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# Resource considerations for integrated planning of railway traffic and maintenance windows 

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#### Abstract

This paper addresses the coordination of railway network maintenance and train traffic. The work extends a previously developed optimization model by considering maintenance resource constraints for crew availability and work time regulations. The aim is to find a long term tactical plan that minimizes the total cost of maintenance and train operations, where train services and train free windows are scheduled such that maintenance can be carried out by a pool of crew resources, which are divided into bases and have limitations on maximum working hours per day and minimum rest time between these working days. A mixed integer linear programming model along with computational experiments are presented which show that these resource considerations can be correctly handled with a moderate increase in model size and solution time.


Keywords: Railway scheduling; Maintenance planning; Optimization

## 1. Introduction

Railway infrastructure maintenance consumes large budgets, is complicated to organize and has numerous challenging planning problems. Specifically, the coordination of maintenance tasks and train traffic is of great importance, since these activities are mutually exclusive. This planning conflict becomes crucial on lines with high traffic density and/or around the clock operation - especially when both traffic demand and maintenance needs are increasing.

We address the problem of how to coordinate network maintenance and train services on a common railway infrastructure. The objective is to achieve a long-term tactical master plan for when and how to perform traffic and maintenance, by scheduling train paths as well as train free time windows where maintenance work can be carried out, such that the total cost for maintenance and train operations is minimized. An aggregated approach (both spatial and temporal) is used, which assumes that the detailed train conflict resolution (meet/pass planning) is handled in a subsequent timetabling process step. A basic model that solves this coordination problem to optimality has been presented in Lidén and Joborn (2017), where a complete description of the background, problem setting, relevant research literature and model details can be found.

This paper extends the previous work by considering maintenance resource constraints and costs. The resource constraints ensure that the obtained scheduling of maintenance windows can be covered by a pool of crew resources, which are divided into bases and have limitations on maximum working hours per day and minimum rest time between these working days.

[^0]The contributions of this work are: (1) showing how some important crew resource considerations can be modelled, and (2) computational experiments demonstrating the effect of including these aspects in the optimization model. To the best of our knowledge, this is the first research publication that jointly schedules both train services and time for network maintenance, while considering crucial maintenance resource limitations.

The paper is organised as follows: Section 2 describes the problem setting and features considered. Section 3 gives a brief overview of related research. The mathematical model is presented in Section 4 , followed by the computational experiments and results in Section 5. Finally, some concluding remarks are given in Section 6

## 2. Problem description

The planning problem we consider applies to organisations that are responsible for coordinating railway traffic and network maintenance slots, such as infrastructure managers, transport administrators or railway companies that own and operate the infrastructure network. The planning horizons of train services and maintenance tasks can differ substantially, which - depending on the planning procedure - may favour early applicants and leave costly or even insufficient track access possibilities for other actors. This situation has been observed in Sweden where the increase in rail traffic together with the current planning regime has forced maintenance to be performed on odd times and/or in shorter time slots which leads to inefficiency and cost increases for the maintenance contractors, potentially even reduced track quality, leading to an increase in governmental spending.

To increase the possibility of suitable work possessions, a new planning regime is being introduced, where the Swedish Transport Administration propose regular, 2-6 h train free maintenance windows before the timetable is constructed. The maintenance windows are given as a prerequisite for: (a) the procurement of multi-year maintenance contracts, and (b) the yearly timetable process, which give stable quotation and planning conditions for the contractors. The overall aim is to increase efficiency, reduce cost and planning burden as well as to improve robustness and punctuality. However, since maintenance windows will reduce the train scheduling possibilities, the window patterns should be designed such that maintenance activities and train operation is coordinated in a well-balanced manner, which is non-trivial.

We address the coordination of maintenance windows and train traffic as an optimization problem for a railway infrastructure network. The purpose of the optimization model is to find a pattern of maintenance windows that allows a desired set of train services to be run and that minimizes the total cost for train operations and maintenance. The train operating cost is measured by total running time, deviation from preferred departure, route choice and cancellations, while the maintenance cost consists of direct work time, indirect setup/overhead time and crew costs. Routing and scheduling of trains respect given minimum travel and dwelling durations as well as the line capacity limitations imposed by the maintenance scheduling. The maintenance window schedule fulfills given work volumes, where the number of windows and their temporal size respect a chosen window option for each network link as well as maintenance resource considerations.

A macroscopic infrastructure model is used, which allows for networks of arbitrary size and granularity. Nodes are placed where train services start, end or may change route, as well as between different maintenance areas. The nodes are connected by links which correspond to single, double or multi-track lines that may contain intermediate stations (meet/pass loops). Traffic capacity restrictions are modelled as limitations on the number of trains that can be scheduled over each link per time period, both in each direction and as a sum of both directions. The traffic capacity is reduced when maintenance windows are scheduled on the links.

Throughout this paper we use the term crew for a potential work group, which may consist of several people and necessary equipment. We only distinguish the crew resources by the links they can service. Thus the model does not consider different resource types and whether a maintenance window should be used for a specific type of maintenance activity. Also, it should be noted that the model is not intended for scheduling actual maintenance tasks - rather it aims at constructing train free windows that give an optimal balance between train operating and network maintenance costs, in a way that can be efficiently utilized by the maintenance contractors.

Although it is the responsibility of the contractor to manage the maintenance resources, the design of the window patterns together with the partitioning into contract areas will determine the possible crew scheduling and subsequently the number of maintenance crew needed. In general, simultaneous windows will increase the crew demand while sequential windows on neighbouring links enable the contractor to cover several windows with the same crew during a working day. Thus the major resource driving aspects should be considered (by the infrastructure manager) when constructing the maintenance windows, since they will impact the maintenance / contractual cost. The resource considerations to include in our model have been discussed and agreed upon with the Swedish Transport Administration and one of the major maintenance contractor companies.

The first type of resource consideration is the number of crew per maintenance base and the links they can service, which constrains the number of concurrent maintenance windows that can be scheduled. The number of crew may also be unlimited along with a cost for each crew utilized, in which case the model should choose the optimal combination of crew, window and train scheduling. Since we have no distinction between resource types, all crew within one base can serve the same set of links. The crew bases may overlap, i.e. certain links may be serviced by more than one base. By varying the base configuration it is possible to evaluate and experiment with different contract areas and partitions.

The second type of maintenance crew resource consideration is work time regulations, given as maximum number of work hours per working day (typically $8-12$ hours) and minimum rest time between working days (typically 10-16 hours). The exact start/end time of the working days need not be given beforehand, but is left for the optimization model to decide.

Finally we note that in this work exactly one crew is assigned to each maintenance window. It might however be interesting to include requirements on the number of crews that should be assigned to each maintenance window.

We end this section by summarizing the problem features and giving some typical properties. First of all, both maintenance windows and train services are to be scheduled over a railway network for a period of one or more days. The tasks are typically one to a couple of hours long, where the train services have continuous (real-valued) start/end times and durations over the links. The track capacity must be controlled according to the maintenance window schedule, where we use discrete one hour time periods. Maintenance crew resources are considered, but only their spatial availability is given beforehand. The temporal crew scheduling and location sequencing shall respect work time regulations regarding maximum work hours per working day and minimum rest time between working days. Different crew types and varying travel time between work locations are not considered.

