

**AN EXPLORATION OF THE MATHEMATICAL
KNOWLEDGE OF PLUMBING APPRENTICES:
A COGNITIVE DEVELOPMENTAL PERSPECTIVE
USING THE SOLO MODEL**

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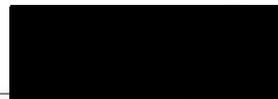
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DEDICATION

For my parents, in thanks for the gifts of life and education.

I certify that I am the sole author and that the substance of the thesis has not already been submitted for any degree and is not currently being submitted for any other degrees.

I certify to the best of my knowledge that any help received in preparing this thesis, and all the sources used, have been acknowledged in this thesis.

A solid black rectangular box redacting the signature of the author.

Signature

ACKNOWLEDGMENTS

Projects of this kind are often undertaken, but rarely completed as a purely individual enterprise. Such is the case with this body of work. It is right and proper, and indeed thoroughly necessary, to thank those who have made it possible for me to have a tangible product for my labours.

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suspect that if my students were to really know the origin, they would never forgive him for having to now suffer the endearing terms of “possums” and “cherubs”.

Earlier, I referred to the undertaking of a thesis as never being one of individual enterprise. This is particularly true when one is a parent. There are many people who need to be thanked here, predominantly those who looked after the children so that I could study. I would like to thank my wonderful husband, Greg for being the plumber who inspired the vocational target for this thesis, and whose patience was stretched as the process wore on. However, he never failed to be supportive and provided the girls with quality “daddy time”. To my parents-in law, Jeff and Dianne, who are the two very special people who always took the girls up to “Manbulloo” for numerous days (and sometimes weeks). I have never regretted not being able to tell mother-in-law jokes, because no one has a better mother in law than I.

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ABSTRACT

This study investigated the skills and understandings of apprentice plumbers from a cognitive developmental perspective. The purpose of the study was two-fold, namely: (i) to determine whether a cognitive developmental framework could be applied to describe workplace performance, and (ii) to use that framework to examine mathematical components of that performance. The design was exploratory in that at the time of inception of the study, use of cognitive theory to examine mathematical workplace practice was a gap in the literature.

The cognitive developmental theory employed in this study is the SOLO Model, which is neo-Piagetian in nature. Numerous studies have verified the SOLO Model as a valid framework upon which to qualitatively describe cognitive performance, using substantial empirical data; however, there have been no studies to date which utilise the SOLO Model to examine workplace practice either holistically, or for mathematical substance. The use of the SOLO Model to examine cognitions within workplace-related contexts is therefore novel.

The present study extends the Mathematics in the Workplace literature and SOLO Model literature in two ways: (i) using the SOLO Model in workplace-related contexts, and (ii) providing a cognitive developmental perspective to mathematics in the workplace.

The study examines the manner in which apprentice plumbers fabricate an offset downpipe to specifications outlined on a Job Sheet, within the context of instruction at a Technical College. The apprentices in this study were taken from the first three years of initial apprenticeship training, with ages ranging from 18 to 22 years. The design was ethnographic in nature, and data collection consisted of researcher immersion, followed by the use of stimulated recall using video technique.

The analysis of data is comprised of two aspects, reflecting the two purposes of the study. The first aspect explores the task as a whole problem-solving event. The second aspect focuses on one component of the task identified as being amenable to mathematical thought: drawing the diagram on the Job Sheet to full size. Each aspect used both descriptive and theoretical analyses to support findings gleaned.

The findings of the study provide support that the SOLO Model shows promise in providing a cognitive developmental perspective to workplace practice. Descriptive levels of quality of performances were obtained for both the task as a whole and for the drawing of a diagram to full size. These descriptions characterised levels of understanding for the task. These levels could be organised using the two-cycle version of the SOLO Model, and within this structure transitional levels were identified.

The study identified levels of student achievement from a holistic problem-solving perspective, as well as within a component of the task amenable to mathematical thought. Whilst the results indicated that cognitive performance on compartmentalised components was higher than that identified for the whole task, knowledge of how the components related to the whole task produced cognitive responses more sophisticated in structure.

Whilst the predominant paradigm for analysis of workplace mathematics has been the socio-cultural perspective, the present research highlights the potential of a cognitive developmental perspective as a powerful teaching and learning tool. The current research effort was successful in determining the applicability of the SOLO Model to provide insight into workplace understandings, including those of a mathematical nature. Consequently, such research opens the door for future studies into workplace mathematics from a cognitive developmental platform, whether it be the SOLO Model, or another similar theory.

