CHAPTER 4

THE RESEARCH DESIGN

Do not worry about tomorrow, for tomorrow will worry about itself.
Each day has enough trouble of its own.
Matt. 7:21 NIV Bible

The previous chapter clarified several important issues which helped elaborate the research themes identified in Chapter 2. Out of this work, eighteen specific research questions were identified related to three themes. Before proceeding to answer these questions, this chapter describes the research design and methodology of the main study. In particular, consideration is given to the type of study that is undertaken, the context, participants, questions to include in the test instruments, and the interview protocol. Mapping diagrams are introduced as they are a means of data analysis.

THE QUALITATIVE RESEARCH PARADIGM

Traditional research has often been associated with quantitative studies that aim to be objective in their findings and methodology. In such research an hypothesis is tested and it is important that the study is conducted so that the population is sampled carefully. The observations that are made are converted into discrete units for comparison with other units by using some form of statistical data analysis. When such a study is done carefully, the issues of validity and reliability are addressed both internally and externally. What is not usually examined in this type of research, however, is the ground work that is associated with the decision making processes in selecting an hypothesis that is to be tested.

In contrast to the quantitative approach to research is the phenomenological approach which focuses "on understanding the meaning of events for the persons being studied" (Maykut & Morehouse 1994, p. 3). One important area that forms part of the phenomenological approach is that of qualitative research. In qualitative research the focus is not on testing a hypothesis, but rather considering a certain part of human understandings, to observe what takes place and to find patterns and underlying structures that give meaning to what is observed. From these findings or outcomes, hypotheses can be made and quantitative research may then take place to test them.

The study described in this thesis is qualitative in nature. It seeks to study the meaning that students give to the concept of speed and to identify strategies that students use to solve questions concerning speed. From the observations, using written tests and interviews, patterns and structure are sought to give meaning to students' perceived notions on speed.

Maykut and Morehouse (1994) identified eight characteristics of qualitative research (pp. 43-47). These characteristics are:

1. An exploratory and descriptive focus: "The outcomes of any of these studies is not the generalisation of results, but a deeper understanding of experience from the perspectives of participants selected for study" (p. 44).

2. Emergent design: As the study proceeds, decisions are made concerning who and what will be considered in the study. This is to be compared with quantitative research where the random sampling of a population is usually fixed before the sample is examined. In qualitative research, however, such decisions are based on earlier observations and analysis that provides a deeper understanding of the issue being studied.

3. A purposive sample: In contrast to quantitative research which attempts to "achieve variation through the use of random selection and large size" (p. 45), qualitative research carefully selects participants to make sure there is variability of outcomes. The sample chosen is relatively small and can be modified as the study progresses.

4. Data collection in the natural setting: There is an interest in "understanding people's experience in context" (p. 45). It is important to meet subjects in their natural setting so that outcomes are more likely to reflect their real ideas.

5. Emphasis on 'human-as-instrument': The researcher has an important role in qualitative research as the culler and interpreter of data, since data are in the form of other peoples words and actions. The researchers background, education, training and personal experience are relevant to how information is selected and analysed.

6. Qualitative methods of data collection: "The data of qualitative inquiry is most often people's words and actions, and thus requires methods that allow the researcher to capture language and behavior" (p. 46). This can be done by observation, questionnaires and interviews.

7. Early and on going inductive analysis: As the study proceeds, analysis takes place that provides opportunity to fine-tune the study into areas that illuminate the phenomena of interest. "*What is important is not predetermined by the researcher*" (italics in the original) (p. 46).

8. A case study approach to reporting research outcomes: "The results of a qualitative research study are most effectively presented within a rich narrative, sometimes referred to as a case study" (p. 47). Case studies provide readers with rich descriptions and information from which they can determine "whether the findings of the study possibly apply to other people or settings" (p. 47).

A major issue in reporting a qualitative study is to make provision for 'trustworthiness'. This idea of trustworthiness was identified by Lincoln and Guba (1985) as a way of referring to the "believability of a researcher's findings" (Maykut & Morehouse 1994). By addressing each of the above characteristics for a qualitative study the trustworthiness of the findings increase. In the sections that follow, each of these characteristics is attended to in order to provide a broader phemenological base for the study.

THE CONTEXT

The research was carried out in two schools (a High School and a Primary School) on the fringes of the City of Brisbane, Queensland. In the immediate region surrounding these schools were five other large state high schools and three independent high schools. The children, who attend the schools used in the research, come from a community that had 20.67% unemployment rate which is the highest in the State of Queensland (the State Mean was 11.34%) and of a low socioeconomic level. This area also had lowest percentage of Degrees/Diplomas in the state with 4.39% (State Mean 12.51%) (from Statistical Profiles of Queensland State Electoral Districts for August 1991, Electoral Commission Queensland 1994). The Queensland Department of Education provided special funding to these schools to enable those that were disadvantaged to have access to 'quality education' as provided by all government high schools in the State. This funding provided extra teachers and operating funds for a support centre at the school for students who had difficulties in school related tasks.

The High School consisted of classes from Years 8 to 12 with an enrollment of approximately 1100 students taught by 80 teachers. The usual entry age to the High school was twelve-to-thirteen years of age and the average leaving age was approximately seventeen-to-eighteen years. A broad general education was offered to students with the variety of subjects increasing as they proceeded through the different Year levels. It was the stated aim of the school to "help each student gain success; that is to achieve to potential and to provide a preparation for employment and life" (Year 9 Course Selection booklet to parents and students, p. 1). In Year 8, all students were offered the same curriculum which covered the core academic areas such as Science, Mathematics, English, History, and Geography; the cultural areas in the arts, music, language; the practical areas such as Manual Arts, Home Science, Health and Physical Education, and typing skills.

In Years 9 and 10, students were required to do seven subjects:

1. English or Communication English
2. Mathematics
3. Science or Core Science
4. A Social Science subject - History, Geography or Citizenship Education

Students could then choose **three** elective subjects from the following: Business Principles, Typing, Speech and Drama, Art, Practical Art, Music, Music in Performance, Shop A, Shop B, Graphics, Home Economics, Health and Physical Education, Commercial Studies, French. In addition to these subjects all students are required to participate in Sport, Core Physical Education, Career Education, Religious Education, Parent-craft and Human Relationships Education.

In Queensland, the Board of Secondary School Studies (BOSSS) accredits and certifies subjects that have been studied for the Junior Certificate at the end of Year 10. Subjects known as Board Subjects have a syllabus and monitoring process operated by the BOSSS, these subjects appear on the Junior Certificate as Board Subjects. The school, however, offers its own subjects to cater for students who would not especially benefit from doing Board Subjects. Such subjects can be registered with the BOSSS and appear on the Junior Certificate as Board Registered School Subjects; these subjects are not monitored by the BOSSS, only the course outline being approved. Board Subjects are prerequisite for further study at Year 11 in a similar subject area. The High School used in the study offered four Board Registered School Subjects at Year 9 and 10 level. These were Communication English, Core Science, Practical Art, and Music in Performance.

The reasons that students continued to pursue more schooling in Years 11 and 12 were many and varied. Ideally, the students would want to gain an education that helps them in personal development and gives them better opportunities for employment as well as preparation for Tertiary Education, if that was required. In reality the motives did not always match these ideals. With the large rate of unemployment in the surrounding community, and the non-availability of unemployment benefits until they reached 18 years of age, a significant number of students appeared to come to school to claim the Austudy allowance of up to $170 a fortnight. Others come because they have to (parent pressure), and also as it is a general meeting place to see friends.

The school tried to cater for all students by offering 24 Board subjects of which five subjects are required for those wishing to enter tertiary institutions. In Years 11 and 12 the school requires that students choose six subjects. For those students who are not preparing for tertiary study, there are another thirteen subjects available that are Registered with the BOSSS. These subjects would appear on the Senior Certificate but could not be used for tertiary entrance requirements. For those students who plan to enter TAFE (Technical and Further Education) after leaving school, some subjects are linked to TAFE to be used for credit later on for TAFE Certificate courses. This gives some status to these subjects as leading to something specific for students after they leave school.

The High School did not practise streaming of students on a large scale. The only exception was one class of music students (those who learned a musical instrument) was created at Year 8 that then moved through to Year 10. These students were often more motivated than their peers and were encouraged and stimulated to work at a high level. All the other classes were a fairly heterogeneous mix of students both in intellectual abilities and in behaviour patterns.

The Primary school, a Year 3 to 7 school, was a feeder to the above High school. The classes were not streamed and the Year 6 class was selected by the school administration as a heterogeneous group, and with a teacher who was willing to cooperate with the research being carried out.

Constraints

More than half the students in the above schools did not value education. Those students who did value their education, had high aspirations but were not encouraged by their peers to strive for those aspirations. The term used by students for those who attempted to do any school work was 'stiffs', which referred to those students who sat up straight in class and took notice of what the teacher was saying and doing. This attitude to education also had an adverse effect on teachers and their teaching as well as students in their learning. Within this climate it was difficult to appeal to students to undertake tests and interviews which did not appear relevant to their needs or desires. To ask students to remain in the classroom during their own time was considered either a punishment or a means of obtaining more 'education' than was required and thus students involved became known as 'stiffs'. For this reason, all testing and interviews were done in regular school time during normal classes. Thus the students were kept in their 'natural settings'.

Since the research had to be carried out during regular school time, the teachers of the school had to be willing to give up their teaching time so that classes could be tested. Also, they had to be prepared to release students from regular class activities for interviewing. The willingness and help that was given by these teachers is again acknowledged.

Since time was a limiting factor, testing had to be kept to a minimum so as not to interrupt the teaching/learning process of normal school life. A test instrument was designed using the speed questions so that the test would take one class period for an average student to complete. The questions in the test included each of the different types of questions that were developed for the pilot study in such a way that most students, at each Year level, could attempt them with some success.

DESIGN OF THE STUDY

In planning the study, it was decided that only three Year Levels would be used, namely, Years 6, 9 and 12. This allowed a range of age groups to be tested and interviewed while restricting the number of students involved to a manageable level. A conscious purpose of the project was to obtain representative responses to test items and interviews, not to produce large quantities of similar data. Such purposive samples is one of the characteristics of typical qualitative research.

The speed test was administered to each group and took approximately thirty-to-forty minutes to complete. After an initial analysis of the responses to the test had been completed, approximately six students from each class were interviewed to clarify responses to the test, and to probe a little deeper into the explanations that were given. Each interview was taped and then transcribed for analysis. This fulfils partly the qualitative research characteristic that the study be emergent.

PARTICIPANTS

Year 6 Students

The Year 6 students consisted of a class of 25 students and were a mixed ability grouping of children. All students did the full range of the curriculum as stipulated by the Queensland Department of Education; no choices of subjects were provided.

Year 9 Students

In the High school there were approximately 250 students in Year 9. The students involved in the study were a heterogeneous group with varying school achievement levels. The actual subject that these students were involved in at the time of testing was Mathematics. This group of students was only together for Mathematics, since they split into other classes for different subjects. For interviews, the students were taken from other classes such as English, Art and Science.

Year 12 Students

At the high school there were approximately 200 Year 12 students. The students in the Year 12 class who did the speed tests all did Mathematics I which was one of the Academic Mathematics courses offered in Queensland as a Board subject. This course consisted of four Semester Units: Preparatory Mathematics, Algebra and Calculus I, Geometry (Analytical Geometry and Trigonometry) and Calculus II and finally Probability and Statistics. The first three of these units follow a fairly rigorous and traditional approach to Mathematics, and there was a large dropout of students from this class before they reached Year 12. There were two Mathematics I classes at Year 12 level with seventeen and eight students, respectively. Eleven of these students also did Mathematics II which consisted of the following units: Matrices and Vectors, Mechanics, Computer Maths, and Geometry and Calculus 3. In the Mechanics unit of Mathematics II, students had a full semester exposure to the kinematic equations and should have been well acquainted with the ideas of speed, velocity and acceleration. Ten of the Mathematics I students also did Physics (of which only one did not do Mathematics II) thus giving them extra experience and practice with concepts of kinematics and dynamics. It was therefore anticipated that this group of Year 12 students would perform better than their peers in academic work.

Table 4.1 provides a summary of the number of students (male and female) who did the tests and participated in the interviews.

Table 4.1
Number of students who did the test and were interviewed

Year level	Gender	Written test	Interviews
6	M	15	3
	F	10	3
Average age 11/8	Total	25	6
9	M	9	2
	F	11	3
Average age 14/6	Total	20	5
12	M	9	3
	F	8	2
Average age 17/1	Total	17	5
	Totals	62	16

TESTING AND INTERVIEWS

THE TEST INSTRUMENT

The final test instruments were made from the questions that were developed for the pilot study, and these have been described in Chapter 3.

The first question in the test asked students to state what they understood by the meaning of the word 'speed'. The remainder of the test instrument was divided into three parts: A, B and C. Part A included the Single Focus Questions of the numerical type (labeled N) and variation type (labeled V). The Part B questions comprised the Dual Focus Closed Comparison Questions (labeled B) with the Dual Focus Open Comparison Questions in Part C (labeled C).

For each question, space was provided for students to place their answer, and additional space was given for them to write their explanation.

Table 4.2 shows the speed questions that were used for each Year level. These questions were chosen in an attempt to try to suit the ability of each Year level in obtaining a range of responses. Also, it was desirable to have questions that overlapped each Year level so that comparisons could be made.

Table 4.2
Speed Questions selected for each Year Level

Year 6	Year 9	Year 12
What is speed?	What is speed?	What is speed?
N1 N3 V1 V2 - -	N1 N3 V1 - V3 -	- N3 V1 - V3 V5
B1 B2 B3 - B5 B8 B10	B1 - B3 - B5 B8 B10	B1 - - B4 B5 B8 B10
C1 C2 - - C7 - -	C1 C2 - - C7 C8 C9	C1 - C3 C5 C7 C8 C9

The survey was designed so that each student's response could be placed on one sheet of paper that could be detached from the rest of the questions. Placing the answers on an answer sheet also had the additional advantage of reducing the amount of paper needed.

After the results of the questionnaires were partly analysed, six students from each year level were selected for interviews. These students were chosen so that two represented those who produced high quality and correct responses, another two represented those who were able to answer fully some of the questions, and the remaining two were chosen from those who appeared to have difficulty with many of the questions. In this way, it was hoped to validate the written responses of students, and to question students more closely as to why they gave the responses they did. This latter point would assist in ascertaining more about their reasoning processes.

INTERVIEW PROTOCOL

The protocol of the interview, provided in Table 4.3, was flexible enough to allow students freedom in giving and explaining their own train of thought, and also permitted the interviewer latitude in following any other interesting points that a particular student might mention. It was anticipated that the investigator would probe any issues that were relevant to the topic that might lead to a better understanding by the investigator and/or the student of the concepts involved.

In most cases the questions used in the interview were the same as those on the test paper. If the student seemed to have full understanding, then the investigator was free to proceed to the next section. Not all questions had to be asked of every student. Table 4.3 provides an outline of the interview protocol, a protocol based on the work of Newman (1977).

The first students interviewed were from Year 12. They had limited time, due to study for final exams after which they would depart from school, four or five weeks before the other Year levels.

It was extremely difficult to arrange appointments with these students for interviews, some forgot and others had school appointments which conflicted with the set time. Eventually, the sixth student interview was abandoned after five appointments were not kept.

Table 4.3 Outline of interview protocol

Instruction	Examples of Question asked by investigator	Process
1. Probe students' understanding of the word speed.	"Can you give a better meaning than the one on the test paper?" "Could you write it mathematically?" "Where have you heard these words before?"	*Amplifying* *Context*
2. The investigator will use questions from the test instrument that the student appeared to have difficulty with or where the student has not given an adequate explanation.	"How will you go about doing this question?" "Work at this question again, and think out loud what you are doing." "Show the steps."	*Strategy* *Process* *Encoding*
3. If student has no problems then move to other questions of a different type.	"Can you check your result?"	*Verification*
4. If student has some problem then explore the question with student. (give help if advisable)	"Read the instructions out loud." "What does the diagram tell you?" "What is being asked for?" "What does it mean?" "Now solve it and show steps." "Now try this one."	*Reading* *Recognition* *Comprehension* *Comprehension* *Process and encoding* *Verification*
5. If students seems to apply inconsistent methods across problems then confront the student with these different methods and see how he/she resolves the conflict.	"Note how you did this previous question: why have you changed your method on this question?"	*Conflict resolution*
6. The above procedure can be repeated until no progress is being made or most of the test questions have been used.	"Can you generalise what you are doing?"	*Strategy*

The Year 9 students were the next students to be interviewed. Again, arranging times and keeping appointments was difficult and eventually only five of these students were interviewed. In most of these interviews questions that were on the test paper were used. The exception was when the interviewer wanted to pursue a certain line of questioning but had exhausted the relevant questions from the test. In these cases, other questions, developed in the pilot study, were used.

The Year 6 students were the last to be interviewed. All six students were interviewed since they were always in the same place with the same teacher, and therefore easy to locate. The interviews followed closely the protocol using the questions in the written test.

The dynamics of the interview did not always allow the interviewer to ask all the questions as outlined in the protocol but, in general, the ideas were followed through.

It was realised that no test instrument would provide information regarding the actual cognitive processes of the student. Since both the answer and the explanation were to be written, students had to formulate their thinking into a language using symbols to try and express their thoughts. Also, in the interviews there was a potential that what students said they had done did not always equate to what actually occurred. This meant that, the response might well "represent the subject's rationalisations of the cognitive processes (i.e., the thoughts he or she feels must or ought to have taken place during performance)" (Rowe 1991, p. 15). Such limitations would need to be kept in mind when analysing the outcomes.

DATA ANALYSIS TOOLS

In dealing with the data in this study, a range of tools were used to organise and analyse the data. In particular the meaning of Quality of Response and the role of Mapping Diagrams needs some clarification.

Meaning of Quality of Response

The term 'Quality of Response' has been referred to in Chapter 2 of this thesis, where other researchers have suggested a scale for measuring the quality in the responses of students (Renner, 1979; Collis and Biggs, 1979). Before presenting the outcomes of the Part A, B, and C questions it is necessary to give a brief explanation of what is meant by the term Quality of Response. The Quality of Response covers two dimensions: Fullness of response and Strategy implemented.