## 3. Related research literature

For a broad overview of problems and research models regarding planning of railway infrastructure maintenance and its coordination with train traffic, we refer to Lidén (2016). The literature review in Lidén and Joborn (2017) analyses publications regarding models for scheduling of (a) network maintenance, (b) train services and (c) combined approaches. Some publications in group (a) consider resource constraints, partly matching the constraint types that we are interested in, but as seen by the problem features description in the previous section, we propose a combined approach (model group (c)) with resource considerations. No such research publications have been found. In the following, we therefore focus on publications in model group (c) - where both maintenance and traffic are variable - and papers which handle maintenance crew resource constraints.

Research references that schedule both network maintenance and train services are found in Albrecht et al. (2013); Forsgren et al. (2013); Lidén and Joborn (2017); Luan et al. 2017), where all papers except the third study how to introduce a limited number of maintenance tasks into an existing traffic plan by making small adjustments to the trains. Albrecht et al. study the real-time operational control case for single-track lines while Forsgren et al. and Luan et al. treat the timetable revision planning case for mixed networks. Lidén and Joborn address a long-term tactical planning case for which no timetable or maintenance plan exists as starting point, and present an optimization model that can solve various multi-day instances to near optimality within 1 h of computation. None of these publications consider maintenance resource constraints.

An existing maintenance plan can also be used as starting point. Boland et al. (2013) adjust a given maintenance plan for a complete transportation chain of coal, so as to maximize the transported throughput. Traffic is handled as flows of (fractional) trains and the maintenance activities impose reductions on the link capacities in the network. Although the trains are not individually scheduled in this model, the traffic volumes are still variable and depend on the maintenance plan. A similar approach is used for another coal freight network in Savelsbergh et al. (2014), who use a given maintenance plan and evaluate it by measuring the optimal throughput. From these plans an assessment of the asset reliability, resource requirements and contract compliance can be made. None of these publications consider the type of resource constraints that we are interested in.

As for resource considerations, such work has only been found in publications concerning maintenance planning and scheduling. The most basic type of resource constraint is to limit the number of crew or work groups. Publications that only have this type of constraint are for example Higgins (1998); Lake et al. (2002); Zhang et al. (2013). Handling of different work group capabilities is a common consideration found in many papers, see e.g. Cheung et al. (1999) for an early example. Several publications study the routing of maintenance teams, in which case home location and travel time becomes important. Some recent examples are Peng and Ouyang (2012); Borraz-Sánchez and Klabjan (2012); Camci (2015); Consilvio et al. (2016). The second and third of these papers also consider work volume limitations. Borraz-Sánchez and Klabjan study multi-day tasks, where any necessary rest time is included in the durations, and schedule start/end days such that no tasks overlap weekends and vacations. Thus the rest time regulations are handled implicitly although the exact start/end time of day remains to be decided. A yearly maximum workload is handled as a constraint while the solution heuristic tries to evenly spread the work among the work groups. Camci considers shorter tasks (some hours long) which do not include night rest. These tasks are assigned to weeks and ordered in routing sequences, while imposing a maximum work load per week (typically 40 h ), which should make it possible to decide a detailed schedule, that is feasible regarding rest time regulations, in a later stage.

To summarize, we could not find any existing literature that considers resource constraints when jointly scheduling network maintenance and train traffic. Furthermore, we have not found any railway maintenance scheduling references that decide exact start and end times while considering maximum work hours per working day and minimum rest time between working days. It should be noted that such work regulations are commonly handled in other crew scheduling fields (i.e. airline crew pairing and rostering).

## 4. Mathematical model

This section consists of two parts. Section 4.1 presents the modelling principles that have been used and the structure of the basic optimization model. Then, Section 4.2 presents the formulation for the resource model. A complete and detailed description of the mathematical model, including definitions, input data and variables is given in Appendix A

### 4.1. Preliminaries

Here we give a simplified formulation of the basic model, which we label ISM as an acronym for the Integrated train Service and railway Maintenance planning problem. This description summarizes the main properties and definitions on a structural level.

ISM is an improved version of the optimization model presented in Lidén and Joborn (2017), which we refer to as "the original model". The original model has cumulative train entry/exit variables and allows for variable length time periods. ISM on the other hand, uses binary entry/exit detection variables and explicit link usage variables for the train scheduling, and adopts a unit time approach such that all time periods have length one. Thus we can directly make use of the strong formulation for $\min / \mathrm{max}$ on/off sequences as presented in Pochet and Wolsey (2006, section $11.4 \mathrm{pp} 341-343$ ), labelled Restricted Length Setup Sequences. The mathematical properties are studied by Queyranne and Wolsey (2017), who show that this formulation describes the convex hull and thus there is no tighter formulation for that set of variables.

The railway network is modelled by a link set $L$. Here we assume that all links have capacity restrictions and shall be maintained, but this can easily be restricted to a subset of the links. The scheduling problem
has a planning horizon of length $H$, divided into a sequence $T=\{1, \ldots, H\}$ of unit size time periods $t \in T$, each with starting time $t-1$.

For the links $l \in L$ to be maintained, there is a set $W_{l}$ of window options, where each window option $o \in W_{l}$ is defined by a required number of occasions $\eta_{o}$ that a maintenance window of length $\theta_{o}$, given as an integer number of time periods, must be scheduled. The scheduling of maintenance windows on a link shall be done according to one of the window options. As an example, $W_{l}=\{(1,3),(2,2)\}$ means that either one window of length three or two windows of length two shall be scheduled on link $l$.

For the train traffic we have a set $S$ of train services. Each train service $s \in S$ has a set $R_{s}$ of possible routes, defined as a sequence of links, which gives the set $L_{s}$ of all possible links that train service $s$ can visit. The scheduling of trains shall be done by selecting one route $r \in R_{s}$ and deciding entry and exit times over each link in that route. All event times for train $s$ must be within a limited train scheduling window, given by $T_{s} \subseteq T$.

The following variables are used:
$z_{s r} \quad$ route choice: whether train service $s$ uses route $r$ or not (binary)
$e_{s l}^{+}, e_{s l}^{-} \quad$ event time: $\operatorname{entry}(+) / \operatorname{exit}(-)$ time for service $s$ on link $l$
$x_{s l t}^{+}, x_{s l t}^{-} \quad \operatorname{link}$ entry/exit: whether train service $s$ enters/exits link $l$ in time period $t$ or not (binary)
$u_{\text {slt }} \quad$ link usage: whether train service $s$ uses link $l$ in time period $t$ or not (binary)
$n_{l t}^{h} \quad$ number of train services traversing link $l$ in direction $h$ during time period $t$
$w_{l o} \quad$ maintenance window option choice: whether link $l$ is maintained with window option or not (binary)
$y_{l t} \quad$ maintenance work: whether link $l$ is maintained in time period $t$ or not (binary)
$v_{l o t} \quad$ work start: whether maintenance on link $l$ according to window option $o$ is started in time period $t$ or not (binary)

