

A statistical analysis of nonlinear regression models for different treatments for layers

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ABSTRACT As the cost of research increases, mathematical models become valuable tools to answer research questions. A major application of mathematical modeling is accurate estimation of production performance, growth, and feed consumption for poultry research and production. There are many ways that a given data set can be analyzed, and different models have been proposed to fit those curves. To explore the models available, data were investigated from a study on the effects of a series of balanced dietary protein levels on egg production and egg quality parameters in laying hens from 18 to 74 wk of age. Forty eight pullets were assigned to each of 3 different protein levels. The results clearly demonstrated that balanced dietary protein level was

the limiting factor for body weight (**BW**), average daily feed intake (**ADFI**), egg weight, and egg production. To test differences of fitted curves, the sum of squared reduction test is used. Using a unique data set with data from individual hens, 6 commonly used models were fitted to hen performance technical data. The resulting statistical inferences from using individual and pooled data were compared. There are only differences in using individual or grouped data in fitting nonlinear models to laying hen response data. For the most important response variables, hen-day egg production, and feed intake, predicted responses are within 0.12 and 0.65%, respectively, throughout the production cycle.

Key words: Adams-Bell, Lokhorst, modified compartment, Gompertz, Richards

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INTRODUCTION

Diets fed to commercial laying hens can vary depending on factors such as the strain of layer, production goals, age, and weather conditions (Schaible and Patrick, 1980). Since laying hens produce eggs depending on the dietary nutrients provided, the quality and formulation of the diet is important to producers, especially considering that 65 to 75% of the cost to produce eggs is due to feed costs (Bell and Weaver, 2002). Mathematical models of egg production should be valuable tools to answer research and production questions. Several models depicting curves have been proposed. For instance, the Adams-Bell (1980) and modified compartmental (Miyoshi et al., 1996) models have been applied to fit egg production; Lokhorst (1996) introduced models to fit egg weight and feed intake. Gompertz (1825) and

Richard (1959) proposed models to fit the body weight curves of hens. Rickfels (1985) showed that the Gompertz equation could also be applied to fit the growth curve of Japanese quail. He demonstrated that the variation in the parameters changes the growth curve and so should be used to compare treatments, and effects of treatments on growth. Finally, the Richards equation (Richard, 1959) was developed from the von Bertalanffy equation (Bertalanffy, 1941). These equations were derived to model response variables without considering if or how the birds or animals were grouped. Modern animal research almost always involves birds kept in groups, usually pens, cages, or groups of cages.

Shim et al. (2013) described an experiment in which different dietary protein levels were feed to commercial layers (Medium, with breeder management guide crude protein (CP) recommendations; High, with Medium +2% CP, and Low, with Medium -2% CP). A number of variables were measured on a weekly basis (19–76 wk) to see how the different diets affected the response outcomes: egg production, egg weight, egg output, average daily feed intake (ADFI), cumulative egg numbers, and hen body weight. The Shim et al. (2013) data set is unique in that the egg weight and feed intake data were

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collected weekly on individually-housed hens, and is therefore very suitable to model production performance. Previous research typically compared protein levels using a one or two-way analysis of variance. Good examples of previous analyses are the studies [Keshavarz and Nakajima \(1995\)](#) and [Mazzuco et al. \(2011\)](#). [Billard et al. \(2013\)](#) discussed the benefits of using repeated measures designs instead of a one or two-way analysis of variance and also considered the impact of the presence of a covariate factor. Thus, the present study focused on comparing treatments (3 protein levels) using nonlinear regression analysis.

In research settings hens are usually grouped for data collection, but in industry no such groups exist. The research groups are normally considered when determining significant differences and modeling responses to various inputs, hence the group becomes the experimental unit. Grouping birds decreases the amount of variation in the dependent variable, but also decreases model degrees of freedom, potentially changing predicted lines and research conclusions. Therefore, the objective of this paper was to perform statistical analyses of a number of regression related models to these experimental data and compare the regression models derived when the independent variables were individual data vs. group means.

MATERIALS AND METHODS

The data analyzed have been described in detail by [Shim et al. \(2013\)](#) ([Table 1](#)). Six functions were included in these analyses because they have been commonly used to fit response variables from egg production research (Adams-Bell model; [Adams and Bell, 1980](#)), modified compartment model ([Miyoshi et al., 1996](#)), and egg weight and feed intake Lokhorst model ([Lokhorst, 1996](#)), Gompertz equation ([Gompertz, 1825](#)) and Richards equation ([Richard, 1959](#)). All models were fitted using the nonlinear procedure (“proc nlin”) of SAS

Table 1. Summary of descriptive statistics for data. H, M and L refer to high, medium, and low protein diets (respectively) fed to 6 groups of 8 hens each from 19 to 76 wk of age.

Variables	Treatment	<i>n</i>	Range	Mean ± SD
Egg production (%) (y_1)	H	48	0 to 100	89.90 ± 22.27
	M	48	0 to 100	87.69 ± 21.93
	L	48	0 to 100	72.17 ± 21.98
Egg output (g) (y_2)	H	48	5.53 to 74.53	55.33 ± 9.57
	M	48	5.16 to 68.79	51.24 ± 9.10
	L	48	5.42 to 64.71	39.28 ± 9.89
Egg weight (g) (y_3)	H	48	38.1 to 74.53	59.09 ± 5.24
	M	48	36.9 to 70.43	56.26 ± 4.27
	L	48	37.75 to 66.06	52.85 ± 3.83
ADFI (g) (y_4)	H	48	12.8 to 135.39	101.19 ± 12.87
	M	48	19.1 to 139.63	99.17 ± 13.98
	L	48	37.75 to 66.06	86.75 ± 17.33
Layer body weight (kg) (y_5)	H	48	1.02 to 2.23	1.56 ± 0.21
	M	48	1.03 to 1.91	1.46 ± 0.16
	L	48	0.93 to 1.71	1.30 ± 0.17
Egg cumulative number (y_6)	H	48	0 to 327	172.59 ± 106.00
	M	48	0 to 375	166.62 ± 105.24
	L	48	0 to 356	143.31 ± 85.93

Abbreviation: ADFI, average daily feed intake.

([SAS Institute, 2006](#)) with the Marquardt algorithm ([Marquardt, 1963](#)). A comparison of the three treatments using these nonlinear regression analyses was performed for individual hens ($n = 48$ individual hens per treatment) and groups ($n = 6$, six groups of 8 hens each for a total of 48).

