Chapter 4

THE COURSE ASSESSMENT TESTS: STUDENTS' USE OF EQUATIONS

The previous chapter provided an overview of the data collection and analysis procedures. This chapter reports students' responses to the assessment tests set by the Physics Department as part of the coursework for this unit. While the researcher had no input into the construction of these tests, they still form a useful source of data for assessment of students' understandings about the area. Analysis of these tests can be seen as opportunistic data collection, in order to confirm and extend the findings of the main set of data collected, results of which are discussed in later chapters. This gives a source of triangulation for the study.

The results reported in this chapter have particular bearing on students' use of equations in this topic. The coursework assessment tests had a strong emphasis on this, which was not present in the main data collection for this study. Hence, consideration of students' responses to these tests gives an additional dimension to the study. A research focus for this chapter is to establish how students' use of equations relates to their ability to use concepts describing electric and magnetic fields and related phenomena. It would be of interest to determine whether any particular equations and concepts present special problems for students. As well as this primary focus, there is also value in an investigation of students' concepts based on their responses to more qualitative questions. This allows a preliminary description of students' understandings of field representations and related phenomena. The resulting description can be compared with results from the main study.

While the broad theme of characterisation of students' understandings of fields and related phenomena guides the analysis in this chapter, a more detailed plan would be of assistance. In order to guide the analysis following this broad theme, three specific research questions seem appropriate:

1) Is it possible to discern a variety of strategies used in students' responses?
2) Did the students display competency within the limits of the course assessment tasks? If not, what was the nature of the errors made?
3) Did their errors on these assessment tasks reveal any alternative conceptions about the subject?

The methodology used in the treatment of data from these tests was broadly phenomenographic, without use of the SOLO Taxonomy. Students' responses to questions have been examined and sorted into groups on the basis of the understanding displayed, and a description of these groups is given here. This gives a contrast to the main study, where the Taxonomy is employed. This approach also gives a picture of the results of the tests as available to the physics department, who were not aware of the SOLO Taxonomy when designing and scoring their assessment tests. Subsidiary to the themes relating to the students' quantitative and qualitative understandings was the aim of characterising the nature of the assessment tests, and the sorts of student responses they seemed to encourage. This could have a bearing on the construction of future tests.

The tests themselves are reproduced in appendices H and I, along with figures and comments regarding the class performance on the test in appendix J. The material in this chapter is concerned with conceptual aspects of students' responses to the test questions. Because of this, the questions have been organised into themes for presentation, rather than simply being presented in the same order as in the tests. Quotes from the students' responses are identified by the reference number of the student, given in brackets. For example, "(s. 22)" describes a quote from student number 22. The students' responses to quantitative questions are dealt with in the first section of this chapter, and the second section deals with their responses to more qualitative questions.

RESPONSES TO QUANTITATIVE QUESTIONS

This section deals with the main aim of the chapter, which is to understand students' responses to quantitative questions involving fields. Responses to each of the questions in the test have been dealt with separately. The questions are described in order of increasing complexity. First, responses to questions requiring numerical substitution are described, and then responses to questions, involving the manipulation of abstract equations to prove results, are considered.

Numerical substitution

Questions requiring only numerical substitution can be answered without any need to manipulate equations and concepts. This sub-section starts with an analysis of a simple question which only requires substitution of numbers into a well-known equation. The sub-section then examines more complex questions, which do require manipulation of equations as well as numerical substitution.

Inverse-square force calculation
(Question 1(c) of year-end test)

Two spheres of equal size and mass are suspended from the same point by fine insulating threads of length 1.0 m. When each sphere carries a positive charge of 1.5 μC, the angle between the threads is 30 degrees as shown in the figure below.
(i) Sketch a diagram showing all forces acting on one of the spheres.
(ii) Calculate the mass of the sphere.

In terms of knowledge of electric field phenomena, this is a standard question. The use of the equation describing the force between the two spheres only requires straightforward substitution. The complexity in the question involves the consideration of components of tension in the string and their relation to the gravity force, all of which is outside the interest of this study, and is not discussed here.

Of the 55 students attempting the year-end test, 46 attempted this question. All of these, except one, drew a diagram of forces on a sphere which included a force due to electrostatic repulsion. Only four students left out the tension force. There was general recognition that the spheres repelled one another.

Four students made no attempt to calculate the magnitude of the repulsive force. Two used equations which seem irrelevant, for example, one student (s. 5) substituted the given value for charge into an equation for work. However, the remaining 35 students answered this question by using the equation $F = k\, Q_1\, Q_2 / d^2$ with no difficulties, save one student who had trouble with the trigonometry involved in calculating d. On the whole, students had little problem dealing with the electrostatic component of this question, which involved substitution into a well-known equation. They were able to answer the question next described by the same sort of substitution, even though there was no evidence that they understood the principles involved.

Calculation of potential from field strength
(Question 5 of mid-semester test)

Atmospheric air breaks down when an electric field greater than 3×10^4 V cm^{-1} is present. Calculate the potential on a conducting sphere of diameter 1 mm when the air around the sphere breaks down.

In this question, students are required to use the relationship between the field strength and the potential around a charged sphere. The question had the interesting feature that it could be answered by use of the equation $V = E\, d$, which is commonly learnt by students in a

completely different situation (parallel plates and integration over length between them), i.e., this question could be answered by substitution of the given data into an equation which relates to a different class of problems.

This question was attempted by 38 of the 51 students sitting the mid-semester test. Seven students used the given value for breakdown voltage and divided it by the area, volume or circumference of the given sphere. This approach appears to be based on a substitution into equations guided by the given units of V cm^{-1} and mm of length, where students divided V cm^{-1} by units of length in an attempt to find an answer. There appears to have been use of the equation without a full understanding of its meaning.

This is similar to the most common response to this question, which was based on the equation $E = V / d$, an equation which relates to constant field situations, rather than this situation of a radial field. Twenty-one of the students assumed this equation or the equivalent $V = E\,d$, and used the diameter of the sphere as d. This leads to an answer twice the correct one, barring arithmetic errors. Two more students used the incorrect equation $V = E / d$, which suggests a lack of understanding of the principles involved. Again, these students have made substitutions into equations. In this case, the substitution seems likely to be based on the recognition of the letter d, which they took to be the diameter of the sphere. Six other students assumed the same equation, but used the radius rather than the diameter in their calculations. This gave them the correct answer, barring arithmetic errors. None of the students derived this equation, all taking it as a given.

Only two students showed justification of their use of equations, working from the radial equations $E = k\,Q / r^2$, $V = k\,Q / r$ which apply to this situation. One of these two students (s. 26) derived the relation $V = E\,r$, as follows,

$$E = k\,Q / r^2$$
$$V = k\,Q / r$$
$$\therefore\ V = E\,r = 3 \times 10^4 \times 5 \times 10^{-4} = 5 \times 10^{-4} = 15 \text{ Volts}$$

In the above derivation of the equation $V = E\,r$, the student has used the radial equations which pertain to this radial field situation. This differed from the bulk of responses, which assumed the similar equation $V = E\,d$, that holds in situations involving constant fields. The numerical error in the above answer is due to failure to convert the given field into V m^{-1}. The other student who used the radial field equations calculated the charge on the sphere for the given electric field, using the equation $E = k\,Q / r^2$. He then used this to find the voltage, applying the equation $V = k\,Q / r$. The use of radial equations, as demonstrated by these two students, was the original intention of this question.

With the exception of those two students, responses to this question generally involved the use of an equation which is only coincidentally right. The equation $V = E\,d$ happens to cover two distinct physical situations: 1) the voltage between two points separated by a distance of d parallel to a constant electric field of magnitude E; 2) the relationship between the voltage and the electric field of a point charge at any point affected by the charge. The students had learnt this equation in the context of constant fields and parallel charged plates. It is doubtful that many of them were aware of the distinction between the constant field situation and the problem in this question. This conclusion is supported by their use of radial field equations in questions concerning parallel plates, discussed below.

<u>Electric fields, voltage, particle motion and energy</u>
(Question 6 of mid-semester test)

An electron is released from rest in a uniform electric field of 1000 N C^{-1}. After it has moved through 2 m, calculate the electron's (i) kinetic energy in electron volts, (ii) velocity.

This question, involving work done on a charge in an electric field, generally showed students having little connection between the concepts of voltage and work.

Of the 51 students attempting the mid-semester test, a total of twelve students made no significant attempt at this question. Three students did not attempt this question at all, five others stopped after writing down some of the given information and four more wrote the given data plus some equations and went no further.

In another example of students' confusing the radial field and constant field situations, four students answered using equations relating to radial fields in spite of the fact that those equations were inappropriate for this question. For example, one of these students (s. 31) calculated a voltage by using the equation $V = k\,q_e / R$, and substituted the given distance of 2 m for R. This is interesting in light of similar responses to the question discussed above, where the equation $V = E\,d$ was used in the radial case, even though it was learnt in the context of constant fields. This supports the hypothesis that students are not aware of the distinction between the two cases. Exacerbating the difficulties which students show in their understanding of field equations is their difficulty with concepts of voltage, work and energy.

It was common for students to have difficulty with voltage all through the test. Seven students showed confusion between voltage and energy in their responses to this question. For example, one of these (s. 22) assumed that the energy gained by the particle would be equal to the voltage, rather than the voltage times the charge of the particle.

These concepts of energy and voltage were avoided altogether in ten students' responses to this question. They calculated the acceleration of the particle, and then calculated its velocity using the equations of constantly accelerated motion. This involved using an approach opposite to that suggested by the question, doing the question in reverse order, and thereby entirely avoids using the idea of potential and work. While this shows a level of problem-solving ability, it does not demonstrate any understanding of the concept of voltage and its relation to work.

Another group of 18 students was able to use, at least to some extent, the concept of work as it related to electric field and kinetic energy. Of those students, only six calculated the voltage across the distance involved. This is surprisingly few, in light of the fact that the question asks for a response in electron volts. It would seem that students have no clear idea of the meaning of an electron volt, and even those calculating the voltage did not make full use of the concept. One student (s. 56) provided an example of a response which calculated voltage but did not realise the implications for the energy in terms of electron volts. He wrote

$V = E\,d = 2000$ V.
$KE = e\,V_0$
$= 3.2 \times 10^{-16}$ J
$= 2000$ eV
[Student goes on to calculate v]

The student has gone to some effort to show that the energy an electron gains by moving across a potential difference of 2000 volts is 2000 electron volts. Such responses gave no clear indication of the equivalence between the voltage across a distance and the energy in electron volts of an electron that covers that distance. This gives another illustration of the difficulty which students had with the idea of voltage.

The relationship between voltage and energy was problematic in responses to this question. A large group of respondents avoided this issue by using the equations of motion at constant acceleration. Few of the respondents showed understanding of the concept of the electron volt. The group which did respond in terms of energy due to electric field generally had problems with the voltage issue, and few indeed were able to use the simple relationship between voltage and energy in electron volts. Similar features can be seen in responses to the next question.

Electric fields, voltage, particle motion and energy (continued)
(Question 1(d) of year-end test)

A pair of parallel conducting plates, separated by a distance of 5.0 cm, have a potential difference of 1.0 kV between them. The plates are horizontal and the upper one is at the higher potential. A particle of mass 1.5 g carrying a charge of 2.0 μC is released from rest at the lower plate. (i) Calculate the velocity of the particle when it reaches the upper plate. (ii) Calculate the potential difference between the plates for which the particle may be suspended in the region between the plates.

This question is reminiscent of the question whose results were just described, in that it deals (in part (i)) with the velocity of particles moving in a constant electric field. It is complicated further by the factor of gravity forces on the particle. Again, the students did not, in general, effectively use energy considerations. Parts (i) and (ii) of this question, while related, are treated separately here.

Part (i)

There are two ways to solve this part of the question. Students could use energy considerations to find the kinetic energy gained by the particle, and hence its velocity. Alternatively, the question could be answered by calculating forces on the particle and using the equations of uniformly accelerated motion.

Of the 55 students attempting the year-end test, 40 attempted this question. Five of these were only able to draw a diagram and list some of the given variables. Nine other students were unable to do the question because of difficulties with fundamental concepts. Of these, three confused voltage with electric field. Two others of these nine students confused voltage with energy, two confused electric field with energy, and two used the radial field equation $F = k\,q\,q'/r^2$.

Of the remaining students, fourteen used arguments about the kinetic energy which the particle would have after traversing the given voltage. They went on to calculate the velocity from this. This was the scheme expected in answers to the question discussed above. However, it was not widely used in responses to that question. None of the students using an energy argument for this question took the role of gravity into account. Interestingly, of those fourteen students using energy considerations, five took the charge on one electron as the charge of the particle, rather than the 2.0 μC which was given. The failure to take these factors of gravity and charge magnitude into account seems to indicate a rote-learned response to this question, where the algorithm for predicting movement of an electron in an

electric field has been blindly applied without consideration of the special features of this situation.

Twelve other students avoided the concept of energy, solving the problem by calculating the force on the particle and proceeding from there by the equations of uniformly accelerated motion, which was quite a successful strategy. Four of these twelve students did not consider the effect of gravity in the problem, and eight did. Students were capable of carrying through the lengthy procedure of use of the equations of motion. It is not clear how much understanding of physical concepts lies behind the use of equations, as the equations of motion are well suited for use by simple substitution, because there are only a few of these equations, and they relate to a small number of well-known variables. This situation does not apply for use of work/energy considerations in connection with voltage.

An overall result from this question was that no students were able to use the work/energy formulation of voltage to come to a complete answer. Those using an approach based on voltage and energy failed to allow for the gravitational potential energy. This points to an incomplete understanding of the energy concept in this situation. A large proportion of those students basing their responses on forces and uniformly accelerated motion were able to come to accurate responses. This tends to indicate that the students' notions of force and motion are better developed than those of energy and voltage. The equations of uniformly accelerated motion can be easily used on a basis of substitution without understanding, being a complete set of equations, well adapted to use by means of looking for equations that relate the right variables. The use of energy considerations depends more on fundamental concepts, and the equations used are not a small, complete set. The fact that students were more effective reasoning in terms of force and acceleration may indicate that this method was more amenable to an approach based on substitution into equations, rather than indicating that the students actually had a better understanding of force and acceleration than of energy and work.

Part (ii)

While linked to part (i), part (ii) of this question was still a distinct entity which could be answered quite independently. Energy considerations did not apply in part (ii). All that students had to consider in this part was force, and that seems to have led to a greater number of complete answers than in part (i).

Of the 55 students attempting the year-end test, 32 attempted part (ii) of this question. Nine students confused fundamental concepts in their answers. Four of these confused voltage with electric field, one confused electric field with charge, another used the radial equation $V = k\,q\,/\,r$ in this constant-field case, one reasoned backwards from the equation $(1\,/\,2)\,m\,v^2 =$

$e\ V_0$ to conclude that $v = 0 \Rightarrow V_0 = 0$ and two reversed equations, i.e., "$E = V\ d$". These students' difficulties with fundamental concepts prevented them from grasping the ideas in this question.

Two other students had the right concept, realising that the gravitational force $m\ g$ on the particle had to be balanced by the electrostatic force $q\ E$. However, they could not complete the reasoning to find E. Five more students took this equation and went on to calculate a numerical value for E, which they used to find the desired voltage. It is interesting to compare this with the group next described, where the students were more confident in algebraically manipulating equations without a need to substitute numbers until the end.

Sixteen students manipulated the equation to find an expression for V in terms of the given data, only then substituting the given numbers. One student (s. 54) wrote:

$E\ q = m\ g$
$V = E\ d$
$\therefore\ V / d = E$
$V\ q / d = m\ g$
$V = m\ g\ d / q$
[Student then substituted and found value for V]

This student has carried the equations through in terms of abstract variables without the need to substitute numbers until the last step, showing an ability to cope with abstraction.

Overall, it seemed that part (ii) was simpler than part (i). The fact that it did not involve energy considerations seems to be the factor that led to a larger number of complete responses. The relationship between energy and voltage appears to have been problematic for students.

<u>Overall comments on questions requiring numerical substitution</u>

Students were most capable when dealing with simple questions which only involved a routine use of substitution into equations. Few students had difficulty substituting numbers into the inverse square force equation (in question 1(c) of the year-end test). When dealing with questions involving concepts, their performance was poorer. In question 5 of the mid-semester test, they assumed the equation $V = E\ d$ without any justification, and this reduced the question to a routine substitution into equations. The questions dealing with fields, voltage, particle motion and energy generally showed a poor understanding of voltage and energy, but an acceptable use of equations of motion. This could well be because the equations of motion are more amenable to use by simple substitution into equations. Overall,

students showed little ability to manipulate equations following high level concepts, particularly those of fields, voltage, and energy. This issue appears is addressed again below in the context of students' attempts to prove results by use of equations.

Quantitative questions involving proof

The questions treated in this section require students to prove results by manipulating equations. Generally speaking, this requires higher levels of understanding than simply substituting numbers into formulas. On the whole, students had significant problems with this, both in understanding the concepts involved, and with the idea of a proof. The first question dealt with in this section involves the concepts of electric fields and voltage. These concepts were important in the questions discussed above. Here, as in the previous questions, students showed great difficulty with manipulating equations guided by higher level concepts.

<u>Electric field and voltage</u>
(Question 2, mid-semester test)

A pair of parallel plates, separated by a distance d, has a potential difference V between them. Show that the electric field between the plates is V/d.

This demonstration requires an ability to manipulate the concepts of voltage and electric field as they relate to force and energy. These are the same concepts which appeared in the questions involving numbers which were just treated. Few students were able to complete this question to the satisfaction of the examiners. Study of the set of responses shows a number of groupings in the students' answers.

Of the 51 students attempting this test, eight students did not attempt the question or stopped after drawing a sketch of parallel plates with an electric field between them. Eight other students wrote equations without arriving at the desired result. Some equations used had no obvious relevance to the question. It is interesting to note that some of those eight students assumed the equation $V = E\,d$, but failed to reason from there to $E = V / d$. $V = E\,d$ needed only to be divided on both sides by d to obtain the desired result. The failure to do so indicates a great lack of sophistication on the part of these students. Dividing the equation by d is a trivial piece of algebra, which it is almost certain that all these students would have been capable of. However, their overview of the system of equations appears to have been so limited that they did not see the need for this simple step. Six other students appear to have done the same thing, i.e., writing unrelated equations, except that they then wrote $E = V / d$ to finish the "proof". The 14 students in two groups described in this

paragraph seem to have had no overall strategy for dealing with the question, and wrote equations which appear not to be linked with each other.

Also showing difficulty with relating equations together, ten other students used mathematically inappropriate methods to "prove" the point. For example, two of these students "showed" that $E = V / d$ by assuming that E is proportional to V and to $1 / d$, and that this proved the result. One student (s. 1) wrote:

$\underline{E} = V / d$
$\underline{E} \propto V$ and $\underline{E} \propto 1 / d$
$\therefore \underline{E} = V / d$

While the above is true, it is mathematically tautological. Six of the ten mathematically inappropriate responses assumed the equation $V = E\,d$, and used this to "prove" the desired result. For example, one student (s. 22) stated:

Electric field is equivalent to gravitational acceleration.
$\therefore$ P.E. = V
= E d
$\therefore$ E = V / d

The reasoning by analogy to the gravitational case above is promising in relation to the physics, but its mathematical sophistication is limited. As an additional error, it contains the statement that $P.E. = V$, an error which has been mentioned before in the context of confusion between energy and voltage. The remaining two of the ten mathematically inappropriate responses assumed the relations $V = k\,Q$ and $E = k\,Q / d$.

While these statements are physically true, they fall short of the mathematical idea of rigourous proof which is a feature of the course. It is interesting to muse whether these ten students have failed to produce a rigorous proof simply because they lack mathematical sophistication about the concept of proof, because their understanding of the subject was too slight to permit them to form the steps of a proof, or a combination of both factors.

As a mirror image of these "proofs" which were mathematically unacceptable, eleven other students produced "proofs" which were mathematically acceptable but unacceptable in terms of physics concepts. They produced the desired relation by assuming equations of the form $V = k\,q / r$ and $E = k\,q / r^2$ and hence showing $E = V / r$, i.e., $E = V / d$, QED. These students have made an acceptable proof in mathematical terms, but they have used equations which do not apply to the physical situation in question. There is a failure to distinguish between the equations relating to the constant field case and the equations related to the

radial field case. This failure has been noted above in reference to students' use of equations, and points to a lack of physical intuition about this situation. One sees a similar effect in responses to question 6 of this same test, which describes a physically similar situation with constant electric field, yet has students using these radial equations.

