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Instructional efficiency: The role of prior knowledge and cognitive load

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Abstract

We used a 2 (prior knowledge: low vs. high) × 4 (instructional approach: unitary vs. unitary-pictorial vs. equation vs. equation-pictorial) ANOVA to examine the relationship between instructional approach, student prior knowledge, and personal belief of best practice for learning of the find-whole percentage problems, which pose a challenge for many middle school students. The unit percentage concept is central to both the *unitary approach* and the *unitary-pictorial* approach, where the latter has a diagram to scaffold the unit percentage concept. Both the *equation approach* and the *equation-pictorial approach*, in contrast, are algebra approaches that integrate relevant information to form an equation, allowing learners to solve for an unknown (e.g., *x*). Furthermore, the equation-pictorial approach relies on the proportional concept, scaffolded by a diagram to form an equation. A student's best practice, reflected by what is known as the 'actual – optimal bests dichotomy', details her belief in capability to perform task complexity (i.e., simple task vs. complex task). The concept of element interactivity within cognitive load theory framework predicts differential instructional efficiency: equation-pictorial > equation approach > unitary-pictorial > unitary. Our findings ($N = 218$ secondary students) showed that performance outcomes favored high prior knowledge students for complex problems and, to a lesser extent, practice problems and simple problems. Low prior knowledge students benefitted most from the equation-pictorial approach, and they invested higher mental effort than high prior knowledge students across three approaches (unitary, unitary-pictorial, equation) but not the equation-pictorial approach. Importantly, cognitive load imposition, by proxy of students' mental effort, was unrelated to students' belief in optimal best.

KEYWORDS
belief in optimal best, cognitive load theory, element interactivity, instructional approaches, prior knowledge

1 | INTRODUCTION

Is there a better way to help middle school students learn how to solve percentage problems such as "If I pay $300 for my weekly rent, which represents 25% of my weekly wage – what is my weekly wage?" Mathematics educators in Australia tend to classify such percentage problems as 'find-whole' (Baratta et al., 2010), whereas educators in Singapore classify these as 'reverse percentages' (Chow, 2007). The mathematics education researchers have concurred that there are three types of percentage problems (i.e., *fine-whole*, *find-part*, and *find-percent*), which interrelate with each other (Baratta et al., 2010; Phan et al., 2017). What is of significance, though, is that there is research evidence to indicate that students differ in their understanding of three percentage problem types. For example, Baratta et al. (2010) found that middle school students performed more poorly on the find-whole percentage problems (e.g., 15% of $x = \$30$, where x represents the whole amount) than the find-part (e.g., 15% of $200) and the find-percent (e.g., What percentage of $200 is $30?).

Concerning mathematics education, there are different instructional approaches, which educators may recommend. The Australian Curriculum: Mathematics, for example, emphasizes the importance of the *unitary approach* for teaching of 'find-whole percentage' problems. The unitary approach highlights the calculation of a unit percentage (i.e., 300/25), which is then multiplied by 100 to obtain the answer (i.e., $300/25 \times 100$). However, educators in Singapore opt to use the *algebra approach* (or the equation approach) – for example: $15\% \times x = \$300$, and solve for x (Chow, 2007). Does this difference in preference of a particular instructional approach (e.g., the unitary approach versus the algebra approach) make any meaningful impact? We need to acknowledge the important fact in which on average, Singaporean middle school students outperform their Western counterparts in international mathematics education studies (Thomson et al., 2010).

More importantly, strengthening middle school students' algebra foundation would pave the way for them to pursue senior mathematics education where algebra is an essential component of its curriculum. For example, consider a percentage increased problem: "A book is sold for $160 which included 10% GST (Goods Services Tax). Find its original price excluding the GST". We can form an equation, $\$160 = x + (x \times 10\%)$, and solve for x. Koedinger et al. (2008) regard such percentage increased problems as "double-reference problems" where the variable appears twice in the equation and therefore poses a challenge to students. Nonetheless, if students have schema to solve the find-whole percentage problems using the algebra approach, they would have prior knowledge of $x \times 10\%$, which should help to reduce cognitive load imposed. Indeed, research has highlighted a constructive alignment between the design of an instructional approach and a learner's level of expertise in a domain (Kalyuga et al., 2003). For example, an instructional approach that is suitable for novice learners may be detrimental for expert learners, and vice versa. The novice learners who possess low prior knowledge requires greater instructional support than expert learners.

The present study, which involves secondary school students ($N = 213$) is significant as it advances the study of contrasting instructional approaches and levels of learner expertise on learning find-whole percentage problems. Our inquiry also considers Phan et al., (2017) conceptualization of optimal best, and how this motivational concept could coincide with cognitive load imposition and explain the comparative instructional effects for learning (i.e., unitary, unitary-pictorial, equation, and equation-pictorial). This examination, we contend, is evolutionary especially in terms of providing empirical evidence, which could help elucidate the association between two distinct theoretical orientations: cognitive processing of information and motivational beliefs for learning.

2 | COGNITIVE LOAD THEORY

According to cognitive load theory (Sweller et al., 2011; Sweller et al., 2019) there are three distinct categories of cognitive load: intrinsic, extraneous and germane cognitive loads. The *intrinsic cognitive load* indicates that cognitive load imposition or cognitive burden is imposed by the element interactivity level that is intrinsic to a unit material. A level of element interactivity arises from the interconnection between different elements (e.g., number, symbol, procedure) that require learning (Chen et al., 2017). Importantly, from cognitive load theory, the level of element interactivity may act as an index for the complexity of material. A high level of element interactivity, reflecting high complexity of material, would impose a high level of intrinsic cognitive load and vice versa. However, levels of learner expertise can alter the level of element interactivity. An increase in expertise allows the learner to incorporate interconnected elements into a schema, thus reducing the level of element interactivity.

The *extraneous cognitive load* indicates that cognitive load imposition is accounted for by a level of element interactivity that is deemed ineffective or unprofitable for learning. For example, integrating relevant information from diverse sources would increase the element interactivity level, and thus increase extraneous cognitive load imposition. To improve such instructional design, educators could place textual material in relevant locations of a diagram, which then would minimize extraneous cognitive load (Sweller et al., 1990).

The *germane cognitive load* indicates that cognitive load imposition is account for by a level of element interactivity, which is intrinsic to the task and thus is essential for learning. Germane cognitive load is considered as part of intrinsic cognitive load and not as an independent source of cognitive load. The variability practice that directs a learner to the same problem structure across different problem contexts (Likourezos et al., 2019; Paas & Van Merriënboer, 1994) increases the element interactivity level, which corresponds to an increase in germane cognitive load.

3 | THE IMPORTANCE OF PRIOR KNOWLEDGE

The study of instructional designs, situated within the framework of cognitive load theory (Sweller et al., 2011; Sweller et al., 2019), contends an important premise – namely, a student's prior knowledge and understanding of a subject matter. How much a student knows may account for his/her appreciation of a particular instructional design. Consider a linear equation of $2x + 8 = 12$, which has four solution steps: (i) $2x = 12–8$, (ii) $2x = 4$, (iii) $x = 4 \div 2$, and (iv) $x = 2$. Assume, likewise, that a student has acquired a schema to solve $3x = 6$. When presented with $2x + 8 = 12$, the student would retrieve his/her acquired schema ($3x = 6$) from long-term memory to solve $2x = 4$. Because the student can process the schema of $2x = 4$ as a single element in working memory, the element interactivity level in $2x + 8 = 12$ is therefore reduced. In other words, prior knowledge of simpler equations ($3x = 6$) is advantageous in helping to reduce the level of element interactivity of more complex problems ($2x + 8 = 12$). From the example above, it can be said that the interconnected elements in steps 2, 3, and 4 have been incorporated into a schema such as $3x = 6$.