With all models, t was the age in weeks of the hens and y was the output variable measured each week. Egg production was the number of eggs per 100 live hens per day. This is also called “hen-day egg production” or HDEP and is expressed as a percentage. Egg output is the grams of egg produced by the average hen per day (HDEP/100 × egg weight).

Adams-Bell Equation

The Adams-Bell equation ([Adams and Bell, 1980](#)) was proposed to fit egg production and egg mass output data. Egg production and egg output are considered as response variables (y_1 and y_2 , respectively) in [Table 1](#):

$$y = 100 \left(\frac{1}{1 + ab^t} - ct + d \right) \quad (1)$$

where response y corresponds, respectively, to y_1 for egg production (%) or y_2 egg output at age t weeks, and a , b , c , and d are parameters. These parameters have no interpretation other than being model parameters.

Modified Compartmental Equation

The modified compartmental equation has been applied to fit egg production y_1 and egg output y_2 by [Miyoshi et al. \(1996\)](#):

$$y = a \left(\frac{\exp(-bt)}{1 + \exp(-c(t-d))} \right) \quad (2)$$

where y corresponds, respectively, to y_1 egg production (%) or y_2 egg output, t is age in weeks and a , b , c , and d are parameters. These parameters have no particular interpretation.

Lokhorst Egg Weight Equation

[Lokhorst \(1996\)](#) also proposed an equation for egg weight y_3 , specifically

$$y = a + br^t \quad (3)$$

In this equation, y ($\equiv y_3$) is egg weight (g/egg), a is the theoretical maximum egg weight, b (< 0) should be added to a to determine the initial egg weight, r ($0 < r < 1$) refers to the growth rate, and t is the age of the hens in weeks.

Lokhorst Feed Intake Equation

[Lokhorst \(1996\)](#) proposed different equations for egg weight (g), second-grade eggs, and feed intake. The Lokhorst feed intake equation is

$$y = \left(\frac{a}{1 + b \exp(-act)} \right) + dt + ft^2 \quad (4)$$

where y ($\equiv y_4$) is feed intake (g/ hen/ day), a is the horizontal asymptote of the restricted growth curve, b represents the feed consumption at the start of the laying period, and c refers to the rate of the increases in feed consumption in the restricted growth phase. Parameters d and f indicate that feed consumption increases or decreases during the rest of the laying period and the age of the hens in weeks.

Gompertz Equation

The Gompertz (1825) equation was proposed to fit growth curves according to

$$y = N e^{[-\alpha e^{(-N\beta t)}]} \quad (5)$$

In the present application, y corresponds to the body weight (kg) (y_5) of the hen, β is the intrinsic growth rate, N is the asymptotic or maximum growth response, and α is a constant.

Richards Equation

Richard (1959) introduced an equation which was applied to fit egg weight as

$$y = \frac{N}{[1 + a \exp(-N\beta t)]^\gamma} \quad (6)$$

where y corresponds to body weight(kg) (y_5) of the hen, β is the intrinsic growth rate, N is the asymptotic or maximum growth response, and α and γ are parameters.

Comparison of Models

To compare treatments using nonlinear regression analysis, the sum of squares reduction test (Galant, 1987) was used. For initial illustrative purposes, suppose there are 2 regression models, one each for high (H) protein and one for low (L) protein levels, viz.,

$$H \quad y_{Hi} = a_H + b_H x_i + e_i, \quad i = 1, \dots, n_H, \quad (7)$$

$$L \quad y_{Li} = a_L + b_L x_i + e_i, \quad i = 1, \dots, n_L, \quad (8)$$

for a response variable y with regressor or predictor variable x , and where there are n_H hens receiving the H diet and n_L hens feed the L diet.

In the full model, the n_H High protein fed hen observations are used to estimate a_H and b_H and hence the predicted response \hat{y}_{Hi} in the usual way; and likewise the n_L Low protein hen observations are used to estimate a_L and b_L and hence the predicted response \hat{y}_{Li} . That is,

$$H \quad \hat{y}_{Hi} = \hat{a}_H + \hat{b}_H x_i, \quad i = 1, \dots, n_H, \quad (9)$$

$$L \quad \hat{y}_{Li} = \hat{a}_L + \hat{b}_L x_i, \quad i = 1, \dots, n_L \quad (10)$$

The sum of squared residuals for this full model is given by

$$SS(\text{Residuals})_{full} = \sum_{i=1}^{n_H} (y_{Hi} - \hat{y}_{Hi})^2 + \sum_{i=1}^{n_L} (y_{Li} - \hat{y}_{Li})^2 \quad (11)$$

and has $n - 4$ degrees of freedom (i.e., $df(\text{Residuals})_{full} = n - \# \text{parameters estimated}$ (here $\# \text{parameters} = 4$)).

The reduced model pertains when it is assumed that the 2 diets are the same, that is, the model is

$$y_i = a + b x_i + e_i, \quad i = 1, \dots, n_H + n_L, \quad (12)$$

where now it is assumed $a_H = a_L = a$ and $b_H = b_L = b$. Now all $n = n_H + n_L$ observations are used to estimate the model parameters a and b . The predicted response is

$$\hat{y}_i = \hat{a} + \hat{b} x_i, \quad i = 1, \dots, n; \quad (13)$$

and the sum of squared residuals for this reduced model is

$$SS(\text{Residuals})_{reduced} = \sum_{i=1}^n (y_i - \hat{y}_i)^2 \quad (14)$$

and has $df(\text{Residuals})_{reduced} = n - 2$ degrees of freedom.

To compare the full model with the reduced model, the test statistic is (Eq. 15)

$$F = \frac{(SS(\text{Residuals})_{reduced} - SS(\text{Residuals})_{full}) / (df((\text{Residual})_{reduced} - df(\text{Residual})_{full}))}{M S Error_{Full}} \quad (15)$$

where $M S Error_{full} = SS(\text{Residuals})_{full} / df(\text{Residuals})_{full}$. This F statistic has an F -distribution with df_1 and df_2 degrees of freedom where

$$df_1 = df(\text{Residuals})_{reduced} - df(\text{Residuals})_{full}, \quad df_2 = df(\text{Residuals})_{full}. \quad (16)$$

Interpretation: If the full model is accepted, the 2 models are significantly different from each other. Or, when the reduced model is accepted, there is no difference in the responses/models for the different diets.

