CHAPTER 1

SETTING THE CONTEXT: MATHEMATICS IN WORK

> Mathematics is not an abstract aggregation of facts and procedures that a few clever people have managed to access; mathematics is something that people do; mathematics is a verb.
>
> (Grier, 1994, p. 316)

Doing mathematics, or using mathematical ideas and techniques, has been identified as a significant factor in effective participation in the workplace. This chapter provides a background to the research upon which the present study rests. It is divided into two sections. Initially, the context of "mathematics in work" is defined. Although on first reading such a phrase appears self-explanatory, a careful definition is required to situate the present study within the research field. The second section outlines briefly the relevant studies that have examined mathematics within vocational contexts, and presents a selection of the major findings.

Clarifying the Context

The term "mathematics in work" requires a qualifying definition. The apparent clarity of the words "mathematics" and "work" belies the debate over their meanings in the literature. This section clarifies the term as it is used in this thesis in order to focus the frame of reference.

Defining Work

Throughout history "work" has been central to human existence: with physical or mental exertion providing the basic needs of existence (i.e., food, clothing and shelter). The Encyclopaedia Britannica (2002) outlined the evolution of work from early Judeo-Christendom to the present day. Within this reference, initial conceptions of work defined it as "punishment sent by God ... to punish human beings for some deviation from the wishes or rules of the divine" (p. 916), later to be modified to be "to fulfil one's duty to God and thus achieve salvation" (p. 916). Such definitions largely related to work being the output of physical labour in the production of the basic goods necessary for human existence. This energy expenditure was predominantly rewarded through the barter of goods or monetarily at various levels of the work hierarchy.

The onset of the industrial age, and subsequent post-industrial and technological periods, greatly reduced the need for mass participation in hard physical labour. The definition of work and the workplace therefore changed during the last century, with automation and machines expending much of the effort previously achieved by human toil. As in earlier times, the action of working to produce goods was, and still is, rewarded at some level by payment for services or goods provided; thus, one can be said to be "in employment". As such, a definition of work can be easily extrapolated as the "the activities by which one earns one's livelihood" (Funk and Wagnalls, 1984, p. 790), or as a person's "occupation or business" (Knight & Nestor, 2000, p. 17).

Within the academic community, there is a great deal of debate regarding the definition of "work" for the purposes of fine-tuning the direction of collective academic research. The term "Societal Work" (Inglis & Lindberg, 2004) was proposed to this effect. Societal work is work that includes all activities necessary for the continuation of society at the present level and to the development of that society according to changing need structures (Illeris, 1994, inspired by O'Negt). Consequently, under such a definition of "work", paid work like in a commercial kitchen, voluntary work like in a soup kitchen for the homeless, and everyday work such as making meals for the family at home could exist. However, the broad definition of "work" as societal work in the context of this thesis is not a workable construct: discussions would become either vast, or scant and superficial in order to cover the field. Therefore, the construct of "paid" work is adopted for the present study.

The decision taken to use such a definition is not without implications with regards to research themes. In the current workplace climate, the drive to make profit from a business places additional constraints and requirements on the worker. The need for a worker to balance efficient production with quality of output, depending on the requirements of the workplace or industry, shapes the nature of the action of that work.

Defining Mathematics

As with the concept of “work” within the context of this thesis, the term “mathematics” also requires explanation. There are two main reasons for a qualification of the word, namely: (i) the existence of differing definitions of “numeracy” within the literature, and (ii) different perceptions of “mathematics”.

“Mathematics” or “Numeracy”

Much of the literature associated with mathematics in the workplace, or indeed adults’ mathematics, uses both the terms “mathematics” and “numeracy” when reporting on observations of tasks that are of a mathematical nature. In some cases no delineation is apparent, with the terms being used interchangeably within the same report.

Baker and Street (1996) provided a detailed account of the historical development of the term “numeracy”: Initially defined as a broad appreciation of statistical concepts and scientific reasoning in the late 1950s, later definitions were that numeracy was a component of literacy where “attainment in reading, writing and mathematics” (p. 82) made it possible for a person to contribute towards the development of their community and to participate actively in the life of the community. The term “numeracy” then evolved, focusing on basic arithmetic calculations; this prompted Cockcroft (1982, p. 11) to argue that the view of numeracy should remain broad and contain two main facets. These are that a numerate person should be:

1. Comfortable with numbers, and to be able to use mathematical skills involved with the “practical mathematical demands of everyday life”; and,

2. Able to understand information which is presented in terms of mathematical structures, such as graphs and tables.

Since the Cockcroft Report, a definition for the term "numeracy" has continued to evolve, as evidenced by Baker and Street (1996, p. 82):

> The dichotomy between the view of numeracy as limited to arithmetic and calculations and broader visions of it as mathematics and ways of thinking mathematically ... exists to this day. A broader vision of numeracy has emerged ... [and] sees mathematics as more than a body of knowledge of facts and skills, and concepts. It also includes ways of thinking mathematically.

These ideas have been explored in international forums such as Adults Learning Mathematics (ALM). One member of this forum, Kaye (2003, p. 198), commented: "the ambiguity of the current meaning of numeracy enables the complex ideas and concepts it represents to be continually redefined and extended. It enables questioning and inclusion, rather than enforcing acceptance and exclusion."

The lack of either a clear definition of numeracy, or a definitive delineation between numeracy and mathematics, does not appear to be of consequence to the continuance of research into the field; it may not even be a desirable enterprise (Withnall, 1995). The ability "to use mathematics appropriately, confidently, meaningfully and effectively" (Coben, 2002) appears to be the over-riding qualifier in the discussion. Therefore, numeracy can be conceived as a subset of mathematics: the practical and functional application of mathematics, including thinking mathematically. Consequently, the focus shifts from a need to define numeracy, towards clarifying what we mean by "mathematics".

Mathematics is regularly regarded as a pure and universal scientific discipline. Benn's (1997, p. 159) view was:

> For many, including or indeed especially mathematicians, mathematics is quintessentially pure reason and beauty. It exists unchanged for all time. It is written in the stars. This ensures a mathematics that is exclusive, elitist and disconnected from everyday

> experiences. The consequent abstraction and alien discourse reduces mathematics to a gift from tutor to learner, a peek into the mind of God, which many see through a glass darkly, if at all.

The typical view that mathematics is transmitted from tutor to learner has developed from experiences within formal schooling, and has therefore shaped the perceptions of mathematics for the majority of people. International school mathematics curricula, particularly in western countries, generally prescribe content which entails number, algebra, space, statistics, and measurement. It is clear that despite a clinical definition provided by looking at mathematics as a scientific discipline, ideas as to what mathematics comprises is predominantly shaped by experiences and perceptions of the individual.

Perceptions of Mathematics

The following discussion explores the term "mathematics" within the context of this thesis by examining the perspectives of the worker and the teacher/researcher.

Frequently workers are not fully aware of the mathematics they are using. The reasons for this are tied to the fact that workplace mathematics is embedded in context (Noss, Hoyles, & Pozzi, 2000; Sträßer, 1996, 2000b). In the workplace, awareness of specific mathematical skills and procedures may not be important to the worker. From their point of view, there is little benefit in knowing where mathematics is interwoven within workplace problems. Workers are more likely to see occupational and professional concerns as being more important (Noss, Hoyles, & Pozzi, 2000), and hence afford more attention to those than to overt mathematics. Furthermore, workplace practice tends "not to distinguish mathematical knowledge from other knowledge helpful to cope with the professional problem" (Sträßer, 1996, p. 432).

If there are mathematical elements to the activity, the workers often do not recognise the mathematics they are using because it is hidden within artefacts or technology (Sträßer, 1996, 2003). For many workers, the most easily recognisable mathematics is that which is learnt in formal schooling, and yet "practitioners' epistemologies of quantity, space and time can differ fundamentally from those anticipated from a school mathematics orientation" (Noss, Hoyles, & Pozzi, 2000, p. 33). Noss et al. (2000, p. 32) labeled these epistemologies as situated abstractions, and defined them as the "practitioner's conceptions of the mathematics they use at work." These situated abstractions are generally limited to the four major mathematical operations of addition, subtraction, multiplication or division (Foyster, 1990; Milroy, 1992) or standard arithmetic algorithms such as working out a percentage of a quantity (Wedege, 2002). It is noteworthy to add that school-learnt mathematical procedures potentially impede workplace activities (Sträßer, 1996), or are abandoned completely (Billett, 1998; Milroy, 1992).

A researcher or teacher schooled in higher mathematical thinking has a perception of what constitutes mathematics that is often very different to that of the worker. As well as looking at mathematics purely as an easily defined set of topics and/or procedures, mathematics can be seen as a way of thinking and of solving problems logically: working mathematically as opposed to simply performing mathematical procedures. Such a perception may also allow a teacher or researcher to find mathematics in all sorts of activity.

Lindenskov (2003) discussed conceptions of mathematics from the perspective of tutors in adult basic education. Whilst these conceptions were derived from the experiences of the mathematics tutors of adults, it is easy to extrapolate them outwards to other educators and researchers. These conceptions can be summarised as:

1. Mathematics is a separate discourse and is separate from life and work;
2. Mathematics is only found in academic and technical work;
3. Mathematics is "everywhere" and can be found in all life and work episodes; and,
4. Mathematics-containing competencies are used intentionally to live and work and are purposeful.

Such epistemological positions of a teacher or researcher regarding a conception of mathematics can potentially be in conflict with the worker's perception of mathematics involved in the task at hand. Sträßer (1996, p. 439) recognised that the world of work contained many instances of mathematics, but that "vocational mathematics ... is interested in solving workplace problems, not disciplinary mathematical problems".

Milroy (1992) and Sträßer (1992b) recognised that one of the difficulties in identifying mathematics in the workplace, was that researchers restricted their definition of mathematics, by excluding non-academic/scientific uses. As such, Milroy (1992, pp. 50-51) stated:

> Rather than dismissing mathematical practices and strategies that develop out of daily activities as lacking in authenticity and rigour, mathematics educators need to study such practices, acknowledging their strengths and seeing their weaknesses as opportunities to negotiate broader understandings in the classroom ... and may begin a process whereby mathematics could be seen as an active experience, accessible to all people.

Definitions used

To summarise, "work" is defined in this thesis as the activities carried out to earn money or goods to live. The definition of "mathematics" is somewhat more problematic. In order to resolve this issue, mathematics is divided into two domains: (i) mathematics content as comprising the areas of number, space, algebra, statistics, and measurement most commonly found in western school curricula of, and (ii) thinking mathematically.

Studies of Mathematics in the Workplace.

Traditionally, studies that inform mathematics education have focused on mathematics as a "scientific discipline, as a social phenomenon, and as a school subject" (Wedege, 2003, p. 38). It is only recently that mathematics, as it is used by people in the course of their everyday or working lives, has become the subject of intense research focus. This section outlines the emergence of this branch of mathematics education research and provides a literature review of previous studies involving mathematics in work.

Based on the definition given in the previous section, a review of the literature for mathematics in work is now provided. This section is presented in two parts. Firstly, a summary of studies into this field is presented, and the section culminates in an outline of the general findings emanating from this research.

Studies into mathematics in and for work

Early accounts of research used simplistic approaches (Foyster, 1990) into what mathematics people used at work. These investigations were largely characterised by the researchers' underlying assumption that the mathematics was "unproblematically visible in workplace settings and that it consisted mainly of calculation" (Hoyles, Noss, & Pozzi, 2001, p. 4).

In the United Kingdom, the Cockcroft Report (1982) was seminal in providing a firm foundation for further investigation of aspects of mathematics in work. The report provided a comprehensive overview of the general state of mathematics teaching and learning in a variety of settings, such as adult life, employment, and school. Examination of workplace mathematics covered a wide range of categories of employment including the construction industry, hotels and catering, work with computers, and nursing. Cockcroft concluded that the range of mathematics required included calculation, fractions, algebra, and estimation.

The bulk of research into mathematics in work has taken place since the investigations undertaken for the Cockcroft Report. It is not surprising that initial studies focused predominantly on identifying the mathematical skills and knowledge that were required and used in certain workplace contexts. Subsequent research promptly evolved to include how this mathematics was being used. Examples of some of these studies include the mathematics of: dairy workers (Schribner, 1984); bookmaking (Schliemann & Acioly, 1989); carpet laying (Masingla, 1992); technical drawing (Sträßer & Bromme, 1992); floor plans (Clarke & Helme, 1993); fishing (Nunes, Schliemann, & Carraher, 1993); banking (Noss & Hoyles, 1996); pool builders (Zevenbergen, 1997); building (Baker, 1998); farming (de Abreu, 1998); pediatric and general nursing (Kvalø, 2003; Noss, Hoyles, & Pozzi, 2000); automobile manufacturing (Smith, 1999); building site plans (Bessot, 2000); jewelry making (Hahn, 2000); working on a shop floor (van der Zwart, 2000); pharmaceutical manufacturing (FitzSimons, 2000), bricklaying (Fioriti & Gorgorio, 2001); carpentry (Milroy, 1992); and painting (de Aguero, 2003). This is not an exhaustive list: As the breadth of vocations available to workers is extremely varied, research directions have mirrored this range.

Outcomes of previous research

There have been a variety of objectives underlying previous research into mathematics in work. The desire to improve the performance outcomes of workers on tasks that involve some element of mathematics can be seen as the intent behind much of this research. Consequently, it has been important to find out what mathematics is being used, and how it is being used. Research foci of studies have been diverse, but include:

1. Studies to inform the school-curricula in order to make the learning experiences more purposeful from the perspective of the students (e.g., Smith, 1999);
2. To examine the differences between how mathematics is learnt in school and how it is used in the workplace (e.g., Masingla, 1992; Nunes, Schliemann, & Carraher, 1993);
3. To improve mathematics education for adult learners (e.g., Fioriti & Gorgorio, 2001); and,
4. To create meaningful learning activities to inform training packages comprising competency-based outcomes, some of which included mathematics (e.g., FitzSimons, 2000).

Significant results have emerged from such studies, either directly from initial research questions, or as tangential and unexpected findings. In addition to the recognition that workers' often do not identify the mathematics they are using, there have been three other findings emanating from the field. These are:

1. Identification of the types of mathematics in use;
2. Differences between mathematics in school and mathematics for work; and
3. Affective factors of learning mathematics.

These three additional findings are outlined in more detail below.

Types of Mathematics in Use

Evidence for the types of mathematics in use in workplace contexts can be found in an expanding body of literature within the research field. General observations of mathematics in practice has been observed as being inclusive of, but not confined to measurement (Masingla, 1992), computational algorithms (Masingla, 1992), geometry (Bessot, 2000; Fioriti & Gorgorio, 2001; Masingla, 1992; Sträßer & Bromme, 1992), ratio and proportion (Masingla, 1992), average and variation (Noss, Hoyles, & Pozzi, 2000), and reading diagrams and plans (Bessot, 2000).

The context of the current study is within the building industry, where mathematical competency in required in order to work soundly in three dimensions (plumbing); calculate accurately in making stairwell and erecting frames and trusses (joinery); and reading site plans and performing accurate measurements (builders) (Cockroft, 1982). In a study into the training of young people in the building industry, Steedman and Hawkins (1994) found similar results.

In examining the building industry, Bessot (2000) looked at the geometrical knowledge required to operate effectively in this domain. Whilst confirming the necessity of measurement and spacial familiarity, it was also noted that a certain amount of elementary geometry was required for practically resolving problems involving space and shape. Of particular note is the following finding:

> … the geometrical properties which are of interest in 'mathematics' and those concerned with solving problems in space are not the same. The resolution of apparently similar problems is motivated by different concerns: the mathematician seeks truth whilst the practitioner looks for maximum efficiency. A builder may well have problems finding solutions to practical problems in a traditional geometry course.
>
> (Bessot, 2000, p. 143)

In Bessot's (2000) study focusing on geometry, she stated that knowledge is developed rather than applied, leading to the conclusion that "geometrical knowledge is working knowledge for developing solutions to problematical tasks" (p. 139). In order to develop such skills the worker requires basic knowledge of elementary geometry.

When faced with a difficult practical task requiring a solution, the building trade practitioner may resort to a "rule of thumb". A brief investigation into the mathematics of building by Baker (1998) went some way to bring this practice into view. He asked a builder whether the rectangular framework he had just constructed was "square". The builder stated that his diagonals were the same length, therefore the framework was a rectangle (i.e., angles 90°); justification of why this rule worked was not seen to be important by the builder. In this case, the mathematical theory underlying the performance was not a necessary component of the performance. More useful in this scenario is when to apply the technique, and that it works.

Substantial earlier research by Milroy (1992) examined the mathematics in use by carpenters. This ethnographic study was insightful, as the researcher became an apprentice to a master carpenter in a workshop, spending a significant number of months being trained in this craft. As such, she was able to gain rich insight into the mathematics used, as well as the acquisition of mathematical skills and knowledge involved.

Emanating from this study was the integral part played in the senses of touch and sight in the efficient execution of practical tasks. Master craftsmen advised Milroy to feel the edges of the wood to establish whether they were the same length. The sense of vision was also important to those in the trade in comparing lengths, or "to check whether a line was straight or a surface was horizontal" (Milroy, 1992, pp. 172-173).

As with the findings of Sträßer (1996), Milroy also experienced situations where school mathematics, or academic techniques, were impinging on the efficient solution to the problem. Milroy (1992, p. 177) provided the following narrative:

> Comparison turned out to be the most direct and accurate way of completing a task. Frequently, the assignment of a number would be an unnecessary step. For example, when squaring the aprons to support the tabletop, it was not necessary to know the lengths of the diagonals. It was enough to know whether they were the same or different. If they were different, the stick being used to compare the diagonals would be cut and used as a rigid bar to push the aprons into the correct 'square' position. Thus measuring to the nearest fraction of an inch or millimetre with the tape would simply be extra work, as well as less accurate.

The studies by Bessot, Baker and Milroy are not limited to merely categorising the types of mathematics evident within workplace practice, but also afforded attention to investigating how that mathematics is operationalised. How mathematical knowledge is actually used in practice is one of the major concerns of vocational training (Bessot, 2000).

The differences between mathematics in school and mathematics in and for work are perhaps the most visible and substantiated finding of the research. Indeed, it could be argued that this result requires little formal research, as these differences have been the topic of conversations in workplaces for years. Mentioned early in the literature by Cockcroft (1982), many researchers have come to the same conclusion (i.e., Bessot & Ridgeway, 2000; Carraher, Carraher, & Schliemann, 1985; Sträßer & Zevenbergen, 1996). Additionally, these studies have also initiated tangential investigations into the issue of "transfer" (Billett, 1998) or "transition" (Beach, 2000) of mathematical knowledge from one context to another.

It is not within the scope of this section to go into depth regarding reasons for a lack of transition of mathematical skills and understandings from either (i) school to the workplace or vis-versa, or (ii) within different workplaces. However, a common explanation is that mathematics in the workplace is highly contextual, whereas school mathematics is devoid of much of the real-life context that is evident in the world of work (Forman & Steen, 2000; Noss, Hoyles, & Pozzi, 2000; Sträßer & Zevenbergen, 1996).

Mathematics is interwoven into the many facets of workplace practice (Noss, Hoyles, & Pozzi, 2000). It rarely exists as a stand-alone entity involving calculations based on clinical and often de-contextualised problems. The workplace consists of a rich environment, where the situations under which a problem needs to be solved can regularly change, requiring different solution processes.

A strong mathematical foundation needs to be constructed in the workplace so that a competency in particular skills can be developed quickly (Sträßer & Thering, 1986). This is predominantly due to additional pressures of the workplace such as the speed and accuracy required in order to make money. "Mathematics for work has (in most cases) a non-mathematical context and purpose, and is related to employment and money" (Sträßer, 1992a, p. 244), and often mathematics may or may not be involved in the problem solving path (Sträßer, 1996).

Affective factors

Another aspect to mathematics in work is the investigation of what influences mathematics learning in the workplace. Wedege (2000) examined the theoretical underpinnings of the relation of mathematical knowledge to workplace skills and argued that the use of mathematics in the workplace is also subject to the influences of attitudes, feelings, experiences and motives. With regard to motives for using mathematics in the workplace, Zevenbergen (1997, p. 94) commented: "the mathematics which is most effectively employed by workers in that which is most amenable to their immediate needs." Affective factors can also include emotional aspects related to mathematics and the workplace. As the following quote illustrates, adults who have had poor experiences with mathematics often harbour negative feelings:

> Mathematics is not seen as something that people actually use, but as a best-forgotten (and often painful) requirement of school. For most members of the public, their lasting memories of school mathematics are unpleasant - since so often the last mathematics course they took convinced them to take no more.
>
> (National Research Council, 1989, p. 10)

There are other factors influencing how mathematics is learnt at work. Some of these factors are associated closely to the set-up of the work environment and the factors impacting on the learning situation. Influences here involve the role of mentors in the learning episode, social and cultural factors of the workplace, and the role of artefacts and tools.

Summary

The preceding discussion has provided a definition of mathematics in and for work as it is operationalised within the present study, as well as an overview of the development and current state of the field. Mathematics has been the object under investigation in many workplaces, and from this examination it is clear that the primary aim had been to first identify the mathematics in use, and secondly, to gain insight into how this mathematics is used. Such studies have greatly contributed to an understanding of the nature and complexities of mathematics in the workplace.

Of the studies that add additional information regarding the influences acting upon how mathematics is learnt, the vast majority of research has been based on understanding the context in which the mathematical meaning has been constructed, rather than on the cognitive process of that construction. The mechanisms by which mathematical understanding is acquired within workplace settings is important to inform teaching and learning processes. Cornford and Bevan (1999, p. 25) stated: “to date, there has been little substantial conceptual analysis from a learning perspective of the problems and issues surrounding workplace learning."

The fact that mathematics can be obscured within workplace practice and is often hidden from the perception of the worker, means that "special attention and force is to be invested to find, identify and describe workplace related mathematics" (Sträßer, 2000a, p. 241). The fact that the mathematics in use may or may not be visible, or important, *from the perspective of the worker* means that surveys and traditional interview methods may not provide authentic data regarding actual workplace thoughts and practices. Careful consideration of the research design and method is required. To this end, Sträßer (2000a, p. 245) maintained the "superiority of ethnographic studies and stimulated response type research".

Consequently, two main issues stand out from this review and require further elaboration through a deeper investigation of the literature. The first is that there is a lack of research into the cognitive processes of learning mathematics in work. The reasons for such a deficiency can in part be clarified, and further direction gained, by looking at theories of how mathematics is operationalised within out-of-school settings. The second is that uncovering mathematical understanding in workplace settings is a complex issue, and as such, further elaboration and investigations of methods are required to establish rich and authentic data.

CHAPTER 2

CONSTRUCTIVISM

> To learn mathematics means to *construct* mathematics. Mathematics is essentially a constructive process. The student learns mathematics not by absorbing concepts, definitions, theorems and proofs, but by constructing them through his or her own intellectual efforts. But individuals usually do not do these things by responding to their own problems and by resorting to their own natural, intellectual means. Our natural behaviour is adapted to the concrete reality in which we live and not to formal constructs governed by formal definitions and rules.
>
> (Fischbein, 1990, p. 7)

Mathematics as it occurs in everyday life, including a working life, is embedded in situations that can be dynamic and ever-changing. All human activity is a response to a situation within a context. The action of responding requires both the recall of knowledge of previous experiences and in the majority of cases, formulation of new knowledge based on those previous experiences to solve the problem at hand. How people learn to perform tasks, and solve problems that may involve some mathematical elements, requires active construction of that knowledge by the person. Essentially, such a perspective can be said to be "constructivist".

The purpose of this chapter is to explore the learning and understanding of mathematics in work from a constructivist perspective. To achieve this, the discussion is presented in three sections. Initially, a broad description of constructivism is provided, along with a summary of five common types of constructivism. The second section examines the work of two theorists, each approaching the topic from opposing ends of the constructivist continuum. The chapter concludes by reviewing some recent research into mathematics as it is used out of school, with a particular focus on the constructivist perspectives underlying these approaches.

General Definition

Constructivism is a type of cognitive theory. If *cognition* is defined as the "internal mental processes and states" (Aik & Tway, 2003, p. 274) involved in learning behaviours, by examining what mental processes are involved, then *constructivism* is the theory that explains how knowledge is accrued within this cognitivist perspective. Constructivism is often seen as the antithesis of the "transmission" or "absorption" model, where knowledge is viewed as a collection of facts (Baroody, 1987) that are conveyed as "sacred truths" (Burton, 1993, p. 8).

Constructivism is generally seen as having evolved from the teachings of Plato, Rosseau and Kant, who stated, respectively, that (i) learning is liberating what is already intrinsic to the learner, (ii) learning is a dynamic process of exploration, and (iii) new epistemological constructions are only possible given pre-existing mental schemas (Winch & Gingell, 1999). The major principle, to which all constructivist positions adhere, is that individuals devise their own meanings and knowledge from their own experiences (Fosnot, 1996; Steffe & Gale, 1995).

Airasian and Walsh (1997) outlined two assumptions fundamental to all constructivist theories of knowledge acquisition, namely (i) that constructivism describes the attainment, development and usage of cognitive processes by the individual, and (ii) that individuals construct knowledge from interactions between their "existing knowledge or beliefs and the new ideas or situations they encounter" (p. 445). Airasian and Walsh stated that constructivism is "an epistemology, a philosophical explanation about the nature of knowledge" (p. 444) and that it is "not an instructional approach" but a "theory about how learners come to know" (p. 445). Despite these general aspects of constructivism, it is acknowledged that the epistemology underlying constructivist thought diverges beyond this point (Arlidge, 2000; Larochelle & Bednarz, 1998).

Types of Constructivism

Many different categories of constructivism have emerged from the literature, some of which include: personal, developmental, radical, social, cultural, socio-cultural, critical and contextual. Such breadth can be explained by acknowledging that "one person's version is likely to differ from another person's version" (Good, 1993, p. 1015). However, despite the "panopoly of theoretical positions" (Cobb, Yackel, & Wood, 1992, p. 3), only a few categories of constructivism are regularly referred to in the literature on mathematics in the workplace. The six predominant types of constructivism are described briefly below, and comprise personal, developmental, radical, social, cultural and socio-cultural.

Personal constructivism (also referred to as *trivial constructivism* (Piaget, 1977; von Glaserfeld, 1990) is founded in the work of Kelly's Personal Construct Theory (Kelly, 1955), and the early writings of Piaget (1950). Personal constructivism is the simplest form of constructivism, and proponents contend that knowledge is actively constructed by the learner and not passively received from the environment. Personal constructivists believe that knowledge is an active internal process, built on prior experiences and conceptions (Piaget, 1977).

Developmental constructivism, also referred to as *cognitive constructivism* (Talja, Tuominen, & Savolainen, 2005), represents a slight diversion from personal constructivism, and is strongly influenced by Piaget. Developmental constructivists believe that (i) the individual is the constructor of their own knowledge, (ii) acquisition of this knowledge is guided by universal and static descriptors, and (iii) knowledge is built on prior conceptions (Inhelder & Piaget, 1964). Of importance to this perspective is the idea that construction is organised hierarchically (c.f. developmental cognition). However, influences on learning such as gender, culture and context are minimally acknowledged, but not afforded detailed attention.

Radical constructivism is built upon the principles of personal constructivism, agreeing that knowledge is actively constructed by the learner. However, in addition to this foundation, radical constructivism also concedes that there exists a relationship between knowledge and the environment (von Glaserfeld, 1990).

Radical constructivists emphasise that “subjective reality [is] informed by interactions with the social and physical environments” (Zevenbergen, 2000, p. 213). The basic principles of this constructivist perspective posit that (i) knowledge is not passively received either through the senses or by way of communication, but is actively built up by the cognising subject, and (ii) the function of cognition is adaptive and serves the subject’s organization of the experiential world, not the discovery of an objective ontological reality.

Social constructivism deals with how the human environment affects learning. It evolved largely out of the work of Vygotsky (1978) who focused on role that society plays in the development of the individual. The major tenet of social constructivism is that the construction of knowledge is based on an agreed upon, socially-constructed reality.

Social constructivists see influences on learning as coming from various stakeholders in all forms of activity, and takes into account the collaborative aspect of learning; Doolittle (1999, p. 4) however, asserted: “social constructivism is more concerned with meaning than structure.” Phillips (2000) added a historical dimension to this category of constructivism: Social constructivism is also seen to be bodies of public knowledge that have been built up throughout history. Knowledge is also essentially a human construct derived from constraints of “politics, ideologies, values, the exertion of power and the preservation of status, religious beliefs and economic self-interest” (p. 6).

The need for cultural constructivism grew out of a requirement to look beyond the social constructivist perspective, to include cultural influences on learning. White (1956) outlined four categories of culture that may form one possible filter through which to see cultural influences on learning. These include ideological (beliefs), sociological (customs), sentimental (attitudes) and technological (tools). Cultural influences may include a set of shared mathematical understandings (Bishop, 1988).

The final predominant category of constructivist thought prevalent in the literature relating to mathematics in work is that of socio-cultural constructivism. As indicated by the conjunction, this type of constructivism "emphasises the appropriation of knowledge through interpersonal interactions as well as through more distant social and cultural sources" (Billett, 1998, p. 4). Although Vygotsky is seen by many researchers to be the "father" of social constructivism, others (e.g., Cole, 1985) also attribute to him the genesis of socio-cultural constructivism.