In the Speed test items, students had to give an answer and an explanation of how they arrived at that answer. Students' answers were either correct or incorrect and the explanations were categorised as full, partial, unknown or inappropriate. A full response produced an explanation that could be easily followed and usually lead to a correct solution (these were coded in the tables as CF and IF for Correct Full and Incorrect Full responses, respectively). A partial response had only a partial explanation of how the answer was arrived at (coded CP and IP for Correct Partial and Incorrect Partial, respectively). Where it was unknown how students arrived at their answer or no explanation was given then this was coded as CU for Correct Unknown and IU for Incorrect Unknown. If no response was given this was also coded as Incorrect Unknown (IU). Some responses had the correct answer but the explanation did not lead to that answer, such responses were coded as Correct Inappropriate (CI) but were included with the incorrect responses since the student did not appear to have a true understanding of the problem.

The Student Difficulty Ratio (SDR) is introduced as a means of ranking the difficulty that students have with questions (see Appendix F where it was used in the Pilot Study). The SDR is given at the bottom of each Fullness of Response Table for each question in Chapter 5. The weights given to different quality responses were CF = 2, CP = 1.5, CU = 1, CI and IF = 0.5, all other incorrect responses were weighted with zero. The SDR was calculated by finding the mean of all weightings for each response on a particular question.

An SDR score of two suggests a correct and full response for all students on that question. An SDR greater than one suggests that responses were of a reasonable standard. An SDR below one indicates that students had some difficulty with that question.

The Quality of Response is further amplified by considering the strategies that students used on different items. Each response was coded according to the strategy that was employed.

Mapping Diagrams

An analytical tool that is used to explore student responses that has been introduced by Biggs and Collis (1982) is the Mapping Diagram. They have been used by Watson and Mulligan (1990) and, Collis and Watson (1991), to analyse the structure of mathematical responses. These mapping diagrams are employed in this present study to elucidate the response structure of student answers.

In this thesis the symbols used are:

x represents inappropriate or incorrect data: In this situation, the student focuses on data that are irrelevant to solving the particular problem. This data may be incorporated in the given problem in order to make it a real-life context or it may be taken from a diagram that is provided.

* data given with potential to cue a response: These data are relevant to solving the problem and may lead to a correct solution if processed correctly.

Φ concepts, processes and/or strategies expected within the "Universe of discourse": The student picks up correct data and processes them. To do this they have used a concept that they believe will lead to a solution.

o abstract concepts, processes and/or strategies within "Universe of discourse" but additional to those expected as part of understanding of the question. Some students are able to bring in outside information or processes from their past experience. This information or process may not have been explicitly implied in the problem, but it helps the student in providing an overview of the problem and clarifying the limitations and constraints.

■ responses - both intermediate and final: The final response is what students believe is the answer to the problem. In most problems, however, students will have produced intermediate responses upon which they will do more processing before reaching their final response.

(Note: these symbols are not all the same as that used by Biggs and Collis.)

This idea has been extended by Collis and Watson (1991) to indicate how the structure can be expanded to show the detail of the strategy for a given response. The lines show the connections between data, processes and responses. Such a structure is called a map of the response. Figure 4.1 illustrates how the symbols are used.

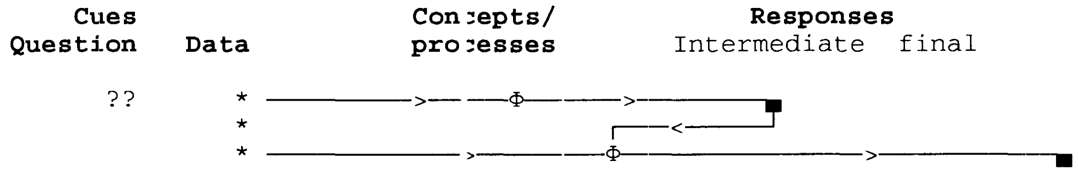

Figure 4.1 Illustration of mapping procedures

These mapping diagrams are used to illustrate and clarify levels of responses in later chapters.

CONCLUSION

To conclude this chapter it is important to address briefly those characteristics of qualitative research that have not yet been be referred to already. The issues that need more clarification are: emergent design, early and on going inductive analysis, a case study approach to reporting research outcomes, and emphasis on 'human-as-instrument'.

Emergent design: From the research design described previously it is apparent that this study is not a purely emergent one. For a study to be purely emergent then at every phase of the investigation the researcher keeps going back to the participants in an endeavour to clarify and expand on the findings that are occurring in the analysis. This study has some of the elements of the emergent study in that the trials and the pilot study described in Chapter 3 gave some indications of how to proceed and provided a set of research questions. Also, the students that were chosen for interviews were carefully chosen from the written test responses to give variability to the outcomes. Thus the study is not fully non-emergent, but lies somewhere between this and a purely emergent design.

Early and on going inductive analysis: The pilot study provided a few early outcomes so that some inductive analysis could take place. The initial analysis of the written responses, before undertaking the interviews, provided another stage for making decisions about how the interviews were conducted. As the interviews proceed it is possible for the investigator to fine tune the questions for better responses as patterns begin to emerge. It is also possible for the investigator to go back over the collected data, both written and transcribed interviews, to clarify patterns that emerge.

A case study approach to reporting research outcomes: One of the major purposes of a case study is to provide the reader with the opportunity to review the outcomes and judge whether they can be applied to other settings or people. In Chapter 9, six case studies are provided for this purpose and to furnish a method of seeing a student's profile in the light of the models that are developed.

Emphasis on 'human-as-instrument': The views and experiences of investigators are an important factor that contributes to how a qualitative study proceeds. Decisions need to be made on what is important and retained, and what is considered unimportant and culled. It is from their perspective that patterns emerge and underlying structures are considered.

The investigator of this study, on students conceptions of speed, has a strong background in pure and applied mathematics based on a very traditional education. The research that he has been involved with in the past was within a positivist paradigm using quantitative and investigative research in physics. He has taught mathematics and physics in the secondary school for over eighteen years and has been involved in developing programs in mathematics and physics education at the tertiary level for five years. With this background, the investigator brings experience concerning students and their learning experiences in the classroom as well as an academic understanding of content in mathematics and physics.

Summary

Clear attempts have been made to fit the main study within the characteristics of a qualitative research project, and it is important that the study be viewed within this paradigm. Since all the characteristics of a qualitative study have been addressed, the foundation has been set for a trustworthy study. To complement this 'provision for trustworthiness', a post-Piagetian framework is used as a basis for analysing the outcomes and searching for patterns and structure. A framework such as this, that has been used successfully in other studies, leads to some validity in the outcomes that are presented. Also, the use of mapping diagrams to highlight the structure of student responses adds more reliability, and thus more trustworthiness, to the study

The next chapter sets the scene by providing a detailed summary of students' attempts on all the test items. Also included are selected episodes of interviews to assist in clarification and exemplification of responses. This is then followed in later chapters with a deeper analysis of the outcomes as a means of addressing the research questions.

CHAPTER 5

THE OUTCOMES ON SPEED ITEMS

There was a young lady named bright,
Whose speed was faster than light,
She set out one day
In a relative way,
And returned home the previous night.
Arthur Henry Reginald Buller (1874-1944)

Introduction to Chapter

It is the purpose of this chapter to provide an overview of students' responses to the questions in the written Speed Test and in the interviews that were conducted with some students soon after the test. In particular, consideration is given to the following Research Questions:

Students' Descriptions of Speed

1. What are the main category groupings of students' descriptions of speed? Are these findings consistent with trends identified in the pilot study?
2. During the interviews what additional categories emerge when students are probed or prompted?
3. Are there broad bands of student descriptions consistent with Year groupings? If so, what are their characteristics?

Responses to Questions on Speed

4. What are the main identifiable features in students' performance on the speed test? How is this influenced by Year level and question type?
5. When students solve speed problems, what is the nature and consequences of the strategies employed to solve these questions?

6. How do the interviews concerning speed problems enhance understanding of strategies used by students?

The Speed Test was divided into four main sections and this mirrors the structure of this chapter. The first section considers the meaning students gave to the term speed. The second section, Part A, deals with Single Focus Questions that involved both Numerical and Variation type problems. The third and fourth sections of this chapter consider the Dual Focus Questions, consisting of the Closed Comparison Questions of Part B and the Open Comparison Questions of Part C, respectively. After each section the outcomes are discussed with examples from the interviews which provide richer information on responses to the questions. The insert sheet reproduces the speed questions from Appendix D that were used in the test. This is provided as a convenience to the reader when reading Chapters 5 to 9.

STUDENTS' DESCRIPTIONS OF SPEED

It is the purpose of this section to report the findings associated with students' responses on giving a description to the term speed. These outcomes are then used to answer Research Questions 1, 2 and 3 developed in the pilot study of Chapter 3 regarding the categories of descriptions that students gave concerning speed.

The categories identified in the main study were found to be the same as those in the pilot study. Each category A to G, is now described with examples from the written responses. When more than one category of response is provided by a student, the coding reflects the more sophisticated aspect of the response. For convenience, students were identified by a number. The first digit indicates the Year level for that student, '1' implies that the student is in Year 12, '9' for Year 9 and, similarly, a '6' for Year 6.

Category A

In Category A, the response is characterised by reference to speed as the quotient of distance and time or as the rate of change of speed with respect to time. In each case the response is similar to that of a formal definition of speed. The following example illustrates the way a Year 12 student responded with a Category A response.

Is how fast you go for a given time for a given distance $s = d/t$
Student 101 (Maths I and Physics)

This student was currently studying Physics and Mathematics I and thus had been taught and practised concepts related to speed and velocity. The student has given a written description of speed that starts by relating speed to the term fast (Category D) then moves through to relating speed to the two variables time and distance (Category B) and finishes with the formula (Category A), thus the student knows that speed is the quotient of distance and time.

Category B

A response that refers to speed being related to both distance and time but where there is no reference to an overall relationship is characteristic of Category B. The following examples illustrate typical Category B responses.

Means how fast or quick you are. The amount of time it takes to get from one point to another.
Student 912

Is the term used for the movement of an object relative to time and distance.
Student 108 (Maths I & II and Physics)

The first example is that of a Year 9 student who began his response by referring to speed as "fast" or "quick" (Category E), and then expanded on his answer, referring explicitly to time and implicitly to distance, i.e., going from "one point to another". Thus he has moved to a Category B response. The second example is from a Year 12 student, who was studying Maths I & II and Physics and who knew that speed was related to distance and time. He has given the additional fact that speed involves motion.

Category C

A Category C response is characterised by students referring to only one of the variables, either distance or time, without having any reference to the other variable. Only two Year 9 students responded in this way and both responses are given below.

Running in a short period of time e g. sprints
Student 913

A drug that makes you weid [sic] / *How fast you go our* [how] *quick* [time] *you move.*

Student 915

Both students referred to time but they did not mention distance. The words "running" and "fast" may imply distance but could also refer to speed. Similar usage of these words occurred in the interviews. In the second example, the student has referred initially to 'speed' as a drug' and then followed on with a more accepted meaning. Responses that referred to drugs occurred a number of times and were not always a 'smart' answer as this one might well have been. Such responses illustrate the changes that occur in our language which eventually can lead to confusion over accepted meanings of words.

Category D

Responses that did not refer to either distance or time seem to fall into another class of responses of which Category D is the first one. Responses in Category D referred to speed as related to an object that could go fast or slow. In this case, the object had motion of some type but this was not related to any particular characteristic. The examples given below come from Year 6, 9 and 12 students.

I think speed is how fast you travel i e. in a car or running.

Student 616

Speed means how fast you go and if you can go faster or slower.

Student 910

Speed is the term used to describe how fast or slow and [sic] object is moving e g. car travelling at 100 km/hr.

Student 106 (Maths I & II and Physics)

In each of the three examples, the students have made it very clear that the term 'speed' can be used to describe the motion of objects that may be going fast or slow, that is, they may have great or small speed.

Category E

In contrast to Category D above, Category E refers to objects that travel 'very fast'. The responses appear only to associate speed with objects that are breaking the 'speed limit', and are therefore 'speeding'. It does not refer to slow moving objects or things considered to be going at ordinary speed. Of the four examples that are given to illustrate this category, two are taken from Year 6 and the other two from Year 9.

When something goes very fast.

Student 604

Speed mens you go over the lemit of that Rodo or track [sic].

(A possible translation of this is: 'Speed means when you go over the [speed] limit of the road or track'.)

Student 610

It is when you are going very fast. If you are speeding in a car you are going over the speed limit.

Student 914

Is to go very fast. Excelerate [sic].

Student 903

That such a large group of students in Year 6 and 9 are in this group (see Table 5.1) indicates that the term 'speed' is commonly associated with the everyday meaning of the word 'speeding' or going too fast, whether as in legal speed limits or as a sensory perception.

Category F

Any response that had some association with speed, but would not fit in the above categories, was placed into Category F. Usually, these responses showed that students had a meaning for speed but it was not associated with distance or time, or going fast or slow. Two examples are given here, one from a Year 6 student and the other from a Year 12 student who was a recent immigrant to Australia and English was her second language.

Well if the speed limite [sic] saide [sic] 60 and you did 120 you are speeding and you can get a ticket froe [sic] speeding.

Student 621

This response does not explain the meaning of speed but gives an example of speed that refers to 'speeding' and is similar in concept to Category E.

Speed means something accelerating faster
Student 109 (Maths I recent immigrant)

This student has confused the concept of speed with that of acceleration. There is a relationship between these concepts but they are not synonymous.

Category G

Any response that was not related to the concept of speed, or where there was no response, was placed into Category G. The two responses coded in this category provided "I don't know" as the response.

Table 5.1 summarises, for the students involved in the main study, the frequency of responses for each category from A to G, for each Year level.

Table 5.1
Categories of description of speed

Categories	Description	Frequency Year level 6	9	12	Total
A	Formal definition	0	0	5	5
B	Used time and distance	0	1	8	9
C	Used time or distance	0	2	0	2
D	Fast & slow	8	8	3	19
E	Very fast or high speed	11	8	0	19
F	Idea of speed (incorrect)	5	0	1	6
G	Incorrect	1	1	0	2

DISCUSSION

Year 6

It is of significance to note that none of the Year 6 students (see Table 5.1) gave a description of speed using the words distance and/or time. It would appear that the Year 6 students had not been exposed to the accepted concept of speed in the classroom and that their knowledge of speed was limited to how it was used in relation to 'speeding' cars or exceeding the speed limit, that is, they were using knowledge from outside the classroom. They had not shown by their responses that they understood which variables determine the speed of an object.

For the students who were interviewed, there was no change in the category descriptions of speed at the beginning of the interview compared to their responses in the written test. In fact, some of them used the identical words that they had used in the test.

After the students completed the speed questions in the interview they were again asked to explain what speed meant. Two students (622 and 606) provided a Category B response by referring to both distance and time correctly. It will be of interest to see how these students perform in the speed questions as compared with their peers. The other students made no improvement in category and just gave the meaning they had given at the beginning of the test. Thus, for most Year 6 students it appears that their ideas of speed are very stable, and these are difficult to modify even after completing and discussing concepts of speed with the interviewer. This indicates a robustness or persistence in the nature of descriptions for these young students.

Year 9

Most Year 9 students responded at Categories D and E. This means, they could not say spontaneously that speed was related to distance or time. Usually they referred to speed as "going fast" with only a vague idea of the formal definition of speed. Their perception of speed has a strong visual element, that is, they could tell if an object has greater speed or not by simply looking at the objects and their relative movements. Often students' ideas of speed were built around the car and speed limits. Clearly, not very many students had thought carefully about what 'km/hr' really means even though it was part of their everyday life experiences.

In the interviews, an attempt was made to see whether or not students could provide better descriptions of speed. If they could not give a clear initial response then the interviewer either probed their understanding or prompted for more information. After the questions on speed had been given and discussion had taken place about their responses, students were asked again for their meaning of speed. Three students could not provide a better description of speed even when prompted. The other two students (915, 906) supplied more complete descriptions as the following transcripts show.

S: *Um. Speed is when you move faster, you know, not necessarily - oh when you move fast, you know, some distance in an amount of time.*
(Student 915)

Here the student began with a Category D response and, as he formulated his thoughts, he responded at Category B.

Student 906 had to be prompted for more information before he mentioned distance and time. (Note: a raised dot (·) implies that about one second of time has elapsed.)

I: *What is speed?*
S: *Um · How fast an object will be going I guess.*
I: *So how would you tell if one was going at a higher speed than another one or not?*
S: *By measuring it.*
I: *What would you measure?*
S: *Um, how much distance it covered um in how much time.*
(Student 906)

Again it will be interesting to note how these two students performed on the speed problems and whether their performance was different to their peers.

Year 12

Since all of the Year 12 students were studying Mathematics and ten of them were studying Physics, it was not surprising that most of them responded within Categories A or B. One of the physics students did respond at Category D, while the other two Year 12 students in Category D were not studying Physics. Four non-Physics students, however, responded at Category B.

In the interview three students gave similar descriptions of speed as in the test. Student 114 who gave a Category A response on the test had to be prompted in the interview before he could give a response in this category again. Another student (112) gave a Category B response in the written test but in the interview, after discussion on the speed questions, seemed confused over the definition and reverted to a Category D explanation.

I: *What is speed?*
S: *Speed? Suppose a ···· I don't know. I know what it is but I don't know how to define it.*
I: *OK, when I use the term speed what do you think of?*

S: *Speed is what you do on the road or something. ··· um ··· speed is ···· a certain velocity. Speed is the um ··· to do with forward motion or any motion.*

(Student 112)

This student could not formulate a suitable response for a description of speed in the interview even when prompted. However, he was the only Year 12 student interviewed who could not give a similar response to the one provided in the test.

The Year 12 students, like the Year 6 and 9 students, were stable, in general terms, in there description of speed, though a few could give fuller descriptions when probed for more information.

SUMMARY - DESCRIPTIONS

In all, seven categories were identified to describe the groups of speed descriptions. These Categories of responses were consistent with those found in the pilot study. In general, as would be expected, the higher the Year level of students the better the category of response. Some of the Year 6 and 9 students supplied a better category of response on prompting in the interview, while none of the Year 6 students provided Categories A, B or C responses in the written test. Only 15% of Year 9 students gave Categories B and C responses. Eighty percent of the Year 12 students gave a response at Categories A, B, C.