The same principles apply to the situation of different regression models where the "full" model consists of three different models (instead of the two as in Eq. (7) and Eq. (8)), and the reduced model sets of 2 protein levels (e.g., H and L) as being equivalent. For example, for the Adams-Bell model, the full model contains twelve parameters (4 parameters \times 3 treatments), that is,

$$y_i = 100 \left(\frac{1}{(1 + a_i b_i^t)} - c_i t + d_i \right), \quad i = 1, 2, 3. \quad (17)$$

where $y_i = 1, 2, 3$, corresponds to the response at level H, M, and L, respectively, of the response variable y . The reduced model contains 8 parameters since under the null hypothesis that there is no difference between 2 of the treatments (e.g., H_0 : High and Medium diets are the

same), those 2 particular treatments share the same a , b , c , and d (analogously to Eq. (12)).

RESULTS AND DISCUSSION

Egg Production (HDEP, %) (y_1)

First, the Adams-Bell (1980) model (Eq. (1)) and the modified compartmental model (Miyoshi et al., 1996, Eq. (2)) were fitted to the individual and to the group data for egg production (y_1 , Table 2). The parameters a , b , c , and d are very similar for each treatment (dietary protein) level so only those for the group data are shown. However, here as for all models and all variables, the values of the squared correlation coefficient R^2 for the group data fits are higher than when the individual hen data are used, since as expected there is more variation

in the individual data than in the group data. When comparing the results of the 2 models, the R^2 values of the M and L treatments in the Adams-Bell model are lower than those in the modified compartmental model, suggesting that the modified compartmental model is a better fit. This is in contrast to results of experiments from previous research (Faridi et al., 2011), in which R^2 in the Adams-Bell model was higher.

Figure 1 shows the 3 fitted curves of the Adams-Bell model with the plots for the group data in Figure 1 (those for the modified compartmental model are comparable). Here, as for all plots for illustrative clarity, the observed group mean data set values are presented. Using the individual points results in an extraordinary number of point markers (144 at each time point, which may be confusing) and do not aid in visualizing the data. The shapes of the 3 curves are very similar to each

Table 2. Model-parameter estimates and goodness of fit for egg production (y_1). Age in weeks is represented by t . H, M, and L refer to high, medium, and low protein diets (respectively) fed to 6 groups of 8 hens each from 19 to 76 wk of age.

Model	Entity	H	M	L
Adams-Bell ¹	a	1.208×10^{-13}	9.29×10^{-16}	5.133×10^{-14}
	b	0.2310	0.1468	0.1834
	c	0.000443	0.000861	0.00247
	d	-0.0429	-0.0492	-0.1365
	R^2	0.9843	0.9762	0.7465
Modified compartment ²	a	95.7401	94.9419	87.0641
	b	0.000475	0.000907	0.00330
	c	1.5874	2.1645	2.3425
	d	20.6513	20.4605	20.2862
	R^2	0.9828	0.9757	0.7414

$$^1y = 100\left(\frac{1}{1+ab^t} - ct + d\right).$$

$$^2y = a\left(\frac{\exp(-bt)}{1 + \exp(-c(t-d))}\right).$$

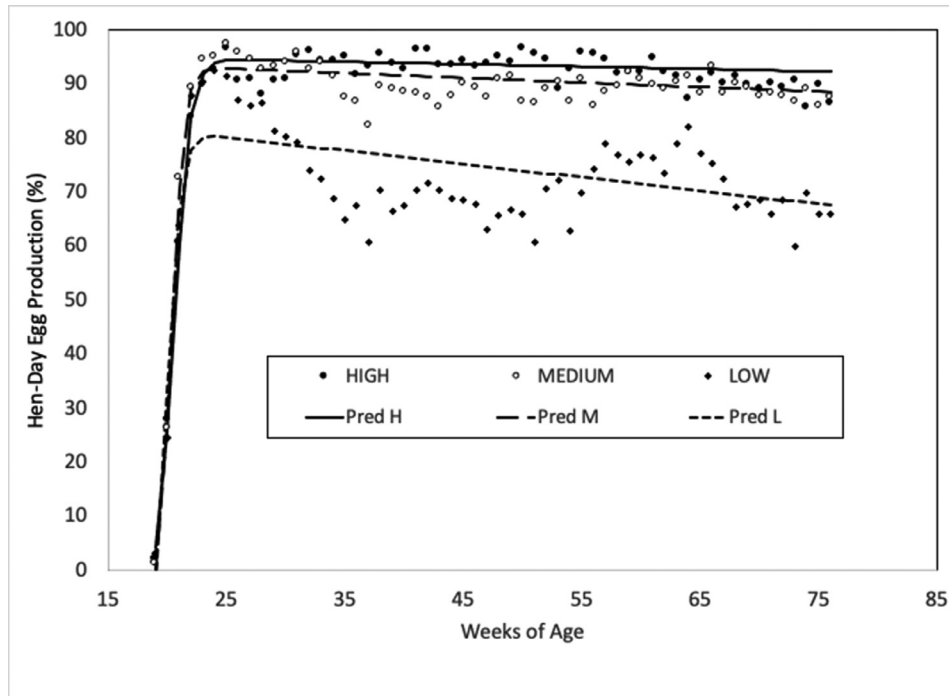


Figure 1. Adams-Bell model for egg production. Each point represents the average of 6 groups of 8 hens each. High, Medium, and Low refer to high, medium, and low protein diets (respectively) fed to 6 groups of 8 hens each from 19 to 76 wk of age. Pred H, Pred M, and Pred L refer to model predicted values.

Table 3. Model-parameter estimates and goodness of fit for egg output (y_2). Age in weeks is represented by t . H, M, and L refer to high, medium, and low protein diets (respectively) fed to 6 groups of 8 hens each from 19 to 76 wk of age.

Model	Entity	H	M	L
Adams-Bell ¹	a	427,849	13,485,517	1.118×10^{-8}
	b	0.5115	0.4246	0.3715
	c	-0.00058	-0.00046	0.000541
	d	-0.4675	-0.5059	-0.5792
	R^2	0.9804	0.9520	0.7252
Modified compartment ²	a	52.0804	41.8585	41.9949
	b	-0.00143	0.00166	0.00131
	c	1.2822	2.3564	2.2727
	d	20.9650	20.4723	20.4033
	R^2	0.9786	0.9659	0.7616

$$^1y = 100 \left(\frac{1}{1+ab^t} - ct + d \right).$$

$$^2y = a \left(\frac{\exp(-bt)}{1 + \exp(-c(t-d))} \right)$$

other. The gap between the curve of the L level and the other two (H and M) is wide, suggesting there are differences between them.