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INTRODUCTION

Competence in a chosen vocation is of importance to the majority of the population. Such competence also goes beyond that of the individual to encompass collectives, therefore impacting on respective workforces. Cornford and Beven (1999) concluded that a skilled workforce is the key to national wealth and economic ascendancy. One fundamental competency for effective participation in work is the use of mathematical ideas and techniques (Mayer Committee, 1992). Not surprisingly, international and domestic research in recent years has more closely examined mathematics in and for work. On the domestic front, the National Centre for Vocational Education Research (2001, p. 2) called for “the practical or functional application and use of mathematics” to be a key area for exploration.

Whilst numerous studies have been afforded in this direction, examination from a cognitive developmental perspective has been a gap in the literature: The predominant paradigm has been socio-cultural. Socio-cultural studies provide a rich tapestry of the factors impacting on the learning environment, however substantial conceptual analyses (Cornford & Beven, 1999) and developmentally-based research (Sträßer, 2001) are still required in order to provide insight into appropriate teaching and learning strategies to underpin skill development amenable to mathematics.

The deficiency of cognitive theory in the target field required investigation into alternative contexts in order to acquire an appropriate framework for analysis within this study. The SOLO Model (Biggs & Collis, 1982) is a neo-Piagetian cognitive theory that has been successful in providing insight into the knowledge and understandings of numerous subject areas. Despite such success in examining cognitions in formal schooling contexts, empirical data regarding its applicability in practical workplace environments remains elusive. Therefore, the focus of this study is an exploratory one to establish the applicability of a cognitive developmental model,

namely SOLO, in providing insight into workplace mathematics. As a consequence, the deficiencies in mathematics in the workplace research may also be addressed.

The following discussion provides an outline of the structure of the thesis consisting of eight chapters. Since the study is concerned with mathematics in the workplace, Chapter 1 provides a background of research directed in this field.

In Chapter 2, literature relating to the knowing and learning of mathematics from a constructivist perspective is reviewed. A more detailed discussion on the historical background of research in the field of mathematics in work is addressed. This discussion highlights the possible reasons underlying the predominance socio-constructivist paradigm in studies in this area.

The SOLO Model has a proven ability to determine the cognitive developmental aspects of students' understandings in a variety of school-based subjects and related concepts. As a result, the SOLO Model has been chosen for its potential to analyse workplace practice, and in particular, tasks amenable to mathematics. A description of the SOLO Model is provided in Chapter 3, as well as a brief literature review of studies incorporating this conceptual framework. A theoretical perspective on the applicability of the Model in workplace contexts completes this Chapter.

Due to the methodological implications inherent in the dual novel nature of this study, initial investigations into appropriate methodology, data gathering techniques and rudimentary examination of the appropriateness of the conceptual framework to be used for analysis, need to be carried out. Subsequently, a Pilot Study was designed to address these issues is outlined in Chapter 4. Implications of the findings as they relate to the Main Study conclude this chapter.

Chapter 5 details the research design utilised in the Main Study. It consists of two sections. The first section outlines the design employed, the data collection protocols, and the data analysis plan. The chapter concludes with an evaluation of the research methodology.

Within Chapter 6, data were analysed from a holistic perspective, examining the responses of each individual participant to provide an overall view of the entire task performance. The first research theme was to assess whether the SOLO Model could be applied to workplace practice, to describe overall cognitive performance irrespective of an academic disciplinary focus. The responses were initially carried out using workplace criteria, and secondly using the theoretical perspective provided by the SOLO Model.

Chapter 7 provides the results of exploring the second research theme, namely, whether the SOLO Model could describe cognitive performance involved in more mathematically oriented components of the task. This chapter follows a similar structure to the previous chapter, and presents the SOLO analysis of participant responses to the component of the task of drawing a scale diagram to full size.

Finally, Chapter 8 identifies some of the limitations emerging during the course of the study, along with a summary of the results as they relate to the two research themes explored. Emerging from these findings are implications in relation to the field of Mathematics in Work, and the theoretical framework. Due to the exploratory nature of this study, both in relation to workplace studies and the SOLO Model, numerous future research directions have been generated, and these conclude the thesis.