Only eight students used considerations of work to answer this question. Proof of the result $V = E / d$ under these circumstances depends on an awareness of the relationship between work, energy and force. As this principle is known to be problematical in itself, it is hardly surprising that its use in the relatively non-intuitive domain of electric fields has not been common in this sample. Two of these eight students mentioned the idea of work in their responses, but did not take the proof to a conclusion. The remaining six of these eight students were able to use the idea of work to complete the proof, for example, one student (s. 21) wrote:

$$V_{ba} = W_{ba} / q$$
$$W = Fd$$
$$\therefore V_{ba} = F\,d / q$$
$$\therefore q = F\,d / V_{ba}$$
$$E = F / q$$
$$\therefore E = F\,V_{ba} / F\,d$$
$$\therefore E = V_{ba} / d$$

The above was a rare display of coherent use of equations relating work and voltage. Even in answers like the above, one wonders if those who were able to complete the question using considerations of work were only enabled to do so by their having seen the question done in class; that is, to what extent are these students simply copying an algorithm they have seen used before, without understanding the algorithm. As those answers show considerable variation in detail, it seems likely that students were showing some level of personal understanding in their responses.

Generally, students had difficulty with this question. The average score was 0.3 out of a possible 2. This seems to result from a combination of lack of mathematical sophistication and lack of understanding of the physical situation. Lack of sophistication about the mathematical idea of proof allowed students to produce answers proving the desired proposition by the assumption of tautologies. Lack of understanding of the physical situation allowed students to use equations which do not hold in this case. Finally, the concept of work/energy, which is required for the proof, is known to pose problems in itself for students at this level and can be expected to pose further problems in this when complicated by consideration of electric fields.

Three dimensional integration to find electric field
(Question 2(a) of year-end test)

A thin circular disc of radius R carries a uniform distribution of charge with a charge density of σ C m^{-2}. (i) Consider a ring section of the disc of width dr at radius r as shown in the diagram. Write down the potential V due to the charge distribution on the ring section at a distance Z from the centre of the ring and on the axis of the ring section. (ii) Using the result of (i) show that the electric potential on the axis of the disc, at a distance Z from the plane of the disc is given by $V = (\sigma / 2\ \varepsilon_0)\{(R^2 + Z^2)^{1/2} - Z\}$. (iii) The disc has a diameter of 1.0 m and has 10^{10} electrons uniformly distributed over its surface. Calculate the electric potential at the centre of the disc.

This question involves a rather complicated integration in three dimensions, and would appear to be of formidable difficulty for students with little experience in integrals of this sort.

Presumably, this difficulty is the reason that only six of the 55 students attempting the year-end test attempted question 2(a). Two students made attempts at this question which lacked consistency. These students assigned data to inappropriate variables, and were unable to come to a correct answer to (iii). Two other students attempted only part (iii), which involves no more than the substitution of the given data in the expression given in (ii). These two students were successful in their calculation, which involved recognising all the given variables in the data, and calculating the charge density, σ. They recognised these important features of the question. Only two students were able to reason through the entire proof, and they had no difficulty with part (iii), completing the difficult integration, and substituting the correct numbers into the given equation. These two students showed an understanding of the nature of the question, as well as a successful proof of the desired result.

One surmises that many students had no clear understanding of what was required in the proof, and that these generally did not attempt the question. Some that did attempt it produced answers lacking clear organisation. Other students produced answers which seemed to show an understanding of what the variables involved in the question were, but did not attempt the proof. Finally, other students showed ability to both comprehend the question and perform the integral manipulations necessary for the proof. There is a progression here. The three-dimensional integration required here is not part of the standard high-school mathematics curriculum in the state, so students would have encountered it for the first time at university, possibly only in passing in physics lectures. It is not surprising that so few students attempted this question.

Torque on a current-carrying coil in a magnetic field
(Question 3(b) of year-end test)

A square coil of one turn of wire and side l is suspended in a uniform magnetic field B with the plane of the coil parallel to B as shown in the diagram. The coil is free to turn about an axis perpendicular to B. At the instant that a current I commences to flow in the coil, (i) determine the magnitude of the force and its direction acting on each side of the coil. (ii) Hence determine the torque on the coil at that instant.

Question 3 of the year-end test had two parts. Part (b) above was the only one concerned with field-related phenomena, and counted for 8 marks out of 20. 51 out of the 55 students doing the test attempted question 3(b).

An answer to this question involved consideration of the forces on the wires making up the coil, and a calculation of the torque as a result of those forces. The division of the question into parts (i) and (ii) seems to be intended to encourage students to follow through these steps. Their answers to these two parts are considered separately below.

Part (i)

This part of the question required students to identify the forces on the coil. Every one of the 51 students answering this part made some sort of statement based on the equation $F = B\,I\,l$. There was a growing level of complexity in these answers. Nine of the answers did not clearly relate the force to any aspect of the physical situation. Ten students made a clear prediction about the coil's behaviour, but the way in which the force related to the coil as a whole remained quite unclear, for example, one student (s. 47) commented,

> F = B I A, The right hand side of the coil will move into the page while the left hand side out of the page.

The above does not make clear how the force of "$B\,I\,A$" relates to the various sides of the coil. Seventeen other students added detail, not only mentioning a force on the coil as a whole, but describing the force on the two sides that would experience a force, e.g., one student (s. 4) wrote:

> F = B I l for both sides. The force on the right hand side will be up out of the page and the force on the left hand side will be down into the page.

No mention was made of the other sides of the coil in the above group of responses, which is in contrast to four other respondents, who described the force on the two sides experiencing

a force, as well as mentioning that the force on the side between them was zero. They did not seem to recognise that the side containing the current feed lines was also a side of the coil. Eleven replies described the force on all four sides of the coil.

In responses to part (i), one can see an increase in the number of elements differentiated. Firstly, the whole coil, then the sides perpendicular to the magnetic field, then all 4 sides. It is true that failure to mention the sides parallel to the magnetic field could reflect a lack of appreciation of the need for rigour in a proof, rather than unawareness of their existence. Even so, mention of all four sides is a clearly observable increase in complexity of the answer.

Part (ii)

This part of the question asked students to use the result of part (i) to prove the required result. Of the 51 students attempting this question, 47 attempted part (ii). Few students seem to have appreciated the importance of the word "hence" in this question, which may reflect a lack of mathematical sophistication. Most have based their answer directly on the equation giving torque on a loop in a magnetic field, $\tau = n\ I\ A\ B$. They have generally not attempted to relate this to the work done in part (i).

Thirty of the students have assumed an equation of the form $\tau = n\ I\ A\ B$ in their answer. They seem to have been unaware that they were required to <u>derive</u> such an equation from their work in part (i) of the question. Alternatively, they may have been unable to do so. Fifteen of these thirty simply wrote down the equation $\tau = n\ I\ A\ B$ or $\tau = I\ A\ B$ and stopped, making no attempt to calculate or simplify further. They did not derive this equation, but simply stated it.

Four of those assuming $\tau = n\ I\ A\ B$ started from this equation, and then went on to find (incorrectly) that the torque on the coil would be zero. Eleven students of the thirty students who started from the assumption of an equation of the form $\tau = n\ I\ A\ B$ went on to substitute the correct variables from these given. These students moved from that equation to the equation $\tau = B\ l^2\ I$, e.g., one student (s. 29) wrote:

$$\tau = N\ \underline{I}\ \underline{A} \times \underline{B}$$
$$A = l^2 \quad N = 1$$
$$\therefore\ \tau = I\ l^2\ B \sin\theta$$
$$= I\ l^2\ B$$

In a similar way to those thirty students starting from assumption of equations of the form $\tau = n\ I\ A\ B$, four other students simply wrote the equation $\tau = B\ l^2\ I$, without any explanation of

their reasoning. It seems likely that their reasoning was also based on assumption of the equation $\tau = n\,I\,A\,B$. There was a strong tendency for students to make assumptions about equations for τ, rather than to prove the required result for torque from first principles.

In fact, only thirteen students attempted to calculate the torque on the coil from first principles, based on the work done in part (i) of the question. Of these, two were not aware of the fact that torque is given by force times distance from axis. Of those thirteen students attempting to calculate from first principles, two others used the correct definition of torque but were still not able to come to a correct answer. Only nine of the thirteen respondents attempting to do so were able to accurately calculate the torque on the coil from first principles, e.g., a student (s. 24) stated:

$$\begin{aligned}\tau &= F\,r \\ &= 2\,I\,B\,l \times (l/2) \\ &= 2\,I\,B\,l^2/2 \\ &= I\,B\,l^2\end{aligned}$$

But l^2 = Area of coil (A)

$\therefore\ \tau = I\,B\,A$

This student has the idea of proof, and has shown the desired result that $\tau = I\,B\,A$.

Most of the students seem to have missed the point of this question, which was intended to lead students through two parts to prove the result $\tau = I\,B\,A$ in the special case presented. Instead, students generally approached the two parts of the question separately, and assumed the general case of the result to prove the special case, or simply stated the general equation. This probably points largely to a lack of mathematical sophistication among the students but may also relate to a lack of understanding of the physical situation.

<u>Overall comments on questions involving proof</u>

Students in general had a poor grasp of the concept of proof, with most assuming equations, or manipulating them in an ineffective manner. Their difficulties with proofs were compounded by their lack of understanding of the physical concepts involved.

Overall comments on quantitative questions

The students' responses to quantitative questions generally showed that they were far more competent at substituting numbers into equations than manipulating equations to obtain or prove results. Students tended to use equations with imperfect understanding of the meaning

behind those equations. A repeated theme which drew attention to this was confusion between equations relating to a radial field and those relating to a field constant across space.

The distinction between situations involving constant fields and those involving radial fields was seldom made by students in their use of equations. This became evident in the answers to questions 2 and 6 of the mid-semester test, which concerned a constant field of the type between two parallel charged plates. In both these questions, a high proportion of students gave answers using the radial field equations $E = k\,q\,/\,r^2$ and $V = k\,q\,/\,r$. This indicates a lack of understanding of the meaning and limits of application of these equations. Similarly, in question 5 of the mid-semester test, where the situation was a radial field, students generally used the equation $E = V\,/\,d$, with no explanation of derivation and incorrect substitution into this equation. This makes it likely that they were simply applying this equation from their work with constant fields, with no understanding of its relation to this physical situation.

The relationship between work and voltage was also poorly developed in student responses. In question 2 of the mid-semester test, few were able to use the idea of work to prove that $E = V\,/\,d$ in a parallel plate capacitor - this was in spite of having seen the proof in lectures. In question 6 of the mid-semester test, very few students went directly to express the work on an electron in electron volts, in spite of the question specifically asking for this. It was more common to calculate work in joules, and then convert to electron volts, in spite of the trivial ease of calculating work done by moving an electron across a given voltage, when done in electron volts. This indicated a lack of familiarity with the meaning of the electron volt, and the underlying relationship between voltage and work which makes it such a natural unit.

In general, the students' approach to these questions seemed to be based on equations without a firm underlying conceptual structure. Their misconceptions mentioned above in connection with radial versus constant fields are a case in point. There, students used equations because they contained the right variables, rather than because they pertained to the physical situation involved. This would have been impossible if they had a firm grasp of the quantities related in the equations. Generally, student answers to quantitative questions in the exam were not based on deep understanding of principles of physics.

RESPONSES TO QUALITATIVE QUESTIONS

This section describes students' responses to the qualitative questions in the physics department's assessment tests for the topic. While these qualitative responses are not the primary focus of this chapter, they still provide useful insights into students' reasoning about the concepts in the questions. These responses are a source of useful triangulation on the data collected in the interview study, as presented in later chapters.

This section first considers the students' attempts to explain phenomena qualitatively, and then their responses to questions requiring them to remember definitions.

Qualitative explanations of causes for phenomena

In these questions, students are required to give arguments for why phenomena should occur, though not in a way requiring equations. This is the qualitative counterpart of their attempts to prove, using equations, that phenomena should occur, as described in the preceding section on quantitative proofs. In general, the same difficulty in applying physics principles can be seen in responses to these qualitative questions.

Force on a charged particle in a magnetic field
(Question 9 of mid-semester test)

(a) Write down a vector relationship for the force acting on a charged particle q moving with velocity $\underline{v}$ *in a magnetic field* $\underline{B}$. *(b) Explain why it is not possible to change the speed of a charged particle by any combination of steady magnetic fields.*

This question, similarly to question 3(b) of the year-end test, dealt with in the previous section, involves force in magnetic fields.

This requires students to have a three-dimensional understanding of the vector equation mentioned in the question. Although the average score of 1.14 out of 3 on this question was relatively high, this was largely due to the allocation of half of the marks for production of a vector equation in part (a). Replies to part (b) of the question, which involves principles, were generally quite poor. Only replies to part (b) are considered below, as part (a), which only requires statement of an equation, generated very repetitive answers.

Of the 51 students attempting the mid-semester test, ten students did not attempt question 9(b) at all. Six other students came up with the tautological response that a magnetic field has to be changing to alter the velocity of a charged particle, e.g., "It is not possible to change the speed of of [sic] a charged particle by any combination of steady magnetic fields because the speed is varied if the fields are not steady and hence the combination of steady fields has no effect on speed" (s. 19). Slightly less tautological was the response of seven students that only the direction of the particle changes, e.g., "Because in combining magnetic fields it is only possible to change the direction of the velocity and not the magnitude" (s. 56). These two groups of responses do not add anything to the information in the question itself and hence hardly count as responses at all. Seven other students gave answers which,

while not tautological, seemed to have little bearing on the physics principles involved, e.g, "because there is no change in magnetic flux" (s. 34).

One fallacious argument, that was popular, and used by eleven students, was to argue essentially in scalar terms from the equation $F = q\ v\ B$. The argument was that because F, q and B were constant, then v must remain the same, e.g., "F = q v B, F / q $\underline{B}$ = v taking the charge to be constant & the magnitude of the force to be constant for a magnetic field of constant magnitude. The only way to change the velocity is to vary the magnetic field!" (s. 6). This reasoning shows a lack of understanding of vector cross products, as $\underline{v} \times \underline{B}$ varies between a maximum of $v\ b$ and a minimum of 0, depending on the angle between $\underline{v}$ and $\underline{B}$. These students have not used the three-dimensional quality of the situation, giving yet another example of the common tendency to use equations rather than principles to attack physics problems.

The principle relevant to answering this question is that force is perpendicular to velocity for a charged particle moving in a constant magnetic field, and hence the speed of the particle is not changed. Two students recognised that some idea involving perpendicularity was appropriate, but they assumed that it was the magnetic field which was perpendicular to the velocity. For example, one of these two students (s. 48) wrote that, "It is not possible to change the speed because the magnetic field doesn't interfere with the velocity of the particle. (The velocity is perpendicular to the magnetic field thus it is not affected by it)". Eight other students used the fact that the force caused by a steady magnetic field is always perpendicular to the charge to argue that speed of the charge should not change, e.g., "Because the force $\underline{F}$ acts at right angles to the plane of $\underline{F}$ and $\underline{B}$" (s. 29). Rather few students were able to come to this answer, which even then may reflect rote-learning of a fact, specific to charged particles in a magnetic field, rather than a full understanding of the mechanics principles involved, which relate to the concepts of uniform circular motion and work/energy.

In summary, an understanding of two concepts is needed to answer this question. Firstly, one must appreciate that the force caused on a charge by a steady magnetic field is always at right angles to the velocity. This three-dimensional picture may be unclear to many students. It is dependent upon, but not guaranteed by, an understanding of the distinction between magnetic and electric fields in their effects on charge. As Maloney (1985) pointed out, this distinction is problematic for many students. Secondly, one must appreciate the reasoning about infinitesimal quantities which means that a force acting perpendicular to the direction of motion will not change the speed of a particle. If a force acts for a *finite* period of time in a direction perpendicular to the *initial* direction of motion of the particle, then it will create a new velocity component perpendicular to the initial velocity of the particle. The two components add together to give a total velocity of greater magnitude than the initial

velocity. So the idea that the speed of the particle will not change is dependent on reasoning about instantaneous forces.

Similar reasoning about forces, momentum and trajectories is required for an understanding of uniform circular motion, which is also known to present difficulties for students. This indicates, that the concepts required to deal with force and motion here are difficult for students in themselves without the addition of magnetic fields to complicate matters. This parallels Galili's (1995) work relating mechanics concepts to electric fields, and is a theme which recurs in later chapters. In the following question, it becomes apparent that students also have trouble reasoning about the movement of charge as it relates to electric fields.

Electric fields and bulk charge movement
(Question 3 of mid-semester test)

Why is the direction of an electric field at the surface of a charged conductor always normal to the surface?

This question presented problems for the students, who did not generally use the desired idea of movement of charge within the conductor to cancel field components parallel to the surface.

Of the 51 students attempting this test, twelve made no attempt to answer the question. Two other students stopped after drawing a sphere with lines radiating in all directions. This assumption that the conductor was spherical also appeared in more sophisticated answers. Four students gave answers that were basically tautological, e.g., "The direction of an electric field at the surface of a charged conductor is always normal to the surface because electric field lines are always perpendicular to the surface" (s. 40). Three students gave answers which were incomprehensible. Two others made the argument that the field would have to be perpendicular to the current which they assumed to be flowing in the conductor, e.g., "The direction of an electric field at the surface of a charged conductor is always normal to the surface because the electric field force $\underline{E}$ is always perpendicular to the flow of the current through the conductor" (s. 20). There were large numbers of students who did not seem to have any reasoning which applied to this question.

Fourteen students did use some sort of reasoning based on "evenness". Five of these students argued that the field would be normal to the surface because it was evenly distributed, for example, "Field is normal because it is evenly distributed and acting away from the surface and is therefore normal to the surface" (s, 29). Another five, of the fourteen students whose argument was based on evenness, believed that the effect was because the charges in the conductor repel each other to a maximum separation, e.g., "The reason is that

the charged particle in side having the same charge repel each other wanting to have maximum separation" (s. 25). The remaining four of these fourteen students argued that the direction perpendicular to the surface was the "most direct" direction, and that therefore the field would go this way, e.g., "The electric field is always going to be affecting things directly, attracting or repelling a charge so ∴ the electric field is direct to the charged conductor ∴ it is normal to the surface" (s. 4). Most of these arguments seemed to either tacitly or overtly rely on a spherical conductor for their justification of the "evenness" in their argument, which shows a failure to grasp the full parameters of the question.

Five other students argued that the phenomenon happened because the effect of all the infinitesimal charges on the sphere cancelled out except in the normal direction at all points on the surface, e.g., "The electric field is the addition of all of the vectors from every point and each point radiates its own electric field in all directions from it the only vector which is not cancelled by an opposing vector is the one perpendicular to that point" (s. 8). While this is true, these respondents failed to argue why this should be true for a charged conductor as opposed to any other configuration of charge; that is, their response is either a rather complicated tautology or assumes "evenness" as in the category above.

In a more useful and apposite way, four respondents based their reply on the fact that a conducting surface is at a uniform potential, and that the field would therefore have to be at right angles to it, e.g., "The direction is always normal because the surface of the conductor is equipotential, hence direction of field must be perpendicular, so all points on the surface are at the same potential" (s. 9). Two others said that the electric field would be zero inside a conductor and used this as an explanation, e.g., "E = 0 in a charged conductor ∴ the electric field at the surface must be normal otherwise E = 0 would not be true" (s. 5). These respondents were reasoning in terms of secondary facts about fields, rather than reasoning from first principles about the essential nature of a conductor as a material in which charge can move freely.

Only three respondents gave the response that the charge in a conductor would be moved by any component of the field which was parallel to the surface, and so cancel this component. This could be stated quite succinctly, as in the statement, "Because if there were any component other than normal, electrons would congregate untill [sic] the field became 0" (s. 52), and is in itself quite a simple answer. Few students seem to have been comfortable enough in this domain to reason from ideas about charge movement.

Responses to this question were generally at a low level. Most of the respondents seemed to have some tacit or overt assumption that the conductor was spherical. However, the reasoning involved in the desired answer does involve a number of points: that charge is free to move in a conductor, and that the movement of this charge in turn affects electric fields.

A similar theme is taken up in the part of chapter 6 dealing with potential in fields, potential difference and circuit phenomena.

Comments on qualitative explanations of causes

Students generally were unable to explain the phenomena in question using qualitative reasoning, which is consistent with their inability to use principles in answering quantitative questions as described in the first section of this chapter. In the next sub-section, students are giving qualitative descriptions without need for causal explanations, and, as such, their answers are somewhat more effective.

Qualitative questions involving description

The answers to questions requiring descriptions appeared to be of a rote-learned nature. Students appeared to be responding to simple cues in the questions, rather than the deeper structure of the issues. They were able to satisfactorily answer routine questions, but the nature of errors made on these questions was of interest. Often, the students' knowledge seemed to be limited to a number of phrases, about various concepts, which sometimes became confused. It was interesting to note the difference in responses between the first time a question was asked, in the mid-semester test, and the second time it was asked, in the year-end test. Once again, students did not generally appear to be able to work from first principles.