In answering the research questions, three findings stand out. First, the categories of responses that were identified in the pilot study are mirrored in the main study (see Table 5.1). Second, after the students had worked on the speed problems and discussion had taken place, no additional categories appeared in the interviews. Third, the different Year levels do cluster about different categories. The Year 6 student responses are generally found in Categories D, E, F, i.e., they can not identify speed as being related to distance and time. Most of the Year 6 responses were found to occur in Category E where the descriptions of speed related to very fast objects. After discussion of speed problems some students identified speed as being related to distance and time. The Year 9 student responses all had a concept of speed that related to Categories C, D, E. With most of them connecting 'speed' to the idea of 'fast'. In the interviews, two students were able to provide Category B responses though one had to be prompted. Most of the Year 12 student responses were in Categories A and B and thus these students were able to state that speed was related to distance and time. The interviews suggested that in general these students were stable in their responses for a description of speed.

PART A: SINGLE FOCUS QUESTIONS

The Single Focus Questions in Part A of the Speed test were of two different types: the Numerical (coded with a N) and Variation (coded with a V). The Numerical questions gave numerical information that students could process quantitatively and in the Variation questions the information required students to determine how one variable would change given that another had changed.

Four main tables are displayed to provide an overview of students' responses. The first table presents the outcomes of the responses and indicates whether the responses are correct or not. The second table details examples of typical responses for different levels of response. The frequency of correct responses that students obtained for all these questions is provided in the third table. The fourth table shows the frequency of the types of strategies that students used to solve the problems. This same structure occurs also in the other sections of this chapter.

To enhance the analysis, a range of fullness of responses (see Chapter 4 for details) were observed for the Part A questions. These data are summarised in Table 5.2 as the frequency of the Fullness of responses for students in Years 6, 9 and 12.

As expected, performance on Question N1 was better than performance on Question N3 for both Year 6 and 9 students. This is reflected in both the number of correct responses and the Student Difficulty Ratio (SDR) listed in Table 5.2. (Note that the SDR for Year 6 students fell below one for Question N3.) The concept of speed in Question N1 is closely related to everyday experiences and most students knew that, by multiplying the speed by the time, distance would result (it appeared to students intuitively as the right thing to do). Question N3 was more difficult in that time was required. Thus, the operation of division had to be used, resulting in a fraction.

Table 5.2

Frequency for fullness of responses for Part A questions

Type of response (code)		**Year 6** (n = 25) Questions				**Year 9** (n = 20) Questions				**Year 12** (n = 17) Questions			
		N1	N3	V1	V2	N1	N3	V1	V3	N3	V1	V3	V5
Correct responses													
- Full	CF	19	8	9	5	19	10	3	5	13	12	11	9
- Partial	CP	1	1	4	3	0	2	3	3	2	1	2	5
- No reason, or guess	CU	0	2	2	1	0	1	8	5	0	3	2	2
Total correct		20	11	15	9	19	13	14	13	16	16	15	16
Answer correct but													
- Inappropriate	CI	0	2	1	0	0	1	1	3	1	0	1	0
Incorrect responses													
- Full	IF	2	2	0	0	1	0	0	0	0	0	0	1
- Partial	IP	1	2	2	2	0	1	1	2	1	0	0	0
- Inappropriate	II	0	1	4	5	0	4	0	2	0	1	0	0
- No reason / not done	IU	2	7	3	9	0	1	4	0	0	0	1	0
Total incorrect		5	14	10	16	1	7	6	7	1	1	2	1
Student difficulty ratio (SDR)		1.66	0.86	1.06	0.62	1.92	1.23	0.95	1.03	1.79	1.68	1.62	1.65

Note: a response that was correct but with an incorrect explanation is grouped with incorrect responses.

Questions V1 and V2 both asked for the time taken. The difference was that in Question V1 the speed was constant and in Question V2 the distance was constant. Both questions were designed to check students' ability to respond to the two different types of variation: direct variation for Question V1 and for inverse variation Question V2. A factor that needed to be considered when comparing these two questions was that Question V2 was a longer and more involved question than Question V1. The SDR scores for these questions reflect the difficulty that students experienced with Question V1 and, particularly, Question V2.

Of the three Year 6 students who did not attempt Question V1, all attempted Question V2 and two students obtained the correct answer using inverse variation.

All but one of the Year 9 students had Question N1 correct with a full response. For Question N3, however, only ten students gave a full response. Even though most Year 9 students gave correct answers to Questions V1 and V3, a large proportion of them could not give an explanation of how they arrived at their answer. Three students answered Question V3 correctly, but gave inappropriate reasons for obtaining their answer. The one Year 9 student that had Question N1 incorrect also had Question V1 incorrect but Questions N3 and V3 correct.

In Part A of the Year 12 speed test, only four questions were given, namely: Questions N3, V1, V3, V5. Question N3 was designed to check for numerical calculation. It was anticipated that these particular Year 12 students would successfully complete Questions N1 and N2 very easily so only Question N3 was included. Questions V1 and V2 were included to observe the strategies that students apply on simple variation-type problems and Question V5 on inverse variation. The above outcomes indicate that most Year 12 students showed facility in answering these questions, with an SDR well above one for all questions.

Table 5.3 provides some examples of these different types of response taken from Year 6 students for Question N3 and V1. These responses reflect the type of question that was presented to students. For example, the responses to the numerical question, N3, are mostly of a calculation type even for those students who had incorrect answers, whereas the responses to the variation question, V1, use language associated with variations such as 'twice', '2 times' or 'double'.

Table 5.3

Examples of fullness of response for Part A
for Questions N3 and V1 from Year 6 students

Question N3	Student Number	Question V1	Student Number
CORRECT FULL (CF) Ans: 30 mins. Exp: 60 mins = 20 30 mins = 10.	602	CORRECT FULL (CF) Ans: 2 times. Exp: Becues [sic] you doublet [sic].	601
CORRECT PARTIAL (CP) Ans: 30 mins. Exp: because he is going 20 km/hr.	622	CORRECT PARTIAL (CP) Ans: Twice. Exp: because the first journey was short.	612
CORRECT UNKNOWN (CU) Ans: 1/2 h. Exp:	613	CORRECT UNKNOWN (CU) Ans: 2x. Exp:	610
CORRECT INAPPROPRIATE (CI) Ans: 30 mins. Exp: 1 hour - 30 mins. [wrong operation]	612	CORRECT INAPPROPRIATE (CI) Ans: twice as far. Exp: [time was required]	614
INCORRECT FULL (IF) Ans: 2 hrs. Exp: 10 x 2 = 20.	623	no incorrect full responses	
INCORRECT PARTIAL (IP) Ans: 30 h. Exp: I took the 20 km and added it to the 10 Km.	619	INCORRECT PARTIAL (IP) Ans: Half. Exp: because the second journey was about 1000km the first journey was 500 KM.	615
INCORRECT UNKNOWN (IU) Ans: ~~5~~ ~~5~~ 2. Exp: I gest [sic].	601	INCORRECT UNKNOWN (IU) Ans: Exp: can I pass.	608

Table 5.4 provides a summary of the frequency of correct responses for the different Year levels. As the Year 12 students undertook different questions from the Year 6 and 9 students, direct comparisons need to be carried out with caution.

Table 5.4
Frequency for the number of correct responses for each student in Part A

	Number of correct responses				
	4	3	2	1	0
Frequency for Year 6	3	10	5	6	1
Frequency for Year 9	6	10	2	2	0
Frequency for Year 12	14	2	1	0	0

From this table it can be seen that 50 percent of the Year 6 students answered three or more questions correctly. Only three Year 6 students had all four questions correct out of this group. This was surprising because the questions were designed to be familiar to students at this level. The one Year 6 student who had all the questions incorrect seemed to find answers quickly, that is, close rapidly on an answer, by using an operation that was inappropriate for the question. He may not have understood the questions since his answers often had the wrong units, for example, when the question asked for time, he found distance.

Six Year 9 students had all four questions correct and no Year 9 student had all questions incorrect. Eighty percent of the Year 9 students answered three or more questions correctly. Most Year 12 students had all questions correct.

STRATEGIES USED IN PART A

The students used a number of strategies to solve the problems. These ranged from more successful attempts using all the information provided, to the not so successful and the guesses. Eight categories were identified. These are: Formula; Inverse variation; Direct variation (without the formula); Proportion; Closure operations - multiplication, addition, subtraction, division; Unclear response; Guess; and No response.

Below are examples of student responses illustrating the different types of strategies based on the written test. Both the answer (Ans) and the explanation (Exp) are provided as well as the code for the fullness of response in the brackets, after the question number is identified.

1. Formula

A response was coded as using the formula strategy if the student used $v = s/t$, that is, speed equals distance divided by time, (v = speed, s = distance, t = time) either stated or implied to solve the problem. Most students who used the formula either substituted into the formula or reorganised it into an appropriate form before making a substitution.

An example from a Year 12 student was:

Ans: *30 minutes, 1/2 hour, 1800 secs.*
Exp: *s = d/t => t = d/s*
therefore s = 10 km ÷ 20 km/hr = 1/2 h.
Student 102 Question N3 (CF)

Some students quoted the formula but did not substitute the values into it. They used the formula as a start to using another method. The first example given below shows the response of a student who used the formula, and then applied either proportion or direct variation to the formula to solve the problem. The second example is that of a student who used the formula as a springboard to using direct variation.

Ans: *twice.*
Exp: *s = d/t and 2s = xd/t t is constant*
=> x = 2.
Student 111 Question V3 (CF)

Ans: *twice.*
Exp: *she went twice as fast ie 2 x speed*
s = d/t t stayed the same so I doubled the distance.
Student 117 Question V3 (CF)

2. Inverse variation

If the response indicated that the student noticed one variable increased while the other variable decreased, then this was coded as an Inverse variation. Some of the questions were designed so that inverse variation had to be used to arrive at the correct answer (e.g., Questions V2, V5) whereas for other questions this strategy would be entirely inappropriate. The examples of this type of response, given below, show a variety of ways students use inverse variation.

Ans: *Halph [sic] the time of Mr Smith.*
Exp: *he was going at twise [sic] the speed so it will take him halph [sic] the time.*

Student 603 Question V2 (CF)

Student 603 recognised inverse variation and stated how it can be applied. He has given this explanation using the names of the variables and how those variables relate to one another.

Ans: *1/2 as long.*
Exp: *becuse [sic] mister Jones was twice as far[st].*

Student 606 Question V2 (CF)

This Year 6 student used inverse variation as he indicated that because speed has doubled, time has halved

Ans: *1/2 hour.*
Exp: *if he travelled at 10 km/hr for 10 km 1 hour, 20 km/hr is faster 1/2 the time.*

Student 112 Question N3 (CF)

The response of Student 112 is interesting because the student has reworded the question to consider first the distance of ten kilometres and then reasoned that if the boy rode this in one hour he would travel at ten kilometres per hour. Then, if he doubled the speed he would halve the time. Thus the response uses the strategy inverse variation, even though this method was not anticipated when the question was first designed.

3. Direct variation

A direct variation response occurs when a student states that as one variable increases then another variable also increases. Questions N3, V1, V3 were appropriate to the direct variation strategy. The examples, in Table 5.5, illustrate the different responses that can be coded in this way.

Table 5.5
Examples of direct variation responses for Part A

Ques.	Stu No.	Quality of Response	Answer	Explanation
V1	106	CF	Twice as long	If the car was travelling at the same speed for both journey and B was twice as far => B would take twice as long.
V2	614	II	Double the amount	Twice as far.
V3	909	CP	2	She doubled her speed.
V3	920	CF	2 times	She is going twice the speed so I times the number she went around the block by 2.
N3	603	CF	30 mins.	because it takes one houre [sic] to do 20 km so 10 km would be halph [sic] the time.
N3	616	CF	1/2 an hour	because if hes [sic] travelling at 20 kph just 1/2 it.
N3	909	CF	30 mins.	Well takes him 20 km an hour so 10 is half of 20, so I halfed [sic] 60 and got 30.

It is of interest to note the different ways that students have used the variables. In Questions V1 and V3 they have used the variable names and not the numerical values given in the problem, whereas in Question N3 they have tended to the numerical values.

Student 614 attempted to use direct variation on Question V2 using the variable names but Question V2 was an inverse variation question not a direct variation. Using the variable names, though, was encouraged by this type of question.

In the examples for Question N3, the students used the numerical values of the variables. This is to be expected since the question was given in this mode. Thus it is apparent that the manner in which the question was presented influenced the way students referred to the variables in their explanation.

The interviews indicated that students who used direct variation were fairly stable in taking this approach, for example, Student 616 used the same strategy in the interview. See Table 5.5 for his written response.

S: *Half an hour.*
I: *How?*
S: *If he is travelling at 20 km/hr and it says how many hours did it take him to ride 10 km then you just halve the time.*

4. Proportion

The proportion strategy was coded when students used numbers and expressed them as a quotient for comparison. The two examples given are both from Year 12 students. They show that the students applied a mathematical technique with which they were familiar.

Ans: *1/2 hr.*
Exp: $\frac{20}{1} = \frac{10}{x} \Rightarrow x = \frac{10}{20} = 1/2.$

Student 107 Question N3 (CF)

Ans: *1/2 hr.*
Exp: *20 km/hr ÷ 10 km = 1/2 hr.*
[the student had cancelled the km]

Student 116 Question N3 (CI)

The answer in the latter example is given as '1/2 hr' but the explanation is '20/10' which is two, and not a half as the student indicated. The student has been able, obviously, to 'see' the answer, then has 'searched' for a method to explain it and he has not seen that the method is inconsistent with the answer. This strategy was categorised as proportion assuming that the student has used this approach first then looked for some numbers to give the required ratio.

5. Closure operations

Closure operations include any arithmetical operation that has been applied to achieve a result. It does not include those operations that are related to the formula $v = d/t$. Sometimes the result is correct and the student may have had some rational reason for such an application but he/she has not thought it important enough to write it down in the explanation. Since it is the response that is being examined and not the student, the focus of the analysis is on what is written. Most of these arithmetical responses appear to be premature closures in order to arrive at an answer. The examples in Table 5.6 illustrate responses for different arithmetical operations.

Table 5.6
Examples of closure operations responses for Part A

Type of closure	Ques No.	Student No.	Quality response	Answer	Explanation
Multiplication	N3	623	IF	2 hrs	10 x 2 = 20
	N3	920	II	200	I just multiplied 20 by 10
	V3	903	IP	15 times around in 5 mins	3 x 5 = 15
	N1	606	CF	180 km	3 x 60 which = 180
Addition	N3	619	II	30 h	I took the 20 km and added it to the 10 km
	V2	605	II	3 times faster	I added it
Subtraction	N3	612	CI	30 mins	1 hour - 30 mins
Division	N3	903	II	2 hs	20 ÷ 10 = 2 h
	V3	912	II	4 laps	5 min divided by twice the normal speed (4)

For most of these responses, the answers were incorrect. In Question N1 the multiplication strategy could be applied and the correct answer obtained. Thirty-nine students used multiplication on Question N1. Whether the use of this technique was fortuitous or intentional was not apparent always from the response.

6. Unclear response

Some responses were inadequate or inappropriate for the question and could not be identified with any particular strategy described previously. Examples of these are given below.

> Ans: *1 hour.*
> Exp: *because the bike is faster than the other.*
>
> Student 620 Question N3 (CU)

Ans: *Twice as long.*
Exp: *because the old car goes slower.*

Student 612 Question V2 (II)

Ans: *60.*
Exp: *I took a time for the Smith family and x2 then added them both.*

Student 619 Question V1 (II)

7. Guess

In this case students have written an answer, that could be coded as correct unknown (CU) or incorrect unknown (IU) for Quality of Response, but they have written "guess" as their explanation. Sometimes the answer may give an indication as to what has been done.

Ans: *twice.*
Exp: *I guessed.*

Student 611 Question V1 (CU)

This answer was correct and indicates that the student may have used a direct variation strategy but she did not know how to, or chose not to, explain it.

8. No response

Some students gave no response or indicated that they could not complete the question, for example Student 615 said that Question N3 was "too hard".

Overview of Strategies

Table 5.7 lists the strategies with the frequency that each strategy was used for each question for the different Year levels. A dash indicates that no students in that Year group used that particular strategy.

As anticipated, Year 12 students used more higher-order mathematical procedures than the Year 6 or 9 students. While students, across all groups, used direct and inverse variations appropriately, some of the Year 6 and 9 students were more likely to use simple mathematical operations.

Table 5.7

Frequency of strategies used in Part A

Strategy	Year 6 (n = 25) Frequency				Year 9 (n = 20) Frequency				Year 12 (n = 17) Frequency			
	N1	N3	V1	V2	N1	N3	V1	V3	N3	V1	V3	V5
1. Formula												
substitution	-	-	-	-	-	-	-	-	5	3	1	5
direct variation	-	-	-	-	-	-	-	-	0	0	2	0
2. Inverse variation	0	0	1	10	0	0	2	0	2	0	0	10
3. Direct variation.	0	8	12	4	0	10	15	16	2	11	8	1
4. Proportion	-	-	-	-	-	-	-	-	6	2	4	1
5. Closure operations												
multiply	21	3	0	2	18	2	0	3	-	-	-	-
add	2	2	2	1	1	0	0	0	-	-	-	-
subtract	0	2	1	0	-	-	-	-	-	-	-	-
divide	-	-	-	-	1	6	0	1	2	0	0	0
6. Unclear response	1	2	4	0	-	-	-	-	0	1	2	0
7. Guess	0	1	1	2	0	2	0	0	-	-	-	-
8. No response	1	7	2	6	0	0	3	0	-	-	-	-

*

The dashes (-) indicate that no student at that Year level used that strategy.

In the numerical question N1, most students (93%) multiplied speed and time to find distance, and gave a suitable explanation. Fewer students (65%) provided a full response to Question N3 where they had to find the time by dividing the distance by the speed.

In solving Question V1 most students who had a correct response used direct variation, the percentages of students who did this in Years 6, 9, 12 were 60%, 75% and 94%, respectively, with an overall rate of 73%.

For most students these questions were familiar. Those who did have difficulty appeared to close rapidly on an answer using a single arithmetical operation. The way the questions were presented did influence the way the explanations were given. For example, the Numerical questions were given in a numerical form and illicited numerical type responses. The Variation questions were not presented in a numerical way and students responded by using variable names in their explanations.

