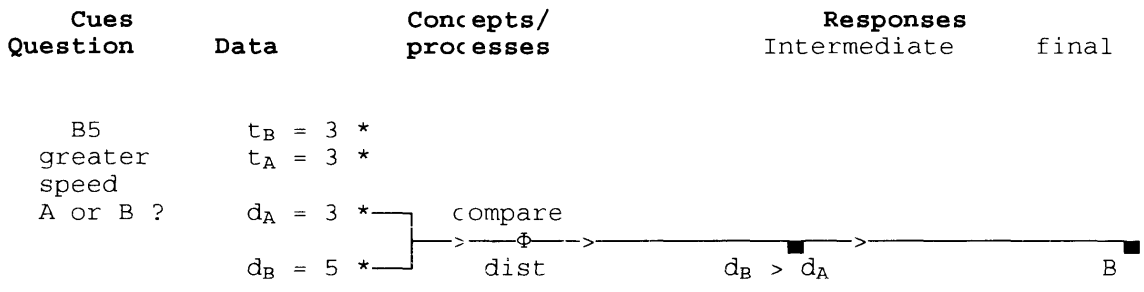


Figure 8.5 Map of unistructural first cycle concrete symbolic mode, using a distance comparison strategy (Correct response but incomplete strategy).

A similar response structure could occur if the student had focused on the times instead of the distance, as in Question B3.

Ans: B.
Exp: B went for 5 secs and A only went 3.

The mapping of this response is provided in Figure 8.6.

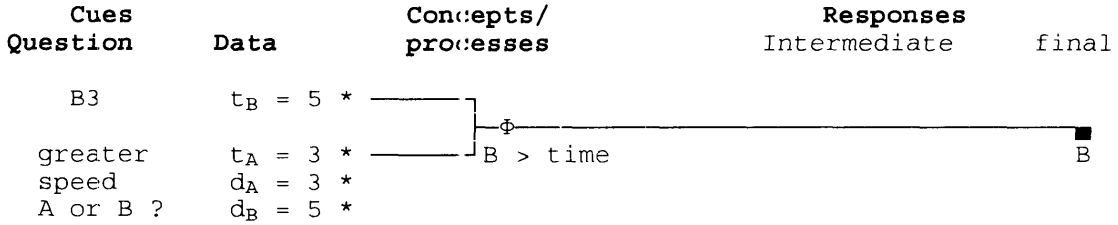


Figure 8.6 Map of first cycle concrete symbolic mode unistructural response, using a time comparison strategy (Correct response but incomplete strategy).

Multistructural

A multistructural response involves a comparison of both variables, that is, time of Trolley A with time of Trolley B as well as distance of Trolley A with the distance of Trolley B. Thus the student focuses on both variables, but has inappropriate connections between those variables. The example provided is a response to Question B3.

Ans: B
Exp: B got more units than A and had more time

The mapping of this response, in Figure 8.7, illustrates the comparisons between the same variables.

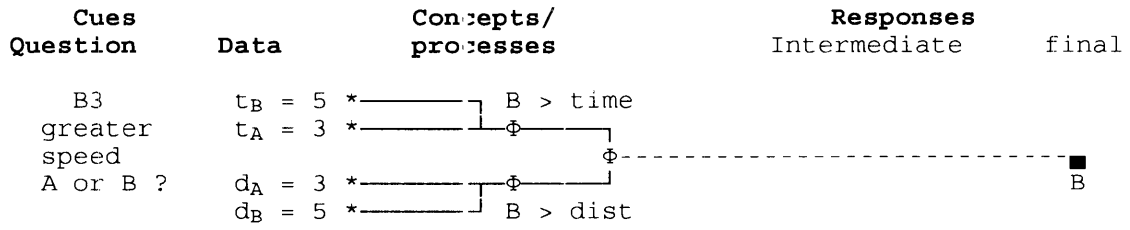


Figure 8.7 Map of multistructural first cycle concrete symbolic mode response, using a time comparison and distance comparison strategy (Correct response but incorrect connections between variables).

One important feature of this structure is the attempted linking of the two concepts to provide one final answer but there is no attempt to indicate how this linking occurs.

Relational

A response to Question B5 at the relational level is:

Ans: *B.*
Exp: *B did 5 units in 3 secs and A did 3 units in 3 secs.*

Distance and time values for both trolleys have been referred to in this response. It could be just a repeating of the variables in the question, but the correct variables have been identified and linked. The response refers to all the relevant data in this problem, however, on this occasion two different processes on the data have taken place - a comparison of time with distance for each trolley. There is no indication in this response as to the process that has taken place to relate the results of these two comparisons. There is some intuitive processing taking place and the answer seems obvious to the individual. Increased working capacity is required at this level compared to the multistructural approach, since the individual must hold a number of processes in mind before closing.

Figure 8.8 illustrates the structure of a first cycle relational response. The dotted line indicates that there is some uncertainty concerning the final processing.

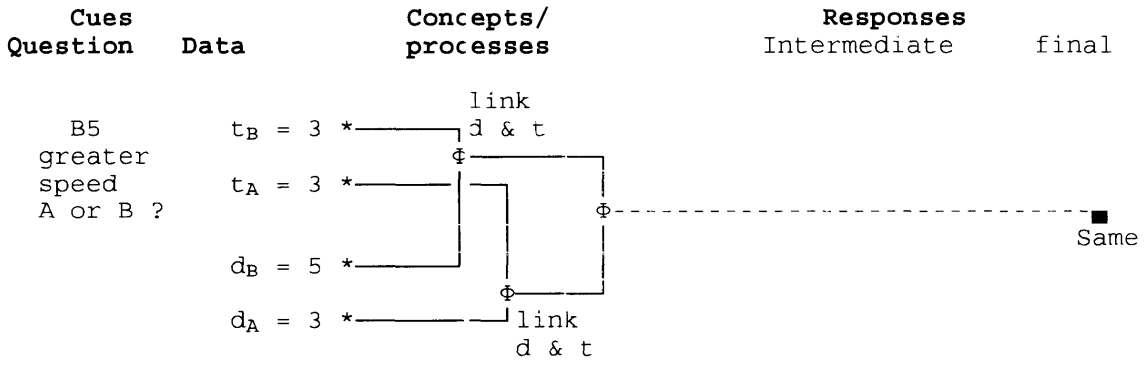


Figure 8.8 Map of relational first cycle concrete symbolic mode response, using a distance/time comparison strategy.

Second Cycle

The second cycle is characterised by the use of a specific strategy to compare the distance and time ratio. To accomplish this the intermediate results must be processed before a final result is given.

Unistructural

For this first level, students select a comparison strategy that does not rely upon the use of the formula or the calculation of each ratio to one number. The strategy used is proportion or equivalent ratios, that is, common mathematical strategies are selected. The example provided concerns Question B3.

Ans: Same.
Exp: A did 3 units in 3 seconds
B did 5 units in 5 seconds they are equivalent.

This response compares both situations, Trolley A and Trolley B, giving an overview of the whole problem. Both the time and distance for each trolley are compared then the distances and times are related to each other. All aspects of the data are taken into consideration and utilised to form a smaller number of higher order concepts, these are further processed to give the final response. Closure takes place only after the final processing and judgement is withheld on the initial processing of data. The working memory capacity is much larger here than in the previous levels as the results of each process are held and used later. This response is to be compared with the relational response in the first cycle where the comparisons are not related explicitly. Figure 8.9 shows the mapping of the structure for this

response. It is interesting to note that multiple processing takes place to arrive at the intermediate responses and then these responses are reprocessed to achieve the final result.

