

**LINGUISTIC POINTERS TO STUDENTS'  
UNDERSTANDING IN INTRODUCTORY ALGEBRA:  
A COGNITIVE APPROACH**

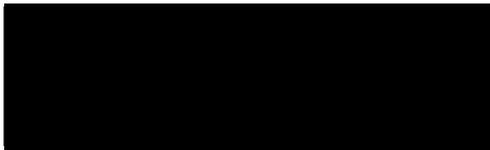
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I certify that the substance of this thesis has not already been submitted for any degree and is not currently being submitted for any other degree or qualification.

I certify that any help received in preparing this thesis, and all sources used, have been acknowledged in this thesis.



Judith L. Falle  
December 2008

*Language is the dress of thought.*

Samuel Johnson

*Sermo hominum mores et celat et indicat idem.*

Speech both conceals and reveals the thoughts of men.

Cato

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My thanks must go to the teachers and students who participated in the study by giving their time and acceptance of disruption to their normal routines. To the students an acknowledgement also of their willingness to talk about their thinking to a stranger. To them I owe many insights that I can only wish that I might have had as a classroom teacher.

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## ABSTRACT

Teachers use “extremely subtle pragmatic interpretive judgements [...] regularly in the course of mathematics teaching and learning...” (Pimm, 1987, p.167). The form of their discourse – the coherence, the structure and modality, characteristics of natural language in use – indicates the commitment of students to the truth-value of their statements. Hence, the listener might infer the extent of students’ confidence in their understanding.

In this study, linguistic features were identified that could be aligned with the conceptual growth of students in the context of introductory algebra. The aim was to devise a model that provided explicit, objective evidence to support the subtle, interpretive judgements made by teachers.

Secondary students in Years 8 and 9 (13-15 year olds) from three schools in a NSW regional centre ( $N=222$ ) participated in the study. The study consisted of two phases of data collection. The first was the collection of quantitative data from students’ responses to a survey (test) of 40 algebra items drawn from the algebra syllabus for the first four years of secondary schooling in NSW.

Survey data provided information about algebra concepts, and conceptual development demonstrated by the students, through Rasch modelling of the responses and an analysis of errors. The Rasch model indicated items and students clustered around significantly different estimates of, respectively, difficulty and ability<sup>1</sup>.

Clustering indicated groups of items requiring similar levels of conceptual development to be addressed successfully, and the corresponding groups of students who demonstrated this development. End-points of clusters indicated where conceptual change was necessary for further success on items, and the students who could achieve this.

In the second phase of the study, the collection of qualitative data, students ( $n=31$ ) were selected, on the basis of their survey results, for interview. Interviews were *contingent* – a pre-determined set of questions was followed, and these questions

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<sup>1</sup> *Ability* is the term used in the literature on Rasch measurement to describe student achievement on the test instrument.

supplemented by further questions where necessary. Interviews were audio-taped and transcribed for analysis. There were two purposes for the interview.

Data obtained from interviews were used to supplement the survey results. Accounts of their thinking provided insight into students' content knowledge and perceptions, as well as their particular approach to survey items – enabling characterisation of stages in conceptual growth.

Interview data were also analysed for ways in which students structured their explanations (i.e., linguistic characteristics). Results of the analysis were mapped to item-difficulty and student-ability clusters identified from the Rasch model. Where students' conceptual understanding was found to develop in complexity and depth, so too did their associated discourse increasingly adopt the register of the discipline. The resultant language-conceptual model demonstrates an association between algebraic understanding and linguistic features of student talk in a developmental hierarchy that uses the SOLO model as a theoretical framework.

The language-conceptual model resulting from this exploratory study focused on linguistic features that are important in a pragmatic analysis of language – pronoun use, response types, and modality indicators – but which constituted a relatively small part of interviews. Their low frequency of occurrence meant that statistical significance could not always be established. However, where patterns of change did emerge, these have been identified and discussed, recognising that further data from a larger sample would be needed.

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