Summary

The six descriptions outline some of the different perspectives within the constructivist paradigm that are prevalent in the literature. They can be seen to sit along a continuum formed between the two contrasting perspectives: psychological (theoretically oriented with a focus on the way individual learners construct knowledge) and anthropological (using "traditionally applied linguistic models, notable classical formal semantics, to classifactory paradigms of general cultural knowledge" (Lave, 1988, p. 9)).

Cole (1985, p. 147) listed the methodological and conceptual contrasts between the psychological and anthropological perspectives, which he saw as representing "a dualistic approach to mind and society". This list is reproduced in Table 1 below:

Table 2.1 Conceptual polarities between anthropology and psychology

Anthropology	Psychology
Culture	Cognition
Higher functions	Elementary functions
Products	Process
Content	Process
Group	Individual
Independent variable	Dependent variable
Observation	Experimentation
Field	Laboratory
Holistic	Analytic
Description	Explanation

Developmental constructivists are predominantly grounded in the psychological paradigm, whereas socio-cultural constructivists identify more with the anthropological perspective. Of the six categories of constructivism outlined previously, radical constructivism sits most closely towards the middle of this continuum. The main difference between these perspectives has to do with the locus of knowledge construction. The developmental constructivists' view is that knowledge is constructed in the head, while socio-cultural constructivists believe that knowledge is constructed in communities of practice through social interaction (Kuhn, 1996; Lave & Wenger, 1991; Vygotsky, 1978).

Fundamental Theories of Socio-cultural and Developmental Constructivism

The following section examines the work of Piaget and Vygotsky, from whom the developmental and socio-cultural constructivist perspectives have predominantly emerged. Such a discussion is important to providing a historical perspective and understanding of the underlying philosophies of both developmental and socio-cultural approaches to studying mathematics in and for the workplace.

Piaget

This sub-section outlines briefly the cognitive theory of Piaget. This section is divided into three parts. Firstly, a background to the development of Piaget's work is given. Secondly, the main constituents of the Piagetian theory are outlined. This discussion concludes with an outline of the strengths and weaknesses of its application for informing assessment of cognitive functioning.

Background

Cognitions are assumed to be mental structures, internally organised wholes, or systems of internal relations, which govern the processing of information or the connecting of events (Kohlberg, 1987). Much of the drive behind this area of educational psychology is the desire to measure knowledge, understanding and achievement by examining changes in cognitive structure.

In order to assess achievement more accurately, early work centred on individually administered and quantifiable achievement tests, with the first test published by Binet in 1905 (Case, 1985). Despite Binet's attempt to equate intelligence with cognitive development by assuming that "intelligence in general could be represented by an average mental age" (Kohlberg, 1987, p. 99), these tests failed to inform classroom instruction and could not identify qualitative differences within responses.

The notion of identifying these qualitative distinctions is popularly attributed to Jean Piaget. However, the Piagetian Model of Cognitive Development emerged out Piaget's examination of research carried out by American psychologist James Baldwin. Baldwin's development of *genetic epistemology* (Case, 1985) provided the first broad view of cognitive development phenomena: the study of how an individual gains knowledge of their world. The underlying tenet of this model was that intellectual functioning moved through a series of "epochs" or higher stages.

Whilst assessing answers to the Binet standardised test for intelligence, Piaget became more interested in incorrect, rather than correct answers: the qualitative analysis of these gave Piaget further insight into the developmental stages first described by Baldwin. Despite the vagueness of Baldwin's theory with regard to intellectual functioning in older children, Piaget determined that the types of mistakes made appeared to be partially dependent on the age of the subjects (Case, 1985). From this emerged his observation that the "thought of younger children is qualitatively different from older ones" (Ginsburg & Opper, 1979, p. 3).

During his investigations, Piaget preserved the majority of Baldwin's theoretical assumptions, such as the attitude of the child to reality (Kohlberg, 1987). However, additional focus was given to two areas that were only hinted at in the original theory, these are (i) the changes that occur in psychological functioning from infancy to adulthood, and (ii) the relationship between changes in psychological functioning and changes in how the child sees the world around them (Case, 1985, p. 13).

Main constituents of Piagetian theory

Piagetian theory comprises two main parts. These parts are stages of development, and how movement within these stages is operationalised. A brief outline of these two aspects are given here, beginning with the stages of intellectual development which Piaget called *sensorimotor*, *operational* (*pre- and concrete-*) and *formal* (Kaplan, 1991).

Sensorimotor Stage. The stage of sensorimotor intelligence covers fully the first two Baldwin epochs with partial intrusion into the epoch of thought; it also expands upon whose underlying assumptions by looking more closely at the levels of the child's understanding. Piaget identified an additional sub-stage (previously not identified by Baldwin), which existed from birth to one month, and consisted mainly of grasping and sucking. For Piaget, the complete sensorimotor stage consists of six successional sub-stages, which exist from birth to 2 years of age and lead to the development of physical motor skills, such as walking and throwing. This sensorimotor stage "terminates in various kinds of behaviour implying the presence of stable imagery", such as solving problems whilst looking at them rather than by physical manipulation (Hunt, 1969). This termination (or transition) ultimately leads into the next developmental Piagetian stage: operational.

Operational Stage. The stage of operational thought begins to deal with the thinking of the child; it consists of two sub-stages and spans the age range of 2-to-11 years. This work follows on from that carried out by Baldwin, who ended his theory at approximately 2 years with the operationally vague and open-ended "Epoch of Thought". Although early research carried out within the representational stage by Piaget was not clear or conclusive, he was able to develop his ideas by using clinical research in order to generalise descriptors for two sub-stages (Ginsburg & Opper, 1979): *preoperational* (2-7 years) and *concrete-operational* (7-11 years).

The preoperational stage is concerned with the development of language and symbolic thought, and terminates with the emergence of concrete (rather than intuitive) reasoning. The concrete-operational stage is denoted by the ability to exhibit conservation properties with concrete materials and to explain their observations clearly and logically. When behaviours such as the "ability to conserve volume and to decode properly propositions involving classes and utilizing the terms *some*, *all* and *none* as related to a given nesting of classes..." (Hunt, 1969, p. 48), functioning in the formal stage starts to be exhibited.

Formal Stage. The formal stage begins approximately at 12 years of age (Biggs & Collis, 1982) where complex problems of reasoning can be dealt with as well as working with abstract thoughts and hypotheses are generated. Kaplan (1991, p. 54) defined this formal thinking as the stage where students can "also think about thinking; that is, they become aware of the processes by which they come to hold a particular opinion".

With regard to movement through the stages, Ginsburg and Opper (1979) identified four factors, which they list as describing development in Piagetian theory: maturation, experience, social interaction, and equilibration. All of these can be said to have potential to affect progression through the stages of intellectual development.

Maturation comprises the physical and psychological growth that occurs at a specific stage. Experience is when the child thinks and interacts with real or concrete objects in the external environment; Case (1985, p. 21) referred to this as *physical experience.* Social interaction involves socializing with others in order to facilitate progression through the stages; this is evident primarily with children. The last factor affecting progression is equilibration, which occurs when the child brings together maturation, experience, and social interaction in order to build mental schema. Equilibration is considered to be the tendency to seek cognitive clarity and stability, and is due to the differences between biologically maturing schemata and the environmental structures of the surrounding physical and social world (Kohlberg, 1987).

Motivation for this drive for equilibration is perceived as being a discrepancy between an existing scheme and something new (Ginsburg & Opper, 1979). More simply, equilibrium is gained when the individual's cognitive schemes are consistent with the information that he/she gets from existing in their physical world. Despite being linguistically similar to equilibrium (implying a stable state), once equilibration is reached, another state of flux emerges due to the desire of humans to seek further stimulation (Inhelder & Piaget, 1964). It is this movement which is said to explain the transition from one stage or sub-stage to the next (Case, 1985).

Strengths and weaknesses of Piagetian theory

As is the case with many theories, Piaget's theory exhibited strengths and weaknesses. Case (1985) identified four main strengths:

1. The theory accounted for the intellectual and certain types of biological development, which gave it a broad scope;
2. Despite having this broad scope, the theory had few constructs and was therefore generalisable to some degree;
3. The theory could be used to describe or analyse higher and lower order intellectual functioning; and,
4. The theory was systematic.

Despite these strengths, Piaget and other researchers were finding discrepancies in their own studies as far back as the 1960s (Flavell & Wohlwill, 1969). As parallel avenues of research were explored (for examples of this research, see Kohlberg, 1987, p. 161), it became clear that there were certain aspects of the theory that were problematic, and not able to inform, or be reconciled completely with, the emerging field results.

An attempt to retain generality in his theory, Piaget created the term *décalage* to be used when exceptions to his theory appeared. Two main inconsistencies were identified and they are referred to as *vertical* and *horizontal décalage*. Vertical décalage was created to explain the lag between the age at which a child can solve a problem through action and the age at which the child can verbally solve or explain the same problem (Ginsburg & Opper, 1979). Horizontal décalage is used to explain lags in time to execute tasks of similar logical structures, such as conservation of number at the age of five years, conservation of volume at seven years and conservation of weight at nine years (Case, 1985). In other words, a horizontal décalage occurs when an individual's results are inconsistent across a range of tasks of similar structure.

Piaget's failure to examine other factors that influenced knowledge acquisition (such as social interaction and culture) is a weakness of his theory, and has therefore provided an avenue for further elaboration and investigation into constructivist pedagogy. In summary, although Piaget is credited with the development of constructivist thought, his work is largely of a cognitive nature, focusing on the mental states of knowledge, whilst merely acknowledging the existence of some external influences.

Vygotsky

This sub-section outlines Vygotsky's views on the social origins of individual mental functioning, and is divided into three parts. Firstly, a brief background is given to the development of Vygotsky's work. This is followed by an outline of the main constituents of his theory. The section concludes with an outline of the strengths and weaknesses of the theory to inform assessment of cognitive functioning.

Background

It is important to make an initial distinction between pure socio-cultural constructivist thought and that upon which Vygotskian theory is based. A more pure form of socio-cultural constructivism is based upon the notion that it "incorporates society and the socio-cultural to develop an analysis of construction of knowledge which is external to the individual" (Arlidge, 2000, p. 34). More recent ideas surrounding socio-cultural constructivism still have their foundations within the anthropological paradigm, but also incorporate the idea that learning is also based on an individual enterprise. Although knowledge may have its "origin beyond the individual ... it is this sociocultural basis that forms the interpretive background of our individual minds" (Hundeide, 1985, p. 311). As such, the following discussion on Vygotsky and the socio-cultural perspective acknowledges a foundation in anthropological thought, but with psychological elements.

Main constituents of Vygotskian theory

Underlying the founding work on socio-cultural theory is the premise that there is a pivotal connection between social interaction and the formation of consciousness (Vygoysky, 1978). In order to explain higher forms of cognitive functioning "one must go beyond the human organism and search for origins of conscious activity ... in the external processes of social life" (Luria, 1981, p. 25).

Wertsch (1985) identified three main aspects to Vygotsky's theory: (i) that higher mental functions occur as a product of social interaction, (ii) that there is a developmental aspect to knowledge construction, and (iii) semiotic mediation is a facet of psychological development.

The first aspect is related to the contention that there exists a firm connection between socialisation and individual thinking that "applies equally to voluntary attention, to logical memory, and to the formation of concepts" (Vygoysky, 1978, p. 57). Development is seen to occur first on social or "interpsychological" level, and then on an individual or "intrapsychological" level.

The second aspect of Vygotskian thought forms the basis of the developmental aspect of his model, and lies with the notion that higher mental functions can only been explained by looking first at the nature of more elementary and pre-existing conceptions. These pre-existing conceptions are seen to develop along biological lines, whereas the "higher functions originate as actual relationships between individuals" (Vygoysky, 1978, p. 57) and develop as a function of culture.

Associated with this second aspect are Vygotsky's "genetic domains". These domains were important in order to provide an adequate explanation of human mental processes (Wertsch, 1985). Vygotsky (1978, pp. 64-65) stated that

> We need to concentrate not on the product of development but on the very process by which higher forms are established ... To encompass in research the process of a given thing's development in all its phases and changes – from birth to death – fundamentally means to discover its nature ... The study of behaviour is not an auxiliary aspect of theoretical study, but rather forms its very base.

The genetic domains encompass four stages. These stages involve firstly the formation of a relationship between the person and the environment with a grouping of things in an apparently random way. Secondly, the individual forms a "chain of concepts" (Boudourides, 1998) and subsequent recognisable collections of groups of objects. Thirdly, the individual finds it difficult to justify actions and generalise to new situations. Finally, the subject should be able to "a focus on at least one attribute accurately and be able to explain the type of chosen grouping by naming that attribute" (Boudourides, 1998).

Out of the concept of genetic domains emerged the "Zone of Proximal Development", which is characterised as "the difference between what the child is capable of himself and what he can become capable of with the help of a teacher" (Leont'ev, 1997, p. 29). Vygotsky (1978) defined the Zone of Proximal Development as the difference between a child's "actual development as determined by independent problem solving" and the higher level of "potential development as determined through problem solving under adult guidance or in collaboration with more capable peers" (p. 86). Consequently, problems come under three categories: (i) ones which the child cannot do, (ii) ones which the child may be able to do, and (iii) ones which the child can do with some help (Vygotsky, 1978). This idea is vital in understanding fully the importance of social support and scaffolding with regard to learning from the perspective of social constructivism.

The final feature of Vygotskian theory relates to his uses of culture to explain aspects of human development and mental functioning: semiotic mediation. Wertsch and Tulviste (1992, p. 550) stated that

> Mind extends beyond the skin ... because human mental functioning, on the instrumental as well as intermental plane, involves cultural tools, or mediational tools ... [where such functioning is] ... inherently

> social, or sociocultural, in that it incorporates socially evolved and socially organised cultural tools.

Semiotic mediation generally refers to the individual's use or appropriation of tools such as language, symbol and counting systems, and diagrams, in psychological development or meaning-making.

Strengths and weaknesses of Vygotskian theory

Vygotsky provided valuable insight into understanding how knowledge is gained particularly with his contributions from a socio-cultural perspective. There is little doubt about the importance of his work in informing recent studies in developmental psychology (Wertsch & Tulviste, 1992); however, this model contains weaknesses.

Wertsch and Tulviste (1992) outlined a major weakness that existed with fundamental Vygotskian theory: that the notion of a developmental model of genetic domains was too general. More importantly, the theory was still ambiguous regarding the role of developmental progression in mental processes, and how to examine mental processes "without falling prey to ungrounded assumptions about the general superiority or inferiority of individuals or groups" (Wertsch & Tulviste, 1992, p. 253). As a consequence of such a lack of detail, there was little information regarding the development, in the first instance, of elementary mental functioning.

Summary

The previous discussion has briefly described the theories of Piaget and Vygotsky, as well as attempting to provide some critique on the strengths and weakness of both to inform how an individual develops and organises knowledge and meanings. Both Piagetian and Vygotskian theories of knowledge development are partially sympathetic to elements of the other. Piaget, seen as the architect of developmental constructivism, acknowledged the importance of socialisation to cognitive development whilst maintaining that the locus of knowledge construction lay primarily with the individual. Vygotsky acknowledged cognitive developmental posits, but maintained that individual processes are determined predominantly by social and cultural influences; from his work grew the socio-cultural constructivist perspective of learning.

As well as influences on learning, a major difference between the two theories relates to stages of development of knowledge within the individual. Although similarities can be observed within the "developmental stages" of each, Vygotsky did not believe that an individual necessarily progressed maturationally within these stages, a position central to Piagetian theory.

To this point, discussion has centred around the founding work of developmental and socio-cultural constructivism in order to provide a basis upon which to view research that has investigated mathematics out of school settings. Research perspectives have illuminated the similarities that exist in both the developmental and socio-cultural constructivist paradigms. Cobb (1994, p. 13) stated that "learning should be viewed as both a process of active individual construction and a process of enculturation into mathematical practices of wider society". Knowledge may be constructed both in a learners mind, and through social interaction, and Vrasdis and Zembylas (2004, p. 326) pointed out: "there is no clear boundary between these two processes."

Research Investigating Mathematics Out-of-school

Within the literature on out of school mathematics, there exist two predominant theories of constructivist thought on how the relevant mathematical knowledge is developed: developmental and socio-cultural. The preceding discussion concluded that although grounded in different views on the locus of construction of knowledge (the individual and society respectively), within each paradigm existed an acknowledgement of the importance of aspects of the other.

The following section examines the seminal work of three constructivist theorists on the subject of mathematical knowledge: namely, Lancy (1983), Lave (1988), and Saxe (1991). These researchers have examined different aspects of mathematical knowledge in contexts out of a school setting, and have made valuable contributions to how mathematical skill and understanding develop within the individual.

Lancy: A developmental perspective

Lancy (1983) used a Piagetian framework to support his data analysis in his investigation of the relationship between a child's cognitive development of mathematics and their cultural background. Despite acknowledging that Piaget's stage theory had been criticised, he still believed that "the heuristic value of stages is still important" (Lancy, 1983, p. 203) and proposed a three-stage model. Consequently, Lancy developed a neo-piagetian model by which to describe cultural mathematical cognitive development.

Stage 1 was seen as corresponding to Piaget's sensori-motor and pre-operational stages, with some minor overlap into the concrete-operational stage. Lancy found that "accomplishments at this stage were shared by all human beings" (1983, p. 203) irrespective of their cultural background, physical or mental disability; achievement at this stage was shared by other primates, with socialisation processes supporting the learning.

Learning in *Stage 2* is attributed to enculturation; it is the environment and culture that shapes the knowledge, as opposed to genetics. Mathematical skills and knowledge developed within this stage take on different characteristics, reflective of the culture in which they are formed. It is within Stage 2 that differences that the mathematical knowledge and understanding of people in different cultures diverges. Regarding similarities within Piagetian theory, Lancy stated that it is not individuals who achieve higher concrete-operational or formal stages, but the societies themselves, which allow such transitions.

Stage 3 concerns how mathematics is culturally developed. This stage primarily concerns the development of metacognition. Interestingly, he stated that this stage "begins in infancy and continues throughout adulthood" (1983, p. 208). Lancy (1983, p. 208) outlined the performance expected of this stage:

> In addition to developing cognitive and linguistic strategies, individuals acquire ‘theories’ of language and cognition. They learn what kinds of knowledge are important for what purposes; they learn the relationship between knowledge and status; they learn the appropriate occasions for knowledge acquisition and display; and so forth.

The use of formal and logical problem solving and inferential techniques equating to the “formal operational” stage of Piaget, is tied to scientific principles. Such an approach can be seen as a “theory of knowledge” that is specific to Western society; other cultures may well have their own theories of knowledge. Lancy defined these theories of knowledge associated with Stage 3 metacognition as the underlying principles of language and symbols developed within a culture.

According to Lancy, it appears that culturally homogeneous cognitive attributes only occur within Stage 1. Such cognitive development relates to the Piagetian Stages of Sensori-motor, pre-operational and partially into the concrete-operational. It is proposed that all qualitative cognitive changes occur within this Stage.

Predominantly quantitative changes are said to occur after this in Stages 2 and 3, with Stage 3 occurring concurrently with Stages 1 and 2, tracking the development of metacognition from infancy to adulthood. This attribute of Stage 3 is at odds with the Piagetian notion that once a Stage is passed through, it is not revisited.

Lave: A socio-cultural perspective

With regard to investigations into mathematics as it occurs in everyday life, Lave (1988) synthesised the dilemmas faced by anthropology and psychology, with an attempt to reconcile the two perspectives within her research. She noted that cognitive anthropologists began to look toward cognitive psychology to provide sophisticated formal models of language, logic and problem solving, whilst those examining practice from a psychological perspective started to see opportunities within the field of anthropology to provide knowledge regarding “real-life” activities and situations (Lave, 1988). The theoretical orientation of psychology used in conjunction with the descriptive insights provided by anthropology gave new insights into the richness and complexities of mathematics-containing activities out of school.

Lave examined the mathematics of shopping and dieting from the domain of arithmetic. Her work has been heavily weighted towards the anthropological rather than the psychological paradigm. Cognition is seen to be "socially-organised" (Lave, 1988, p. 1) as being "constituted in dialectical relations among people acting, in the contexts of their activity, and of the activity itself" (p. 148).

The work of Lave (1988) and Lave and Wenger (1991) has provided invaluable contributions to understanding the influences on how mathematics is operationalised in out-of-school settings. Important constructs are those of "situated cognition" and "communities of practice". Situated cognition has evolved largely out of a socio-cultural constructivist perspective into a "strong theoretical orientation" (Noss, Hoyles, & Pozzi, 2000, p. 17), and is defined as providing a "view of intellectual work (including mathematical work) which is inseparable from socio-cultural contexts" (Noss, Hoyles, & Pozzi, 2000, p. 17). It focuses predominantly on the "mathematical activities and practices rather than on the written corpus of mathematics as a cultural construct" (Barton, 1996, p. 1035).

Within this theory of situated learning, Lave and Wenger (1991) coined the term "community of practice". This term has been defined as "particular socio-cultural practice shaped by circumstantial social factors, norms and values which embody the community" (Billett, 1998, p. 12).

Lave and Wenger (1991) affirmed that all forms of knowledge, with the exception of higher-order procedures, have their origins in social practice. Full participation in a community of practice, and hence an identity within that community, involves the acquisition of expertise.

Lave described mathematical activity whilst taking cognition into account, but stops short of exploring developmental cognition within the psychological paradigm. This reflects her position on the continuum that links the dichotomies of the socio-cultural and developmental constructivism, as being more grounded in the anthropological/socio-cultural paradigm.

Saxe: A combined perspective

Another researcher who examined the relationships between culture and cognition using Piagetian theory was Saxe (1990, 1999). His epistemological position on cognition is grounded on both Piagetian and Vygotskian ideas relating to the construction of knowledge. With regard to developmental aspects of mathematical knowledge formation, Saxe recognised failings within the developmental work of Piaget. He stated that “Piaget’s focus on universals does not well afford a differentiated treatment of history and culture in treatments of cognitive development” (Saxe, 1999, p. 254). Despite the weaknesses of Piagetian theory, Saxe (1999, p. 254) upheld the developmental notion of cognition as a “process of undergoing transformation”.

Underlying Saxe’s research, are three related types of cognitive transformation that are linked to activity in a cultural context. These are firmly grounded within the genetic domains of Vygotsky, and are outlined below.

Microgenesis refers to the transformation that occurs when an individual adjusts cultural forms (such as currency and/or number words) to enable them to use these conceptions to solve a problem in an immediate activity. Saxe (1999) distinguished this definition from other conceptions of the term, which refer to a methodological approach to the study of changes in children’s cognitive structures or problem-solving strategies.

Ontogenesis is defined as the “developmental shifts in the structure of individuals’ repeated efforts to create and accomplish recurrent goals in practices” (Saxe, 1999, p. 254). These shifts can occur either with age, or with additional participation within the community of practice.

The final type of cognitive transformation is *sociogenesis*. This involves the “spread and evolution of means for solving and accomplishing goals” (Saxe, 1999, p. 264). It is defined largely by the way in which knowledge develops to become valued by individuals and within communities of practice.

These three categories of genetic domains form a process of cyclical development of cognitive change. Saxe (1999, p. 264) explained the dynamic as follows:

> In individuals' appropriation of forms to accomplish emerging goals in practices, they adapted and repurposed earlier sociogenetic developments through microgenetic processes and built on earlier constructions in ontogenesis. In turn, the microgenetic transformation on forms into means in activity served as a basis for new processes of sociogenesis involving the spread and institutionalisation of new cultural forms.

Saxe asserted that the relationship between these developmental processes is intertwined in cultural practices.

Summary

The preceding section outlined that the foundations of the "opposing" constructivist paradigms of developmental and socio-cultural, but emphasised that within each perspective there was a place for aspects of the other. A synthesis of the examples of research into mathematics out-of-school presented in this section thus far, concludes that there is a predominant disposition to approach the study on mathematics out-of-school from a socio-cultural perspective. Of the three studies presented, the later work of Lave, and more recently Saxe, focused on social and cultural factors following from this paradigm. Studies in a similar vein are those by Carraher, Carraher, and Schliemann (1985), Lave, Murtaugh, and de la Rocha (1984), and D'Ambrosio (1985). The efforts of these researchers have paved the way for the development of a substantial body of work relating to the social and cultural aspects of mathematics, particularly in out-of-school settings.

Lave asserted the importance of such studies over developmental constructivist approaches that isolated "the culturally and socially constructed activities and settings of everyday life and their economic and political structures and cyclical routines from the study of thinking" (Lave, 1988, p. 76). Additionally, Lave (1988, p. 170) was equivocal regarding predominantly cognitive-developmental approaches, and stated that:

> I have argued against the view that cognitive holdings of a person are stable, constant and theorisable, while their contexts are specific, variable and untheorisable. Instead of a person acting, arenas and settings appear to be implicated together in the very constitution of activity.

This perspective has largely been shaped by pure cognitive science, and its position that cognitive structures need to be generalisable. Lave (1988, p. 76) commented that there are other ways of generalising theory across settings other than "the mind, with its durable cognitive tools".

More specifically, Hundeide (1985, pp. 310-311) was critical of predominantly developmental ideas relating age-associations to movement through stages. As such:

> When we study other cultures with different institutions and episode structuring of reality, we may find that the definition required for the proper execution of certain mental operations that are of interest to us are outside the episodic repertoire of the culture. In such cases, an orthodox Piagetian diagnostician runs the risk of diagnosing an entire culture as pre-operational.

It appears that such positions on where on the constructivist continuum such research should lie, has shaped the majority of investigations into the nature of mathematics out-of-school. The complex nature of the world provides for many tangential avenues of research from the socio-cultural perspective, as such the importance of these investigations should not be discounted or trivialised.

Whilst socio-culturally derived research continues to be built upon, modified and developed, research from the perspective of developmental constructivism remains notably absent from the literature. The work of Lancy (1983) attempted to reconcile the developmental and cultural aspects of mathematics knowledge, yet from the opposite paradigm to the other studies mentioned previously. Whilst an attempt to provide a neo-Piagetian perspective was made, the model was necessarily vague regarding descriptions of Stage development in order to provide for not only cultural and developmental aspects, but also for the inherent weaknesses with Piagetian theory such as the décalage.

Conclusion and Research Directions

The manner in which mathematical meaning and knowledge is accrued by an individual is widely accepted, particularly within research communities involved in investigations into mathematics out-of-school, as being situated within the constructivist paradigm. As such, this Chapter began by providing a general overview of Constructivism.

It is important to note that whilst constructivism appears to be a dominant force in modern education, there are criticisms. Simpson (2002) argued that within the sciences, although some bodies of knowledge are constructed as a social, cultural and political process, a large portion is based on evidence gathered from the environment. Hypotheses are only accepted after "careful replication of the knowledge claims" (Simpson, 2002, p. 350). There are inconsistencies with reconciling the constructivist viewpoint that knowledge formation involves "synthesising new information within existing structures" (Arlidge, 2000, p. 34), and that often "scientific knowledge flies in the face of common sense" (Hodson & Hodson, 1998, p. 34). It is the position here that although the constructivist perspective on learning is important to the field of research into mathematics in and for work, the co-location of mathematics within the scientific community means that there do exist fundamental and universal truths. As such, learning in this situation requires consideration of elements of a non-constructivist set of truths.

Closer examination of the literature related to this field elucidated that there existed two prominent perspectives on constructivism through which research could be conceived in order inform how mathematical knowledge and meanings were obtained by the individual. These two types were developmental and socio-cultural, and were seen by numerous authors to exist at either end of the constructivist continuum.

In order to provide sufficient background upon which to situate initial and contemporary research perspectives regarding the out-of-school mathematics on this continuum, an outline of the basic theories of Piaget and Vygotsky were provided, as well as their strengths and weaknesses. From these brief descriptions, it was deduced that proponents of either theory acknowledged the importance of aspects provided by the other. Piagetian theorists conceded that social influences were at work in learning, and Vygotskian theorists recognised the need for developmental aspects to their descriptive studies.

The next section of the Chapter provided an overview of some of the seminal work into what mathematics '"looks like" out of a classroom, with the work of Lave being the most influential. The bulk of subsequent research followed from these studies, and approached examination of every-day mathematical practices from a predominantly strong socio-cultural perspective.

Lave was not the only early pioneer into mathematics in such settings. Lancy also attempted to reconcile social and cultural aspects with a developmental constructivist perspective based on Piagetian theory. Such neo-Piagetian work has notably been absent from subsequent literature.

The discussion has not yet provided a specific treatment of the state of mathematics in and for work; a predominant reason for this is that this area is a new research field. The following paragraph synthesises the small body of growing and evolving research.

In describing the current position of research into mathematics in and for work, Sträßer (2001) commented that there are three significant conclusions that have emerged from research into this field:

1. There is significant work which outlines the facts and arguments surrounding the role mathematics plays in vocational contexts;
2. Substantial investigations have been carried out into why the mathematics learnt in school differs from the mathematics required for the workplace; and
3. Determination of the role played by artefacts (such as man-made tools or machinery, ways to organise and redistribute goods, and/or human competencies) within the workplace.

These conclusions are important to understanding mathematics in the workplace. English and Halford (1995, p. 117) acknowledged "the importance of sociocultural and other related factors, both within and beyond the classroom"; however, they concluded that there is still a need to investigate how mathematical concepts "map into relevant cognitive processes."

Sträßer (2001, p. 4) identified that this area is still in need of significant research:

> Research on the actual learning processes (of individuals) is badly needed. In this respect, descriptive studies of classroom and individual learning processes have to be complemented by research-supported developmental efforts to learn more about these processes.