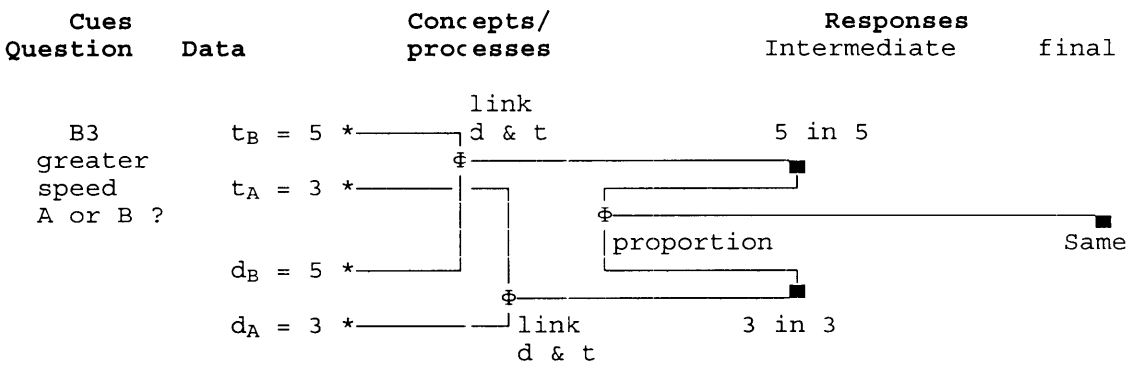


Figure 8.9 Map of unistructural second cycle concrete symbolic mode response, using a proportion type strategy.

Multistructural

A student who does not have access to the formula but uses a calculation approach to compare the two speeds has made a transition into the next level. They have calculated the speed, without using the formula. A response given to Question B3 illustrates this level.

Ans: Same
Exp: A went 5 in 5 which is 1
B went 3 in 3 which is 1 unit per second

In this response the actual speed of one unit per second is calculated for each trolley, then these values are compared. The student has not stated explicitly that these values are the speed of each trolley, but it is implied. Responses to other questions indicate that this student was not familiar with the formula. The mapping of this response is provided in Figure 8.10.

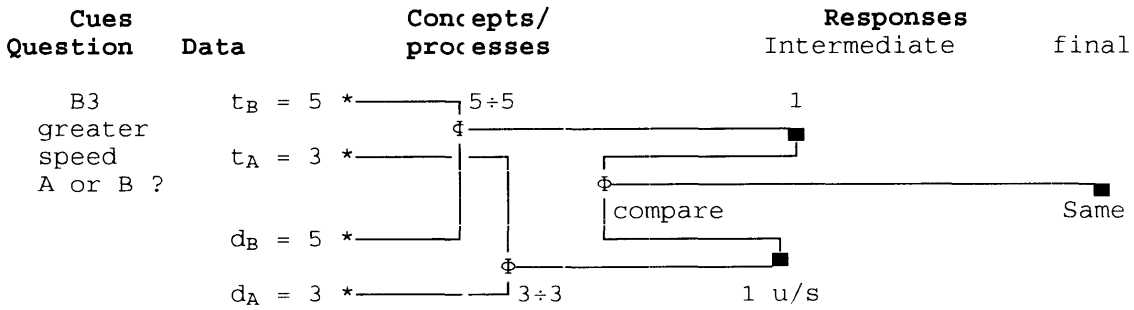


Figure 8.10 Map of multistructural second cycle concrete symbolic mode, using a calculation strategy.

Relational

The next example considers a response that employs the formula to calculate speed. A well known formula is used that some students have at their disposal from prior learning. Since these students can use such a procedure it indicates that they have synthesised the variables distance and time into one relationship and suggests that they have reached a new level of understanding. This level has been identified as the relational level within the second cycle of the concrete symbolic mode. This example is based on Question B5.

Ans: *B.*
Exp: *Speed of A = d/t = 3/3 = 1*
speed of B = d/t = 5/3 = 1.6
B has greater speed.

This response takes into account all the data points, relating distance and time by extracting the formula for speed from within the given universe of discourse. Its use is triggered by the occurrence of the word 'speed'. Each result is then compared with the other and a decision made as to which is the appropriate answer. There is high probability that students will be consistent on similar questions at a later time since they have an overall strategy that is successful on most problems. Figure 8.11 shows the mapping for this response.

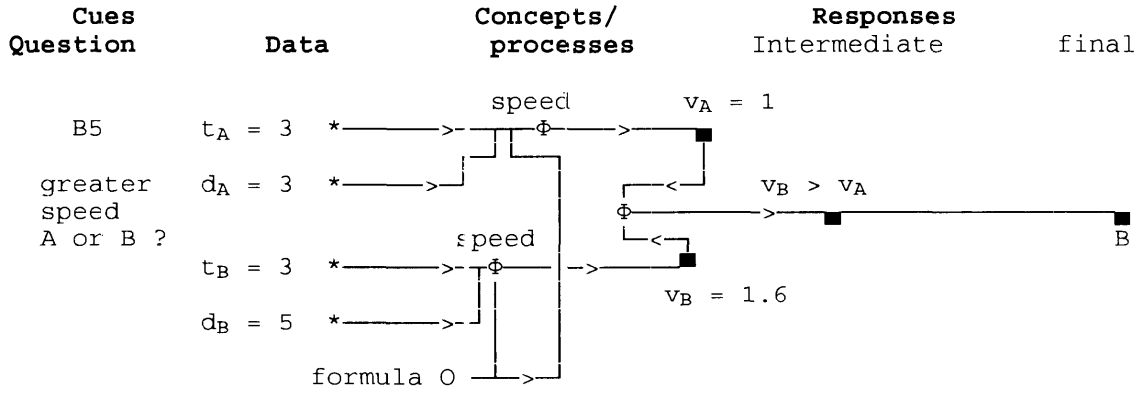


Figure 8.11 Map of relational second cycle concrete symbolic mode, using a formula strategy.

SUMMARY

Table 8.2 provides a summary of the three cycles as detailed above for Questions B3 and B5 of the Speed Questions. This model indicates a cycle of levels within the ikonic mode and two cycles within the concrete symbolic mode.

Table 8.2
Levels of the SOLO Taxonomy applied
to the Dual Focus Closed Questions of speed

	IKONIC MODE	CONCRETE SYMBOLIC MODE	
LEVELS		**First Cycle**	**Second cycle**
Transitional	Focus on diagram and uses limited data.	---------	----------
Relational	Front or fast and counting units on diagram	Link time and distance but no method provided for comparison.	Formula $v = d/t$ employed explicitly
Multi-structural	One in front of other, i.e., both trolleys are considered.	Time compared and distance compared. But not with each other. Wrong connection	Calculation of speed but without the use of the formula.
Uni-structural	Answer provided A guess.	Makes a comparison based on one type of variable.	Link time and distance and compare ratios

From this table it is apparent that when solving problems, the level of response, especially in the concrete symbolic mode, does not depend solely on the number of data elements that are chosen but also on the way the data are processed. This is to be compared with the previous section in this chapter, where students were asked to give a description of speed. In that situation, students were presented with an open ended question with neither cues nor data points from which to choose. They were only asked for a description of speed, so any data used by students would determine the level of response. In the case of problem solving, cues (for example, diagrams) and data were already available and an individual must select the most relevant and combine it in a suitable way. During an analysis of the data, it became clear that their was a difference in the way levels of responses are determined depending on the type of question that is being asked, that is, by either recall or problem solving. Such differences have previously been noted by other researchers, for example Biggs and Collis (1980, 1982). An information gathering question is quite different from a problem-solving question and appears to require two different demands on working memory. In the former the actual information provided by the student determines the level, whereas in the latter, the complexity of the processing also needs to be considered to determine the level.