Graphing electric field and electric potential
(Question 1(a) of year-end test)

Sketch graphs to show how the electric field and electric potential depend on the radial distance from the centre of a charged conducting sphere, both inside and outside the sphere.

To answer this question from first principles would appear to demand a good grasp of the concepts of electric field and potential, and the behaviour of each inside and near a charged conductor. However, many students appear to have produced a rote-learned answer to this question, and those who tried to produce an answer from their own reasoning about the concepts appear to have been less successful in terms of scores for this question.

Of the 55 students sitting this test, 38 attempted this question. Three of them stopped after drawing a circle, which presumably was intended to represent a sphere. It may also have been an attempt to draw the diagram (figure 4.1) which was reproduced by a number of the students, as discussed below.

Five students drew a sphere with some sort of field lines around it, rather than drawing graphs as the question requests. This may result from a lack of clarity about the difference between a graph and a diagram, or it may be simply that the students felt incapable of producing the graphs. It also seems possible that students were uncertain which of several pictures that they had seen in class were applicable.

Five other students produced the diagram shown in figure 4.1. This diagram shows a positively charged sphere, with graphs of electric field and electric potential, in that order, below the circle. Both these graphs are symmetrical about the centre of the circle. Two more students produced the same diagram, with minor alterations, which did not affect the correctness of the graphs (respectively, the graph of electric field before the graph of potential, omission of circle at top). Six more students had a version of the same diagram, but with changes which made it physically inaccurate. Two of these had exchanged the graphs of electric field and potential, two had changed the graphs slightly, and two had included asymptotes in the electric field or voltage at the edge of the sphere.

The widespread reproduction of the diagram in figure 4.1 by students indicates that it was rote-learned as a picture. It is not *a priori* obvious to draw a diagram with a sphere at the top and lines showing the correspondence with graphs of field strength and potential at positive and negative sides of the origin. Students took the time to include these unnecessary features while under time pressure in an exam situation, which indicates that they were unable to separate the essential features of the concepts from what were non-essential stylistic features of the diagram they had been shown. The students appear to have rote-learned the entire diagram, rather than isolating its conceptual message.

As well as the large number of responses which seemed to be attempts to copy figure 4.1, seventeen other students produced graphs which were of standard x-y format, with no symmetry about $x = 0$ or picture of a sphere. Most students who produced graphs in this format gave incorrect responses, presumably because they were thinking independently, rather than copying a figure they had seen previously.

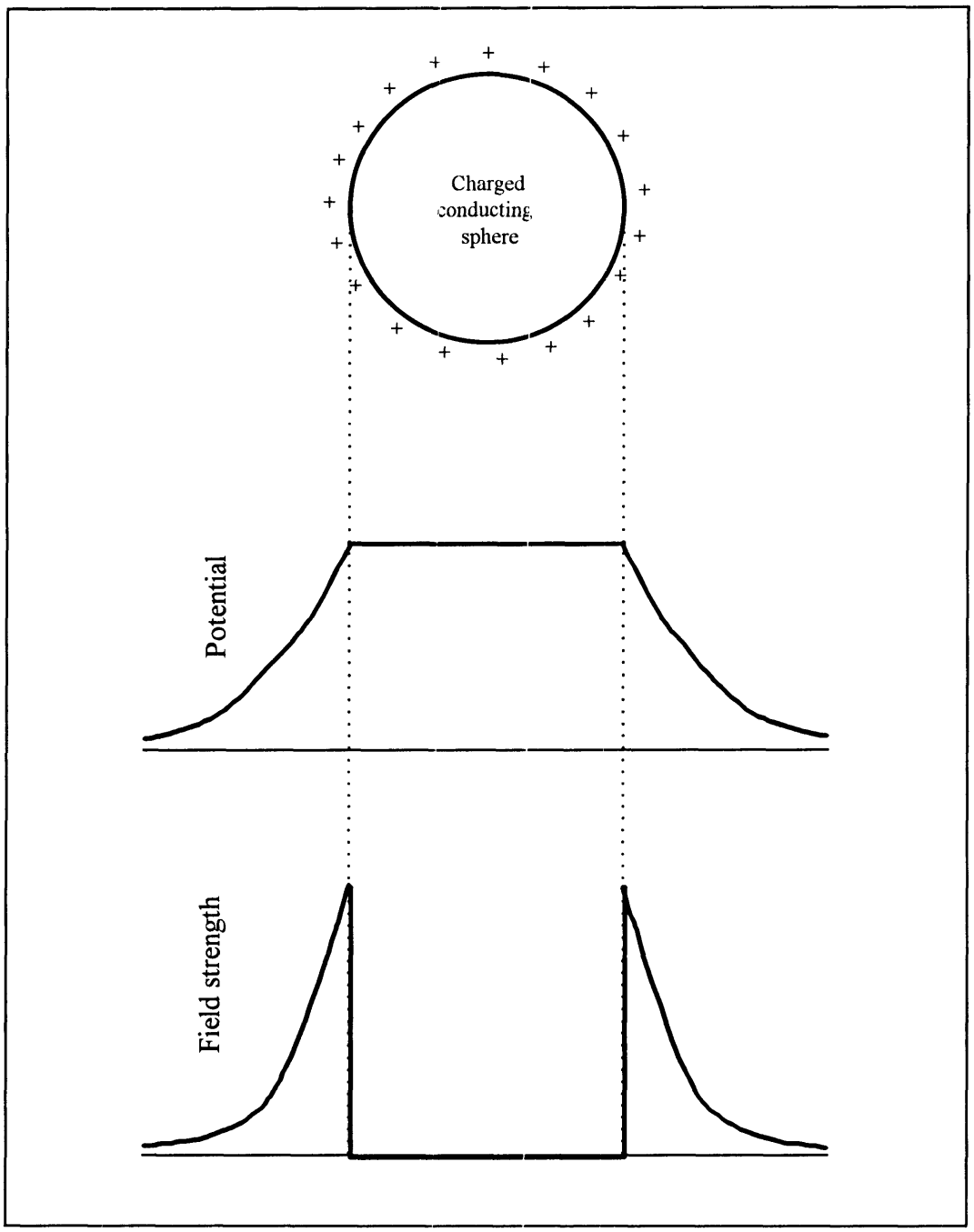

Figure 4.1 Graph of potential and field strength near a charged conducting sphere

The distinction between electric field and potential was seldom clear in these *x-y* graph answers. Two of them said explicitly that the electric field and potential would be the same, and two more had separate but indistinguishable answers for field and potential. Three had potential increasing and electric field decreasing with increasing distance from the sphere. Another had potential and electric field as two parts of the same curve, with potential the part inside the sphere and zero outside, with electric field vice-versa. Finally, six students distinguished between E and V by reference to their $1 / r^2$ vs $1 / r$ respective dependencies,

but drew otherwise similar graphs for both. The impression from responses including *x-y* graphs was that students could not usefully express a difference between potential and field strength. Only three students produced correct graphs in *x-y* format. These were the only correct graphs which did not appear to be direct copies of figure 4.1.

The overall impression from responses to this question is that the students appear to have a poor understanding of the concepts of voltage and field involved. A large group of students seem to have directly copied the answer from a diagram they had seen in class. Those who used independent reasoning on the problem came to a less accurate answer than those who appear to have copied lecture information verbatim. The answers to the question discussed below also emphasised the use of lecture information, rather than reasoning.

Description of a macroscopic phenomenon
(Question 4 of mid-semester test)

What is St Elmo's Fire?

This question did not seem to encourage responses with complicated reasoning. It calls for recall of a phenomenon described in lectures, and does not specifically ask for an explanation of the mechanism behind the phenomenon.

Of the 51 students attempting the mid-semester test, six did not answer this question. Five answers restricted themselves to a description of the observable phenomena with no description of the process involved, e.g., "St Elmos Fire is a form of lightning which creates a cloud of light in the sky" (s. 35). Four others said only that it was a "corona discharge", which is correct, but limited.

Eight students said that the process was caused by "charge" or "electricity" with no real elaboration, for example: "St Elmos Fire is a corona that forms on top of high objects ie ships masts, church steeples, etc. It is formed by static electrical charges from the atmosphere and object" (s. 1). Six others said that it was caused by charge or electricity caused by friction, e.g., "St. Elmo's Fire:- Static discharge of a heat due to friction betwen the heat & the water. is known a [sic] St. Elmo's Fire" (s. 2). These answers introduce some electrical terminology, but no more than that.

Going further, fourteen students said that St Elmo's Fire was caused by the breakdown or ionization of air, but said little about the macroscopic conditions causing this breakdown, for example, "St. Elmo's Fire is the phenomenon that appears around ships masts at sea when the air becomes ionised and charge starts flowing producing a faint glow" (s. 47). There is a mention of ionisation here, but no explanation of why the ionisation occurs.

Eight responses did describe the process where charge in clouds causes induced charging of high points, e.g., "St Elmos Fire occurs when a build up of electrons occur. During rain storms when lighting [sic] occurs the clouds become very negatively charged, because the lightning [sic] draws the negatively charged particles from the earth, making the earth very positivly [sic] charged. St Elmos fire occurs when this build up is released" (s. 49). Of these eight responses, five went on to relate this to breakdown of air, which is responsible for the glow seen in St Elmo's Fire, e.g., "When a thunderstorm is immininet [sic], the masts of ships tend to have a 'light' appearing on top of them. This is due to the clouds being highly charged with respect to earth and the mast of the ship being close to earth have the ionisation of air around them due to the large difference in electric fields" (s. 21). This combines mention of ionisation with an explanation of the macroscopic phenomena behind it.

Responses to this question were not particularly interesting, but it was noteworthy that some students concentrated on the concrete visible phenomena, others mentioned aspects of the physics, and still others tied them all together.

Definition of a technical term
(Question 1 of mid-semester test, 1(b) of year-end test)

Define the dielectric constant of a material.

This question appeared in both the mid-semester test and the year-end test, and it is interesting to note the development of student responses between the two tests.

Responses from the first (mid-semester) test

Of the 51 students attempting the test, twelve did not attempt this question.

Sixteen students related the dielectric constant to phenomena involving electric circuits, namely, conductivity, resistance, electrons or current. Nine of these students said that the dielectric constant described the conductivity of a material, e.g., "The dielectric constant is a measure of the conductivity of a material" (s. 22), with another two of these students relating it to the resistance, and two to the number of free electrons in the material. Three of the nine students relating the dielectric constant to circuit concepts gave answers relating to the amount of current that could flow, or the response to a flow of electricity, e.g., "The dielectric constant of a material is a measure of how the material responds to a flow of electricity through the material. The strength of current generated in the material due to the amount of free electrons present" (s. 19). This set of responses used circuit ideas with which

students were more familiar in order to explain the dielectric constant, and seem likely to have been guesses.

There were other responses which appeared to be guesses, but with a heavier emphasis on equation use. Three students gave an answer identical to each other, that the dielectric constant was "a constant of proportionality which relates the resistance of a given material to its temperature" (s. 50, s. 8, s. 2). It seems likely that they memorised the definition of the thermoelectric constant, and simply repeated it here. Two more students gave replies that seemed to represent simple guesses - one said "electric potential" (s. 6), the other "F / A / q^2" (s.56). Four students confused the dielectric constant, κ, with the electrostatic constant, k, and said that it was equal to $1 / (4 \pi \varepsilon)$. Another two students said that ε itself was the dielectric constant. These equations have been used in a way which does not suggest the presence of a qualitative understanding behind them.

More relevantly, five students related the dielectric constant to its use in describing capacitor behaviour. Three of these gave the equation $C = \kappa C_0$, of whom two explained it, e.g., "Dielectric constant is a constant by which amount capacitance will be increased by that material if it is used as a dielectric as opposed to a vacuum. i.e, C_0 = capacitance with vacuum dielectric, C = capacitance with new material, κ = dielectric constant. $C = \kappa C_0$" (s. 4). This is an effective operational definition of the dielectric constant.

Finally, seven students gave answers in terms of the very abstract idea of ratio of permittivity of the medium to free space, e.g., "The dielectric constant is the relative permativity [sic]. It is therefore defined as $\varepsilon / \varepsilon_0$ where ε = permativity of the medium, ε_0 = permativity of free space" (s. 11). While this is an accurate answer, and received full marks in terms of the department's marking scheme, it is not obvious what meaning, if any, the students attached to the words in this response.

The question discussed here asks for the definition of an abstract concept. This first time this question was asked, there were a large number of equation-based answers, coupled with more or less plausible guesses. The answers based on capacitor behaviour seem to be most clear in their relation to reality, in that they give an operational definition of the quantity, rather than referring to an undefined "permittivity". It is interesting to note the change when the question appears in the second test.

Responses from the second (year-end) test

The identical question appeared as question 1(b) in the year-end test, and there was a shift in the nature of student responses. In general, the second set of responses to this question do not show the same variety as in the mid-semester test.

Of the 55 students sitting this test, 35 attempted this question. Only five students appear to have made the sort of guesses which dominated the responses to the mid-semester test, e.g., "The dielectric constant of a material in definition can be broken down into 2 parts. di meaning 2 and electric meaning, the use of particle flow to create energy. Therefore dielectric constant of a material is the two different particles being electrons and protons added together to give a constant" (s. 49). The rest of the students cited equations, with more or less success.

Four students made what appeared to be guesses, based on equations. Of these, three students identified the dielectric constant with the electrostatic constant, and wrote that "dielectric constant = $k = 1 / 4 \pi \varepsilon$" (s. 17, s. 31, s. 37), and the fourth wrote "$\kappa = C / \varepsilon_0$" (s. 9). This indicates an equation-centred way of thinking. There is a certain lack of deep structure related to the meaning of this constant in the answer. The real meaning of the term to these students remains unclear.

Eleven other students used the accurate equation $C = \kappa C_0$, with varying degrees of sophistication. Of these, four showed no clear understanding of the physical situation involved, e.g., "The dielectric constant of a material. $C = N C_0$ (F). C = Capacitance of conducting material with insulating material. N = Constant value of insulating material. C_0 = Capacitance of conducting material in air" (s. 53). In this response, it is not made clear what the roles of the conducting and insulating materials are. Three of the eleven students who cited this equation mentioned increasing capacitance of a capacitor, and four explained how one increases capacitance by placing material between the plates of the capacitor, e.g., "The dielectric constant κ of a material is defined as the capacitance C of the capacitor when the space between the plates is filled by the dielectric medium divided by the capacitance where the plates are separated by a vacuum. ie, $\kappa = C / C_0$" (s. 12). This student has a full description of the physical meaning of the dielectric constant in terms of physical observables.

Fifteen students used the equation $\kappa = \varepsilon / \varepsilon_0$, which is also accurate, but more abstract than the equation based on capacitance. Three of these students gave this equation and no further details. Five of them added that κ was the relative permittivity, but did not make clear what it was relative to. Seven of the fifteen students citing this equation went on to say that it was the permittivity relative to free space or vacuum, e.g., "$\kappa = \varepsilon / \varepsilon_0$. The dielectric constant of a material is the ratio of the permittivity of that material to that of free space" (s. 13). While this answer is correct, it is impossible to tell whether the student has any understanding of what the permittivity of material means in this sense. The students' ability to repeat this abstract definition does not greatly inform us as to the their understanding of this topic.

It is interesting to note the change in responses to this item since the mid-semester test. There has been an increase in "correct" responses, but one wonders how well these answers have been understood by the students, as they consist of a stringing together of technical terms without explanation in ordinary language. These responses define jargon with jargon, which does not indicate (or disprove, for that matter) any link to students' everyday thinking. The concept of dielectric constant, particularly when phrased in terms of the concept "permittivity", is of a rather abstract nature.

Overall comments on qualitative questions

Generally, students were not able to give qualitative explanations of phenomena. Their most effective answers to qualitative questions appear to have been based on rote-learning, with students including information which was not essential to the issues in the question, and defining technical terms with technical terms.

As an example of this rote-learning, students' drawings of graphs of electric field around a charged object seemed to replicate a diagram they had seen previously, rather than represent their own ideas. The development between tests of their response to the question asking them to "define the dielectric constant" was interesting, in that it showed an increase in simple jargon responses that were correct but did not demonstrate understanding. This indicated an emphasis on memorisation.

Linked to this emphasis on memorisation, the students had difficulty with a number of specific concepts, when required to reason rather than simply remember facts. Students had difficulty with the idea of charge moving within an electric field and thereby changing the field. This feedback between the charge movement and the field was not clear in the minds of most of the students. They had problems also with the three-dimensional behaviour of a charged particle moving in a magnetic field, where they were generally able to give the equation $F = q\,V\,B$, but were not able to describe, in simple terms, how this would lead to a particle having a constant velocity in a steady magnetic field. In general, a high level of qualitative understanding of concepts was not displayed.

Responses to qualitative questions, like the responses to quantitative questions, showed the students having difficulty using and explaining concepts in the topic area of fields. The overall impression from the responses to qualitative questions is one of poor understandings and responses based on rote-learning. This is a similar finding to that leading out of an investigation of students' understanding of their quantitative work in the first section of this chapter.

CONCLUSION

Students' manipulation of equations in their course assessment tests was poor overall. Most students were able to substitute numbers into equations with little difficulty, but few were able to manipulate these equations following general principles relating to electric and magnetic fields. This difficulty with general principles was also seen in their answers to more qualitative questions.

At the beginning of this chapter, three specific research questions were identified in order to provide structure to investigation of the general theme of students' understandings in the analysis of the responses to these coursework tests. These research questions have been answered, as set out in point form below:

1) It was indeed possible to discern a variety of strategies in students' responses. There was a range from simple recall of individual facts, through use of algorithms, up to a small number of responses which appeared to be able to use principles to manipulate equations. This variety in responses is worthy of investigation in the main study described in the next chapters. This investigation was guided by use of the theoretical frameworks of the current study.

2) Even within the limits of the assessment tasks, students' responses had many weaknesses. This was a recurring theme in the detailed analysis of responses in this chapter, and is also illustrated by the average scores of 39.2% for the mid-semester test and 47.7% for the year-end test. Students' work was plagued by errors, in terms of the marking scheme used by the Physics Department.

3) However, these errors did not, by and large, provide a large insight into students' alternative conceptions about the topic. The errors tended to indicate use of equations which were not appropriate to the topic, or failure to remember definitions. However, the relatively "closed" nature of these questions made it difficult to discern the nature of students' own conceptions. This issue of the students' own conceptions about the topic is worthy of attention in the main study, where instruments have been designed especially to fill this need.

While the responses to these assessment tests were not highly infomative about the details of students' conceptions, a number of specific concepts were seen to provide problems. One instance of this was the tendency for students to confuse equations relating to the radial field and the constant field cases. This occurred in a number of questions, and seems to show a use of equations which contain the right variables without concern for the physical principles applying in the situation. This approach of substitution into equations without regard for

physical principles has been seen across student problem solving in many problem domains in physics, so it is no surprise to see it here.

This lack of knowledge of physical principles to guide the use of equations was also behind students' trouble with the relationship between voltage, electric field and energy. They had trouble with clear reasoning involving these quantities. While they could cite the equations involving them, they were unable to manipulate the equations to obtain the desired results. This seems to show a lack of ability to use these concepts to guide manipulation of the equations; the concepts of voltage, energy and electric field are investigated in detail in the following two chapters. Lack of knowledge of physical principles was particularly damaging to students' ability to produce proofs.

Students had trouble with the very idea of proof. Where proofs were called for, they often simply cited equations with no justification, and no indication that they realised that this was not adequate. This indicates that they are not comfortable with the idea of manipulating equations following guiding concepts, which is the essence of a mathematical proof.

This lack of ability to use guiding concepts to explain was also evident in responses to qualitative questions, where students' answers often seemed to be learned phrases or diagrams. Their answers to questions involving qualititive explanations were of the same low general level as their responses to questions requiring quantitative proof by equation manipulation, and displayed the same lack of use of physical principles.

Overall, students' responses to the Physics Department's assessment tests showed an inability to use concepts effectively to guide answers. This was noticeable both in the students' attempts to manipulate equations and in their answers to more qualitative questions. The analysis of the Physics Department tests has shown the nature of students' attempts to manipulate equations in this problem domain. The following chapters investigate the qualitative ideas that students have, without emphasis on equations.

Chapter 5

STUDENTS' REPRESENTATIONS OF ELECTRIC AND MAGNETIC FIELDS

The previous chapter has shown that the sample of students in this study had difficulties using qualitative concepts in their manipulation of equations when responding to the course assessment tests. The current chapter explores the nature of their qualitative understandings of electric and magnetic field representations. This investigation has a particular focus on the structure of understandings, which is explored using the framework of the SOLO Taxonomy. To assist with this, the test questions are dealt with according to theme, rather than order of appearance, giving a more integrated picture of the students' ideas about this topic.