The example of the linear equation of $2x + 8 = 12$ showcases the importance of prior knowledge. Prior knowledge (e.g., novice versus expertise) may explain an instructor's use of a particular instructional approach, and/or account for the complexity of a unit material. A student who is relatively weak in knowledge would appreciate a teacher's scaffolding (e.g., solution steps 1, 2, 3 and 4). In contrast, a student who is knowledgeable would regard step 3 and step 4 as redundant information, given that he/she already has prior knowledge of these two steps. Processing redundant information inevitably consumes cognitive resources, which would impose extraneous cognitive load (Chandler & Sweller, 1991) and limit quality learning experiences. Aside from identifying a student's level of expertise, it is interesting to note that the concept of element interactivity may also determine the efficiency of an instructional approach (Ngu, Phan, et al., 2018; Ngu, Yeung, et al. 2018; Phan et al., 2017).

4 | ELEMENT INTERACTIVITY AND INSTRUCTIONAL APPROACHES

Existing research has supported the efficacy of learning from the use of 'worked examples' (Sweller et al., 2011; van Gog et al., 2019). As an instructional tool, a worked example provides a step-by-step solution procedure to facilitate learning. However, the effectiveness of a worked example depends, in part, on its design. From the perspective of instructional design, the concept of element interactivity may serve as a point of reference to discern and differentiate the efficiency of a particular instructional approach (Ngu & Phan, 2022; Ngu, Phan, et al., 2018; Ngu, Yeung, et al. 2018, Ngu et al., 2014). For example, an instructional approach that aligns with a high level of element interactivity would impose high cognitive load and, likewise, vice versa.

How shall we estimate the level of element interactivity that is associated with a particular instructional design? According to cognitive load theory (Sweller et al., 2011), anything that requires learning constitutes an element. Chen et al. (2017) regard a symbol, a number and a concept as an element. In their study, Leahy and Sweller (2008) examined how students learned how to use bus timetable. They used the element interactivity count to estimate the complexity of test materials (i.e., in terms of the number of elements). They consider, 'Look up to route numbers' as one element, for example. The element interactivity count, in this case, is proportional to the number of elements that exist in the materials.

In line with prior studies (e.g., Leahy & Sweller, 2008; Ngu & Phan, 2022; Ngu, Phan, et al., 2018; Ngu, Yeung, et al., 2018), we used the number of elements and their interactions as a point of reference to estimate the level of element interactivity that is associated with a particular instructional approach (i.e., unitary, unitary-pictorial, equation and equation-pictorial approaches) (Phan et al., 2017). Consider a percentage problem such as "A worker pays \$350 in tax per month, which is 8% of her monthly income. What is her monthly income?". We analyse the number of elements and their relations in the solution procedure of each instructional approach in order to estimate the element interactivity count. We regard the following as elements:

- a number (e.g., 8%)
- relevant information (e.g., monthly income)
- a symbol (e.g., a variable such as x)
- a concept (e.g., the sub-goal is 1% of monthly income)

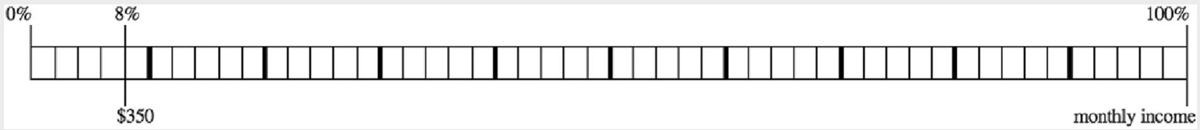
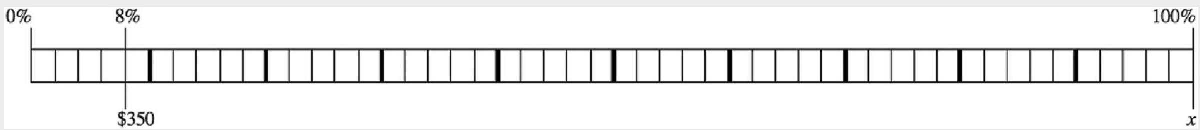
4.1 | The unitary approach

The *unitary approach*, as shown in Table 1, emphasizes the concept of 'unit percentage'. This description depicts a need to calculate the unit percent of a quantity (i.e., 1%) and then a multiple of this to solve the problem. We estimate the level of element interactivity that is imposed by the unitary approach on learning the percentage problem above:

> *Step 1* has four elements: 8%, monthly income, \$350, and one concept (i.e., the relationship between 8%, monthly income, and \$350 where the left side of the equation equals to its right side).
>
> *Step 2* has five elements: \$350, 8, \$43.75, and two concepts (i.e., \$350 represents 8% of monthly income, and the sub-goal is 1% of monthly income).
>
> *Step 3* has five elements: \$43.75, 100, \$4375, and two concepts (i.e., \$43.75 is equivalent to 1% of monthly income, and \$43.75 × 100 is equivalent to 100% of monthly income).

Manipulating multiple elements within and between the steps involved would impose a high level of element interactivity, and, thus a high level of intrinsic cognitive load. Calculating the sub-goal of 1% would be problematic without a point of reference, such as "8% is

TABLE 1 Unitary, unitary-pictorial, equation and equation-pictorial approaches for solving a find-whole percentage problem.

Approach	Problem: *A worker pays $350 in tax per month, which is 8% of her monthly income. What is her monthly income?*		
Unitary approach	Solution Procedure		
	Step 1		8% of monthly income = $350
	Step 2	1% of monthly income	$350 ÷ 8 = $43.75
	Step 3	100% of monthly income	$43.75 × 100 = $4,375
	Answer: The monthly income is $4,375.		
Unitary-pictorial approach			
	Note: The solution procedure is identical to the unitary approach.		
Equation approach	Solution Procedure		
	Step 1	Create an unknown	Let x be the monthly income
	Step 2	8% of monthly income is $350	$8\% \times x = \$350$
	Step 3	Solve the equation	$x = \$350 \div 8\%$
			$x = \$4{,}375$
	Answer: The monthly income is $4,375.		
Equation-pictorial approach			
	Step 1 Create an unknown		
	Step 2 From the diagram, form an equation		Let x be the monthly income
	Step 3 Solve for x		$\frac{8}{350} = \frac{100}{x}$ [$\div x$ becomes $\times x$, $\div 350$ becomes $\times 350$]
	Answer: The monthly income $4375.		$8x = 100 \times 350$
			$x = \frac{100 \times 350}{8} = \$4{,}375$

equivalent to \$350". Moreover, investing cognitive resources for the purpose of searching and integrating discrete sources of information (i.e., 8% in step 1 and \$350 in step 2) is likely to impose a level of element interactivity, which would constitute extraneous cognitive load (Yeung et al., 1998). Therefore, the unitary approach may not yield effectiveness, consequently because of the combined effects of both intrinsic cognitive load and extraneous cognitive load.

4.2 | The unitary-pictorial approach

The *unitary-pictorial approach* shares similar solution steps as those found in the unitary approach with the exception, though, that the former has a diagram (Table 1), which depicts the underlying problem structure such as the proportion concept. The alignment between 8% and \$350 is proportional to the alignment between 100% and monthly income (which will be greater than \$350). Investing germane cognitive load to activate prior knowledge of a proportional concept would enable a student to process fewer elements in the working memory (Carlson et al., 2003). Importantly, aligning 8% with \$350 not only eliminates the split-attention effect (Yeung et al., 1998) but also serves as a point of reference for the calculation of the unit percentage – the sub-goal of 1% of monthly income. Overall, scaffolding provided by the unitary-pictorial approach, which eliminates the split-attention effect as well as eliciting proportional schemas would assist to lower the level of element interactivity. As such, unlike that of the unitary approach, the unitary-pictorial approach would impose a lower level of element interactivity for learning.