Another important consequence is that the mapping of different strategies used by students also indicates the level of response within the SOLO Taxonomy and may even show that different strategies are more likely to occur at one level than another.

Conclusion

The questions used to illustrate the cycles in this section have been limited to Questions B3 and B5 which occur in Dual Focus Closed Comparison Questions of Part B. There seems to be strong indications that responses concerning speed problems can be categorised into two modes of functioning, ikonic and concrete symbolic, with the later having two cycles (see Table 8.2 for an overview). In the next section, consideration is given to the application of this model to the Dual Focus Open Comparison Questions.

SOLO AND THE DUAL FOCUS OPEN QUESTIONS

The ideas that have been discussed relating to descriptions of speed and to the responses given for the Dual Focus Closed Questions appear to be fairly stable. Two modes with two cycles in the concrete symbolic mode occur both with descriptions of speed and also in the responses to the Dual Focus Closed Questions. It is the purpose of this section to determine if the responses of the Dual Focus Open Comparison Questions also follow a similar pattern.

Examples of typical student responses are given below to illustrate each level and the map associated with each response is provided also. The examples come from a range of Dual Focus Open Questions.

IKONIC MODE

As in the previous sections the ikonic mode is characterised by responses that rely heavily on the visual aspects of the problem and evoke intuitive type responses

Unistructural

The example of this level of response is based on Question C2, the answer is incorrect.

Ans: *3.*
Exp: *I just Guessed!!!!*

Figure 8.12 shows the mapping of this response with the dotted line showing the uncertainty as to how this student obtained the answer and that the student started with no known data information.

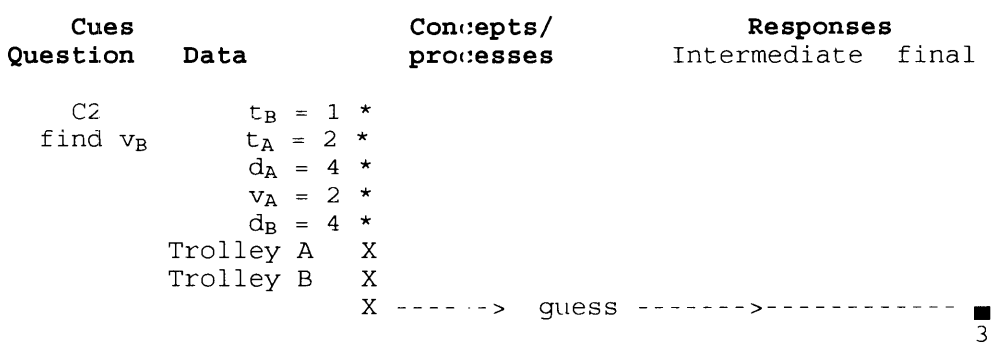


Figure 8.12 Map of unistructural ikonic response. A guess strategy.

Multistructural

The example of a multistructural response is that of a Year 9 student on Question C8.

Ans: *4.*
Exp: *because its faster than A.*

This response gives more detail than the unistructural response, as to the type of strategy the student was using. The student has compared the two trolleys and noted that Trolley B was faster than Trolley A. But there is no indication as to how she obtained the answer '4'. It might be that since Trolley A was at three units then Trolley B had to be in front, so four units was a suitable place. The mapping diagram in Figure 8.13 shows the tentativeness of the processing by the dotted lines. This student is relying heavily on the visual aspects of the diagram.

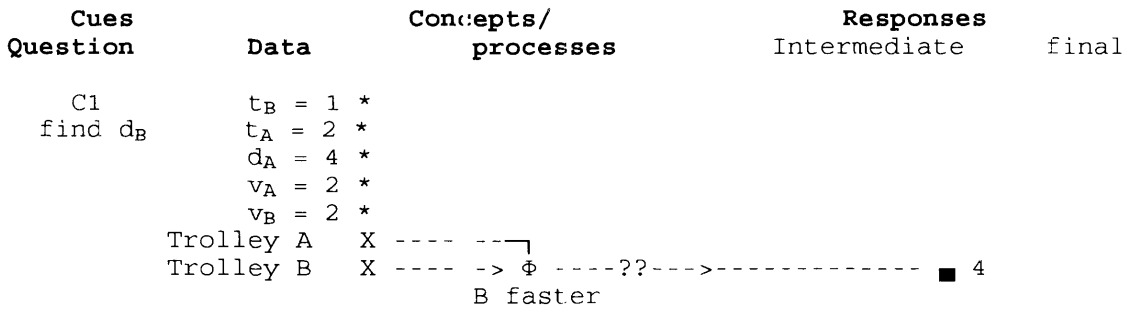


Figure 8.13 Map of multistructural ikonic response.

Relational

The response for this level is provided by a Year 6 student to Question C7.

Ans: *6.*
Exp: *Because it is 3 secs faster.*

This response picks up on one aspect of the data, three seconds, but it focuses on the 'faster' trolley in the diagram, that is, the one in front. It appears that the student then noted that Trolley B was at six units and decided that the speed was six. Thus there is an attempt to use the data while concentration is on the diagram. The mapping of this response is provided in Figure 8.14. The dotted line indicates the uncertainty of how the student was able to link the three seconds to the final result.

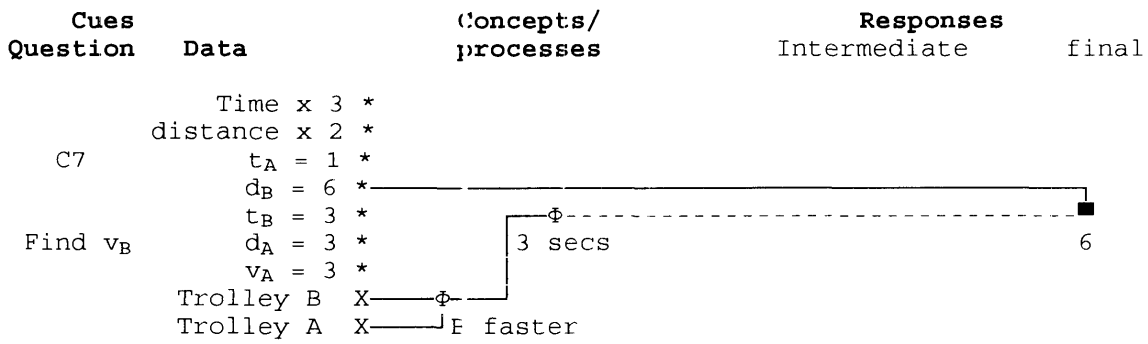


Figure 8.14 Map of relational ikonic response.

CONCRETE SYMBOLIC MODE

First cycle

This cycle is characterised by responses that refer to the data (variable value or variable name), and process them to yield a result. Responses do not rely on the diagram alone, though the diagrams may have provided some ikonic support.

Unistructural

A unistructural response refers to only one or two data points of the same type and only one concept or process is used. The example is based on Question C1.