By comparing the coefficient of determination (R^2) values given in Table 2 and the plotted fits of Figure 1, it may be concluded that both models fit the data from hens fed the H and M series very well, but not those fed the L series of diets. The Adams-Bell model was designed to fit usual egg production curves well, but the hen’s response to the L diets was not usual: Egg production of hens fed the L series of diets increased normally in the early weeks but then decreased much more quickly than usually expected and did not decrease in a linear manner (Figure 1). We believe this was the result of cold weather: The low protein-fed hens could not cope with cold weather as well as those fed higher protein levels (Shim et al., 2013).

Egg Output (y_2)

The egg output parameter estimates for b , c , and d Adams-Bell (1980) model of Eq. (1) are very similar at each protein level, but those for a are not (Table 3). A

Table 4. Lokhorst¹ egg weight model-parameter estimates and goodness of fit for egg weight (y_3). Age in weeks is represented by t . H, M, and L refer to high, medium, and low protein diets (respectively) fed to 6 groups of 8 hens each from 19 to 76 wk of age.

Entity	H	M	L
a	60.7176	57.3861	54.1269
b	-370.8	-825.0	-109.1
r	0.8504	0.8114	0.8828
R^2	0.9356	0.8329	0.8685

$$^1y = \left(\frac{a}{1+b \exp(-act)} \right) + dt + ft^2.$$

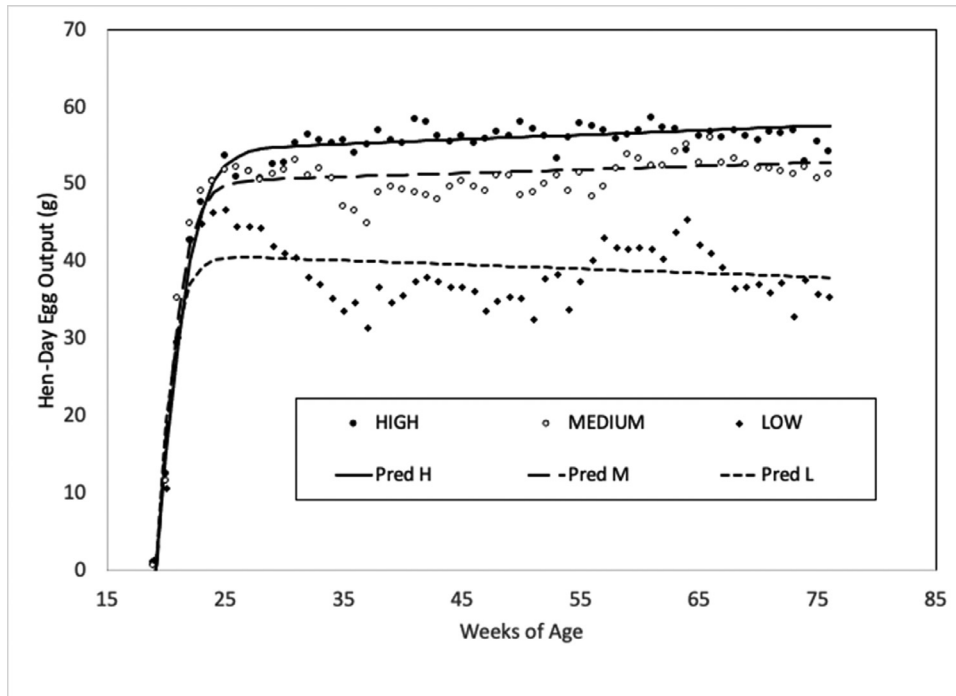


Figure 2. Adams-Bell model for egg output. Each point represents the average of 6 groups of 8 hens each. High, Medium, and Low refer to high, medium, and low protein diets (respectively) fed to 6 groups of 8 hens each from 19 to 76 wk of age. Pred H, Pred M, and Pred L refer to model predicted values.

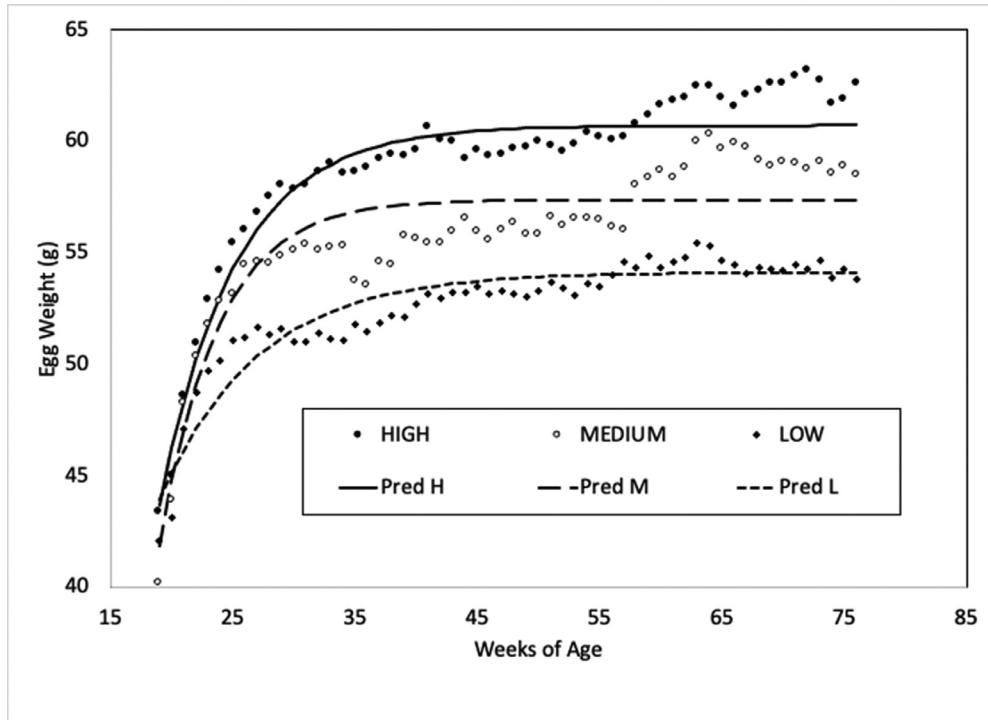


Figure 3. Lokhorst egg weight model for egg weight. Each point represents the average of 6 groups of 8 hens each. High, Medium, and Low refer to high, medium, and low protein diets (respectively) fed to 6 groups of 8 hens each from 19 to 76 wk of age. Pred H, Pred M, and Pred L refer to model predicted values.

consequence of this dissimilarity is observed in [Figure 2](#) for the Adams-Bell model (those for the modified compartmental model are similar). The gaps among the 3 fitted curves are wider than those in [Figure 1](#) for egg

production ([Figure 2](#)). As age increased, the 2 egg output curves corresponding to the H and M levels both increased. However, the curve from the L diet fed hens decreased.