It is interesting to note the number of responses that included the formula $v = d/t$ to solve the problems. All students who used the formula had completed Year 11 Physics and mechanics in Maths II, therefore they had access to the formula, but not all of the Maths II and Physics students used the formula.

Summary

There were distinct differences in applying strategies across the three Year groups. Some of the Year 6 students did not have a proper understanding of the problem in Question N3. Without a good understanding of what '20 kilometres per hour' means, they could not unravel the question. Those students who attempted the problem, closed quickly on a solution by applying an arithmetical operation to the numbers and did not always choose the correct operation. The Year 6 students in Question V2 knew there had to be a change in the variables but more than half of them could not relate to that change being an inverse variation. Four students assumed that the change was direct. When the question did involve direct variation, as in Question V1, half the students used that strategy, and no students attempted an inverse variation strategy.

Most Year 9 students managed these questions adequately by using an appropriate strategy, except in Question N3 six students attempted to subtract instead of dividing. This is to be compared with the Year 6 students who did not subtract on any question. The Year 9 students seemed to know that as speed increases so distance increases for the same time, thus they could use direct variation appropriately. Those students who had difficulty with these questions appeared to close quickly on an answer by using an operation on the numbers available. Some of those students who had problems showed poor concepts of the notion of speed and what is meant by such things as twenty kilometres per hour.

The number of misconceptions displayed in this section by the Year 12 students were very few, as most of the students were able to complete the questions with suitable explanations using appropriate strategies. The strategies used were mostly related to higher order mathematical procedures, that means, only two students used closure operations on three questions.

INTERVIEWS AND WRITTEN RESPONSES.

The interviews provided students with the opportunity to present additional information and explain how they arrived at their answer by talking about it, instead of writing. The interview also permitted the interviewer to ask students to amplify their explanation of how they solved the problem. Many students used the same type of response and strategy in both the written test and the interview. Other students, however, used different responses and strategies in the interview compared to their responses in the test. Consideration is now given to these issues.

Variation of responses was compared in two ways: 1. A comparison between **answer** in the test and in the interview; and, 2. A comparison between **strategy** in the test and in the interview. For the questions in Part A of the test, the interviews revealed over 75% of student responses provided the same answers compared to the written test. Over 50% of student responses also used the identical strategy in both the test and interview.

Variations in approach

Not all students provided the same responses and/or strategies across the test and interviews. Three examples, one from each Year level, are provided to illustrate this.

Student 112, was referred to previously under the inverse strategy, had an unusual way of doing Question N3 in the written test in that he re-worded the question and then employed an inverse variation approach. This was not the easiest way of doing the problem. In the interview, this student was asked this question again and arrived at the same answer but his explanation used the direct variation strategy.

S: *I just um ·· If he is riding at twenty kilometres per hour and he is only got to ride ten kilometres, ten kilometres is half of twenty so it is going to take him half an hour.*

Responses that differed in this way suggest that the student was not tied to any one method which was a learned or taught algorithmic approach, though the opposite is not necessarily true, that is, if students have stable approaches this did not imply that they were algorithmic or taught. It is of interest to note that this student has continued to use a variation type of approach in both responses. This commonalty in type of approach is explored in later chapters.

The second example concerns a student in Year 9 who used multiplication in the written test for Question N3. In the interview he tried initially to close quickly but on prompting provided a more considered response.

S: [read question out loud] *um ··· oh! how many hours did he ride?*
I: *Yes I want to know how many hours did he ride for?*
S: *How far did the boy ride the push bike at a speed of twenty kilometres oh! per hour um about two.*
I: *How did you get that?*
S: *··· ten.*
[It appears that he is guessing]
I: *So it took him ten hours?*
S: *NO! it took him about half an hour.*
[Now he has given the correct answer]

I: *Where did you get a half from?*
S: *I just ·· one hour in half an hour.*
I: *Where did half in one hour come from?*
S: *because um ·· we want to know like that ten kilometres is half of twenty so how many hours would you ride in ten kilometres so you just divide the hour by half.*
[he really means divide the time by two]

This student had to read the question a number of times to select the right information and decide on what was being asked. There was some confusion in trying to explain how he obtained his answer. This suggests that an intuitive approach was taken in arriving at an answer and this was followed by a search for a more formal explanation when he was asked for a reason. This issue is examined in more detail in the next chapter.

The final example concerns Student 623, who chose to write a simple multiplication for the reason in Question N3. It appears that the student has arrived at the answer of two hours and then in seeking an explanation has searched for an answer to the problem 'ten times what equals 20?' and she deduced the answer to be two.

Written test:

Ans: *2 hrs.*
Exp: *10 x 2 = 20.*

The interview confirms the belief that she was closing quickly on an answer without considering the problem properly.

S: *···· um ··· thirty.*
I: *thirty what?*
S: *Kilometres.*
I: *But I want to know how many hours?*
S: *um ··.*
I: *See he rode twenty kilometres in one hour didn't he?*
S: *Yes.*
I: *How long would it take him to ride ten kilometres?*
S: *··· five hours.*
I: *How did you get five?*
S: *mm ····.*

Other attempts were made to help her solve similar problems without success. Eventually a diagram was drawn of 20, 40 and 60 kilometres, and she gave the correct times. She was then asked for the time for ten kilometres and she obtained an answer of half an hour. Thus when she was given problems that were simple multiples of twenty, she deduced the correct result and once she saw the pattern she saw what would happen with ten kilometres.

In summary, a comparison between test responses and interview responses indicates that most students could provide the same answer using similar strategies. Those responses that differ indicate some issues, such as, similar strategies and intuitive answers that need further exploration. These issues are addressed in later chapters.

SUMMARY PART A

The Single Focus Questions in Part A were composed of the Numerical and Variation type questions. Most students were able to successfully answer the numerical questions and the direct variation question V1. The Year 6 students were not so successful on the inverse variation questions. A range of strategies were identified in the students' responses, from those, such as, the formula, that were always successful to those that were just arithmetical closure operations that, generally, were unsuccessful. The interviews confirmed that the majority of students were fairly stable in their approaches to the questions. Those that differed raised a number of issues that are explored in later chapters. In particular, the discussion focuses upon students' use of intuitive methods to solve problems and the effect of complexity of questions on students' responses.

PART B: THE DUAL FOCUS CLOSED COMPARISON QUESTIONS

The Questions in Part B of the test consisted of the Dual Focus Closed Comparison Questions. As in the Single Focus Questions of Part A, different questions were given to each Year group with some questions being common to all. The responses given by students had a range of quality from the Full Response to that of No Response. Table 5.8 provides a summary of the frequency for the Fullness of Response for the questions at each Year level and indicates the number of students with correct and incorrect responses. Also, the Student Difficulty Ratio (SDR) is included for each question.

It was expected that all students would answer Question B1 correctly since all the variables were the same. This question was included to test students ability to comprehend the diagram. Many students gave a full explanation for their answer to B1 but not as many could do so on the questions that followed. It is as if students were intuitively able to balance out the variables in their minds as they read the question. Hence they could be successful by just noticing each similarity. Such a strategy was not possible on any other question.

The SDR score also shows, as anticipated, that the level of quality of response increased with Year level. In general the Year 6 and 9 students' performance on these questions suggest that the questions increased in difficulty. However, the SDR scores suggest that their are small anomalies with the responses to Questions B8 and B10. Students appear to have more difficulty with Question B8 compared to Question B10. The reasons for this are interesting and are discussed later.

Table 5.9 illustrates different Fullness of Response answers taken from Year 9 students using Questions B8 and B10. In these responses it is valuable to note the same student's responses for each question. For example, Student 905 in Question B8 only referred to the time but in Question B10 he mentioned both variables, time and distance. Student 904, on the other hand, had both questions incorrect. In Question B8 he mentioned the difference in starting positions, as well as the time and distance of travel, but in Question B10 he did not mention the variable time. Students 914 and 911 were incorrect on both questions.

Table 5.8

Frequency for fullness of response for Part B questions

Type of response	(code)	**Year 6** Questions (n = 25)						**Year 9** Questions (n = 20)					**Year 12** Questions (n = 17)				
		B1	B2	B3	B5	B8	B10	B1	B3	B5	B8	B10	B1	B4	B5	B8	B10
Correct responses																	
- Full explanation	CF	6	3	7	8	5	2	12	5	9	7	7	15	12	14	14	13
- Partial reason	CP	15	6	3	4	2	6	5	5	4	3	4	0	0	0	0	0
- No reason, or guess	CU	4	1	0	2	1	4	3	2	2	0	1	2	1	1	1	1
Total correct		25	10	10	14	8	12	20	12	15	10	12	17	13	15	15	14
Correct answer																	
- Inappropriate reason	CI	0	0	0	5	2	6	0	2	3	0	4	0	0	1	1	1
Incorrect responses																	
- Full explanation	IF	0	2	0	0	3	0	0	0	0	1	1	0	2	0	0	0
- Partial reason	IP	0	13	15	5	10	3	0	2	2	4	1	0	2	0	0	0
- Inappropriate	II	0	0	0	0	2	2	0	4	0	3	1	0	0	1	1	1
- No reason / not done	IU	0	0	0	1	0	2	0	0	0	2	1	0	0	0	0	1
Total incorrect		0	15	15	11	17	13	0	8	5	10	8	0	4	2	2	3
Student difficulty ratio (SDR)		1.54	0.68	0.74	1.06	0.66	0.8	1.73	1.02	1.37	0.92	1.22	1.9	1.5	1.7	1.7	1.6

Table 5.9
Examples of fullness of response
from Year 9 students on Questions B8 and B10

Question B8	Student Number	**Question B10**	Student Number
CORRECT FULL (CF) Ans: B. Exp: because A took 5 sec to go 3 units & it took B 3 secs to go 3 units.	906	CORRECT FULL (CF) Ans: A. Exp: quicker time for the distance Travelled.	905
CORRECT PARTIAL (CP) Ans: B. Exp: faster time.	905	CORRECT PARTIAL (CP) Ans: A. Exp: got it further than B.	902
CORRECT UNKNOWN (CU) (none identified)		CORRECT UNKNOWN (CU) Ans: A. Exp: guessed.	908
CORRECT INAPPROPRIATE (CI) (none identified)		CORRECT INAPPROPRIATE (CI) Ans: A Exp: because it doesn't have far to go.	901
INCORRECT FULL (IF) Ans: Same. Exp: although A starts further ahead they still go the same distance in the same time.	904	INCORRECT FULL (IF) Ans: B. Exp: Its going faster and further in 3 secs.	903
INCORRECT PARTIAL (IP) Ans: same. Exp: they made it the same distance.	902	INCORRECT PARTIAL (IP) Ans: B. Exp: Although B got a head start A went further.	904
INCORRECT INAPPROPRIATE (II) Ans: A. Exp: they have two trains so they are faster.	914	INCORRECT INAPPROPRIATE (II) Ans: Same Exp: they have two trains each and they have covered the same amount between them.	914
INCORRECT UNKNOWN (IU) Ans: Same. Exp: they are the same.	911	INCORRECT UNKNOWN (IU) Ans: same. Exp: they are the same.	911

Table 5.10 shows a summary of the frequency of the number of correct responses obtained by students.

Table 5.10
Frequency of the number of correct responses for each student in Part B

	Number of correct responses 6	5	4	3	2	1	0
Freq Year 6	3	4	2	6	6	4	0
Freq Year 9	-	7	5	3	4	1	0
Freq Year 12	-	11	4	1	0	1	0

More than 50 percent of the Year 6 students had three or more correct responses. The Year 6 students had greater difficulty with these questions compared to the questions in Part A. Sixty percent of the Year 9 students had four or more questions correct. Of the five Year 9 students, who had only one question wrong, four of them had Question B8 incorrect and the other had Question B3 incorrect. Eleven of the seventeen Year 12 students were correct for all five questions. Again, this was expected given the background of the class.

Questions B8 and B10

When reporting the Fullness of Response outcomes it was noticed that for students in Years 6 and 9, more of them arrived at a correct answer for Question B10 than for Question B8. This was a little surprising since Question B10 had a greater number of variables differing than Question B8, and thus it was expected that Question B10 would have been more of a challenge to students.

The contingency tables in Table 5.11 for Year 6 and Year 9 students show more clearly the relationship between quality of the responses for the two questions. Only two Year 6 students had a full and Correct response for both Questions B8 and B10. Of the ten other students that had a correct response for Question B10 eight had Question B8 incorrect. For the responses of these students to Question B10, three had no explanation, five focused on the distance, one on the time and one said "that it looks like A has done more speed than B". Such incomplete strategies gave the correct answer but were based on insufficient data to conclusively reach that answer. The interviews were consistent with this finding except for Student 610 (see student profile on Luke in Chapter 9) who gave full responses for both Questions B8 and B10 in the interview but in the test could give no reason for either questions.

Table 5.11
Contingency tables for Questions B8 and B10

Year 6

		Q. B10 CF	C	I
Q. B8	CF	2	2	1
	C	0	0	3
	I	0	8	9

Year 9

		Q. B10 CF	C	I
Q. B8	CF	4	2	1
	C	3	0	0
	I	0	3	7

CF = Correct Full Response
C = correct Partial or Unknown
I = Incorrect (including Inappropriate)

In Year 9, four students gave correct and full responses to both Questions B8 and B10. Three students gave a full response to Question B8 but not to Question B10. One student gave a partial response to Question B10, one just "guessed", and the third student used an inappropriate method by saying that "trolley A was 1 sec in front". Three students who had a partial response for Question B8 gave a full response to Question B10; in Question B8 two focused on time and one guessed. For the three students who had Question B8 incorrect yet had the correct answer for Question B10, two focused on the distance in both and one focused on the time but mentioned that Trolley A in Question B10 was "behind".

Therefore, students who had correct answers for Questions B8 and B10 but not a full response were basing their answers on incomplete data. Such reasoning sometimes yielded a correct answer but at other times an incorrect answer. Thus, in looking at responses to compare correct answers it is more useful to consider those responses that are coded as Correct Full. In this way it can be seen, in Table 5.8, that the same number of students had Question B8 and Question B10 correct in Year 9 and, for Year 6, less students had Question B10 correct compared to Question B8. Thus, the original conjecture that Question B10 should be more difficult to solve than Question B8 holds.

STRATEGIES USED IN PART B

The strategies used by students in solving the Part B questions were identified under three major groupings. Within each group there were a number of categories. The first group of strategies employed both the variables distance and time to compare the speeds and were considered under four categories, namely: Formula; Calculation; Proportion; and, Written variables. The first three were mostly numerical in nature and the last one was a written statement involving distance and time. The second group of strategies involved the use of one variable and was classified as: Time only; and, Distance only. The third group, of four strategies, relied on the visual aspect of the problem, namely: the Trolley in front; Faster; Speed; and, 'Guess'.

Each strategy is now described with examples of student responses to the Part B Questions from the written test.

Two Variables - Distance and Time

The following four categories of responses incorporated both variables, distance and time.

1. Formula

A response was coded as using the formula strategy if the student used $v = s/t$ (v = speed, s = distance, t = time), or some equivalent, to solve the problem, that is, speed equals distance divided by time was either stated or implied. Most students, who used the formula, either substituted into the formula or reorganised it into an appropriate form before substitution. In the example given below the formula is stated fully.

Ans: *same speed.*
Exp: $Speed_1 = distance/time = 3/3 = 1$ *m/s*
$S_2 = d/t = 4/4 = 1.$

Student 104 Question B4 (CF)

The above example suggests that some students were aware of the variables needed since they could quote a formula which has the symbols in place of the variables. These students then used the numerical values of the variables to substitute into the formula. However, it does not necessarily follow that the presence of the formula is sufficient to guarantee the correct answer.

2. Calculation

This strategy is similar to the previous method in that students have divided the distance by the time, except there is no evidence that they know the formula. This seems to be more of an intuitive notion to use a method to compare the ratios of distance and time rather than a learned method to calculate speed. In the responses to the questions preceding Question B10, these students used other strategies with no suggestion of a learned or rote response that would indicate that they were aware of a formula. The following example from a Year 9 student, illustrates this strategy.

> Ans: *A.*
> Exp: *A = 5 ÷ 4 secs = 1.25 units per sec.*
> *B = 3 ÷ 3 secs = 1 unit per sec so A went faster.*
> Student 920 Question B10 (CF)

This student has not used the formula and it is not apparent that she is using the actual variable name or symbol even though she has used the numerical values appropriately. Her responses to other questions suggest that she is not aware of the formula.

3. Proportion

The proportion strategy resulted from responses that used the values of the variables and compared these values in some way. The easiest way to do this was by comparing the ratios. For ratios that had either the same denominator or numerator, this was clear and did not require any further explanation. In the more difficult questions, it is not always clear what operation was used to decide on which trolley had the greater speed, but it appears to be associated with proportion-type thinking. The first three examples below illustrate typical responses across Year 6, 9 and 12 that were coded as proportion.

> Ans: *Same.*
> Exp: *Because B went to 3 in 3 secs and A went to 5 in 5 seconds.*
> Student 606 Question B3 (CF)

> Ans: *B.*
> Exp: *Because A took 5 secs to go 3 units & it took B 3 secs to go 3 units.*
> Student 906 Question B8 (CF)

> Ans: *Cart A.*
> Exp: *A has travelled 5 km in 4 secs cart B has gone 3 km in 3 secs.*
> Student 112 Question B10 (CF)

Each of these responses shows a comparison between the two variables, distance and time, though there is no indication as to how students compared the ratios. It appears that students considered it to be obvious that one was larger than the other just by stating it. Interestingly, Student 906 wrote the ratio in the order, time to distance whereas others have it in the order, distance to time. The accepted way of using the ratio is distance to time so a student who uses the ratio in the reverse order has probably had little experience with these type of questions. In the descriptions of speed, Student 112 knew that speed was related to distance and time, whereas the other two students described speed as being 'fast'.

4. Written (Time and Distance)

All of the above strategies involve the use of the numerical values associated with the variables, distance and time. Some responses, however, used the variable names and did not always mention the numerical values. The examples below illustrate this type of strategy.

Ans: *A goes farest* [sic] [fastest].
Exp: *Both go the same distance. A goes slower* [More time].
Student 615 Question B8 (IF)

Although this student has the wrong answer, she referred to both the distance and the time as "slower" in her explanation. It appears in this question that she identified the trolley with the greatest speed as also taking the greatest time.