Ans: *2.*
Exp: *because it has only 1 sec.*

The mapping of this response is shown in Figure 8.15 with a '?' indicating that the processing that was done is unknown.

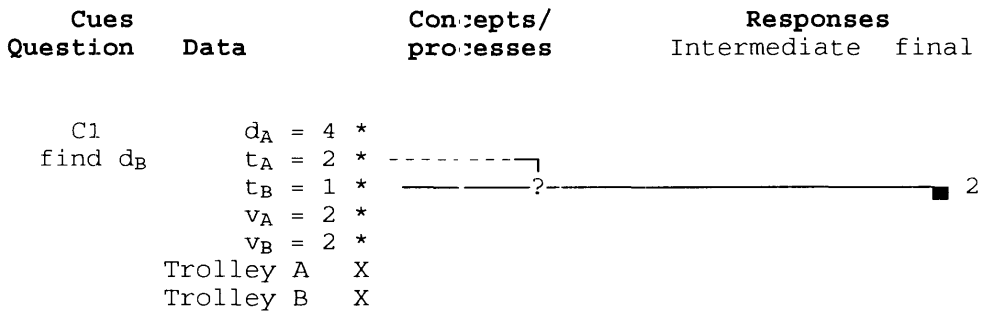


Figure 8.15 Map of unistructural first cycle concrete symbolic mode, comparing times.

This response is classed as unistructural since the student has only referred to one aspect of the problem (that is, time) but he has obviously seen both times and compared them and perceived that Trolley B has taken less time than Trolley A (that is, one process). The method of comparison and how the answer is obtained was not provided.

Multistructural

At this level, the responses would refer to two variables but there would be no attempt to link them appropriately. For example, the response would compare the two distances and the two times in an attempt to find the speed. However, there were no responses provided by students at this level. The questions did not encourage students to respond by focusing on two similar variables without linking them. At the end of each question, a statement was provided as to how the variables changed and which variable was to be found thus students tended to try and link these two different variables, if they could.

Relational

The relational level response links time and distance and uses a suitable strategy to reach a conclusion. The example provided here uses direct variation as the strategy on Question C1.

> Ans: *2.*
> Exp: *Trolley A is at 4 in 2 secs so trolley B has to be around 2 in 1 sec.*

This response, with the mapping shown in Figure 8.16, is clearly relational in that more than one process has taken place linking both the distance and the time appropriately but there is no reference to how the final result was processed. By the way the explanation is written, it may be assumed that the student has used a direct variation approach.

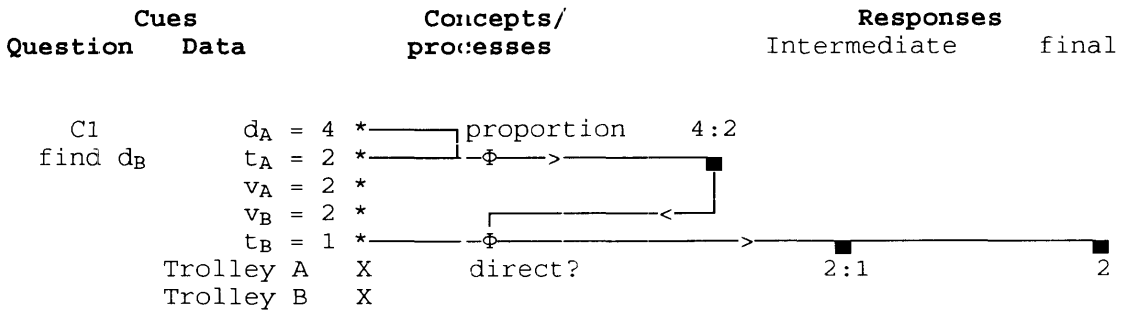


Figure 8.16 Map of relational first cycle concrete symbolic mode using direct variation.

Second cycle

Unistructural

The unistructural level response given below is based on Question C3 from a Year 12 student.

Ans: *It will go to 8 cm.*
Exp: *If it goes twice as fast, it will go twice as far in the same time.*

This student has paid attention to all three variables and their interrelationships. He has seen that it is important that the time remain constant if the direct variation of distance with speed is to hold. Such a response shows an overview of the correct relationships between the variables, and uses a suitable strategy to reach a conclusion. This indicates a second cycle response. The response, however, does not employ the formula nor does it use a calculation resembling the formula, this response is coded as unistructural. Figure 8.17 shows the associated map for this response.

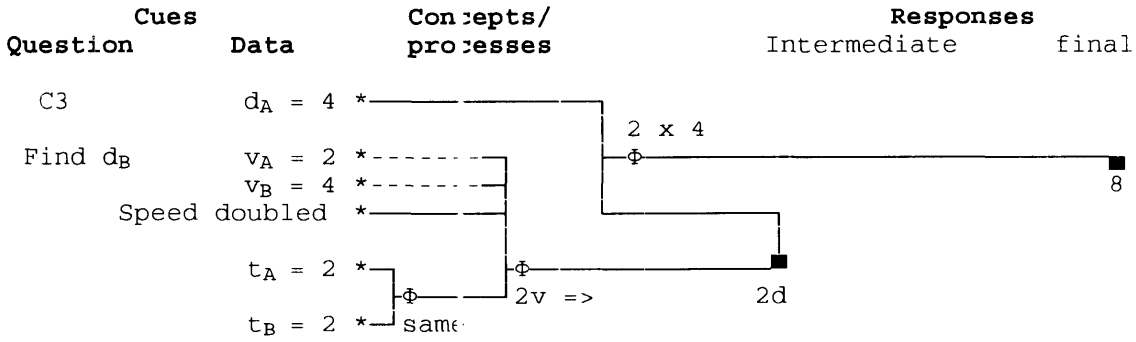


Figure 8.17 Map of second cycle unistructural concrete symbolic mode. Direct variation with condition stated.

Multistructural

At this level students focused on Trolley B and employ a logical calculation to solve the problem. This calculation indicates a 'common sense' approach and does not show that the formula for speed has been used. The following example is a response to Question C2.

Ans: *4.*
Exp: *cause if B got to 4 in 1 sec then 4 ÷ 1 = 4. So speed is 4 per sec.*

This response shows the correct calculation and answer where the student has divided the distance by the time. There is no indication that this student has used the formula and she has not used it in any previous problem either. She has seen that Trolley B has travelled four units in one second and realised that this is the speed. The mapping diagram in Figure 8.18 illustrates this response with only one processing of data taking place, but a link occurs between the correct variables.

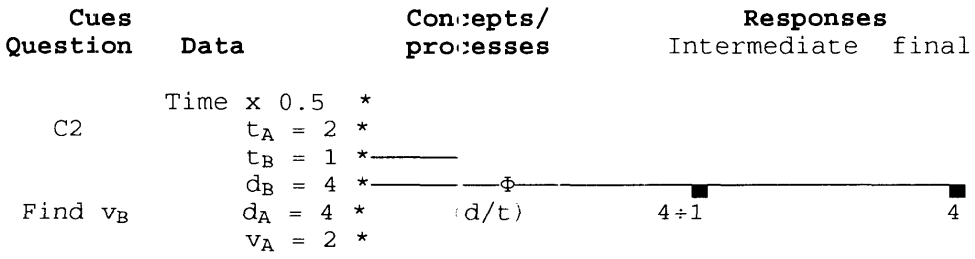


Figure 8.18 Map of multistructural second cycle response concrete symbolic mode. Using a calculation.