4.3 | The equation approach

The *equation approach* is analogous to the algebra approach of learning, highlighting the integration of relevant information (i.e., "monthly income", 8%, \$350) to form an equation (Hegarty et al., 1995), and solve for the variable (x) (Table 1). There are three solution steps involved:

> *Step 1* has one element: the variable (x) may pose a challenge if the student is not familiar with the concept of a variable in the context of linear equations.
> *Step 2* has four elements: 8%, x, \$350, and one concept (i.e., the relationship between the percentage, quantity, and the variable, where the left side of the equation equals to the right side).
> *Step 3* has four elements: x, \$350, 8%, and one concept (i.e., conceptualize $\times$ as an inverse operation of $\div$ so as to solve the equation).

We speculate that the element interactivity level within and between the solution steps of the equation approach would be lower than the element interactivity level for the unitary and/or the unitary-pictorial approach. This observation is warranted, given that the solution steps in the equation approach have fewer elements than the corresponding solution steps in the unitary approach and/or the unitary-pictorial approach.

4.4 | The equation-pictorial approach

The *equation-pictorial approach* is an alternative algebra approach, where it shares the same diagram as the unitary-pictorial approach except for the fact that x replaces "monthly income" (Table 1). Similar to the unitary-pictorial approach, a student is expected to invest germane cognitive load to activate prior knowledge of proportional reasoning, which could result in the processing of fewer elements in the working memory (Carlson et al., 2003). From proportional reasoning, a student can transfer relevant information in the diagram to form an equation such as $8/350 = 100/x$, and solve for x. The equation-pictorial approach and the equation approach differs in the format of the equation ($8/350 = 100/x$ versus $8\% \times x = 350$), but they share the same number of solution steps.

The equation-pictorial approach may impose a high level of element interactivity for a student who has limited knowledge of solving linear equations with fractions. Nevertheless, unlike that of the equation approach, the equation-pictorial approach would impose a lower level of element interactivity. We expect the diagram to assist students in their learning (e.g., to elicit a familiar schema such as proportional reasoning), setting up the equation such as $8/350 = 100/x$, and solve for x.

5 | MEASURE OF COGNITIVE LOAD

Paas (1992) suggested that a learner can retrospect his/her cognitive process during learning and indicate the magnitude of his/her mental effort on a Likert scale, which ranges from 1 (extremely low) to 9 (extremely high). This Likert scale designed by Paas (1992) has been tested in previous studies (Carlson et al., 2003; Ngu & Phan, 2022). Researchers have used multiple items (Leppink et al., 2013) to measure three types of cognitive load separately (Naismith et al., 2015). Importantly, they found a positive association between Paas's scale and intrinsic cognitive load. Moreover, students who had higher prior knowledge rated lower for intrinsic cognitive load. In the present study, we estimated the level of element interactivity, reflecting intrinsic cognitive load to distinguish the efficiency of the four instructional approaches. Thus, we used Paas's scale to measure mental effort, indicating intrinsic cognitive load imposed during learning.

6 | SUMMARY

Overall, then, from the preceding sections, which of the four instructional approaches would benefit students' learning most? Of course, the effectiveness of a particular instructional approach may interrelate with the student's prior learning experience. From the perspective of cognitive load imposition (e.g., Sweller et al., 2011), however, we speculate that the four instructional approaches would differ in terms of element interactivity level and, consequently, their order of efficiency would follow: the equation-pictorial approach > the equation approach > the unitary-pictorial approach > the unitary approach.

7 | OPTIMAL STATE OF FUNCTIONING AND MOTIVATIONAL BELIEFS

The study of optimal best within the context of academic learning is emerging, reflecting the *paradigm of positive psychology* (Seligman, 2010), which emphasizes the importance of motivation, personal resolute, confidence, and inner strength (Seligman et al., 2009). The underlying nature of optimal best, as Fraillon (2004) explains, is intimately linked to the nature of a related concept known as 'actual best'. In terms of secondary school mathematics, consider two examples, which may elucidate and clarify the nature of actual best, denoted as 'L_1', and the nature of optimal best, denoted as 'L_2':

i. Actual best, which refers to a student's testament of his/her ability to understand and/or to perform a simple task as a result of his/her learning experience – for example: "12% of my weekly pay is $120; therefore, what is my weekly pay?"
ii. Optimal best, which refers to a student's indication of belief and conviction of his/her optimal ability to understand and/or to perform a complex task as a result of their learning experience – for example: "Lucy and John both won the first price of a maths competition. Lucy's share of the price is $140, which represents 20% of the total price. How much does John get?"

According to Fraillon's (2004) explanatory account, which Phan et al., (2016) subsequently expanded on, a person may use his/her actual best as a point of reference to structure his/her optimal best. Moreover, successful experience of optimal best from actual best, denoted as '$+\Delta_{(L2 - L1)}$', is positive and proactive, requiring some form of 'optimization' or motivation (Phan et al., 2016, Phan et al., 2020, Phan et al., 2021). We argue that the design of an instructional approach can, in fact, influence a student's learning experience of a subject matter. An optimal instructional approach, in this case, can act as 'an optimizing agent', which would optimize and/or facilitate a student's successful progress from L_1 to L_2. Thus, an interesting question for consideration, arising from this proposed premise relates to the extent to which an instructional approach would assist to motivate and/or to facilitate a student's progress from L_1 to L_2.

Our research focus for development, as detailed, is insightful and innovative, linking two independent lines of inquiry: the design of an instructional approach, which considers the impact of cognitive load imposition (Sweller et al., 2011) versus the achievement of optimal best in a subject matter, which may reflect a person's state of motivation (Phan et al., 2021). We seek to understand the relationship between an instructional approach and the two levels of best practice. For example, does a particular instructional approach align more closely with, say, L_1 than L_2? In relation to our previous mentioning (e.g., the equation-pictorial approach > the equation approach > the unitary-pictorial approach > the unitary approach), for example, we rationalize that the equation-pictorial approach would coincide with L_2, whereas the unitary approach would coincide with L_1. In other words, in terms of comparison, the equation-pictorial approach is more effective as this approach would facilitate the successful achievement of L_2 whereas, in contrast, the unitary approach would align with the achievement of L_1.

An interesting focus of inquiry, which closely associates with the study of instructional approaches is the measurement of cognitive load imposition (Sweller et al., 2011). A determined level of cognitive load imposition, as we discussed, may account for the design of a particular instructional approach. By the same token, however, in accordance with our rationalization, a determined level of cognitive load imposition may coincide with a state of L_1 or L_2. For example, in terms of comparison, we contend that a high level of perceived cognitive load is likely to associate with a state of L_1, whereas a low level of perceived cognitive load would relate to a state of L_2.

8 | THE PRESENT STUDY

The present study considers a particular instructional design against a student's level of prior knowledge, cognitive processing of information, and his/her subsequent motivational beliefs for learning (Feldon et al., 2019; Likourezos et al., 2019). Feldon et al. (2018) found that the cognitive load imposition during learning negated students' academic self-efficacy beliefs. Moreover, Phan et al., (2017) provided a theoretical framework, which explores the nature of optimal best or optimal functioning (Fraillon, 2004; Phan et al., 2016). According to the authors, the nature of optimal best, reflecting the maximization of a learner's internal state of functioning (e.g., optimal cognitive functioning in mathematics) is motivational and proactive. A learner's successful accomplishment of optimal best in a subject matter would indicate his/her motivational beliefs for learning (Phan & Ngu, 2021).

Overall, the present study seeks to advance the study of appropriate instructional approaches for learning find-whole percentage problems that exhibit two levels of complexity: simple problems and complex problems. Students would need to adapt the solution procedure of simple problems in order to solve complex problems (Appendix A). We posit that the complexity of find-whole percentage problems, determined by the level of element interactivity, would be lower for students who have a high level of prior knowledge.