The next two examples show a mixture of variable name and value.

Ans: *B.*
Exp: *it reaches the farthest* [sic] *in only 3 secs.*
Student 903 Question B5 (CF)

Ans: *B.*
Exp: *A started at 2 B started at 0 and B got the same distance in less time.*
Student 908 Question B8 (CF)

Student 908 summarised correctly the essence of the problem in stating "the same distance in less time" in the response. The following two examples illustrate an economy in explanation that incorporates the correct ideas about speed.

Ans: *A greater.*
Exp: *Quicker time for distance.*
Student 915 Question B10 (CF)

Ans: *A.*
Exp: *Travelled the greater distance in the shortest time.*
Student 106 Question B10 (CI)

The method used by Student 915, was expanded by other students in the interviews and may be summarised as follows: pull back Trolley B one unit so that it starts at the same place as A. It then goes to 3 units in 3 seconds. So A should reach 4 units in 4 seconds but it went 5 units so it must be going faster, or it should get to 5 units in 5 seconds but it only took 4 seconds, that is, "it had quicker [shorter] time for its [longer] distance".

In Question B10, Trolley A did travel the greater distance but not in the shorter time, Student 106 seemed to be a little confused. Hence the Correct Inappropriate (CI) category for fullness of response.

These students stated a general principle, which is true for determining the relative speed of objects. They could identify the trolley with the greater speed by comparing the distance and the time variables. Apparently, this was achieved without doing a calculation and indicates that these students had a better concept of speed than those students who did not compare distance and time.

One Variable - Distance or Time

The following strategies focus on one variable without reference to the other factors that are involved.

5. Time only

In this strategy, students referred to the time taken, but did not refer to distance at all. There may be an implication of the use of distance but it was not stated, and so these students did not see the importance of referring to all variables in their responses.

Ans: *A.*
Exp: *Becaus* [sic] *A took less time than B. B took 5 seconds and A only 3 seconds.*
Student 619 Question B3 (IP)

This student used the 'time only' strategy on five occasions out of the six questions in Part B as the method for determining the greatest speed.

The following response from a Year 9 student referred only to time using the value of the variable.

Ans: *B.*
Exp: *Because its taking 3 sec for A.*

Student 913 Question B3 (II)

Student 114, however, referred to 'speed' and time. However, it is not clear what she meant by the term 'speed'. It could be distance or speed. (English is her second language).

Ans: *B.*
Exp: *B has greater speed more time.*

Student 114 Question B5 (II)

Nevertheless, since the time for both trolleys was the same this response appears to be inappropriate for the question.

6. Distance only

This strategy is similar to the previous one except students referred to distance as the sole criteria for determining greatest speed instead of time. The responses of both the examples given below refer to variable names. Time might be implied but it is not stated in the response.

Ans: *B.*
Exp: *B has faster distances* [sic] *then A.*

Student 623 Question B5 (CP)

Ans: *B.*
Exp: *cause B got further than A.*

Student 904 Question B3 (IP)

Visual

The following strategies have been grouped under the term 'visual' since students focused on an aspect of the diagrams to determine their answer. A 'guess' was also incorporated in the responses of this group.

7. In Front

In this strategy, students' responses indicated that they thought that the trolley in front was the one with greater speed. Such responses were coded as 'in front'.

Ans: *B.*
Exp: *because its* [sic] *in front.*
Student 616 Question B2 (IP)

Ans: *A.*
Exp: *A is one second in front.*
Student 917 Question B10 (CI)

Time is also referred to in the second response but it is used as a measure of how far 'in front' one trolley is compared to the other.

8. Faster

If the student's response indicated that the term "fast" or "faster" was the predominate feature, then this response was categorized under the strategy 'faster'. Such strategies often led to incorrect answers and poor explanations.

Two examples are given of this strategy. The student who gave the first response used similar strategies in five of the six questions in the Part B questions.

Ans: *B.*
Exp: *because the cars where* [sic] *travelling faster than A.*
Student 612 Question B2 (IP)

The following response indicates that the student saw that Trolley A was in the lead and as a result it was faster.

Ans: *A.*
Exp: *Because it is faster than B.*
Student 901 Question B3 (II)

This student referred the term 'faster' to the trolley that had the lower time. If the trolley had travelled the same distance, this would have been correct, however, Trolley B had moved a greater distance.

9. Speed

Some students justified their answer by using the term 'speed'. This term could mean the one with the greater distance or the one with the shorter time and is similar to the 'faster' strategy referred to previously.

> Ans: *Same.*
> Exp: *they go the same speed & cover the same, the same speed.*
> Student 602 Question B8 (IP)

This student has not used the terms correctly and confused speed and time. His use of the incomplete phrase "cover the same" may indicate that he is referring to the distance.

Overview of Strategies

Table 5.12 summarises the frequencies of the Types of Strategies that were used by students in the different questions.

For those Year 6 students, who focussed on distance and/or time, only one third considered distance <u>and</u> time. The other students used only distance or time alone, with distance being used more than time as the determining variable. None of the Year 6 students used anything resembling the formula or the quotient of distance to time. A third of the students used the "in front" strategy to determine the greatest speed for Question B2, but only one student used the same strategy for Questions B8 and B10.

Similar results to those observed with some of the younger students in this study have been cited by Piaget (1970). In his work he found that some students (in Stage 1 and 2) thought that the object that was in front would have the greater speed. Student 615 identified the trolley with the greatest speed as taking the greatest time. This was not a common strategy for students tested in this project, but one which Piaget, in his study of students' intuition of speed, had identified as significant.

Strategies used by the Year 12 students were restricted mainly to the use of the Formula, Proportion and comparing Distance and Time variables in written form, that is, they were taking into account all of the variables. In Question B10 most Year 12 students reverted to the formula to compare the speeds of the two trolleys.

From the work done in the pilot study and confirmed in the main study, it was realised that words like fast or faster had more than one meaning for students. It could indicate greater speed or it could indicate shorter time. There was a conscious avoidance of these terms in the question paper and when any explanations were given to students. Thus, when a student used these terms it was from their own repertoire of words which they associated with speed. This confusion in terminology occurred more often with the Year 6 students and two of the Year 9 students.

The Year 9 students used a range of strategies with the most common being that of a written explanation of the changing variables. A number of students focused on one variable to solve the problems. It was only when the more able Year 9 students reached Question B10 that some of them reverted to a calculation-type of approach. An analysis of the strategies employed by Year 9 students has been reported previously by Cuthbert and Pegg (1993).

Table 5.12
Frequency of strategies for Part B Questions

	Strategy	Year 6 (n = 25) Frequency						Year 9 (n = 20) Frequency					Year 12 (n = 17) Frequency				
		B1	B2	B3	B5	B8	B10	B1	B3	B5	B8	B10	B1	B4	B5	B8	B10
Two Vari-able	1. Formula	-*	-	-	-	-	-	-	-	-	-	-	1	7	6	7	10
	2. Calculation	-	-	-	-	-	-	0	1	0	0	3	-	-	-	-	-
	3. Proportion	0	0	7	1	1	0	0	4	1	3	1	0	6	0	3	2
	4. Written	7	5	1	7	8	3	11	6	10	6	4	14	2	8	4	2
One Vari-able	5. Time only	6	2	6	3	6	6	2	4	1	3	1	0	0	1	1	0
	6. Distance only	5	8	10	5	4	5	0	2	5	6	5	-	-	-	-	-
Vis-ual	7. One in front	0	9	-	0	1	1	0	0	1	0	3	-	-	-	-	-
	8. Faster	0	-	1	4	2	3	}3	1	0	0	0	0	0	0	1	1
	9. speed	4	0	0	2	1	2	}					-	-	-	-	-
	10 Guess	0	1	0	1	0	0	1	1	1	0	1	-	-	-	-	-
	11 Unknown	3	0	0	2	2	5	3	1	1	2	2	2	2	2	1	2

* The dashes (-) indicate that no student at that Year level used that strategy.

Relationship between strategy and response

In the discussion above, the issues of fullness of response given by students and strategies employed by students have been reported for the Dual Focus Closed Comparison Questions of Part B. Consideration is now given to the relationship between these two issues.

Table 5.13 is a cross tabulation of the Fullness of Response and strategies of students. Over all the students participating and all the questions, there were a total of 324 responses.

The most common strategy for those who gave Correct Full (CF) responses was that of comparing the distance and the time using the variable names. Fifty-seven percent of the correct full responses involved this strategy with about equal numbers of responses across all Year levels. For those students who gave correct full responses the proportion strategy was used by 20 percent of students and the formula and calculation was also 20 percent, mostly by Year 12 students.

Table 5.13
Cross tabulation of Fullness of Response
with Strategy for Part B questions

	Fullness of response							
Strategy	CF	CP	CU	CI	IF	IP	IU	Total
Formula	22							22
Calculation	5							5
Proportion	27	1				1		29
Written	78	8		1	5	4		96
Time only		18		13	2	20		53
Distance only		27		2		18		47
Front				2		9		11
Faster				12		4		16
Speed		3		3		2		8
Unknown			28	3			6	37
Totals	132	57	28	36	8	57	6	324

The strategy of using distance or time by itself, was used in nearly a third of all responses but only a half of these responses were correct. This strategy was used mostly by Year 6 students and a few Year 9 students, but no Year 12 students.

When responses included a consideration of both time and distance (the first four rows of the table) they were more likely to be correct. Responses that referred to time or distance only were correct by default, since just as many had incorrect and/or inappropriate responses.

There is a close relationship between strategy and fullness of response. The use of both variables was more likely to produce a full response, and the use of a full response implied that students must refer to both variables. A student referring to both variables was more likely to produce the correct answer.

INTERVIEWS AND WRITTEN RESPONSES

The interviews indicated that most students provided the same answer in the written test and as they did in the interview. However, there was more variation between rather than in the types of strategies employed in the written test and the interview. This variation in types of responses provided insight into relationships between strategies and gave better understanding to some strategies.

Variations in Responses

The first example provided, illustrates a link that exists between two strategies that have been identified as different.

Written response:

> Ans: *B.*
> Exp: *it reaches the farthest* [sic] *in only 3 secs.*
>
> Student 903 Question B5

This was categorised as a written distance and time strategy, however, in the interview the student attempted a proportion type strategy by comparing the two variables for both trolleys, which led him to conclude that the bigger distance was the important variable.

The interview:

S: *um ·· B because it reached five in three seconds and A only reached three in three seconds and five is going the fastest.*
[same answer, different strategy]
I: *Why?*
S: *Because it only took three seconds to go to five.*
I: *But A took three seconds to get to three.*
S: *·· well I don't know um um the seconds are shorter to get to a lower number.*
I: *Are you saying it went a bigger distance?*
S: *Yea in fewer seconds.*
I: *Righto? But the seconds are the same.*
S: *Yea but ·· but in three seconds A only stopped at three.*
I: *Yes so A stopped at three.*
S: *And B took three seconds and it got to five.*

This student made an effort to explain why there was a change but could not express the reasoning in terms of the variable names, he kept referring to the numerical values given to the variables. He was successful, with considerable prompting, in indicating that the bigger distance for the same time implied a greater speed which was similar to the test answer. Thus, in this case, there is a link between the proportion response he gave initially in the interview and how he explained it in terms of distance and time.

An example of how the interview clarified some strategies given in the test is found in the responses of Student 114 on Question B5.

Written response:

Ans: *B.*
Exp: *B has greater speed more time.*

The answer was correct but this student had misread the times, as each trolley had the same time. In the interview, she recognised that the times were the same but used speed as equivalent to the distance.

The interview:

S: *They have the same time and different speed ····· B.*
I: *Why?*
S: *Um. It is ····· guess.*
I: *Just a guess?*
S: *An intelligent guess. I used a formula.*
I: *Oh! Did you use a formula to get your answer?*

S: *No. But I can explain it to you using the formula, speed equals distance over time.*
I: *Can you explain how you did it originally?*
S: *Same time A travelled three and B travelled more therefore B.*

Thus speed, in her initial answer, referred to distance. She was well aware that an object that travels a greater distance in the same time has greater speed. It is interesting to note that when this student was asked to give an explanation she was prepared to use the formula as her strategy but admitted that this was not how she had arrived at her initial answer. This occurrence may not be isolated among students who 'know' the answer then have to think up a method to justify it. Similar situations are discussed in the next chapter. This student found the interview very stressful and eventually she could not explain any of her answers, and the interview was abandoned.

The interview with Student 616 on Question B2 confirmed the 'in front' strategy that was used by some students. This student gave a better response in the interview though there is still no mention of time.

Written response

Ans: *B.*
Exp: *Because it is in front.*

The interview:

I: *Which has greater speed?*
S: *.. the same.*
I: *Why?*
S: *Because that one* [A] *is back there and that one* [B] *is in front to start with, so it is one notch in front of it* [A] *so then so really they are at the same place, because if you moved it [A] forward it would be at 4.*

The interviewer then confronted him with the answer he gave in the test.

The interview:

I: *Now in the test you told me that B was going the at greater speed. Why do you think that you might have done that?*
S: *Just looked at it and it was in the front.*

The test response suggests that the student was taking a visual approach to the problem. In the interview he was able to imagine that if Trolley A was moved 'one notch' forward then both trolleys travelled the same distance. He then interpreted this as both trolleys having the same speed. His final answer in the interview confirms what was implied from the test result, namely, he had "just looked at" the diagrams. (This interview was given two weeks after the test so he might not have remembered his actual answer to this question but he did know his possible alternative strategy.)

In summary, the interviews have clarified some points that have already been alluded to in Part A. Some of the strategies that have been identified as different can be grouped together. This means that some procedures have similar features, and students who use one strategy within a cluster are likely to choose either that strategy again or a similar one within the same cluster. This issue is explored in more detail in Chapter 7.

Two additional outcomes that came from the interviews are that: First, some of the obscure explanations given by students in the test have now been clarified; and, second, strategies chosen by students could be confirmed in the interview even when the student had chosen, initially, a different strategy.

SUMMARY - PART B

In the Dual Focus Closed Comparison Questions of Part B, all students were able to obtain a correct response to Question B1 (where all variables were unchanged) but only fifty-two percent gave a full correct response. Fifteen Year 6 students and five Year 9 students gave only partial explanations, focusing on either time or distance but not on both. As the Questions increased in complexity, so the number of full correct responses decreased. The number of variables changing played a key role in determining the difficulty of the item. Often this resulted in students focusing on the variable(s) changing to the neglect of those which were constant. The apparent anomaly to this pattern for Question B10 was explained by students being able to arrive at a correct answer by using incomplete strategies.

A wide range of strategies were used to solve the questions in Part B, but only three of these strategies would produce a correct result consistently, that is, using calculation, proportion and time/distance comparisons. Those who had access to the formula found it the easiest strategy to apply, especially as the problems became more complex. The proportion approach was common but it was not always possible to determine how students compared the two ratios.

Another issue that arose concerned students referring to the variables in two ways. They either, used the name of the variable or the numerical value. An analysis of these ways of referring to the variable is considered in Chapter 6.

PART C: DUAL FOCUS OPEN COMPARISON

Part C of the test consisted of the Dual Focus Open Comparison Questions on speed. Questions C1, C2, C3 and C5 contained only one variable that was shown as different on the diagram, and students had to identify a value for the variable that was unknown. (Questions C3 and C5 were given to Year 12 students only.) In the other questions, two variables were changed, and students had to find or calculate a value for the unknown variable.

Once again the fullness of responses varied from Full to Unknown. A summary of the frequency of the Fullness of Response is provided in Table 5.14. The table indicates the number of correct and incorrect responses for each question for the different Year levels as well as the Student Difficulty Ratio (SDR) at the bottom of the table for each question. It should be noted that in the Year 12 speed test, Question C1 was placed after Question C9.

As expected the direct variation Question C1 provided the least difficulty for all students, followed by the inverse variation Question C2. However, the SDR score suggests that Year 6 students had some difficulty with Question C2. None of the Year 6 students were able to deal with questions in which two variables changed.

The Year 9 students did find the questions increasing in difficulty as they went from Questions C1 to C9 (see the SDR). The one student who did have Question C9 correct was confused in his explanation. He had Question C7 correct also but Question C8 was incorrect. The number of students who gave inappropriate responses or no response to the last three questions was large and is reflected in the very low SDR scores for these questions. Consideration is given later as to why some of the Year 9 students answered correctly Question C7 but found difficulty with Questions C8 and C9.

The Year 12 students did most of these questions correctly with full responses but they also had more strategies available to them when attempting these problems. Questions C7 and C9 caused a number of students to falter as they tried questions with two variables differing. Responses to Question C8 had more full correct answers when the students were required to find the distance the trolley had travelled.

Table 5.15 provides typical responses that illustrate the different fullness of responses from Years 6, 9 and 12. In general, the Year 12 students provided responses that are mathematical in nature whereas other students provided responses that use written language to explain how they arrived at their answer. There are exceptions to this, for example, Student 905 on Question C1 gave a very cryptic explanation (see Table 5.15).

Table 5.14

Frequency for fullness of response for the Part C questions.