The basic model can now be summarized as follows:

$$
\begin{align*}
& \text { ISM }:= \text { minimize } \quad c(\mathbf{z}, \mathbf{e}, \mathbf{y}, \mathbf{v})  \tag{1}\\
& \text { subject to } \mathbf{A}(\mathbf{z}, \mathbf{e}, \mathbf{x}, \mathbf{u})^{\text {route }}  \tag{2}\\
& \mathbf{A}(\mathbf{z}, \mathbf{e})^{\text {trains }}  \tag{3}\\
& \mathbf{A}(\mathbf{w}, \mathbf{y}, \mathbf{v})^{\text {maint }}  \tag{4}\\
& \mathbf{A}(\mathbf{u}, \mathbf{n}, \mathbf{y})^{\text {cap }}  \tag{5}\\
& \mathbf{z}, \mathbf{x}, \mathbf{u}, \mathbf{w}, \mathbf{y}, \mathbf{v} \text { binary }  \tag{6}\\
& \mathbf{e}, \mathbf{n} \text { non-negative } \tag{7}
\end{align*}
$$

where $c(.$.$) is a linear objective function and \mathbf{A}(.$.$) are linear constraints over the indicated variables.$
The objective (1) is a linear combination of the route choice, train and maintenance scheduling variables. The constraints enforce: (2) correct (feasible) bounds on the train events and linking of entry / exit and usage variables according to the selected route, (3) sufficient travel durations and dwell times along the chosen route, (4) sufficient maintenance windows scheduled according to the chosen option and (5) that the available network capacity is respected, both during normal conditions and when maintenance is scheduled. The constraint matrices in $\sqrt{2}-5)$ are sparse.

The structure of this model is illustrated in Figure 1 as a constraint (or co-occurrence) graph, with vertices for the variables and edges connecting variables that occur in the same constraint. The constraints correspond to cliques in the graph as indicated in the figure. For example, there are no constraints connecting the variables $z_{s r}$ or $e_{s l}^{a}$ with $u_{s l}$. Hence, the corresponding blocks in the constraint matrix of (2) are zerovalued. Similarly, the $u_{s l t}$ and $y_{l t}$ variables are not connected by any constraints which means that the constraint matrix of (5) is banded.
[Figure 1 about here.]

### 4.2. Resource model formulation

The resource constraints to handle are: (a) a limited number of maintenance crew, with (b) work time limitations enforcing a maximum length $\Omega$ on the work day and a minimum rest time $\Psi$ before and after the work day. The values $\Omega$ and $\Psi$ are in integer time units. As input we need a set $K$ of maintenance crews and subsets $K_{l} \subseteq K$ stating which crews can maintain link $l$. Typically the crews are partitioned into crew bases, such that all crew belonging to a base can maintain the same set of links. Some links may be covered by several crew bases. In the following we make use of the definition $L_{k}^{K}:=\left\{l \in L: k \in K_{l}\right\}$, which gives the set of links that can be maintained by crew $k \in K$.

The resource model uses the following variable definitions:
$q_{k} \quad$ crew usage: whether crew $k$ is used/assigned to any work or not (binary)
$\breve{y}_{k t} \quad$ crew availability: whether crew $k$ is available for maintenance work in time period $t$ or not (binary)
$\check{v}_{k t} \quad$ start of work day: whether time period $t$ is the first in a working day for crew $k$ or not (binary)
$d_{l k t} \quad$ crew assignment: whether maintenance on link $l$ in time period $t$ is done by crew $k$ or not (binary)

We define $D_{k}^{\min }$ as the shortest work day length, and $\Delta_{k}^{\max }$ as the largest possible separation between two work days, for each crew $k$. These values can either come from input data or be calculated from the window options while considering which links crew $k$ can be assigned to. Here we use the latter approach as detailed in Appendix B

With these definitions, the restricted length sequence formulation of the problem becomes:

$$
\begin{array}{cl}
\sum_{k \in K_{l}} d_{l k t}=y_{l t} & \forall l \in L, t \in T \\
\sum_{l \in L_{k}^{K}} d_{l k t} \leq \check{y}_{k t} & \forall k \in K, t \in T \\
\check{v}_{k t} \geq \check{y}_{k t}-\check{y}_{k, t-1} & \forall k \in K, t \in T \\
\check{v}_{k t} \leq \check{y}_{k t} & \forall k \in K, t \in T \\
\sum_{t^{\prime}=t+1-D_{k}^{\min }}^{t} \check{v}_{k t^{\prime}} \leq \check{y}_{k t} & \forall k \in K, t \in T \\
\sum_{t^{\prime}=t+1-\Omega}^{t} \check{v}_{k t^{\prime}} \geq \check{y}_{k t} & \forall k \in K, t \in T \\
\sum_{t^{\prime}=t+1}^{t+\Psi} \check{v}_{k t^{\prime}} \leq q_{k}-\check{y}_{k t} & \forall k \in K, t \in T \\
\sum_{t+\Delta_{k}^{\max }} \check{v}_{k t^{\prime}} \geq q_{k}-\check{y}_{k t} & \forall k \in K, t=1, \ldots, H-\Delta_{k}^{\max } \tag{15}
\end{array}
$$

Here constraint (8) ensures that each time period of a maintenance window is assigned exactly one crew, (9) that a crew can have at most one assignment per time period for which it also must be available, 10 11 coupling between crew availability and start of work day, 12 / (13) minimum / maximum work day length, and (14) / 15) minimum / maximum rest time.

When including the above resource constraints we obtain the constraint graph shown in Figure 2
[Figure 2 about here.]

There are however some further additions needed. First we may want to minimise the crew usage. To do so, we add the following term to the objective function:

$$
\lambda^{u s e} \sum_{k \in K} q_{k}
$$

where the weight factor $\lambda^{\text {use }}$ scales the crew usage versus the train and maintenance scheduling costs. To remove symmetries an ordering of the crew variables (within each crew base) should be imposed. Assuming all crew are partitioned into $B$ bases $b=1, \ldots, B$ consisting of disjoint ordered sets of base crew $K_{b}^{\text {base }}$ then the ordering constraints are:

$$
\begin{equation*}
q_{k} \leq q_{k-1} \quad \forall b=1, \ldots, B ; k \in K_{b}^{\text {base }} \backslash \text { first }\left(K_{b}^{\text {base }}\right) \tag{16}
\end{equation*}
$$

It may also be of interest to minimise the crew availability (e.g. if salary cost depends on scheduled availability). This is achieved by adding the following term to the objective function:

$$
\lambda^{\text {avail }} \sum_{k \in K} \sum_{t \in T} \check{y}_{k t}
$$

It should be noted that $\sqrt[89]{9}$ allow for crew swaps within a maintenance window. If the movement times between links are negligible and the main purpose of the model is to ensure that work time regulations can be respected, then the swaps are not a problem since the actual crew rosters can be constructed in a post processing step. To some extent, we can discourage unnecessary swaps between links by reducing the number of links assigned to each crew. This is achieved by the objective term

$$
\lambda^{l i n k} \sum_{k \in K} \sum_{l \in L_{k}^{K}} q_{k l}^{L}
$$

which relies on the variables:
$q_{k l}^{L} \quad$ crew link usage: whether crew $k$ is used/assigned to any work on link $l$ or not (binary)
and the constraints:

$$
\begin{equation*}
d_{l k t} \leq q_{k l}^{L} \quad \forall k \in K, l \in L_{k}^{K}, t \in T \tag{17}
\end{equation*}
$$

If there are limitations on how soon a crew can be assigned to another link $l^{\prime}$ after having finished an assignment on a link $l$, either the required window times should be increased so as to include the average relocation time or additional constraints introduced for separation of the $\check{y}_{l k t} / d_{l k t}$ variables. Link to link dependent values will make the problem considerably harder to solve, as compared to fixed separation values.