Therefore, questions must arise regarding how insight into developmental processes can be derived. As such, there is a need to look back to the foundations of developmental constructivism and to the work of Piaget. Despite thorough searches of the literature, no evidence for research into mathematics in the workplace using a Piagetian cognitive developmental framework was found.

CHAPTER 3

THE SOLO MODEL

The preceding chapter outlined constructivism, both in general terms and with respect to research investigating mathematics out-of-school. Evident within mathematics in work literature are two predominant paradigms of constructivist thought: developmental and socio-cultural. Judging from subsequent exploration of contemporary research in the present field of investigation, it is evident that the emphasis has been on socio-culturally based work, with little attention given to cognitive development of knowledge within workplace contexts. As such, it was proposed that one potentially valuable approach to addressing such a deficiency could be to incorporate the perspectives of neo-Piagetians into workplace research.

Piaget's theory was described briefly in the previous Chapter. This theory has been subjected to rigorous investigation. Emerging from these analyses was a view of general agreement but accepting of many weaknesses. Contemporary neo-Piagetian theorists have attempted to address the weaknesses of the original Piagetian position on cognitive development, and use the strengths to form new understandings.

One such neo-Piagetian approach is called the SOLO (the acronym SOLO stands for the **S**tructure of the **O**bserved **L**earning **O**utcome) Model. This framework assists in identifying cognitive developmental pathways of knowledge accrual by analysing the qualitative differences evident in the structure of an individual's responses to stimuli. The SOLO Model is now, through a wide body of research, acknowledged as a theoretical model by which the cognitive structure of students' understandings, especially in classroom settings, can be assessed.

This chapter consists of two sections. The first section provides a detailed description of the SOLO Model. The second section furnishes a short literature review regarding previous research incorporating its use.

A neo-Piagetian Model: SOLO

This section discusses the SOLO Model in three parts. Firstly, the SOLO Model's derivation from the work of Piaget is outlined. Secondly, the constituent elements of the Model are described, followed by an overview of the currently accepted form of the SOLO Model. This section concludes with a summary of previous SOLO-based research.

The evolution of the SOLO Model

The SOLO Model (Biggs & Collis, 1982, 1991) evolved as a response to problems encountered by applying the Piagetian theory of cognitive development to classroom contexts. Van Hiele (1986, p. viii) stated that to have criticisms regarding a theory is "only meaningful if one agrees with the greater part of it"; as such, this statement gives substance to the importance of Piaget's work in the development of the SOLO Model.

Collis and Biggs (1979) carried out a research project aimed at constructing a general cognitive framework that could be used by teachers across subject areas to aid assessment and instruction. By utilising the Piagetian idea that exploration of students' errors could possibly identify "natural development phenomena" (1979, p. 2), Collis and Biggs examined previous work in the areas of History, Mathematics, Geography, English and Modern Languages, as well as carrying out their own fieldwork. As the research progressed, certain discrepancies became apparent, making their aims difficult to achieve. These were:

1. As the content of each particular discipline was so different, it became almost impossible to generalise the development of intelligence in set Piagetian

stages across these subjects; this was attributed in part to appropriate task selection;

2. As the collection of student responses grew, it became apparent that the décalage of Piaget became the rule rather than the exception (see Biggs & Collis, 1991, p. 60), both across subjects and across topics within subjects. Piagetian theory outlined that an individual could only exist in one developmental phase at a time; however, some students were observed to give responses that could be categorised in up to four of the traditional Piagetian stages for the same subject area; and
3. Occasionally, responses of a particular student on the same question at different moments in time (up to weeks apart) generated responses at different levels.

Since Collis and Biggs (1979) were examining classroom development phenomena, many of these discrepancies are not surprising, as “Piaget’s work was not developed in the service of education, but rather in the service of genetic epistemology” (Sigel, 1969, p. 465). In support of Piagetian applications to education, Sigel (1969, p. 467) stated that educators “must have a conceptual framework within which to establish programs [and] devise teaching programs ... [and] ... assess the child’s developmental level and establish relevant levels of curriculum content”.

Cognitive frameworks have been validated by other neo-Piagetians such as van Hiele. The van Hiele theory of cognitive development in geometry was to also aim to provide a framework able “to provide teaching that is appropriate to the level of the children’s thinking” (van Hiele, 1999, p. 311) in that topic area.

Rather than attempt to replace Piagetian theory due to these three apparent inconsistencies, Collis and Biggs (1979) focused on using their results to create an alternative framework, maintaining and refining as many of the original concepts as possible. They kept the constructivist approach and focus on the quality of students’ responses that is evident in Piaget’s theory of intelligence. However, where Piaget maintained that stages are sequential, universal and invariant, Collis and Biggs (1979) developed a more flexible approach. Where Piaget focused on categorising students into developmental stages, Collis and Biggs (1979, p. 3) placed the emphasis on the responses of the student: "... we could see that to achieve our purposes we needed to shift the focus away from the response implying a stage of development to a consideration of the quality of each individual response."

By focusing on the response rather than the student, external factors acting on cognitive development, such as "emotional state, physical state and motivation" (Collis & Biggs, 1979, p. 3) could now be taken into account. This de-emphasis on the response indicating the developmental stage of the child also allows for explanation of the occurrence of variation in the apparent Piagetian stages of the student's response. Sigel (1969, p. 471) hinted at this discontinuity as he attempted to apply Piaget's theory more closely to the system of education:

> A further important consideration in applying [Piagetian] principles is the fact that the child does not move on all fronts simultaneously. The child may be in one stage in one conceptual area and another in another area. In effect, the rates of growth may vary among children as well as for a particular child.

In order to provide appropriate descriptions to enable teachers and researchers to assess whether learning that had taken place, Collis and Biggs (1979) identified certain aspects of Piagetian theory as having quantifiable aspects. Subsequently, SOLO was able to deliver to educators the framework sought in the original research effort: The assessment of a student's cognitive response to a particular task that would facilitate the use of such information to inform more appropriate instructional strategies to suit the student's current level of knowledge and understanding.

The framework of the SOLO Model

The framework of the SOLO Model can be defined in terms of *modes* and *levels*. This section describes both of these terms. More recent research developments have identified cycles of levels within the modes; these are also discussed, with concluding remarks made regarding the currently accepted evolutionary stage of the SOLO Model.

SOLO modes

The basic sequence of cognitive development of the SOLO Model is linked closely with the Piagetian stages (Pegg, 1992), and is distinctly characterised by five modes These modes are listed in increasing levels of abstraction as: *sensorimotor*, *ikonic*, *concrete symbolic*, *formal* and *post-formal*.

The Sensorimotor mode. The sensorimotor mode is available from birth, and relates to knowing how to perform particular physical skills: such as grasping an object in the early years, to complicated gymnastic routines. This is the mode that deals with the development of motor skills (Biggs & Collis, 1982).

Adult performance in the sensorimotor mode has not been examined in depth using the SOLO Model. However, it is hypothesised that advanced cognitive performance in this mode could be aligned with athletes competing at the highest levels of their sport, or similarly with any skilled practical task execution.

Sensorimotor performance is identified predominantly through observation, as performance in this mode is not easily verbalisable. Biggs and Collis (1991) illustrate this point by referring to a quotation of the dancer Isadora Duncan in Gardiner (op. cit., 1985, p. 225): When she was asked to explain the meaning of a dance, she replied "If I could tell you what it is, I would not have danced it."

The Ikonic mode. The ikonic mode is approximately available from 18 months of age, and is where intuitive thinking is developed. This mode is pre-symbolic as it is where a child "develops words and images which can stand for objects and events" (Pegg, 1992, p. 369); in these early years it is where imagery and imagination are evident. Later applications comprise the appreciation of the visual and dramatic arts, and expanded functioning based in intuitive knowledge where tasks are performed on a sub-conscious level (Biggs & Collis, 1982).

Ikonic modal functioning is generally said to be intuitive in nature. The existence of intuition can be said to be the summative culmination of formal and informal learning, and the point at which an individual can make an intuitive judgement once all the necessary knowledge has been gained. Intuition is the ability for an individual to make choices in order to carry out a certain behavioural path, as well to have a "feeling for what is going to work and what is not going to work" (Sternberg et al., 2000, p. 58). However, Wagner and Sternberg (1986) attribute such judgemental attributes as tacit.

Collis, Watson, and Campbell (1993) stated that individuals using an ikonic approach to solving problems saw little value in supporting their solution by using mathematics, and that individuals were more likely to move from the concrete symbolic mode to the ikonic mode when school-based methods of solution failed. However, this particular study related to school-based learning and has yet to be verified by research into actual workplace practice. Care must also be taken not to define "geometrical, graphical, or pictorial illustration" (Collis & Romberg, 1991) as being indicators of functioning in this mode, as these form part of the concrete symbolic modal characteristics.

The Concrete Symbolic mode. The concrete symbolic mode is usually available from around six years of age, and coincides with the start of a child's school life for the general population. The concrete symbolic mode is where symbolic systems, such as musical notation, mathematics and written language are developed. Achievement in this mode is required for satisfactory functioning in modern Western society (Biggs & Collis, 1991), and hence, it is the focus of primary and secondary school curricula.

The Formal mode. Formal mode functioning becomes available from about 16 years of age, and is where cues from the concrete world are no longer necessary in order to understand higher-level cognitive concepts. When working in this mode, abstractions from the "real" world are possible, along with the ability to work with theories and hypotheses.

The Post-formal mode. The post-formal mode becomes available at approximately 20 years of age. Although operations in this mode are not well documented (Biggs & Collis, 1991), it is where "a person is able to question or challenge the fundamental structure of theories or disciplines" (Pegg, 1992, p. 369).

Modal strategies

Fundamental to the underlying philosophy of the SOLO Model, and in direct opposition to Piaget (Ginsburg & Opper, 1979), is the notion that acquisition of a later acquired mode does not entirely replace the mode before; a person is able to operate in more than one mode for the same or different tasks at any particular point in time. As more modes become accessible, different learning strategies become available.

Biggs and Collis (1991) outlined four modal strategies as *unimodal*, *top-down*, *bottom-up*, and *two-way.* Unimodal functioning refers to working in one mode due to other modes either being inappropriate for the task, or not available to the individual. Such functioning is most evident with the sensorimotor mode in infancy, where the child has not yet acquired higher modes of learning.

Top-down functioning refers to working in the target mode and using later acquired modes to supplement the instruction. For an expert golf player, functioning in the sensorimotor mode is not equivalent to an infant grasping a stick. Although the development of sensorimotor proficiency appears to be unimodal, the expert is able to use skills from later acquired modes such as watching videos of other experts (ikonic), reading books on technique (concrete symbolic), or coaching others (formal) in order to supplement activity in the sensorimotor mode.

Bottom-up functioning refers to working in the target mode using earlier acquired modes to support instruction. Such behaviour strongly supports constructivist theories of learning, where students learn by creating their own learning environment rather than accreting knowledge. This notion was supported by Dienes and Golding (1971) in their levels of mathematical learning which identified previous experiences in earlier acquired modes as "provid(ing) a peg on which to hang what has been abstracted" (Reys, Suydam, & Smith, 1995, p. 20).

Two-way functioning refers to working in the target mode and incorporating both later acquired and earlier acquired modes to support instruction. Biggs and Collis (1991) used music appreciation to describe two-way learning. In order to appreciate music, which is an ikonic activity, students are encouraged to play musical instruments (sensorimotor) and read music (concrete symbolic) so that learning in the target mode is increased or optimised. These multi-modal learning strategies are contextual and individual.

Associated with these strategies is the concept of *level reduction,* which can occur in two ways. In the first instance it can occur where the mode of the task at hand is not yet accessible to the individual. In order to be able to provide an answer to the problem, the individual relies on rote-learned strategies or tricks. These tricks involve a set of procedures or instructions to follow, and are often given by the classroom teacher to reduce apparent complexity of the task.

Collis and Romberg (1991) discussed this concept in terms of unimodal learning with reference to algebra, where students are taught this abstract idea of symbol manipulation without the support of any learning in the concrete symbolic mode. This leads to rote-learning of facts and procedures, without a deep understanding of the concepts involved.

In the second way, and more positively, it can occur when a student has "understood a concept at a higher level and [has] chosen to routinise it in a certain way" (Pegg, 1997, p. 35). This leads to faster processing of information and "insight", which van Hiele (1986, p. 159) defined as an ability to act "adequately with intention in a new situation".

SOLO levels

Within each mode there exists a series of three levels: *unistructural, multistructural* and *relational.* These are listed in increasing order of structural complexity as:

- Unistructural (U) – these responses are characterised by a focus on one relevant aspect.

- Multistructural (M) – these responses focus on two or more important aspects of the problem as independent items.

- Relational (R) – these responses show an integration of the identified independent aspects into a coherent whole, and show an overall understanding of the concept.

In the first stage of the evolution of the SOLO Model (Biggs & Collis, 1982), these were identified along with two additional levels: *prestructural* and *extended abstract*. Prestructural responses did not show that functioning in the target mode had been reached, with extended-abstract responses being those that exceeded functioning in the target mode. These five levels, shown in figure 3.1, are valuable in order to simplify the model in practical circumstances when dealing with responses centred predominantly in one mode.

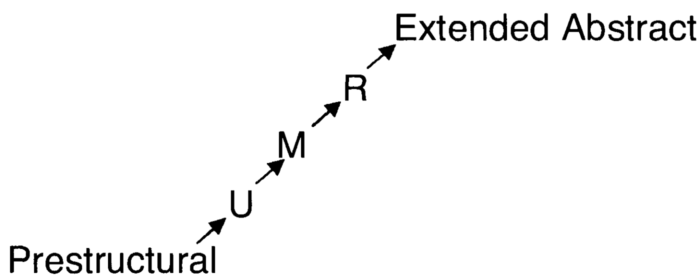

Figure 3.1 Learning cycles of development incorporating the levels, within the concrete symbolic modes in the SOLO Model.

Because the original work of Biggs and Collis was directed at assessment practices related to schooling, the target mode for the five levels was the concrete symbolic mode. In this case, the prestructural level refers to any functioning preceding the concrete symbolic mode, and extended abstract level refers to any functioning beyond concrete symbolic mode.

A similar logic follows for the target mode being any of the other four modes of operation. As a child passes through the SOLO levels of cognitive development each of the preceding levels remain accessible to them (Biggs & Collis, 1982). Table 3.1 shows the relationship between the pre-structural and extended abstract levels, with the modes either side of the target mode.

Table 3.1 The relationship between cycles and modes (adapted from Biggs & Collis, 1982, p. 216).

Mode of Functioning	Structure of Response	
Sensorimotor	Unistructural Multistructural Relational	 (or Prestructural for Ikonic)
Ikonic	Unistructural Multistructural Relational	(or Extended Abstract for Sensorimotor) (or Prestructural for Concrete Symbolic)
Concrete Symbolic	Unistructural Multistructural Relational	(or Extended Abstract for Ikonic) (or Prestructural for Formal)
Formal	Unistructural Multistructural Relational	(or Extended Abstract for Concrete Symbolic) (or Prestructural for Post-formal)
Post-formal	Unistructural Multistructural Relational	(or Extended Abstract for Formal)

Responses sometimes contain elements of different modes or levels. These responses are called "transitional" and are marked by "confusion and inconsistency", with an overload of working memory leading to losing "track of [the] argument" (Biggs & Collis, 1982, p. 29). These responses may contain elements of the next SOLO level but are marked by an inability to close, or by supporting the argument from a lower level.

Biggs and Collis (1982) outlined four features associated with the levels, which govern progression through them. These features are *working memory capacity, logical operations*, *consistency,* and *closure*. These elements are embedded within the SOLO levels, necessary in order to identify more clearly appropriate levels of understanding in students' responses. The properties are outlined below.

Working memory. Working memory refers to the capacity to handle information. As the demand for a higher level of response increases, the load on the working memory increases. McNamara and Scott (2001) acknowledged the role of working memory capacity in learning by stating that in freeing working memory capacity from an excessive focus on basic or routine tasks, students are better resourced to undertake higher order mental processing.

Logical operations. Logical operations refer to how the stimulus and response interrelate. At one end of the spectrum, responses coded as prestructural in the concrete symbolic mode consist of "denial, tautology, transduction, [and being] bound to specifics" (Collis & Biggs, 1979, p. 67); at the other end, responses beyond the concrete symbolic mode show the ability to generalise beyond the experience of the respondent.

Consistency and closure. Consistency can be explained as a high compatibility between the response and the initial data encountered. In order to maintain this consistency, students often would hold off on "closing" their response. Closure refers to the need of the individual to come to a conclusion to the problem. Should an individual close quickly in a task, there is a greater chance of the response being inconsistent with the data.

Cycles of levels within modes

Research into the SOLO Model since the early 1990s, has identified more than one cycle of development within the modes (Pegg, 1992). This cyclical development of levels defines the second major evolutionary phase of the SOLO Model.

As the target mode for much of secondary schooling is the concrete symbolic mode, the bulk of research has been focused on understanding functioning in this domain. Consequently, one of the most recent developments in the model occurred when researchers observed the presence of more than one unistructural-multistructural-relational (U-M-R) cycle in the concrete symbolic mode (K. Campbell, Watson, & Collis, 1992; Pegg, 1992). The first U-M-R cycle is where the development of a 'concept' takes place, consequently forming a U_2 in the second cycle. Numerous developments of this type form the 'elements' or 'components' that form an M_2 in the second cycle. Once the second cycle is reached, the first cycle becomes intrinsic.

With regards to the concrete symbolic mode in geometry, the notion of individual properties of two-dimensional shapes is developed in the first cycle, with the application of those properties occurring in the second cycle. Whilst there is no research to support the existence of more than two cycles within each mode, Pegg (1992) suggested that it may depend upon the nature of the concept being studied. Figure 3.2 incorporates the sub-scripts 1 and 2 to denote the first and second cycles, respectively.

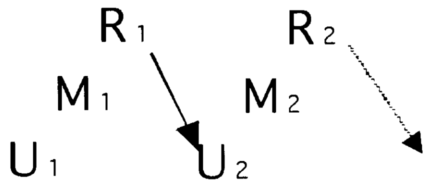

Figure 3.2 Two cycles of development incorporating the levels, within modes in the SOLO Model (Pegg, 1992).

This cyclical development within the modes was previously evident (although not instructionally adequate) in the writings of Baldwin and Piaget in their underlying notions of primary, secondary and tertiary circular reactions (Case, 1985; Ginsburg & Opper, 1979). Case (1985, p. 15) stated that "... the ability to coordinate two primary circular reactions" was a necessary prerequisite for the construction of the concept of invisible movement. Ginsburg and Opper (1979) outlined the Piagetian circular reactions in the sensorimotor stage in similar fashion, with the six-sub-stages falling into a crude two cycle form.

Summary

The SOLO Model can be represented in its current accepted form in Figure 3.3.

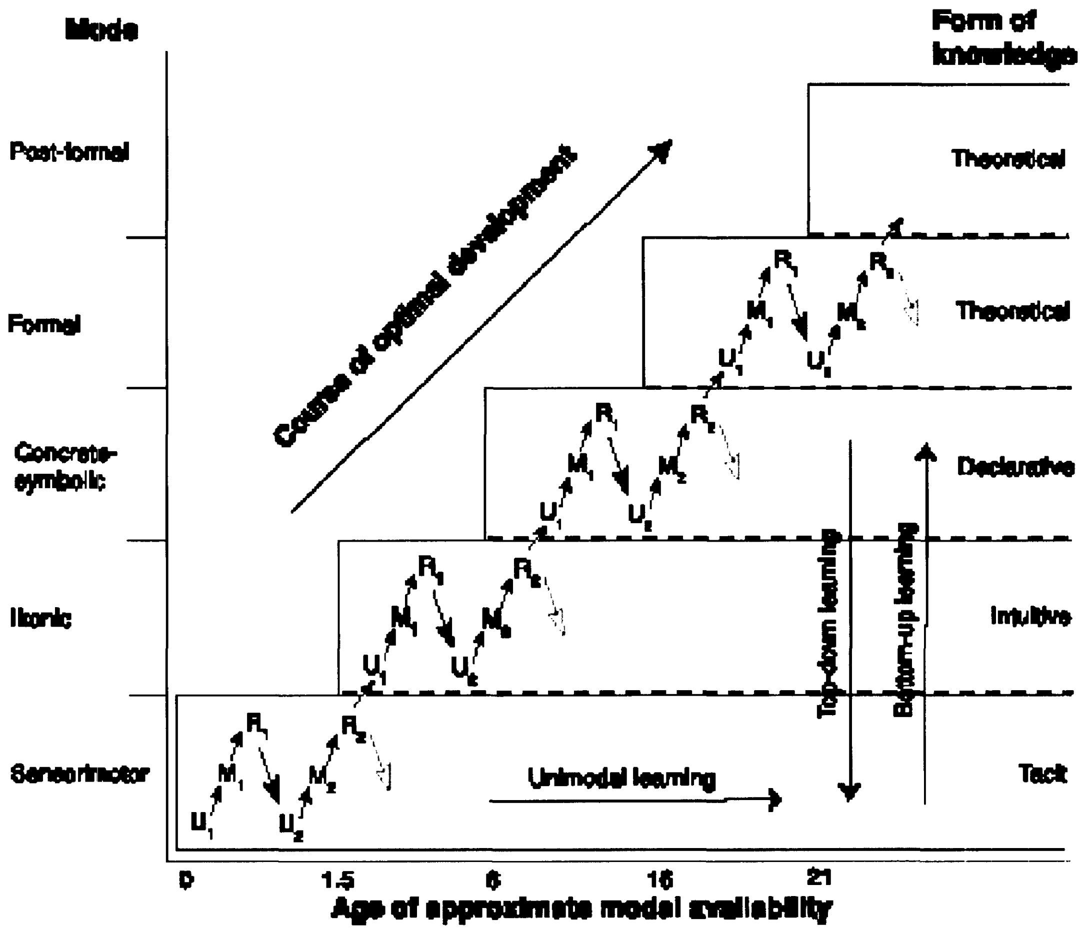

Figure 3.3 Modes, learning cycles and forms of knowledge of the SOLO Model (adapted from Biggs & Collis, 1991; Pegg, 1992).

The advantages of the use of this model lie in its application across qualitative and quantitative aspects of cognitive development, while maintaining flexibility that matches the uniqueness of the human population. Researchers from across a broad range of disciplines have adopted the SOLO Model as a framework to scaffold analysis of cognitive functioning, testing and refining the model in the process to arrive at the current model. However, it has retained its original structure of the five modes and U-M-R levels throughout this evolution, which is testament to its strengths as an assessment tool to measure cognitive functioning.

Previous Research Using the SOLO Model

Previous studies using the SOLO Model as the underlying theoretical framework have encompassed a range of educational situations. These fall mainly into the broad categories of tertiary and school-based education, with curriculum development and general assessment techniques spanning both. This section reviews the literature in two sections, (i) an overview of studies using the SOLO Model as a tool to analyse responses, and (ii) alternative mechanisms within the Model to provide insight into cognitive functioning.

The SOLO Model: previous research

Research using SOLO in the tertiary sector consists of involvement of the students in knowledge of their own learning (Boulton-Lewis, 1994; Boulton-Lewis, Wilss, & Mutch, 1996; Dart, 1994; Trevitt & Pettigrove, 1995), essay writing and construction (Anderson, Walker, & Neilsen, 1994; J. Campbell, Smith, & Booker, 1998), and computer assisted learning (McAlpine, 1996). Many studies incorporate tertiary students' responses along with secondary students' responses in order to explain formal mode functioning (Coady, 1994; Panizzon & Pegg, 1997).

The greater part of the research into the SOLO Model has been carried out in the subject areas of mathematics and science in the secondary school. In mathematics, the SOLO Model has been applied to a diverse range of topics including problem solving (Collis, Watson, & Campbell, 1992; Stillman, 1996; Taplin, 1994; Wilson, 1990), statistics and probability (Callingham, 1994; Collis, Watson, & Pereira-Mendoza, 1996; Reading & Pegg, 1996; Watson & Collis, 1994; Watson, Collis, & Moritz, 1994), volume (K. Campbell, Watson, & Collis, 1992), LOGO (Hawkins & Hedberg, 1986), number (Gorrie, 1997; Watson & Mulligan, 1990), algebra (Chick, 1988; Coady & Pegg, 1993; Redden, 1994; Robertson & Taplin, 1994), geometry (Pegg & Davey, 1998; Pegg & Woolley, 1994; Serow, 2002; Watson, 1988), fractions (Hayman, 1998; Watson, Campbell, & Collis, 1993), and area (Watson, Chick, & Collis, 1998).

In the case of science education using the SOLO Model, topics have included diffusion and osmosis (Panizzon & Pegg, 1997), evaporation (Levins, 1992), plant growth (Levins & Pegg, 1993), chemistry (Stanbridge, 1993), magnetic fields (Guth, 1995), as well as various studies into assessment practices in science (Biggs & Collis, 1989; Collis & Davey, 1984; Lake, 1999). Other studies in SOLO research have been directed towards english (Collis, 1982), music (Scott, 1999), geography (Biggs & Collis, 1982; Courtney, 1986), development of vision (Jones, Sprod, Collis, & Watson, 1996), and information technology (Hoddinott, 1998; Jackson, 1998; McLaughlan & Kirkpatrick, 1999; Thomas, n.d.).

Assessment of student outcomes can be generalised to a broad spectrum of subject areas (Collis & Courtney, 1986; Collis & Romberg, 1991; Tham-Ng & Lam, 1996) leading to the strengthening of curriculum development (Alagumalai, 1996; Biggs, 1986; Biggs & Collis, 1989; Pegg, 1998). The SOLO Model "provides one means of making it possible to discuss, and test, the quality [of understanding] required at different levels" (Collis & Biggs, 1986, p. 5). As such, curriculum development is an area that can benefit greatly from insight gained through this cognitive developmental approach. Despite this, many school syllabi are still lacking a sound theoretical framework to underpin new outcomes-based performance descriptors (Pegg, 1999).

Recent innovations have included the incorporation of the SOLO Model with professional teacher in-service programs. An example of one such program is Developmental-based Assessment and Instruction, developed by Pegg (Pegg, Panizzon, & Inglis, 2002), which aimed to give a stable structure to existing pedagogical knowledge about the performance bands into which the quality of students' understanding is expected to be categorised.

The SOLO Model and workplace contexts

The SOLO Model as previously described earlier in this chapter focused on cognitive functioning from birth to adulthood along a "course of optimal knowledge accrual". To date, counselling (Burnett, 1999) is the only vocational area that has examined cognitive processes using the SOLO Model. Burnett, using a single-cycle version of SOLO, investigated the way in which social workers developed their counselling skills. Use of the SOLO Model as an analysis tool for cognitive processing in practical or skill-based professions remains unexamined.

Missing from investigations using the SOLO Model is how cognitive functioning operationalises in workplace situations; this is the focus of the present study. Fundamentally, adult cognitive behaviour is not restricted to the higher levels of cognitive functioning. It is expected that the lower modes could manifest in adulthood and continue "to grow in both power and complexity into adulthood" (Collis, Watson, & Campbell, 1993, p. 109).

Performance in the workplace often involves more than merely automated tasks. Solving problems often forms the bulk of workplace activity. Collis, Watson, and Campbell (1993) researched problem solving in early adolescence, showing that the basic factor determining solutions using ikonic or concrete symbolic functioning, is the nature of the task. Figure 3.4 shows possible problem-solving paths formed by decision-making in the problem-solving process.

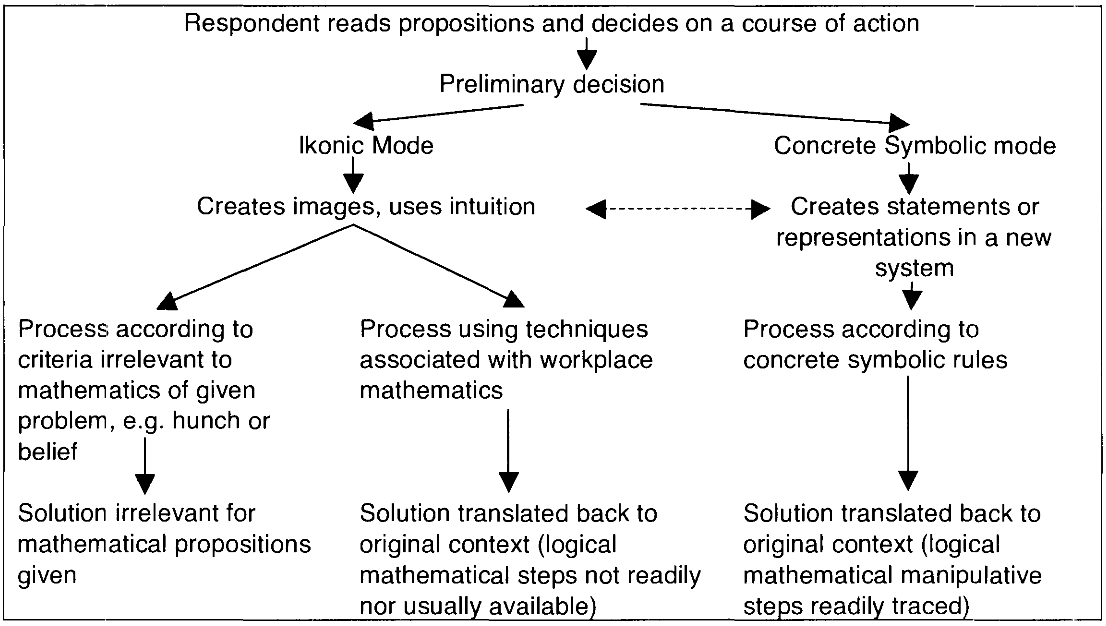

Figure 3.4 The problem-solving path: adapted from Collis and Romberg, (1991, in Collis, Watson & Campbell, 1993, p.111)

Collis and Romberg described the problem-solving path for a mathematical example in the following manner:

> Let us take for example the area of measurement: the Field A may represent the initial problem of measuring, predicting, or recording a measure of some empirical phenomenon. Consider the typical question that involves finding "How many?" The individual that has to solve the problem has basically two options. One is to use the ikonic mode of functioning and solve the problem by intuition and imaging, perhaps supported by some sensorimotor activity. The other is to translate the relevant aspects to the number field of concrete symbolic mathematics to operate upon them according to the model which appears appropriate in the field, and then map the result of the calculation back onto the empirical field.
>
> (Collis & Romberg, 1991, p. 93)

This situation describes a decision made by a student in early adolescence, in the problem-solving context, to either follow an ikonic or concrete symbolic solution path. However, adult functioning may be somewhat more complex, due to the fact that "as the individual matures physiologically, the mode(s) developed earlier continue to develop on the basis of the increasingly mature physical and intellectual background" (Collis & Romberg, 1991, p. 87). The sophistication of the earlier acquired modes increases with maturation, as does the modal selection available to the adult; the acquisition of higher modes does not replace, but coexist, with those obtained previously.