Type of response	(code)	**Year 6** (n=25) **Questions** C1	C2	C7	**Year 9** (n = 20) **Questions** C1	C2	C7	C8	C9	**Year 12** (n = 17) **Questions** C3	C5	C7	C8	C9	C1
Correct responses															
- Full explanation	CF	8	3	0	13	6	7	0	0	15	13	8	12	8	11
- Partial reason	CP	9	2	0	2	3	1	2	1	0	1	4	1	1	0
- No reason, or guess	CU	1	0	0	1	4	0	0	0	1	1	0	1	1	4
Total correct		18	5	0	16	13	8	2	1	16	14	12	14	10	15
Correct answer															
- Inappropriate reason	CI	1	1	0	0	0	0	0	0	0	0	0	0	1	0
Incorrect responses															
- Full explanation	IF	0	1	5	0	1	0	0	0	0	0	1	0	0	1
- Partial reason	IP	1	8	4	1	2	2	3	2	1	0	2	1	0	0
- Inappropriate	II	3	2	7	2	2	4	9	9	0	0	0	0	3	0
- No reason / not done	IU	2	8	9	1	2	6	6	8	0	2	2	2	3	1
Total incorrect		7	20	25	4	7	12	18	19	1	2	5	3	7	2
Student difficulty ratio (SDR)		1.24	0.40	0.10	1.50	1.05	0.62	0.17	0.06	1.80	1.67	1.32	1.55	1.12	1.60

Table 5.15
Examples of fullness of responses
in Part C using Questions C1 and C2

Question C1 Fullness of response	Student Number	**Question C2** Fullness of response	Student Number
CORRECT FULL (CF) Ans: 2 cm v = d/t. Exp: v = d/t = 2d = 2 cm.	101	CORRECT FULL (CF) Ans: 4. Exp: Distance is the same but time is halfed [sic] so B would have to be going 2*x* as fast.	906
CORRECT PARTIAL (CP) Ans: 2. Exp: half 4.	905	CORRECT PARTIAL (CP) Ans: 4. Exp: Because the sec are different. Because it is 1 sec to get to 4.	913
CORRECT UNKNOWN (CU) Ans: 2 m. Exp:	105	CORRECT UNKNOWN (CU) Ans: 4 units per sec. Exp: I guessed it.	105
CORRECT INAPPROPRIATE (CI) Ans: 2. Exp: it will get 1 secs faster.	605	CORRECT INAPPROPRIATE (CI) Ans: 2. Exp: it's on the one as A/B.	620
INCORRECT FULL (IF) Ans: 4 cm. Exp: s = d/t d = st = 4*x*.	114	INCORRECT FULL (IF) Ans: B. Exp: B is faster because he has done the same distance in half the time.	914
INCORRECT PARTIAL (IP) Ans: 3. Exp: B went 1 secs slowy [sic] than A.	624	INCORRECT PARTIAL (IP) Ans: 1. Exp: because it is half of two.	901
INCORRECT INAPPROPRIATE (II) Ans: 5. Exp: Because it is travelling faster than A.	901	INCORRECT INAPPROPRIATE (II) Ans: A 1 sec B 1/2 sec. Exp: tack [sic] 1/2.	911
INCORRECT UNKNOWN (IU) Ans: Exp:	913	INCORRECT UNKNOWN (IU) Ans: 2. Exp:	904

Table 5.16 shows the frequency of the number of correct responses for students.

Table 5.16
Frequency of the number of correct responses for each student in Part C.

	Number of correct responses 6	5	4	3	2	1	0
Year 6	-	-	-	0	5	13	7
Year 9	-	0	3	4	6	4	3
Year 12	10	1	3	1	1	1	0

As expected, Table 5.16 indicates that none of the Year 6 and 9 students answered all their questions correctly and a number of students had all answers incorrect. Eighty-five percent of Year 9 students gave more than two incorrect responses. Most of the Year 12 students answered all these question correctly. Compared to Part A and B questions, these Part C questions presented a greater challenge to students.

Question C7 and C8

Consideration is now given to the following question: Why were some Year 9 students able to answer correctly Question C7 yet were unable to do Questions C8 and C9? Questions C7, C8 and C9 are similar in that two variables are stated to be changing, however, Question C7 had a higher number of correct responses.

Seven Year 9 students had Question C7 correct with a full response. Below are the responses for Question C7, C8, C9 (in that order) of two of these students to illustrate the type of responses that were provided.

Written responses of Student 915:

Ans: *2.*
Exp: *esay [sic] 2 = 6 ÷ 3.*

Question C7 (CF)

Ans: *2.*
Exp: *esay [sic] 2 = 6 ÷ 3.*

Question C8 (II)

Ans: *2 sec.*
Exp: *1/3 of 6 seconds.*

Question C9 (II)

Written responses of Student 917:

Ans: *1.*
Exp: *B trolley makes 2 metres in one second.*

Question C7 (CF)

[The answer is incorrect but this looks like a slip when the explanation contains the correct result.]

Ans: *18.*
Exp: *speed doubled and time increased by 3.*

Question C8 (CP)

Ans: *1.*
Exp: *the trolley B has double the distance and speed gone down by one third.*

Question C9 (CP)

Question C7 appears to be easier because the question asks for speed, and students seem to have an intuitive sense that speed can be found by using distance divided by time. They do not refer to how the variables are changing as given in the question but refer to the Trolley B and divided six by three. In the other questions, Question C8 asks for the distance and Question C9 for the time, this appears to be a less intuitive notion because students either continue to use the division operation they used in Question C7 or consider the variables suggested in the question. Two students had the correct answer for Question C8 and used the exact same wording as given in the question for the changing variables in their explanation. It appears that they were copying the information provided in the question.

In summary, it appears that not all variables in speed problems carry the same cognitive load. This results in some students having a better intuitive approach to finding speed than they have of finding the other variables.

STRATEGIES USED IN PART C

Nine different categories of strategies were identified for the Dual Focus Open Comparison Questions that can be clustered into two groups. The first group includes those strategies that used both distance and time, these were: Variation - complex, Formula, Variation - direct and inverse, Calculation. The second group of strategies were informal. These were: Closure operations, Variable the same, Faster/slower, speed and guess. Each of these strategies is now discussed with examples from the test.

1. Variation - complex

A response that used the variations as they were given in the problem for those questions that had two variables changing were coded as Variation - complex, that is, these responses could not be coded as simple direct or inverse variation as they were a mixture of both. This type of strategy was attempted by seven Year 12 students and two Year 9 students. The first two examples illustrate this strategy in a straight forward way but the third is different.

Ans: *18.*
Exp: *(2* x *A's distance)* x *3.*
Student 108 Question C8 (CF)

Ans: *6.*
Exp: *1/3 speed* x *distance = 6.*
Student 108 Question C9 (CP)

Ans: *18 cm.*
Exp: *B would go 6 cm in 1 sec and it would go 18 cm in 3 secs.*
Student 103 Question C8 (CF)

In this last response, the variations used by the student were not those given in the problem but the student had deduced that the speed is six units per second and has then used direct variation to find the distance.

The next example illustrates this complex variation but the explanation mirrors the way the variations are referred to in the question.

Ans: *18.*
Exp: *Speed has doubled and the time increased by three.*
Students 919 Question C8 (CP)

This response (and a similar one given by Student 917) used the following strategy: 2 x speed (i.e., 2 x 3 = 6). The final step involved increasing the distance by a factor of three giving eighteen (18 = 3 x 6) because the time was increased by a factor of three.

2. Formula

A response was coded as 'formula' if it included the formula $v = d/t$ or it was implied by the response. The following example illustrates this from a Year 12 student who studies Physics.

Ans: *2 m/s.*
Exp: $d = 6,\ t = 3,$ *velocity* $= 2$ *m/s*
$v = d/t.$

Student 101 Question C7 (CF)

Three students quoted the formula but used another strategy as well to complete the question. They had used ratios or a variation type strategy. The example below illustrates the response of a student who accessed the formula, and used it to find the speed of Trolley B but included in his response the ratio of 2:3, i.e., distance : time.

Ans: *2.*
Exp: $v = d/t \quad B = 2 \quad A = 3$
$2 : 3.$

Student 108 Question C7 (CP)

The next example is from a student who used the formula but mentioned the way the variables were changing as part of the response.

Ans: *18 cm.*
Exp: $A - \ s = d/t = 3/1 = 3$ *doubled and increased by 3*
$=> B \quad d = s \times t = 6 \times 3 = 18$ *cm.*

Student 117 Question C8 (CF)

3. Variation

The questions in Part C were designed to find methods students would use with variation type problems, though they were not limited to using the two major strategies, direct and inverse variation.

Direct: In this strategy the student could apply the rule: as one variable increased then the other variable increased in the same ratio. Questions C1 and C3 were suited especially to the direct variation strategy. Three examples are given here, one from each Year level.

Ans: *Just before the two*
Exp: *It was half the time half the distance.*
Student 606 Question C1 (CF)

Ans: *2.*
Exp: *Trolley A is at 4 in 2 secs, so trolley B has to be around 2 in 1 sec.*
Student 903 Question C1 (CF)

Ans: *8 cms.*
Exp: *It took the trolley 2 secs travelling at 2 to travell [sic] 4. Therefore double speed double distance.*
Student 106 Question C3 (CF)

The Year 6 student referred to variable names but the Year 9 student focused on the values. The Year 12 student used the values and then summarized the explanation by using the variable names. A progression such as this suggests some ordering of processes. Students at a lower level of cognitive development use the names of the objects that they refer to. Those who are at a higher level could use the values of the variables but were not able to tie together all the elements presented. Students at a level above this used both variable values and refer to the names of the variable. They have an overview of the elements in the question. This helps to explain why Year 6 and Year 12 students used the names of the variables and appear to be at the same level. The Year 6 student is not giving an overview but indicating how the values have changed whereas the Year 12 student has understood the concept to give the general case. Both responses look similar but the underlying cognitive processes are clearly different.

Inverse: In using this strategy the student saw that increasing one variable meant that another variable would decrease in the same ratio. Questions C2 and C5 were suited to this strategy. Again three examples are given, one from each Year level.

Ans: *4.*
Exp: *If its half the time and twice the speed.*
Student 606 Question C2 (CF)

This student (606) used the variable names in the explanations of this question and also for Question C1 (see example above under direct variation). This was typical of those students who applied inverse variation and of some who used direct variation. Such responses may be triggered by the way the question was posed, that is, the variables that change are stated explicitly.

Ans: *4.*
Exp: *cause if its half the just time speed A by 2.*
[Possible translation: Because if it is half (the time), then just multiply the speed of A by 2.]
Student 920 Question C2 (CF)

Ans: *1/2 sec.*
Exp: *A doing 3 km/hr 1 secs and 3 km, B doing 6 km/hr 2 times faster half the time.*

Student 112 Question C5 (CF)

This last response showed some confusion with the units, nevertheless, the final phrase which sums up the problem, indicates clearly that the strategy used was inverse variation.

4. Calculation

As in Dual Focus Closed Comparison Questions in Part B a response is coded as calculation when the correct calculation was used but did not refer to a formula such as $v = d/t$. The usual responses of students indicated that they were not aware of a formula approach to solving speed problems.

(a) d/t: In the first category of calculation, the distance is divided by the time to calculate the speed of Trolley B. In most of these cases Trolley A was ignored. For example:

Ans: *2.*
Exp: *esay* [sic] $2 = 6 \div 3$.
Student 915 Question C7 (CF)

Ans: *Speed = 2.*
Exp: *if A went 3 cm in 1 sec then B would go 2 cm in 1 sec if it went for 3 secs & 6 cm*
$3 \div 6 = 2$.
Student 103 Question C7 (CF)

These students do not have a formal idea of the formula. The latter student, who does not study Physics, made use of it intuitively by reducing the ratio of distance and time back to distance per second. For Trolley A, it is not clear whether she is looking at the speed given on the trolley or deducing the speed from the distance and time. For Trolley B she has seen that it went six centimetres in three seconds. From this she deduced the speed as two centimetres in one second. She then tried to justify her answer by using an operation, and chose to divide three by six and obtained two which was the correct answer to the question but not with three divided by six. This seems to be a re-occurring procedure for students who do not have formal training in handling speed problems. They can do the problem 'in their heads' and then they have to search for an explanation and often write the explanation down incorrectly without noticing the consequences.

(b) d x v: In the second category of calculation, students multiplied distance and speed to find the time.

> Ans: *2 sec.*
> Exp: *Take the speed and units and times* [multiply].
> Student 912 Question C9 (IP)

This is similar to an example of using an arithmetical operation. However, this student, as well as Student 915, (see example above) has in some of the previous problems been dividing distance and time to obtain speed. This seems to be a case of taking two variables and operating on them to obtain a reasonable outcome. Sometimes this method yields the correct answer. The student attempted to work with the variables and not the numerical values. If the student had actually done what was stated in the explanation, and multiplied the 'speed' of one unit per second by the six 'units', then he would have obtained the correct answer. This response could have been placed in the closure strategy but because of the correct explanation by using the right operation on the variables it has been coded as calculation.

(c) Non-formal: The next strategy coded as a calculation occurs when responses refer to speed in non-formal ways. Usually, speed is measured as the distance travelled in unit time, however, these students have not used unit time.

> *Ans: 2 units/0.5 or 4 units/sec.*
> *Exp: it would be 2 units per half a second because it did.*
> Student 907 Question C2 (CF)

This student possibly used the following thought process: The time is halved and A was going two units in 1 second so that means B will go two units in half a second (the time is halved) which gives '4 units/sec'.

A second example:

Ans: *3 units/1.5 secs.*
Exp: *If car a makes 3 units in 1 sec & car B makes 3 units in 1.5 secs.*
Student 918 Question C7 (CF)

Here the student saw that Trolley A has moved three units in one second and has found how much time it would take for Trolley B to travel the same distance.

(d) 1/3 of a variable: The last strategy, coded as a calculation, occurred only in Question C9 where five students used a strategy of multiplying a value by one third. They had ignored the fact that the distance was doubled.

Ans: *2 sec.*
Exp: *1/3 of 6.*
Student 905 Question C9 (II)

This student took one third of the distance six units and obtained two seconds, ignoring the other variations.

5. Closure operations

When a response indicates that an operation has been carried out that is not relevant to the problem, then it has been coded as a closure operation. However, this could have happened at other times for other students' responses but it has not been identified easily as the student has chosen to use the correct operation. The two operations, addition and multiplication, were identified as simple closures. The first two examples illustrate addition closures.

Ans: *3.*
Exp: *because it's 1 sec.*
Student 608 Question C2 (IP)

Here the student added either the two units per second or the two seconds to the one second.

Ans: *5.*
Exp: *I guessed.*
Student 909 Question C7 (IU)

This student could have added the three and the two that were given as the way the variables were changing.

The next two examples illustrate multiplication closures.

Ans: *9.*
Exp: *3* x *A sped [sic] and you will get B speed.*
Student 606 Question C7 (IF)

Ans: *6.*
Exp: *Becase* [sic] *it is 2 secs faster then A and the speedo is 3.*
Student 618 Question C7 (IF)

The last student saw that the change in time is two seconds and multiplied this increase by the speed.

6. Variable the same

This strategy is not easily understood, in that the student may have become confused as to which variable is changing and has stated that one of the variables is the same in the explanation to the answer. The following example illustrates this for Question C2 where the distances were the same but the student focused on the time as the variable that is the same and thus concluded that the speeds must be the same.

Ans: *2.*
Exp: *because B travelled at the same time as A.*
Student 619, Question C2 (IP)

7. Faster/slower

Some students in Years 6 and 9 used the terms 'fast(er)' and 'slow(er)' to explain their answer. Such an explanation was usually inappropriate and led to incorrect answers. It was not possible always to know what a student meant by the word fast. Two examples are given of this type of response one from Year 6 and the other from Year 9.

Ans: *4.*
Exp: *becaus [sic] it was very fast.*
Student 623 Question C1 (II)

Ans: *4.*
Exp: *because it is faster than A.*

Student 901 Question C8 (II)

In the former example it is not clear which trolley is being referred to. In the latter example, Trolley B is said to be faster than Trolley A, but no reason is given for the statement.

8. Speed

In this strategy students focused on what happened to the speed and ignored other factors.

Ans: 6.
Exp: because the speed has doubled.

Student 907 Question C8 (II)

This student doubled the speed of Trolley A (2 x 3) as suggested by the problem. She has ignored the fact that the time also increased by three and that it was the distance that was required, and not the speed.

9. Guess/unknown

In this case, students stated that they guessed the answer or had not given a response.

Overview of strategies

A range of strategies were employed by students in solving the problems and these are listed in Table 5.17 with their frequencies for the different questions for each Year group.

Most Year 6 students applied direct variation correctly to solve Question C1 but only a third of these students used inverse variation on Question C2. These students found it difficult to use inverse variation-type strategies. With the two-variable type Question C7, five students tried the direct variation while the other students could not give any response or used inappropriate strategies.

Table 5.17

Frequency of strategies for the Part C questions

	Strategy	Year 6 (n = 25) Frequency			Year 9 (n = 20) Frequency					Year 12 (n = 17) Frequency					
		C1	C2	C7	C1	C2	C7	C8	C9	C3	C5	C7	C8	C9	C1
Dist & time	1. Variation - complex	-	-	-	0	0	0	2	0	0	0	1	4	2	0
	2. Formula	-	-	-	-	-	-	-	-	8	6	12	10	10	8
	3. Variation														
	Direct	17	2	5	11	2	1	4	1	7	1	0	0	0	4
	Inverse	0	6	0	0	6	0	0	1	0	8	0	0	0	0
	4. Calculations														
	(a) d/t	-	-	-	4	3	7	3	0	0	0	2	0	0	0
	(b) d x v	-	-	-	0	0	0	0	2	-	-	-	-	-	-
	(c) Non-formal	-	-	-	0	2	2	2	0	-	-	-	-	-	-
	(d) 1/3 of a variable	-	-	-	0	0	0	0	5	-	-	-	-	-	-
Informal	5 Closure operations														
	Added	0	2	0	-	-	-	-	-	-	-	-	-	-	-
	Multiplied	0	1	3	0	0	1	0	0	-	-	-	-	-	-
	6. Variable the same	0	2	1	-	-	-	-	-	-	-	-	-	-	-
	7 Faster/slower	3	1	2	1	0	0	1	0	-	-	-	-	-	-
	8 speed	0	1	1	0	2	2	4	0	-	-	-	-	-	-
	9 Guess	2	2	2	1	4	2	2	2	-	-	-	-	-	-
	10 Unknown	3	8	11	3	3	7	6	9	2	2	2	3	5	5

Most Year 9 students used direct variation appropriately on Question C1, but only a half of these students were able to use inverse variation. Again this suggests that inverse variation is a more difficult method to use. With the two-variable type Questions C7, C8, C9, most students were unsuccessful.

The Year 9 students who used the calculation strategy did not state the formula for speed but they seemed to know intuitively that dividing distance by time would yield speed. This was not done consistently by students - seven students did this for Question C7 but none used it for Question C9. The two students who did use a calculation to find the time in Question C9 did so inappropriately.

The one Year 9 student who had Question C9 correct wrote down a wrong strategy by multiplying the distance and the speed instead of distance divided by speed. It seems as though he knew what the correct solution was and had found an operation that appeared appropriate to him. That six times one gives the same solution as six divided by one was a little unfortunate for the student but it does highlight the fact that the student was looking for an operation that worked and not the way in which the variables were actually related. Responses that are correct but with the wrong explanations, may suggest that the student knows the answer but has some difficulty in communicating how to arrive at the particular result. Such responses are analysed further in Chapter 6.