A final concern is that there might not exist any feasible solution, which happens if there are too few crew available or the combination of resource limitations, maintenance needs and train services is impossible to fulfil. There are various ways of handling such cases, either by having a pool of extra crew (which have a higher cost) or by introducing variables that make it possible to override the resource limitations or leave certain maintenance tasks unassigned (at a high cost). We have not included such possibilities in the current model.

The complete model for solving the integrated planning problem with resource considerations is now given by

$$
\begin{array}{rlrl}
\text { ISMR }:= & \text { minimize } & & (1)+\lambda^{u s e} \sum_{k \in K} q_{k}+\lambda^{\text {avail }} \sum_{k \in K} \sum_{t \in T} \check{y}_{k t}+\lambda^{l i n k} \sum_{k \in K} \sum_{l \in L_{k}^{K}} q_{k l}^{L} \\
& \text { subject to } & (2-17 \\
& \mathbf{q}, \check{\mathbf{y}}, \check{\mathbf{v}}, \mathbf{d}, \mathbf{q}^{L} \text { binary }
\end{array}
$$

## 5. Computational experiments

This section contains two parts. Section 5.1 describes the experimental setup and the data instances, while 5.2 presents the results along with the conclusions drawn.

### 5.1. Setup and data instances

The same set of synthetic problem instances as in Lidén and Joborn (2017) have been used for testing the various resource constraint formulations. An additional set of network instances N6-N9 have been created which are essentially copies of N2-N5 but with a different traffic pattern, such that there are origindestination relations between all peripheral nodes which interact during the planning horizon. Thus there are nine line instances and nine network instances, having a planning horizon of five hours to one week divided into 1 h periods and with 20 to 350 train services. All line instances except one (L4) are single track, while the network instances have a mixture of single and double track links. The properties of the problem instances are listed in Appendix C For the links $l \in L$ the maintenance volume $V_{l}$ to be covered by maintenance windows, ranges from 1 to 3 h per day.

Five different crew base configurations have been created, ranging from one base covering all links to several small bases with varying levels of overlap. The configurations are as follows:

NR - no resource considerations (used as comparison)
BL - one large base for all links $\left(\left|L_{k}^{K}\right|=|L|\right)$
BM - medium sized bases with no overlap $\left(1 / 3 \leq\left|L_{k}^{K}\right| /|L| \leq 1 / 2\right)$
BS - small bases with no overlap ( $1 / 6 \leq\left|L_{k}^{K}\right| /|L| \leq 1 / 4$ )
BSoS - several bases, small overlap (1 link)
BSoL - several bases, large overlap (2-3 links)
The overlap configurations BSoS and BSoL are illustrated in Figure 3.

$$
\text { [Figure } 3 \text { about here.] }
$$

The number of crews for configuration BL is determined by the expression

$$
|K|=\left\lceil\frac{1}{\Omega p} \sum_{l \in L} V_{l}\right\rceil
$$

where $p=|T| /(\Omega+\Psi)$ estimates the number of work days in the schedule. The amount of crew for the other configurations have been scaled such that each base has the same number (at least one). All the instances and crew configuration data are detailed in Appendix C

Finally the cost factors for the crew resources have been set to $\lambda^{\text {use }}=1, \lambda^{\text {avail }}=0.1$ and $\lambda^{\text {link }}=0.01$. These are arbitrary values that roughly give a hierarchical crew cost structure. For real life instances the factors should either be normalised or have a common monetary meaning which is correctly scaled towards the train operating and maintenance window costs.

### 5.2. Results

The computations have been run on a Dell PowerEdge R710 rack server with dual hex core 3.06 GHz Intel Xeon X5675 processors and 96GB RAM running Red Hat Enterprise Linux 6. Gurobi 6.5 was used as the MIP solver with four threads, a relative MIP gap of $0.001(0.1 \%)$, and a maximum computation time of 3600 seconds. Otherwise the default options were used. All solutions obtained have been plotted and visually inspected to make sure that the work time regulations as well as the crew base partitioning have been respected.

Table 1 gives the solution time (in seconds) to reach the optimal solution or else the remaining MIP gap after 3600 seconds for all instances. The double track instance L4 is easily solved no matter the crew configuration. We note that all instances except L9 are solved to optimality or a gap of $<1 \%$ when no resource considerations are included. This is still achieved when adding the resource limitations for all the network instances and most line instances except L5, L7 and L8 where the optimality gap becomes larger than $1 \%$. Also we see that it takes longer to reach optimality or that optimality is not reached within the time limit. However, we find it quite encouraging - perhaps even surprising - that so many instances can still be solved to optimality relatively quickly, or to a relative optimality gap of a few percent or better within the time limit.

## [Table 1 about here.]

We now take a closer look at how the solutions differ depending on the crew configurations. Figure 4 shows the different solutions obtained for instance L6, where crew assignments are shown with a coloured horizontal line on the windows. The NR solution would require nine crews to perform work on all maintenance windows. The BL solution on the other hand only needs two crew, but with long travel distances between the maintenance windows. BM and BSoL use three crews while BS and BSoS need four crews.
[Figure 4 about here.]
It is also interesting to study how the different cost components are affected by the resource constraints, which is shown in Table 2. We see that the train and maintenance window costs have only minor changes, while the crew cost follows the number of crews used. The total cost is the lowest for BL (as expected), followed by BM, BSoL, BSoS and BS. Note that other weight factors would change the outcome. However, the model is clearly capable of adjusting the train and window schedules depending on the crew configurations and cost settings.
[Table 2 about here.]
Next we compare how the different crew configuration settings affect the results over all the test instances. To do so, performance profile plots are used which show the accumulated number of instances (on the vertical axis) reaching a certain level of quality measure (on the logarithmic horizontal axis) - normalised as a factor of the best outcome for all configuration settings. In Figure 5 the time for reaching the required optimality criteria (gap $<0.1 \%$ ) is the quality measure. It is clear that BS has the shortest solution times in general, while BL and BSoL are the slowest (and have the fewest instances reaching optimality within one hour of computing). BM and BSoS lie in between, with BM solving as many instances to optimality as BS , and BSoS performing well for the small instances but taking longer to solve the large ones. The conclusion is that small bases and small overlaps solve faster, while large bases and large overlaps - although giving lower objective values - take longer to solve.
[Figure 5 about here.]
Next we plot the performance profiles for the final MIP gap, which is shown in Figure 6. Since NR has the best MIP gap for all instances its profile is a straight line. For the other crew configurations we find the number of instances solved to optimality at the far left of the horizontal axis (same as the value at the far right of Figure 5). The number of instances with a feasible solution is given by the far right
value. We see that all configurations find feasible solutions to all instances except BL which fails to find a solution to one of the instances (L9). Apart from this, it is hard to see any clear differences between the crew configuration settings. Thus, it seems the improvement of the solution quality achieved by the MIP solver is largely independent of the crew resource configurations tested here.
[Figure 6 about here.]
Finally we study how the problem size is affected, by looking at the number of constraints, variables and non-zero entries. Without the resource model the line instances range from $1.4 \times 1.5$ to $188 \times 201$ thousand constraints $\times$ variables, with 6.5 to 1200 thousand non-zero entries. The network instances go from $1.8 \times 2.0$ to $48 \times 45$ with 8.6 to 299 thousand non-zeros. The Gurobi presolve method reduces the number of constraints and variables by about $10 \%$ and the number of non-zeros by $0-10 \%$. Figure 7 shows how the problem size changes relative to NR for the different crew configurations, both for the original number of constraints and variables, and after presolve. First we note that, in general the number of constraints and variables increase about $5-10 \%$. All configurations except BL have larger growth in constraints than variables. BSoL has roughly the same growth in original number of constraints and variables as after presolve, but for the other configurations the relative change after presolve varies considerably for the different instances. Hence it is not clear whether the resource considerations make it harder for the solver to reduce the problem size during presolve or not. In some cases the number of constraints and/or variables after presolve are even reduced when including the resource model.
[Figure 7 about here.]