Workplace performance by adults is likely to be a compilation of functioning in different modes, including sensorimotor, ikonic and concrete symbolic, in order to complete the given task. Whilst the problem-solving path outlined by Collis and Romberg provides structure upon which to gain insight into the cognitive functioning or solving practical tasks, little empirical data exists to confirm its validity in contextual environments.

The workplace is a highly contextual environment where the solution to the problem must be considered in conjunction with many other factors; solving the problem itself often does not exist as a primary aim, but forms a part of a larger challenge. Within workplaces exist many salient cues, which can be used in the solution path, such as sounds, artefacts, and the knowledge of colleagues. Therefore, the degree to which a worker moves between the sensorimotor, ikonic and concrete symbolic modes depends largely on their environment, as well as their previous experiences.

The concept of intelligence is a key construct in a range of educational domains, and is highly debated (e.g., Gardner, 1999; Guilford, 1971; Spearman, 1927; Sternberg, 1985). Biggs and Collis (1991) discussed intelligent behaviour in order to attempt to give a possible structure to investigations into adult functioning in everyday situations.

Biggs and Collis (1991) list four areas that need to be investigated simultaneously with research into intelligent behaviour in this context. These are: (i) the structure of the behaviour in terms of quantitative and qualitative measures, (ii) the mode of cognitive operation within which the structure exists, (iii) the determination of uni-modal or multi-modal functioning, and (iv) an identification of in which content areas the behaviour occurs in.

To this point, everyday intelligence has been mentioned but not defined in depth. Biggs and Collis (1991) described a display of competence in non-academic contexts as "everyday intelligence". Together with definitions of workplace mathematics given by Collis and Romberg (1991) a comprehensive list can be compiled outlining characteristics of what is defined as *everyday mathematical intelligence*:

1. Problem solving is socio-contextual;
2. Skills are generally only demonstrated in the context in which they are acquired and practised;
3. Little manipulation is made of algorithms or symbol systems, with predominant usage of concrete aids enabling solution of problems;
4. Possible extraneous factors such as cultural, emotional, and economic can impinge on the use of mathematics;

5. Motivation to solve a directly experienced task is based on the importance of the task;
6. Intuitive reasoning is acceptable, however, concrete logic is required;
7. Information needed to solve the problem must be sought and selected on the basis of its ability to aid in solving the problem; and
8. The accuracy of the solution is defined by the needs of the situation.

Levels of operation of these characteristics can be used to describe effective functioning in day-to-day situations, with the manifestation of procedural knowledge in practical trades (such as plumbing, carpentry) often exhibited by various levels of proficiency. Biggs (1994) defined performance at the relational level as *competent*, and extended abstract performance as *expertise*. Table 3.2 compares the various components of the development of expertise, with Biggs (1994, p. 15) adding that expertise cannot be accessed by every individual, as it requires "a highly specialised and well structured knowledge base plus certain high-level cognitive abilities enabling radical restructuring of that knowledge base."

Table 3.2. The development of expertise (adapted from Biggs, 1994)

SOLO Descriptors	Assessment paradigm	Practical implications	Procedural skills	Performance
Unistructural	Quantitative	Learning one skill		
Multistructural	Quantitative	Accrual of more than one skill.	First-order procedural	
Relational	Qualitative	Placing skills on to a meaningful, integrated and functionally working whole	Second-order procedural	Competent
Extended abstract	Qualitative	Generalising from a known domain to a different domain resulting in a novel synthesis	Third-order procedural	Expert

The quality of intelligent behaviour appears to be affected by certain factors (Biggs & Collis, 1991, p. 67). These factors are *physical maturation*, having passed through the *relational level in the previous mode*, *available working memory*, *social support* and *confrontation with a problem*. Because this research is concerned with adult functioning, issues relating to physical maturation can be discounted as a significant factor in modal progression. Secondly, new modal functioning is reliant on adequate functioning at the relational level in the previous mode.

Increased competence with a singular mode can be described as high competence (or expertise) before generalising to a new domain. An apprentice takes time to develop new skills, and therefore must begin to focus on single aspects of a task (unistructural) so that they can accrue skills (multistructural), before generalisation of those skills to a complete activity (relational) can be achieved. At the multistructural level these independent skills require significant memory capacity for processing, and it is the ability to organise this information into mental schemas, that defines the capacity of working memory required to complete a task. At a relational level, organization of knowledge becomes clearer; however, "any increase after relational requires a change in the basis of organisation ... a modal shift" (Biggs & Collis, 1991, p. 68).

Biggs and Collis (1991) discussed confrontation with a novel problem as being a possible factor in modal shifts. In terms of workplace practice, the drive for money and a need to complete a task quickly and efficiently impact on an individual's motivation for completing a task that is different from what is already known. Therefore, the individual may attempt to solve the problem using previous experience and attempt to reorganise their knowledge for the new situation (Biggs & Collis, 1991).

This generalisation of the SOLO Model has not been discussed in terms of the discovery of two cycles of levels within the modes, nor has it been experimentally tested for practical skills.

Summary

Biggs and Collis (1991) stated that the SOLO Model has the potential to be a theory of intelligence able to describe competence in both academic and everyday contexts. To date, SOLO has given insight into many academic or traditional school-based subjects (as outlined in the previous section); however, application to everyday contexts has been highly anecdotal, with little empirical data existing with regard to the workplace.

Conclusion and Research Directions

In conclusion, it is important to state that the SOLO Model is focused on observations of the outcome of the learning. As such, its premise is not to account for external factors affecting the learned outcome, but merely to identify possible paths for optimal accrual of knowledge of a particular task. While it is important to acknowledge the existence of external factors outlined by social-constructivists and researchers into workplace learning, the SOLO Model only attempts to inform the teaching and learning practices involved by analysing the *responses* to tasks.

The SOLO Model is still evolving. Despite being increasingly validated for assessing the quality of responses to school-based tasks, there are areas that require further investigation. The lack of empirical data regarding cognitive functioning in workplace contexts, particularly involving tasks of a practical nature, remains an ongoing challenge for the research. If the Model is to evolve into a generalisable cognitive framework across educational sectors, it is important that this omission is addressed.

In the present study, the SOLO Model will be used as the conceptual framework upon which to analyse participant's responses to a practical task. This study has the potential to further strengthen the SOLO Model by giving deeper insight into cognitive aspects of workplace-related activity, and cognitive functioning of adults.

The present study primarily explores whether the SOLO Model is an appropriate framework to identify developmental pathways of skill accrual in the workplace. In particular, the following research questions drive the study:

> **Research Question 1: Can the SOLO Model be applied as a framework to describe cognitive performance in workplace practice?**
>
> **Research Question 2: Can the SOLO Model be applied as a framework to describe cognitive performance of mathematics in workplace practice?**

As a means of addressing the research questions above, a study involving plumbing apprentices was designed to explore cognitive functioning during the execution of a practical task. In particular, a decision was taken to explore downpipe fabrication with a group of apprentices who were in one of the first three years of training for the trade.

An important point garnered from the literature reviewed in Chapter 1 emphasised the importance of attention to research method in gathering data to inform understandings of mathematics in work. In the context of the present study, such an endeavour takes on additional significance. As noted previously, the cognitive-developmental focus of this thesis evolved due to the vast majority of research originating from the socio-cultural perspective. Hence, methodologies in existing literature in this field have not been shown to produce valid and authentic data for cognitive analysis.

In conclusion, there is a need to determine whether existing research techniques that have evolved from the socio-cultural perspective are sufficient and appropriate to gather data that is rich and comprehensive enough to support analysis using a cognitive developmental framework such as SOLO. Additionally, preliminary analyses are required to assess the whether it is at all possible to analyse workplace practice using the SOLO Model. The following chapter reviews research methodologies and outlines a Pilot Study employed to (i) test and refine the procedure for data collection to be used in the Main Study, and (ii) to perform a preliminary analysis on the data using the SOLO Model.

CHAPTER 4

RESEARCH METHOD

The previous chapters have reviewed both the literature relating to studies examining mathematics in work as well as constructive perspectives on how people learn mathematics out-of-school. An important theme that emerged concerned the potential of the use of a neo-Piagetian framework, with which to address the lack of cognitive developmental research into mathematics in and for work. It is evident from the concerns emphasised by researchers on past studies that careful planning is required to support data collection in this field.

This chapter addresses the methodological issues related to the present study, and is divided into two sections. The first section reviews a possible method and data collection technique. The second section describes the Pilot Study employed to test and refine the data collection technique to be used in the Main Study.

Research Background

Two avenues for gaining authentic data from vocational contexts with regard to mathematics have been identified as ethnomethodology and stimulated response interviews (Sträßer, 2000a). This section takes up these ideas in three parts. The first part briefly describes ethnomethodology and its underlying philosophy. This is followed by a discussion of possible impediments to collecting rich and authentic data in the context of the present study. The section concludes with a detailed account of the use of stimulated response interviews as a data collection technique within this methodological approach and in the context of this research.

Ethnomethodology

The major type of methodology utilised in examining mathematics in the workplace is ethnomethodology. Zevenbergen (2000, p. 222) discussed the importance of this approach and stated that

> ... [ethnomethodology] permits the documentation of the approaches, skills and knowledge used within particular contexts in order to understand the reasoning and problem solving skills employed by the participants. Within workplace settings, this is a particularly powerful methodology for deconstructing the ways in which participants understand and undertake their daily work practices.

In general, ethnomethodology is "concerned with how people make sense of their everyday world" (Cohen & Manion, 1994, p. 31) and relates to the study of a population for the "purpose of describing their socio-cultural activities and patterns" (Burns, 1994, p. 245). Within workplace contexts, ethnography has been found to be useful in documenting "approaches, skills, and knowledge ... in order to understand reasoning and problem solving skills" (Zevenbergen, 2000, p. 222).

There are no set procedures to follow when employing this type of methodology, and it has been acknowledged as being difficult to describe (Milroy, 1992). Despite the difficulty in definition, researchers who use ethnomethodology appear to be flexible in their planning, sensitive to contextual cues, and are at ease with change (Burns, 1994).

Additionally, Burns (1994, p. 249) listed six "general commitments" to which ethnographers should conform. These are:

1. The problem of understanding social action; that knowledge is socially constructed;
2. The emphasis on process; that meanings and interpretations are dynamic and changing;

3. The investigation of natural settings; what people say and do depends on their social context;
4. The study of social phenomena in their context; actions cannot be studied outside the whole environment in which they occur;
5. The assumption that there are always multiple perspectives; how different people define an event through their actions, perceptions, interpretations and beliefs; and
6. The use of multiple techniques, with emphasis on participant observation and interviewing.

The predominant focus of the following discussion is to provide an overview of what Zevenbergen (2000, p. 216) called the "key tools" for data collection which are used within this approach. These tools are participation-observation, interviews, and artefacts.

Participant observation

In the purest sense, participation-observation within ethnography requires total immersion of the researcher into the context of the study. Zevenbergen (2000, p. 218) stated that this often requires the researcher to "work in the field all day and ... spend the evening writing field notes". However, she also recognised that this is not always a "practical or sustainable" approach. Researcher participation is more often seen as existing at any position between full participation and observing (Ball, 1988).

Certain factors need to be considered when engaging in participation in a workplace. Zevenbergen (2000) outlined some pertinent considerations for ethnomethodologists in these circumstances, with an initial issue being the granting of access from the "gatekeepers" (Burns, 1994, p. 254) who allow access to the workplace, and may impose certain restrictions on that access. Considerations must also be given to assigning appropriate status to the researcher within the community of practice; to response effects, and to gender issues.

Interviews

Interviews are seen to be complementary to data collected through observation (Burns, 1994; Zevenbergen, 2000). The process of observation and the analysis of subsequent data may illicit questions that require clarification. Zevenbergen (2000, p. 220) stated: "asking questions about an incident or item allows the researcher to gain access to more information."

Interviewing is an integral aspect of the present study as it is the main method by which insight into participant cognitions are gained. Due to the importance of designing the most appropriate style of interview for the present study, this aspect is discussed further in the next part of this section, particularly regarding impediments to gaining rich and authentic data.

Artefacts

Within the context of mathematics in work, artefacts are described as potentially consisting of mathematics-containing technologies or products of work. However, they may also consist of representations of practice produced by mechanically-recorded means such as tape recordings, video recordings or photographs.

As such, artefacts may be used to supplement observations, or "provide the catalyst for interviews" (Zevenbergen, 2000, p. 211). Additionally, in circumstances where participants have trouble verbalising performance, artefacts have been found to provide stimulus for discussion (Zevenbergen, 2000).

Summary

The preceding discussion has provided a brief description of what comprises an ethnnomethodological approach to uncovering mathematics in work. The pre-text of this study is to examine workers mathematical cognition from a developmental perspective. Therefore, it may seem contradictory that the underlying methodology chosen is grounded in socio-cultural constructivism. However, the contribution of socio-cultural research to examining mathematics in work is to be acknowledged. The ethnographic approach to acquiring rich and authentic data is well documented as being successful in this regard.

Impediments to gathering rich and authentic data

When dealing with workplace contexts, there are two major impediments to gathering rich and authentic qualitative data by which to analyse worker's mathematical cognitions. An overview of these two issues is provided in the following discussion. These issues are (i) that workers' are often not able to identify the mathematics that they are using, and (ii) that their mathematical cognitions are not directly available to the researcher and have to be accessed through carefully constructed data collection techniques.

Perceptions of the mathematics in use

There are two possible explanations for workers sometimes not recognising the mathematics they are using, both of which are related to the fact that workplace mathematics is embedded in context (e.g., Noss, Hoyles, & Pozzi, 2000; Sträßer, 1996, 2000a; Sträßer & Zevenbergen, 1996). This anomaly has already been discussed at length in Chapter 1, with the underlying reasons being that the mathematics in use may be hidden within artefacts or technology, and that the workers situated abstractions often differ greatly from those of the researcher.

The workers' access to mathematics is also less likely in circumstances where efficiency implies automaticity. Cognition that is automated is generally not accessible to the individual (Ericsson & Simon, 1980). Hence, it is not surprising that in cases where the automated task involves mathematics, then the workers do not recognise it.

Ericsson and Simon (1980, p. 222) stated the following with regard to interviews:

> The request for a certain type of information, may serve as a hint to subjects about what aspects of the task are important. Subjects may also alter their normal mode of processing in order to be able to give the requested information to the experimenter on subsequent trials.

From the perspective of research design, interview questions that ask the worker "What mathematics do you use?" may not be able to provide useful or authentic data. Such questions can also give undesirable response effects leading to inauthentic data on actual workplace practice.

Access to cognition

The second major methodological challenge faced by researchers attempting to explore the quality of worker's mathematical cognition is that cognitive behaviour cannot be measured directly by the researcher. It must instead be gathered by observing actions (Biggs & Collis, 1982) and gaining access to the thoughts of the participant. Consequently, in order to assess the quality of workers' mathematical cognitions, two aspects need to be considered. The researcher needs to gather information using techniques such as field notes and observations, as well as interviews, to gain access the worker's cognitive perspective. Secondly, the most common method of gaining access is by interviewing the participant. White and Gunstone (1992, p. 65) stated:

> An interview about an instance is a deep probe of the students' understanding of a single concept, that checks whether the student can not only recognise whether the concept is present in specific instances, but also whether the student can explain his or her decision. The explanation reveals the quality of the students' understanding.

Interviewing is common in studies of workplace mathematics to gain insight into participants' experiences and points of view. There appears to be three main types of interview method: (i) participant verbalisations of practice in "real-time", (ii) participant reflections on action after the event, and (iii) participant reflection on action after the event stimulated by reflective prompts.

In the first type of interview, the researcher is often immersed in the workplace, with explanations of practice given by the worker during the task performance. This procedure has received some criticism, particularly because of the potential for concurrent verbalisation to cause "an unacceptable degree of interference with task performance" (Marland, 1984, p. 157). Thinking-aloud strategies have also been documented as placing additional strain on working memory and impeding "best-practice". Studies such as those by Dickson, McLennan, and Omodei (2000) proposed that using concurrent verbalisation to study cognitive processes in task execution could have "reactive effects on task processes and downgrade performance". However, despite the limitations, such strategies have the benefit of providing the participant with the opportunity to explicate practice in the presence of salient cues in real-time.

The second type of interview involves obtaining reflections on a lived event. These data are collected after the event to provide potential insight into the participants' point of view. Generally, such interviews are semi-structured and seek to clarify issues that arose during the observation (e.g., Hogan & Morony, 2000, p. 104), whilst "eliminating any possibility that the probing will affect the "real" data of the experiment" (Ericsson & Simon, 1980, p. 220). What such interviews cannot guarantee is accurate recall, due the absence of appropriate stimuli.

The final type of interview involves the participant providing thoughts regarding their performance after the event, with these thoughts being stimulated by appropriate cues. Initially, the task is performed and then followed by an interview session; static artefacts (such as photos) and targeted verbal probes are on hand to provide the cues and stimulate recall. In support of such methods, Sträßer (2000a, p. 245) was convinced that stimulated recall research was superior to traditional interview and survey studies. He determined that such methods enabled the researcher to find the mathematics hidden within the workplace practice. An example of such a study examined the mathematics in use by investment bank employees, paediatric nurses and commercial pilots (Noss, Hoyles, & Pozzi, 2000, p. 21). In relation to the data-collection methodology, Noss et al. (2000) stated the following:

> A series of task simulation interviews was devised, based on a selection of workplace activities and contexts. The majority of the simulations recreated the breakdown episodes witnessed during the ethnographic observation. Each scenario was made as 'real' as possible, by the use of context-rich descriptions of the activity, with supplementary information provided in the form of familiar resources and visual displays: e.g. copies of blood pressure charts, screen shots of instrumentation.

These interviews provided insight into the participants' reasoning while allowing the researchers to manufacture novel situations. However, this method does not provide information regarding actual practice, as such, "simulations" cannot be said to provide the participant with salient cues in real time. Contrived episodes of simulated workplace practice are to some extent decontextualised, devoid of the affective factors influencing performance in an actual real-time task.

For interviews that rely on participant reflection on a certain task, the quality and authenticity of the data gathered is directly related to the appropriateness of the cues provided to stimulate recall. The accuracy of either explanations of practice, or thought-in-action, rely heavily on the provision of salient cues to stimulate such recall.

Summary

In order to gain insight into human cognitive processes, interviewing is the most logical and easily applied technique. However, there are many different ways in which interviews can be carried out. In circumstances where cognitive processes involved real-time decision-making, particular care should be taken in order to ensure the validity of the data collected.

When the interviewing relates to a completed task, the use of salient cues from that task provide the participant with assistance to recall their thoughts at a particular time. One way of providing salient cues to stimulate recall is by using videotaped recordings of task performance.

Traditionally, video recordings have been used by researchers in triangulation with other data sources such as field notes and artefacts, providing additional real-time data in context. Video data have the ability to (i) preserve subtle aspects of performance (eye gestures, body posture, intonation, environmental noises and subsequent interactions), (ii) allow for repeated examination of data, (iii) contain no intrinsic bias, and (iv) are flexible with regard to setting types. Although such data increase the validity of analysis when used within mixed-methodological studies, issues of researcher subjectivity must still be addressed. Using video-data of the lived event as a stimulus for an interview, however, can provide data from the participant's point of view, enabling some minimisation of researcher subjectivity in analysis.

With regards to interviews, previous discussions so far in this thesis have pointed to stimulated recall (or 'stimulated response') style interviews as being useful in uncovering mathematics in the workplace. As such, the following discussion looks more closely at this particular manifestation of the traditional interview.

Stimulated Recall

Stimulated Recall is a data gathering technique that has the potential to provide insight into a worker's mathematical cognitions through the collection of rich and authentic data. The following discussion outlines the underlying rationale of the Stimulated Recall method of data collection as well as possible limitations in its use.

Overview

Stimulated Recall (also referred to in behavioural sciences as Interpersonal Process Recall (Kagan & Kagan, 1991)) is a technique designed specifically to provide access to the cognitive processes of human problem solvers during performance tasks (Marland, 1984). First pioneered by Bloom in 1954 to study the thought processes involved in university students' discussions, the most common instances of Stimulated Recall as a defined genre of methodology have been to examine the "in-flight" (Paterson, 2000, p. 44) thoughts of teachers (e.g., Marland, 1984; Paterson, 2000; Stough & Palmer, 2003), and the reflective practice in the teaching and learning of physical activities (Lee, 2002; Parker & Pittney, 2003; Solomon & Lee, 1997). Although these areas of research are not exclusive, this researcher has found no overt use of this technique in existing literature on mathematics in vocational contexts.

Stimulated Recall predominantly involves the use of mechanically recorded visual and/or aural data, consisting of a participant's behaviour whilst they are carrying out a particular task, to “stimulate recall of simultaneously occurring thought processes” (Marland, 1984, p. 156). The common underlying rationale for research using a Stimulated Recall method is that it potentially provides the participant with appropriate stimuli to be able to relive an original situation.

Marland (1984, p. 157) described the use of Stimulated Recall in circumstances where think-aloud methods would cause “an unacceptable degree of interference with task performance”. More recently, Dickson, McLennen, and Omodei (2000, p. 220) cautioned against the use of concurrent verbalisation regarding in-task execution as it may have “reactive effects on task processes and downgrade performance”. To maintain the internal validity of the data, these concerns must be taken into account, particularly when the day-to-day execution of tasks being observed are executed without such concurrent verbalisations.

Limitations of using a Stimulated Recall as a data collection technique

Stimulated recall is a powerful methodology for data collection (Paterson, 2000); however, special care must be taken in its application. To date, no singular optimum way for applying the technique has been determined, with individual research circumstances dictating the particular methodology required. Keith (1988, p. 8) raised three issues that a researcher should pay attention when adopting the Stimulated Recall technique: (i) possible bias in questioning, (ii) the nature of the stimulus, and (iii) the availability of interactive thoughts for recall.

Bias in questioning. Bias in questioning appears to manifest mainly within the nature of the probes being directed at the participants during the interview. These probes may cue the participants to certain areas that the researcher may wish to focus on, and must be carefully monitored to reduce response effects.

Keith (1988) suggested that one way by which to reduce these response effects was to use unstructured interviewing techniques. Questions such as “stop the video when you want to discuss what you see” or “tell me what is going through your mind” can be used to direct the participants to what is required from the interview. In order to reduce distortion of the participants’ verbal data, the probes must remain neutral. This makes it less likely that the participant is prompted and their self-reporting data distorted from what they otherwise would have said (Yinger, 1986).

Nature of the stimulus. Marland (1984) made two assumptions regarding aural or visual nature of the stimulus. Firstly, such stimuli should provide sufficient cues to "stimulate accurate and near-complete recall of mental processes" (p. 158). Secondly the presence of the recording hardware should not change the nature of the performance in a research setting.

To address the first point, Stimulated Recall proponents generally assume that the up-close and uninterrupted record of performance prompts participants to recall as accurately and realistically as could be expected, providing in-flight thinking at the time of the event. Additionally, provision of optimum recall conditions require that the recorded data must be as close as possible to the perceived view of the participant.

Despite these assumptions regarding the technique, the extent to which retrospective reports on in-flight thinking can be considered accurate sources of data fluctuates depending on the researcher. Critics such as Yinger (1986, p. 273) argued that at best, Stimulated Recall is only "tangentially related to actual thinking during the recorded event and at worst be entirely fabricated". He added that it is more likely to be "reflection-on-action" rather than "reflection-in-action". However, close examination of the transcripts of the interviews can provide useful clues to filter such data.

The second point to be raised is associated with the stimuli, and refers to the effect of the videotaping itself on the initial task performance. Marland (1984) confirmed the importance of keeping the setting naturalistic. This can be done by (i) establishing a rapport with the participants prior to the data-collection sessions whereby making the participants feel at ease with the researcher, (ii) establishing a comfortable precedence with the presence of the video camera by recording the everyday goings on in the context, (iii) making sure that the participants understand the focus of the research clearly, and (iv) that the videotaping does not unduly disrupt the naturalistic setting.

An additional complication with regards to the nature of a video stimulus deals with the effect of viewing oneself. Keith (1988) and Yinger (1986) both stated that such an experience was likely to have an impact on the authenticity of reports of perceived in-flight thinking.

Availability of interactive thoughts for recall. As mentioned earlier in this section, when performance becomes automated, these behaviours are often unavailable for verbalisation on a cognitive level. This means that the availability of interactive thoughts for recall is potentially decreased. Such a notion is supported by Keith (1988, p. 12) who stated that in studies of classroom teachers, tasks of an automated nature were "not available for verbal reports because [participants] are not aware of them".

Consequently, it may well be impossible for teachers to describe their highly automated interactive decisions and the cues used to make them, even if they wanted to do so. Therefore, Stimulated Recall interviews need to be carried out no more than 48 hours following the event. The theory underlying this restriction is related to short and long-term memory (see the following for in-depth examples of this work: Ericsson & Kintsch, 1995; Ericsson & Simon, 1993). Ericsson and Simon (1980, p. 218) stated: "what is remembered, and how well, will generally depend critically on the interval between the moment of acquisition and the moment of recall."

The availability of interactive thoughts for recall are closely related to the idea that cognitive processes cannot be measured directly (Biggs & Collis, 1982; Paterson, 2000), but observed by associated processes. As such, interviews carried out by Stimulated Recall using videotape potentially suggest that participants are "responding not only to their memory of the viewed situation, but also to a set of cues supplied ..." (Yinger, 1986, p. 269). Therefore, data can only be interpreted as a combination of recall and construction of thought, as the participant is not likely to be able to discern between the two (Yinger, 1986, p. 270).

Such criticisms can be overcome by accumulating knowledge of the event prior to the main data collection in "order to present the 'right' stimuli for the worker" (Sträßer, 2000a, p. 245). Additionally, criticisms of Stimulated Recall techniques that propose that the participant responds primarily to the stimulus and only secondarily to the thoughts that occurred at the time of the event (Yinger, 1986) can be partially addressed by careful analysis of the discourses collected.

Of importance is the need to make the participant aware of the purposes for the creation of the stimulus material (video) immediately. The participant needs to be aware of what sorts of things they need to be thinking about during the task performance. Ericsson and Simon (1980, p. 218) stated with respect to retrospective verbalisation: "The subject's performance may depend heavily in how much incidental memorizing he or she does whilst performing the initial task." If a participant is going to be expected to recall the thoughts they had at the time, they need advance warning of this.

Summary

Stimulated Recall as a sole data collection instrument within a research design would be unlikely (Calderhead, 1981). Recent research methodologies using Stimulated Recall have involved mixed-methodological constructs (Corrigan, 2001; Ethell & McMeniman, 2000). The validity of the data collected was strengthened by the triangulation of complementary data collection techniques such as researcher field notes and observations, stimulated recall techniques, and semi-structured interviews.

Despite the limitations of the technique as outlined previously, it is proposed that a Stimulated Recall methodology using video, in triangulation with other forms of data collection, provides the most authentic data gathering technique for participants in the present study.

Conclusion

The predominance of socio-cultural studies has provided research techniques that have been developed, tested and refined constantly for a number of years. In the field of mathematics in and for work, one methodology that is commonly utilised is Ethnomethodology. Ethnomethodology can be particularly useful when looked at more as an underlying philosophy as opposed to a set of steps to follow when attempting to carry out a study in this field.

The underlying philosophy of ethnomethodology can be generally explicated through the following key points: (i) that it has a socio-constructivist disposition, (ii) that understanding that meanings and interpretations are dynamic, (iii) that social context influences verbalisations and actions, (iv) that actions must be studied in context, (v) that there are always multiple perspectives, and (vi) that it must incorporate multiple sources of data with a focus on observation, interviews and artefact collection.

Analysis of cognitive processes involved in the mathematics in and for work derives more from a developmental constructivist perspective. However, such research is not well documented. The SOLO Model is the developmentally-based theoretical framework proposed to underpin the data analysis in this thesis. This framework largely requires coding of qualitative data, a requirement that is satisfied by the use of an ethnomethodological research design.