This mapping is very simple indicating the correct linking of variables (distance and time) to obtain the speed. This mapping is to be compared with the second cycle multistructural mapping of the Dual Focus Closed Comparison Question of the previous section. In that mapping, the response had to take into account both trolleys hence the necessity of comparing the two intermediate responses. For the Dual Comparison Open Focus Questions this comparison was not required if the student focused solely on Trolley B.

Relational

At the relational level the formula is used to solve the problem. Both the variable name and value are used. The name, in the form of a symbol and the

substitution of the value into the formula. The example is based on Question C7 from a Year 12 student.

Ans: *2 m/s.*
Exp: $d = 6$ $t = 3$ *velocity* $= 2$ *m/s*
$v = d/t$

This student has identified the variables that are necessary to solve the problem and then calculated the speed (*velocity*). All other cues and data are ignored. She has indicated that the formula was used to do the calculation. The question asked for the speed to be found and this has acted as a trigger to use the formula for a student who is familiar with the formula. Figure 8.19 shows a map of this response.

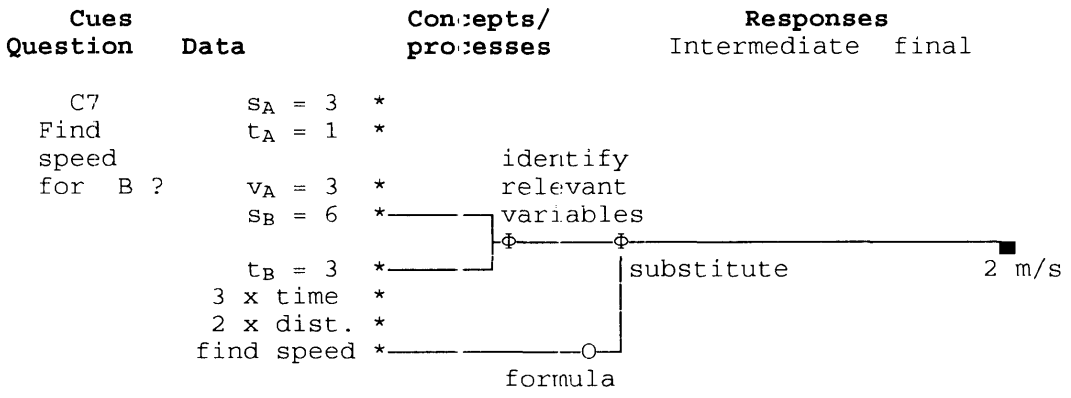


Figure 8.19 Map of relational second cycle concrete symbolic mode, using the formula.

SUMMARY

The ikonic mode is identified by students using the visual aspects of the diagram as an important part of the response. The use of words like 'fast' or 'in front' indicate a visual approach either from the diagram or in the 'minds eye'.

In the first cycle of the concrete symbolic mode the response is not focused on the diagram (though it could be used for support) but rather the data displayed are of importance. The major difference in the levels in this cycle is linking of the data and the processes used to obtain the required result. The emphasis is not on the number of data points that are used, though this is important. The second cycle is characterized by the use of mathematical procedures that link all the data in a coherent way. Table 8.3 provides a summary of the cycles and their levels.

Table 8.3
Levels of the SOLO Taxonomy applied to Dual Focus Open Speed Question

	IKONIC MODE	CONCRETE SYMBOLIC MODE	
LEVELS		**First Cycle**	**Second cycle**
Relational	Uses diagrams as basis for counting up units.	Links the relevant variables. Comparison not explained.	Use of formula $v = d/t$.
Multistructural	Comparison using 'faster' or 'front'	Refers to two different types of data but does not link them.	Relevant calculation that takes account of all variables.
Unistructural	Guess.	Comparison of data of same type. One major process	Links all variables with constraints

SOLO AND THE SINGLE FOCUS QUESTIONS.

The Numerical questions of the Single Focus Questions were designed so that the responses would require a simple calculation since the questions were given in a numerical form. The Variation questions, however, were given in a non-numerical form thus influencing responses away from simple arithmetic operations on the data. The analysis of these outcomes was discussed in some detail in Chapter 6. The outcomes of Chapter 5 reported a range of strategies and quality of responses that were used for these questions. Since there were no diagrams given with these questions there was no direct ikonic support. This, however, did not stop children from forming images of the situation in their minds especially if the question was related to familiar real-world contexts.

By their very nature, the Single Focus Questions do not have the same variety of levels in the responses as do the Dual Focus Questions. However, the responses do range from the ikonic to the relational in the concrete symbolic mode though not all levels are readily apparent.

The data provided in the Single Focus Questions are not as rich as that provided in the Dual Focus Questions. Thus the mapping diagrams for the Single Focus Questions are difficult to compare with the mappings of the Dual Focus Questions and therefore identification of levels is not straight-forward. It is an easier task to compare the characteristics of the evels as provided in Table 8.2, for the Dual Focus Questions, as a means of identifyir g levels in the Single Focus responses.

To illustrate responses from the Single Focus Questions, examples are considered from responses to Questions N3 and V1.

Questions N3 and V1 as given to students in the test to all Year levels are:

Question N3

A boy on a pushbike rides at a speed of 20 km/hr, how many hours will it take him to ride 10 km?

Question V1

A car goes on two journeys at the same speed. The second journey was twice as far as the first. How much more time would the second journey take?

The important feature of Question N3 is that it requires an operation on the numbers by dividing the distance *10 km* by the speed *20 km/hr* giving the correct answer of half an hour or 30 minutes. However, it could be answered also by considering how far the bike went in one hour (*20 km*) and then deducing that the time taken for 10 km is half an hour. This latter approach uses a direct variation strategy, that is, half the distance implies half the time for the same speed.

Question V1 requires that students recognise direct variation between the two variables, that is, double the distance implies double the time for the same speed.

IKONIC

The ikonic mode is characterised by visual images being created in students' minds that are reflected in the type of response they give. They visualise the context of the problem and work within this context.

Unistructural

The first example is a response to Question N3.

Ans: *2 h.*
Exp: *Guessed.*

This answer is incorrect with the student indicating that she guessed the answer. The answer she gave could have been obtained by dividing the *'20 km/hr'* by the *'10 km'*. Since the response does not indicate this, it appears that the student has arrived at a result intuitively.

Multistructural

At this level the response indicates an important aspect of the problem based on a 'visual' comparison. In the following example the student realised that the first journey was shorter than the second journey in Question V1.

Ans: *twice.*
Exp: *The first journey was short.*

Relational

Table 8.2 characterises the ikonic relational level with responses that refer to 'fast' or 'front' with some use of the data provided, that is, there is high ikonic reasoning with some use of data. For the Single Focus Questions no responses of this type were found for the students used in this investigation.

CONCRETE SYMBOLIC

First cycle

The first cycle is characterised by students' responses focusing on relevant data and processing that data.

Unistructural

The following example is a response to Question N3:

Ans: *30 h* [the zero in the 30 was crossed out].
Exp: *I took the 20 km and added it to the 10 km.*

This student has taken the two data points of the same 'type' and processed them using one incorrect arithmetical operation. He has closed on an operation without due regard to the problem. In this case, however, the student has realised that the answer was intuitively wrong and simply removed the zero to make it more appropriate. Using the characteristics of levels of responses as provided in Table 8.2 then this response is in the Unistructural level since the same type of variable (distance) was used in the explanation. Figure 8.20 shows a mapping of this response showing the one incorrect processing of the data.