## 6. Conclusions

The inclusion of maintenance resource considerations is important in order to achieve plans that are reasonable and possible to utilize for maintenance contractors in an efficient way.

We have shown that limitations regarding number of crew, localisation and work as well as rest time regulations can be handled in an exact optimization model. The number of constraints and variables grow by about $5-10 \%$ and the solution time increases moderately. Further we have shown that small crew bases with no or small overlaps are easier to handle while large bases and large overlaps makes the problem harder.

The results of this work form another step towards solving joint train and maintenance scheduling problems which are of practical use. A natural continuation is to validate these models on some real life instances. In such a study the sensitivity of the cost factors will be interesting to investigate.

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Figure 1: Variable and constraint graph - basic model (ISM)


Figure 2: Variable and constraint graph - resource model


Figure 3: Crew base configurations BSoS and BSoL . The bases are shown by blue and red lines for the line instances and by the encircled areas for the network instances.


Figure 4: Solutions to instance L6
(a) No resources (NR)

(c) Two bases (BM)

(e) Four bases with 1 link overlap (BSoS)

(b) One large base (BL)

(d) Four bases (BS)

(f) Three bases with 2 link overlap (BSoL)


Figure 5: Performance profile - time to reach MIP gap 0.1\%



Figure 7: Size changes relative to NR, given as Tukey box plots. (Whiskers at lowest/highest value within 1.5 times the interquartile range below/above the lower/upper quartile, and with outliers removed - about $2-4$ per crew base configuration.)


Table 1: Results - time (seconds) to reach $0.1 \%$ or else remaining gap after 3600 seconds. (L4 is a small double track instance, which is easily solved for all the crew configurations.)

| Case | NR | BL | BM | BS | BSoS | BSoL |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| L1 | 3 | 54 | 10 | 3 | 3 | 31 |
| L2 | 3 | 34 | 18 | 3 | 2 | 20 |
| L3 | 172 | $0.11 \%$ | 358 | 52 | 980 | 1197 |
| L4 | 1 | 1 | 1 | 1 | 1 | 1 |
| L5 | 1108 | $0.37 \%$ | $1.09 \%$ | $1.04 \%$ | $1.24 \%$ | $2.52 \%$ |
| L6 | $0.13 \%$ | $0.75 \%$ | $0.99 \%$ | $0.87 \%$ | $0.61 \%$ | $0.92 \%$ |
| L7 | $0.70 \%$ | $16.7 \%$ | $20.0 \%$ | $66.0 \%$ | $29.7 \%$ | $3.78 \%$ |
| L8 | $0.99 \%$ | $166 \%$ | $118 \%$ | $8.07 \%$ | $90.2 \%$ | $2.74 \%$ |
| L9 | $14.8 \%$ | no IP | $202 \%$ | $65.4 \%$ | $69.8 \%$ | $667 \%$ |
| N1 | 2 | 5 | 4 | 2 | 2 | 5 |
| N2 | 4 | 155 | 50 | 26 | 100 | 84 |
| N3 | 25 | 90 | 50 | 59 | 43 | 219 |
| N4 | 326 | $0.14 \%$ | $0.13 \%$ | $0.11 \%$ | $0.13 \%$ | $0.20 \%$ |
| N5 | 542 | $0.15 \%$ | 2725 | 1458 | $0.14 \%$ | $0.18 \%$ |
| N6 | 4 | 101 | 51 | 25 | 58 | 24 |
| N7 | 21 | 66 | 67 | 37 | 39 | 446 |
| N8 | 116 | $0.12 \%$ | 592 | 594 | 807 | $0.16 \%$ |
| N9 | 252 | 1040 | 489 | 643 | 470 | $0.10 \%$ |

Table 2: Costs for instance L6

|  | NR | BL | BM | BS | BSoS | BSoL |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| Train cost | 160.7 | 161.6 | 161.6 | 161.2 | 161.5 | 161.4 |
| Window cost | 3.8 | 4.0 | 4.0 | 4.0 | 3.9 | 4.0 |
| Sum | 164.5 | 165.6 | 165.6 | 165.2 | 165.4 | 165.4 |
| Crew cost | 0.0 | 5.2 | 6.5 | 7.9 | 7.2 | 6.8 |
| Total cost | 164.5 | 170.8 | 172.1 | 173.1 | 172.6 | 172.2 |

## Appendices

## A. Detailed model description

In this appendix we give the complete and detailed description of the mathematical model. The equation numbering is consistent with the rest of the paper.

## A.1. Sets and parameters

The scheduling shall be done within a planning period defined by
$H \quad$ the length of the planning horizon
$T \quad$ a sequence $\{1, \ldots, H\}$ of unit size time periods $t$, each with starting time $t-1$

## Railway network

The network is defined by
$N \quad$ a set of network nodes
$L \quad$ a set of network links $l=(i, j): i, j \in N \wedge i \neq j \wedge i \prec j$
$C_{l}^{\text {Nom,d }} \quad$ nominal track capacity (number of trains per time unit) in each direction for link $l \in L$
$C_{l}^{\text {Red,d }} \quad$ reduced track capacity in each direction for $\operatorname{link} l \in L$, when maintenance is carried out
$C_{l}^{\text {Nom }} \quad$ nominal link capacity (sum for both directions) for $\operatorname{link} l \in L$
$C_{l}^{\text {Red }} \quad$ reduced link capacity (sum for both directions) for $\operatorname{link} l \in L$

## Train services

The scheduling of train services is defined by
$R \quad$ a set of routes $r$, each defined as an ordered set of link-direction pairs $(l, h) \in L \times\{0,1\}$, where $h=1$ if the route traverses link $l$ in the $i, j$ (forward) direction, and 0 otherwise. One route alternative may be a cancellation - consisting of no links.
$L_{r}, N_{r} \quad$ the links and nodes in route $r \in R$, given by $L_{r}=\{l \in L:(l, h) \in r\}$ and $N_{r}=\{i, j \in N:$ $\left.(i, j) \in L_{r}\right\}$
$S \quad$ a set of train services $s$ to be scheduled
$R_{s} \subset R \quad$ the set of possible routes $r$ for train service $s \in S$
$L_{s}, N_{s} \quad$ all links and nodes that can be visited by train service $s$, given by $L_{s}=\bigcup_{r \in R_{s}} L_{r} ; N_{s}=\bigcup_{r \in R_{s}} N_{r}$
$N_{s}^{O}, N_{s}^{D} \quad$ the origin and destination node for train service $s$
$T_{s} \quad$ the allowed time periods for train service $s$
$\tau_{s} \quad$ the preferred departure time for train service $s$
$\pi_{s r l} \quad$ minimum duration over link $l \in L_{r}$ when train service $s$ uses route $r \in R_{s}$
$\epsilon_{s n} \quad$ minimum duration (dwell time) at node $n \in N_{s}$ for train service $s$
$\sigma_{s}^{\text {time }} \quad$ cost per travel time for train service $s$
$\sigma_{s}^{\text {dev }} \quad$ cost per time unit for train service $s$, when deviating from the preferred departure time $\tau_{s}$
$\sigma_{s r}^{\text {route }} \quad$ cost for using route $r$ for train service $s$