The method of Stimulated Recall for data collection has been identified as being a useful tool within the research field under consideration. This section discussed various styles of interviews, with the most appropriate in the present context involving a video of performance as the stimulus for the interview sessions.

Three issues arise from the previous discussion; these are all linked to the collection and nature of data:

1. There have been no reported papers on workplace mathematics that have overtly utilised a Stimulated Recall technique;

2. Stimulated Recall using video has not been mentioned in the literature regarding the use of this particular data collection method. Hence, there are no protocols for the set-up of video within a practical setting, nor the use of such visual data in conjunction with interview; and
3. There exists no empirical evidence that the interview transcripts based on a Stimulated Recall using video technique provides data that are able to be coded using the proposed cognitive-developmental framework of SOLO.

It appears that a Stimulated Recall Technique using video, based within an ethnomethodological research design, is the most appropriate in the present study. There are grounds for a preliminary study to investigate the extent to which such a research design provides data able to be analysed by the proposed theoretical framework.

Pilot Study

The previous section presented a review of the research methods. A theoretical perspective on the potential of an ethnographic research design utilising a Stimulated Recall using video data collection technique was given. However, the use of such a design to examine workplace practice has not been explicitly reported, nor have empirical data been gathered as to the feasibility of the SOLO Model in determining cognitive developmental pathways for (and assessment of) practical workplace skills; this final point being the main theme of this thesis.

This section addresses four specific questions relating to a proposed research method:

1. Can a practical task be appropriately devised and clearly explicated to maximise participant understanding of requirements?
2. Do any changes need to be made to the data collection technique in terms of video data recording and interview length and procedure?

3. Did the data contain evidence of cognitions?

4. Was the practical problem, sample selection and data-collection technique sufficient to provide rich and authentic data that could be potentially coded using the SOLO Model?

These questions are addressed in this section, which is divided into two parts. The first part furnishes the design of the Pilot Study employed to confirm a suitable practical task and data collection technique. This section concludes with the results of the Pilot Study and its implications for the Main Study.

Design

This sub-section deals with the design and execution of a Pilot Study designed to address issues arising from the literature review, and is divided into three parts. Firstly, details concerning the education of plumbers, geographical location of the research, and demographic of participants is given. In the second part, the set-up and methodology employed to carry out the data collection is provided. Finally, results of the Pilot Study are presented and the preliminary questions addressed.

Context

The plumbing trade was chosen as the focus for this present study for two main reasons. Firstly, the analysis of trade course material and plumbing practice contained large amounts of overt and covert mathematical components. Despite only a minor part of the course being devoted to 'calculations', apprentices required mathematics that was hidden within actual practice. Secondly, research involving the mathematical understanding of plumbing apprentices and tradesmen was largely absent from the literature.

Due to the exploratory nature of this study, Technical and Further Education (TAFE) colleges were chosen as the context. For the building trades (e.g., carpenters, plumbers, electricians) these colleges are the intermediary step between school and the workplace.

The context for the pilot study is given in three parts: (i) an overview of the education of plumbers in New South Wales, (ii) the geographic location of the TAFE college, and (iii) the demographics of the cohort from which the participants were chosen.

Overview of the education of plumbers in New South Wales. Enrolment into the Plumbing Trade Certificate III TAFE course has no formal educational entry requirements. The only stipulation is that the students must be employed as apprentice plumber with a firm engaged in the plumbing trade. General educational and performance outcomes are consistent throughout New South Wales. Students are trained to:

1. State and demonstrate the safe working practices required to work efficiently in the plumbing industry;
2. Select and use equipment, hand and power tools common to the plumbing industry;
3. Install, alter and repair pipe work services for cold and hot water supply, gas supply, sewage, waste and rain water disposal;
4. Install metal roofing, gutters, downpipes and flashings, etc. to produce a waterproof finish; and,
5. Apply relevant regulations and safe practices.

(TAFE, 2007)

Apprentices undertake paid block release from their employer to attend the course. This block release takes the form of one week per month at the nearest TAFE College offering a plumbing course. Apprentices are expected to attend for 16 hours per week per month for the first three years, and 24 hours per week per month for the fourth year, although this may vary slightly from college to college.

All apprentices are encouraged to develop links with other trades as well as maintain a high standard of work. Assessment of performance is by written assignments, evaluation of practical skills, and satisfactory performance in end of course examinations. Teachers of these courses use experience and professional judgement to deliver the course outcomes in the most appropriate manner to students. There appears to be no cognitive theory guiding teaching, learning or assessment practice.

Geographic location. The participants involved in the pilot study were plumbing trade apprentices enrolled at the TAFE College in Tamworth, NSW. Tamworth is a rural city located in north-western New South Wales and has a population of approximately 35,000 and is the largest population centre in the region. The main economic support of the town is agriculture, with tourism the next largest income generator for the town. The city also has a large number of high schools and a university access centre.

The participants. Personal communications with Ian Winter (owner of a large metropolitan plumbing firm) and Greg Inglis (self-employed plumber employing apprentices and tradesmen in the rural sector) confirmed that apprentices in small businesses have a wider variety of plumbing experiences, as opposed to those employed in large companies where they may only focus on a few specific tasks at any one time and perform these tasks numerous times. A similar phenomenon was also observed in a study of hairdressers: in smaller salons, apprentices had a wider variety of experiences earlier than those in larger workplaces (Billet, 2001). An additional measure was taken in this study to limit participants to those who worked only for small plumbing firms.

Following consultation with TAFE teachers, apprentices were approached for participation in this pilot study. These apprentices were perceived by their teachers to be average students for their level of study. Three of these apprentices were approached and agreed to participate, one for each of the first three years of the plumbing course: Viv (first year apprentice), Wally (second year apprentice), and Xavier (third year apprentice).

These apprentices worked in Tamworth for small plumbing firms (between 2 and 10 employees), and were aged between 18 and 20 years of age. However, the cohort from which they were drawn enrolled students between 16 and 22 years of age. A smaller number of "apprentices" enrolled in the plumbing course were mature age. These people had worked in the industry for some time but were expected to gain formal qualifications for varying reasons.

Practical task selection

The underlying rationale for the practical task selection was to provide the participants with a dynamic and realistic plumbing task. The task needed to be (i) an activity that could be attempted by apprentices at any stage of their learning in the TAFE Plumbing Syllabus, and (ii) require some use of mathematics.

Preliminary work in devising an appropriate task involved consultation with two qualified and practicing plumbers, each with more than 10 years experience in the trade. A number of tasks were analysed for mathematical content; from these, three tasks were proposed for vetting by TAFE plumbing teachers.

The second stage of task selection was to ask current TAFE plumbing teachers which of the three tasks would be most appropriate to pose to their students in a TAFE setting. The following two provisos were given to the head teacher in the TAFE college for his consideration in selecting the most suitable task: (i) the task needed to be authentic from the perspective of the participants, and (ii) the task context had to be familiar to any plumbing student in each of the first three years of TAFE instruction.

The head teacher was knowledgeable regarding the capabilities of his students and was helpful in the appropriate selection. The task was formatted, published and presented in a way that mirrored normal instructional practice at the college.

This process resulted in the selection of a task that involved fabricating an offset rectangular section downpipe (75mm x 50mm). This task was given to the participants in a Job Sheet format similar to the type that they would be given during a TAFE practical class. A copy of the Job Sheet used in the Pilot Study is given in Appendix A.

Data collection

The purpose for undertaking the Pilot Study was to minimise irregularities within the overall methodology, in order to ensure the production of reliable data for use in the larger investigation proposed for the Main Study. To allow for the small number of participants, the design needed to be cyclical in nature, with the data collected acting as a catalyst for change in procedure for the next participant. This was facilitated by the structure of attendance at the TAFE course: each participant could be interviewed one week apart. This allowed for significant time to modify techniques for data gathering and analysis of transcripts, videos and artefacts.

Prior to the first interview, a data collection protocol was produced based on ethnographic and stimulated recall guidelines, as well as taking into account the context and nature of the task (Appendix B). This protocol was followed and modified following each of the three data collection episodes.

The following steps outline the process for data collection in this exploratory study:

1. The participant is provided with a job sheet and workstation;
2. Participant is videoed whilst carrying out the practical task;
3. The participant undergoes a Stimulated Recall interview using video;
4. The participant is interviewed regarding the process of data collection;
5. The collected data are analysed; and
6. Data collection protocols are modified for next participant.

The apprentices involved in the Pilot Study were given the Job Sheet, a workbench in the practical area of their department, and access to any tools or materials that they needed to complete this task. The TAFE teachers oversaw the set-up and appropriateness of the task so as to ensure that it was familiar to the students.

Each apprentice was asked to complete the task outlined on the job sheet using any materials and by any method he chose. On average, these tasks took less than 40 minutes to complete.

The nature of the environment and participants within this study meant that it was important to be sensitive to the needs of the participants, especially with regard to any feelings of embarrassment in front of peers and anxiety regarding participation in research related to academic activity. The researcher therefore maintained a low profile within and around both the participant and the cohort of the class during the data-gathering phase.

Additional factors impacting on the implementation procedures was that the environment was male dominated, and the researcher was female. This element was largely overcome by having credibility within the profession: the researcher knew enough about the profession, as she was a partner in a large plumbing firm in a local town. This had the added benefit of ensuring that the researcher and participants shared a common language. Additionally, it was important to recognise and acknowledge the participants rather than the researcher as being the knowledge-holders, and that the purpose of this study was to obtain some insight from them.

The participants were videoed during the task execution. Immediately following the completion of the task, the apprentice was taken to a separate room and asked to watch the video and recall, as accurately as possible, any thoughts they had whilst they were carrying out the task. These thoughts were audio-taped and the interview protocol followed stimulated recall guidelines as outlined in Marland (1984).

During the interview, the apprentice was given the opportunity to recall what they were thinking during the performance of the actual task. Probes and prompts were interposed as per the stimulated recall guidelines. Clarifications were prompted by questions such as "why did you do ________?" and "I noticed you did _________, can you tell me why?"

To assess how the participants felt about the data collection process, additional questions relating to the act of data collection were asked after the stimulated recall interview. These questions probed how the participants felt while they were being videoed, what it was like to watch themselves on video, was the camera at an angle where they felt they could discuss their thoughts on performance, and would they change anything regarding the data collection if they were to undertake the process a second time.

Findings

Following the completion of each of the three participants' performances and subsequent interviews, the results were analysed. An evaluation was made of the (i) the appropriateness of the practical task, (ii) the implementation procedures for data collection, and (iii) whether the data collected contained evidence of cognitions of sufficient quality and variability to potentially analyse using the SOLO Model. These findings were used to refine the final design of the Main Study.

The practical task.

The practical task of fabricating an offset downpipe was completed by each of the three participants; no additional information was required and the materials and equipment were sufficient. As such, the task was deemed to be appropriate. However, there were some elements of the overall context of the task that needed to be refined. The participants questioned the authenticity of the materials provided, and sought clarification of procedural methods for task completion.

Viv commented that the downpipe material provided for fabrication, a cross-section of 75mm by 50mm, was not common on the job site. He stated that he usually worked with a cross-section of 100mm by 75mm. This observation was confirmed by discussions with Wally as well as with tradesmen working in the profession. As such, the downpipe supplied for the Main Study was changed to 100mm by 75mm. A copy of this modified Job Sheet is provided in Appendix C, and was incorporated into the Main Study.

Although there were no other issues regarding the Job Sheet or materials, there arose one issue regarding the manner in which the task was to be completed. Viv and Wally asked whether they were to carry out the task as they would on the job site, or whether they were to do it the way “it was done at TAFE”. This comment posed new considerations for the data-collection protocol, and the way in which the task was set up for the participants. Further discussions with the participants and TAFE teachers found that there were contextual, and teaching and learning differences between TAFE and the workplace.

The three participants each stated that they were instructed by their employers to do what they had to do at TAFE to pass the course, but that TAFE instruction conflicted with the way that “you do it in the real world”. Hence, the participants wanted clarification to which way the researcher expected the task to be done: was it to reflect workplace practice or TAFE practice?

When completing tasks at TAFE, the majority of salient job site cues that the participants would utilise in actual fabrication were removed. If apprentices had to construct a downpipe on a work site there would be a building from which to take measurements. Additionally, experienced plumbers used a set angle for their downpipes, either by having a sliding bevel set permanently at a pre-ordained angle or by knowing the set cut-out measurement. If a new downpipe of unusual bend angle was required, there was usually another to which it needed to be matched: measurements could be taken from the existing downpipe and a trace made to establish required fabrication measurements. All of these cues were absent in the TAFE setting, and therefore absent from this present task.

It was not possible in this study to provide such salient cues, but it was necessary to retain as much authenticity in task completion as possible. In order to provide some meaningful context, and therefore some sense of motivation through relevance, a monologue was included prior to commencement of the task:

> It is a rainy day and there is no work to do outside. Your boss has left you in the shed to make up some downpipes for a job to save time. He has been out to the job site and taken measurements from the existing downpipes so that the ones you make will be the same. He has left you this job sheet. Your task is to make this offset downpipe to these specifications. You can complete the task using any method you choose.

This monologue was used in the data collection for Xavier. It provided a realistic context to the task under consideration by the participants despite the absence of the everyday operational cues on which they would typically rely.

Data collection.

The use of a stimulated recall data collection technique involves the creation of an appropriate stimulus, as well as carrying out an interview session using that stimulus. These two complementary aspects of the data collection are reviewed in the following discussion with relation to the Pilot Study.

The preparation of a video record that provides an appropriate stimulus to enable recall of thoughts is an important element of the Stimulated Recall technique proposed for this study. In order to prepare the video record, two aspects need to be considered, namely: (i) that the presence of the video camera should not affect the nature of the performance of the task, and (ii) that the stimulus should provide adequate cues to enable the participant to recall their thoughts-in-action accurately.

To minimise the impact and unsettling nature of videoing the apprentices, a video camera was set up in the workshop each morning on the day of data collection. The presence of the video camera was a novelty initially, with all the apprentices wanting to see themselves on tape. The recording of the class as a whole appeared to dispel much of the anxiety felt by the individual being recorded later in the day.

Viv was the first to be videoed and interviewed. The recording of the task was carried out by the researcher using a hand-held video camera in close proximity to the participant. No tripod was used. The close proximity of both the camera and the researcher made the participant somewhat nervous and conscious of their performance, as he made constant apologies for mistakes or time delays in completion. The subsequent two participants were left alone to complete the task, with the video-camera on a tripod. Despite initial anxiety, once the participants began to concentrate on understanding the task, overt nervous behaviour disappeared. Xavier specifically stated that he had forgotten that the camera was there during the completion of his task.

There was some experimentation with various taping positions involving Viv, but the participant's direct line of sight was ruled out due to the dynamic nature of the task. The camera needed to be set up in a position that would not impede the movement of the participant. Wally was distracted in the interview sessions regarding how he appeared from the chest up. Therefore a camera angle directly in front of the participant and filming only from the chest down, and including the associated workspace, was considered most appropriate.

The videoing of Xavier took these factors into account, with an addition of asking the participant to look through the camera view-finder with the researcher standing at the workspace. Although the recorded picture took in quite a wide expanse, it was important that the participant knew the limitations. Xavier was also shown how to turn off the camera when he was finished. This meant that the researcher did not have to keep interrupting the performance to check if it had been completed.

The practical area at TAFE was not initially conducive to a private workspace. Hence, Viv was videoed within sight of the rest of his cohort and his teacher. This proved to be distracting for him and it appeared that his concentration on the task was not maximised. His peers were constantly going over to the workspace and talking to him. He also stated that he felt embarrassed with his peers watching him.

To overcome this problem with Wally and Xavier, a separate space with a dividing curtain was created. Thus, the participant was able to complete the task privately without peer or researcher presence. Wally made no specific comments relating to this arrangement, but Xavier stated that he was grateful that "his mates" did not see what he did.

Use of a stimulus in an interview session. The second issue associated with the preparation of the video record related to its ability to provide enough cues to stimulate recall. This issue is discussed below in relation to the stimulated recall interview.

The interviews were carried out in a separate classroom within 30 minutes of the completion of the practical task. The participant was informed as to the purposes of the interview and the information required (see Appendix B). The interviews relating to the task took no longer than the duration of the task. This was due to the fact that although each of the three participants were informed that they could stop the tape whenever they wanted to, none took up this offer. Following the completion of the Stimulated Recall interviews, additional questions were asked regarding the data collection method.

The interviewer/participant rapport was very important in each of the interviews. This relationship, nurtured prior to the task execution, was essential in overcoming feelings of anxiety mentioned earlier; highly influential here were the assurances of confidentiality of performance. Interestingly, all the participants were unsure as to what they could possibly say that would be important; they saw their performance as nothing out of the ordinary. As such, the interviewer re-stated the importance of their knowledge, and that they were certainly informing the interviewer who really was not knowledgeable on the subject. The interviewer's insistence that the participants were the holders of the knowledge was crucial to achieve the participant's full disclosure. Few changes were needed at this point regarding interviewing using a video record as a stimulus.

Quality of the data.

Obtaining rich and authentic data useful in gaining insight into the participants' mathematical understanding relating to the task proved to be the most difficult factor in the Pilot Study. Two dilemmas arose here; firstly how to obtain evidence of mathematical cognitions without response effects, and secondly, how to gain access to cognitive thought.

In the first instance, to have revealed the mathematical focus to the participants may have biased their responses. This bias potentially meant that data would contain mathematical references, whereas in actual practice such recognitions may or may not have been present in the minds of the participants. Therefore, the decision was made to withhold from the participants that the focus of the research was on mathematical knowledge and skills.

Consequently, data were collected that could also determine whether the participants were overtly or covertly using mathematics in their solution strategy. A consequence of this approach was that responses in the interview sessions were varied and often did not contain references to mathematical concepts. Often, comments were made with relation to other issues of importance, such as how much easier it would have been to have the use of hydraulic tools.

The focus of the present study is the analysis of mathematical cognitions in workplace contexts. As such, data collected need to contain evidence of cognitive behaviour in this domain. Cognitions are thought processes, and a necessity in assuring rich and authentic data was the *availability of interactive thoughts for recall.*

The use of open-ended interviews using the video as the stimulus was a successful approach in providing a level of verbalisation of thinking or explanations about the task for these participants. All interviewed described something about their thinking processes, but most of the responses involved explanations of why certain actions were taken. In many cases, participants gave running commentary of what they saw on the video.

Ericsson and Simon (1980, p. 221) made specific reference to explanations of practice:

> ... the subjects may be drawing on a variety of kinds of prior information, such as general knowledge on how one ought to do these tasks, to generate a verbal report describing a general procedure or strategy. In this case, the verbal reports may not bear any close relation to the actual cognitive processes used in the tasks.

However, these descriptions of action cannot be discounted entirely, as they are connected in some way to the participants' thoughts due to the automated nature of the task (see Fitts, 1964). Indeed, despite the "pure" vision of a Stimulated Recall technique assuming that subjects recall only what they were thinking at the time of the event by viewing a stimulus, more liberal Stimulated Recall researchers agree "that what people think influences directly what they do" (Marland, 1984, p. 158). As such, these apprentices' explanations or reflection-*on*-action (as opposed to reflection-*in*-action) could be a direct influence on their performance at the time.

In terms of a Stimulated Recall methodology for this study, both types of verbalisations are important for understanding cognitive functioning and decision-making. Such data "will better provide understanding of the information processing skills and styles that characterize ... performance" (Marland, 1984, p.158). Although there remain criticisms of this data collection technique, Marland (1984, p. 164) allowed that

> In spite of ... problems and limitations, there is a generally held view among those who have used stimulated recall that the technique does offer fruitful means of exploring cognitive concomitants of human behaviour in the classroom and other task settings, and of developing new insights into important enterprises ...

There appeared to be three types of data evident in the transcripts of the participants: (i) verbalisation of thoughts they were having at the time of task fabrication (reflection-in-action), (ii) verbalisation of thoughts at the time of watching the video (reflection-on-action), and (iii) explanations of why they performed in a certain way.

Due to the exploratory nature of this study, it is not expected that the analysis of qualitative data gathered in this manner requires fine distinction between reflection-on-action (typically verbalisations in past tense) and reflection-in-action (typically verbalisations in present tense). With the exclusion of blatant prompts, all of these verbalisations are included for the objective of enriching the data for analysis using the proposed cognitive framework.

Suitability of the data for cognitive analysis using the SOLO Model.

The use of a Stimulated Recall technique needed to provide data that provided insights into cognitive processes, and hence had the potential to be coded using the SOLO Model. Changes were made to the data collection method following Viv and Wally's performances, and subsequently employed for use with Xavier. As such, the data gathered from Xavier was then subjected to analysis using the SOLO Model.

The SOLO Model was only used to code the responses of one participant. Therefore, instead of attempting to provide a fine-grained analysis, the initial coding of the data from Xavier was carried out by attempting to identify various elements of performance within SOLO modes and/or levels.

Xavier: Descriptive Vignette

Xavier was the only participant in the Pilot who did not utilise a diagram drawn to scale from the Job Sheet. He constructed a 120° angle from two set-squares: a 90° and a 30° angle. This angle combination was used to set the sliding bevel to this fixed angle, and this placed on the downpipe to begin to mark out the cut-out 'V' as illustrated in Figure 4.1 below.

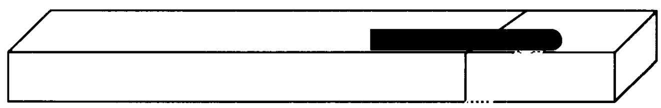

Figure 4.1 Xavier: marking the angle for the first bend

Xavier proceeded to cut out the 'V', and commented that it was important to stick to the lines that were marked on the downpipe otherwise they "would look crooked" and that "straight cuts look better". Once the cut-out was complete and the bend folded up, he was not concerned with checking that the angle of the first bend was 120° prior to fixing in place with the rivets because "if the bevel was right to start with then it would be alright".

Following completion of the first bend, Xavier measured the offset "150mm (sic) straight across" as shown in Figure 4.2, and incorporating the use of a set-square to identify where the offset mark was to be placed at A.

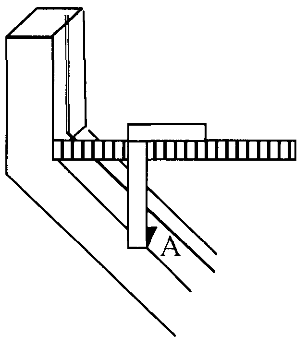

Figure 4.2. Xavier: Establishing the offset mark.

Once this mark was established, Xavier commented that "the bend is going the other way, so you've got to start on the other corner on the other side and do exactly the same thing ... do it sort of like an 'S'". The second bend was completed in a similar manner to the first bend.

In performing this task, there were few moments of hesitation; however, the whole performance was characterised by lots of time spent getting the cuts straight and neat. He did mention that he was only concentrating on one aspect of performance at a time, and not thinking ahead or about the job as a whole.

Xavier: Theoretical perspective

The response of Xavier in the above vignette shows a procedural nature to his performance, with concentration on one aspect at a time before consideration is given to what is required next. From a holistic perspective, such a response can tentatively be classified as a multistructural.

Xavier was asked about the similarities and differences between the first and second bend on his downpipe. He responded that "to do it sort of like an 'S'", and "straight cuts are important because they look neat". This response is characteristic of the ikonic mode as it is heavily tied to what the job looks like. However, an additional statement that "the bend is going the other way, so you've got to start on the other corner on the other side and do exactly the same thing", could mean that his performance exists within the concrete symbolic mode as it is concerned with procedural aspects of task completion. Hence, possibilities exist for coding responses in various modes, with the potential for multi-modal functioning being present.

This preliminary analysis showed that there exists potential for the cognitions of the participants to be analysed using the SOLO Model.

Key Performance Stages.

In addition to the findings addressed above, a particular structure became evident within the performance of each of the participants. This structure has an important bearing on the main study.

When looking at the performance of all of the apprentices, there appeared to be common points of fabrication. These points were often where the apprentices made decisions about the next steps to take in the process, and were characterised by pauses and information being gathered from the Job Sheet. These points have been named here as Key Performance Stages (KPSs), and are defined for this particular task as:

1. The *set-up* of the task prior to initial fabrication, i.e., the scale drawings required to establish cut-out measurements for the downpipe;
2. Making the *first bend*;
3. Establishing the *offset* (how far between the first and second bends); and
4. Making the *second bend*.

These indicators provided the researcher with connections made between certain elements of the task, and whether they were relational in their thinking/execution of the task, or whether they were procedural and/or thought a few steps ahead. They also enabled cognitive analysis to be carried out for each part of the task, as well as providing an overarching view of the job as a whole.

The two sources of verbal data, being the Stimulated Recall interview and Semi-structured interview, enabled SOLO analysis of the mathematical cognition by providing both rich and authentic data. If the participant missed stages that the interviewer deemed important in investigating cognitive functioning, these could be pinpointed and addressed in a semi-structured interview later. An interview checklist was devised using the KPSs, to allow the researcher to easily and consistently identify points for clarification or which needed to be addressed in the semi-structured interview. Therefore, by allowing the stimulated recall interview to continue unimpeded in conjunction with an additional follow-up interview, data could be collected containing the participant's point of view without response effects whilst also providing additional information needed to obtain insight into mathematical cognitions.

Summary

The Pilot Study was devised primarily to fine-tune the data collection technique, and secondly to examine the potential of the data to be qualitatively analysed using the SOLO Model. As such, it was important to investigate (i) the practical task, (ii) construction of the video record and stimulated recall technique, (iii) the quality of the data, and (iv) the utility of the SOLO Model in coding that data.

The practical task was appropriate for plumbing apprentices. However, changes were needed in order to maximise the authenticity of the task in the semi-decontextualised setting of a TAFE practical room. The change of downpipe measurements and contextual relevance were incorporated into the data collection for Xavier. The semi-structured interview session with Xavier showed that he felt that the contextual statement provided him with a necessary sense of relevance and motivation to complete the task. Xavier was also comfortable with the dimensions of the downpipe as "it was okay – that's what we use a lot at work".

The preparation of the video record and subsequent Stimulated Recall interview required a detailed and careful evaluation in the context of this study. In the first instance, it is important to dispel feelings of anxiety related to the presence of the video camera. This was done by videoing the class as a whole and allowing them all to watch the tape; this was not included in the final data-collection involving the individual, nor was it used to inform any part of the analysis.

Some overall discomfort with both the researcher and the action of videoing was still apparent within the cohort and also with the individual. This was due to the fact that the data collection was carried out for one day for each of the three participants. The unassuming manner displayed by the researcher, the researcher's stated respect for the knowledge held by the apprentices, and the fact that the researcher was a partner in a plumbing company, appeared to help to minimise anxiety felt by the participants. It is proposed that the preparation of the video record in the Main Study should occur over a longer period of immersion so as to minimise such discomfort and anxiety with the video camera, as well as improving interviewer/participant rapport.

The process for making the video record consists of (i) videoing the class as a whole over a period of days, (ii) providing a space for task completion that is apart from the participant's peers, (iii) setting up the video on a tripod so that the process of videoing can be run by the participant with minimal input from the researcher, and (iii) videoing the participant and workspace from the chest down.

To maintain the integrity of the Stimulated Recall method, the interview sessions resulted in a delicate balance of probes and general conversation. There were times when additional information was required, yet questioning at that instant was undesirable, as it could have initiated response effects. The solution to this predicament was to separate the interview session into two parts: (i) the Stimulated Recall session using video, and (ii) an additional semi-structured interview session. This structure was successful, and therefore included in the Interview Protocol for the Main Study.

The stimulated recall session was carried out according to the initial guidelines proposed for the Pilot Study. On completion of the video-viewing session participants were then asked questions devised to probe deeper into the participants mathematical understanding. These questions were not pre-determined, but related to points in the stimulated recall session where the researcher required some, or further, clarification of cognitive processes. This method assured that the focal data collected had minimal exposure to response effects.

The inclusion of the semi-structured interview session allowed researcher interjections within the Stimulated Recall session to be minimised. This had the effect of facilitating the flow of the participant's thoughts. Although verbalisations contained explanations of practice as opposed to thoughts-in-action, these were included in analysis as they enriched the view of the performance as a whole.

It was the aim of the researcher to examine cognitive mathematical processes occurring during completion of a workplace-related task, and analyse such behaviour using the SOLO Model outlined in Chapter 3. In a retrospective analysis, the data gathered from all three of the participants in the Pilot Study showed diversity in both fabrication techniques and underlying cognitive performance. Each of the participants solved the task using different approaches. They had different methods for completing the different Key Performance Stages; however, analysis of cognitive mathematical performance could be achieved despite differences in approach to solving the problem.

While a thorough coding of this initial data was not the focus of this study, it was clear that the responses were diverse, and that these responses could be coded using the SOLO Model. The Key Performance Stages provided crucial structure for the organisation of the semi-structured interview session, as well as providing a framework for the subsequent cognitive analysis.

Conclusion

The Pilot Study revealed that despite being derived from a socio-cultural constructivist paradigm, ethnomethodology was a satisfactory research methodology to support collection of qualitative data for analysis within a predominantly cognitive-developmental theory. However, stimulated recall methodology alone was not sufficient to elicit responses from the participants, nor was it conducive to an informed cognitive analysis.