Most of the Year 12 students were able to access strategies that involved both the variables, distance and time. Having access to the formula gave them the opportunity to solve the two-variable type question more successfully than either the Year 6 or 9 students. Some of the Year 12 students, who used direct and inverse variation strategies in Questions C3 and C5, changed to a formula strategy in Questions C7, C8, C9. In proceeding from Question C9 to Question C1 some students continued to use the formula while four students changed back to direct variation. Five students, however, were coded 'unknown' for Question C1 even though three of them had successfully completed Question C3. Since the questions were becoming more difficult, they could have thought that Question C1 would be the most difficult since it was the last question on the written test.

STRATEGY AND LEVEL OF RESPONSE

The relationship between strategy and level of response is shown in Table 5.18.

Of the eighty-six unknown responses, fifty-five were incorrect, thirteen were correct and eighteen were inappropriate. Thus, students that did not give explanations, or guessed by some method, were very likely to give an incorrect response. Of the fifty-two responses that used the formula, forty-eight were correct and forty-four gave full responses There were fifty-two direct variation responses of which forty-one were correct. It is interesting to note that nearly 40 percent of responses were given correctly and fully, whereas only 53 percent of responses were correct, that is, 75 percent of correct responses were given as full responses.

Table 5.18
Cross tabulation of fullness of response
and strategy for Part C questions

	Fullness of response							
Strategy	CF	CP	CU	CI	IF	IP	IU	Total
Formula	44	4		2	1	1		52
Direct	30	11		3		8		52
Inverse	14	3	1		3	2		23
Operation				1		3		4
Calculation	12	4		3	1	1		21
Proportion	1					1		2
Variables	5	3		10	2	2		22
Faster			0	6		3		9
Unknown			13	18		4	51	86
Total	106	25	14	43	7	25	51	271

As with the other sections of the test, formula and direct variation responses were more likely to be correct and to have a full response. Such responses were dependent on the type of question.

INTERVIEWS AND WRITTEN RESPONSES

The responses to the questions in the interviews indicate that over seventy percent of answers were the same as in the test. There was, however, less than fifty percent of responses with the same strategy. Consideration is now given to this variability in the responses, both between test and interview and within the interview.

Variation in responses

The first example is that of a Year 6 student (606) who closes on an answer, after each prompt, in Question C7.

Interview:

S: *Um ··· six.*
I: *Why?*
S: *Because it is three times that ··· it is three more seconds.*
I: *But the distance has doubled?*
S: *The distance has doubled ··· six ·· three I still reckon six.*
I: *You reckon the speed is six?*
S: *Yep because that ·· Oh no its nine actually.*
I: *It's nine? Where did you get the nine from?*
S: *Because three times that speed will be nine and because it is going one second and this one [B] is three seconds.*
I: *But it doubled its distance though, its got a bigger distance.*
S: *OK that speed is nine greater than the other one.*

This student shows a variation of strategy within the interview. For the test this student wrote:

Ans: *9.*
Exp: *3* x *A sped* [sic] *and you will get B speed.*

This shows some similarity to his second closure in the interview. Here he has focused on the speed, ignoring all other variables.

In contrast, the next example considers a student who, in the interview, realised that she was wrong as she tried to explain her answer. This seemed to occur with those students who have better concepts of speed, especially with the questions that concern inverse variation, such as, Question C2. This Year 9 student, in the written responses, gave the correct answer with a full explanation.

Written response:

> Ans: *4.*
> Exp: *Distance is the same but time is halfed* [sic] *so B would have to be going 2* x *as fast*
>
> Student 906 Question C2 (CF).

Interview on same question:

> S: *That is travelling at speed one.*
> I: *Why?*
> S: *Well if half the time to get to the same distance. I mean if A gets to four in two seconds whereas B gets to four in one second so B would be going twice as fast. Oh! hang on ⋯.*
> *It would have to be going twice as fast, yes speed is four ⋯.*
> I: *What happened?*
> S: *I got a bit mixed up with the speed. I halved the speed.*

Thus, as she spoke, she realised that an inverse variation strategy was required and she changed her answer.

In summarising the variation of responses, it was found that some students had very similar approaches to solving problems, while others showed differences both in strategy and answers. Some of the differing approaches indicate that students could use more than one strategy to solve a problem successfully. Where students had different answers in the test and interview, this usually indicated that they were closing rapidly on answers and did not have stable strategies for solving the problems.

DISCUSSION

Year 6 students are familiar with the direct variation concept that if one variable is halved then another variable is also halved. For example, when the time is halved then the distance is also halved when the speed is kept constant (though students do not mention this constraint). When the variation is not direct variation, as in Question C2, only ten Year 6 students used strategies that increased the speed; six of these students could be identified as using inverse variation correctly. The other students used a variety of strategies, which were not appropriate to the problem, guessed or gave no response. It is of interest to note that two students used a direct variation approach on Question C2. Most students were aware that they could not use the same strategy in Question C2 that they had used in Question C1. None of the Year 6 students were successful when two variables changed in the problem.

Most Year 9 students employed a successful strategy to solve Questions C1 and C2 with only one variable changing using either direct, inverse variation techniques or a calculation involving distance and time. Some students were confused as to how the variables related to one another. When more than one variable changed most students could not find a suitable strategy to deal with the problem. Those that ignored the complications of the problem, dealt with only Trolley B and successfully used a calculation to solve the problem. Some of these students who used a calculation operation of multiplying speed and time to find distance showed their lack of understanding of the problems when they continued to use multiplication of two numbers in Question C9, and multiplied the speed and the distance when the time was required.

The Year 12 physics students often quoted a formula and then substituted into it, sometimes without being concerned as to the way the variables were changing. It was the non-physics students who gave the more unconventional answers with a variety of approaches and methods of dealing with variables.

SUMMARY - PART C

In the Dual Focus Open Comparison Questions the use of direct variation when one variable changed, as with Question C1 was the simplest strategy to use. When the most appropriate strategy would have been the inverse variation strategy, fewer students were able to use it compared with those students who used direct variation in appropriate questions. Instead of using inverse variation, often they used another type of strategy, for example, Year 12 students used the formula and Year 6 and 9 students were more likely to use a simple arithmetical operation to obtain an answer.

When more than one variable changed, those students with access to the formula were more successful, though some Year 9 students used a calculation to obtain a correct response. A few students employed the variation of the variables as suggested in the question to obtain correct responses but such responses seemed to taper off as the questions continued.

Some students from all Year levels responded successfully by using the variable names and they understood how those variables were related to one another.

CONCLUSION

This chapter has provided an overview of the types of responses students gave for a description of speed, and also their responses and explanations to the different types of questions that were given to them in both the test and in the interviews.

Seven different categories were identified for students description of speed, these were: A. a formal definition; B. both the variables time and distance were referred to; C. only one of the variables was used; D. objects that can travel both fast and slow; E. objects that only go very fast; F. use terms related to speed; and, G. incorrect idea or unknown. These categories were consistent with the trends identified in the pilot study and no additional categories emerged when the students were interviewed. Broad bands of groupings that relate to the different Year levels were identified. The Year 6 students tended to use Categories D, E and F that is, there meanings of speed were associated with the movement of real objects. The Year 9 students mostly used Categories D and E with two students using Category C. These students had a more informed idea concerning the concept of speed than the Year 6 students. Most of the Year 12 students gave descriptions in Categories A and B with three giving a Category D response. Thus, the Year 12 students' concepts of speed were linked to the variables associated with speed.

The quality of students' responses was related to their performance on the speed test. Students who performed well had high quality responses in that their explanations were clear and followed to the correct answer with all the variables associated with the problem being identified clearly. Students with other levels of response did not always have clear explanations and the variables were not always referred to, if at all. In general, the Quality of Response improved with Year level but on simpler questions some students in the lower Year levels were able to give Full Correct responses. The type of question presented influenced performance. When the question was set in familiar contexts with operations that were straight forward to implement, students tended to give quality responses. The number of variables that were changing in a problem was related to the complexity perceived by students.

Thus, the overall performance of students on such problems dropped quickly, especially for the Year 6 students.

The strategies that students employed ranged from the sophisticated use of the formula down to simple arithmetical closure operations. Those strategies that incorporated all the variables resulted consistently in more correct responses. In general, across the different type of questions, strategies were found to cluster into three groups. The first group were those strategies that took into account all the variables and referred to distance and time in some way. The second group of strategies included those that referred to only one of the variables. These strategies could produce correct answers if one of the variables was kept constant and the student focused on the changing variable, otherwise the answer would be incorrect. The third group of strategies relied on visual aspects of the problem. This could be from the diagram provided in the question or in the 'minds eye', as imagined by the student.

The interviews gave insight into the strategies that were employed by students and confirmed that similar strategies could be grouped together. In general, if a student changed strategy it appeared to be within the same group. Students' choice of strategy depended on the following factors: the information they read first in the question; what they think provides an answer quickly; strategies that they have at their disposal; previous experience with the questions (including experience gained while doing the test); their mood at the time of doing the question; their understanding and skills at reading and writing English. This issue of strategies employed by students is explored in more detail in Chapter 7. Another consequence of the interviews is the verification of the notion that many students used intuitive methods to solve the problems. This issue is addressed in Chapter 6.

A number of other issues that have arisen from this chapter also need further discussion. In particular, further analysis is required on: the way students use variable values and variable names in giving explanations; the effect of question complexity on students' responses; and, trends that occur in strategies chosen by students. The next two chapters address these issues in more detail.

CHAPTER 6

VARIABLES, VARIATION AND INTUITIVE THINKING

"To speed today, to be put back tomorrow"
Spencer Complaints, Mother Hubbard's Tales 1. p. 895

The previous chapter provided an overview of: student responses to questions asked in the speed tests; a list of strategies for each type of question at each Year level; and, a brief analysis of student responses within each section of the test. This chapter continues to analyse the outcomes reported in Chapter 5 and to answer more of the research questions arising from the pilot study in Chapter 3. In particular, the focus is on: the way students refer to variables in their explanations (Research Question 7); the methods used by students in solving questions involving direct and inverse variation (Research Question 8); and, the role of intuitive thinking in solving speed problems (Research Question 9).

STUDENTS' REFERENCE TO VARIABLES

It was noted in Chapter 5 that in the explanations given by students, some referred to the variable by name while others referred to the numerical value of the variable. These two ideas are referred to as **variable name** and **variable value**, respectively. In addition, some Year 12 students could access the formula and solve problems using **variable symbols**. While this might be considered as a subset of 'variable name', it has been identified separately in the analysis which follows.

Consideration is now given to students use of variables for each part of the test.

SINGLE FOCUS QUESTIONS

The questions in Part A of the test were of two types: Numerical (labeled with a N) which involved using simple calculations and Variation (labeled with a V) which concerned direct and inverse variation. Table 6.1 indicates the frequency of the different ways students referred to the variables in their responses for the Part A questions. Students who did not give a response or, who did give a response but gave no indication as to how they might have used the variables, have been coded as 'unknown'.

Table 6.1
Frequency of use of variables in Part A questions

Type of Variable Used	Year 6 (n = 25)				Year 9 (n = 20)				Year 12 (n = 17)			
	N1	N3	V1	V2	N1	N3	V1	V3	N3	V1	V3	V5
Symbol	-	-	-	-	-	-	-	-	4	4	3	5
Name	-	2	15	12	-	-	15	11	3	10	7	8
Value	22	14	3	2	20	17	-	4	10	1	4	4
Unknown	3	9	7	11	-	3	-	5	-	2	3	-

The table above shows that students did respond in the way the problems suggested, that is, Questions N1 and N3 have a high incidence of numerical responses and Questions V1, V2, V3 have a high incidence of variable names in the responses. Some Year 12 students did not use the variable name but expressed the relationship in mathematical ways that suggest they were looking at the variables in their symbolic form rather than the actual value of the variable. Such responses were coded as 'symbol'.

Students who used the formula showed that they could name the variables with a symbol (e.g., *d* represents distance) and then they could use the numerical information to solve the problem. Thus, they identified both the variable by using its name or symbol and the variable value given to that variable in the substitution.

THE DUAL FOCUS CLOSED COMPARISON QUESTIONS

The questions in Part B of the test consisted of displaying two trolleys with the distance of travel and the time taken indicated on diagrams. The students had to indicate which trolley had the greater speed. There was no particular method of solution embedded in the question as in Part A.

Table 6.2 provides a summary of the frequency for responses that used variable name, variable values or symbols for the Dual Focus Closed Comparison Questions. The last column (headed CO) for each Year group indicates the number of students who consistently used that mode (where 'consistent' is defined to be using it four or more times).

Table 6.2

Frequency of use of variables for Part B questions

Type of Variable Used	Year 6 (n = 25)							Year 9 (n = 20)						Year 12 (n = 17)					
	B1	B2	B3	B5	B8	B10	CO ≥4	B1	B3	B5	B8	B10	CO ≥4	B1	B4	B5	B8	B10	CO ≥4
Symbol	-	-	-	-	-	-	-	-	-	-	-	-	-	1	6	6	7	8	5
Name	17	15	7	15	15	10	11	7	10	8	9	6	4	13	3	7	5	3	2
Values	4	5	15	5	7	8	5	8	7	7	6	8	5	2	7	3	4	4	1
Unknown	4	5	3	5	3	7	-	5	3	5	5	6	-	1	1	1	1	2	-

From Table 6.2 it can be seen that Year 6 students used the variable name more than Year 9 and 12 students, and they did so more consistently. This occurred even though there were six questions given to Year 6 students compared to five questions given to students in Years 9 and 12.

Most students used the variable name in Question B1 to describe how they chose their answer. This question had all the variables the same, so it was not necessary to do any calculation but just to mention that the distance and the time were the same, and hence the speed must be the same also. Nearly half of the Year 9 students, however, answered this question by referring to numerical information.

There was a change in the type of responses given as Year 6 students proceeded from Questions B2 to B3. They started by using the variable name, but in Question B3 they reverted to variable values. In this question, the trolleys were going at the same speed, with both distance and time different, so the numbers became more important to the student. This shift is due to the less experience that Year 6 students have with ratios as they attempted to unravel the question and reverted to numerical values. For Year 9 students, there was a slight shift in the reverse direction, that is, the number of students using variable values to using variable names increased. Some Year 9 students were able to see the ratios of the numbers in a clearer way and referred to the variable name rather than the numbers.

Students in Years 9 and 12 were not as consistent as Year 6 students in their use of variable name or value, though five Year 12 students were consistent in using symbols in the formula. They used variable name (symbols) to give the formula then substituted values to do the calculation. The low consistency figures for Year 9 and 12 students indicate that some students could shift from one type of variable mode to another as they progressed through the questions.

THE DUAL FOCUS OPEN COMPARISON QUESTIONS

The questions in Part C were presented in a diagrammatic form. Students were required to find the missing variable on the second trolley. The variables that had changed were indicated under the diagrams. Often students' explanations used either the variable names or the variable values displayed. A summary of the frequency for each question is shown in Table 6.3. The last column for each Year group headed with 'CO' (for consistent) is the same as that used in Table 6.2. The number under the 'CO' refers to the number of times students needed to use it to be considered consistent.

Table 6.3

Frequency of use of variables for Part C questions

Type of Variable Used	Year 6 (n = 25)				Year 9 (n = 20)						Year 12 (n = 17)						
	C1	C2	C7	CO =3	C1	C2	C7	C8	C9	CO ≥4	C3	C5	C7	C8	C9	C1	CO ≥5
Symbols	-	-	-	-	-	-	-	-	-	-	6	5	10	9	10	8)	8
Name	13	8	4	4	7	7	3	6	2	3	6	7	0	1	0	3)	
Value	6	6	9	2	9	7	10	5	11	4	4	3	5	4	3	1	2
Unknown	6	11	12	4	4	6	7	9	7	4	1	2	2	3	4	5	1

There was a decrease in the number of students using the variable name when they went from questions with only one variable changing to questions with two variables changing. For example, from Questions C2 to C7 in Years 6 and 9 and from Questions C5 to C7 in Year 12. A number of students used the variable value in the two-variable type problems, though some Year 12 students reverted to the formula to solve these questions.

Half the Year 6 students could refer to the variable names in Question C1. This type of response was triggered by the way the question was presented with the variable that was changing being given and named.

Fewer Year 12 students used the variable name on Question C1 than Questions C3 and C5, which are all one-variable-differing problems. Question C1 was placed at the end of the test instrument, that is, Year 12 students did Question C1 after they had completed the questions with two variables differing. Of interest was that many of these students, already in 'a formula mode of operation', continued in this mode when they did Question C1. All four students who were coded as 'unknown' for Question C9 were also coded as 'unknown in Question C1. These students could see that the questions were becoming more difficult and possibly they assumed that the last question would be difficult also and therefore they did not analyse it properly. Thus the ordering of the questions does influence the outcomes of responses in two ways: first, in most tests, students are familiar with questions becoming more difficult as they proceed, so they assume that the latter questions of a test are more difficult; and secondly, when students have started a certain way of operating in one question they are more likely to continue to use the same approach in the next question even if simpler methods are available.

Overview

Research Question 7: How do students refer to variables in their explanations? What influences the way these variables are used?

It was found that the way the question is presented can trigger a response as to whether students use variable symbols, variable names or variable values to explain how they solved the problem (see Part A above). The range of strategies students have available determines also the way in which students refer to the variable. In the responses to the Part B questions students used a variety of methods to refer to the variables, even though the questions were given in a similar manner. The different ways the variables were referred to was not consistent across student responses within a question or within responses of students across questions. It appears that students change their strategy and the way they referred to the variable at will. When the number of variables changing was increased from one to two, fewer students would refer to the variable name. They were not able to relate the variables to each other. There appears to be a link between the number of variables that differ and the way students chose to refer to the variables. The effect of the number of differing variables on the strategy employed by students is examined further in Chapter 7.