Chapter 5 and chapter 6 report the results from the main body of tests and interviews conducted for this project. Unlike the previous chapter, these chapters both incorporate the SOLO Taxonomy. As in the preceding chapter, qualitative groupings were found in students' understandings of a question, and these are presented as results. However, the present chapters go on to describe these groupings in terms of the Taxonomy. Descriptions within the Taxonomy focus on the structure of the responses. Responses have been classified into the levels of the Taxonomy (unistructural, multistructural and relational) according to the amount of connection shown between ideas in the response. These levels provided a useful framework for describing responses. The levels also fit within the context of modes of functioning. The concrete symbolic and formal modes are of interest for this study. These modes contain the levels mentioned above.

The results presented here and in chapter 6 are the result of analysis of approximately 60 half-hour interviews, which produced approximately 400 pages of typed transcripts. Analysis of the results included phenomenographic analysis into groupings, and SOLO analysis of those groupings. This SOLO analysis included informal use of SOLO concept maps, as discussed in chapter 3. This concept mapping used a technique developed by other authors to aid in SOLO analysis of understandings; maps had relatively few components, arranged in a hierarchical fashion which reflected the SOLO coding. The nature of the technique was fully explained in chapter 2.

One aspect of the SOLO analysis has also been discussed in chapter 2, but bears repetition here. In the earlier version of the SOLO Taxonomy (Biggs & Collis, 1982), there was no

recognition of multiple learning cycles within a single mode of functioning. In the present work, following more recent publications on the Taxonomy (Campbell, Watson & Collis 1992; Pegg, 1992b; Levins & Pegg, 1994), these multiple cycles have been used, and this involves recognition of fine structure in learning cycles which was not recognised in the 1982 version of the Taxonomy. The findings in this work reflect the more recent formulations.

Report of findings in these chapters involves a significant number of quotes from the students. These quotes are given with the identification number of the student, in brackets, e.g., "(s. 17)". Quotes from the student's written work are in plain type, and quotes from the interviews are in italics. These reports are made in the context of the organisation of this chapter into sections centred on aspects of the topic.

This chapter, and chapter 6, have been structured according to the organisation of research questions in table 3.1. That table divided the issues of the topic area into those connected with representation of fields on the one hand, and those connected with understandings of related phenomena on the other. Chapter 5 deals with the issues of field representation, and chapter 6 with those of understandings of related phenomena. These chapters have been further divided into sections corresponding to the rows of table 3.1. In table 5.1 below, the titles of the three relevant rows of table 3.1 have been reproduced, along with a summary of the headings in this chapter which deal with them. Each of these three rows corresponds to one of the three sections of this chapter. After each chapter heading in table 5.1, questions from the instruments are listed which deal with the topics of that heading. In order to provide a tighter focus to this chapter, only one question is described in detail for each category. Where more than one relevant question exists, the additional questions are discussed at length in appendix O. Those questions described within the chapter are in bold type in table 5.1, and those dealt with in appendix O are in plain type. The roman numeral refers to the number of the test/interview, and the number after that refers to the number of the question in that interview. For example, "II6" would indicate the sixth question in interview two.

In the following table, it can be noted that there are many headings, with only a few questions under each heading. This reflects a deliberate decision during the design of the study, to cover the topic area of field representation with a range of questions, rather than focussing on a narrow context within that topic area.

TABLE 5.1

Aspects of field representation

Field lines:

Field lines and lines in iron filings **I1**, I2 (see appendix)

Field lines and trajectories of charged particles in fields I5 (see appendix), **II7**

Field lines and induced current **III1(iii)**

Field vector:

Connection between the field vector and field lines I3 (see appendix), II1, 2, 3 (see appendix), **II4, 5, 6**

Reasoning about quantitative aspects of the field vector **II8**

Students' use of the magnetic field vector **III2, 3, 4**

Flux in fields:

General conceptions of flux **I7, 8**

Use of flux with induced current **III1(viii)**

The topic-based research questions, stated in chapter 3, couple with the theoretical framework used, to give a specific research question to be answered in course of this chapter: What is the nature of students' understandings of these topics, in terms of the theoretical framework used? Within the framework of the SOLO Taxonomy, this involves an investigation of elements of understanding within students' responses, and the relationships between them. Findings about these elements are results in terms of that framework, and will form part of the report about students' representations of fields. There is also value in characterising the difference between the modes of functioning in the Taxonomy, as seen in these students' responses.

Student representations of fields are an important consideration, as reasoning about any topic is based on representations in the student's mind. The information in this chapter about understandings of fields leads on, in chapter 6, to information about students' understandings of phenomena related to fields, where the students use their representations of fields to explain phenomena.

UNDERSTANDINGS OF FIELD LINES

Field lines are an important aspect of students' understanding of fields. Recent literature (Galili, 1995; Törnkvist, Pettersen & Tranströmer, 1993) has pointed out student misconceptions related to field lines. The students described in those papers showed a grasp

of field lines based on particle trajectories in the field. This idea of field lines as paths that particles will follow is a more concrete idea than the scientist's concept of field lines.

The issue of concrete versus abstract leads to the SOLO Taxonomy and, in particular, to its concrete symbolic and formal modes, which relate particularly to the notion of abstractness. The distinction between these modes has been explored in a general way in chapter 2, but it is valuable to explore what the modes mean in the specific context of field lines. A hypothesis is: concrete symbolic responses refer to field lines as representing things which are directly observable in the real world, such as particle trajectories, whereas a formal understanding is reflected by responses which can describe field lines without referring to direct concrete referents, and in relation to the abstract notion of the field strength vector.

Students' understandings of field lines were investigated in separate questions involving iron filings around a magnet, particle trajectories in an electric field, and induced current in a conductor moving in an electric field, respectively. This gives a range of situations in which to investigate students' understanding of field lines. This chapter section deals with the student responses to each of these questions in turn. An important theme through the investigation was the difference between concrete and abstract representations of field lines in students' minds.

Field lines and lines in iron filings*

The following is question 1 from test/interview 1.

If you hold a magnet under a sheet of paper covered with iron filings, the iron filings form a pattern as shown [see figure 5.1].

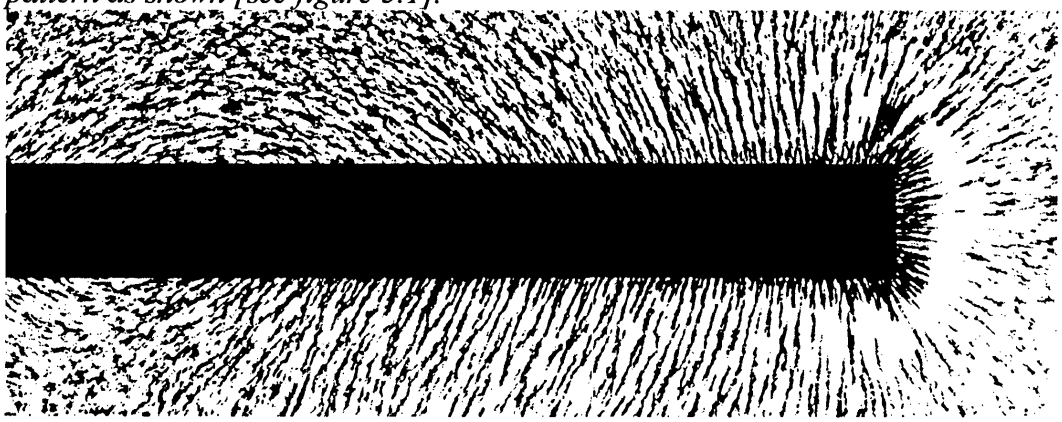

Figure 5.1 Iron filings around a magnet

* The findings reported under this heading have already been the subject of a publication: Guth, J. & Pegg, J., (1994). First-year tertiary students' understandings of iron filing patterns around a magnet, Research in Science Education, **24**, pp. 137-146

(i) Explain why this pattern forms. We know large pieces of iron would go to the magnet. Why do the iron filings show the magnetic field of the magnet instead of simply going to the magnet?
(ii) What will happen if an iron filing is dropped into an empty space in the pattern? Why?
(iii) The pattern is now swept away, leaving the magnet by itself. What will happen if an iron filing is dropped where there used to be an empty space in the pattern? Explain why.

This question deals with the well-known, but generally poorly understood, phenomenon of formation of lines in iron filings surrounding a magnet. Explanation of this phenomenon involves some concept of field lines and the nature of the field around a magnet. Rather than phrasing a question directly in terms of abstract concepts, this practical example was chosen in order to see what use students made of the concepts in their explanation. This question was used in the pilot testing, in a slightly different form. Students' responses to this question can be seen in the context of their answers to the second question of the first test/interview, which are discussed in appendix O. Briefly speaking, their answers to that question, also involving magnets, showed that most had difficulty reasoning about magnets.

In the current question, students did not show a very complex understanding of the process which causes the phenomenon in the question. Most tended to give simple descriptions, and many were aware that their explanations were incomplete, or even self-contradictory. The students' responses tended towards the concrete symbolic mode (n=13) of the SOLO Taxonomy as opposed to the formal mode (n=11) with four responses being transitional between modes. The levels of description which appeared are discussed below.

Concrete symbolic responses

Characteristic of this mode was the idea that the filings were moving to directly map items. At the unistructural level, the element mapped was the undifferentiated "field of the magnet". In the multistructural level, students started to differentiate various aspects of the field: field lines, field strength, magnetisation of filings and forces on the filings. A relational level was reached in this mode of functioning when students related these aspects into a consistent picture where filings moved to distinct "field lines" because these were where the field was strongest. Students responding in this mode of functioning generally believed that there were distinct lines of magnetic field separated by spaces. The spaces between these lines were often considered to be areas of lesser or even zero magnetic field. The concrete elements of the field become more complicated with increasing level.

The sole student at the unistructural level described the filings as mapping out some sort of undifferentiated "field" of the magnet, which was judged to be a single structure in the student's mind, e.g., "The iron filings follow the magnetic fields [sic] patterns of the magnet"

(s. 8). This level was only seen once in the sample, probably because of the amount of exposure the students have had to field concepts, particularly that of field lines. There is no mention of field lines or the other concepts which were evident in the multistructural responses.

The multistructural level of response was shown by eight students, who were beginning to apply the ideas of field lines, field strength, magnetisation of filings and forces on the filings. They lacked, however, an integrating principle, e.g., one student (s. 38) wrote:

> (i) The iron filings are small and light enough to be attracted to the individual lines of magnetic force, thus displaying the pattern.
> (ii) It will be attracted to one of the lines of filings. The direction it moves is dependent on the various strengths of magnetic attraction.
> (iii) It will be attracted to where there was a line of filings, for the same reason as above.

This student has used the ideas of lines and attraction, but has not mentioned a mechanism causing the movement.

The four students at the relational level related the concepts using a unifying structure based on ideas about field lines. Field strength is part of this structure, and explains the movement of the filings. For example, one student (s. 56) stated,

> (i) The iron filings situate themselves in a position oriented according to the magnetic field.
> *Interviewer: [reads from written response] They "situate themselves in a position oriented according to the magnetic field". Why do they do that?*
> *Student: Because that's where the field lines are.*
> *Interviewer: So what are you saying, what do you mean by "oriented according to"?*
> *Student: Like the filings, you know, the lines, the field lines go like that, sort of curved in. They just gather along the field lines.*
> (ii) The iron filing will situate itself oriented on the lines of force of the magnetic field.

Use of the word "oriented" here seems to correspond to the idea of "just gather along the field lines". This would seem to be a naïve conception that has survived, dressed in more appropriate terminology.

(iii) [No written answer]

Interviewer: You didn't answer it, but if the pattern - you may as well read it. [long pause]

Student: It'll just go back into the, move into the pattern.

Interviewer: So it'll move to where one of the lines was?

Student: Yeah.

Interviewer: O.K., so would you say why it might do that?

Student: Coz that's where the strength of the field is. Or it's more concentrated or something.

This student's conception was based on the simple assumption that if we see a finite number of lines, they must be real. The student has elaborated that assumption by associating these lines with greater field strength. There is a relationship seen between the movement of the filings and these lines of greater field strength. This response is still solidly based on the assumption of a finite number of concrete lines in the field, which are mapped out by the filings. In the concrete symbolic mode, there was an acceptance of this assumption of tangible lines, without more formal abstraction being considered.

Formal responses

In the formal mode, students realised that the patterns in the filings were caused by some process more abstract than simply mapping out of a finite number of concrete "field lines". To explain this, students used two ideas. First, that filings became magnetised and hence aligned their long axis parallel to the field at that point. Second, that these magnetised filings affected each other by their magnetic fields, which explained the spacing of the lines. A relational level was achieved where these two ideas were related to each other and to the formal definition of field lines.

These ideas have been considered formal as they show signs of using abstract propositions in reasoning. The idea that each filing becomes a magnet is not a directly observable idea - it is an abstraction, especially when the students come to reason about the poles and fields of the magnetised filings. It is the use of this abstract idea in describing physical phenomena that indicates formal reasoning.

Four of the responses were transitional between the formal mode and the concrete symbolic mode. As mentioned above, the ideas of magnetic alignment of filings and magnetic interaction of filings are required to explain the formation of the pattern around the magnet without postulating a finite number of concrete lines. The four students in this category applied one of these ideas, but still remained attached to the idea of a finite number of

concrete lines, thereby combining concrete and abstract aspects in their response, e.g., one student (s. 1) wrote,

> (i) This pattern forms because of the magnetic field lines cause the iron fillings [sic] to arrange themselves in this order. Because the fillings [sic] are small and movable this allows them to arrange themselves in this order ...
> *Interviewer: How do the magnetic field lines cause the iron filings to arrange themselves into this order?*
> *Student: I would imagine ...like a little magnet, arranged, with the north and the south and there's the magnetic field lines. [pause] Something like that I think ... They arrange themselves along that line*
> *Interviewer: O.K., why should they do that?*
> *Student: Because of this magnetic force which is attracted to this north to this south and this south to this north.*

As can be seen in the above, the student used the idea that the iron filings form chains due to their induced magnetism, which is an idea with a unistructural character in the formal mode.

> (ii) It will join the pattern because it is not in an [sic] line of electric field.
> *Interviewer: Well, what would you say is the difference between a point on a line and a point right next to it?*
> *Student: Stronger, stronger on the line.*
> ...
> *Student: I'm going on what the lines are, a physical thing, they're not just, um, something we draw in our books. I'm, um, suggesting that they're actually there.*

This is a clear statement that the student still feels a need to assume a concrete cause for the visible lines, and hence is still linked to the concrete symbolic mode, in spite of his use of abstract concepts about induction of magnetism. This is the essence of his transitional level of response, which contains aspects of both concrete symbolic and formal modes.

Fully in the formal mode, although lacking in sophistication, were the unistructural formal responses. Five students stated that the pattern formed because the filings became magnetised and aligned themselves like tiny compasses. They used the single idea that magnetisation of filings causes them to rotate parallel to the local field direction. Using this reasoning, they were unable to account for the formation of distinctly spaced lines, as seen in the student's (s. 22) response below,

> (i) The filings are big enough to align with the field, but are too small to be pulled by it.

(ii) The magnetic field will magnetise the filing, and it will thus align itself with the field.
(iii) Same as in part (ii), and for same reasons.

In the above, the student made no comment on the formation of distinct lines, so this was taken up in the interview:

Interviewer: If you look at the picture closely, it looks like there are distinct lines of iron filings there with distinct spaces between them. Now, would that happen if all that happened was they just swivelled round?
Student: Yeah, I see - um depends whether the filings themselves are just round or whether they've got a bit of length to them, coz if they're like scattered about and they align themselves, then they're going to form lines and there's going to be some space in between them - it'll vary but there will be some.
Interviewer: Why will there be space between them? Like, if you imagine a crowd where everyone is looking the same way, it doesn't mean that they'll form lines.
Student: Yeah, I see what you mean - dunno, forgotten. I knew once.
Interviewer: Could you guess?
Student: Um, [pause] I couldn't even hazard a guess.

A unistructural level of response was not sufficient to fully explain the phenomenon, as it focussed solely on the alignment of the filings. Students at the multistructural level focussed on this idea plus one other, without integrating them. Five students responded at this level. Here, we still have mention of alignment of filings in the magnetic field, as was seen in the unistructural formal responses. At the same time, another concept is used, where the lines are explained by interaction between the magnetised filings. At the multistructural level, these understandings are incomplete, essentially saying that there some sort of ill-defined interaction which moves the filings relative to one another. Even so, students are working with two aspects of the abstract model of invisible magnetisation of the filings, and using it to explain the phenomenon, as did the student (s. 30) below:

(i) Because every piece of iron has a +ive and -ive end like the magnet. Therefore, the positive ends will form closer to the +ive end and lie in a line in the same direction the field does.

The "+ive and -ive" above shows a confusion between electric and magnetic fields which was common in the sample. This theme was taken up in other questions in the study, but was not pursued here.

(ii) It will turn it self [sic] until it to [sic] lies as the other will. They all will lay almost equidistant from one another because the filings will have their own little magnetic fields.
(iii) Will align to a magnetic field line.
Interviewer: O.K., why are they in line? You can see that looks like a line, that looks like a line, why should they form lines?
Student: Because that's the direction of the magnetic field - if um - well, it's gotta go - um - well, the field's sorta more denser close to the magnet than it is out, so it makes sense they've got lines, I guess.
Interviewer: Why should they form these gaps between them?
Student: Coz the filings repel one another.
Interviewer: And why should they repel one another?
Student: Well, all the filings, the ones in lines, they're going to be positive to negative, all the way around. And - so the filings next door to 'em, they're going to be positive to negative all around ... And likewise in the next line, there's going to be the same, ions, positive and negative, so the two positives are going to repel and the negatives are going to repel.

This student has the ideas of magnetic alignment of filings and magnetic interaction between filings, but still has a poorly-defined idea of the nature of the interaction between filings, and lacks a clear overview of the process forming the lines. This lack of overview and appreciation of interactions is typical of multistructural forms of response.

By contrast, the single student (s. 32) at the relational level had the ideas about interactions between the iron filings fitting into a framework involving the idea of field vectors, and had a clearer picture of field lines, saying,

(i) The filings align themselves in the direction of the magnetic field because the field magnetizes [sic] <u>them</u> most effectively along their length...
(ii) The new filing, too, will be magnetized with its own little N and S poles and align itself in the prevailing field ... The new filing may well be attracted to, and join, a "chain" of filings along a field line. The fields of each of these tiny magnets reinforce one another.
(iii) The solitary filing will behave almost exactly as if the pattern were still there, though the tendency of the filings to link up may have decreased their motion, ie, the new filing is likely to stick to the magnet.
Student: ... if you have a north pole on its own, with no south pole to the magnet, then it will act in a magnetic field as a proton would in an electric field. And by following the path of that pole, you'd be able to see the direction of the lines ... the initial direction of its motion would be a tangent ... if it does have mass then its

initial direction is the direction of the field at that point. So if you were to place it a little further along in that direction, but once again with no inertia so it's not already moving, then each place that you put it, it will move off in the direction of the magnetic field and so you could trace out a field line ...

In the above, this student has included the points about magnetisation and alignment seen in the multistructural formal responses, but he has also discussed these in the context of the relation of the lines to the forces on a particle at any point in the field. This has allowed him to state the difference between field lines and particle trajectories, with some consideration of the effect of particle mass, a theme that is dealt with in greater depth when examining students' responses to the next question discussed. The description of placing the particle "a little further along in that direction" seems to be a hint at infinitesimal reasoning, and is a way to describe this difference. In this context, it is mentioned that the filings, as little magnets, form chains and align to the field. The response shows an overview of the question, though seems to lack conciseness. This conciseness would probably occur in a later cycle of growth, where this student's current relational understanding formed a single unit in a larger picture.

Conclusion about student understandings of lines in iron filings

These tertiary physics students generally had difficulty explaining the pattern in iron filings surrounding a magnet. There was a strong tendency for the students to reason in terms of a finite number of concrete lines to which the filings were attracted. This seemed to be their concept of "field line" in the context of this phenomenon. Twelve of the twenty-eight students used this model of concrete lines.

The students' understandings of the specific phenomenon of patterns in iron filings around a magnet have implications for broader issues regarding student representation of magnetic and electric fields. Students generally did not use the idea of an abstract field vector associated with each point in space, preferring to answer in terms of concrete elements when describing field lines. The common misconception that the field of the magnet had non-uniform "lines" which attracted the iron filings to map them seems to be worthy of attention when teaching. Full understanding of the iron filing patterns and their relation to field lines requires quite a high level of understanding.