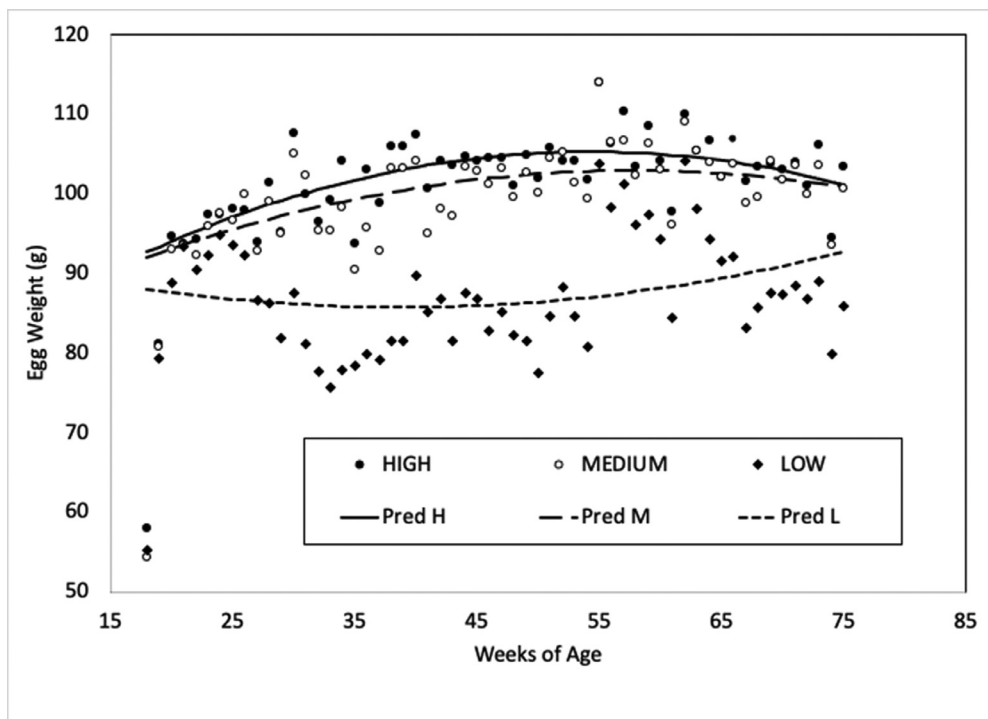


Figure 4. Lokhorst feed intake model for ADFI. Each point represents the average of 6 groups of 8 hens each. High, Medium, and Low refer to high, medium, and low protein diets (respectively) fed to 6 groups of 8 hens each from 19 to 76 wk of age. Pred H, Pred M, and Pred L refer to model predicted values.

Table 5. Lokhorst¹ feed intake model-parameter estimates and goodness of fit for ADFI (y_4). Age in weeks is represented by t . H, M, and L refer to high, medium, and low protein diets (respectively) fed to 6 groups of 8 hens each from 19 to 76 wk of age.

Entity	Group data		
	H	M	L
a	77.0445	79.9941	93.7631
b	4.763×10^{12}	4.145×10^{13}	2.546×10^{14}
c	0.0212	0.0218	0.0200
d	1.0414	0.7924	-0.4127
f	-0.00959	-0.00684	0.00532
R^2	0.8277	0.7887	0.3475

$$^1y = \left(\frac{a}{1+b \exp(-act)} \right) + dt + ft^2.$$

Egg Weight (y_3)

The egg weight coefficient estimates (Lokhorst model Eq. (3)) of hens fed each diet are slightly different among the parameter estimates of s , a and r (Table 4). However, the parameter estimates of b for hens fed the M and L levels are quite different from those fed the H level, and are also quite different for the individual and the group data sets (Table 4). The effects of the differences in coefficient estimates are obvious in the fitted curves (Figure 3). The increase in egg weight in hens fed the M diet is responsible for the larger r compared to when diets H and L were fed. The meaning of parameter a is the theoretical maximum egg weight. Even though the value of a for each diet (Table 4) is not the same as for the corresponding values for average maximum egg

Table 6. Model-parameter estimates and goodness of fit for body weight (y_5) using group data. H, M, and L refer to high, medium, and low protein diets (respectively) fed to 6 groups of 8 hens each from 19 to 76 weeks of age.

Model	Entity	H	M	L
Gompertz ¹	α	13.5127	187.2	3.660
	β	0.1257	0.2459	0.135
	N	1.6215	1.4888	1.310
Richards ²	R^2	0.9757	0.9578	0.6248
	α	0.5671	-1.0374	6.433×10^{-42}
	β	0.1263	0.00132	4.6283
	N	1.6214	1.5474	1.3100
	γ	24.4221	-0.0139	5218.7
	R^2	0.9756	0.9720	0.6248

$$^1y = N e^{[-\alpha e^{(-N\beta t)}]}$$

$$^2y = \frac{N}{[1+\alpha \exp(-N\beta t)]^\gamma}$$

weight (Table 1), the diet level of the highest and lowest maximum egg weight is the same.

Average Daily Feed Intake (y_4)

For the Lokhorst (1996) feed intake model (Eq. (4), Table 5), fitted parameters a , b , c , and d are very similar for each treatment. From Figure 4 it is clear that the gap between the curve of L level and the curves for H and M levels is wider than that between the curve for H and M levels, thus showing these represent different patterns after the peak. As age increased, the curves of the H and M levels increase and then decrease. However, the