Our focus of inquiry also examines optimal best, reflecting a student's state of motivation and how this consideration could relate to the varying efficiencies of four instructional approaches. We speculate, for example, that the four approaches would differ on the achievement of optimal best but not necessarily actual best, irrespective of students' prior knowledge levels. We formulate the following hypotheses for statistical testing:

1. Considering differential efficiency of the four instructional approaches: (i) they would differ on practice problems for low rather than high prior knowledge students, (ii) they would not differ on the simple problems regardless of students' levels of prior knowledge in the post-test, and (iii) they would differ for the complex problems regardless of students' levels of prior knowledge in the post-test.
2. Performance on the practice problems, simple problems and complex problems in the post-test would favor high prior knowledge students across the four instructional approaches.
3. Low prior knowledge students, and not high prior knowledge students, would attest to differential mental effort and, similarly, they would invest higher mental effort than high prior knowledge students across the four instructional approaches.
4. Regardless of students' levels of prior knowledge, the four instructional approaches would differ on the Optimal Best (L_2) but not on the Actual Best (L_1) [Corrections added on 17 August 2023, after first online publication: '(L_1)' and '(L_2)' have been changed to '(L_2)' and '(L_1)', respectively, in the preceding sentence.]. Moreover, drawing from previous research evidence, we speculate that mental effort would not associate with the Optimal Best.

9 | METHOD

9.1 | Participants

We invited 218 Chinese students (boys = 47%) who consented to participate in the study from two private secondary schools located in a city in Asia. The students whose mean age was 15.00 (SD = 0.18) were drawn from three classes of each school. The ethnic composition of one school: Hokkian = 18.21%, Foochow = 43.33% and Hakka = 38.46%. In another school, the ethnic composition: Foochow = 56.07% and Hakka = 43.93%. In both schools, English Language was the medium of instruction for Mathematics and Science subjects. Students followed the National Curriculum for Secondary School Mathematics. According to mathematics teachers, students had learned the percentage problems a year ago prior to data collection.

9.2 | Materials

The materials consist of: (1) a pre-test that shares similar content as the post-test, (2) an instruction sheet, (3) acquisition problems plus a Likert scale to rate cognitive load, and (4) Optimal Outcomes Questionnaire. The pre-test or post-test has 10 simple problems and two complex problems (Appendix A). The simple problems share similar problem structure as the practice problems (Reed, 1987). The first complex problem has two parts and the second complex problems has three parts. The learner needed to adapt the solution procedure of the simple problems in order to solve the complex problems.

One mark was assigned for a correct solution for each simple problem, resulting in a total of 10 marks. Each part of a complex problem was awarded one mark. One of the complex problems had two parts and the other complex problem had three parts, resulting in a total of five marks. The solution was scored correct with or without including the solution steps. We ignored the computational errors. However, no mark would be awarded if students did not show correct procedure. For example, to calculate the unit percentage for question 2 (Appendix A), a student wrote 12 ÷ 3 (incorrect step), and thus no mark was assigned for such calculation.

The instruction sheet pertaining to a particular approach presents the definition of percentage, which is common across the four approaches, a review of prior knowledge (i.e., unit concept, proportional reasoning, and equation solving skills), and a worked example (Table 1). In line with prior research (van Gog et al., 2011), we implemented multiple worked example – practice problem pairs during the intervention. The acquisition problems for each approach comprises six worked example – practice problem pairs. Each pair consists of a worked example and a practice problem that shares a similar problem structure. Thus, students were required to study the solution procedure of a worked example, and then transferred their understanding to solve a practice problem.

In previous worked example research, the length of acquisition phase varies across different domains (Carroll, 1994; Chen et al., 2015; Ngu, Phan, et al., 2018, Ngu, Yeung, et al., 2018). For example, Carroll (1994) and Ngu, Phan, et al., (2018), Ngu, Yeung, et al., (2018) implemented worked examples intervention in 20 min. Likewise, we postulate that 20 min acquisition phase (i.e., study an instruction sheet + complete six acquisition problems) would be sufficient for students to learn how to solve find-whole percentage problems. We anticipate that schema acquisition would predominately occur when students completed the acquisition problems; nonetheless, they may also benefit from studying the instruction sheet. Overall, students in each approach would have been exposed to relevant prior knowledge pertaining to a specific approach (e.g., the unit concept for the unitary approach) and one worked example in the instruction sheet plus 12 acquisition problems. Students were required to indicate their mental effort on a 9-point Likert scale once they had completed the acquisition problems.

The Optimal Outcomes Questionnaire consists of two subscales: (1) Actual Best, and (2) Optimal Best. There are 12 items for each subscale ranging from 1 (always false) to 5 (always true). Of the 12 items, 8 items are related to personal attributes associated with actual – optimal bests dichotomy, and 4 items are related to the impact of the instructional approach upon learning. In regard to Actual Best, sample of items include: (1) *I am content with what I have accomplished so far for the topic of percentage problems* and (2) *The practice exercise is not effective in helping me learn percentage problems*. For the Optimal Best, sample of items include: (1) *I can achieve much more for the topic of percentage problems than I have indicated through my work so far*, and (2) *The practice exercise is very effective in helping me learn percentage problems*. The reliability estimates for the Actual Best and Optimal Best that comprised 16 items (excluded 4 items that are related to the instructional approach) were .81 and .79, respectively (Phan et al. 2018).

9.3 | Procedure

Two researchers and three mathematics teachers from each school administered group testing. In one school, 105 students from three classes assembled in a seminar room. Random assignment of students resulted in: unitary approach ($n = 27$), unitary-pictorial approach ($n = 27$), equation approach ($n = 26$) and equation-pictorial approach ($n = 25$). In another school, 113 students from three classes assembled in a school hall. Random assignment of students resulted in: unitary approach ($n = 28$), unitary-pictorial approach ($n = 28$), equation approach ($n = 28$) and equation-pictorial approach ($n = 29$). We eliminated five students who did not complete all the test materials in the final data analysis.

The students from the two schools undertook the same experimental procedure. We provided a briefing to students: (1) a pre-test (10 min), (2) a learning phase that comprised an instruction sheet (5 min), acquisition problems and a Likert scale of mental effort (15 min), (3) a post-test (10 min), and (4) Optimal Outcomes Questionnaire (10 min). We advised the students: to read the instruction on the first page of each task before they begun, not to discuss with their classmates while completing each task, and to seek help during the learning phase, if needed. We distributed each task and collected it after the allocated time had elapsed with the exception of the instruction sheet – it was collected after students had completed the learning phase.

First, all students sat for a pre-test. Second, they studied their respective instruction sheets, completed acquisition problems and indicated their mental effort on a Likert scale. Third, students completed a post-test. Lastly, they completed Optimal Outcomes Questionnaire. In sum, the four groups were matched with the same materials and time to complete a pre-test, an acquisition phase, a post-test, and an Optimal Outcomes Questionnaire. The main difference between the four groups (unitary, unitary-pictorial, equation, equation-pictorial) was the design of the worked examples.

10 | RESULTS AND DISCUSSION

The Cronbach's alpha value for the pre-test was .91. There were six practice problems and most students either did not attempt or made mistake for the 6th practice problem. Thus, after deleting the 6th practice problem, the Cronbach's alpha value for the practice equations was .69. For the post-test, the Cronbach's alpha values for the simple problems and complex problems were .83 and .87, respectively.

Researchers have used differential scores on the pre-test (Blayney et al., 2016), and students who studied in different year levels (Year 8 vs. Year 9) (Bokosmaty et al., 2015) to differentiate their prior knowledge. In the present study, one-way ANOVA indicated nonsignificant difference between four approaches on the pre-test, $F(3, 213) = 0.85$, $p = .46$, partial $\eta^2 = 0.01$. We used the mean scores of the pre-test as a point of reference to allocate students to low prior knowledge group (pre-test < .25, $n = 85$), and high prior knowledge group (pre-test > .25, $n = 128$).