A hierarchy of levels of understanding of the phenomenon was found, within the concrete symbolic mode and formal mode of the SOLO Taxonomy, with a transitional level between the modes. The formal mode was characterised here by student use of an abstract model to explain the phenomenon involved, and a willingness to accept that the pattern seen was not simply a reflection of a finite number of concrete field lines. Similar themes can be seen in students' responses to the questions discussed next in this chapter.

Field lines and trajectories of charged particles in fields

The relationship between field lines and trajectories in fields is more complicated than it might at first appear, requiring consideration of ideas of force, momentum and acceleration. Two questions probed students' understanding of these concepts.

The first question, described in detail in appendix O, was the fifth question of the first test. It was designed to probe students' ideas about field lines as the lines relate to movement of charged particles in the field. Two points were of specific interest. Firstly, to find whether students predicted that the finite number of "field lines" drawn in the diagram had a qualitative effect on the charge. This relates to the findings of the question about iron filings and field lines discussed directly above. Secondly, to find whether students believed that field lines were the exact trajectories that charged particles would follow in the field - this is in fact a misconception.

An analysis of results for the first question revealed that students fell short of the formal mode in their description of field lines and trajectories in their answers to this question. They commonly believed that field lines represent the concrete paths taken by particles in the field, rather than the more abstract notion that trajectories do not generally follow field lines. This may tie to an intuitive mechanical idea that movement is always in the direction of total force. In an effort to encourage formal reasoning, the question described below was included in the second interview. This second question was similar to the first question, but it also presented students with an argument which said why particle trajectories should not follow field lines. The second question is shown below, and was question 7 of test/interview 2.

A small positively charged particle is placed at point A [see figure 5.2]. Draw its path. Why should it follow that path?

What forces are acting on the particle at a point half-way along its path to the negative charge? Draw them. Draw the total force acting on the particle at this point.

Consider the velocity at this point. What forces are acting to make the particle follow the field line?

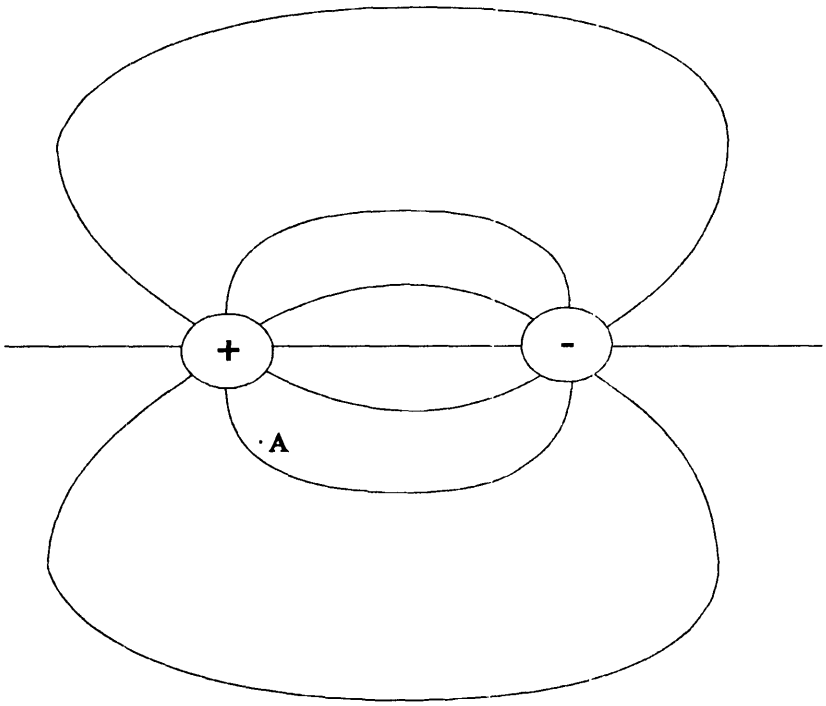

Figure 5.2 Charged particle in an electric field

The question is intended to confront students with the concept that as force is parallel to the field line, which implies that the particle cannot follow the curve of a field line, because there is no centripetal force to cause it to follow the curve. This argument relates to the mechanics of circular motion. In general, students did not appreciate the argument, which involves vector reasoning about instantaneous acceleration.

Concrete symbolic responses

This mode was characterised by a concrete view of the field lines, which followed through in the predictions of trajectories. There was an increasing amount of structure in the students' picture of these lines as the level of response increased.

The two unistructural answers provided, predicted that the particle would go to the "field line" drawn on the diagram, because point A was in the "space between field lines". They seemed to be thinking of these drawn lines as the only items worthy of concern, and the effect of these lines as being simply to attract the particles, e.g., in the interview, one student (s. 25) said,

> *Student: ... It'll just move [draws line from point A to nearest "field line" of diagram] there*
> *Interviewer: And stop?*
> *Student: [pause] Yeah.*

Interviewer: O.K., why does it do that?
Student: Move there?
Interviewer: Yeah.
Student: Coz, um, it's attached to the field, like the iron filing was.
Interviewer: O.K., why does it then stop?
Student: [pause] I dunno, why it stops, but [pause] I dunno.

This student has a single idea, as with the iron filings, that the particle is attracted to a "field line" and stops there. The idea of a field line is extremely simple for this student, being essentially the line drawn on the page. This is less developed than the ideas of field lines which tie together a relational concrete symbolic view of field representation. This student's statement about a field line can reasonably be described as focussing on a single element.

Adding to this single idea of a field line, multistructural responses, provided by five students, mentioned that the particle might go straight to the negative, or along the field lines. These two ideas did not seem to be integrated in their minds, and their idea of field lines was still firmly concrete, in terms of paths. There did not appear to be any clear link between the concepts of field lines and force in these multistructural answers, which focussed on the field lines as paths, competing with the "natural" path of a positive test charge, which is straight towards a negative charge, as stated in the example below (s. 1):

Student: That's positive, it'll go towards the negative.

The idea of positive charges going to negative is the first idea the student applied. However, it conflicted with the second idea used, namely, of particles following field lines, as seen below:

Interviewer: O.K., you want to draw that in?
Student: [draws line from point A straight to negative charge, draws second line from point A to negative charge along field line through A]. I'm not too sure. I think if it crosses it there'd have to be energy involved in crossing the field.
...
Interviewer: O.K., so why would it want to follow a field line?
Student: ... It'd follow the electric field [pause] rather than crossing the electric field.

Here is the second idea, of not "crossing the electric field". There appears to have been no clear justification for the idea in the student's mind, and it would seem to be a unistructural unit for him. In the interview excerpt below, there is a repeated probing of the idea that the positive charge goes straight to the negative, crossing the field lines.

Interviewer: Well, why should it want to go straight?
Student: To get to that charge, because it's a positive charge, so this would be the easiest way to do it, around between the two.
Interviewer: O.K., why would it take more energy for it to go straight?
Student: Coz it's having to cross these lines.
Interviewer: O.K., and what happens when it crosses the lines?
Student: [pause] uh [pause] I'm not sure [pause] Um, it'd be like crossing - a train trying to go onto different tracks. You have to pick up the train and move it.

There is a distinct vagueness in the above response about the mechanism that prevents the particle from "crossing the field". The idea of "train tracks" is intuitively appealing, but not detailed in terms of mechanisms.

Interviewer: O.K., and would you expect any force involved in keeping it on the line?
Student: [pause] Uh [pause] there would [pause] there would be a small force, but not a large force. Not as large as it is to cross it.
Interviewer: O.K., could you draw in the forces you expect then?
Student: [Draws two arrows parallel to line, one in front of and one behind X] That way, that way.

In the above, there is a mention of force, but it is not detailed. Adding a further unresolved concept, he described another force on the charge, caused by movement in an electric field, "is it force caused by the right hand rule somehow?" There was no ability to relate these ideas in this multistructural response.

At a relational level, five students stated that the particle would follow the field line, and related this to the concept of force, e g., one student (s. 28) responded to the question,

Interviewer: O.K., so why would you expect it to follow that path?
Student: [pause] Coz they're the lines of, the field, between the positive and negative charge.

This is a simple identification of field lines with path, as in other responses of all levels. However, there is an ability here to relate this path to force, as seen in the extract below:

...
Interviewer: O.K., if you consider a point sort of halfway along there, call it X, what forces are acting on the thing when it's at X?

Student: [pause] There'd be the force of these, of the electric field. [draws single arrow, along field line]

There is no consideration of the forces caused by each of the charges, but force has been related to the field line, with a statement that the force is parallel to the field line.

...
Interviewer: ... What direction is the thing moving in when it's at X?
Student: What direction? Along the lines of the field - towards the negative.
Interviewer: O.K., so if the force is in that direction and the velocity is also in that direction, what's actually going to hold the thing onto that line ...
Student: The lines of force.
Interviewer: How are they going to do that?
Student: [pause] Hmm. [pause] It's this line here that it's on, so that if it's load enough to shift it, then it's load enough to keep it, the ones around it would be load enough to keep it on that path.
Interviewer: So would they be applying forces on it as well?
Student: Yeah.
Interviewer: Which way?
Student: They'd be running that way and if that was coming that way, it'd be pushed back onto the line.

This concept of "the lines around it" keeping the charge moving along the field line is interesting, in that it recognises a need for some sort of force causing the particle to follow that path, but at the same time, finding an answer purely in terms of these concrete "field lines". The closure and consistency here make the answer relational, but it is still reasoning in terms of concrete symbolic concepts, the "field lines", rather than more abstract, formal ideas.

Formal responses

Seven students answered in the formal mode. Their answers are characterised by an ability to see the vector picture of the forces on the moving charge. However, the students in this study were not able to reach a consistent picture of the particle's motion relative to the field lines.

The three students at the unistructural formal level appreciated the vector picture of forces on the charge, being able to draw these, and discuss the instantaneous force on the charge. However, they still believed, as shown by the student (s. 44) quoted below, that the force and motion could both be parallel along the curved line:

Student: [Draws path along field line] ... Like, coz the electric field's going that way. [pause] That's positive, so it's just going like that.
Interviewer: O.K., and why would you predict that?
Student: Coz the electric field's going that way, and coz that's [the test charge is] positive, they're going to attract, it's going to be attracted to that, so it goes ... to that [the negative charge].
...
Interviewer: O.K., if we consider a point halfway along, call it point X, what forces are acting on the thing at point X?
Student: Um, an attractive force, towards there [negative charge]. A repulsive force, from there [positive charge]... It goes to there [along field line]
...
Interviewer: O.K., which direction is it [the particle] going?
Student: That way, towards the negative charge.
Interviewer: The same as the force?
Student: [pause] Yeah.
Interviewer: O.K., so if the velocity is in the same direction as the force, why should it change direction?
Student: Change direction where?
Interviewer: Well, if the velocity is heading the same way as the force, why shouldn't it carry on straight, if there's nothing to move it round into a curved path.
Student: But there'll be this moving it around, your force'd be there. I mean, your force isn't just straight through there, it's sort of different at different points, sort of - aiming towards the negative charge.

This extract provides the essence of his response. There is a belief that the particle continues along this path, parallel to the force acting on it. The curvature of the path somehow occurs as a result of the fact that the force is "sort of different at different points ... aiming towards the negative charge".

This was a unistructural formal response, as the sole abstract element involved was the use of vectors to describe the force on the particle, which can be compared to the greater use of elements in the multistructural response described below.

Three students gave multistructual formal responses, which were characterised by a confusion between the factors of movement parallel to the field line, and the forces on the particle at a given point, e.g., one student (s. 4), after an initial description of movement parallel to the field line, was questioned as to the forces acting on the particle:

Interviewer: O.K., if we consider - what force or forces would you expect to be acting on it at point X?
Student: [pause] There'd be some small force that way, away from the positive, and a greater force - this way, attracting it to the [points at negative] [pause]

The above extract describes forces acting directly on the test charge from the two stationary charges. The next line of questioning was intended to relate this to the total force on the test charge.

Interviewer: O.K., and what would you expect the total force to be?
Student: Pretty much in that direction, somewhere.
Interviewer: How does that relate to the line it's following?
Student: Um [pause] It's like, parallel to the line, at that point.
...
Interviewer: O.K., if the total force is in the same direction as the velocity, why should it change direction?
Student: [pause] Coz as it moves on a bit, the force'll change. As it gets further from the positive, or closer to the negative charge.

This is the same idea as seen in the previous unistructural formal answer, but there is also an appreciation of the extra point that if velocity is parallel to force, then direction cannot change. This extra point was not apparent to those responding in the concrete symbolic mode, or the unistructural level of the formal mode.

Interviewer: O.K., so your final statement about the forces acting on the thing at X?
Student: Um [long pause] I guess the force would be parallel to the field line at that point, but the velocity'd be slightly [pause] off.
Interviewer: If the velocity is not parallel to that line, is it following that line?
Student: [pause] Yeah [pause] hmm, yes, it's following that line, I'm just [pause] hmm [pause].
Interviewer: Can the velocity of a particle moving along a curve be anything but tangent to that curve?
Student: [pause] I guess not. No, um, no [long pause] So I guess the velocity has to be a tangent to the curve [pause] um, and the force'd be more off here, somewhere.

This student has an idea of the problems involved in the assumption that the particle will follow the curve, but cannot integrate them into a consistent picture, even when prompted in this direction. He has changed between the force and velocity both being parallel to the curve, to the force being parallel to the curve, and ending, rather tentatively, on the

conclusion that the velocity follows the curve and the force does not. There was no evidence of a relationship between the force on the particle and the curve it follows in his answer.

The sole student (s. 32) to give a relational formal response was able to appreciate the difficulties of the component forces and their relationship to the trajectory of the particle, particularly the relationhip between force and direction.

> *Interviewer: If you imagine a small positive test charge, at point a, what path will it follow?*
> *Student: It will go along a field line - so if you start there, you will have a line which is [pause] more or less parallel to the field lines which are already drawn.*
> *Interviewer: O.K., you say more or less?*
> *Student: I'm not drawing it straight enough. It is parallel.*

In the above statement, there was a clear prediction that the path is parallel to the field line.

> ...
> *Interviewer: ... could you draw the force or forces acting on it at that point? [point X]*
> *Student: In vectors?*
> *Interviewer: Yes, please.*
> *Student: Well, the easiest way to do it is to split it up, and say that there is a [pause] small force from the positive point charge there, repelling it, and stronger force from the negative point charge attracting it, which gives you a net force which is, lo and behold, a tangent to the field line ...*

This student was able to consider the component forces involved, as seen in the above, and also had an ability to describe the relationship between force and direction, as in the excerpt below:

> ...
> *Interviewer: So is the force parallel to the velocity at that point?*
> *Student: [pause] Hmm, if it wasn't then the charge would not move along that line - I think we've had a discussion about this earlier, about um whether the heavier object would [pause] move on a wider curve than the field line, but we worked out that it would still follow the field line, but I don't remember why. The force on it is definitely [pause] along the field line, I don't know if it's - if the line of motion will be exactly along the field line or not. So, we decided it was, I can't remember why. [pause] If our moving point charge doesn't have any motion, then its initial motion will go along with the initial force on it.*

This last sentence is true. The student goes on to consider the complication of the movement of the particle.

> *Student: It will have some momentum, but the force still changes as it moves. [pause] I don't want to think that it goes along the field line, but we decided that it would, I can't remember why, that's a dreadful admission to make on tape. I'll say it is in the same direction as the motion, yeah.*

Here, the student has a conflict between the logic of the situation, and the remembered result that the particle ought to follow the field lines. However, as the following statements show, the student has a good understanding of the relationships in the argument.

> *Interviewer: O.K., if it is in the same direction as the motion, that means that at that point, you wouldn't expect the thing to change direction?*
> *Student: [pause] Good point, and that's what I was trying to think about and not trying to say because it sounds too silly. That path that it is following looks circular, so you would expect that any force on it would be towards the centre of the circle or near circle that it's following, but it isn't.*

This last statement is in fact the essence of the argument. The absence of any centripetal force makes it impossible for the particle to follow a curved line. The student appreciates this argument fully, but is unable to reconcile it with the remembered "fact" that the particle would follow the field line.

> *Student: [pause] It's not moving at a constant speed though, it is accelerating. I don't know that that helps though. [pause] No, something wrong here, and I don't understand what it is, sorry.*

The student is unable to come to a firm conclusion, in spite of an understanding of the argument that was presented. There is a conflict between the argument and a remembered "fact", that particles follow field lines. Even the most cognitively advanced answer had problems with this ingrained concept. Full appreciation of the relation between field lines and trajectories would appear in a later cycle of growth in the formal mode. Responses in this later cycle of growth would resolve the conflict.

<u>Conclusion about understandings of field lines and trajectories of charged particles</u>

These questions have revealed information about the way students see field lines relating to particle trajectories in the field. In responses to both questions, the majority of students

believed that the particle trajectories would follow the field lines exactly (21 out of 28 respondents to the first question, as described in appendix O, and 13 out of 19 respondents to the second question, described above). The large number of such answers to the first question was responsible for the design of the second question, which was intended to encourage higher-level understandings of the same situation. A number of students did respond at higher level to the second question, being able to reason formally in terms of the instantaneous forces on the moving particle. They were still unable to come to a completely satisfactory picture of the motion of the particle in the field, due largely to their belief that the particle would follow field lines.

There is a natural tendency for students to see connections between field lines and particle trajectories. It is of interest to investigate whether students use field lines in less obvious contexts, such as in considerations of induced current.

Field lines and induced current

Field lines can be used as part of a description of the phenomenon of induced current. The first question of the third interview was centred on induced current, and is treated in depth in the second section of chapter 7. The question concerned a number of aspects of the situation represented in the diagram of figure 5.3. Below, only responses to that part of the question dealing with field lines are considered. This part of the question was intended to find out whether students saw concrete "field lines" as having a major role to play in the induction of current.

What causes the current? How do the field lines of the magnetic field relate to the current?

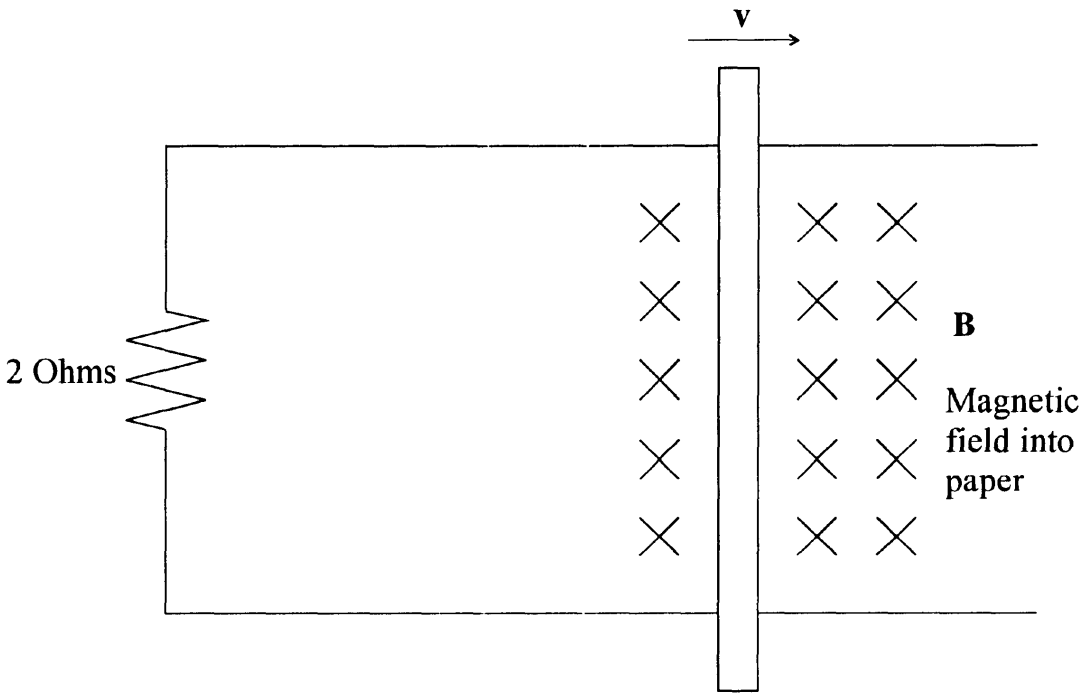

Figure 5.3 Situation causing an induced current

The above question was part (iii) of question 1 in interview 3. In general, field lines as such were not central to students' responses about induced current. No student seemed to have a picture where concrete field lines were the cause of current. A number of students did, however, phrase the cause of current in terms of field lines, though they knew that this was representational rather than causal. One student (s. 33), when asked about the cause of the current, replied as follows:

> *Interviewer: O.K., so what actually causes the current?*
> *Student: What actually causes the current, uh [pause] it's the movement of the conductor across the magnetic field.*
> *Interviewer: O.K., do the lines of the magnetic field have anything to do with the current?*
> *Student: Uh, yeah, if you [pause] if the conductor is - if the conductor travels parallel to the lines of the magnetic field, i.e., doesn't actually cut any lines of magnetic field, you don't get any um [pause] any induced voltage in the conductor.*

Generally, field lines were not central to the students' view of induced current. No student seemed to use a model involving concrete field lines to explain induced current. It is interesting to compare this result with those from the questions about iron filings near a magnet and particle movement in an electric field, earlier in this chapter, where field lines were central to the reasoning of many students. Their failure to use field lines in this

situation seems to indicate a certain compartmentalisation and situational dependency of the students' knowledge.