## Maintenance windows

The scheduling of maintenance windows is defined by
$W \quad$ a set of maintenance window options
$W_{l} \subseteq W \quad$ the set of possible maintenance window options for $\operatorname{link} l \in L$
$V_{l} \quad$ the required maintenance volume (total time) for $\operatorname{link} l \in L$
$\eta_{o}, \theta_{o} \quad$ number of occasions and required number of time periods for maintenance window option $o \in W$
$M T_{o} \quad$ the minimum consecutive maintenance time for option o
$\lambda_{l t}^{t i m e} \quad$ cost per time unit when performing maintenance work on link $l$ in time period $t$
$\lambda_{\text {lot }}^{\text {start }} \quad$ cost for starting maintenance work on link $l$ with window option $o$ in time period $t$

## Crew resources

The availability and assignment of crew resources is defined by
$K \quad$ a set of maintenance crews
$B \quad$ the number of crew bases
$K_{l} \subseteq K \quad$ the maintenance crews that can maintain link $l \in L$
$L_{k}^{K} \quad$ the set of links that can be maintained by crew $k$, given by $L_{k}^{K}:=\left\{l \in L: k \in K_{l}\right\}$
$K_{b}^{\text {base }} \subseteq K$ the ordered set of crew belonging to crew base $b$, with the requirement that no crew can belong to more than one crew base, i.e. $K_{b}^{\text {base }} \cap K_{b+i}^{\text {base }}=\emptyset, \forall b=1, \ldots, B-1 ; i=1, \ldots, B-b$
$\Omega, \Psi \quad$ maximum length for, and minimum rest after, a crew work day (number of periods)
$D_{k}^{\min } \quad$ the shortest possible work day length for crew $k$
$\Delta_{k}^{\max } \quad$ largest possible separation crew $k$ can have between two work days
$\lambda^{\text {use }} \quad$ cost for using a crew
$\lambda^{\text {avail }} \quad$ cost per time unit for having crew available for work
$\lambda^{\text {link }} \quad$ cost for assigning one crew to do maintenance on one link
Appendix B describes how the separation values $M T_{o}, D_{k}^{\min }$, and $\Delta_{k}^{\max }$ have been determined.

## A.2. Variables and bounds

The variables and bounds are defined as follows:
$z_{s r} \quad$ route choice: $=1$ if train service $s$ uses route $r, 0 \quad \in\{0,1\} \quad \forall s \in S, r \in R_{s}$ otherwise
$e_{s l}^{+}, e_{s l}^{-} \quad$ event time: entry $(+) / \operatorname{exit}(-)$ time for service $s$ on
$\in[0, H] \quad \forall s \in S, l \in L_{s}$
$\operatorname{link} l,=0$ if not using link $l$
$e_{s}^{O}, e_{s}^{D} \quad$ departure/arrival time at the origin/destination for
$\in[0, H] \quad \forall s \in S$
$f_{s} \quad$ departure time deviation for train service $s$, i.e.
$\in[0, H] \quad \forall s \in S$
how much $e_{s}^{O}$ deviates from $\tau_{s}$
$x_{s l t}^{+}, x_{s l t}^{-} \quad \begin{array}{ll}\text { link entry/exit: }=1 \text { if train service } s \text { enters/exits } \quad \in\{0,1\} \quad \forall s \in S, l \in L_{s}, t \in T_{s} \\ & \operatorname{link} l \text { in time period } t, 0 \text { otherwise }\end{array}$
$u_{s l t} \quad$ link usage: $=1$ if train service $s$ uses link $l$ in time $\quad \in\{0,1\} \quad \forall s \in S, l \in L_{s}, t \in T_{s}$
$n_{l t}^{h} \quad \begin{aligned} & \text { period } t, 0 \text { otherwise } \\ & \text { number of train services traversing link } l \text { in }\end{aligned}$
$\in[0,|S|] \quad \forall l \in L, t \in T, h \in\{0,1\}$
direction $h$ during time period $t$
$w_{l o} \quad$ maintenance window option choice: $=1$ if link $l$ is
$\in\{0,1\} \quad \forall l \in L, o \in W_{l}$
maintained with window option $o, 0$ otherwise
$y_{l t} \quad$ maintenance work: $=1$ if link $l$ is maintained in time period $t, 0$ otherwise
$v_{l o t} \quad$ work start: $=1$ if maintenance on link $l$ according
$\in\{0,1\} \quad \forall l \in L, t \in T$
to window option $o$ is started in time period $t, 0$ otherwise
$q_{k} \quad$ crew usage: $=1$ if crew $k$ is used/assigned to an work, 0 otherwise
$\check{y}_{k t} \quad$ crew availability: $=1$ if crew $k$ is available for maintenance work in time period $t, 0$ otherwise
$\check{v}_{k t} \quad$ start of work day: $=1$ if time period $t$ is the first in a working day for crew $k, 0$ otherwise
$d_{l k t} \quad$ crew assignment: $=1$ if maintenance on link $l$ in time period $t$ is done by crew $k, 0$ otherwise
$q_{k l}^{L} \quad$ crew link usage: $=1$ if crew $k$ is used/assigned to
$\in\{0,1\} \quad \forall k \in K$
$\in\{0,1\} \quad \forall k \in K, t \in T$
$\in\{0,1\} \quad \forall k \in K, t \in T$
$\in\{0,1\} \quad \forall k \in K, l \in L_{k}^{K}, t \in T$
$\in\{0,1\} \quad \forall k \in K, l \in L_{k}^{K}$

## A.3. Objective function

The objective function, to be minimized, is formulated as follows:

$$
\begin{align*}
& \sum_{s \in S}\left[\sigma_{s}^{\text {time }}\left(e_{s}^{D}-e_{s}^{O}\right)+\sigma_{s}^{\text {dev }} f_{s}+\sum_{r \in R_{s}} \sigma_{s r}^{\text {route }} z_{s r}\right] \\
& +\sum_{l \in L}\left[\sum_{t \in T} \lambda_{l t}^{\text {time }} y_{l t}+\sum_{o \in W_{l}} \sum_{t \in T} \lambda_{l o t}^{\text {start }} v_{l o t}\right] \\
& +\lambda^{\text {use }} \sum_{k \in K} q_{k}+\lambda^{\text {avail }} \sum_{k \in K} \sum_{t \in T} \check{y}_{k t}+\lambda^{\text {link }} \sum_{k \in K} \sum_{l \in L_{k}^{K}} q_{k l}^{L} \tag{A.1}
\end{align*}
$$

The function consists of three traffic costs (total running time, deviation from the preferred departure time and route choice), two maintenance window costs (work time and setup/overhead time) and three crew costs (number of crew used, crew availability time and number of links scheduled per crew).