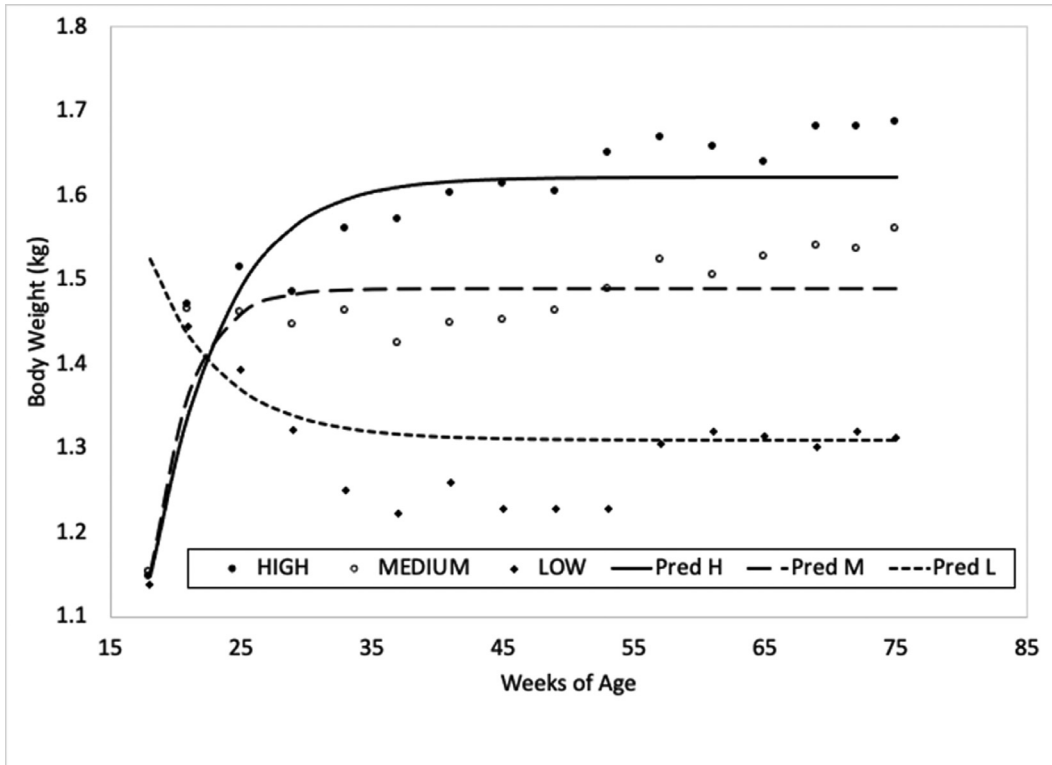


Figure 5. Gompertz model for body weight. Each point represents the average of 6 groups of 8 hens each. High, Medium, and Low refer to high, medium, and low protein diets (respectively) fed to 6 groups of 8 hens each from 19 to 76 wk of age. Pred H, Pred M, and Pred L refer to model predicted values.

Table 7. Regression goodness of fit for body weight (y_5) with group data. H, M, and L refer to high, medium and low protein diets (respectively) fed to 6 groups of 8 hens each from 19 to 76 wk of age.

Group	Regression type	H		M		L	
		<i>P</i> -value	R ²	<i>P</i> -value	R ²	<i>P</i> -value	R ²
Post-peak	Linear	0.007	0.8651	0.045	0.676	0.440	0.031
Post-peak	Quadratic	0.022	0.9257	0.081	0.813	<0.001	0.724
Post-peak	Cubic	<0.001	0.9311	<0.001	0.928	<0.001	0.873

curve of the L level decreases and then increases. These patterns are affected by the sign of the parameters d and f , since d and f indicate that feed consumption increases or decreases after the starting of the laying period. The estimated values of parameter a , the horizontal asymptote of the restricted growth curve, was highest for the H diet series fed hens (Figure 4). However, the highest estimated value of a is actually for the L level diet (Table 5). is for the M level. The highest feed consumption at the start is in the M level resulting in the highest estimated value of b (which represents the feed consumption at the start of the laying period). However, the highest estimated value of c (which refers to the speed of the increases in feed consumption in the restricted growth phase) is in the M level.

The Lokhorst feed intake model very nicely fitted both types of responses observed here. For hens fed the H and M series of diets, ADFI increased quickly and then increased slowly and decreased. For hens fed the L series of diets, ADFI increased quickly but then decreased slowly before increasing.

Laying Hens Body Weight (y_5)

The Gompertz model (Eq. (5)) and the Richard's model (Eq. (6)), are not as good as other models, specifically, the fitted line of the L level fed hens is flat (Table 6, Figure 5). Since there is high variation in the L level fed hens due to variations in environmental temperature, the Gompertz equation used in previous research (Faridi et al., 2011) did not give nicely fitted curves. Rogers et al. (1987) demonstrated that polynomial regression models may more accurately predict responses for specific parts of the data. Thus, linear/quadratic/cubic regression models were considered for body weight.

Before fitting basic linear regression curves, the shape of the curves based on those of Figure 5 was considered. There was an initial linear trend up to 21 wk of age. There appear to be linear, quadratic, or cubic trends after age 21 wk. Thus, the data was divided into 2 phases. The first 2 points (18 and 21 wk of age) are in the first phase (pre-peak), and the other points (ages 21–75 wk) are in the second (post-peak) phase. The equations for linear, quadratic, and cubic responses are

$$y = a + b \times t, \quad (18)$$

$$y = a + b \times t + c \times t^2, \quad (19)$$

$$y = a + b \times t + c \times t^2 + d \times t^3 \quad (20)$$

where t is age in weeks and a , b , c , and d are model parameters. Since there are only 2 time points ($t = 18$ and 21) in the pre-peak group, clearly only a linear regression (connecting the 2 observations) can be used to fit 2 observed values of the body weight.

Fits for the quadratic and cubic regression models are very good and reasonably comparable for the H and M diets (Table 7, Figure 6). The cubic model is better for the L protein level diet for the group data (Table 8) due to the sigmoidal nature of the response.

Comparison of Treatments

For each of the egg production response fits using the Adams-Bell and the modified compartmental models, and for each of the individual and group data comparisons (Table 9), H_0 is rejected at the 2 % level of significance ($P < 0.012$). Except for the H and M comparison with the group data, H_0 is rejected at the 0.1% level of significance. That is, the three nonlinear curves are significantly different from each other. The different protein levels produce significantly different egg production responses.

For egg output, egg weight, average daily feed intake, and hen body weight, like HDEP, the 3 nonlinear curves for each of the individual and group data sets are significantly different from each other at least at the 0.5% level of probability ($P < 0.004$, Table 9).

Cumulative Egg Production (y_6)

There is no “peak” in cumulative egg numbers (y_6) possible (except at the end, Figure 7). Since the analysis shows there is a statistically significant difference across the three diets ($P < 0.001$). However, there is now a significant difference in cumulative egg numbers between the high and medium protein content diets ($P = 0.162$). Hens fed the low protein diet were initially better egg producers but as they age the loss of body protein, compared to the high and medium protein content diet hens, becomes a factor with a resulting loss of overall egg production (Figure 7).