Conclusion about students' understandings of field lines

Students did not, in general, demonstrate clear understanding of field lines. They had a strong tendency to concrete pictures of field lines, whereas field lines are basically an abstract representation. In SOLO terms, some students' responses did reach into the formal mode, but their understandings were still incomplete.

The question regarding the lines visible in iron filings around a magnet was generally difficult for students. There was a strong tendency for them to see the pattern in the iron filings as being due to a finite number of discretely spaced "lines" of field. Some students were able to consider a more abstract mechanism causing the pattern, but few of these had a complete picture. Concrete pictures of discrete "lines" being mapped by filings were clearer for students than their abstract concepts of the situation, which were generally incomplete and poorly defined.

Concrete pictures also came out when the subjects were asked to consider the relationship between field lines and trajectories of charged particles in the field. Almost all of the subjects saw the field lines and trajectories as being identical, even after the interviewer suggested otherwise. It was meaningful for them to identify field lines with the concrete trajectories of charged particles in the field.

Use of concrete pictures of field lines was not universal for all situations. In response to a question relating field lines to induced current, students did not explain the induced current in terms of field lines. This seems to reflect a compartmentalisation of knowledge for the students, who did not learn about induced current in the context of field lines.

An analogy which explains the difference between concrete and formal understandings of lines can be seen in figure 5.4 below, which shows a road map and a weather map. In the case of the road map, lines on the map represent concrete objects - roads, rivers and so forth. In the weather map, the isobar lines have an abstract meaning, and are drawn following a convention. If one examines a point corresponding to a line on a road map, then one will find a concrete object. If one examines a point on an isobar line of a weather map, there is nothing to distinguish it from neighbouring points. In terms of this analogy, the students responding to the question in the concrete symbolic mode were thinking of concrete field lines which correspond to roads on a road map, rather than abstract lines corresponding to the isobars of a weather map.

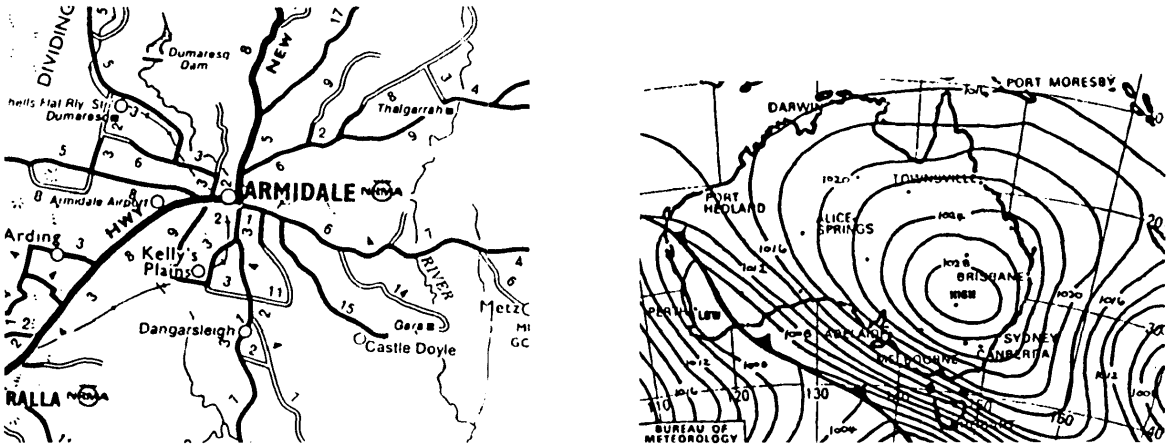

Figure 5.4 Concrete road map and abstract weather map

Within the context of this difference between the concrete and the abstract, a number of more detailed findings emerged about the nature of elements in students' understanding. However, these elements tended to be highly specific to the particular questions used to probe students' knowledge. This context-specificity is a feature of the SOLO Taxonomy's classification of learning outcomes. While the elements found were consistent across all questions, different ones were required, depending on the stimulus material presented to students. Field lines could be taken as a single element for students, as could field strength, or a number of more specific units, depending on the context of the question. Students were often able to use field lines in order to relate other elements of thought within the concrete symbolic mode of response. A formal understanding of field lines is related to understanding of the field vector, and the idea of field at a point. Understandings of the field vector are investigated in the next section of this chapter.

UNDERSTANDINGS OF THE FIELD VECTOR

As the previous section of this chapter has shown, all students could represent fields in terms of field lines. These field lines often had a distinctly concrete character in students' minds. However, the formalisms describing electric and magnetic fields are all phrased in terms of the field vector and its values at given points (the field vector describes the direction and strength of the field at a given point). This section is concerned with whether students can use the abstract concept of the field vector, and whether they can relate it to their concrete ideas about fields which are based on field lines.

The concept of the field vector is central to further study of fields and related phenomena in physics. The field vector includes the idea of a mathematical variable in its scalar aspect, but involves extra complications when direction has to be considered. As a result of this complexity of the concept of the field vector, it should come as no great surprise to see students having difficulties in using it. One could hypothesise that the field vector is

inherently an abstract concept which belongs in the formal mode of the SOLO Taxonomy. To explore the idea of the field vector in various contexts, this chapter section is broken into subsections dealing with students' understandings of the field vector in relation to field lines, algebraic calculation and magnetic fields, respectively.

Connection between the field vector and field lines

When considering the interaction between fields, it is possible for students to base their reasoning on field lines alone, or to make the connection between field lines and the field vector. Three of the test/interview questions dealt with this situation of interaction between fields. The first of these was question 3 of test/interview 1. This asked students to predict the field resulting from two point positive charges. An outline of students' responses to that question is given below, with details appearing in appendix O. The second question was broken into parts, and appeared as questions 1, 2 and 3 of the second interview. It asked students to predict the field in the same situation as the question previously discussed, but specifically requested them to use vectors in their answer. An outline of responses to this question is also given below, with details in appendix O. The third question concerning interaction between fields was also broken up into parts, and appeared as questions 4, 5 and 6 of interview 2. Details of responses to this question make up the bulk of the current sub-section, and follow the summary of responses to the other questions.

The first question concerning interaction between fields was question 3 of the first test/interview. It asked students to predict the field caused by two positive charges, and to point out minima and maxima in that field. Responses to this question showed that students were generally unable to use vectors and equations in their considerations of field strength near two point charges. A large number (12 out of 28) of respondents made a consistent prediction based on concentration of field lines. The field lines were the unifying feature for those students unable to use the abstract system of the field vector and field strength at points.

In a similar question, students were specifically asked to use vectors to predict the same field. This was question appeared, in parts, as questions 1, 2 and 3 of the second interview. Even when asked to use vectors, few were able to do so. The overall impression from responses to those questions (see appendix O) is that only a few students are able to use vectors in their reasoning about this topic, even when asked specifically to consider them. This point is also illustrated by the responses to the question discussed in detail below.

This question appeared, in parts, as questions 4, 5 and 6 of interview 2. This question was concerned with a non-routine interaction between two fields. The question had two concurrent goals. Firstly, to investigate students' differentiation between electric and

magnetic fields in the context of combining these fields. Secondly, to investigate students' ideas about combination of fields in an unfamiliar context. The novelty of the fields in this question was intended to preclude, as much as possible, rote-learned responses about known combinations of fields. This created a true problem-solving situation in which students had to apply their knowledge. The question appears below:

This is a resistor with a current flowing through it [top diagram, figure 5.5]. Can you draw the field near the resistor?
This is a capacitor charged by a battery [middle diagram, figure 5.5]. Can you draw the field inside the capacitor?
The capacitor has a resistor between the two plates. The battery keeps the capacitor charged and there is a current flowing in the resistor [bottom diagram, figure 5.5]. How would you expect the field to look?

Only three students focussed their answer on the fact that the capacitor has an electric field and the resistor has a magnetic field. Even these students were not able to come to any clear explanation of the way the electric field and magnetic field would combine, or whether these fields would combine at all. In light of the few students who focussed on the distinction between electric and magnetic fields here, student distinction between these two types of fields has not been pursued in the analysis of this question. All but those three students were willing to add the fields together as if there were no differences between the two. As with responses to other questions, it was also the case here that those students responding in the formal mode used a vector picture to predict the interaction, while others who responded in the concrete symbolic mode based their answers on field lines and imprecise ideas of fields.

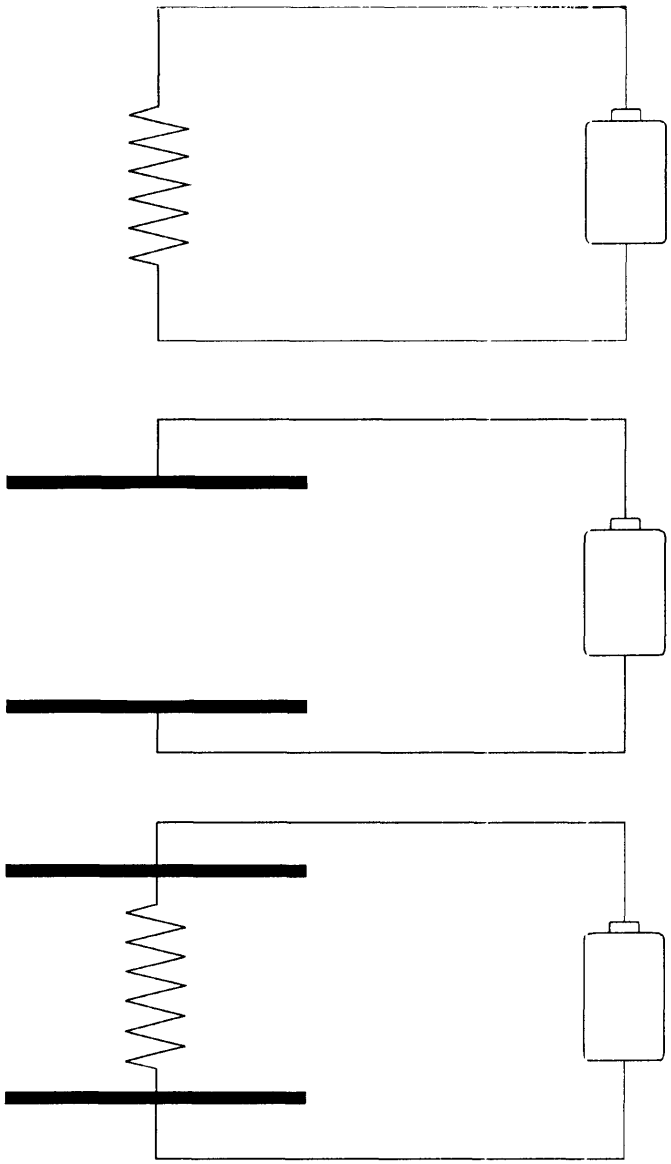

Figure 5.5 Situations for prediction of field

Concrete symbolic responses

The vast majority of students (16 out of 19) responded in this mode. These responses did not use an abstract model to answer the question. Some answers were based on field lines, others focussed on other concrete aspects of the problem. A large number believed that a capacitor could not be charged with a resistor between the plates. There was a range of responses possible at the multistructural concrete symbolic level, depending on the units students chose to focus on. Possible units included field, field lines, current and repulsion. Eleven students responded at this level. The response quoted below was provided by a student (s. 25) who concentrated on the ideas of field and current. This tied in with a misconceived idea of capacitor action:

Interviewer: ... What does the field look like?

Student: Probably just around there, [draws circles around the resistor] it's probably just the same as before, ["before", ie, around a resistor by itself] this way.

...

Interviewer: Does the fact that the capacitor is there make any difference?

Student: No, it doesn't.

Interviewer: O.K., why not?

Student: Because the connection, the resistor'd make a connection, so [pause] the current just keeps flowing through, so the field'd just be around, there.

This student's idea of a capacitor was that the current was flowing through the capacitor, along the path of the field lines, as seen in this excerpt from the same interview with this student:

Interviewer: That's a capacitor which is charged by a battery. What does the field look like inside the capacitor?

Student: [draws parallel lines between plates] I dunno, probably like that.

Interviewer: What do those lines mean?

Student: The direction of the field.

Interviewer: O.K., what does it mean to say the field is acting in that direction?

Student: [long pause] It acts from positive to negative, that's the direction of current.

Interviewer: So is there a current flowing there?

Student: Well, it's a current discharge.

Interviewer: What's the difference?

Student: Well, there's no actual connection between the two points but the current is flowing, the discharge, particles, I dunno [pause] charged particles.

Interviewer: And you're saying that that happens all the time?

Student: What do you mean?

Interviewer: Well, it's happening constantly?

Student: No, it builds up, alternatively, discharges.

Interviewer: And then does that again?

Student: Yeah.

In this students' description above, it seems that the student regards the electric field in the capacitor as being an electric current passing through that capacitor. In other words, he regards the field and the current as the same thing. As a result, he predicted that if the current was allowed to flow through a resistor, then there could be no field in the capacitor. This is particularly interesting in light of previous findings in the literature about students confusing current, voltage, charge and energy into one vague idea of "electricity". Those previous findings referred to students' ideas of electric circuits, and it seems that the findings

here about fields in capacitors indicate that students are also confusing electric field into this same vague "electricity" concept. Five of the multistructural responses had this same idea about current flowing through a capacitor.

The five relational concrete symbolic responses were based on a integrated picture where field lines repel one another, allowing for consistent predictions. The field vector was not used in this level of reasoning, as seen in one student's (s. 39) drawing (figure 5.6) and response below.

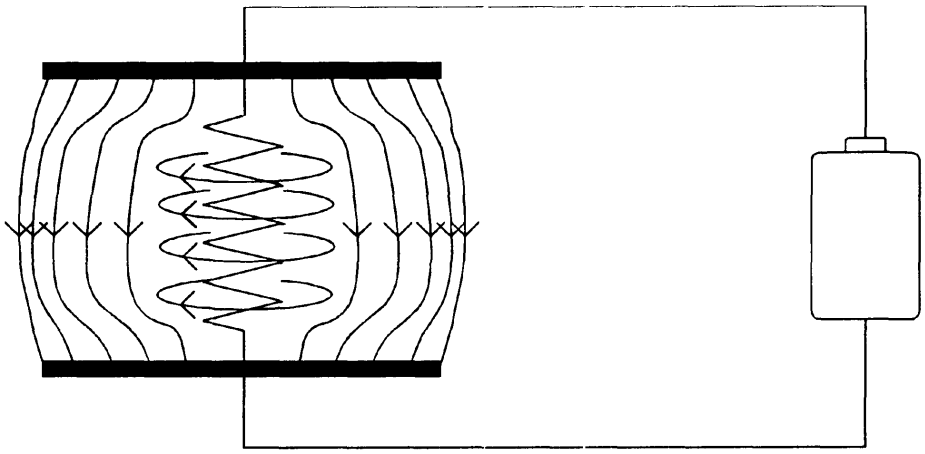

Figure 5.6 Student 39's prediction for field

> *Interviewer: Well, here, there's a current flowing through the resistor, but the battery keeps the capacitor charged. What fields would you expect in there?*
> *Student: [pause while drawing] This is sorta coming straight out at ya [pause].*
> *Interviewer: So these ones near the resistor are coming straight out?*
> *Student: Yeah, so they're [pause] so, sorta, I dunno, like a bird cage around the thing.*
> *Interviewer: O.K., why would you expect that?*
> *Student: Well, just one field having an effect on the other field ... repelling each other.*

This notion of repulsion between fields is implemented in terms of the field lines and their effect on each other. It allows for a consistent picture of field interaction without recourse to the more abstract picture of field vectors which is used in the formal mode.

Formal responses

There were only three students responding in the formal mode to this question, by using the vector field. They used a unistructural formal concept of vector superposition to predict the interaction between the two fields, e.g.,

> *Interviewer: O.K., if we take [page] the capacitor and string a resistor between the plates, there is a current through the resistor, but the capacitor is kept charged by the battery. What does the field look like in there now?*
> *Student: [pause] It's the addition of them, probably. I wouldn't have a clue.*

This student (s. 28) showed a lack of confidence, but further questioning revealed that he *did* "have a clue":

> *Interviewer: Well, how would you add them?*
> *Student: [long pause] Using vectors, I'd say, like, if that's going round that way, and if you took a point say across that way, you've got that one coming down this way. Out of them two specific vectors then you'd have a vector of magnetic field in that direction, so possibly be coming across in that direction, maybe [see figure 5.7 for his drawing].*
> *Interviewer: O.K., would you expect the fields to repel each other?*
> *Student: No, coz they're not going like in opposite directions, that one's going directly around, that one's coming down that way.*

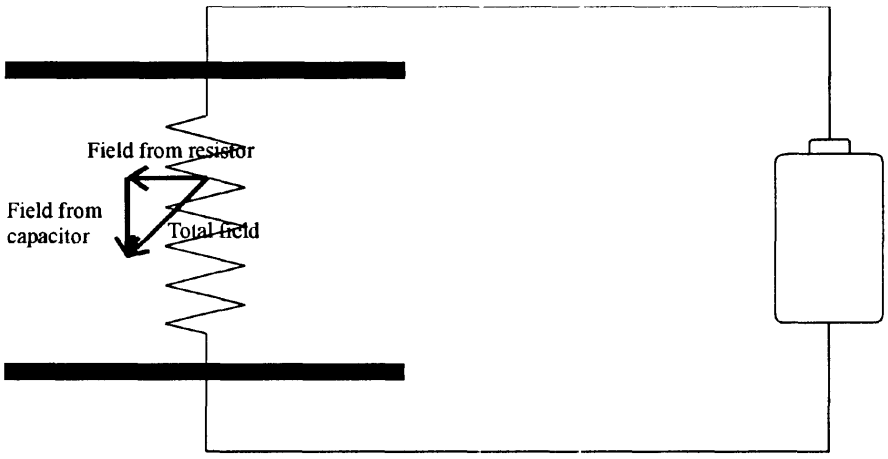

Figure 5.7 Student 28's prediction for field

As mentioned before, most students were unable to distinguish meaningfully between electric and magnetic fields for the purposes of this question. Even this student had the same problem, as the following line of questioning indicates:

Interviewer: O.K., when you draw these, are you drawing electric fields, magnetic fields?
Student: Magnetic fields.
Interviewer: O.K., so the field in the capacitor is also a magnetic field?
Student: [pause] Yeah.
Interviewer: O.K., what's the difference between an electric and a magnetic field?
Student: [long pause]
Interviewer: Is there any difference?
Student: There is, but [pause].
Interviewer: Is there any difference in the effects they have on things?
Student: [long pause]
Interviewer: No?
Student: No, they both apply force on [pause] different things.
Interviewer: Different things?
Student: Like, particles or charges.

This general vagueness about the difference between electric and magnetic fields was typical of the students' responses to this question. The three students in the formal mode were able to use vectors in this non-routine situation of adding fields, but such responses were rare. The non-routine nature of the question forced them to consider first principles, and few were able to connect this to a formal system. Only three students focussed on the difference between the electric field of the capacitor and the magnetic field of the resistor, and none of these three were able to come to any conclusion about the effect of this difference.

Summary for the connection between the field vector and field lines

There were a number of frameworks which students used to model the interaction between fields. At the lowest level of understanding, they used ideas about fields repelling and clashing with one another. These ideas were not fully related into any consistent system, and were generally classed as multistructural concrete symbolic in SOLO terms. A higher level of understanding was reached where students consistently saw the interaction between fields in terms of field lines, and related field strength and direction to the concentration and direction of these lines. Those students who had a consistent view where field lines were used to relate phenomena were classed as relational concrete symbolic. Students who were able to use the abstract idea of the field vector were in the formal mode of understanding. Not all students who mentioned field vectors were able to use them. There was particular trouble relating vectors to points in the field, and those students who could not do so were unable to use the abstract idea of the field vector. Consideration of the interaction between

fields was a useful probe for the relationship between the field vector and field lines in students' understandings.

While field lines are generally a concrete concept in students' minds, corresponding to paths of particles or other direct observables, the idea of a field vector at every point in space is less accessible to students. This was seen in the tendency of students to use field-line based pictures rather than the idea of the field vector in describing field interactions. Field vectors are a more abstract system, and it was not common for students to be able to relate these to field lines in their answers. They generally preferred to use the field lines to make their predictions without reference to vectors. Many students had consistent pictures of field phenomena based on field lines.