DIRECT AND INVERSE VARIATION

From the outcomes of Chapter 5 it was observed that students could use the strategy of direct variation to solve problems more effectively than they could use inverse variation. The questions that attempted to evoke a direct or inverse variation approach were in the Variation Speed Questions in Part A (coded with a V) and the Dual Focus Open Comparison Questions of Part C with one-variable differing. In particular, these questions, which relate specifically to direct or inverse variation, that were given to different Year levels are shown in Table 6.4

Table 6.4
Questions that invoke a direct or inverse strategy

	Year level		
Type of Variation	6	9	12
Direct	V1 C1	V1, V3 C1	V1, V3 C1, C3
Inverse	V2 C2	- C2	V5 C5

Table 6.5 presents a contingency table for the results as a percentage of responses with correct answers as well as for correct full responses (CF). The leading diagonal (in bold print) indicates the percentage of students who had that particular question correct. For example, eighty-one percent of responses for the direct variation questions in Part C were correct and fifty-eight percent were correct full responses. Whereas, only sixty percent and twenty-nine percent of responses had both Part C and Part A direct variation questions correct.

Table 6.5
Contingency table (in percentages) of correct responses
for direct and inverse variation questions

Variation		Direct		Inverse	
	Part	A	C	A	C
Direct	A All	**89**	60	50	44
	CF	**39**	29	24	23
	C All		**81**	50	48
	CF		**58**	26	31
Inverse	A All			**60**	38
	CF	-------		**33**	24
	C All				**52**
	CF				**35**

The percentages on the leading diagonal decrease down the table. In particular, students had less inverse questions correct than direct questions, and they had less Part C questions correct than Part A questions. The latter finding was anticipated when the questions first were designed. The drop in percentages across the table indicates that only some students who used successfully direct variation could use also inverse variation in the appropriate questions. The decrease in percentages going up the table in the last two columns, indicates that having an inverse question correct does not guarantee a correct response to the direct variation question. This issue is examined later in this section.

The direct questions in Part C had a better Correct Full response rate than the direct question in Part A (see the leading diagonal). Two factors that could influence this were the diagram and the written statement of the how the variables had changed. The number of correct responses decreased a little in Part C implying that it was either more difficult to guess correctly or that those who had poor strategies could not continue to give reasonable explanations. This did not seem to be the situation, however, with the inverse variation questions, with both Part A and Part C having about the same correct response rate.

There were only two Year 6 students who had correct answers for both the inverse variations of Part A and Part C, they were both full responses. Similar outcomes were observed with inverse and direct variation of Part C and also with the inverse variation of Part C and the direct variation with Part A. Though in these cases, there were two other students who had them both correct but not with a full response. It appeared that for Year 6 students to have both variation-type problems correct, it was necessary to have a full response. This means that students, in general, must know what strategy they are going to use and how to use it, if the question is to be answered correctly and fully. Of the ten Year 6 students who used inverse strategies in Question V2, three of these students could not use direct variation in Question V1.

There were three students in Year 6 who had the inverse Question V2 correct yet had the direct Question V1 incorrect. Student 603 answered Question V2 with a correct and full response but he did not attempt Question V1. This student, however, completed the direct Question C1 but did not attempt the inverse Question C2. Student 613 used a subtraction approach to Question V1:

Ans: *10 times.*
Exp: *I said to my self [sic] take away ten from 20.*

For Question V2 he had the correct answer but his explanation shows a more intuitive approach that depended on his past experience.

Ans: *half times [sic].*
Exp: *i fig erd [sic: I figured] that anew [sic] car would go alot [sic] faster.*

This student could do neither of the Questions C1 or C2. His explanation indicates that he misunderstood Question C1.

Ans: *B*
Exp: *the b car is new because it is feather [sic] in front.*

Student 621 stated in Question V1 that she could "not understand so I left it". However, for Question V2, she gave a correct and full response using the inverse strategy. For Question C1 she had the correct answer and a partial explanation, even though she could not complete Question C2.

These responses of the Year 6 students indicate the complexity of trying to generalise across questions and students in finding trends and overviews. That some students could use inverse or direct variation in one situation but not in another can not be explained easily although it is not surprising. It was responses from students such as these that influenced the percentages in Table 6.5. This means that students, who used an inverse operation correctly, would not be guaranteed to have the direct variation question correct. Such students were the exception rather than the rule.

The outcomes of Chapter 5 Table 5.17, indicate that six Year 9 students used the inverse variation strategy on Question C2 and all of these students used direct variation correctly for Question C1. Five of these students referred to the way the variables changed, for example:

> *Distance is the same but the time is halfed [sic] so B would have to be going 2x* [two times] *as fast.*

These students used the same kind of response in Question C1 when dealing with direct variation. This manner of thinking could be triggered by the way the question was asked, that is, the question stated how the variables change. Not all students were influenced in this way as other explanations, given by students, do not refer to the change in the variable as stated in the problem. Students, who used inverse variation in Question C2, referred most often to the variable name, but in the Part B questions these students referred more often to the variable values than the variable name.

Two Year 9 students used the direct variation strategy for Question C2 which appears to be a follow-on from the direct variation of Question C1. Two students who used direct variation for Question C1 guessed an answer to Question C2, and another two students used a calculation in Question C2. Student 913 had Question C2 correct but he had difficulty with direct variation questions. He made no attempt at Questions C1 and V1.

Year 12 students who used direct variation in Question C3 used inverse variation also in Question C5. In Questions V3 and V5, however, there was a tendency for students to use the formula and a proportion approach as well as variation. In Questions C3 and C5 the question could have triggered a variation type of approach whereas in Question V5, no such trigger was provided, so they chose a strategy that suited them. There was no student in Year 12 who could complete the inverse variation questions and who also had difficulty with the direct variation questions.

Overview

Research Question 8: How do students react to direct and inverse variation questions?

Across all Year levels, the information given in the question could trigger a particular type of response that involved variable name. Students who were successful in the inverse variation questions of Part C were more likely to use variable name in their explanation. Also, these students were likely to have answered direct variation questions correctly using variable name.

Students' performance on direct variation problems was better than on inverse variation problems. Those students who had suitable strategies for solving these problems, solved questions from Parts A and C. Year 6 students were able to solve direct variation questions but did not always offer an explanation. In the inverse questions, they did not perform very well. Year 12 students employed both techniques satisfactorily to solve the problems. For the younger students, it was more of an intuitive approach to use direct variation, whereas older students seemed to be more aware of specific relationships between the variables.

INTUITIVE THINKING

It was noted in Chapter 5 that some students appeared to use an intuitive approach. Intuitive responses are provided by some students who had a written answer followed by an explanation which was not consistent with the solution. The explanation appeared to be a justification of the result rather than the actual way the answer was found. Either the calculation they gave did not give their answer or their reasoning should have yielded a different result. Some of these types of responses were referred to in the discussion of the outcomes in Chapter 5.

Examples

Below are some examples taken from Part A of the Speed test that illustrate how responses are inconsistent with the given explanation.

Ans: *180 km .*
Exp: *I just added 60 km.*

Student 623 Question N1 (CP)

This student has the correct result but the explanation is not complete and at face value does not give the correct answer. He may have meant that he added sixty, three times.

Ans: *30 min.*
Exp: ~~*1 hr*~~ *1 hour - 30 mins.*

Student 612 Question N3 (CI)

This student has the correct answer but has given an inappropriate calculation to derive that answer. Here, the student knows what the correct result is, then appears to search for an operation which gives that answer.

Ans: *1/2 hour.*
Exp: *10/20 = 0.5*   *d/s = t.*

Student 106 Question N3 (CF)

This response is correct, and the calculation is correct. However, the formula appears to be added on to the end to try and justify the calculation.

Ans: *1/2 hr.*
Exp: $\frac{20\ km/hr}{10\ km}$ = 1/2 hr.

Student 116 Question N3 (CP)

This calculation does not give the correct solution, but the answer is correct. It is as if the student knows the correct result but needs to find a method. In this response it involves writing down two numbers that are in the right ratio of 1:2 without thinking about the actual calculation involved.

Ans: ~~*twice as much*~~.
Exp: ~~*It took them*~~ ---- ~~*I don't know*~~
She would slow down.

Student 902 Question V1 (CI)

This student stated the correct answer, started an explanation, then crossed it all out. She then wrote an inappropriate explanation for the question. It appears that in trying to write her explanation she had confused herself sufficiently to then reverse her intuitive notion for the answer.

> Ans: *2 times more.*
> Exp: *$d = vt$ while letting the first journey be the value (9) rearrange and subs [sic].*
>
> Student 113 Question V1 (CP)

Student 113 suggested a method to find the solution, then he did not follow this method. The result is correct, and it appears that this student has arrived at the solution intuitively. An attempt to justify how this answer could be obtained was given by providing the formula and a possible method.

Discussion

For Question V1, a number of students from all Year levels gave the correct answer but for an explanation wrote that it was "obvious" or "common sense". They made no further attempt to explain why it was "obvious" or "common sense". Such responses could be interpreted as being intuitive but without the students' own explanation it is difficult to be sure of this. The interviews indicated that some students knew that if an object went twice the distance it would take twice the time, but they had to be prompted for an explanation.

From such responses as these it appears that some students can work out the answer in their 'heads' and write it down but they have difficulty justifying that answer verbally or in writing. A typical example of this is discussed in the student profiles of Chapter 8 where Heather (Student 110) was interviewed on Question B10 and gave the correct answer but when asked to explain how she had found her answer said: "I don't know, I can just see it".

Such situations are well known to junior secondary teachers where some problems given to students are 'one liners' that students think they can solve in 'their heads'. For students, however, to actually write their explanation on paper and to show how they solved the problem is difficult. To do this they must be able to analyse the steps taken in the thinking process, which requires language competence, as well as the ability to tag or identify the appropriate steps.

The above discussion was based on the responses of those students whose answers did not match their explanations. This gave clues as to possible ways that students had solved the problems. From the written responses alone it was not possible to make a distinction between those who have used this process and those who have not. A similar problem occurred in the interviews, when students gave a correct and fluent reason for their answer, it was not always possible to detect if that was the original way they did the problem unless challenged about it.

This intuitive approach could be used also by other students, or by the same student on other questions, when they arrived at their answer first, then they have been able to express their explanation in a coherent way that was consistent with their result.

The intuitive approach is usually successful in the situation where the problem is relatively simple and only one variable is involved. The availability of a diagram also encourages an intuitive approach using visual cues. When the question is more complicated, students intuitive answers are not always correct, such as, when students attempted questions in which two or more variables have been altered. Focusing on the diagram in such a situation, without regard for stated variables, usually leads to inappropriate strategies for that problem, and often incorrect answers. Students with more experience have often learned that the intuitive approach leads to wrong conclusions and if they have been taught an algorithmic approach they would rather take that route, even doing so on problems which may only require a more simple approach.

Most students in Years 6 and 9 used intuition to solve the speed problems. It played a prominent role in solving speed problems; sometimes the approach did not give the correct solution since it was based on only one variable or on visual aspects of the problem. For example, those students who focused on the trolley that was in front may have assumed it had the greater speed while neglecting other factors. Sometimes, by chance, they were correct. Such a strategy, which may appear intuitively correct, was not reliable.

Another aspect of intuitive reasoning occurs when students, who do not have an immediate answer available, and also do not have a learned strategy for dealing with the problem, devise their own approach to solve the problem. In this case, students take an **intuitive approach** in solving problems and have some insight as to how the variables are related.

Some Year 9 students provided an example of this intuitive strategy when they paid attention to the variables that were changing in Question B10 and developed a strategy to compare speeds. Such strategies were not learned ones, and they may be classified as intuitive. These students reached a high level of intuitive thinking by finding a way to compare the two ratios and often arrived at a correct result. Some of these students then explained what they had tried to do, which places them in a higher level of cognitive development than those who could not do this. This approach is to be contrasted with the use of the formula by some Year 12 students. These students used the formula in an automatic way to solve the problem, and did not think through the actual changes that occurred to the variables in the problem.

An example of a response that shows how a student reasoned through the problem without a formula was given by Student 103. She found her own approach to the two-variable changing Question C9. She had the correct answer but reasoned through the question in her own fashion.

> Ans: *6 secs.*
> Exp: *It would go 1/3 the distance so it would go 1 cm in 1 sec => it would go 6 cm* [in 6 sec].

Whether she had the answer first and then found a method of justifying the answer (intuitive thinking) or used the reasoning to produce the result (intuitive approach) is not clear. She was able to consider, however, each variable and work to an answer. Students who had done physics would approach this question using the formula, which is a learned approach.

Processes such as this can be interpreted within the SOLO Taxonomy. Here, responses which are based solely on 'that it looks right' can be identified within the ikonic mode. Responses that can have appropriate steps and use the written language of the subject are within the concrete symbolic mode.

Collis and Watson (1992) suggested that the **ikonic** mode of operation:

> ... means appeal to common sense, everyday life and visualization and the use of intuitive (aha!) experiences. It is interesting to note that students at this level (advanced maths Years 9 and 10), who see what they believe to be a solution using ikonic approaches, see little point in backing up their solution by an appeal to mathematics.
>
> (p. 14)

Thus, what has been referred to as the intuitive approach can be associated with the ikonic mode within the SOLO Taxonomy. When answers appear to be intuitive and/or rely on visual imaging and an explanation is given for that answer, then for students of this age group, who can use some aspects of the concrete symbolic mode, it seems appropriate to refer to that response as using ikonic support in the concrete symbolic mode.

The possible course of action' schema of Collis and Romberg (1991) (see Figure 2.4) reinforces this view of student responses. That students change their mode of functioning as they proceed through the problem is a very likely scenario. The above discussion, however, provides greater insight into students' approaches to cognitive tasks as they answer the question using intuition (ikonic mode) but seek explanations using mathematical procedures, sometimes inappropriately (concrete symbolic mode). Hence, the concrete symbolic mode may have more structure to it than first envisioned by Biggs and Collis (1982). This will be explored further in Chapter 8.

Overview

Research Question 9: What role does intuitive thinking have in solving problems? Can these be identified? What methods do students use to explain their answers?

In summary, a number of students used intuition (the ikonic mode) either in obtaining a spontaneous answer or developing their own strategy. These are indicated in at least three ways: first, those students who have the correct answer but their explanation does not agree with the result. They arrive at the answer but can not give a reason for it. Second, those responses that have explanations such as 'obvious', 'just looks like it', or maybe even 'guess', may indicate an intuitive insight to the problem. Finally, some students have given an explanation that is not routine and devised a strategy of their own.

For those responses classed as intuitive, students gave an explanation which was not consistent with their answer. Other students may have used an intuitive method to arrive at their answer but found a suitable explanation for this answer which was not the same as they had originally employed; such responses are difficult to class as intuitive in the written test. Some students in the interviews, however, specifically stated that the explanation they gave was not the method which had been used to solve the problem.

A COMPARISON WITH SOME OF PIAGET'S RESULTS

Before concluding this chapter a brief mention needs to be made concerning some of the results of Piaget's work and the outcomes that are reported here.

In Chapter 1 consideration was given to Piaget's work on children's concepts of speed. In particular, Piaget's chapter dealing with "Speeds of Movement in Succession Travelling Unequal Distances in Unequal Times" is of interest here since the concept is similar to the Dual Focus Closed Comparison Questions of Part B. Piaget was dealing with younger children and used concrete models to represent the concepts being studied. The children in his study were able to follow the motion with their eyes and make decisions about the speed of the objects based on relative motion. In contrast, the Dual Focus Questions, given to students in the present study, used diagrams only to represent the initial and final positions of the trolley. Students could not rely on the actual motions of the objects to determine the relative speeds. This did not prevent them, however, from imagining what the motion might have been like.

As students participated in the speed experiments conducted by Piaget, he found that two types of errors occurred in students' reasoning. Firstly, when distances are unequal and times are the same students deduced the same speed. Similar outcomes occurred in some of the responses in this study where students focused on time to determine which object had the greater speed.

Secondly, Piaget found that students thought that when distance is equal, and times differ then the one with greater time was the faster. The outcomes in this study did not support this error in student responses as found by Piaget. In fact there were more students who identified the object with the shorter time as the one with greater speed, than those students who identified the greater speed as the one with larger time. One reason for this difference might be that students today are exposed to sports events in school and the media (especially television), where the winner is the one with the shortest time, whereas in Piaget's day children might not have ready access to these ideas early in life. Hence, this change in results observed may be explained by the different cultural experiences now available to students as they formulate their ideas.

CONCLUSION

In this chapter three issues have been dealt with: students reference to variables, direct and inverse variation, and the role of intuitive thinking.

The focus of Research Question 7 is on the way students referred to the variables. They did this in three different ways: variable name, variable value, and through the use of symbols. There was no precise way of predicting which way students would refer to variables but it was influenced by the type of question that was asked, the range of strategies they had at their disposal, and the number of variables which were shown as different in the problem. Of particular interest is the finding that if no variables changed in the question, most students referred to both the distance and time by name. The Year 12 students did this even though they had access to the formula. As the number of variables that differed increased then more students used the variable values in their explanations. Some Year 6 and 9 students used the variable value but only on one variable, usually the one that differed. Year 12 students used the symbols in the formula, especially when all the variables differed.

Research Question 8 concerned the effect of direct and inverse variation questions on students' responses. Direct variation questions elicited better responses from all students than inverse variation questions. Year 6 students, however, could not always explain their reasons for using direct variation. A student who could do inverse variation questions was more likely to complete direct variation questions successfully but not vice versa. For some students, direct variation may have been an intuitive-type approach to the question.

The role of intuitive thinking was the focus of Research Question 9. Students use of intuition to solve problems was employed extensively as a means to solve given problems. Intuitive thinking was often apparent when the process of reaching an answer differed from the explanation provided. The interviews indicated that most students who did use intuitive methods had access to formal methods of applying mathematics (to varying degrees) and could employ them to solve simple problems when asked to do so. Thus it is preferable to refer to these students as not only capable of employing the symbolic language of mathematics but using intuitive thinking to support their responses. The amount of intuitive support used by these students depended on the level of sophistication of their mathematical skills. For example, a student who relied wholly on the formula for a solution used very little intuitive thinking to give a response. A typical example of this is provided in the student profile of Richard in Chapter 9.

Intuitive responses are not always apparent from a student's written response. It only becomes evident when there is an inconsistency between the explanation and the answer. On the other hand, some correct answers have been provided by students using intuitive methods before they gave suitable explanations. The method that was employed by these students for the initial solution could not be deduced in the written test or the interviews, the students 'just know' or 'see' it as obvious. The explanations provided to justify answers covered the full range of the strategies listed in the previous chapter.

This chapter has considered some of the issues that arose out of the outcomes in Chapter Five and addressed the Research Questions 7, 8 and 9. The next chapter continues to discuss these issues with the focus being on the strategies employed by students in solving speed problems.