These models all fitted the data very well and models based on both individual hens and groups of hens (Table 9). The models chosen were capable of finding significant differences between the treatments to a very high degree, usually $P < 0.0001$. Many of the specific

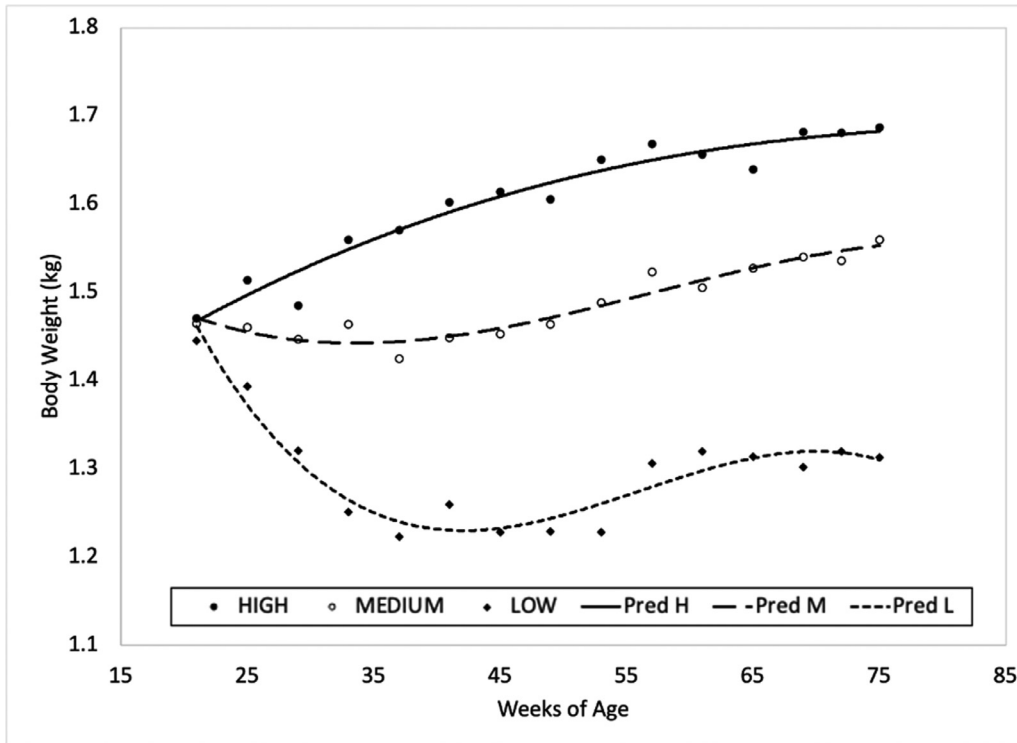


Figure 6. Cubic regression for body weight. Each point represents the average of 6 groups of 8 hens each. High, Medium, and Low refer to high, medium, and low protein diets (respectively) fed to 6 groups of 8 hens each from 19 to 76 wk of age. Pred H, Pred M, and Pred L refer to model predicted values.

Table 8. Cubic regression-parameter estimates for post-peak body weight (y_5) using group data. H, M, and L refer to high, medium, and low protein diets (respectively) fed to 6 groups of 8 hens each from 19 to 76 wk of age.

Diet	a	b	c	d
H	1.2643	0.0117	-0.0001	0.0000003
M	1.6950	-0.0174	0.0004	-0.000002
L	2.6446	-0.0866	0.0017	-0.00001

models developed to predict hen egg production, egg weight, and intake were published some decades ago. The amount and efficiency of egg production are much greater than in the past. Still the underlying biology and

production patterns of hens remained similar despite intense genetic selection,

The pertinent question is then “Did one independent variable, individual or group data, better model the responses?” Predictions from using the 2 independent variables were within 0.12% for egg production and 0.65% for feed intake, the 2 most important variables in economic modeling (Table 10). The models for egg weight and egg output were within 1.38% to 50 wk and 3.5% to 76 wk of age. Changes in one parameter offset changes in another parameter(s) to make very similar predictions (Tables 2–6, 8). It therefore appears that modeling average data from hens kept in groups should

Table 9. Comparison of model fits from individual and group data using various models for different response variables. H, M, and L refer to high, medium, and low protein diets (respectively) fed to 6 groups of 8 hens each from 19 to 76 wk of age.

Variable	Model	Data	H and M		H and L		M and L	
			F-value	P-value	F-value	P-value	F-value	P-value
Egg production	Adams-Bell	Individual	0.746	<0.001	406.993	<0.001	303.520	<0.001
		Group	3.338	0.012	126.027	<0.001	93.601	<0.001
	Modified compartment	Individual	9.527	<0.001	334.975	<0.001	253.614	<0.001
		Group	3.365	0.012	127.566	<0.001	96.693	<0.001
Egg output	Adams-Bell	Individual	9.961	<0.001	142.699	<0.001	78.405	<0.001
		Group	18.797	<0.001	245.395	<0.001	131.180	<0.001
	Modified compartment	Individual	78.848	<0.001	1,055.393	<0.001	572.374	<0.001
		Group	20.420	<0.001	289.033	<0.001	158.789	<0.001
Egg weight	Lokhorst	Individual	375.639	<0.001	1,800.290	<0.001	540.793	<0.001
		Group	37.244	<0.001	229.739	<0.001	79.593	<0.001
ADFI	Lokhorst	Individual	50.995	<0.001	340.676	<0.001	281.264	<0.001
		Group	3.617	0.004	54.056	<0.001	35.688	<0.001
Body weight	Cubic	Individual	202.110	<0.001	773.494	<0.001	1,039.641	<0.001
		Group	5.970	0.030	88.623	<0.001	10.595	0.002

Abbreviation: ADFI, average daily feed intake.