The inter-scorer agreement was above .90 for the pre-test, practice problems, and post-test. The means and standard deviations for the practice problems, simple problems and complex problems in the post-test, mental effort and Actual Best and Optimal Best are presented in Table 2. We used ANOVA fixed effects, main effects and interactions in G*Power analysis to estimate the minimum sample size (Faul et al., 2009). The sample size (i.e., 213 students) exceeds the minimum requirement of 179 participants, based on a priori power calculation for an effect size, $f = 0.25$ for power = 80% and Type I error rate = 5%. We acknowledge that the effect sizes for the results were relatively small. As will be discussed later, for example, the effect size for the main effect on prior knowledge for the complex problems was 0.16.

We used 2 (prior knowledge: low vs. high) × 4 (instructional approach: unitary vs. unitary-pictorial vs. equation vs. equation-pictorial) ANOVA to analyse performance outcomes (i.e., practice problems, simple problems and complex problems in the post-test, Actual Best and Optimal Best) and the rating on the mental effort. A follow-up pairwise comparisons with Bonferroni correction were used to examine between-group differences on performance outcomes and the rating on the mental effort. We used correlational analysis to determine the relationship between mental effort and the Optimal Best.

10.1 | Practice problems

Significant differences were found for the main effect on approach, $F(3, 205) = 5.02$, $p < .001$, partial $\eta^2 = 0.07$, the main effect on prior knowledge, $F(1, 205) = 22.87$, $p < .001$, partial $\eta^2 = 0.10$, and the approach × prior knowledge interaction effect, $F(3, 205) = 2.76$, $p = .043$, partial $\eta^2 = 0.04$. As indicated in Figure 1, performance on practice problems favored high prior knowledge students especially for the unitary approach and unitary-pictorial approaches.

In support of hypothesis 1 (i), the simple main effects test revealed a significant difference among the four approaches for low prior knowledge students, $F(3, 205) = 5.94$, $p < .001$, partial $\eta^2 = 0.08$, but not for high prior knowledge students, $F(3, 205) = 0.51$, $p = .68$, partial $\eta^2 = 0.01$. For low prior knowledge students, both the equation approach ($p < .001$) and equation-pictorial approach ($p = .01$) were significantly better than the unitary-pictorial approach, and no difference was found between other pairwise comparisons.

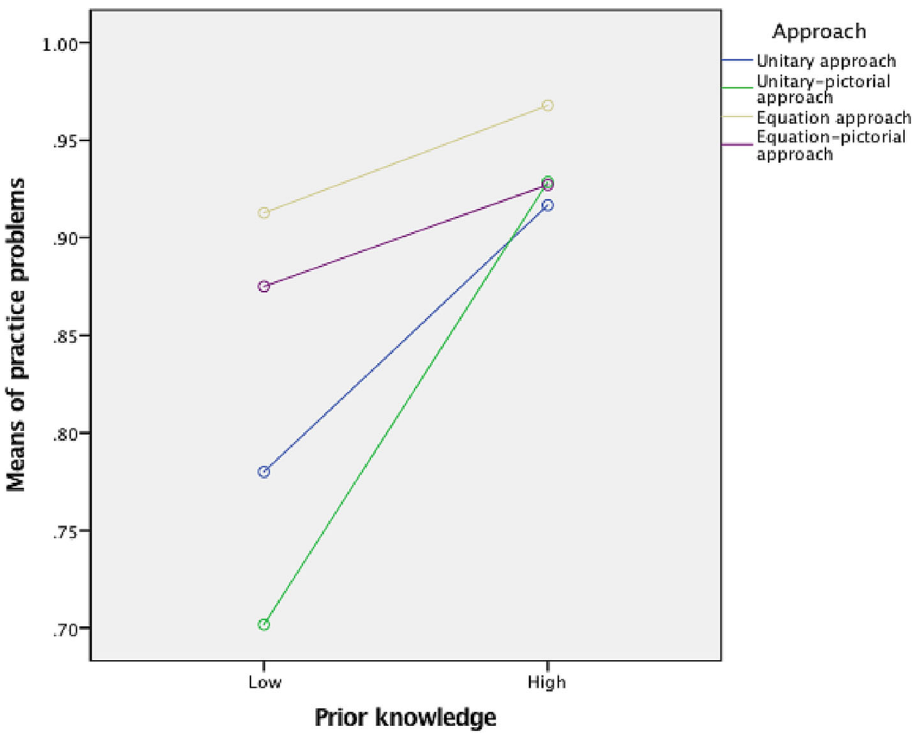
We examined the main effect of prior knowledge (low vs. high) on the four approaches. Significance simple main effects were found between two levels (low vs. high) of students' prior knowledge for only the unitary approach, $F(1, 205) = 8.30$, $p < .001$, partial $\eta^2 = 0.04$, and the unitary-pictorial approach, $F(1, 205) = 20.65$, $p < .001$, partial $\eta^2 = 0.09$. Indeed, high prior knowledge students outperformed low prior knowledge students for the unitary approach ($p < .001$) and the unitary-pictorial approach ($p < .001$) only. Therefore, the results partially support hypothesis 2.

TABLE 2 Means and standard deviations of scores on practice problems, post-test, mental effort and actual best and optimal best.

	Unitary approach		Unitary-pictorial approach		Equation approach		Equation-pictorial approach	
	Low prior knowledge $n = 25$ M (SD)	High prior knowledge $n = 30$ M (SD)	Low prior knowledge $n = 19$ M (SD)	High prior knowledge $n = 35$ M (SD)	Low prior knowledge $n = 21$ M (SD)	High prior knowledge $n = 31$ M (SD)	Low prior knowledge $n = 20$ M (SD)	High prior knowledge $n = 32$ M (SD)
Practice problems (proportion)	0.78 (0.22)	0.92 (0.14)*	0.70 (0.30)	0.93 (0.15)*	0.91 (0.15)	0.97 (0.10)	0.88 (0.22)	0.93 (0.12)
Post-test (proportion)								
Simple problems	0.73 (0.29)	0.95 (0.17)*	0.66 (0.33)	0.91 (0.13)*	0.76 (0.32)	0.91 (0.21)*	0.87 (0.24)	0.95 (0.11)
Complex problems	0.08 (0.19)	0.31 (0.34)*	0.08 (0.22)	0.38 (0.32)*	0.13 (0.21)	0.38 (0.30)*	0.10 (0.14)	0.33 (0.34)*
Mental effort	5.04 (2.51)*	3.87 (2.53)	5.13 (0.96)*	3.79 (2.10)	5.14 (1.98)*	3.33 (1.83)	4.60 (1.76)	3.94 (1.48)
Actual Best	2.74 (0.28)	2.83 (0.51)	2.65 (0.47)	2.68 (0.35)	2.88 (0.32)	2.81 (0.48)	2.85 (0.24)	2.75 (0.41)
Optimal Best	3.62 (0.52)	3.69 (0.49)	3.75 (0.48)	3.64 (0.45)	3.55 (0.35)	3.48 (0.70)	3.60 (0.47)	3.46 (0.48)

Note: There were 6 practice problems. The post-test comprised 10 simple problems and 2 complex problems. The Actual Best and Optimal Best had 12 items each. *$p < .05$.

FIGURE 1 2 (prior knowledge) × 4 (approach) ANOVA on practice problems.



Differential performance favored high prior knowledge students for the unitary and unitary-pictorial approaches only. Taken together, the results support the prediction that the equation approach and equation-pictorial approach were more efficient than the unitary approach and unitary-pictorial approach for low prior knowledge students.

10.2 | Simple problems

The approach × prior knowledge interaction effect was not significant, $F(3, 205) = 1.36$, $p = .26$, partial $\eta^2 = 0.02$. A significant main effect on approach was observed, $F(3, 205) = 2.59$, $p = .054$, partial $\eta^2 = 0.04$. Also, a significant main effect on prior knowledge was found, $F(1, 205) = 31.47$, $p < .001$, partial $\eta^2 = 0.13$. As revealed in Figure 2, performance outcomes favored the high prior knowledge student particularly for the unitary, unitary-pictorial, and equation approaches.