## A.4. Constraints

The constraints are grouped into categories as explained in Section 4.1 and 4.2 .
Train routing, link usage and event bounds

$$
\begin{array}{cl}
\sum_{r \in R_{s}} z_{s r}=1 & \forall s \in S \\
\sum_{t \in T_{s}} x_{s l t}^{a} \geq \sum_{r \in R_{l}} z_{s r} & \forall s \in S, l \in L_{s} \\
u_{s l t}=\sum_{t^{\prime} \in T_{s}: t^{\prime} \leq t} x_{s l t^{\prime}}^{+}-\sum_{t^{\prime} \in T_{s}: t^{\prime} \leq t-1} x_{s l t^{\prime}}^{-} & \forall s \in S, l \in L_{s}, t \in T_{s} \\
\sum_{t \in T_{s}}(t-1) x_{s l t}^{+} \leq e_{s l}^{+} \leq \sum_{t \in T_{s}}(t-\xi) x_{s l t}^{+} & \forall s \in S, l \in L_{s} \\
\sum_{t \in T_{s}}(t-1+\xi) x_{s l t}^{-} \leq e_{s l}^{-} \leq \sum_{t \in T_{s}} t x_{s l t}^{-} & \forall s \in S, l \in L_{s} \tag{25}
\end{array}
$$

This group of constraints ensures correctness of: 2, 1] route choice, (2,2) linking of entry / exit variables to route choice, 233 calculation of usage values, and $\sqrt{2 / 4} 2 \sqrt{2} 5$ bounds for the event variables. Here $\xi$ is a small positive number, which ensures that $x_{s l t}^{+}=1 \Longleftrightarrow t-1 \leq e_{s l}^{+}<t$ and $x_{s l t}^{-}=1 \Longleftrightarrow t-1<e_{s l}^{-} \leq t$.

## Train scheduling and durations

$$
\begin{align*}
e_{s}^{O} & =\sum_{l \in O U T_{s, N} O} e_{s l}^{+} & \forall s \in S  \tag{3}\\
e_{s}^{D} & =\sum_{l \in I N_{s, N_{s}^{D}}} e_{s l}^{-} & \forall s \in S  \tag{3}\\
f_{s} & \geq e_{s}^{O}-\tau_{s} & \forall s \in S  \tag{3}\\
f_{s} & \geq \tau_{s}-e_{s}^{O} & \forall s \in S  \tag{3}\\
e_{s l}^{-}-e_{s l}^{+} & \geq \pi_{s r l} z_{s r} & \forall s \in S, r \in R_{s}, l \in L_{r}  \tag{3,5}\\
\epsilon_{s n} \sum_{r \in R_{s}: n \in N_{r}} z_{s r} & \leq \sum_{l \in O U T_{s n}} e_{s l}^{+}-\sum_{l \in I N_{s n}} e_{s l}^{-} & \forall s \in S, n \in N_{s} \backslash\left\{N_{s}^{O}, N_{s}^{D}\right\} \tag{3}
\end{align*}
$$

This group of constraints handles: (3.13.2) calculation of departure and arrival times, 3.3 .3 .44 deviation from preferred departure time, (35) sufficient travel durations, and (3|6) dwell times along the chosen route. Here $I N_{s n}$ and $O U T_{s n}$ are functions returning all possible incoming and outgoing links for train service $s$ at node $n$.

Maintenance window scheduling

$$
\begin{align*}
& \sum_{o \in W_{l}} w_{l o}=1 \quad \forall l \in L  \tag{4.1}\\
& \sum_{t \in T} y_{l t} \geq V_{l} \quad \forall l \in L  \tag{4}\\
& \sum_{t \in T} v_{l o t} \geq \eta_{o} w_{l o} \quad \forall l \in L, o \in W_{l}  \tag{43}\\
& \sum_{o \in W_{l}} v_{l o t} \geq y_{l t}-y_{l, t-1} \quad \forall l \in L, t \in T  \tag{44}\\
& v_{l o t} \leq w_{l o} \quad \forall l \in L, o \in W_{l}, t \in T  \tag{4.5}\\
& \sum_{o \in W_{l}} v_{l o t} \leq y_{l t} \quad \forall l \in L, t \in T  \tag{46}\\
& \sum_{o \in W_{l}} v_{l o t} \leq 1-y_{l, t-1} \quad \forall l \in L, t \in T  \tag{47}\\
& \sum_{o \in W_{l}}\left[\sum_{t^{\prime}=t+1-\theta_{o}}^{t} v_{l o t^{\prime}}\right] \leq y_{l t} \quad \forall l \in L, t \in T  \tag{4}\\
& \sum_{t^{\prime}=t+1-\theta_{o}}^{t} v_{l o t^{\prime}}+1 \geq y_{l t}+w_{l o} \quad \forall l \in L, o \in W_{l}, t \in T  \tag{4}\\
& \sum_{t^{\prime}=t+1}^{t+H-M T_{o}} v_{l o t^{\prime}} \geq w_{l o}-y_{l t} \quad \forall l \in L, o \in W_{l}, t=1, \ldots, M T_{o} \tag{4.10}
\end{align*}
$$

This constraint group concerns: (4.1) choice of maintenance window option; (4.2.4.3) work volume and number of window occasions; 4.4 4.7) consistency of work start variables $v_{l o t}$, window choice $w_{l o}$ and work variables $y_{l t}$; and 4/84 10) length of maintenance windows and separation between them.

Train counting and link capacity

$$
\begin{array}{rlr}
n_{l t}^{h}=\sum_{s \in S_{l h}^{d i r}: t \in T_{s}} u_{s l t} & \forall l \in L, t \in T, h \in\{0,1\} \\
n_{l t}^{h} \leq\left(1-y_{l t}\right) C_{l}^{\text {Nom }, d}+y_{l t} C_{l}^{\text {Red,d }}=\left\{s \in S:(l, h) \in r \forall r \in R_{s}\right\} \\
n_{l t}^{0}+n_{l t}^{1} \leq\left(1-y_{l t}\right) C_{l}^{\text {Nom }}+y_{l t} C_{l}^{\text {Red }} & \forall l \in L, t \in T, h \in\{0,1\}  \tag{5}\\
& \forall l \in L, t \in T
\end{array}
$$

These constraints control: (5,1) train counting for each link and direction, and $5,25,3$ capacity usage under normal and maintenance conditions.

## Crew resources

The remaining part of the model concerns the crew resource constraints (8, 17), which are explained in Section 4.2.

$$
\begin{array}{rlrl}
\sum_{k \in K_{l}} d_{l k t} & =y_{l t} & & \forall l \in L, t \in T \\
\sum_{l \in L_{k}^{K}} d_{l k t} \leq \check{y}_{k t} & & \forall k \in K, t \in T \\
\check{v}_{k t} \geq \check{y}_{k t}-\check{y}_{k, t-1} & \forall k \in K, t \in T \\
\check{v}_{k t} \leq \check{y}_{k t} & \forall k \in K, t \in T \\
\sum_{t^{\prime}=t+1-D_{k}^{\min }}^{t} \check{v}_{k t^{\prime}} \leq \check{y}_{k t} & \forall k \in K, t \in T \\
\sum_{t^{\prime}=t+1-\Omega}^{t} \check{v}_{k t^{\prime}} \geq \check{y}_{k t} & \forall k \in K, t \in T \\
\sum_{t^{\prime}=t+1}^{t+\Psi} \check{v}_{k t^{\prime}} \leq q_{k}-\check{y}_{k t} & \forall k \in K, t \in T \\
t+\Delta_{k}^{\max } & & \\
\sum_{t^{\prime}=t+1} & \forall k \in K, t=1, \ldots, H-\Delta_{k}^{\max } \\
q_{k} & \geq q_{k}-\check{y}_{k t} & & \forall b=1, \ldots, B ; k \in K_{k-1}^{\text {base }} \backslash \text { first }\left(K_{b}^{\text {base }}\right) \\
d_{l k t} \leq q_{k l}^{L} & & \forall k \in K, l \in L_{k}^{K}, t \in T \tag{17}
\end{array}
$$