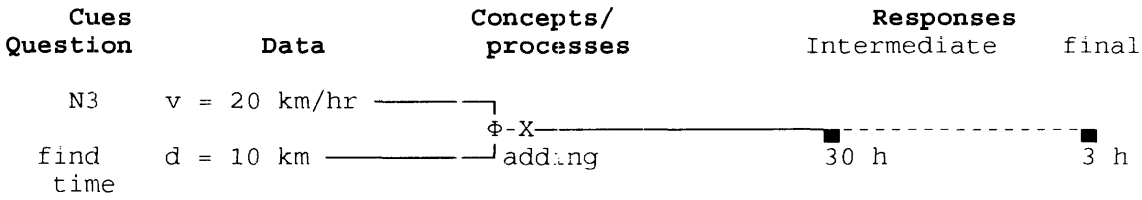


Figure 8.20 Map of Unistructural first cycle concrete symbolic mode using adding, inappropriate operation.

This mapping reflects a typical unistructural mapping when it is compared to Figure 8.5.

Multistructural

A multistructural response for the Single Focus Questions is not likely to occur since such a response requires the student to do two or more processes on different

types of data. The Single Focus Questions did not lend themselves to such comparisons.

Relational

The relational level is characterised by responses including the relevant variables linked in an appropriate way. The example below is a response to Question N3.

Ans: *30 mins.*
Exp: *60 mins = 20*
30 mins = 10.

This student appears to have used a proportion type response to solve the problem to obtain the correct answer. The relevant variables have been identified, but the strategy to reach the result has not been specifically supplied. Responses such as this, which link the data correctly, indicate a relational level response. A mapping of this response is provided in Figure 8.21.

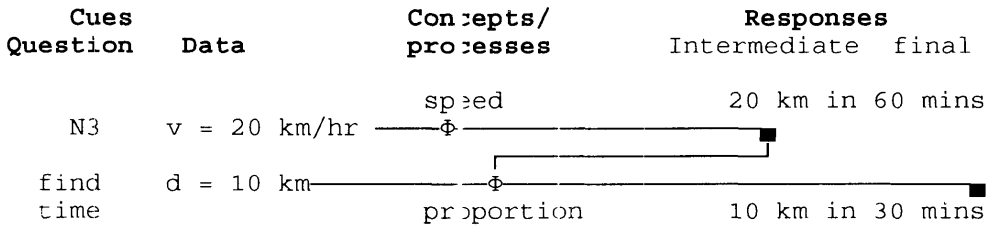


Figure 8.21 Map of relational first cycle response in the concrete symbolic mode, linking data.

This mapping does not have all the features of the first cycle relational mappings for the Dual Focus Questions but it does have the essential characteristics of this level, that is, linking speed and distance in an appropriate way.

Second cycle

The responses in the second cycle provide an overview of the whole problem with all variables been taken into account, but their is sophisticated use of how the variables are related. The examples provided in this cycle are all taken from Question V1.

Unistructural

Ans: *Twice the time.*
Exp: *Using direct proportion, if you double distance with constant speed then the time must increase by* 2x.

This response states the strategy used to double the time and links all the variables appropriately including the fact that for this to occur the speed must remain constant. The mapping of this response is provided in Figure 8.22.

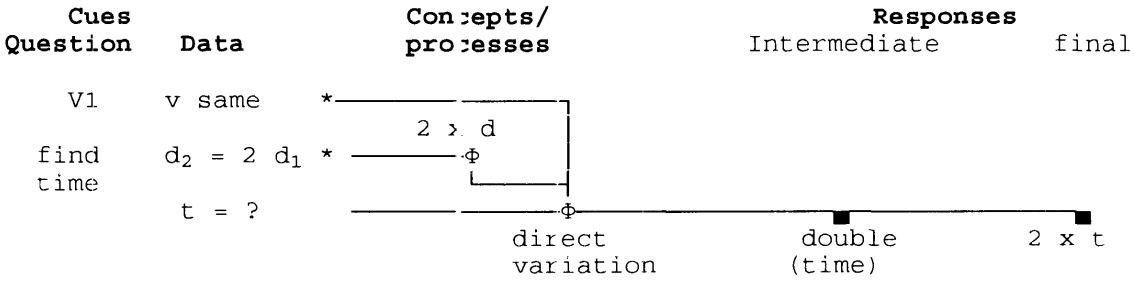


Figure 8.22 Map of unistructural second cycle concrete symbolic mode using direct variation taking into account all the variables.

Multistructural

Again this level is not easily recognisable due to the nature of the Questions.

Relational

When the formula is used to solve the problem this is characteristic of a relational response. In this problem, however, the formula can not be used in its defined state, and must be manipu ated or rearranged to solve the problem.

Ans: *It will take as twice as long.*
Exp: j_1 *distance* x 2 = j_2 *distance*

$$Speed = d/t \Rightarrow t = d/speed$$

$$j_1 t = \frac{j_2/2}{s} = \frac{j_2}{2s}$$

$$j_2 t = \frac{j_2}{s} = j_2/s.$$

This student has an overview of the whole problem and can use the formula and rearrange it to suit the question. (Note: the "j" in the explanation refers to journey). Figure 8.23 shows the mapping of this response.

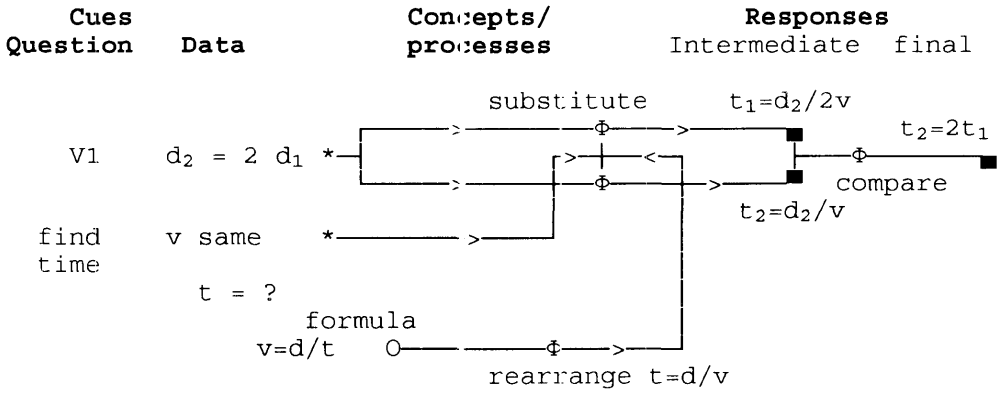


Figure 8.23 Map of relational second cycle concrete symbolic mode, using the formula.

SUMMARY

The responses to the Single Focus Questions did not provide all the levels, as obtained for the Dual Focus Questions. However, the levels that did occur ranged from the ikonic to the higher levels of the concrete symbolic mode. The mapping diagrams of these responses at the different levels did not have the same detailed structure as the Dual Focus Questions and this is expected due to the simple nature of the questions. Using the general characteristics that were developed for the levels of the Dual Focus Questions the responses to the Single Focus Questions could be placed within this framework.

DISCUSSION

This chapter has successfully interpreted the outcomes within a post-Piagetian framework, the SOLO Taxonomy, using mapping diagrams to illustrate the structure of the responses. It is the purpose of this section to discuss the implications of these outcomes in relation to the SOLO Taxonomy. A number of features stand out, but in particular three are considered. First, an overview of the structure of the mapping diagrams in relation to the modes and cycles within the SOLO Taxonomy. Second, a summary of the model presented in this chapter is presented. This is compared to the strategies model developed in Chapter 7. Finally, a suggested modification is given to the SOLO model presented by Campbell, Watson and Collis (1992).