For the main effect on approach, the equation-pictorial approach was significantly better than the unitary-pictorial approach for low prior knowledge students ($p = .03$). All other pairwise comparisons were not significant. Such results partially support hypothesis 1 (ii) because not all pairwise comparisons were significant. For the main effect on prior knowledge, performance on simple problems

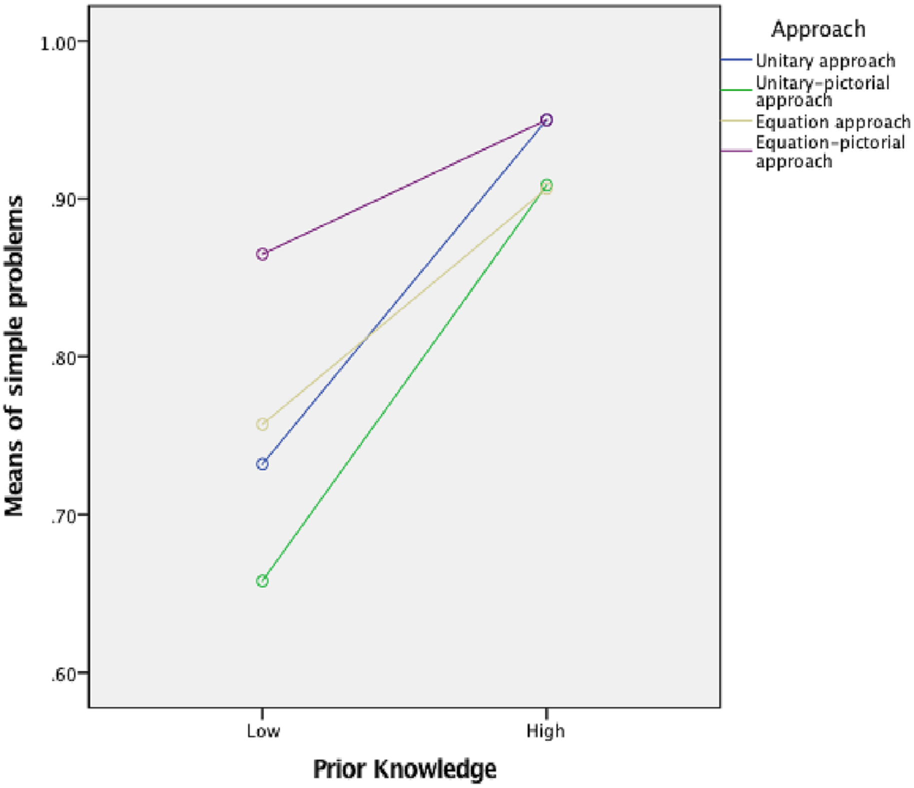


FIGURE 2 2 (prior knowledge) × 4 (approach) ANOVA on simple problems.

favored high prior knowledge students for the unitary approach ($p < .001$), unitary-pictorial approach ($p < .001$) and equation approach ($p = .02$), but not the equation-pictorial approach ($p = .18$). Once again, the results partially support hypothesis 2 – the high prior knowledge students were better than low prior knowledge students across three approaches instead of four approaches.

Overall, a similar pattern of results emerged for the simple problems. As hypothesized, the four approaches did not differ for high prior knowledge students, but the equation-pictorial approach was better than the unitary-pictorial approach for low prior knowledge students. Moreover, differential performance favored high prior knowledge students for the unitary approach, unitary-pictorial approach and equation approach but not the equation-pictorial approach. Hence, the results support the superiority of the equation-pictorial approach over other approaches for low prior knowledge students.

10.3 | Complex problems

A significant main effect on prior knowledge was observed, $F(1, 205) = 39.93$, $p < .001$, partial $\eta^2 = 0.16$. Indeed, as shown in Figure 3, performance on complex problems favored high prior knowledge students across the four approaches. Neither the main effect on approach, $F(3, 205) = 0.44$, $p = .73$, partial $\eta^2 = 0.01$, nor the approach × prior knowledge interaction effect, $F(3, 205) = 0.14$, $p = .93$, partial $\eta^2 = 0.00$ was significant. The nonsignificant main effect on approach contradicts hypothesis 1 (iii).

For the main effect on prior knowledge, high prior knowledge students outperformed low prior knowledge students on complex problems across all the four approaches: (1) unitary approach ($p < .001$), (2) unitary-pictorial approach ($p < .001$), (3) equation approach ($p < .001$), and (4) equation-pictorial approach ($p = .01$). Such results support hypothesis 2.

In sum, for complex problems, it was a surprise that the four approaches did not differ irrespective of students' levels of prior knowledge. Presumably, the low mean proportion scores (Table 2) for the complex problems revealed that these problems were too challenging for all students irrespective of their prior knowledge. The high prior knowledge students outperformed low prior knowledge students across the four approaches, supporting the hypothesis.

10.4 | Mental effort

A significant main effect on prior knowledge was found, $F(1, 205) = 18.96$, $p < .001$, partial $\eta^2 = 0.09$. Figure 4 shows that low prior knowledge student invested higher mental effort than high prior knowledge students across the four approaches. Neither the main effect on approach, $F(3, 205) = 0.17$, $p = .92$, partial $\eta^2 = 0.00$, nor the approach × prior knowledge interaction effect, $F(3, 205) = 0.69$, $p = .56$, partial $\eta^2 = 0.01$ was significant. The nonsignificant effect on approach supports hypothesis 3 for high prior knowledge students but not for low prior knowledge students. For the main effect on prior knowledge, low prior knowledge students invested significantly higher mental effort than high prior knowledge students for the unitary approach ($p = .03$), unitary-pictorial approach ($p = .03$), equation approach ($p < .001$), but not the equation-pictorial approach ($p = .25$). Such results support hypothesis 3 with the exception of the equation-pictorial approach.

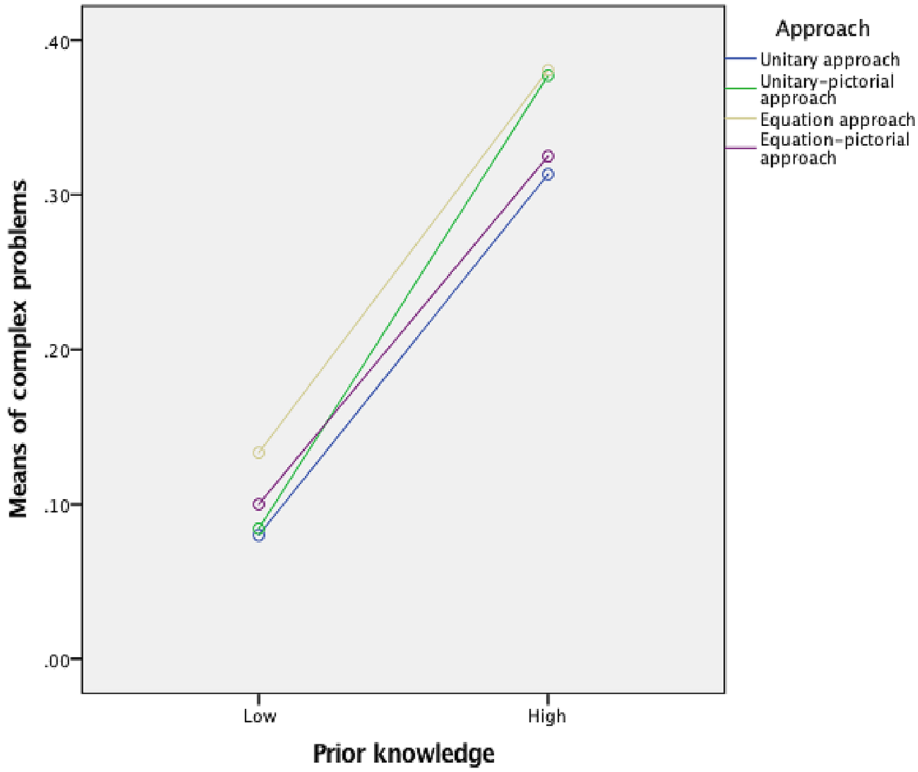


FIGURE 3 2 (prior knowledge) × 4 (approach) ANOVA on complex problems.