Reasoning about quantitative aspects of the field vector

While few students were able to use the field vector in the qualitative problems discussed above, it was also of interest to see if they had any ability to reason with its equation-based aspects. The magnitude of the field vector at a given point is the strength of the field at that point, which represents the amount of force on a unit charge placed at that point. Alternatively, it can be said to represent the ratio of the force on the charge to the magnitude of the charge for any charge placed at that point. Students were asked the following question (question 8 of interview 2) to investigate their level of insight into the reasoning involving these issues, particularly the issue of whether students could cope with this concept of a ratio between the two quantities of force and charge.

> *A charge in an electric field experiences a force of three newtons. What happens to the force if the strength of the electric field and the magnitude of the charge are both doubled?*

The question was given in written form, with space for working. In addition, students were asked to draw a diagram to cast more light on their picture of the situation. Overall, student responses to the question were correct, although their means of answering the question differed. The categories of response are discussed below.

<u>Concrete symbolic responses</u>

Ten of the nineteen responses were in the concrete symbolic mode. Responses in this mode were characterised by a failure to use the concept of field strength in the abstract terms required by the question.

The four responses at the unistructural concrete symbolic level focussed on one of the aspects of the given information, such as the fact that the charge doubled, or the field doubled. For example:

> *Interviewer: A charge in an electric field experiences a force of three newtons. What happens to the force if the strength of the electric field and the magnitude of the charge are both doubled?*
> *Student: [long pause] Doubles?*
> *Interviewer: And why?*
> ...
> *Student: The electric field [long pause] Well, coz it is proportional to q [pause] e doubles, q doubles, q's proportional to f, so if q doubles, f doubles.*

This student (s. 56) has focussed only on the doubling of charge, which he used to predict a doubling of the force. By contrast, the three students responding at the multistructural concrete symbolic level focussed on both the field and the charge, but were not able to integrate them in their answer. One student (s. 19) stated in reply to the interview question:

> *Interviewer: A charge in an electric field experiences a force of three newtons. What happens to the force if the strength of the electric field and the magnitude of the charge are both doubled?*
> *Student: [pause] A charge in an electric field [pause] O.K. [pause] What happens to the strength of the electric field and the magnitude of the charge if both are doubled? [pause] The force, what do you mean, what happens to the force? Of what? Oh, the force experienced by the charge, because of the electric field?*
> *Interviewer: Yeah.*
> *Student: I'd say the force would [pause] I'd say the force would increase as well.*
> ...
> *Interviewer: O.K., when you say that the force would increase, could you say how much it would increase by?*
> *Student: Well, if they're both doubled, I could only guess that that'd be doubled as well, but [pause]*
> *Interviewer: You wouldn't have any reason for guessing that?*
> *Student: No.*

In this answer, the doubling of both the charge and the strength of the field have both been mentioned, but there has been no way to integrate both of them, leading to the guess that the force also doubles.

The three students who responded in the relational concrete symbolic level were able to draw a correct conclusion, based on a consideration of the idea that the effects were due to another charge, using the equation $F = k Q_1 Q_2 / d^2$. They did not reason directly with the idea of the field strength at the point, which is a more abstract concept, e.g. (s. 29),

> *Student: ... Well, sort of a point charge [pause] there, is our little charge Q [pause] and it experiences a 3 newton force, in the first instance, and if [pause] we [pause] double the electric field [pause] like [pause] and this is 2 Q and this is [pause] double whatever that was [pause] [produces diagram shown in figure 5.8] then, it should experience [long pause] hmm [long pause] and [long pause] this would be like, being [pause] four times closer.*
> *Interviewer: Hmm?*
> *Student: This particle would experience, like, from this, this would be like having this four times closer, which means it would be [pause] twelve newtons.*

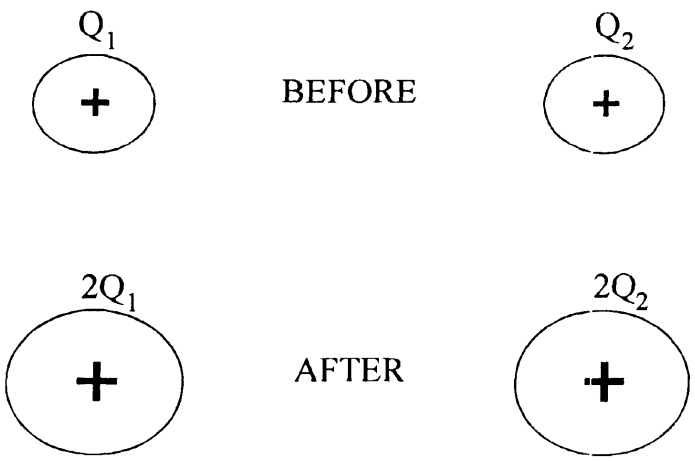

Figure 5.8 Student 29's reasoning about numerical field strength

This student (s.29) has correctly predicted a fourfold increase of the force. There is reasoning about the magnitude of two charges involved, one making the field and the other experiencing the force. This has been translated into distance in his verbal reasoning. As it happens, a fourfold increase of force would follow from a halving of the distance, as the dependence of force on distance is an inverse-square relationship. The student appears to have consistently treated the dependence as inverse-linear, which has led to the prediction. His reasoning was probed in further questioning:

> *Interviewer: O.K., so what's your reasoning there?*
> *Student: Mm [long pause] sort of [pause] there, now, that inverse square law, d_1, proportional to d_2 squared on four [pause]*
> *Interviewer: O.K., so how's the distance relate to the problem?*

Student: Well, I haven't really changed the distance, it's just that by doubling [long pause] um [pause] Have to do that twice, coz I doubled this and this.

The reference to "doubled this and this" above refers to the two charges shown in figure 5.8. This student has been able to reach a correct answer through consideration of charges producing the field. There has been no consideration of the field vector as an entity in itself. It was not necessary for him to use the field vector in his thinking, as a correct answer could be reached at a lower level of abstraction.

Formal responses

Nine responses were characterised by an ability to reason quantitatively using the field vector, using all the data given within this wider framework. Students either manipulated equations or gave verbal arguments, assuming the concept of the field vector in itself. The student (s. 32) quoted below did both:

Student: Well, presuming that the force is entirely due to the electric field, and I don't suppose that a charge can experience any other sort of force, then you're going to have a force of twelve newtons on it.
Interviewer: Can you draw a picture of the situation?
Student: [long pause while writing][wrote Eq = 3 N, 2E2q = 12 N]

This is an answer based on manipulation of equations. Further questioning established that there was a verbal argument underlying this:

Interviewer: Is this actually the way you first reasoned about the problem, or is this just what you thought it would be most defensible to write down?
...
Student: I was thinking the equations [pause] oh, no, I was just waffling still.
Interviewer: O.K., so was that waffling to buy time, or
Student: Yes
Interviewer: O.K., and when you'd finished buying time, how did you actually come to the number twelve?
Student: Well, you double the electric field, and you've doubled your force. And you double the charge, and you've doubled your force again. And [pause] you double three twice and you get twelve.

This student has been able to work from the definition where force is equal to the charge times the field strength. He has been able to do this by verbal reasoning, and also by expressing the situation in an equation. The use of both these forms places his response at a

multistructural level in the formal mode. The other students who gave unistructural responses in this mode were only capable of providing one of the above. At a relational level, there would be a clear statement of the relationship between equations and verbal reasoning, which appear to be seen as alternatives in the above statement.

Overall comments on quantitative use of the field vector

Well over half of the students answered this question correctly, both in the formal mode and the relational concrete symbolic. This is considerably better than the class performance on the earlier questions about interactions between fields, which did not give students the cue to use the field vector. Some students were able to answer the question correctly by the relational concrete symbolic response which considered the magnitude of charges. Others were able to answer in terms of the formal concept of the field vector at the test charge. The fact that a majority of students were able to reach a correct answer for this question is consistent with the overall findings from other questions in this thesis, that students were more able at manipulating equations than using physical principles. This finding was confirmed by their responses dealing with qualitative and quantitative aspects of the magnetic field vector.

Students' use of the magnetic field vector

It was known from the literature (Maloney, 1985) that students tend to confuse the effects of electric and magnetic fields with one another. A series of questions in the third interview was intended to discover the role of the magnetic field vector in students' thinking about magnetic fields, particularly as these compare to electric fields. The questions all concerned the movement of individual charged particles in electric and magnetic fields.

The question set concerned the behaviour of charged particles in the situations pictured below (figure 5.9) and was broken into three major parts, appearing as questions 2, 3 and 4 of the third interview. Fifteen students responded to this third interview. Responses to the first part, question 2 are discussed below, followed by a discussion of responses to the latter parts, questions 3 and 4, in that order.

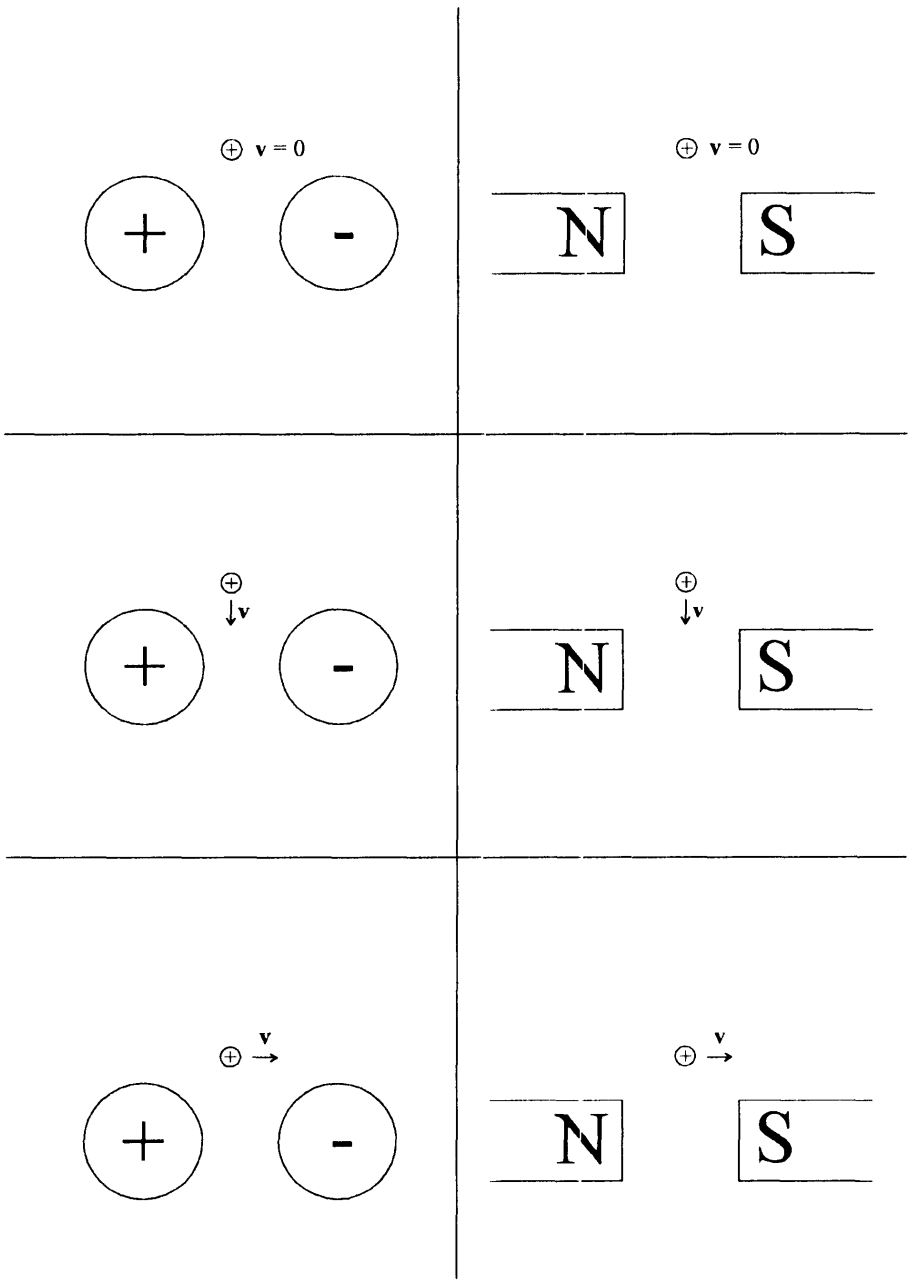

Figure 5.9 Particles moving in electric and magnetic fields

In question 2, a charged particle with some initial velocity is between either a negative and a positive charge or a north and a south pole, as shown in figure 5.9. Students were asked to describe any force on the charge and movement of the charge. Comparison between the electric field situations and the analogous magnetic field situations is the object of these questions.

In the first part of question 2, which relates to the top-right picture in figure 5.9, students were asked to predict the behaviour of a charged particle placed between a positive and negative charge. This is the same situation as they dealt with in questions discussed earlier in this section, even though there is less emphasis here on the exact relation of the path to the

field line. This first part of question 2 is not of great interest in itself, although it did serve to confirm the responses to previous questions about particle movement in electric fields, treated earlier in this chapter in the section on field lines. Details of responses to this first part of the question are in appendix O. The main function of this first part of the question was to serve as a contrast to the second part, where students predicted the behaviour of charged particles in magnetic fields.

The results below refer to the students' predictions about the movement of the charged particle in the three pictures on the right-hand side of figure 5.9. Overall, students were inaccurate in their predictions of movement for the charged particle in a magnetic field. This inaccuracy stemmed from a failure to use the magnetic field vector in their reasoning about the situations. Only three of the fifteen students gave correct predictions.

Concrete symbolic responses

Eight students made a multistructural concrete symbolic prediction that the effect of magnetic fields would be exactly the same as that of electric fields. These students drew the same forces acting on the particles in the magnetic field as on the particles in the electric field. They also predicted that the particles would follow the same paths in the magnetic fields as the electric fields. These students did not bring facts about electric and magnetic fields into relationship with one another, and hence were unable to distinguish between electric and magnetic fields in this situation.

By contrast, four students, replying at the relational concrete symbolic level, did relate electric fields to magnetic fields, which were the two units used in their responses. However, they did not provide evidence of an abstract system which would allow them to express this difference. As a result, they were limited to concrete symbolic explanations of the difference, which did not suffice. Two of these four students suggested that the difference was that magnetic fields acted in the opposite direction to electric fields, and the remaining two suggested that magnetic fields have no effect on charge at all. While there was a distinction between electric and magnetic fields, these students' displayed understandings of the relationship between the two types of field which were lacking in abstraction, and hence inaccurate.

Formal responses

Three students responded with a unistructural formal action by bringing in the concept of a vector field. They did this in relation to the magnetic field at the test charge, using this in the rule governing force on charged particles in a magnetic field. These students were able to use this rule in the qualitative situations presented, e.g.,

[referring to the stationary test charge in a magnetic field]

Student: ... We have magnetic field [long pause] that thing's still and [pause] there's nothing happening, so it's going to stay where it is.

Interviewer: O.K., and why is it going to stay where it is?

Student: Coz it doesn't have a velocity, so there's [pause] it's not moving through the field.

...

[referring to a charge moving perpendicular to the magnetic field]

Student: Force acting on this charge goes out of the page.. because on this diagram the velocity might still be moving this way, but the force out of the page [pause] will um [pause] cause it to [pause] move out of the page.

This student (s. 29) related the direction of the magnetic field at the particle in terms of the abstract definition of force on a particle in a magnetic field, rather than needing to see it as identical to the direction the charge would move. The direction the charge moves is a concrete observable, but the magnetic field direction has a more abstract relation to the direction of movement than simply being identical.

Only a few students were able to use the vector model to explain particle movement when they were presented with diagrams involving magnetic fields. The following question followed in the interview directly after the question above, and can be regarded as another part of this same question. The question above presented students with a charge is moving between two magnets, and required them to realise for themselves that the magnetic field vector was relevant. The below question (question 3 of interview 3, see figure 5.10 below) presented students with the magnetic field at a point in a given form, as they would commonly meet it in questions involving equations.

<u>Related quantitative question</u>

If the magnetic field at some point is 2 T upwards and a particle with charge of 2 C is moving at 3 ms^{-1} then what force will the magnetic field cause on the particle? [see figure 5.10]

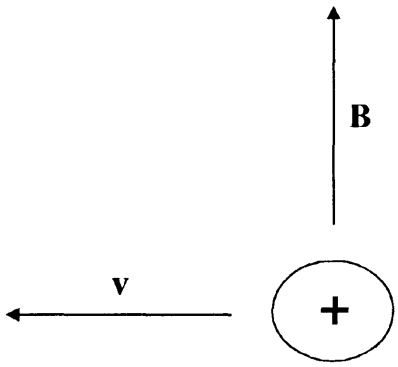

Figure 5.10 Given data for a particle moving in a magnetic field

This question was intended to give information about students' responses in a more familiar situation, involving given variables. Students performed better on this question than the preceding one (question 2 of interview 3), which was more qualitative in its use of magnetic fields. The current question is of such a nature that it does not require an overview of the phenomenon involved, and can be answered on a rote-learned basis. The magnetic field vector has been supplied to students, so that they are not required to make the connection between this field vector and a given situation, as they were in the previous question. In this question, students only have to note all the appropriate data and combine them in a learnt way.

Students were all able to make predictions for the force on this particle, which involved use of hand rules for direction and equations for the magnitude of the force. Only one student predicted that the magnetic field, as shown in this question, would have the same effect on the particle as an electric field. The majority of students were able to use the "hand rules" to predict the direction of force on the particle, e.g., one student (s. 1) stated,

> *Student: ... I'm not sure if my axes is right, but, that'll be the force, [pause] and the direction [pause] again would be the um [pause] it'd be cross in there somewhere, but it depends on how you write it, if that rule will work.*
> *Interviewer: So what would you say about the direction?*
> *Student: [long pause] The thumb is the magnetic field, so the force'd probably be [pause] that's a bit harder, that one, I could take a guess at it, but [pause] the force would cause it to move into the page, circle, into the page [pause] so the charge going like that, circle like that.*

It was interesting that the students gave overwhelmingly more correct answers for this third part of the question than for the previous part of the question involving pictures of magnetic fields. This brings out the theme that students were generally more competent in quantitative than qualitative work, being particularly competent where work is based on remembering

equations for familiar problem situations. The students' answers for this question 3, dealing with calculations when given the magnetic field vector were not generally consistent with their answers for question 2, discussed above, which required predictions of movement of a charged particle in a magnetic field when the field vector was not given. The next question (question 4) in this third interview can be thought of as an extension of those two questions. It pointed out this inconsistency to students, to test their reaction.

Questioning about consistency

Are your answers to question 3 consistent with your answers to question 2?

As the large majority of students had inconsistent answers to the two preceding questions, they were asked to explain their inconsistency in the above, which was question 4 of this second interview. This inconsistency was between their responses to a question phrased in terms of pictures of magnetic fields in question 2 and to a question phrased in terms of a given magnetic field vector in question 3. Four students were not asked about inconsistency, as their answers were already consistent for both parts of the question. Of the eleven students who were asked, the majority (seven) were unable to relate their answers to the two questions, as the quote from one student (s. 19) exemplifies:

> *Interviewer: O.K., so if the force is out of the page [referring to the previous question], is that consistent with what you said would happen here? [referring to the question before the previous question, where he predicted a force parallel to the magnetic field]*
> *Student: Um [pause] with, sort of [pause] um [pause] here's the positive charge, magnetic field [pause] so no, not consistent. I can't work it out.*
> ...
> *Student: Um [pause] I don't know how to relate the two. Like, you've got - in this one [the diagram of magnets], you've got the field, you've got field going this way.*
> *Interviewer: Mm, from the magnet, yeah.*
> *Student: Like [pause] with this one [the diagram of magnets], I've done it from my practical experience and that.. but with this one [a given magnetic field direction] [pause] I've always thought that the force would be [pause] perpendicular to both the um, the current. Oh, that's a, that's a charge, [pause] well, I learnt it was perpendicular to current and the field, but with a charge, um [pause] I don't know how it works ...*

This student was unable to integrate the two different predictions, and remained in the concrete symbolic mode of response. By contrast, the student (s. 54) in the following quote

was able to come to connect the two questions, and to revise his predictions accordingly, coming into the unistructural level of the formal mode, with the help of prompting.