This small preliminary study resulted in the design of a specific data-gathering procedure to enhance the quality and validity of the data collected. The following procedure potentially provided avenues by which to gather data elucidating the participants' points of view, while also providing data rich enough to inform a predetermined cognitive analysis. Specifically, this four-step data collection procedure consists of:

1. Derivation of "Key Performance Stages". Such indicators of the mathematical components of the task performance need to be pre-determined by some means;
2. Mechanical recording (in this case video) of the task in accordance with Stimulated Recall guidelines (Marland, 1984), while also being sympathetic to the nature of the task;
3. Collection of verbal data comprising in-flight thinking or explanations of action by following Stimulated Recall guidelines (Marland, 1984), whilst noting whether the participant has addressed all points within the Key Performance Stages; and
4. A semi-structured interview based on what the participant has not addressed, or where further clarification is required by using the structure of Key Performance Stages.

These initial findings based on the use of this proposed stimulated recall protocol in the setting described were promising. Overall, the Stimulated Recall method provides a tool to gain insight into workers' mathematical cognitions, but must be set up carefully in order to assure the provision of appropriate cues to stimulate recall. As such, the interview protocol used for the Pilot Study was modified (Appendix D) for use in the Main Study, which is detailed in Chapter 5.

Interestingly, Straßer (2000) commented that in an active methodology, the first requirement is to identify the various components of the task so as to provide appropriate stimulus for the interview sessions. The use of video as a stimulus negates the necessity to provide the participant with a collection of appropriate alternative stimulus artefacts to enable recall; however, the present study included the scale drawing, if provided by the participant, as well as the completed downpipe in the Stimulated Recall session. More importantly, this four-step data collection procedure afforded the researcher with rich and authentic data from the participants' perspective as well as providing a more structured response to be coded using a cognitive model.

CHAPTER 5

MAIN STUDY DESIGN

In the previous chapter, aspects of the design were piloted with the view to improving the authenticity of the data collected. The results of the pilot resulted in some changes to the main design. These were that (i) the environment for the research required fine-tuning of techniques and circumstances surrounding the creation of the video stimulus, (ii) a standard stimulated recall interview based on the video stimulus needed to be supplemented with the identification of key performance stages of the task, and (iii) to carefully devise an interview protocol to enable consistency in the data collected for the Main Study.

This chapter presents the research design employed in addressing the main research theme via these sub-questions, and is made up of two sections. The first section provides the details of the design employed in the Main Study, with the Chapter concluding with an evaluation of the methodology.

The Main Study

The main aim of the research study is to determine whether discernable differences are apparent in mathematical skills of plumbing trade apprentices; specifically, tasks which have been identified as containing mathematical concepts. This section addresses the broad choice of methodology, the overall research design of the present study, and culminates in a description of the methodological constraints relating to research in the workplace.

Context of the study

The context for the present study mirrors that of the pilot. Details concerning the geographical location of the research, reasons for selection of the participants, and their learning setting within TAFE are discussed.

Geographical Location

All of the participants involved in the present study were plumbing trade apprentices enrolled at the TAFE College in Orange, NSW. Orange is a rural city located in central-western New South Wales and has a population of approximately 35 000. As such, it has a large number of high schools and further education facilities: two government high schools, one Catholic high school, one independent K-12 school, a K-10 Christian school, TAFE College and University of Sydney agricultural campus.

The participants

Plumbing apprentices involved in the main study attended the Western Institute of Technical and Further Education (TAFE) in Orange for one week every month during the academic year for a period of four years. Apprentices were male and enrolled in the Plumbing Trade Course. They were drawn from a variety of areas across the Central West of New South Wales. Most students not working and living in Orange stayed in shared accommodation for the week in which they attended TAFE.

The TAFE College at Orange was chosen as a site from which to take this sample of students was predominantly as it was similar environment to the Pilot Study. Orange and Tamworth have similar rural contexts and population centres.

The sample for the main study was eight apprentices. Details including names and year of study are provided in Table 5.1. The actual names of the participants have been changed to reflect the confidentiality of the data.

Table 5.1. The participants in the Main Study

Year of Study	Name
1	Anthony, Bruce, Chris
2	David, Eddie, Fred
3	Gerry, Howard

Data collection

The data collection for the Main Study follows the interview protocol devised from the Pilot Study and given in Appendix C. The data collection method for the Main Study is outlined in three parts: (i) the nature of the environment, cohort and preliminary work prior to practical task execution by individual participants, (ii) the process of creating a suitable video stimulus, and (iii) the stimulated interview session.

Preliminary set-up prior to data collection

It was clear from the Pilot Study that apprentices feel anxiety over the data collection process. Therefore, the Main Study involved immersion by the researcher in the TAFE environment with each cohort for the full week of instruction. The timetable was such that the whole data collection was carried out over a period of three weeks: one week for each year of students.

Each week began with an introductory class where the researcher was introduced to the cohort by the TAFE teachers, followed by a presentation by the researcher of the reasons for her presence. An overview of the project was given, as well as explication of task performance requirements, data collection procedures, ethical issues, informed consent procedures, and confidentiality of data. The students were then given an Information Sheet for Participants and Consent Forms for Participants (Appendix E).

The first four days consisted of the researcher being immersed in the teaching and learning environment of the students, as well as participating in all theory classes and tasks. The researcher often asked the students to assist her in completion of some of these tasks, and asked for their advice on plumbing questions. Some sessions were videoed with students; no students had issues with being videoed in a class setting in this way. As these video sessions only served the purpose of minimising anxiety regarding the process, the videos were later destroyed and the students informed of this. The fourth day was where students were asked individually if they wished to participate in the study, and to complete their consent forms the next day. The final day of the week was the day set aside for the formal data collection using stimulated recall with video.

The practical task and preparation of the video stimulus

The practical task was to fabricate an offset downpipe from a rectangular section of 100mm by 75mm colourbond material. The setting for the practical task was apart from the rest of the class: upstairs in the mezzanine classroom. The set-up and equipment for the practical task was identical to that provided for in the Pilot Study, with the video camera being positioned to the front of the work area.

The task was video-taped in the absence of the researcher, with no participant taking longer than 45 minutes for completion. A break of approximately fifteen minutes was provided between the completion of the downpipe fabrication and the start of the interview session.

Interviews

The participant was taken into a classroom equipped with a television and a video recorder. The participant also was provided with their scale diagram and downpipe. The interviews followed the steps and structure provided in the Data Collection Protocol provided in Appendix C. The Stimulated Recall interviews using video were immediately followed by a semi-structured interview regarding clarification of issues arising from the stimulated recall session.

Data Analysis Plan

As outlined in previous Chapters, the research design for this study was based on an ethnomethodological method. As such, the study incorporated the "key tools" for data collection within this approach, namely: participant observation, interviews and artefacts (Zevenbergen, 2000, p. 216). This section details the methods used to analyse data collected from these instruments.

Preparation of data for analysis

The SOLO Model, as described in Chapter 3, can be used to code student responses to set stimuli in order to determine the cognitive structure of an individual's understanding. The responses to the practical task, which was undertaken by the participants in this study, consisted of the three key components identified as important to data collection in an ethnomethodological study. These are participant observation (video-taped performance of the downpipe fabrication process), interviews (Stimulated Recall and semi-structured follow-up interviews), and artefacts (the completed downpipe section).

Data analysis was facilitated by the construction of a single source for each participant, consisting of the videoed performance with time-sequenced tape-recorded audio overlay of the Stimulated Recall interview. The follow-up semi-structured interview was continued as audio data on the end of the performance. A transcript of the entire data was created (see Appendix F for an excerpt), which included the interview, diagrams of performance, and annotations of physical movements. The artefact provided additional cues to the participant to facilitate verbalisation in the interview sessions as well as existing to document the accuracy of the final product.

Qualitative analysis using the SOLO Model

In previous studies utilising the SOLO Model, the common practice for analysis was to first place the responses in "like" groups, and then code these groups using the SOLO Model. A distinction between the majority of such research and the present study, is that the number of participants in this study is small (i.e., $n = 8$). This limitation provided difficulties when attempting to obtain groups of responses of sufficient size. Therefore, the data analysis plan was altered from previous practices to take into account the sample size.

The data analysis in this study was facilitated by initially considering each participant's response in a holistic manner. The data were reduced to provide an overview of appropriate length, followed by a content analysis of the data. Content analysis is a qualitative method used to examine and code responses to a given set of stimuli, therefore providing the scope of participants' responses to the task.

From the content analysis, similarities and differences of response types begin to emerge out of close inspection of the holistic data. The responses were then compared against the SOLO Model for resemblance in structure, resulting in determination of the level and mode of the response. A fine-grained analysis was then carried out in a like manner by utilising the performance aspects within the Key Performance Stages.

The novel nature of this study meant that there was no precedent literature upon which to base direction of the analysis. Therefore, prior to initiation of the coding process, decisions relating to the analysis had to be determined. It was decided that the researcher carry out a preliminary SOLO coding on the initial holistic analysis; these same responses were then examined by an experienced researcher in use of the SOLO Model. Although this process also allowed for preliminary inter-rater reliability, the primary purpose in this instance was to obtain clarity of coding definitions in an area where no precedent for cognitive developmental analysis could be consulted. After discussion between the two researchers, general descriptions were determined for different response types, and direction gleaned for a more detailed analysis within the Key Performance Stages.

The analysis within the Key Performance stages utilised a more formal determination of inter-rater reliability. Inter-rater reliability is defined as the ability of two or more coders to agree on coding of a set of responses. Changes in perception of responses from the perspective of the researcher occur over time as the researcher becomes more familiar with the data. This, in conjunction with monitoring of careless errors and incorporation of circuitous coding, improved the reliability of the results obtained through this process.

An examination of the Pilot Study data produced clear areas for possible SOLO analysis. In the first instance, there was diversity in the manner in which the three apprentices drew and used their scale diagrams. There was also diversity in the degree to which they used this information, and the information on the original job sheets in their fabrication process. Within these two themes, it was relatively easy to identify mathematical components. However, a third theme emerged, and that was the strategies employed by the participants in completing the whole task, which also showed diversity despite not being overtly mathematical in content. The data analysis examined these three themes firstly from the perspective of the SOLO Model. Once this preliminary analysis was complete, each participant's performance was explored as a whole across these three themes to determine whether patterns exist. These analyses are presented in Chapters 6 and 7.

Evaluation of Design Method

The previous section discussed the methodology of the present study, including derivation of the practical tasks, data collection and analysis, and reasons for their use in the three stages of this study. This section reviews the effectiveness of the design in terms of its validity and reliability, as well as any ethical considerations pertinent to the present study in terms of human research.

Kerlinger (1969, p. 429) discussed the issue of validity and reliability of educational research, stating: "if one does not know the reliability and validity of one's data little faith can be put in the results obtained and the conclusions drawn from the results." As such, issues pertaining to the reliability and validity of data must be determined and addressed in order to make informed statements based on results obtained from these data.

Reliability

LeCompte & Goetz (1982, p. 2) defined reliability as the “degree of replicability” of the study. There exist two types of reliability: internal and external. Internal reliability deals with being able to measure accurately and reliably within the study, whereas external reliability is concerned more with the replicability of the study.

Internal reliability

Threats to internal reliability can be reduced by the incorporation of five strategies. These have been outlined by LeCompte and Goetz (1982) as low inference descriptors, multiple researchers, participant researchers, peer examination, and mechanically recorded data.

Low inference descriptors. The present study incorporated video data and tape-recorded conversations as a data gathering technique. These data were later transcribed to provide a written and accurate record. This documentation is defined as low inference descriptors, which then are supporting evidence for conclusions based on the analysis of the data.

Multiple, participant and peer researchers. The use of multiple or peer researchers is an avenue by which to reduce threats to internal reliability. These additional researchers examine the data and subsequent analysis in order to provide “insight as well as to provide information regarding its predictions, biases, and possible influences” (LeCompte & Goetz, 1982, p. 22). In order to obtain consistency in analysis, an additional researcher with considerable expertise in SOLO coding was utilised to obtain interrater reliability. This additional researcher had worked with the SOLO Model for many years having coordinated numerous workshops for researchers and teachers in schools, as well as gaining significant research grants using the Model.

Consistency in the coding process was gained through two means. These were (i) independent coding of a selected group of responses by the additional researcher, and (ii) co-analysis of the few responses that proved difficult to code.

Mechanically-recorded data. The use of mechanical recording to "record and preserve data" (LeCompte & Goetz, 1982, p. 44) is a way to reduce threats to internal reliability. Mechanical recording can be defined as any media aids such as tape recorders, videotapes, photographs, digital pictures and film. This method of recording data can have benefits in that it allows the researcher to focus fully on the responses of the participant (verbal and behavioural) with the knowledge that all the information can be drawn upon at a later date for transcription and analysis.

This present study employed the use of video footage to record the procedural nature of the task being performed with audiotape being used to record the stimulated recall interview after the performance task. Although the presence of both the video and tape recorder has the possibility of altering the responses of the subjects, this was minimised by videoing and recording sessions with the participants prior to the main study taking place.

External reliability

The present study employs a predominantly qualitative methodology in that it is attempting to examine the phenomena from the participants' point of view, and as such, the extent to which this can be replicated is determined by the handling of five factors. These were outlined by LeCompte and Goetz (1982, p. 37) as "researcher status position, informant choices, social situations and conditions, analytic constructs and premises, and methods of data collection and analysis". These factors have been addressed at various points in this chapter, with careful consideration being given to minimisation of threats to reliability as much as possible.

With regards to research, reliability is a "precondition for validity as a study that is unreliable cannot possess validity" (Guba & Lincon, 1989, pp. 234-235). It follows that to be able to establish validity in qualitative research is enough to establish reliability by association (Lincoln & Guba, 1985). To this end, Maxwell (2002) proposed that poor reliability can be seen as a threat to validity as opposed to an independent aspect of its own. The consequence of this discussion is that if reliability is taken as the only criterion for the "goodness" of qualitative research, then there the potential for the research effort to be "no good" (Stenbacka, 2001, p. 552). She goes as far as to assert that the issue of reliability is extraneous to qualitative research.

Validity

The present study is essentially qualitative, and emerging from the previous discussion is the importance of validity over reliability in this type of research. Validity in qualitative research can be viewed from two different perspectives: (i) the two main traditional types of validity of internal and external, and (ii) the more qualitative-specific of descriptive, interpretive, theoretical, generalisability and evaluative (Maxwell, 2002).

Generalistic views on validity: internal and external

LeCompte and Goetz (1982) outlined two types of validity: internal validity and external validity. Internal validity is defined as the extent to which results accurately represent the phenomena being investigated, whilst external validity is concerned with generalisation of results across the population studied. These two types of validity are not mutually exclusive, with a research design exhibiting high internal validity not necessarily also having high external validity and vice versa.

Many researchers (e.g., Cohen & Manion, 1994; LeCompte & Goetz, 1982; Leedy, 1997) have outlined threats to internal validity. These threats include history, maturation, testing, selection, experimental mortality, observer effects and spurious conclusions.

History and maturation. Issues pertaining to history and maturation relate to external factors influencing events between two test situations. To minimise the effects of history and maturation, individuals were given each the practical skill task as close as possible in time to their peers, as well as carrying out the Stimulated Recall interview directly after the execution of the practical problem.

Selection. Selection as a threat to internal validity can exist in a research design when intact groups of participants are used in the data gathering process (Cohen & Manion, 1994; LeCompte & Goetz, 1982). As such, it is possible that these groups of participants do not accurately represent plumbing apprentices state-wide. This study attempts to minimise these effects by clearly identifying the nature of the region where the apprentices work. Additionally, it was proposed that the nature of the plumbing work encountered by the apprentices would reflect the diversity of the profession as the plumbing trade consists of different types of work. As such, constraints were placed upon the participants in that they should be from small firms where they would have been exposed to the widest variety of plumbing experiences.

Observer effects. Observer effects are noted as a major threat to the internal validity of this study. The presence of an unfamiliar, female observer in a male dominated trade situation provides many issues relevant to the gathering of data in this study. Burns (1994, p. 365) stated that "the sex, age, dress, race, social class, and attractiveness of the interviewer are all known to influence the responses to and rapport with the interviewers". While these factors could not be addressed specifically within this study, care was taken in the setting to minimise any potential negative effects.

Threats to internal validity due to observer effects were minimised by frequent visits of the researcher to the TAFE classroom prior to the commencement of the main study. Rapport was established between the researcher and the participants in order to reduce possible threats by (i) becoming involved in classroom the setting and theoretical and practical activities, (ii) asking assistance and advice of the participants in order to complete tasks, and (iii) informally discussing their profession with them. Additionally, the researcher was also involved in the plumbing industry by being a long-time partner in a plumbing firm. This meant that not only did the researcher have credibility by being connected to the profession, she could also discuss general issues about the trade using the participants' own terminology.

Mortality. As data were gathered as singular events, mortality was not observed within tasks. Due to the non-comparative nature of the study between tasks, it was not seen necessary to make the associated deletions of responses in the respective tasks.

Spurious conclusions. Calls for subjectivity on the part of the researcher can increase the incidence of inaccurate conclusions being drawn by the researcher on the data. This is defined as spurious conclusions (LeCompte & Goetz, 1982), and can occur in two separate stages of the research: (i) data gathering, and (ii) data analysis.

By including interviews based on a Stimulated Recall methodology, participants describe their thoughts in a unstructured interview situation to the researcher following the skill completion. Marland (1984) stated that the goals of the researcher must be communicated fully to the participants to avoid the participants constructing their own theories regarding the study. This reduces the incidence of these self-derived theories distorting the data, which in turn may undermine the goals of the research. However, as a part of the focus of the research is to determine whether the participants use school-learnt mathematical techniques in their problem solving, this aspect of the research was withheld form the participants so as to reduce response effects in this manner.

Judgements made on coding these responses with respect to SOLO have the potential to contain a certain element of subjectivity. The threat to internal validity on the data analysis level was reduced by establishing interrater reliability by coding the responses with the aid of an established practice coder, as well as the incorporation of a validity study at the end of the data analysis of the main study.

Internal validity of the chosen stimulated recall methodology. Additional factors affecting the internal validity of the present study lie with the use of a stimulated recall methodology as a data gathering technique. Two main criticisms of the use of stimulated recall techniques are: (i) the degree to which retrospective personal accounts of thought processes be accurate and authentic sources of data, and (ii) the potential of extensive cues provided in visual stimuli to distort the legitimacy of the participant's verbalisations (Yinger, 1986).

Marland (1984) suggested three guidelines to be used in stimulated recall methodology in order to reduce these threats to internal validity:

1. To hold the interview session as soon as possible after the event, not exceeding 24 hours after;
2. To inform the participants of the research goals; and
3. For the participants to verbalise and relive the events of the task whilst viewing the tape, as well as discerning between thoughts of the task and those construed after the event during the viewing session.

Some limitations exist with regards to the internal validity of a stimulated recall methodology. These include issues related to whether factors such as anxiety and ego-protection, influence bias or distort the participant's recall of thoughts. In this study, the Stimulated Recall technique was used in triangulation with the observation of the participant completing the task. The Stimulated Recall interviews supplied additional subjective and qualitative data that increased the quality of the data gathered. Marland (1984, p. 164) concluded that:

> In spite of these problems and limitations, there is a generally held view among those who have used stimulated recall that the technique does offer fruitful means of exploring the cognitive concomitants of human behaviour in the classroom and other task settings, and of developing new insights into the important enterprises like teaching.

The focus of the present study is the building trades; functioning within this context predominantly manifest in performance-based skills, resulting in the production of concrete products and structures. It follows that functioning in the sensorimotor and ikonic modes is often exhibited in practical workplace situations. Problematically, the analysis of cognition within these modes during skilled performance is not directly measurable. In addressing this fact, Courtney (1986, p. 50) stated that

> ... the SOLO taxonomy makes a clear distinction between the generalised cognitive structure of the individual, which is a hypothetical concept not directly measurable, and the structure of the actual responses that the student makes to specific learning tasks. This distinction makes it a valuable evaluative and instructional tool.

External validity. Cohen and Manion (1994, p. 171) defined threats to external validity as "likely to limit the degree to which generalisations can be made from the particular experimental conditions to other populations or settings", and then go on to list setting effects, history effects and construct effects as three threats to external validity.

A major impact on this study was the decision to carry out the research in a TAFE setting. Although this is not perceived to be an authentic workplace setting, it is the environment in which semi-formal instruction takes place in the plumbing trade. Therefore, the apprentices are used to carrying out performance-based tasks in this setting. In order to further minimise setting effects, the apprentices were provided with a realistic context within which to perform the task that would take into account the absence of a house in the process of making a downpipe. This context is provided in the Interview Protocol in Appendix D.

The demographics of the sample were gathered separately in order to reduce history effects as a threat to external validity. The variety of demographics for the sample ensured that these effects were minimal. The external validity of the present study can be enhanced by clarifying that the data gathered using the stimulated recall methodology are respective cognitions of the event, rather than an accurate record of the decision making processes at the time of the event (Keith, 1988).

Construct effects refer to "the extent to which a particular test can be shown to measure a hypothetical construct" (Borg & Gall, 1989, p. 255), which is in this present study, the construct of cognitive development. In order to minimise this threat to external validity, the practical skill tasks were vetted by experienced plumbers and plumbing TAFE teachers as providing accurate representations of workplace practice. Additionally, the researcher was the only presenter of the tasks so as to ensure continuity in presentation. All participants were presented with the task in the same manner.

Qualitative-specific views on validity

Researchers (e.g., Davies & Dodd, 2002; Lincoln & Guba, 1985; Stenbacka, 2001) in the qualitative vein have sought out alternatives to the traditional forms of validity summarised previously. Lincoln and Guba (1985), Maxwell (2002) and Kvale (1996) considered that validity refers to whether the inferences drawn from the data are "worth paying attention to" (Lincoln & Guba, 1985, p. 290).

The need for additional clarification of validity within the qualitative paradigm has arisen due to the lack of controls and variables in qualitative research. This generally prohibits dealing with threats to validity in the design phase, meaning that the "prior elimination of threats is less possible" (Maxwell, 2002, p. 56) in qualitative research. Therefore when the strengthening of validity in qualitative studies largely rests on the transparency of the researcher's point of view, as well as "how a particular researcher's values and expectations influence the conduct and conclusions of the study" (Maxwell, 2005, p. 108).

The following discussion presents an avenue by which to understand validity in qualitative research as developed by Maxwell (2002). As opposed to the more traditional types of validity mentioned earlier, these five types refer not to the data or method of research, but more to the accounts of the research. These types are descriptive validity, interpretive validity, theoretical validity, generalisability, and evaluative validity. Maxwell stated that for most qualitative research, evaluative validity, which refers to the degree to which findings based on the study are legitimate, is not directly relevant. This is the case in the present study. The remaining four types are described below.

Descriptive validity. This type of qualitative validity refers to the extent to which the descriptions of actions as viewed and related by the researcher, represent reality. It is important to ensure that the recording of events is consistent between two or more possible researchers. Winter (2000) queried the suitability of this type of validity as the qualitative paradigm allows for different truths to exist. Therefore, it is rational to expect that the recording of events is prepared with the viewpoint of minimising threats to descriptive validity, but that it is not realistic or appropriate within the qualitative paradigm to sterilise the data completely.

Minimising threats to descriptive validity occurred during the data collection and transcription phase. In the present study, the data to be analysed was mechanically recorded and transcribed to preserve factual accuracy. Researcher inferences were withheld from the transcription phase; these included annotations to smell, perception, touch and so on. All of the verbal and visual data was combined in a single data source (an example of this is included in Appendix F). Additionally, another single data source comprising the video record and tape-recorded record was also derived as a reference. Transcriptions were re-checked numerous times against these two single-data sources to identify inaccuracies prior to the data analysis.

Interpretive validity. Interpretive validity deals with the degree to which researchers interpret what the behaviours of the participants mean to the participants themselves. Maxwell (2002, p. 49) described the inference of meaning from participants' actions as follows:

> The development of accounts of these participants' meanings is usually based to a large extent on the participants' own accounts, but it is essential not to treat these latter accounts as incorrigible; participants may be unaware of their own feelings or views, may recall these inaccurately, and may consciously or unconsciously distort or conceal their views. Accounts of participants' meanings are never a matter of direct access, but are always *constructed* by the researcher(s) on the basis of participants' accounts and other evidence.

In order to minimise threats to interpretive validity of the present study, the mathematical focus of the study was withheld; this also addresses the issues associated with response effects. Great care was taken with the recording of interviews and associated transcriptions so as to present the data in a way that represented the perspectives of the participants. In this way, the transcription of data was also held true to the language and concepts of the participants. When verbalisations were not understood by the researcher, probes in the interview sessions were utilised to gain further insight from the participants as to what they meant.

The use of Stimulated Recall as a data collection method aided in the minimisation of threats to interpretive validity, by having the participants verbalise their in-flight thoughts, feelings and actions. This method was useful in obtaining, as much as possible, the perspective of the participants. The data collected maximised descriptive validity by aiming to capture the visual and aural accounts from the perspective of the participant; the transcribed data did not contain researchers inferences of action. Therefore, it was deemed an acceptable omission to not feedback the transcribed data (as outlined in Annexure F) to the participants.

Theoretical validity. Maxwell (2002, pp. 50-51) described theoretical validity as "going beyond the concrete description and interpretation and explicitly addresses the theoretical constructions that the researcher brings to, or develops during, the study ... [and] ... refers to an account's validity as a *theory* of some phenomenon". In critiquing theoretical validity, Winter (2000) argued that such frameworks predetermine the methods of recording and interpretation of the data. However, Maxwell (2005) proposed that minimising threats to researcher bias does not necessitate elimination of a researcher's beliefs or theories, and that it is more important to have clarity of the position of the researcher.

The present study is not attempting to develop a new theory or idea from the accounts of the participants, but to determine whether an existing theoretical framework is appropriate to analyse the data. Additional measures were incorporated into the data collection by having a Stimulated Recall interview, followed by a semi-structured interview. The former allowed the researcher to obtain data in which response effects were minimised, and the later provided avenues to utilise further probing to obtain rich and purposeful data to inform the analysis using the theoretical framework. This type of validity also has comparisons to construct validity, which has been dealt with in an earlier section.

Generalisability. Generalisability is said to refer to the extent to which the results of the study can be expanded to the wider population. It is therefore possible that comparisons can be made to the traditional concepts of external validity. However, generalisability, as it is referred to in a qualitative framework, can be described as having two aspects: internal and external (Maxwell, 2002). The former is concerned with whether the results can be applied within the immediate community of practice, and the later deals with generalising to other communities of practice.

Whilst it is possible that the results of the present study may provide internally generalisable results, the ability to generalise to other populations and communities of practice may not be achievable. The localised scale of this exploratory study limits its external generalisability; however, "the value of a qualitative study may depend on its *lack* of external generalisability" (Maxwell, 2005, p. 115).

Generalisability, in either of its two forms, can at most be seen to encompass whether the SOLO Model is useful in assessing responses to workplace practical problems involving elements of mathematics. Claims of applicability of these results to other communities of practice external to the ones studied in this present research, need to be evidenced by larger numbers of participants in such communities of practice.

Despite the claim made by LeCompte and Goetz (1982, p. 56) that "attaining absolute validity and reliability is an impossible goal for any research model", the present study has utilised triangulation, especially in the videoing and subsequent stimulated recall interviews, in order to "maximise the strengths and minimise the weaknesses" of the dichotomies of evaluation of educational research (Bowden, 1996, p. 3).

Ethical considerations

The present study is based in an ethnographic research method to obtain qualitative data. The close relationship between the researcher and the participants in such ethnographic studies means that ethical considerations must be paramount in the design. Burgess (1989, p. 60) stated that this relationship "implies a respect for the rights of the individual whose privacy is not invaded and who is not harmed, deceived, betrayed or exploited". He conceded that in ethnographic studies ethical dilemmas occur frequently. Therefore, it was important to consider and predict where such ethical dilemmas may occur in the research, and identify avenues to overcome and account for such problems.

In addition to the validity of the research design, which establishes that the participants are not being used for insignificant purposes, Sieber (1996) identified a number of categories through which to view research involving human subjects. These included, agreement and communication between the researcher and the participant, as well as the acquisition and use of data. These two categories are discussed below, within the context of the present study.

Agreement and communication between the researcher and the participant

This category subsumes the issues of voluntary informed consent and deception. Informed consent requires that the potential participants are fully aware of the nature and purposes of the research, what will be required of them and for how long, and that their explicit agreement has been given. Additionally, "clear communication, not complex technical explanations or legal jargon" (Sieber, 1996, p. 26) beyond the comprehension of the potential participant needs to be facilitated. In order to ensure consent is voluntary, requests are required to remain free from inducements or threats.

Voluntary informed consent was obtained on three levels for the Pilot and Main Studies study through plain language Information Sheets and Consent Forms (see Annexure E for an example): The Directors of the TAFE Institutes, the Heads Teachers of the Plumbing Sections at each TAFE following approval from the Institute Directors, and the plumbing students following approval from the Head Teachers. Participants in the final data collection consisted only of those who had given such consent. Additionally, informed consent of a verbal nature was sought at intervals throughout the data collection process.

Deception in the context of the present study relates to the withholding of the mathematical purpose of the study. Participants in this study were told about the general purposes of the research and their educational nature. However, as discussed in previous chapters, the mathematical focus of the study was withheld due to issues relating to response effects.

Acquisition and use of data

Covered within this category are issues pertaining to the participants' right to privacy, and confidentiality of data collected. For the present study, the participants were engaged in non-verbal fabrication as well as an interview session related to that fabrication. Questions of a personal nature were not asked, with the participants being free to answer questions in as much or as little detail as they wished.