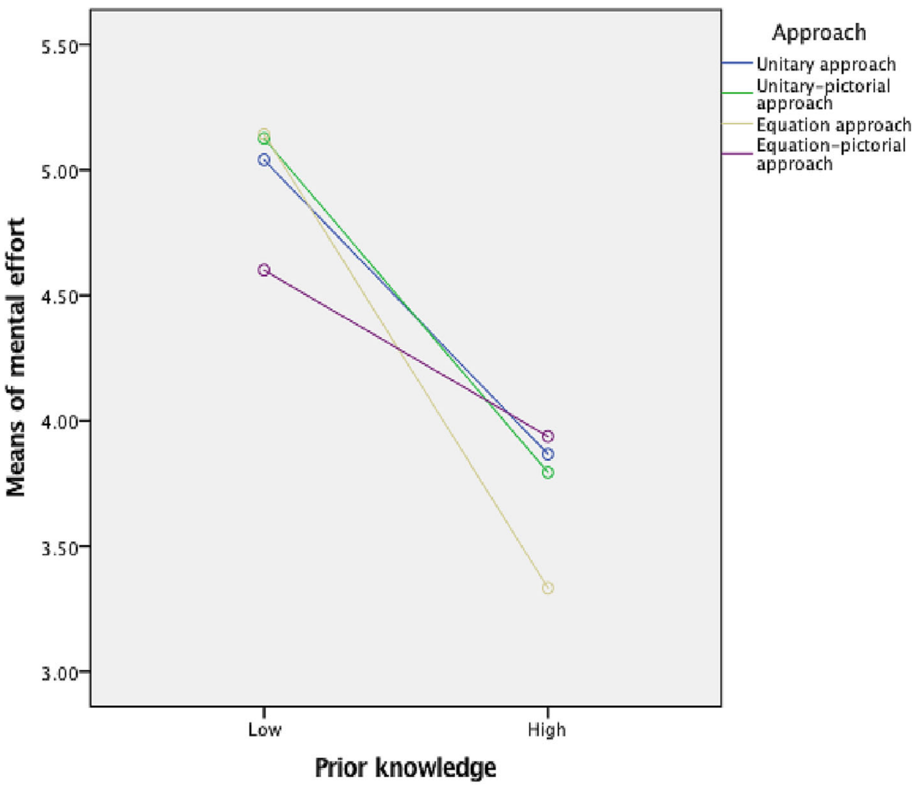


FIGURE 4 2 (prior knowledge) × 4 (approach) ANOVA on mental effort.

In sum, neither high prior knowledge students (support the hypothesis) nor low prior knowledge students (contrary to hypothesis) invested differential mental effort across the four instructional approaches. As hypothesized, low prior knowledge students invested higher mental effort than high prior knowledge students across all approaches except the equation-pictorial approach. Viewing the results together with the performance on practice problems and post-test, the equation-pictorial approach was the most efficient approach for low prior knowledge students.

10.5 | Actual best and optimal best

For the Actual Best, in support of the hypothesis 4, nonsignificant differences were found on the main effect on prior knowledge, $F(1, 205) = 0.04$, $p = .84$, partial $\eta^2 = 0.00$, the main effect on approach, $F(3, 205) = 1.77$, $p = .15$, partial $\eta^2 = 0.03$, and the approach × prior knowledge interaction effect, $F(3, 205) = 0.64$, $p = .59$, partial $\eta^2 = 0.01$. Contrary to hypothesis 4, for the Optimal Best, nonsignificant differences were found for the main effect on prior knowledge, $F(1, 205) = 0.80$, $p = .37$, partial $\eta^2 = 0.00$, the main effect on approach, $F(3, 205) = 1.52$, $p = .21$, partial $\eta^2 = 0.02$, and the approach × prior knowledge interaction effect, $F(3, 205) = 0.43$, $p = .74$, partial $\eta^2 = 0.01$.

It was a surprise that both low and high prior knowledge students had similar belief in achieving optimal best irrespective of the instructional approaches. As shown in Table 1, both low and high prior knowledge students across the four approaches struggled to solve the complex problems; consequently, this may affect their belief in the optimal best.

10.6 | Mental effort and optimal best

In support of hypothesis 4, the correlations between mental effort and the Optimal Best for low prior knowledge students were nonsignificant for the unitary approach, $r(25) = .18$, $p = .39$, unitary-pictorial approach, $r(19) = .02$, $p = .93$, equation approach, $r\ (21) = .26$, $p = .25$, and equation-pictorial approach, $r(20) = .05$, $p = .85$. Thus, the evidence suggested that cognitive load imposition, by proxy of a student's mental effort, was unrelated to belief in optimal achievement best.

Similarly, as hypothesized, for high prior knowledge students, nonsignificant correlations between mental effort and the Optimal Best were found for the unitary approach, $r(30) = .28$, $p = .14$, unitary-pictorial approach, $r(35) = .20$, $p = .27$, and equation-pictorial approach, $r(32) = .02$, $p = .94$. However, it was a surprise that the correlation between mental effort and the Optimal Best for the equation approach was positive, $r\ (31) = .60$, $p < .001$, suggesting that experienced of low mental effort may not contribute towards greater optimism to excel in mathematics learning and vice versa. Overall, the evidence implied that cognitive load imposed during learning via the proxy of a student's mental effort, was not associated to belief in optimal achievement best.

In sum, irrespective of students' levels of prior knowledge, differential efficiency of the four instructional approaches did not influence their belief in actual-optimal bests dichotomy. Such results support the hypothesis for the belief in actual best but not optimal best. The results for the relation between mental effort and Optimal Best were as hypothesized with the exception of the equation approach for high prior knowledge students. Therefore, cognitive load imposition during learning was not associated with belief in achieving optimal best in mathematics.

11 | DISCUSSION

The objective of the present study was to investigate the relation between instructional efficiency, level of students' prior knowledge, and motivation (reflected by indication of actual – optimal bests dichotomy) on learning to solve find-whole percentage problems. We used the concept of element interactivity as a basis to distinguish the relative efficiency of the four instructional approaches: equation-pictorial > equation > unitary-pictorial > unitary. In line with the hypothesis, the equation approach and the equation-pictorial approach (i.e., algebra approaches) were more effective than the unitary-pictorial approach across the practice problems and simple problems for low prior knowledge students. As hypothesized, for high prior knowledge students, the four instructional approaches neither differed on the practice problems nor on the simple problems. Contrary to the hypothesis, irrespective of the levels of student prior knowledge, the four instructional approaches did not differ on the complex problems. In support of hypothesis, performance outcomes favored high prior knowledge students for the practice problems (i.e., the unitary and unitary-pictorial approaches), simple problems (i.e., the unitary, unitary-pictorial and equation approaches), and complex problems (i.e., all four instructional approaches). Substantiating our hypothesis, the results show that low prior knowledge students invested higher mental effort than high prior knowledge students across three of the four instructional approaches (i.e., the unitary, unitary-pictorial, equation), but not the equation-pictorial approach.

Taken the results together, we may conclude that the equation-pictorial approach, given its low level of element interactivity (i.e., the lowest in terms of comparison) and thus low cognitive load imposition, is the most efficient approach for low prior knowledge students. We also note that differential performance outcomes (i.e., especially the complex problems) tend to favor high prior knowledge students across the four instructional approaches, in line with our hypothesis. Such finding points to the importance of prior knowledge upon learning. The high prior knowledge students could process the complex problems with fewer elements and thus impose lower cognitive load then low prior knowledge students irrespective of the instructional approaches. This line of evidence is valuable, adding theoretical understanding into the comparative nature and effectiveness of different instructional practices (Ngu & Phan, 2022; Ngu, Phan, et al., 2018; Ngu, Yeung, et al., 2018).