MAPPINGS AND LEVELS

Mapping diagrams have been used to illustrate the levels of students' responses to questions on speed in the Single and Dual Focus Questions. The structure of these mappings has been significant in identifying the levels of the responses.

Table 8.4 provides a summary overview of the mapping diagrams to illustrate the structure for the different levels as used in this chapter. The mappings in this table represent a combination of the mapping diagrams from all three types of questions. These structures may therefore be more general than just restricting them to problems dealing with distance, time and speed. The variables given on these mappings may well represent other simple relationships that occur for other variables.

The characteristic feature of the mapping in the ikonic mode is that the response is related to the visual aspects of the diagram (X) and thus the mapping does not begin on the numerical data cues (*) provided in the problem.

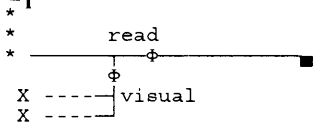
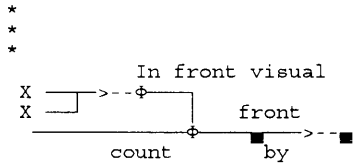
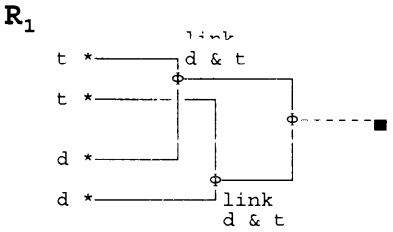
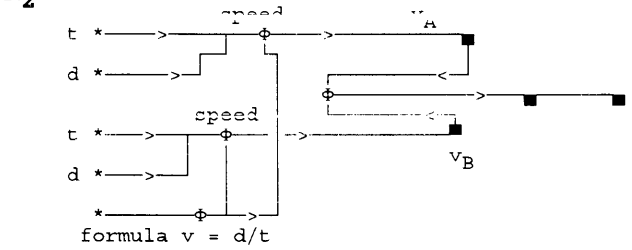
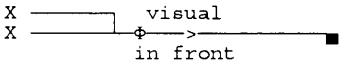
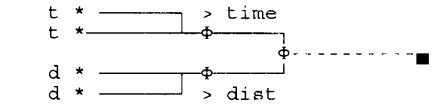
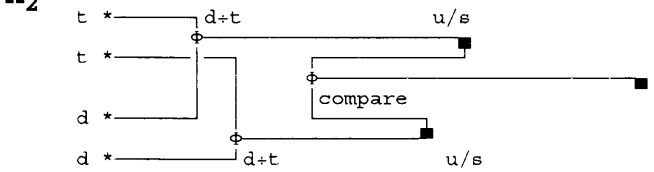
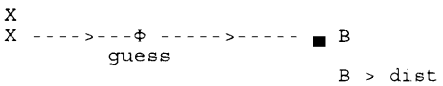
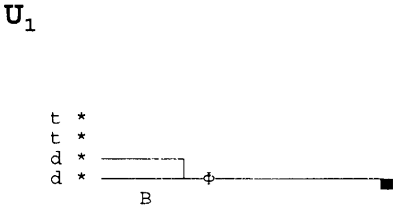
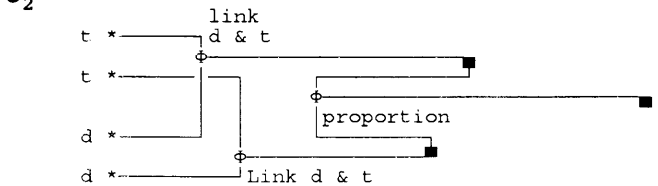
As the level of response increases, then the mapping diagrams indicate a more complex processing until some of the data is used as a transition is made to the next mode, that is, there is a linking of visual data to numerical data starting to take place.

The first cycle in the concrete symbolic mode shows mappings that process the numerical data in increasingly sophisticated ways. The unistructural level shows only one process (Φ) occurring on the same type of data. When this process is repeated with different pairs of data, but not cross linking the variables, the mapping indicates multistructural responses. A relational mapping indicates a cross-linking of the different variables in an appropriate way to arrive at an answer but there is no explanation as to how the comparison is made between the links.

The second cycle of the concrete symbolic mode takes the same initial structure of the relational level of the first cycle, but continues to take the intermediate results of the comparison of the variables and processes those results before reaching the final result. The closer the processing on the intermediate results is to using the formula the higher up the levels is the response.

Thus a significant outcome of this research has been to highlight the mapping diagrams as means of clarifying the level of a response through the structural pattern of the mapping. An important consequence of this is that the strategy chosen by the student is related to the type of mapping and hence the level of the response.

Table 8.4 Overview of mapping structure

Ikonic	Concrete Symbolic Mode	
	First Cycle	Second cycle
Transitional T_i		
Relational R_i		 
Multistructural M_i		 
Unistructural U_i		 

COMPARISON OF THE SPEED RESPONSE MODEL (SRM) AND THE STRATEGIES MODEL

This chapter has presented a more detailed model for the SOLO Taxonomy than originally given by Biggs and Collis (1982). Their emphasis was on the modes of development and a cycle of growth that occurs within each mode. For a particular mode the Transitional level reaches into the following mode while the prestructural level comes from the preceding mode. In this chapter two modes have been identified and it has been established that within the concrete symbolic mode two cycles occur with regard to students' understanding of speed, this model is referred to as the Speed Response Model (SRM). In Chapter 7, four levels of students' strategies were identified for solving the problems. The levels that make up this model is referred to as the Strategies Model. It is of interest to consider if relationships exist between the levels found in the Strategies Model and the cycles and levels within the SRM provided in this chapter. This comparison is made using the concept of speed as a basis.

For ease of reference the levels in the SRM are given the first letter of their level and a subscript to indicate the cycle or mode. For example, U_1 is the first cycle unistructural level in concrete symbolic mode and U_i is the unistructural level in the ikonic mode.

Those responses that use strategies which do not incorporate the relevant variables and rely strongly on visual cues are characteristic of Level 1 of the Strategies Model. They are reflected in the SRM in the first two levels of the ikonic mode, U_i and M_i and maybe extends into R_i.

At Level 2, of the Strategies Model, the response indicates a one-variable strategy. In the SRM this is similar to the relational level in ikonic mode R_i and the unistructural level of the first cycle U_1. In both these levels the response refers to one data point or process. In the case of R_i, the data is referred to, but not used in a significant way. The student relies on the visual impact of the diagram to reach a conclusion rather than the data provided.

