13th Asia Pacific Microscopy Congress
Brisbane, Australia, February 2-7, 2025

The Fokker–Planck Equation for Speckle-based X-ray Microscopy

Samantha J. Alloo[1*], David M. Paganin[1], Konstantin M. Pavlov[2,1,3] and Kaye S. Morgan[1]

[1] School of Physics and Astronomy, Monash University, Victoria, Australia.
[2] School of Physical and Chemical Sciences, University of Canterbury, Christchurch, New Zealand.
[3] School of Science and Technology, University of New England, New South Wales, Australia.
*** Corresponding author: samantha.alloo@monash.edu**

https://orcid.org/0000-0002-9502-6128, https://orcid.org/0000-0002-7758-7140, https://orcid.org/0000-0002-1756-4406, and https://orcid.org/0000-0001-6997-7977

X-rays are attenuated, refracted, and diffused as they pass through an object. In the context of X-ray microscopy, these three mechanisms can be individually exploited to render three complementary images of a sample. The attenuation image reveals sample density, the phase image (based on X-ray refraction) distinguishes weakly attenuating structures, and the dark-field image (using diffusion-induced contrast) highlights structures that are invisible to attenuation and phase imaging. In dark-field imaging, the contrast arises from small-scale structures that induce intensity variations below the spatial resolution of the imaging system, effectively diffusing the X-ray beam. These three imaging modes can be experimentally captured using a speckle-based X-ray imaging (SBXI) technique [1, 2] and the subsequent multimodal (attenuation, phase, and dark-field) image retrieval can be performed using an algorithm derived from the X-ray Fokker–Planck equation [3, 4]. These two concepts form the basis of our theoretical approach, termed "Multimodal Intrinsic Speckle-Tracking" (MIST) [5]. The experimental set-up for SBXI is simple (shown in Fig. 1), introducing a random membrane (like sandpaper) into the X-ray beam. The membrane generates speckles of light, which are subsequently altered upon passing through a sample. The SBXI Fokker–Planck equation models these sample-induced speckle modifications, while MIST solves the associated inverse problem, retrieving high-quality multimodal images from SBXI intensity data. SBXI is an appealing imaging technique as it is experimentally simple in terms of its set-up and image acquisition procedure. Our MIST algorithm enhances SBXI's appeal by being computationally rapid and robust, retrieving images with comprehensive sample information. This is one of the points-of-difference of our developed algorithm compared to other speckle-tracking approaches [6] which recover multimodal images by performing pixel-by-pixel analysis to quantify how reference speckles have changed.

Our published research has (i) further developed the theoretical framework of MIST, broadening its applicability to a wider range of sample types [7, 8, 9], (ii)

stabilized the naturally ill-posed algorithm, establishing regularization techniques that enable accurate, error-minimized image retrieval [7, 9], and (iii) demonstrated that SBXI Fokker–Planck-derived algorithms uniquely separate edge-induced dark-field contrast from unresolved sample microstructure dark-field contrast [10]. This presentation will provide an overview of our works, explaining SBXI image acquisition, data processing within MIST, along with examples of retrieved multimodal images from various samples that demonstrate the robustness of our approach. Figure 2 shows recovered multimodal X-ray images from SBXI data of a test sample comprising four distinct rods. The attenuation and phase signals of the four-rod sample, shown in Figs. 2a and 2b, are qualitatively similar. However, similar to many real-world samples that exhibit complex structures across different length scales, the dark-field image (shown in Fig. 2c) reveals additional structures, such as delicate fibers, that are not visible in the other images.

We anticipate that the basic theoretical framework of our MIST algorithm for X-ray microscopy can be translated to other types of microscopy, such as visible-light and electron microscopy.

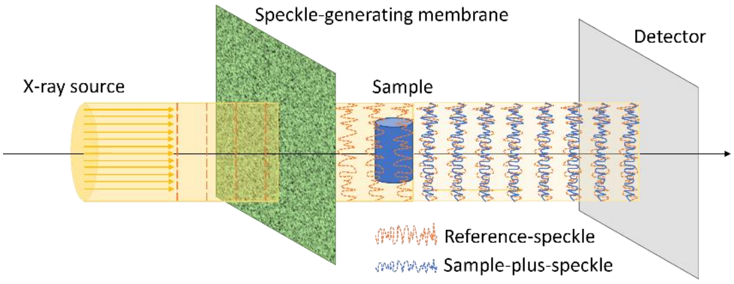

Fig. 1. Experimental set-up of speckle-based X-ray imaging (SBXI) using a monochromatic X-ray source.

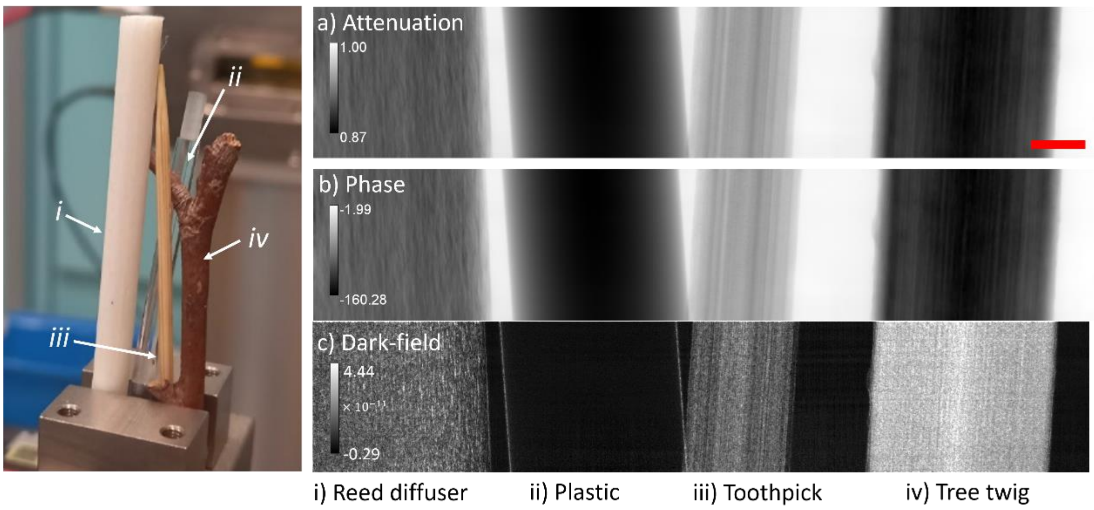

Fig. 2. (left) The test four-rod sample imaged using SBXI. (right) Multimodal images of the four-rod sample retrieved using our Multimodal Intrinsic Speckle-Tracking (MIST) algorithm [9]. The red scale bar in a) is 1mm.

References

[1] S. Bérujon et al., *Phys. Rev. Lett.* **108**, 158102 (2013)
[2] K. S. Morgan, et al., *Appl. Phys. Lett.* **100**, 124102 (2012)
[3] D. M. Paganin and K. S. Morgan, *Sci. Reports* **9**, 17537798 (2019)
[4] K. S. Morgan and D. M. Paganin, *Sci. Reports* **9**, 17465 (2019)
[5] K. M. Pavlov et al., *J. Opt.* **22**, 125604795 (2020)
[6] M.-C. Zdora, *J. Imaging* **4**, 60 (2018)
[7] S. J. Alloo, et al., *J. Med. Imaging* **9**, 031502 (2022)
[8] K. M. Pavlov, et al., *Phys. Rev. A* **104**, 053505 (2021)
[9] S. J. Alloo, et al., *Sci. Reports* **13**, 5424 (2023)
[10] S. J. Alloo, et al. *arXiv* **2410.18317** (2024)