Irrespective of students' prior knowledge levels, the four instructional approaches did not differ with reference to L_1 and/or L_2. We find this 'absence' in statistical significance for L_2, in particular, somewhat unexpected. We speculated that a student's perception of difficulty and, hence, challenge of solving complex problems (Table 1) would influence his/her indication of optimal best belief or lack thereof. For example, a perception of ease would associate with a high level of optimal best (i.e., a student would, in this case, attest to a high level of L_2) whereas, in contrast, a perception of difficulty would associate with a low level of optimal best. The fact that we did not find evidence to substantiate our claim makes this inquiry more intriguing, providing grounding for future research development. For example, to reduce a perception of the difficulty for the complex problems, future research could narrow the gap between the complex problems and simple problems in terms of complexity. Importantly, consistent with prior studies (Feldon et al., 2018; Likourezos & Kalyuga, 2017), a perceived level of cognitive load imposition, requiring an exertion of mental effort negatively impacted on students' belief in achieving optimal best for mathematics learning.

The present study has expanded on previous studies (Ngu & Phan, 2022; Ngu, Phan, et al., 2018; Ngu, Yeung, et al., 2018; Pollock et al., 2002), in which we used the level of element interactivity as a point of reference to identify relevant sources of cognitive load associated with the four approaches (unitary, unitary-pictorial, equation, equation-pictorial). Focusing on the concept of expertise reversal effect (Kalyuga et al., 2003; Kalyuga & Renkl, 2010), the relative efficiency of the four approaches was a function of learning outcomes for low prior knowledge students, but not for high prior knowledge students.

The study of optimal best has recently gained research traction (Appleton, 2021; Phan et al., 2016; Tikoft, 2021), resulting in the development of different types of conceptualizations and empirical inquiries. Optimal best, denoted as 'L_2', and its corresponding level of best practice, actual best, denoted as 'L_1', both serve to indicate a person's state of motivation, aspiration, and self-belief (Phan & Ngu, 2021). Our inquiry as reported here, regardless of its limited evidence, has provided new theoretical insights into the operational nature of optimal best belief and its association with perceived cognitive load imposition. This recognition, indeed, is significant as it supports and coincides with recent emphases, which detail the importance of association between cognitive and non-cognitive processes of learning (Feldon et al., 2019; Phan et al., 2017).

11.1 | Practical implications for consideration

Our focus of inquiry sought to clarify the efficiency and inefficiency of contrasting instructional designs for the purpose of in-class implementation. The equation-pictorial approach, reflecting the intricacy of the algebra approach is most effective for the learning of find-whole percentage problems especially for low prior knowledge students. On this basis, we encourage mathematics educators in Western countries (e.g., Australia) to consider the use of the algebra approach in their teaching. Complementary use of the algebra approach with other popular pedagogical approaches may assist to improve students' learning experiences (e.g., PISA).

Arising from our research investigation, we encourage educators to consider using different visual representations to scaffold mathematical concepts. The use of diagrams, for example, may scaffold and support students' understanding of the proportional concept by reducing the level of element interactivity imposed on working memory. In a similar vein, we encourage educators to consider students' motivational beliefs (e.g., a student's optimal best belief) and their learning needs when considering specific instructional approaches for usage. Drawing from previous conceptualization (Phan et al., 2017), we contend that an instructional approach (e.g., unitary approach) that poses a high level of cognitive load would likely serve to convolute less knowledgeable student understanding, potentially causing a state of demotivation and task-avoidance.

11.2 | Limitations and future directions

There is a need for 'diversity' or generalization – that is, the applicability of the four mentioned instructional approaches for different topics of percentage (e.g., 'find-percentage', 'find-part percentage'). This line of inquiry would provide clarity into the operational nature of a particular instructional approach – for example, does the effectiveness of the equation-pictorial approach apply to different topical contexts?

Another caveat involved our use of the pre-test score to categorize students into two groups: low versus high prior knowledge. This emphasis, upon reflection, does pose a limitation given that prior knowledge associated with a particular instructional approach may instill a student's understanding and assist his/her performance outcome. Therefore, it would be of interest to explore the potential impact of a student's prior knowledge of equation solving skills on his/her understanding and appreciation for the equation approach or the equation-pictorial approach.

We acknowledge that there are other ways to design an experimental study that could enable researchers to investigate the relationship between students' prior knowledge and instructional approaches for learning (i.e., unitary, unitary-pictorial, equation, equation-pictorial). For example, we could consider a factorial design, such as 2 (procedure: equation vs. unitary) $\times$ 2 (pictorial: unitary-pictorial vs. equation-pictorial) $\times$ 2 prior knowledge (experts vs. novices) in future enquiry.

We acknowledge that the relationship between personal beliefs of best practice (Fraillon, 2004; Phan et al., 2016, Phan et al., 2017) and varying efficiencies of contrasting instructional approaches is inconclusive. A 'one-off' cross-sectional design is somewhat limited as it does not offer any meaningful insight into growth patterns (Bollen & Curran, 2006) of progression of skills, and students' beliefs of their best practice (e.g., an increase in belief of optimal best from L_1 to L_2). Aside from considering a longitudinal research design, we could re-design the complex problems in order to reduce their complexity relative to the simple problems. This consideration could assist more students to acquire the skill to successfully solve the complex problems, which would result in differential optimal best beliefs across the four contrasting instructional approaches.

12 | CONCLUSION

The underlying premise of our research inquiry, as reported in this article, is grounded in the observation in which mathematics textbooks (e.g., Vincent et al., 2012), in general, only recommend one preferred way (i.e., the unitary approach) to learn find-whole percentage problems. This preference and recommendation for the use of one particular instructional approach is somewhat limited. Based on the nature of cognitive load theory (Sweller et al., 2011), contrasting instructional approaches and levels of expertise are important considerations to facilitate enriched learning experiences. One way to affirm this plausibility (i.e., the use of more than one instructional approach) is to use the concept of 'element interactivity', which may help to discern the relative effectiveness of contrasting instructional approaches for students with varying levels of expertise.

Overall, the present study has provided evidence, which we contend could help to elucidate theoretical understanding into the relationship between levels of learner expertise and instructional approach (Kalyuga et al., 2003). Furthermore, as one of the very first research studies, our study also highlighted the potential association between cognitive (e.g., cognitive load imposition) and non-cognitive (e.g., belief in optimal best) processes of learning (e.g., the negative impact of cognitive load imposition on a student's belief in optimal achievement best for future mathematic learning).

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CONFLICT OF INTEREST STATEMENT

The authors declare no conflicts of interest.

DATA AVAILABILITY STATEMENT

Research data are not shared.

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[Correction added on 17 August 2023, after first online publication: the Reference 'Ngu, Phan, et al., 2018' has been corrected.]

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APPENDIX A: Sample of post-test

A.1 | Simple problems

1. A tank contained 120 L of water, which was 5% of its total capacity. What is the total capacity?
2. There were 3 students absent from mathematics class on Monday, which is 12% of the total students in the class. What is total number of students in the class?

A.2 | Complex problems

1. Mary and Tom won the first price for an outstanding art work. Mary's share of the price is $150, which represents 20% of the total price.

   (a) What is the total price?
   (b) Tom's share of the price is 80% of the total price. How much does Tom get?

2. When Year 8 students at Jaya Secondary School were surveyed about how they travelled to school each day, it was found that 25% walk, 59% come by bus, and the remaining 25 students were driven to school.

   (a) If 39 students walk to school, how many Year 8 students are there at the Jaya Secondary School?
   (b) Calculate the number of students who come by bus.
   (c) What percentage of students were driven to school?