Table 8.5
Comparison between Speed Response Model (SRM) and the Strategies Model

Mode cycle	SRM Levels	Characteristic of Response	Strategies Model Chapter 7	Characteristics of Strategies and examples
	P	Can not solve question/ no response / irrelevant response.		
Ikonic Mode	U_i	The student provides an answer Explanation is a guess or obvious.	Level 1	Strategy selected that does not use appropriate variables.
	M_i	Refers to two aspects of problem or to the two trolleys with a visual link, e.g. Faster or front one.		E.g. "A is in front".
	R_i	Visual focus by counting units on diagram. Still uses 'fast' or 'front one'		A one-variable strategy is chosen
	T_i	Refers to a given data point, visual cues predominate.	Level 2	E.g. "A went bigger distance".
Concrete Symbolic Mode First Cycle	U_1	One process on basis of one type of data, e.g., distance or time simple arithmetic operations.		
	M_1	Two processes done on different types of data e.g., distance and time, but does not link them.		A two-variable strategy is selected.
	R_1	An overview of problem, all relevant variables considered with correct links, but no detail provided on processes used to arrive at final result.	Level 3	E.g. "A went further than B in less time".
Concrete Symbolic Mode Second cycle	U_2	An overview of problem with all variables linked correctly and an appropriate strategy provided to arrive at final result. e.g uses ratio, proportion and variation type strategies.		A two-variable strategy is selected with an overview. E.g. 1. It will go twice the distance in twice the time if the speed the same.
	M_2	Systematically uses a calculation to find speed but does not explicitly use formula.	Level 4	E.g. 2. $v = d/t \Rightarrow d = vt$ $\Rightarrow d = 2 \times 3 = 6m.$
	R_2	Uses the formula $v = d/t$ to solve the problem.		

In the Strategies Model, a Level 3 response uses a two-variable strategy to solve the problem but does not provide an overview of the whole problem. This level resembles the multistructural and relational level of the first cycle, M_1 and R_1. At these levels the student attempts to link data but does not provide an explanation of how any comparisons are made. In the case of M_1, however, the student does not cross link the variables, but links similar types of data.

Level 4 of the Strategies Model includes those strategies that take into account all the variables, and indicates an overall view of the problem that links the variables to speed. Such responses correspond closely to the second cycle U_2, M_2 and R_2 of the SRM where responses take into account all the variables and use the formula or a calculation like the formula. The SRM, however, provides finer detail across the two variable strategies of Level 3 and 4 in the Strategies Model. Responses range from those that do not link the two variables through to the use of the formula. Table 8.5 provides an overview of the relationships between the two Models. This table also provides a summary of the characteristics of each level within the SRM and the Strategies Model.

A MODIFIED MODEL FOR SOLO

The studies of Campbell, Watson and Collis (1992), Levins (1992), and Levins and Pegg (1993a, 1993b) were referred to in Chapter 2, as suggesting that cycles do occur within modes. Campbell et al. noted that responses at the relational level of one cycle (R_1) were identical to the responses at the unistructural level of the next cycle (U_2) (see Figure 2.3). Levins and Pegg questioned whether responses categorised at these levels were equivalent or different.

The preceding discussion on the speed problems indicates that there is a difference between R_1 and U_2, though there is a link between these cycles where the relational level of one cycle matches into the unistructural level of the next cycle. A new model must also show the possibility of parallel development, at least in the first cycle where students can respond either using variable names or values. The issue of responses referring to variable value or variable name is discussed in Chapter 6.

Figure 8.24 provides a model of intra modal development that is consistent with the discussion of this chapter. This model shows that the unistructural mode of a

cycle is not equivalent to the relational mode of the preceding cycle. The first cycle also includes the two streams that have been identified in responses where students refer to either the variable names (VN) or the variable value (VV).

KEY
U Unistructural level of target mode
M Multistructural level of target mode
R Relational level of target mode
1, 2, 3 the number of cycle

R_p Relational level of preceding mode
VN Variable name
VV Variable value

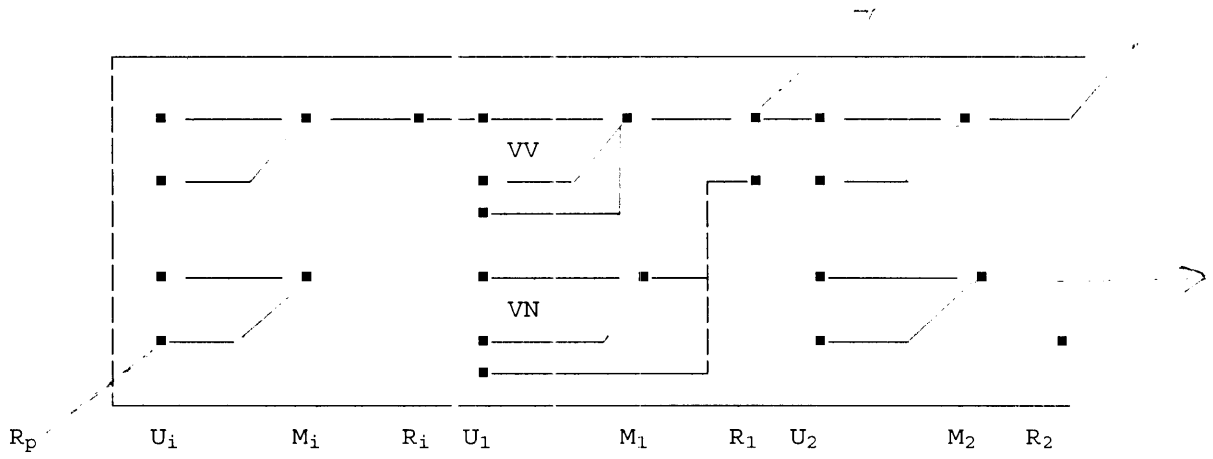


Figure 8.24 A model for intra-modal development consistent with outcomes in this chapter. (Adapted from Campbell, Watson and Collis (1992), see Figure 2.3.)

CONCLUSION

The discussion in this chapter has addressed Research Questions 13, 14 and 15. The SOLO Taxonomy does provide a suitable framework within which to understand responses of students regarding speed (Research Question 13). In particular, the mapping diagrams give insight to the structure of responses and the levels that they occur within the SOLO Taxonomy. The manner in which the processes are used in solving problems are indicative of the level of response. The types of processes that are used are related to the type of strategy that students employ. Thus, the level of the response is related to the type of strategy that students use.

Different aspects of the SOLO Taxonomy are brought out in this study (Research Question 14). In particular, the tables in this chapter indicate that two modes are present in the SOLO taxonomy for the concept of speed as tested by these questions. The ikonic mode with one cycle and the concrete symbolic mode with at least two cycles. The unistructural, multistructural and relational levels occur in each cycle. The ikonic mode has responses that focus on the visual aspects of the problem while the concrete symbolic mode has responses that are related to the data. Within the first cycle, two different streams are identified in which a student can respond, these are: the variable name and values of the variables. It is only in the second cycle that students employ mathematically consistent strategies.

The presentation of a refined model for the concrete symbolic mode of the SOLO taxonomy as presented in Figure 8.24 addresses Research Question 15. The major differences are the increased detail provided for the cycles and levels within each cycle, more detail about the concept of speed than has been previously reported and in this model, compared to previously reported models, the relational level of one cycle is not identical to the unistructural level of the next cycle.

Table 8.5 provides an overview of the comparisons between the Strategies Model, formulated in Chapter 7, and that of the SRM developed in this chapter. The original levels in the concrete symbolic mode, as given by Biggs and Collis (1982), do not have a one-to-one correspondence with the SRM, because of the two cycles that are present in the SRM compared to only one cycle in the 1982 Model. However, the 1982 Model can be seen within the SRM and has a relationship to the Strategies model. Thus the general features of the original SOLO model still provide a basis for interpretation of the levels within the cycles in the SRM.

The SRM, using the ideas of the SOLO Taxonomy, provides a very useful tool for examining the responses of students with regard to speed. In the next chapter this model is used as a suitable post-Piagetian framework to identify the levels of responses of students while examining the profiles of six students on their responses to the speed questions.