Confidentiality of the data was considered to be of vital importance to the well-being of the participants. The participants were assured of confidentiality of performance attributes and interview data, particularly from their peers and their teachers. Real names of the participants were changed for the purposes of publication. With the exception of the researcher, participant responses were viewed by one other individual with the purposes of determining inter-rater reliability.

These ethical issues needed to be considered either during the research design, or during the process of data collection. As such, permission to carry out this research was sought and gained (Approval No. HE02/155), through a submission to the University of New England Human Research Ethics Committee.

Sieber (1996) maintained that ethical standards should also be upheld during the dissemination process of the methodology and results of the research effort. Therefore, communications of such should be done in a manner such that the confidentiality of the participants is protected: individual performance of identified participants should not be divulged.

The following two chapters are concerned with the data analysis. Chapter 6 presents the descriptive results and a holistic analysis of the task performance. Chapter 7 provides an analysis of a component of the task performance.

CHAPTER 6

DESCRIPTIVE RESULTS AND HOLISTIC ANALYSIS

The purpose of this chapter is to present the results and analysis directed at addressing the first Research Question: *Can the SOLO Model be applied as a framework to describe cognitive performance in workplace practice?* Data portrayed represent the performance from a holistic perspective, and comprise both objective visual information collected by the researcher using video and the verbal data collected using the specified stimulated recall technique detailed in Chapter 4.

This Chapter has four sections. The first section is introductory in nature, and describes material, tools and terms used by the participants. These are largely unique to the plumbing profession, and are necessary to comprehend the task performances within the analysis chapters of this thesis. The second section provides an overview of the performance of each participant in the form of short individual vignettes. In the third section, the vignettes are examined for the quality of the performance: efficiency, accuracy and time taken to complete the task. In the final section, the individual vignettes are analysed using the SOLO Model. The result of this analysis is to determine the cognitive structure of the individual response for fabricating an offset downpipe.

Materials, Tools and Terms

The following section provides detail regarding the verbal and visual data collected for this study. Such information is deemed important as a guide to the reader. This section is broken into two parts, namely; (i) a description of the material and tools involved, and (ii) plumbing terms referred to within the remainder of the thesis.

Materials and tools

The material used for the fabrication of the downpipe was a length of 100mm by 75mm Colourbond downpipe (Figure 6.1).

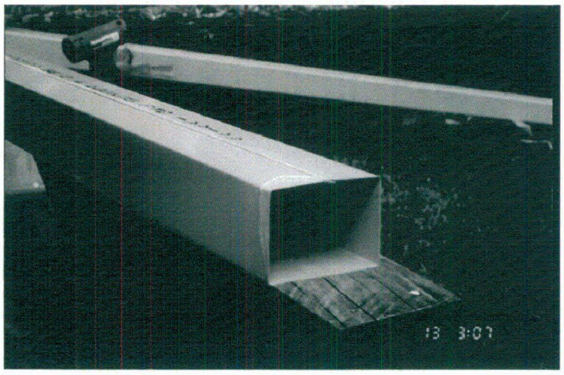

Figure 6.1. 75mm x 100mm Colourbond downpipe

The downpipe is fabricated whilst still covered in plastic to prevent scratching of the surface prior to its use on a house. The plastic is removed once the bends are fabricated. One end of the downpipe is slightly narrower than the other to enable two lengths to be slid together to create a longer section if needed. A seam runs down the length of the downpipe where the folded metal is joined together. Conventionally, this seam faces towards the wall of the house and is often described as the "back". The side facing outwards from the house is called the "face".

Figure 6.2 depicts the tools used by the participants to fabricate an offset downpipe.

Figure 6.2. Tools used in downpipe fabrication.

They are (clockwise from top): duck bill pliers, rivet gun, square (set at 90°), sliding bevel (able to be set to any angle), left- and right-handed snips, hammer, hacksaw and drill. Participants were provided also with a standard mathematical protractor, a 40cm ruler and three large metal set-squares (two of these had angles 90°/60°/30° and one had angles 90°/45°/45°).

Terms

The terms used in downpipe fabrication by the participants are shown in Figures 6.3 and 6.4. Figure 6.3 illustrates the marking and cutting out of the section to enable the first bend to be made.

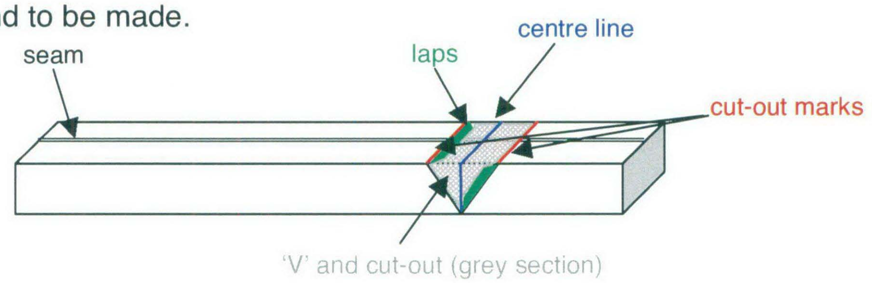

Figure 6.3. 'Marking out' terms for a downpipe bend.

The six terms used in Figure 6.3 are:

- *Centre line*: A line drawn on a downpipe to enable construction of the cut-out. Sometimes spoken of as 'on the fold'.
- *Cut-out marks*: lines drawn parallel to, and equidistant from, the centre line.
- *Laps*: Sections drawn inside the cut-out that fit either inside or outside the bend to channel and restrict the flow of water to the inside of the downpipe. Placement of these laps differed between participants in this study.
- *'V'*: The shape made on the sides of the downpipe by the cut-out.
- *Cut-out:* The (shaded) section removed from the downpipe to enable the bend to be folded together.
- *Seam:* The thicker section where the metal is joined to make a hollow downpipe.

A particular technique is used by many plumbers in obtaining the measurement between the centre line and the cut-out lines. This technique involves drawing a section of the Job Sheet to full size. Figure 6.4 is an example of such a drawing made from the Job Sheet.

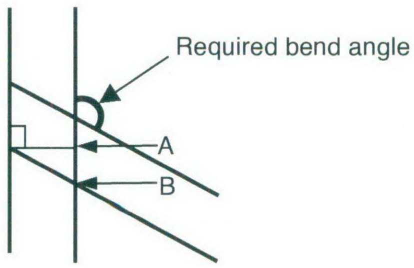

Figure 6.4. General scale diagram to obtain cut-out measurements.

The distance AB provided the cut-out measurement in the fabrication process. The cut-out measurement is transferred to the rectangular-section downpipe (see Figure 6.5).

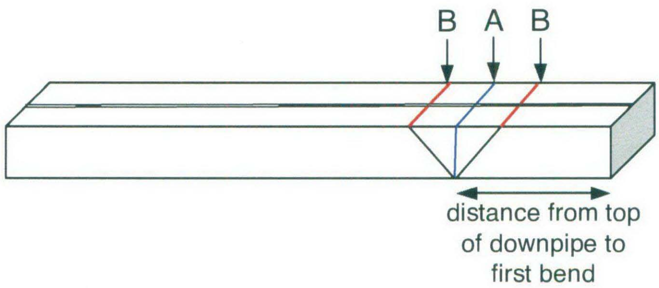

Figure 6.5. Transferring measurements from the scale diagram to downpipe.

The centre line is marked as a starting point for the cut-out measurement, with the centre line at A placed a prescribed distance from the top of the downpipe. When this V-section is removed, and the bend folded together, the required bend angle is achieved. Care must be taken as to the orientation of the V to achieve the requirement of the seam to face the wall of the building.

In establishing the placement of the second bend, specific terms are used. Figure 6.6 relates to establishing the offset distance for the second bend of the downpipe.

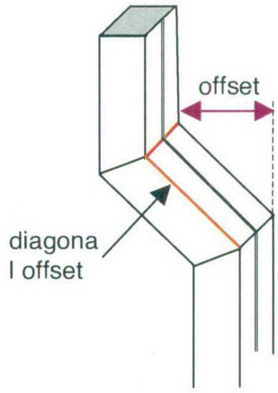

Figure 6.6. Offset diagram for downpipe fabrication.

The two terms used in Figure 6.6 are:

- *Offset*: Perpendicular distance from the back of the first vertical section, to the back of the second vertical section.
- *Diagonal offset*: Diagonal distance along the downpipe from the back of the first vertical section, to the back of the second vertical section.

Measurements used in the plumbing trade are generally carried out in millimetres. The final term identified within the participant vignettes is a common abbreviation used by plumbers for this unit of measurement: "mills".

Vignettes of Individual Performance

This section provides a general overall description of each participant's performance. This is presented in the sequence it occurred during data collection. However, data are identified within the Key Performance Stages (KPS) defined in Chapter 4. These Key Performance Stages are colour-coded to facilitate progression through the vignettes, and are defined as:

- **KPS 1: Setup**
- **KPS 2: First bend fabrication**
- **KPS 3: Offset positioning**
- **KPS 4: Second bend fabrication**

The vignettes describing each participant's performance are given below, in order of the year of learning at TAFE, beginning with Anthony who is a first-year apprentice. Comments made by participants are included where appropriate. This section concludes by describing groups of similar responses.

Anthony

Anthony began by positioning his hands on the pipe in the shape of a V approximating 90° as a means of visualising **the first bend**. This action was followed by using a 90° angle from a set-square to assist in visualising the cut-out angle. After approximately three minutes he "decided to measure it out ... how [it was done] at TAFE" because "it seemed to always work".

He drew the first bend to scale (**setup**). The cut-out marks C and D for **the first bend** were obtained by positioning the downpipe on his scale diagram (Figure 6.7). Anthony stated that this was not the method taught at TAFE, and that he "only just figured that out".

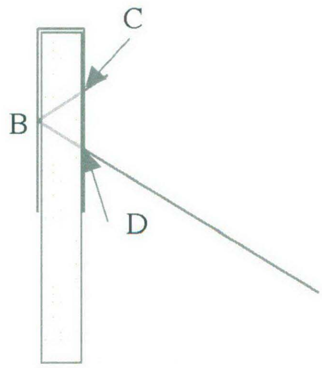

Figure 6.7. Anthony marking the cut-out measurements.

The section for the cut-out of the first bend was removed, and the accuracy of the folded angle checked by placing the downpipe on the scale diagram because he wanted to "measure it up on the lines". Once the bend was fixed in place, it was again placed on the scale diagram to "just check it again". Laps were positioned so that they would not "let all the water out".

Anthony made sure that the rough corners were tapped down to improve presentation as well as minimising leaks. Before he fixed each bend in place, Anthony checked that the angle matched the scale diagram by placing the downpipe on his drawing.

Determination of the **offset** measurement was “bit of a guess” because he “didn’t really know how to measure it”. There were many moments of hesitation in using a ruler to mark the 150mm offset on the downpipe with the measurement taken from the Job Sheet. Anthony was assisted in determining the offset measurement by holding up the job and visualising it against a brick wall. He stated that he was “just trying to line it up”.

The **second bend** was drawn on his scale diagram because he was unsure of how to fabricate this section of the downpipe. In order to draw the second bend, Anthony commenced using inappropriate measurements taken from his drawing of the downpipe. Anthony realised his error and corrected this on the plan. His performance at this stage in fabrication was lengthy and focused entirely on the offset and getting it correct:

> ... that’s where I stuffed up, I drew the lines in the wrong spot. I had those lines off the centre line, instead of either side of the centre line.

Once Anthony was satisfied that his scale diagram was accurate, he placed the downpipe on the scale diagram and marked the cut-out lines for the second bend as he did with the first bend. His whole method for completing the second bend was identical to that used on the first bend.

Bruce

Bruce completed the **setup** by drawing the whole scale diagram first before beginning the fabrication process. This was because he needed to “draw them up”. The reason given was the 120° angle was not what was normally used at work. He stated that the second angle is “different ... you’ve got to measure it properly and then cut to your lines”, indicating a possible reason for drawing the whole diagram to scale rather than just one bend.

He gained the cut-out measurements for the **first bend** from the scale diagram, completed the cut-out and folded the bend. A sliding bevel was then set at 120° from the scale diagram "just to transfer the angle from the paper and check it on the downpipe". After the folded bend angle was checked with the sliding bevel, the bend was then fixed in place with rivets.

The diagonal **offset** measurement was gained from the scale diagram and transferred to the downpipe. The positioning of this mark was then checked visually using a ruler placed perpendicular (using visual estimation) to the vertical edge of the first bend. The **second bend** was then completed and the angle checked with little hesitation, in the same manner as the first bend. After the second bend fabrication he stated that both bends are "the same".

Comments made by Bruce in the semi-structured interview indicated that he did not have much experience with this task, only having "done three or four of them at work".

Chris

In his **setup**, Chris drew the first bend only with a ruler and protractor before beginning his fabrication. He did this in order to "make a template ... so [he] had something to go off". Statements made in the interview sessions indicated that he knew in advance which parts of the diagram were going to provide useful measurements for his cut-out. All of these measurements and angles were double-checked "to make sure [they were] the right measurement before [the] cut-out".

Chris used the measurements from his scale diagram to draw the cut-out lines on the **first bend**. He made a small error in the placement of his centre line, which he quickly corrected. The cut-out was made and the bend folded and riveted. The angle was not checked prior to riveting because "it's going to be 120° because you've cut it out" indicating his confidence in his previous measurements. Additionally, on completion of the task, Chris commented: "there's not much point checking everything all the time, otherwise you'd be there for ages ... you've got to trust yourself."

He returned to his scale diagram to draw the **offset** and second bend, and visualised which part of his diagram was against the wall: "that's my back angle that's going to be up against the wall." He measured the cut-out section for the second bend on the scale diagram "to make sure the cut-out's going to be the same ... make sure [he] had them both right".

The second measurement was slightly different to the first, and therefore he re-used the measurements from the first bend. It was noteworthy that he did not use the offset or diagonal offset distance on the scale diagram to inform his job. To get the offset distance, he placed a ruler perpendicular to the vertical edge of the first bend, and marked 150mm across on the downpipe. There was some hesitation in this strategy, and Chris stated that he would "try to figure out where [he] was going to mark the 150" millimetres.

Chris used the cut-out measurements from the first bend to inform his **second bend**, and competently completed the second bend. When finished, he checked the angles with a protractor and the offset measurement with a ruler.

Chris completed the task quickly. He verified during the interview that he did a lot of downpipes in the course of his work, and stated: "make sure you get the first [bend] right and then worry about the next one."

David

David began his task by **setting up** the first bend on paper, with the reasoning that he needed to "get [his] angles right". However, once the bend was completed, he was unsure how to derive the required cut-out measurement: "I'm thinking how I am going to get my measurement, my cut-out mark with my angles." Even though the drawing of the first bend was correct, he was unsure and re-combined the set-squares, giving an angle of 135° instead of the 120° previously used. This caused confusion, but he eventually returned to the first set of drawn lines to obtain the cut-out measurement. These measurements were re-checked on a number of occasions.

At the beginning of the **first bend** fabrication, David took twice as long as other participants to mark the cut-out lines on the downpipe. The time taken was largely due to hesitation and inaccurate decisions on his part caused by the fact that he set the bevel at the required angle and tried to reconcile this with drawing the V on the side of the downpipe. He eventually drew the lines in with a straight edge, as opposed to the angle set on the bevel. During this lengthy process, he neglected the requirement of the seam to face the back of the job, and had to re-draw all the lines for the cut-out to "switch it around to the other side and do exactly the same thing".

It also took David at least twice as long as other participants to mark his cut-out lines on his downpipe, a process about which he stated:

> It pays to take five-to-ten minutes before you start the job to work it out and have a look at it instead of rushing straight in and mucking about, I find ... make sure ... you're measurements [are] right, your cut-out and that sort of thing. Just like double-check it.

Once the section was cut from the downpipe to enable the bend to be folded up, David spent time trimming and re-folding the bend. He did not check the angles, but relied on how well the cut-section folded together.

To get measurements for his **offset**, David drew the remainder of the diagram to scale because he commented that he did not have the diagonal offset measurement: "it doesn't have the measurement coming straight down the angle ... so I'll draw it up." He completed the whole diagram to scale, and then measured and transferred the diagonal offset measurement to his downpipe.

Once this was done, he then re-applied the set-squares at 135° to the second angle. The action precipitated a set of plans with multiple overlapping pencil lines. These plans were discarded after lengthy deliberations, and the first bend re-drawn on a blank piece of paper. This action seemed to make the situation clearer. David stated in the stimulated-recall interview that: "I had it right the first time" and if he had gone "with [his] instinct" he would not have got himself so confused; he "wasn't thinking right".

David then returned to the downpipe and continued to draw the cut-out marks for the **second bend** using his measurements from the first bend. The second bend was completed without checking the accuracy of the angle because they would be right "by [his] measurements on the paper. It all comes back to your paper."

Eddie

Eddie's **setup** consisted of drawing the first bend to scale on his diagram in a concise manner. There was evidence of a clear purpose and knowledge of which were the required measurements. Subsequently, for the **first bend** fabrication the cut-out measurements were drawn on the downpipe quickly. This Key Performance Stage was closely followed by cutting, folding, and riveting the bend with no hesitation. The accuracy of the first bend angle was not checked until he marked the diagonal offset later from the diagram.

Eddie returned to the drawing to complete the **offset** and second bend because: "150 is out where the wall is, and then I can put my downpipe on there and just put the mark on there." Consequently, using the scale drawing as a template under the downpipe, he transferred the diagonal offset mark to his downpipe.

The **second bend** cut-out marks were drawn quickly using the same measurements obtained from the first bend on the scale diagram. Eddie commented during this KPS that both bends were "basically the same, but opposite", and used the initial cut-out measurements with no hesitation. The second bend was completed quickly, but Eddie did not check the accuracy of his second angle. He did comment that the drawing was useful to gauge visually the accuracy of the finished product.

Fred

In **setting up** his job, Fred completed the scale drawing in its entirety before fabrication of the first bend. There was a high level of precision in the technical drawing aspect of the production of the scale diagram, validated by the requirement that he did not "want thousands of lines going everywhere "because it just gets confusing". Despite the precision in measurements and execution, it was not clear how he obtained his measurements. However, during the drawing process, Fred constantly referred to visualising what it would look like on the job to aid in this procedure: "I just imagined, say, that was a brick wall you were putting it up on to the right hand side there."

The cut-out lines took some time to be drawn on the downpipe for the **first bend**. Reasons for this include making the same error as Chris with the placement of his centre and cut-out lines, and spending some time "visualising where the cut [was] going to end up". The process of cutting out the section for the V was lengthy due to constant folding and re-trimming of the edges of the section. Each time the bend was folded, the accuracy of the angle was checked against the scale diagram. When Fred was finally happy with the angle against his scale diagram, the bend was riveted. He did comment that it was a little bit out, the reasons being: "the pen line might have been to far one way, or [he] cut a little bit too much out of one side."

Although he began by using the ruler on his downpipe to determine the 150mm offset, the practical option chosen was to place the downpipe on the scale diagram and transpose the diagonal **offset** mark directly to the downpipe. Fred deliberated for some time because his first bend angle did not match the scale diagram. This led to some inaccuracies regarding the placement of the diagonal offset mark on the downpipe.

The completion of the cut-out marks for the **second bend** was delayed for approximately two minutes due checking that his offset measurements were correct. Before cutting, Fred checked all measurements on the downpipe and scale diagram with a ruler in case he "made a mistake somewhere". The second bend was completed, and riveted before checking the accuracy of the angle, and he was confident that they were correct because he had "worked off the same angles both sides".

General observations of Fred's task execution were that he was very intent and focused on checking all measurements made with rulers, and with the accuracy of his scale diagram. This practice did not extend, however, to verifying the accuracy of the bend angle on the downpipe. In the semi-structured interview, he commented that "the main thing is keeping the job square", meaning the perpendicular and parallel aspects. The interview contained constant references to visualising either the finished product or what it would be like on the job site.

Gerry

Gerry drew the whole diagram to scale in his **setup** as he "thought [he had] better draw it up". He was hesitant and stated that the underlying reason behind drawing the diagram was because he "didn't know what 120° was for *x* without drawing it up"; *x* being the distance AB in Figure 6.5. The drawing of the scale diagram elicited few comments regarding relationships between it and a real-life scenario. The scale drawing was done using a ruler and set-squares; the downpipe was not used in the construction. At the time of drawing the scale diagram, Gerry's performance did not indicate certainty regarding how to obtain appropriate measurements to inform the actual job, with numerous measurements being taken and re-taken. He was trying to be accurate with his bend measurements – "measure twice and cut once" – but in the process misread the offset distance as 250mm.

Such attention to accuracy relied upon making sure both bend cut-out measurements (distance AB) being the same value. He spent time making sure his diagram was exact in this respect; however, despite these efforts, the measurements were not the same. Gerry knew instinctively that they needed to be the same, but such knowledge needed to be verified in this context. He commented in the stimulated-recall interview: "they're *normally* both the same but you can draw both to check."

The **first bend** was marked and cut-out carefully. Gerry made statements that he had “to cut along the lines to get [his] angle right”. The bend was folded, trimmed, and checked with the scale diagram repeatedly before fixing in place with rivets. He obtained the diagonal **offset** distance from the scale diagram and immediately transferred the measurement to the downpipe because “that’s what [he] did the drawing up for”.

The **second bend** was fabricated by “doing the same procedure as before”, including constant bending, trimming and checking of the angle. He reverted to using the same cut-out measurements on the second bend that he used for the first. After the cut-out was completed, the second bend was made to fit the diagram. He did this rather than concentrate on it being the same as the first bend on his downpipe. In the interview, Gerry stated: “I had the picture drawn of what [the downpipe] would look like, and made [the job] look like that.”

Howard

Howard **set up** his job by drawing the first bend to scale on the paper; however, it took him some time to achieve the required bend angle with his choice of combinations of set-squares. He was sure about what measurement he needed from the scale diagram “to get the measurements to cut out”.

The marking of the cut-out lines for the **first bend** were done in a highly proficient manner; however, a mistake was discovered half-way through his cut-out on the first bend. Howard had marked his cut-out lines upside down, resulting in the seam being on the front of the downpipe “because [he] never looked at the picture”; he had only gained one piece of information from the diagram (angle bend 120°) and gone straight to the job.

The **first bend** was repeated on a new piece of downpipe, taking into account the information regarding the seam at the back of the downpipe. It was marked, cut-out and fixed with rivets. He consciously did not check the angle, relying on his measurements gained from the scale diagram because: "it should be the right angle [on the downpipe] if it's all got the right bend [on the diagram]."

Howard established the **offset** on his job by orienting his job in the same manner as the job sheet. He spent some time in achieving the correct position of the centre line using his method, later stating that "this was where [he] made mistakes". On the video Howard hesitated, and he commented in the stimulated-recall interview that he "knew it didn't look right ... it should have been right".

From this point in the task, Howard rushed through the downpipe execution, taking little time to ensure the accuracy of the job. This was very different from his efforts at the beginning of the task. In the stimulated recall interview, he commented that he "shouldn't have done" those mistakes, and knew it was not correct: "you'd have to throw it out ... you would if you were on a job. You'd have to throw it out because it wouldn't fit." Additionally, he stated that the camera made him nervous and that his performance would have been different if the camera was not there.

The **second bend** was marked and cut out quickly using measurements from the first bend. The sliding bevel was set from the first bend angle on the downpipe, and used to "make sure the [second bend] was the same ... they both have to be the same ... there's no point having one right and one wrong". During the stimulated-recall interview, he was intent in clarifying where, and how, he had made his mistakes; he also noted what impact these mistakes had on his job.

Descriptive Analysis

All participants completed the task of fabricating an offset downpipe. However, there were similarities and differences between each individual performance. It was noteworthy that the data collected were rich and contained many aspects of performance that could be investigated. Therefore, to maintain the holistic approach of this chapter, it was decided to return to the workplace literature to underpin discussion of the quality of the participants' performance.

As mentioned in earlier chapters, one of the aims of employment is to minimise time taken on task so as to maximise monetary gain. Therefore, an aspect of good performance is not only whether the product is accurate for the needs of its destination, but also whether production has been efficient. By utilising this notion of accurate and efficient execution, responses could be assigned to groups depending on whether participants had made certain connections regarding aspects of the job. Therefore, the descriptive analysis is separated into these two distinct parts: strategies employed to complete the task efficiently, and accuracy of the end product.

Efficient strategy

Within this cohort of apprentices, there were three distinct groupings relating to strategies employed to fabricate the downpipe. These groupings are hierarchical, with the least efficient strategy being Category 1.

Category 1

The first group consisted of *Anthony*, *David*, and *Fred*. These apprentices completed the task by dealing with each Key Performance Stage, one at a time. At no point was there evidence that they were "thinking ahead" or considering consequences that may have arisen from their actions. These performances were not efficient because the apprentices often repeated actions that were unnecessary.

One example of a repeated action was to measure aspects of the scale diagram on the downpipe numerous times. Another example was to draw two bends when only one was required. Participants in Category 1 drew each bend to scale because they did not recognise that both bends were the same and that one measurement would suffice.

Category 2

The second group consisted of *Gerry*, *Bruce,* and *Chris*. The participants in Category 2 completed each Key Performance Stage without considering similar aspects of the task, such as the cut-out measurements being the same for both bends.

These responses stood apart from those in the first Category because at some stage during the task they recognised that both bends were the same and that this information could be useful in streamlining performance. For Gerry and Chris, this realisation came too late to impact on the current task. However, they showed that they were at least aware of the potential of this action for later applications. Bruce used his first bend cut-out measurement for the second bend, but this information may have come to him after drawing the whole diagram to scale, as he still drew the second bend, although not using it to inform his job.

Category 3

The responses produced by *Howard* and *Eddie* made up the third category. Their performances were distinguished by decisions made towards the beginning of the fabrication that had the effect of maximising efficiency of task completion.

Howard and Eddie recognised that both bends were the same, and that the measurement for the first bend would suffice for the second. They actively chose to draw only one bend to scale. Comments from these participants indicated that their reason for drawing one bend was because both bends were the same. Therefore, the cut-out measurement was the same for both bends in the fabrication process. This insight lead to a more efficient performance.

Overview

Strategic decisions made by the participants impacted on the time taken to complete the task. The overall time taken to complete each of the Key Performance Stage, and consequently, the overall task, is provided in Figure 6.8.

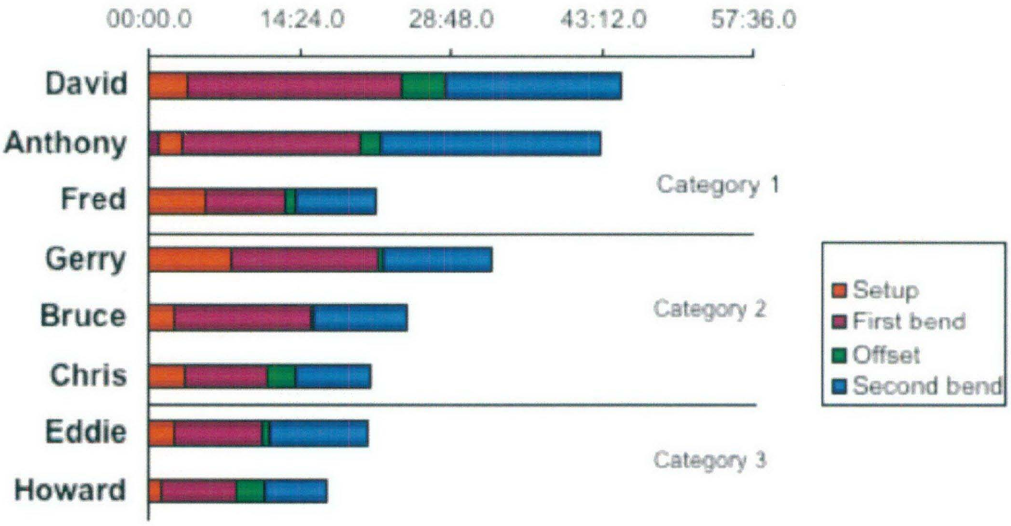

Figure 6.8. Overall key performance stage completion times.

David, Anthony, and *Gerry* took the longest to complete the task, with the maximum time being approximately 45 minutes by David. The time taken to complete the task for these three participants was deemed to be *lengthy* in relation to others. The remainder of the participants could be considered, for the purposes of this study, to have spent a *reasonable* amount of time to complete the task. Nevertheless, the length of time taken to complete the downpipes was not an indicator of accuracy of the final product.

Accuracy

The second measure of the quality of the response was the accuracy of the final product. Of interest here was whether the downpipe produced by the participant adhered to the required specifications, and therefore could be used in a workplace context. Photographs of the completed downpipes are included in Appendix G. An expert plumber with 15 years experience in the trade was asked to appraise each downpipe. The complete appraisals are included in Appendix H.

There were two groupings for accuracy of the final product. These groupings are driven by whether the downpipe was suitable for workplace use based on the specifications provided on the Job Sheet.

Group 1

The downpipes from *Anthony*, *David*, *Fred*, *Gerry,* and *Howard* were in the first group, and could not be used. Anthony's downpipe looked satisfactory from the front view, but the top view showed a "twist" (Figure 6.9).

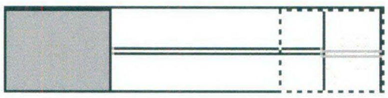

Figure 6.9a. Top view of a usable downpipe

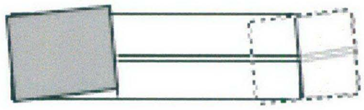

Figure 6.9b. Top view of a twisted downpipe

A twist would mean that a downpipe would not line up with the other joints such as the gutter at the top, or the drain on the ground.