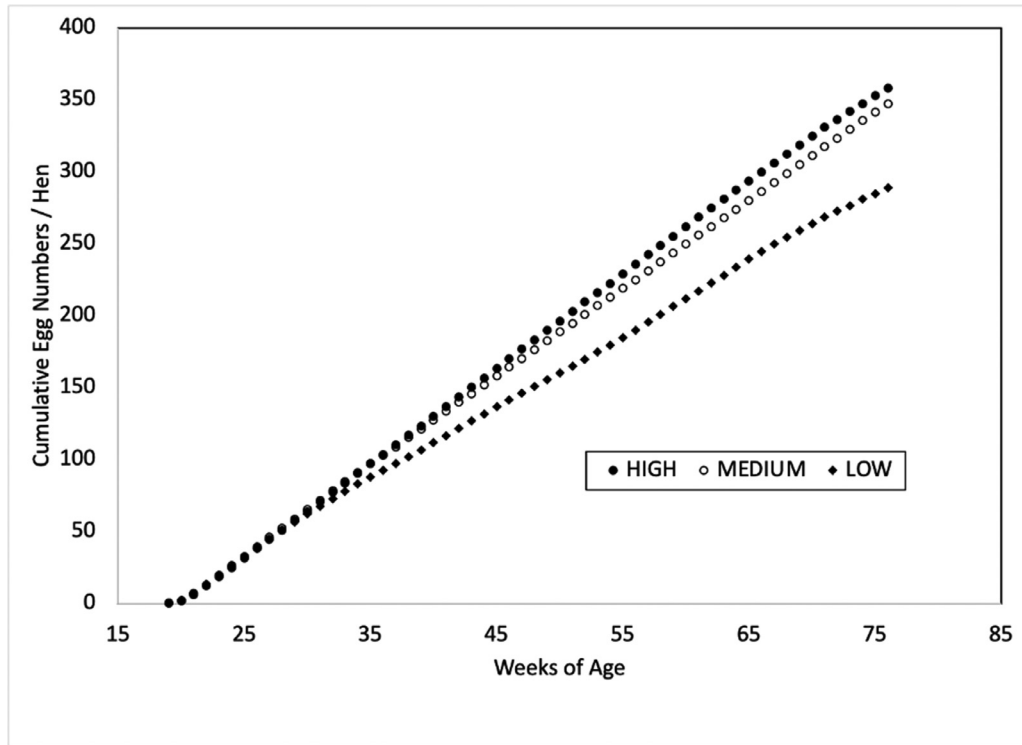


Figure 7. Cumulative egg numbers. Each point represents the average of 6 groups of 8 hens each. High, Medium, and Low refer to high, medium, and low protein diets (respectively) fed to 6 groups of 8 hens each from 19 to 76 wk of age.

Table 10. Comparison of predicted responses from various models for laying hen performance using individual hen responses versus group means for hens fed high, medium or low dietary protein levels. H, M, and L refer to high, medium and low protein diets (respectively) fed to 6 groups of 8 hens each from 19 to 76 wk of age.

Response (Y) variable	Age (wk)	X variable	Predicted values			Units	Individual vs. Group		
			High	Medium	Low		High	Medium	Low
Egg production	25	Individual	94.44	92.95	80.13	HD%	-0.01	0.04	-0.02
		Group	94.45	92.91	80.16	% Δ	-0.01	0.04	-0.03
	50	Individual	93.55	90.83	74.00	HD%	0.05	0.05	0.00
		Group	93.50	90.78	74.00	% Δ	0.05	0.06	0.00
	76	Individual	92.46	88.60	67.60	HD%	0.11	0.06	0.03
		Group	92.34	88.54	67.58	% Δ	0.12	0.07	0.04
Egg output	25	Individual	52.88	49.78	40.68	g/day	0.38	-0.11	0.15
		Group	52.50	49.89	40.53	% Δ	0.72	-0.22	0.37
	50	Individual	56.71	52.17	39.39	g/day	0.56	0.46	0.01
		Group	56.15	51.71	39.37	% Δ	1.00	0.89	0.03
	76	Individual	58.22	54.38	38.01	g/day	0.56	1.47	0.05
		Group	57.66	52.91	37.97	% Δ	0.97	2.79	0.12
Egg weight	25	Individual	54.24	53.63	49.60	g	-0.03	0.69	0.30
		Group	54.27	52.95	49.29	% Δ	-0.05	1.30	0.61
	50	Individual	60.61	58.15	53.82	g	0.00	0.79	-0.10
		Group	60.61	57.36	53.91	% Δ	0.00	1.38	-0.18
	76	Individual	60.77	59.37	54.65	g	0.05	1.98	0.54
		Group	60.72	57.39	54.12	% Δ	0.08	3.45	0.99
ADFI	25	Individual	97.12	95.77	86.81	g	0.04	0.24	0.04
		Group	97.09	95.53	86.77	% Δ	0.04	0.25	0.04
	50	Individual	105.14	102.94	86.38	g	0.00	0.43	-0.05
		Group	105.14	102.51	86.43	% Δ	0.00	0.42	-0.06
	76	Individual	100.89	101.36	93.56	g	0.09	0.65	0.44
		Group	100.80	100.71	93.13	% Δ	0.09	0.65	0.47
BW	25	Individual	1.51	1.46	1.39	kg	0.01	-0.02	0.01
		Group	1.50	1.48	1.39	% Δ	0.96	-1.23	0.51
	49	Individual	1.50	1.48	1.39	kg	-0.13	-0.09	0.08
		Group	1.63	1.57	1.31	% Δ	-8.19	-5.66	6.08
	76	Individual	1.69	1.56	1.31	kg	-0.02	-0.24	-0.18
		Group	1.71	1.80	1.49	% Δ	-1.14	-13.16	-12.12

Abbreviation: ADFI, average daily feed intake.

give similar and very useful equations for modeling the responses of laying hens.

The original analyses of the Shim et al. (2013) dataset was based on 14 individual 4-wk periods: “*Although daily and weekly data were recorded, statistical data were analyzed and presented as the mean of each 4-wk period. The PROC GLM statement of SAS Institute Inc. (2006) used a completely Randomized design for all 5 variables*”. Thus each monthly analysis had a total of 144 degrees of freedom. In contrast, the present repeated measures analyses included the data from all 14 wk and thus had a total of $(14 \times 144 =)$ 2016 degrees of freedom for the individual hen data and $(14 \times 48 =)$ 672 degrees of freedom for the grouped data. Thus the repeated measures design is not only more appropriate, but also a much more powerful way to analyze data. Especially from the perspective of economic analyses and projecting experimental results on future production, the modeling approach used here is superior. It has the advantages of being more analytically powerful and providing mathematical relationships for future profit maximizing models. The models facilitate the calculation of the costs of dietary protein consumed and value of eggs produced (major determinants of profits).

CONCLUSIONS

In this study, nonlinear regression models were fitted to each of several response variables for measurements obtained on hens fed 3 diets (high, medium, and low protein content) over a sequence of weeks. Further, analyses based on repeated measures designs were conducted on these response variables to ascertain if there were any statistically different model fits between the 3 diets. Note that standard analysis of variance calculations are inappropriate since measurements are made on the same hens over time, but with different hens being used for the different diets. The results obtained when comparing the 3 dietary levels using the nonlinear regression analysis and repeated measures design showed that the protein levels make significant differences in the responses for egg production, egg output, egg weight, and body weight, with higher protein diets producing higher response values. Thus, the choice lies between a higher cost (for more protein content) for higher production (for egg production, egg output, egg weight, and body weight) or a lower cost for a lower production. When applying mathematical models previously developed to predict egg production, egg weight, egg output, and feed intake, there is little difference in using individual hens

or using group averages as the independent variable for the most economically important responses.

DISCLOSURES

The authors have no conflicts of interest to disclose.

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