> *Interviewer: If you compare that to what you said would happen here with the magnets, is that consistent?*
> *Student: [long pause] Yeah, it is there, but not [pointing].*
> *Interviewer: [describing pointing] You reckon it is for the middle one but not for the last one?*
> *Student: Yeah, well, depending, coz it'll have some slight effect, [pause] magnetic field sort of acts like, between there like that, since it sort of acts on a curve, it'll have some considerable effect - because of the magnetic field strength at that point, but really, it won't have much of an effect, it'll keep travelling in that direction [pause] should keep travelling in that direction.*
> *Interviewer: O.K., so what is the magnetic field strength at that point - or which direction is the magnetic field at that point?*
> *Student: [pause] It's the resultant, going towards the south pole?*
> *Interviewer: And what about in the one above that?*
> *Student: At this point?*
> *Interviewer: Yeah.*
> *Student: [pause] Be much the same, I think, down, towards your south pole.*
> *Interviewer: O.K., so if the magnetic field is in that direction, and the velocity is in that direction, which direction is the force in?*
> *Student: Ah-hah, hand convention, um [pause] either up or down, into the page or out of the page.*
> ...
> *Student: ... So it'll move [pause] out, up.*

It was interesting that the majority of these students were unable to make the connection between the movement of particles between magnets and the movement of a particle in some given 'magnetic field'. Even when they were encouraged to make this connection, they were generally unable to do so. This relates to their general inability to use the magnetic field vector in relation to non-routine situations involving magnets. The inability to do so is probably caused by the abstract nature of the field vector, which is particularly true for magnetic fields, where field direction does not correspond with the concrete-observable of force on a charged particle.

Conclusion about students' understandings of the field vector

This section has investigated students' understandings of the field vector in a variety of situations involving qualitative and quantitative aspects, in both electric and magnetic fields. The relationships between these two aspects were seen to be weak for the majority of these students. While students were often able to perform calculations using learnt equations, few of them were able to relate these equations to given situations simply involving magnets. The weakness of this relationship was a recurring theme in their attempts to address questions relating the field vector to field lines.

Field lines, as reported in the previous section of this chapter, are generally referred to in a concrete sense by students, and seemed to form a basis for their understanding. Where students were asked qualititative questions relating to the interaction between fields, their reasoning was dominated by consideration of field lines, and the repulsion between field lines. Interaction between fields was the basis for a number of questions which had bearing on students' understandings of field lines and the field vector.

A few students were able to describe interaction between fields in terms of the field vector, but this was rare. The ability to use the field vector in this particular situation required a formal level of understanding which was in advance of that held by the majority of these students.

While students had difficulty using the field vector in description of given fields, they were considerably more competent when given questions where calculations, involving the electric field vector, were required. While these quantitative questions also involved difficulty for some students, they were handled considerably more capably than the qualitative questions.

Again, with magnetic fields, the same was observed. Students were generally able to answer a question which was phrased in terms of given quantities, including the magnetic field vector. However, they were unable to apply the same line of reasoning in a qualitative situation involving a diagram of magnets. Generally, students were unable to connect these two situations, even where it was pointed out that their predictions in the two situations were inconsistent; that is, their knowledge was heavily compartmentalised in this case. In summary, this failure to use the magnetic field vector in qualitative situations is parallel to their failure to use the electric field vector under similar circumstances, and is exacerbated by the more abstract nature of the magnetic field vector.

In terms of the SOLO Taxonomy, the elements of students' understanding of fields tended towards the concrete symbolic mode. Some elements which appeared to have validity in the contexts of these specific questions were field lines, field strength, electric fields, magnetic

fields. However, the idea of field line can be understood at a number of levels, from simplistic to highly abstract. Within the context of these questions, it was commonly used as unistructural unit in the concrete symbolic mode, or as an part of an integrated picture of field phenomena within this mode.

Unlike field lines, which have an intuitive appeal, field vectors are a more abstract form of representation. Their use requires the student to work within an abstract system less linked to everyday reality. It seems difficult for students to learn to associate a field vector with each point in space. Students can rote-learn the use of "vectors" as straight lines that represent field direction without ever becoming aware of the essential association between field vectors and points in space - such rote learning is essentially useless. A meaningful grasp of the field vector requires an acceptance of abstraction.

Students were, in general, unable to use the field vector in situations involving electric and magnetic fields, unless they were presented with a question which asked them to calculate using it; this finding was consistent across their understandings of electric and magnetic fields. A use of the field vector is tied to an ability to use an abstract system to answer questions; this need for abstraction could be a key reason for students' failure to use the system.

UNDERSTANDINGS OF FLUX IN FIELDS

Flux is quite an abstract aspect of field representation. For understanding, it requires an ability to use the field vector in connection with a surface area. Students displayed very poor understandings of flux in the interviews, with the concept apparently having little meaning of any kind for them. There was interest in the relation which students saw between flux, the field vector and field lines.

Two questions were asked concerning flux. The first question, which came from the first interview, asked for general conceptions of flux as well as asking students to calculate flux. The results from this were analysed at length, and categories of response from this question are presented in this section. The second question, from the third interview, asked students to relate flux to a situation involving induced current. Students were unable to do so, which was a confirmation of their difficulties with the concept of flux. Some examples of responses to the second question are also provided below.

General conceptions of flux

This sub-section is concerned with answers to questions 7 and 8 from test/interview 1, as given below:

Explain what 'flux' is in a magnetic field.
Given a cube with sides 1 m long in a uniform magnetic field of 1 T, what is the total flux out of the cube?

This question was intended to have both a qualitative aspect, the explanation of flux, as well as a quantitative aspect, the calculation of flux. The calculation has a number of deliberate ambiguities, namely, flux "out of" as opposed to flux through the cube, and the question of the orientation of the cube in the magnetic field. Neither of these aspects were noticed by the students.

The students' answers to these questions were poorly developed. They were unable to clearly explain the idea of flux, and, generally, also unable to calculate it. Twenty-eight students were asked this question and many (seven of them) refused to attempt it. Of those who did answer, the responses were all in the concrete symbolic mode of the SOLO Taxonomy, lacking in abstract formal-mode organisation. Seven students declined to answer the question, e.g.,

> [no written answer]
> *Interviewer: 'Explain what flux is' - briefly.*
> *Student: Flux uh [pause] I don't know. Like, that's a word that I never come to terms with.*
> *Interviewer: Have you been exposed to it much?*
> *Student: Oh, yeah, last year I did a bit, and this year I have again, but like here, I wouldn't really be sure.*
> *Interviewer: So you have no idea at all?*
> *Student: No.*

While this student (s. 19) was unwilling to even guess at the meaning of flux, most students gave some response.

<u>Concrete symbolic responses</u>

Thirteen students provided responses that were coded as unistructural concrete symbolic. Their responses emphasised field strength only in their explanations of flux, e.g., one student (s. 39) stated,

[Explain what 'flux' is in a magnetic field.]
The density of magnetic field
[Given a cube with sides 1m long in a uniform magnetic field of 1 T, what is the total flux out of the cube?]
zero
Interviewer: O.K., well, if you explain what flux is as density of magnetic field what would that actually mean, the density?
Student: [pause] uh [pause] It would - if [pause]. Well, like I said before [in reference to a magnet], the density is not going to be as close, like as thick, close to the magnet in the middle, but it'd be greater to either the positive or the negative end.

In the above extract it seems that "density" is being used very much like field strength. Further questioning explored this:

Interviewer: So, is density the same thing as field strength?
Student: Uh, yeah.
Interviewer: O.K., so does that mean that flux is the same thing as field strength?
Student: I guess so.
Interviewer: O.K., can you think of any difference?
Student: [long pause] Jeez, I dunno.

There is no evidence in the above that the student (s. 39) had any developed ideas about flux beyond his original description of it as "the density of magnetic field". Further questioning about the use of flux confirmed this:

Interviewer: O.K., where would you use the idea of flux?
Student: [pause] Um [pause] I suppose in electric motors and stuff like that. I dunno, um, using the field to do something.
Interviewer: Why is it important then?
Student: Well, if you want to move something you're going to put it, you're going to fix it where the field strength is greatest.

So in this, the student only uses flux in finding "where the field strength is greatest"; there is no other meaning evident here. He described his answer of zero for the calculation as a "stab in the dark". This student's unistructural answer focussed on the field strength as the meaning for the flux. Multistructural answers included more.

Six students, responding at the multistructural level, added the idea of area to the idea of field strength in their descriptions of flux, but were not able to relate these clearly to each other. An example of this is provided by one student's (s. 6) answer:

> [Explain what 'flux' is in a magnetic field.]
> The magnetic field density at a point charge. Vs^{-} = Weber (Wb)
> [Given a cube with sides 1m long in a uniform magnetic field of 1 T, what is the total flux out of the cube?]
> 1/6 = 0.16 T/m^2
> *Student: ... Well that's a standard definition [referring to above explanation of flux], I think that's correct. In explaining it, I would expect that closer to a point or something that's giving off a magnetic field, then closer to that point there's gonna be more lines of force, and there's gonna be more, um, a stronger magnetic field. As those lines of force part and get away further from each other, the further you come back, the magnetic field density is gonna drop, there's gonna be less lines of force, there's gonna be more distance that they have to cover.*

This statement gives a description of field strength based on field lines. Further questioning confirms that this is the student's idea of flux:

> *Interviewer: O.K., so is flux the same thing as strength of the magnetic field?*
> *Student: Yeah, the way I see it, yes.*
> *Interviewer: So would you talk about flux at a given point?*
> *Student: Yes.*

Here, the student has confirmed the idea that flux is the same as field strength, but when pressed for clarification, he changed his mind:

> *Interviewer: So how would you say it?*
> *Student: Oh, no, I wouldn't - magnetic flux is over a given area.*

The student has brought in the second idea of area, as well as field strength. However, as we see below, there is no clear relationship between field strength and area for this student's idea of flux:

> *Interviewer: And how does that work?*
> *Student: Well, given an area, and given the strength of the magnetic field - oh God, um [pause] um, really, should think more about this, um, thinking about what the equation says, the greater the area, um, the more, the stronger the flux, I'm not a*

hundred percent on this actually, I know it's over an area, so - I'm not a hundred percent.

Here, this multistructural answer is showing an inability to relate the field strength and area aspects of flux. In the relational answers, a connection was made between these.

The two students responding at the relational concrete symbolic level were able to relate field strength to area in their concept of flux, multiplying the two together. However, there was a failure to consider the concept of field direction in their answer, which relates to the failure to use the formal concept of a vector field that describes field direction. They have related field strength to area, but without consideration of the effect of direction and the abstract definition of flux, as in the student's (s. 24) response below,

> [Explain what 'flux' is in a magnetic field.]
> Flux are the lines of strength in a magnetic field.
> [Given a cube with sides 1m long in a uniform magnetic field of 1 T, what is the total flux out of the cube?]
> [phi] = B A = 1 x 1^2 = 1

This calculation is reasonable, but further questioning showed that the students' knowledge was incomplete:

> *Interviewer: Could you explain what flux is?*
> *Student: Um, well, I mean, again it's not something I'm really sure about it's just one of those things that you talk about and to me it's like the lines of, like it is, like the strength of magnetic field, like going through the thing - it's like they talk about actual magnetic field strength being flux density, like being the number of lines of force or something in the area probably.*

This description of field strength as flux density seems promising, but does not emerge clearly in subsequent questioning:

> *Interviewer: O.K., so how would you relate flux density to field strength?*
> *Student: Um, to magnetic field strength? I think it's the same thing. Coz the flux is like, I just think about it as lines of, like, the amount of magnetic field strength or something leaving an area, it's like the magnetic field. I don't really know what flux is, so that's a little bit hard to think about.*

There is still a lack of certainty in this response as to the nature of flux. In further development of reasoning, achieving the formal mode of the Taxonomy, students would be

able to explain the relationship between flux, field strength, area and direction. This would involve an understanding of the field vector in the calculation of flux, and, particularly, the importance of direction in the calculation of flux.

Comments on general conceptions of flux

Students generally had little clear idea as to the meaning, nature or calculation of flux. This indicates that the concept of flux was largely inaccessable to these students. While there was a general association of flux with field strength, and it was common to also consider area as an aspect of flux, there was a universal inability to make the considerations of vector field direction which are a crucial aspect of the concept of flux, and appear to have been at a level of abstraction beyond that accessed by these students. As another probe of students' understanding of flux, students were asked to use the concept in another context, namely, that of induction of current.

Use of flux with induced current

One question asked students to relate flux to a situation with an induced current. In keeping with their poorly developed understandings of flux, as shown in their answers discussed above, the students were unable to link flux to this situation. This was question one of the third interview, which is dealt with at more length in the section of chapter 6 dealing with understandings of current. A diagram was given (figure 5.11).

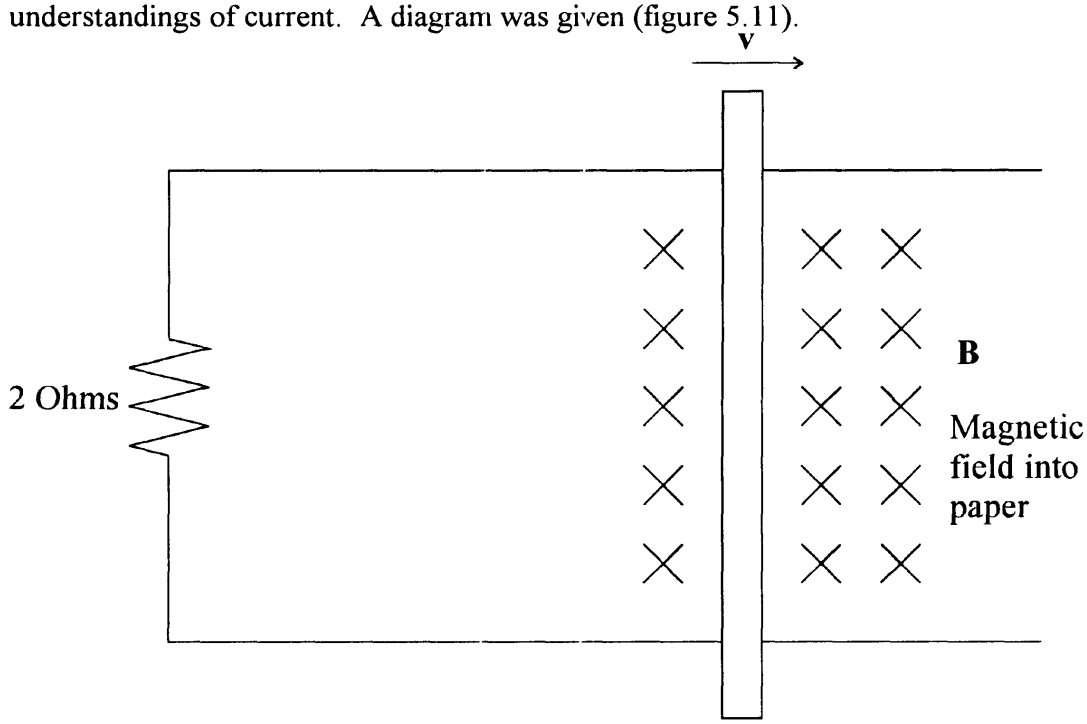

Figure 5.11 Situation causing an induced current

As part of the range of questions which students were asked about figure 5.11 (see chapter 6 for the full set) they were asked (as question 1(iv)) to relate flux to the situation. In fact, of course, the flux enclosed by the circuit is increasing as the size of the circuit increases. As illustrated by one student (s. 17), quoted below, students were not able to use flux in this instance:

> *Interviewer: O.K., going back to this first bit, can you talk about what happens in terms of changing magnetic flux?*
> *Student: Changing magnetic flux [pause] now there's something that rings a dark and distant bell in my head - no, I couldn't, because I've got no idea what changing magnetic flux is - well, I do, have a slight idea, but I couldn't describe this particular situation.*

The student has expressed a lack of knowledge about this situation. The interviewer pressed for detail:

> *Interviewer: Well, what is changing magnetic flux?*
> *Student: Uh, I suppose it's just change in magnetic field [pause] I'm not positive of that one at all.*

Here, the student does not obviously distinguish between flux and field. He continues talking, but this is a discussion of induced current rather than flux. Flux does not enter into the following statement:

> *Student: ... I think in a constant magnetic field, you've either got to have positive [pause] have movement in a constant magnetic field to have current but - or you've got to have a changing magnetic field and something stationary.*

This student, like the others, was not able to relate flux to this situation. This was consistent with the low level of understanding which they displayed when questioned directly about flux in the question last described. Their responses to this question involving induced current confirmed that they were not able to use the concept of flux.

Conclusion about students' understandings of flux

Students generally had little meaning for flux, and were unable to use or explain this concept in the given situations. It would appear that flux was excessively abstract for the students to appreciate the concept given their level of exposure to it. The elements of their thought, in terms of the SOLO Taxonomy, were field strength and area, either singly or together, sometimes related by the idea that they should be multiplied with one another. However, all

of the students responses to questions about flux were in the concrete symbolic mode of response. Use of the concept of flux was not common for students, and it seemed to have little explanatory power for them.

CONCLUSION ABOUT AND IMPLICATIONS FOR FIELD REPRESENTATION

One theme runs strongly through the three sections of this chapter: students' understandings of field representation tended to the concrete over the abstract. Only a few students were capable of using abstract systems to represent fields. This occured in spite of the fact that the subjects were university students who were likely to pursue physics in some form in their future career.

In the section on field lines, it was seen that students tended to concrete views of these lines. In the context of iron filings around a magnet, students often thought of concrete lines being mapped by the filings. In the context of particle movement in fields, students generally saw the field lines as the concrete tracks of the particles in the field.

The idea that particle trajectories follow field lines is appealingly simple and clear. It is easy to explain, and easy to grasp. It is, however, a slight misconception. Is it important that this misconception be confronted? There are arguments both for and against this.

Supporting the approach of confronting the misconception is the fact that this misconception may cause confusion in predictions of trajectories, including those where the particle starts with some initial velocity. In addition, the misconception is very damaging to understanding of trajectories of charged particles in magnetic fields as these do not even approximately follow the lines of the field. Students are known to have difficulties distinguishing electric and magnetic fields. Any vagueness about the relation of force on particles to electric field lines seems likely to carry on into understandings of magnetic field lines. Confronting this misconception also gives students an exposure to reasoning about force and acceleration. This is valuable in itself. The misconception may be linked to the misconception that the force acting on an object is always in the direction of its motion at that time (Galili, 1995).

On the other hand, it could be argued that the misconception should be ignored, as the reasoning involved in forming a true picture of the relationship between field lines and trajectories seems to present problems for the majority of students. Attempts to teach this picture may simply result in students losing their clear idea that particles follow field lines, and having no way to relate to field lines. Further, the fact that the relation between trajectories and field lines is complicated means that any attempt to teach it would probably involve unacceptable amounts of time.

The misconception does not appear to cause problems in predictions of trajectories involving initial velocity - see for comparison the results given in the later section of this chapter on the magnetic field vector, where there is no suggestion by students that particles with initial velocity ought to follow electric or magnetic field lines.

Perhaps the best compromise, in introductory courses, would be to say that the field lines are "approximately" the paths that charges would take in the field, without going into detail on the complications involved. There is an obvious link between field lines and trajectories, which is visible in students' responses. This idea of field lines as particle trajectories does give a unifying picture for the students. More complicated details of the relationship between force, motion, and field lines could be dealt with after students had some grounding in fields. Students were not generally able to make the more difficult link between field lines and the field vector.

As the second section of the chapter, dealing with the field vector, showed, students were generally unable to use the abstract concept of the field vector in their thinking. In considering interactions between fields, students preferred to reason in terms of concrete field lines rather than abstract field vectors. Students were more capable of using the field vector in questions involving given quantities, in the context in which they had learnt it. However, there was generally little understanding behind their use of the field vector, and there was a failure among students to use the abstract system involving the field vector.

The concept of flux is related to, and derived from, the field vector, so it is not surprising that students demonstrated an inability to describe or use flux in field situations. Flux appears to be on a high level of abstraction.

Within the topic areas described above, SOLO Taxonomic analysis was performed, leading to statements about elements of understanding. While this analysis was relative to the context of the specific questions, some broad statements can be made about the nature of elements used in students' responses. Field lines could be used as a single element in the concrete symbolic mode, but they could also be the central element in a relational view of field representation in the same mode. Field strength was also seen used as a single concrete symbolic element in students' responses, but can form a part of the abstract concept of the field vector. The existence of a keyword in a students' response is not sufficient to allow classification of that response in SOLO terms. One cannot identify elements on a one-to-one basis with English words. Consideration must be at a deeper level, with full attention to the context in which the word is used, and the related concepts which are involved. This was the procedure followed in describing students' understandings of field representation.

Overall, students tried to use concrete ideas to represent fields wherever possible, even when this was not appropriate. Their use of abstract ideas was generally based on rote learning of local points rather than an overall abstract view of the system. This tendency to concrete representations of fields has repercussions for the students' understandings of phenomena related to fields, as discussed in the next chapter.