The downpipes made by David, Fred, Gerry, and Howard had an incorrect offset distance. In each case, the offset was larger than specified. This would mean that the top of the downpipe would not fit with the connection inserted in the gutter (Figure 6.10)

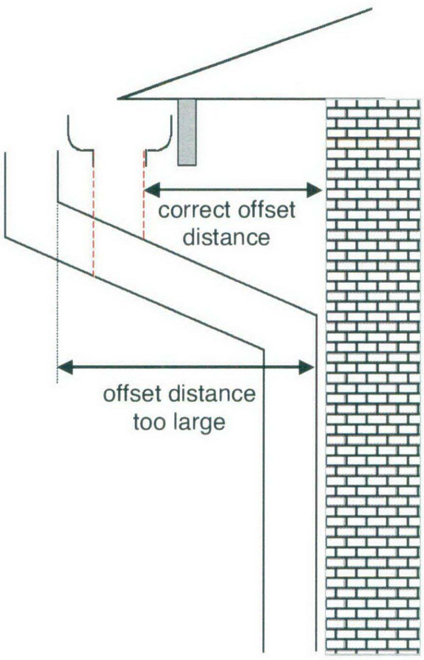

Figure 6.10. Offset distance incorrect

In cases where the offset distance was too large, or similarly too small, these products would have to be discarded and the process begun again. This would be wasteful in terms of materials and time.

Group 2

The second group consisted of downpipes that could be used in the workplace, and these were made by *Bruce*, *Chris*, and *Eddie*. However, two sub-groups were identified: with Chris and Eddie belonging to Group 2A and Bruce to Group 2B.

The downpipes made by Chris and Eddie were not precise to specifications. However, they were deemed satisfactory for workplace use because the nature of the error would not effect the downpipe's use in a workplace context.

Chris measured the incorrect distance from the top of the downpipe, as well as having different angles (116° and 118°). The qualified plumber stated that it would be acceptable to cut down the top of the downpipe to amend this error, and the angle discrepancy could be accounted for due to the flexibility of the joins in order to make it fit to specifications.

Eddie's downpipe had the correct offset distance, but the angles of 122° were deemed to be "a bit out" by the qualified plumber. Despite this discrepancy, the downpipe was judged to be within acceptable limits for workplace use.

The downpipe fabricated by Bruce was technically accurate according to the appraisal by the qualified plumber. All angles and dimensions were exact to the specifications on the Job Sheet.

Summary

The efficient strategy choices used, time taken for completion, and accuracy of the final product, provide insight into the quality of responses in fabricating an offset downpipe for a group of plumbing apprentices. These results are summarised in Table 6.1.

Table 6.1. Overview of descriptive quality of holistic performance

	Efficient strategy	Time taken	Workplace use
David	Category 1	lengthy	no
Anthony	Category 1	lengthy	no
Fred	Category 1	reasonable	no
Gerry	Category 2	lengthy	no
Chris	Category 2	reasonable	Yes 2A
Bruce	Category 2	reasonable	Yes 2B
Howard	Category 3	reasonable	No
Eddie	Category 3	reasonable	Yes 2A

From this table, some tentative results relating to quality of performance can be obtained. *David* and *Anthony* exhibited performances that were lengthy, showed no evidence of an efficient strategy, and could not be used. These two responses were considered to be of the poorest quality. *Fred's* response also existed in Category 1, and although his downpipe could not be used, he took significantly less time to complete the task. Therefore, Fred's performance is considered to be better than that of Anthony and David.

In terms if efficient strategy use, *Gerry*, *Bruce,* and *Chris* produced Category 2 responses, considered to be qualitatively higher than those in Category 1. Gerry took longer to complete the task than Bruce or Chris, and his downpipe could not be used. Bruce took slightly longer to complete the task than Chris, however, both of their performances were deemed to take a reasonable length of time. The downpipe produced by Chris was not as accurate as the one made by Bruce, but both were deemed satisfactory by the qualified plumber (see Appendix H) to be used on a house. Therefore, due to a more precise product, Bruce's performance is judged to be better than that of Chris.

Howard and *Eddie* took approximately the same time to complete their downpipes, and both performances existed in Category 3 for efficient strategy use. The predominant difference between these two performances was that the offset distance on Howard's downpipe was incorrect: the downpipe could not be used. Overall, Eddie delivered the best performance within the cohort: he was efficient, took a reasonable time in fabrication, and produced an accurate product.

By using efficiency and accuracy to support the descriptive analysis, performances could be ranked from lowest to highest quality. However, such grading can be contested depending on the aspect to which the greatest weighting is given. For example, the predominant qualifier for quality of performance used in this analysis was the use of efficient strategy, with the accuracy of the final product the second qualifier. If the order of the qualifier was reversed, a different ranking would result.

Therefore, the notion of 'quality' of performance as described above can be seen to be subjective: dependent on the perspective of the person carrying out the analysis. Additionally, there is little insight gained into the cognitions of the participant, with the exception of looking at the strategy choices made during the task.

Theoretical Perspective

This section considers the quality of responses by utilising a theoretical framework, and is divided into two parts. The first part presents a task analysis of the performance, leading to the expected SOLO levels for drawing a diagram to full size. In the second part, the descriptive vignettes are analysed for cognitive performance using the SOLO Model.

Task analysis and predicted SOLO descriptors

A task analysis of the activity of fabricating an offset downpipe was carried out following the Pilot Study. This can be considered under three Key Performance Stages (KPS); these stages also form the main aspects of fabrication, and are the *set-up* where a cut-out distance for fabrication of obtained, fabricating the *first bend*, establishing the *offset*, and fabricating the *second bend*.

To complete the task, the participants needed to utilise a number of concepts. In the Set-up KPS, the participants had to establish a cut-out distance which would result in the required bend angle of 120°. The method taught at TAFE college to obtain this measurement, is by drawing the diagram to full size. Ultimately, only one bend is required to obtain the cut-out distance. All other fabrication specifications can be gleaned directly from the Job Sheet.

In the first bend KPS, apprentices needed to determine the distance of the bend from the top of the downpipe, transfer the cut-out measurement to the correct position on the downpipe, and draw the V to then cut-out of the downpipe. Drawing the V required careful planning, as the laps also need to be accounted for. These laps enable the bend to be fixed at the desired angle by rivets, but they also need to be placed so that they do not interfere with the flow of water, whilst minimising leaks. In addition to these considerations, the seam on the downpipe is required to face the wall of the house.

In the offset KPS, the participants needed to determine the offset distance correctly. This can be done in many ways, including transferring the offset distance from the full-size diagram, or using a ruler on the downpipe itself. In particular, apprentices were required to measure from the left of the first set of verticals to the left of the second set of verticals.

For the second bend KPS, participants needed to build the bend from the correct place identified from the offset distance measurements, and the V needs to be drawn in such a way so that the downpipe heads towards the ground level, and not up again to make a "U" shape. Apart from this consideration, and the direction of the laps, its fabrication then mirrors that of the first bend.

Overall, the cut-out distance is required to be the same for both bends. Hence, the angles for both bends will also be the same, leading to the vertical sections of the downpipe being parallel to each other, as well as the wall of the house. An additional requirement relates to the nature of the original lengths of downpipes. One end of the downpipe is thinner than the other, enabling multiple lengths to be inserted into each other if a longer downpipe is required. Therefore, on making the first bend, the wider section should be towards the top.

In considering the development of possible descriptors for SOLO levels, it is necessary to consider two major factors: (i) the mode of operation, and (ii) characteristics of levels within these modes. The focus of the performance of fabricating an offset downpipe is the concrete symbolic mode. Functioning in the concrete symbolic mode involves the use of a symbol system that can be related to experiences drawn from the participants' world. Serow (2002, p. 66), stated that operating in this mode

> ... enables concepts and operations that are applied to the environment to be manipulated through the medium of symbolic systems ... [and] such responses indicate logic and ordering between symbols, and between elements of the world they represent.

In the context of the practical task of fabricating a downpipe, examples of this may include the interpretation and use of scale diagrams and written specifications. The existence of a Job Sheet meant that for the participant to complete the task to the desired requirements, they must be able to operate within the symbol system: i.e., concrete symbolic mode.

In looking within the concrete symbolic mode, two cycles of structural development are possible. The first cycle can be described as being the development of a concept, with the second cycle being the application of that concept. In the present task, the fundamental concept of the performance may be the construction of one bend. If the first cycle in the concrete symbolic mode consists of coming to know how to make one bend, then the second cycle would be being able to use that bend in further fabrication: two bends in an offset downpipe.

In terms of the SOLO Model, the level structure is composed of individual elements. Increasing degrees of sophistication relate to how these elements are accrued and related to one another. Each of these two cycles requires component elements in order to achieve functionality. Based on this cyclical framework, descriptors of the levels within each cycle can be hypothesised. These are detailed in Table 6.2.

Table 6.2 SOLO levels of hypothesised concrete symbolic responses

Level	Descriptor
Unistructural (U_1)	Focus on one element only: bend angle (size, marking, cutting out), laps, seam correctly placed, correct way up.
Multistructural (M_1)	Elements that make up the bend are dealt with independently.
Relational (R_1)	Can complete one bend, but this takes time due to the number of elements that need to be taken into account such as water flow (laps, rivets, silicon), bend angle, seam placement, narrow end to bottom. Task often lengthy, with a focus on small elements rather than on the 'big picture'.
Unistructural (U_2)	Can complete one bend efficiently; taking all elements into account. Focus is clearly on one bend, and problems arise with the incorporation of this component into other, larger, tasks.
Multistructural (M_2)	Completion of the downpipe through completion of component elements, namely: fabricating one bend, establishing the offset, and fabricating the second bend. No connection between these elements of is evident.
Relational (R_2)	Understanding of links between the KPSs prior to task being commenced. Such knowledge influences decision making and strategy choices. Choices made here are connected to ways in which task efficiency can be obtained.

In general, there is a cycle of levels in coming to know how to construct one bend. Once this concept is achieved, it forms an element in the second cycle. In order to enable the completion of the second cycle, additional elements also are required, namely: drawing the diagram to scale, establishing the offset, and making the second bend. Each of these elements can be considered as requiring its own first-cycle developmental structure to obtain U_2 status, therefore making it available for incorporation into M_2 grouping. Figure 6.11 provides further detail regarding the elements in the second cycle.

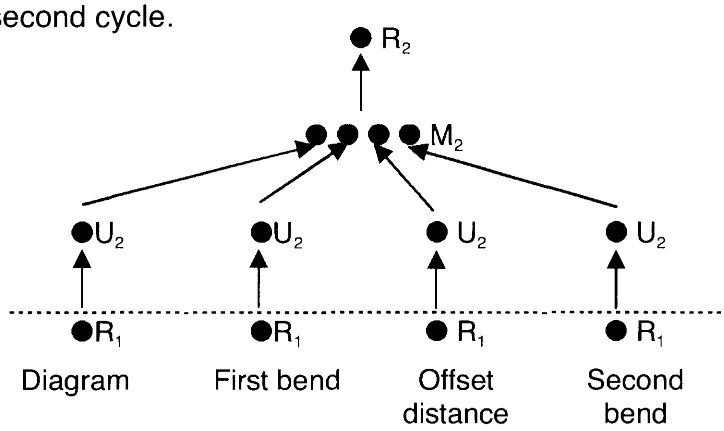

Figure 6.11. Growth of elemental knowledge

In summary, production of an offset downpipe relies on being able to make one bend. The skill of fabricating a single bend is one of the components of an offset downpipe, the others being establishment of the offset distance, and the inclusion of a second bend in the correct direction.

As determined in the previous section, completion of a final product in itself does not guarantee a useable product; such performance can be explained in cognitive terms using the SOLO Model. It is possible for a response to exhibit characteristics similar to those at a higher level, but component elements may contain errors, alternative conceptions, or be absent altogether. In the present context, a downpipe may be fabricated, but a lack of understanding of the elements in either the first or second cycle may result in a faulty or ineffective product. Therefore, the responses in the following discussion need to be examined closely for particular indicators of achievement.

Theoretical analysis

In this part, SOLO Model is applied to the descriptive vignettes to provide a cognitive developmental aspect to the performance. They are presented in order of 'quality' as derived in the previous section, with the student exhibiting the lowest quality performance being discussed first.

David

David completed each Key Performance Stage exhibiting no evidence that he was thinking ahead to solve the task. Each Key Performance Stage was dealt with separately and when the need arose. For example, David drew the first bend to scale to get the measurements to fabricate the first bend. As he required measurements for the offset and second bend, these aspects of the diagram were also constructed. He did not recognise that the first bend measurement would suffice.

His fabrication time overall was lengthy. Much of his time was spent in hesitation over the scale diagrams, marking the sections on the downpipe to cut out, or in trimming the cut sections. With regards to the scale diagram, he had difficulty in finding correct combinations of the set-squares to provide 120°: 90° + 30° and 90° + 45°. He reverted eventually to using the 120° combination for the first bend. Additional time on the scale diagram was spent in drawing the offset and second bend. Even though the offset was established correctly, the second bend was initially drawn using the 135° set-square combination. Due to the first bend being drawn at 120°, the second bend made the vertical lines askew. A convoluted set of plans with multiple lines resulted. Eventually, David re-drew the first bend on paper and used that measurement for the second bend. David had attempted to draw the whole diagram to scale, and could not complete this aspect of the performance.

Efficiency was further compromised, by David spending lengthy periods trimming the cut-out sections prior to folding the bends. Despite such attention to detail, the qualified plumber stated that the cut sections produced joins that were not neat. The accuracy of the final product was not precise. The whole job had a twist, but the fact that the offset measurement was not correct, would mean that it would not be usable in the workplace.

In terms of SOLO, David's focus on precise cuts and numerous measurements for making a bend are characteristic of a relational response in the first cycle (R_1). His fabrication was not efficient, and despite producing an offset downpipe, there were times when David was at risk of not completing the task. Some of the elements of first and second cycle performance were not competently executed, and contained periods of hesitation. These included drawing of the downpipe to full size, positioning of the seam, establishment of the offset distance, and marking and cutting out the bend angle.

Anthony

Anthony completed the downpipe fabrication without showing knowledge of any connections between the Key Performance Stages. Particularly, he did not recognise that the measurements from the first bend had any relation to, or use in, the second bend. Verbalisation of thoughts during the stimulated-recall sessions indicated guessing and "playing around" with procedures "trying to figure out what to do next". Success in strategy choice appeared to come as a surprise to Anthony on many occasions, with statements such as "I only just figured it out ... we didn't learn that" arising throughout the interview. It is noteworthy that the method used by Anthony to establish the cut-out marks for the first bend was different to other participants. However, the procedure was effective.

Long periods of hesitation and changes in decisions indicated a lack of strategy and clear view of what would provide an appropriate solution. In particular, the time spent reconciling the offset position with the orientation of the second bend contributed to lengthy performance overall. Errors here were a result of a lack of congruence between measurements taken from the downpipe, and measurements taken from the inaccurate scale diagram that had been drawn. Eventually, the error was noticed and corrected, leading to the second bend being fabricated and the downpipe completed.

Despite attention to detail regarding the neatness of joins, accuracy of bend angles, and direction of laps, the completed downpipe had a twist that the qualified plumber appraising the products deemed too prominent. Therefore the downpipe could not be used in the workplace.

In terms of SOLO, Anthony was competent at completing a single bend. He exhibited a high degree of attention to detail in fabricating this bend, evidenced by his care and time taken in all aspects of its production. There were no periods of hesitation indicating uncertainty, although he was particular regarding the precision and finish of the bend. This response would be coded at U_2.

In looking at the remainder of the response, Anthony's comments in the interviews confirmed that much of his performance consisted of successful 'guess and check' strategies. This approach was successful in the establishment of the offset distance, where he conceded that he "didn't really know how to measure it so had a bit of a guess and then tried to check it a bit more". However, difficulties arose in reconciling the offset distance with the position and orientation of the second bend, indicating that these elements were not yet fully acquired. This strengthens the case for a U_2 coding.

Fred

In fabricating the offset downpipe, Fred completed each Key Performance Stage as independent items. The whole diagram was drawn to scale as a first step, followed by fabrication of the first bend, establishment of the offset, and fabrication of the second bend. There was no evidence of Fred making connections between the measurements for the first and second bends.

Three factors contributed to the lengthy nature of the fabrication. The first factor was the scale diagram. Fred spent effort creating a technically accurate and clear diagram. The second factor was the numerous checking of measurements, both on the scale diagram and also in marking the cut-out marks on the downpipe. The final factor contributing to the lengthy performance was the hesitation surrounding the accuracy of his offset measurement.

Although Fred spent time ensuring an accurate representation of the job on his scale drawing, his offset measurement was too large. Since his establishment of the offset on the downpipe was based on his scale diagram, this resulted in a product that could not be used.

In terms of SOLO, Fred could fabricate at least one bend competently, placing his performance in the second cycle. He was able to orientate and fabricate the second bend; however, his difficulties with establishing a correct offset distance indicated a lack of understanding of this element. Therefore, he is close to competence in the other component elements required to fabricate an offset downpipe. This determination, in conjunction with a lack of connection between the elements, results in a coding of unistructural transitional, $U_2 \rightarrow M_2$.

Gerry

Gerry completed the task, but did not recognise elements of performance that were similar. No connections were made between either the Key Performance Stage, or the first and second bends. Although he drew both bends to scale, his drawing was inaccurate. Gerry knew that something was not right, but at the time of fabrication, he was unable to resolve this dilemma. Eventually, the measurements from the first bend were used for the second bend, but he was unclear why he made this decision. In the interview session he stated that "normally" they were both the same.

Gerry's performance was highlighted by a high degree of care in assuring accuracy of the completed product. All aspects of the fabrication exhibited attention to detail: Gerry stated that he gets "in trouble from the boss sometimes ... for taking too long and making sure things were right all the time." His performance was lengthy, but this time was not spent in hesitation, rather in taking his time to get each aspect as precise as he could make it.

Gerry's attention to detail resulted in errors, both in the scale diagram, and also in the establishment of the offset. It was this element of establishing the offset that resulted in a product that could not be used.

In terms of SOLO, Gerry could clearly complete a single bend. His performance of this fundamental element was characterised by a drive for accuracy as well as attention to the fine detail required. Incorporation of a single bend into the larger task of the offset downpipe did not diminish the performance of making one bend; however, overall efficiency was greatly reduced due to his focus on this element.

Although Gerry was able to complete the downpipe, there was no clear overview of the task as a whole, and each Key Performance Stage was treated as an individual entity; such a response is coded as M_2. At the end of the performance, Gerry indicated that he had some recognition of the measurements for both bends being the same, indicated his potential to make sound connections between the Key Performance Stages. However, errors in establishing the offset distance strengthen this determination for a weak multistructural response, M_2.

Chris

Chris' performance was very similar to that of Gerry. Each Key Performance Stage was done consecutively, with little evidence of connections between the stages. He stated that you do one bend then deal with the next.

There were few periods of hesitation, and the fabrication process contained no multiple checking of measurements. However, Chris paused when establishing the offset distance, as he was unsure where to mark this distance on the downpipe. Although Chris was quick in his fabrication, he produced a downpipe that was adequate for workplace use. In appraising the downpipe, the qualified plumber pointed out that the top section was too long and the angles were incorrect, but were within reasonable limits for use; he stated that it "was made well".

Chris drew both bends to scale on his diagram to make sure that he had the cut-out measurement for both bends the same. Chris was aware of this link between the

Key Performance Stages of the first and second bends: that the measurement of the first bend is the same as the measurement for the second bend.

In terms if the SOLO Model, Chris was able to make and use one bend, placing this performance in the second cycle of the concrete symbolic mode. Each element in the fabrication process was treated independently on the other, and competently achieved. However, there were moments of hesitation and rechecking of measurements to reassure him of his original decisions. Such an analysis results in a multistructural determination, M_2.

Bruce

Bruce completed the downpipe by treating each Key Performance Stage as separate entities. Each stage was carried out, with the addition that the second bend incorporated the measurements from the first bend. This information was not utilised to minimise time taken in drawing the whole diagram to full size, as this realisation made the potential data gained from the second bend unnecessary. Each element of the performance was carried out quickly and with little hesitation.

The appraisal by the qualified plumber indicated that Bruce had produced a good product, with a neat appearance and correct dimensions. It would therefore be able to be used in the workplace.

In terms of SOLO, the treatment of the Key Performance Stages as independent elements, with a late realisation that the first bend data could be used in the second bend. Bruce's final product was precise to specifications. Therefore, this response is coded as multistructural transitional, $M_2 \rightarrow R_2$.

Howard

Howard's performance exhibited efficient strategy use. He knew that the measurement for the first bend would be the same as that for the second bend. Therefore, he only drew one bend to scale. Despite this connection, Howard had difficulty in establishing the correct combination of set-squares to provide a 120°

angle for his scale diagram. Once this was overcome, he was clear on exactly what measurements were required to begin the fabrication process; these being the cut-out measurement.

Howard worked quickly to mark the cut-out section for the first bend, but during the cutting out process, realised that he had the seam on the incorrect side. This error meant that he had to begin the process again, contributing to the overall time taken.

Although Howard recognised very quickly that he had made an error in establishing a correct offset distance, he made the decision to complete the task regardless. The qualified plumber appraised the downpipe favourably with regard to presentation, but acknowledged that the angles and offset being incorrect would result in it not being able to use the downpipe in the workplace.

In terms of SOLO, the recognition by Howard that the fact that both bends were the same, and that this knowledge could be used to streamline performance, places the response as relational in the second cycle: R_2. However, there are elements of the overall performance that were not competently addressed, such as the placement of the seam, the angle size and offset distance. This characteristic of the response denotes a transitional aspect to the performance. Whilst cognitively Howard is moving towards a relational performance, the absence of a correct use of all elements reduced the quality of the performance. Howard's response is coded as multistructural transitional $M_2 \rightarrow R_2$.

Eddie

Eddie's performance was similar to that of Howard and exhibited efficient strategy use. He only drew one bend to scale knowing that this measurement would suffice for the fabrication of both bends. His diagram was drawn quickly and to the required specifications. All data, other than the cut-out measurement and diagonal offset distance, was gained from the measurements outlined on the job sheet.

Eddie's performance was purposeful. He completed accurately each Key Performance Stage in quick succession and with no hesitation. All elements of the performance were carried out competently and to the standards required.

In terms of SOLO, Eddie's strategy use was similar to that of Howard, and places his response as relational in the second cycle, R_2. This coding is strengthened by the fact that all elements of the overall performance, such as the fabrication of the bends, the seam facing the back of the job, and the offset distance, were carried out competently and with no errors in performance.

Synthesis

In summary, each of the holistic performances for the practical task could be analysed using the SOLO Model. Each response was examined for its cognitive structure using this model. A comparison between the task analysis and the coded responses illustrated strong congruence. The exception was additional levels of response: unistructural transitional $U_2 \rightarrow M_2$ and multistructural transitional $M_2 \rightarrow R_2$. Within the cohort of the Main Study, all participants completed the offset downpipe. A summary of the results are provided in Table 6.3.

Table 6.3 SOLO levels evident in the practical task.

Level	Descriptor	Participant
Relational (R_1)	Completes one bend, but this takes time due to the number of elements that need to be taken into account such as water flow (laps, rivets, silicon), bend angle, seam placement, narrow end to bottom. Task often lengthy, with a focus on small elements rather than on the 'big picture'.	David
Unistructural (U_2)	Completes one bend efficiently; taking all elements into account. Focus is clearly on one bend, and problems arise with the incorporation of this component into other, larger, tasks.	Anthony
Unistructural transitional ($U_2 \rightarrow M_2$)	Competently completes one bend, and uses it competently in larger tasks. However, other elements that make up the larger task may be incomplete or missing.	Fred

Multistructural (M_2)	Completion of the downpipe through completion of component elements, namely: fabricating one bend, establishing the offset, and fabricating the second bend. No connection between these elements of is evident.	Gerry Chris
Multistructural transitional ($M_2 \rightarrow R_2$)	Completes the downpipe, and exhibits R_2 behaviour in terms of connections between elements. However, attributes of the components elements may contain errors or be problematic.	Bruce Howard
Relational (R_2)	Understanding of links between the KPSs prior to task being commenced. Such knowledge influences decision making and strategy choices. Choices made here are connected to ways in which task efficiency can be obtained.	Eddie

The previous discussion focused on the essential element of coming to know how to make one bend as being the pivotal factor in discriminating between first and second cycle responses. However, for a performance to exist at the multistructural level, more than one element needs to be in existence (as outlined in Figure 6.10). At some stage, the participants need to acquire each of these elements in order to use them. Although a downpipe can be fabricated by knowing how to fabricate bends, the lack of the other elements result in a faulty and/or ineffective product.

Summary

This section considers the performance of each participant from a descriptive and theoretical perspective. The congruence between both of these analyses are provided in Table 6.4.

Table 6.4 Congruence between descriptive and theoretical analysis of the practical task.

Participant	Descriptive analysis			Theoretical analysis
	Strategy use	Time taken	Workplace use	
David	Category 1	lengthy	no	R_1
Anthony	Category 1	lengthy	no	U_2
Fred	Category 1	reasonable	no	$U_2 \rightarrow M_2$
Gerry	Category 2	lengthy	no	M_2 (weak)
Chris	Category 2	reasonable	Yes 2A	M_2
Bruce	Category 2	reasonable	Yes 2B	$M_2 \rightarrow R_2$
Howard	Category 3	reasonable	No	$M_2 \rightarrow R_2$
Eddie	Category 3	reasonable	Yes 2A	R_2

Although the descriptive analysis provided a framework upon which to place the performance of the participants, it was three-fold and hence did not produce definitive groupings of performance. Of the eight participants, only two had similar traits using such analyses. The hierarchical arrangement of the performances was based predominantly on strategy use, and gave little insight into the mechanisms that enabled quality production of the artefact.

The theoretical perspective provided by the SOLO Model mirrored the hierarchical nature of the descriptive analysis. Greater efficiency and accurate workplace performance reported in the second section, were congruent to higher-order cognitive functioning identified by using SOLO. By carrying out the analysis with an underlying framework, groups of types of performance could be obtained, with clear descriptions of performance at each level.

In this way, reconciliation of the differences between efficient strategy, time taken and workplace use could be actualised, therefore leading to a greater understanding of the quality of the cognitive performance, despite mistakes in the fabrication process, or with the end product. For example, although the downpipe made by Howard could not be used, the quality of his overall performance in terms of the SOLO Model would indicate that his potential to produce an accurate downpipe would be greater in the near future than, for example, as produced by Fred.

Conclusion and Further Directions

In this Chapter, the performances were characterised by the utilisation of the Key Performance Stages, which then provided a framework upon which to examine more subtle similarities and differences in approach to the task. Analyses of the responses were carried out from two perspectives: descriptive and theoretical.

The descriptive analysis consisted of looking at the responses for efficiency (strategy and time taken) and accuracy of the final product. Despite the similarities evident across responses, the quality of performance could be derived: lowest quality to highest. The lowest quality performance consisted of no linking between elements of the task, took a long time to complete, and was not accurate against the specifications on the job sheet. The highest quality performance exhibited an overall view of the task prior to fabrication and made decisions relating to minimisation of time on task. The product was completed quickly and accurately.

The theoretical analysis was pre-empted by a task analysis based upon the results from the Pilot Study. Following this, each vignette was examined for cognitive structure based on the SOLO Model. In brief, each of the responses from the apprentices in this research sample can be described as existing in the concrete symbolic mode. Levels from two cycles were identified, with transitional responses also evident, reflective of movement between levels within this mode. The responses were identified as R_1, U_2, $U_2 \rightarrow M_2$, $M_2 \rightarrow R_2$, and R_2.

A first cycle response exhibits difficulties with the fundamental component of fabricating the first bend. In the context of the present task, the whole task may be completed, but the difficulties existing with this fundamental component prohibit adequate completion. Responses in the second cycle showed sound skill in fabricating one significant aspect, usually the first bend, as a precursor to being able to establishing the offset distance and then fabricating the second bend independently, and fully being able to see relationships between the different tasks.

Research Question 1 stated, *Can the SOLO Model be applied as a framework to describe cognitive performance in workplace practice?* The similarity of the cognitive analysis to the 'expected' levels of quality underpinned by efficiency and accuracy which permeate workplace expectations, is evidence that Research Theme 1 can be answered in the positive. The descriptive analysis used a subjective framework where the performance could be categorised according to what elements were considered and what weighting were given to the elements. Use of the SOLO Model provided a much clearer objective approach where the quality of performances could be ascertained. Further, because the SOLO framework is more generally applicable it provides links with the performance on other tasks.

Although this initial analysis on the overall performances provided substantiation that the SOLO Model is a useful tool to examine workplace practice, such performances cannot themselves by described as uniquely 'mathematical'. Subsequently, the Research Question 2 remains to be investigated: *Can the SOLO Model be applied as a framework to describe cognitive performance of mathematics in workplace practice?* By examining the transcripts of each participant, there were common elements of performance that were amenable to utilising mathematical knowledge. The foremost manifestation of mathematics in the task was in the first Key Performance Stage, where the participants were setting up the task by drawing the scale diagram to full size. The next Chapter investigates Research Theme 2 by examining the use of scale diagrams to support fabrication of an offset downpipe.