## Variable types

$\mathbf{z}, \mathbf{x}, \mathbf{w}, \mathbf{v}, \mathbf{y}$ binary
$\mathbf{e}, \mathbf{n}$ non-negative
$\mathbf{q}, \check{\mathbf{y}}, \check{\mathbf{v}}, \mathbf{d}, \mathbf{q}^{L}$ binary

## B. Calculation of length and separation values

In this appendix we describe how the following separation values are calculated: $M T_{o}$ (minimum consecutive maintenance for window option $o, D_{k}^{\min }$ (shortest possible work day length for crew $k$ ), and $\Delta_{k}^{\max }$ (largest possible separation between two work days for crew $k$ ). These values are used in the constraints for separating maintenance windows and enforcing minimum work day length and maximum rest time length.

First we make the observation that the largest possible separation $\Delta_{l}^{\mathrm{K}, \max }$ between two work days for any crew on $\operatorname{link} l$ is given by $\Delta_{l}^{\mathrm{K}, \max }=H-M T_{l}^{\min }$, where $M T_{l}^{\text {min }}$ is the minimum consecutive maintenance time, which in turn is given by $M T_{o}$ as follows:

$$
M T_{l}^{\min }=\min _{o \in W_{l}}\left(M T_{o}\right)
$$

To calculate $M T_{o}$ we use the following:

$$
M T_{o}=\eta_{o} \theta_{o}+\max \left[0,\left(\eta_{o}-2\right)\right]
$$

where the first term calculates the maintenance window time and the last term (max[..]) counts the necessary number of time periods between maintenance windows. This is illustrated in Figure B. 1 for a case where $\eta=3$ and $\theta=2$. Now we get the largest possible separation $\Delta_{k}^{\max }$ crew $k$ can have, by considering all links crew $k$ can be assigned to, as follows:

$$
\Delta_{k}^{\max }=\max _{l \in L_{k}^{K}}\left(\Delta_{l}^{\mathrm{K}, \max }\right)
$$

Finally, we need the value $D_{k}^{\min }$ for the length of the shortest work day for each crew. If we assume the shortest possible work day consists of a single maintenance window of minimum size, the value is calculated as follows:

$$
D_{k}^{\min }=\min _{l \in L_{k}^{K}, o \in W_{l}}\left(\theta_{o}\right)
$$

This is a rather conservative value and it might be more reasonable to enforce that a working day should not be started unless a certain amount of working hours (e.g. 4-6 h or max $\operatorname{mow}_{l}\left(\theta_{o}\right)$ ) are scheduled.
[Figure 8 about here.]

Figure B.1: Calculation of $M T_{l}^{\min }$ and $\Delta_{l}^{\mathrm{K}, \max }$


## C. Instance data and crew configuration details

Table C. 1 summarizes the data instances and crew configurations. The leftmost part gives the number of links, time periods, train services, max work day and min rest time length. The rightmost part lists the number of bases, the number of crew and the overlaps - given as a tuple ( $n_{1}, n_{2}$ ) where $n_{i}$ is the number of links covered by $i$ bases. Thus $(6,3)$ means that 6 links are covered by only one base (i.e. has no overlap) and 3 links are covered by two bases.
[Table 3 about here.]

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Table C.1: Instances, properties and crew configurations

|  | Properties |  |  |  |  |  | $\|B\|,\|K\|$, overlap |  |  |  |  |
| :---: | ---: | ---: | ---: | ---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Case | $\|L\|$ | $\|T\|$ | $\|S\|$ | $\Omega$ | $\Psi$ | BL | BM | BS | BSoS | BSoL |  |
| L1 | 4 | 5 | 20 | 2 | 3 | 1,4 | 2,4 | 4,4 | $4,4,(1,3)$ | $2,4,(2,2)$ |  |
| L2 | 4 | 5 | 20 | 2 | 3 | 1,4 | 2,4 | 4,4 | $4,4,(1,3)$ | $2,4,(2,2)$ |  |
| L3 | 4 | 12 | 40 | 5 | 7 | 1,3 | 2,4 | 4,4 | $4,4,(1,3)$ | $2,4,(2,2)$ |  |
| L4 | 4 | 12 | 40 | 5 | 7 | 1,3 | 2,4 | 4,4 | $4,4,(1,3)$ | $2,4,(2,2)$ |  |
| L5 | 9 | 24 | 40 | 8 | 16 | 1,3 | 2,4 | 4,4 | $4,4,(6,3)$ | $3,3,(5,4)$ |  |
| L6 | 9 | 48 | 80 | 8 | 16 | 1,2 | 2,4 | 4,4 | $4,4,(6,3)$ | $3,3,(5,4)$ |  |
| L7 | 18 | 24 | 80 | 8 | 16 | 1,6 | 2,6 | 6,6 | $4,8,(15,3)$ | $5,10,(10,8)$ |  |
| L8 | 18 | 96 | 160 | 8 | 16 | 1,4 | 2,4 | 5,5 | $4,4,(15,3)$ | $5,5,(10,8)$ |  |
| L9 | 25 | 168 | 350 | 8 | 16 | 1,4 | 3,6 | 7,7 | $8,8,(18,7)$ | $6,12,(10,15)$ |  |
| N1 | 9 | 5 | 20 | 2 | 3 | 1,4 | 2,4 | 4,4 | $4,4,(1,3)$ | $2,4,(2,2)$ |  |
| N2 | 9 | 24 | 50 | 8 | 16 | 1,3 | 2,4 | 3,3 | $2,4,(8,1)$ | $4,4,(2,7)$ |  |
| N3 | 9 | 48 | 100 | 8 | 16 | 1,2 | 2,2 | 3,3 | $2,2,(8,1)$ | $4,4,(2,7)$ |  |
| N4 | 9 | 96 | 200 | 8 | 16 | 1,2 | 2,2 | 3,3 | $2,2,(8,1)$ | $4,4,(2,7)$ |  |
| N5 | 9 | 168 | 350 | 8 | 16 | 1,2 | 2,2 | 3,3 | $2,2,(8,1)$ | $4,4,(2,7)$ |  |
| N6 | 9 | 24 | 50 | 8 | 16 | 1,3 | 2,4 | 3,3 | $2,4,(8,1)$ | $4,4,(2,7)$ |  |
| N7 | 9 | 48 | 100 | 8 | 16 | 1,2 | 2,2 | 3,3 | $2,2,(8,1)$ | $4,4,(2,7)$ |  |
| N8 | 9 | 96 | 200 | 8 | 16 | 1,2 | 2,2 | 3,3 | $2,2,(8,1)$ | $4,4,(2,7)$ |  |
| N9 | 9 | 168 | 350 | 8 | 16 | 1,2 | 2,2 | 3,3 | $2,2,(8,1)$ | $4,4,(2,7)$ |  |


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