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**On the profile of solutions for an elliptic problem arising in nonlinear optics. (English summary)**

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Summary: "We study the profile of solutions of the problem

$$\begin{cases} -\Delta u + (\lambda - h(x))u = g(x)(u^{p-1} + f(u)) \text{ in } \mathbb{R}^N, \\ u > 0 \text{ in } \mathbb{R}^N,\ u \in H^1(\mathbb{R}^N), \end{cases}$$

where $\lambda > 0$ is a parameter, $h$ and $g$ are nonnegative functions in $L^\infty(\mathbb{R}^N)$. We obtain the asymptotic behaviour of the least energy solutions or solutions obtained by the minimax principle. From the asymptotic behaviour we conclude that those solutions are asymmetric for $\lambda$ large even if $h$ and $g$ are radially symmetric."

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*