

## APPENDIX I

### SETS TEST - TRIAL RESULTS (Study 3)

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VARIMAX COMPONENT LOADINGS OF THE FIVE VARIABLES  
NUMBER SPAN, LETTER SPAN, WORD SPAN, PAPER FOLDING AND SETS,  
ON THE TWO COMPONENTS OBTAINED FROM THE PRINCIPAL COMPONENT  
ANALYSIS OF RAW SCORE DATA (N=32) - TRIAL STUDY

	COMPONENT 1	COMPONENT 2
NUMBER SPAN	.87	-.13
LETTER SPAN	.68	.37
WORD SPAN	.83	.23
PAPER FOLDING	-.04	.81
SETS	.27	.78

ITEM ANALYSES OF STUDY 3 TRIAL OF SETS TEST - UNION OF SETS (TYPE A)

TYPE	QUESTION	DIFFICULTY PARAMETER	ITEM - TOTAL CORRELATION [ $r_{i(t-1)}$ ]	ALPHA RELIABILITY COEFFICIENT
A	1	.44	.72	.72
	3	.41	.35	
	5	.44	.67	
	8	.53	.42	
	10	.28	.13	

From the above analysis of Type A items, namely Set manipulation - union of sets', three questions were selected for use in the main study, namely questions 1, 5 and 8. Three additional questions of this same type and potentially of the same level of difficulty were created, to make a total of six questions of the type union of sets.

ITEM ANALYSES OF STUDY 3 TRIAL OF SETS TEST - INTERSECTION OF SETS (TYPE B)

TYPE	QUESTION	DIFFICULTY PARAMETER	ITEM - TOTAL CORRELATION [ $r_{i(t-1)}$ ]	ALPHA RELIABILITY COEFFICIENT
B	2	.78	.47	.69
	4	.47	.53	
	6	.44	.61	
	7	.41	.43	
	9	.56	.53	
	11	.19	.21	
	12	.09	.18	
	13	.09	.50	
	14	.16	.40	
	15	.13	.32	
	16	.00	.00	

From the above analyses of Type B items, namely 'Set manipulation - intersection of sets', five questions were selected for use in the main study, namely questions 2, 4, 6, 7, and 9. One additional question of the same type as Question 9 and potentially of a similar level of difficulty was created, to make a total of six questions of the type intersection of sets.

ITEM ANALYSES OF STUDY 3 TRIAL OF SETS TEST - SUB-SETS (TYPE C)

TYPE	QUESTION	DIFFICULTY PARAMETER	ITEM - TOTAL CORRELATION [ $r_{i(t-1)}$ ]	ALPHA RELIABILITY COEFFICIENT
C	17	.97	.09	.54
	18	.13	.28	
	19	.22	.16	
	20	.59	.27	
	21	.56	.07	
	22	.19	.23	
	23	.41	.15	
	24	.88	.46	
	25	.66	.35	
	26	.72	.02	
	27	.47	.42	
	28	.78	.39	

From the above item analyses of Type C, namely 'Set manipulation - sub-sets', four questions were selected for use in the main study, namely questions 24, 25, 27 and 28, leaving a total of four questions of the type intersection of sets.

REVISED ALPHA COEFFICIENTS - SETS TEST

QUESTION TYPE	ALPHA COEFFICIENT
A UNION	.72
B INTERSECTION	.78
C SUB-SET	.70

## APPENDIX J

### TEST MATERIALS (Study 3)

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NUMBER AND LETTER SPAN TEST ITEMSNUMBER SPAN TEST ITEMS

1	8	1	9	5	7	2					
2	4	6	2	9							
3	3	7	1	4	9	2	5	8	1	6	
4	9	2	6	2	8	6	1				
5	7	9	5	3	8						
6	5	2	9	4	1	6	8	3	7		
7	2	6	3	1	5						
8	2	4	8	5	1						

LETTER SPAN TEST ITEMS

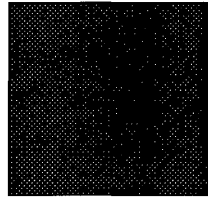
1	K	F	C								
2	H	S	L	Y	G						
3	P	F	R	C	W	S	G	K	Y		
4	P	L	S	C	W	K	R	F			
5	R	G	S								
6	L	W	C	Y	K	R	P				
7	F	S	Y	L	C	H					
8	S	C	F	K	W	L	P				



WORD SPAN TEST ITEMS

1. : MAN, LETTER, STRING
2. : COW, PEN, THORN, BABY, TIN
3. : HALF, TABLE, CAMP, PARTY, LAKE, BODY
4. : PAPER, MOSS, RIVER, DREAM, SKIN
5. : GRASS, CUP, HOUSE, DOOR
6. : AIR, CHIN, CORN, ICE, SOCK
7. : RUN, GOLD, BOOK, TAP, DRESS, HILL
8. : SALT, CLAW
9. : KISS, FOX, ARM, FISH, LIP, CARD, BELL

SHAPES TEST - TEST ITEMS



**A**



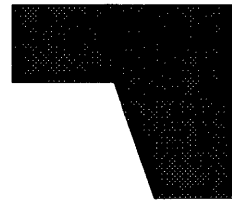
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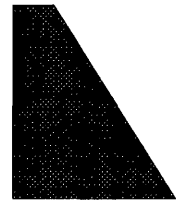
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3

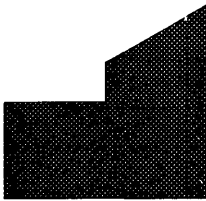


4



5

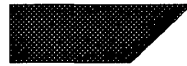
**B**



1



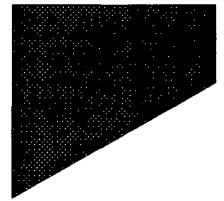
2



3



4



5

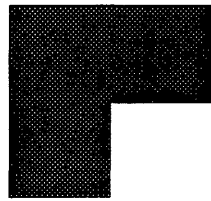
**C**



1



2



3

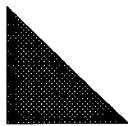


4



5

**D**



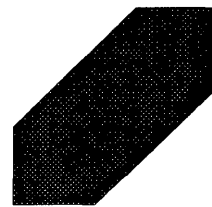
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2



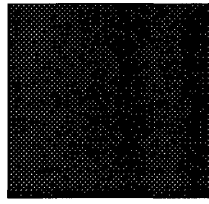
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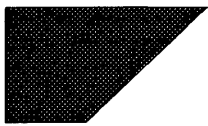
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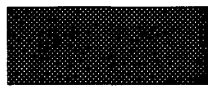
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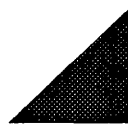
**E**



1



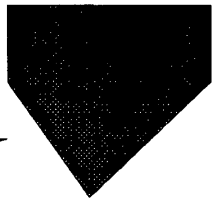
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3

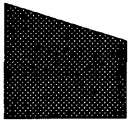


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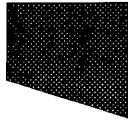


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**F**



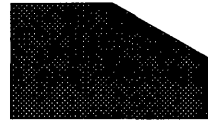
1



2



3



4

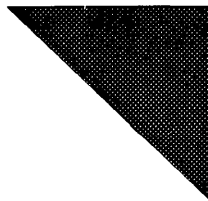


5

**G**



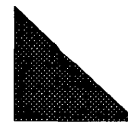
1



2



3



4

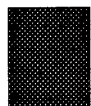


5

**H**



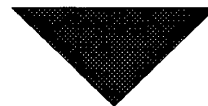
1



2



3

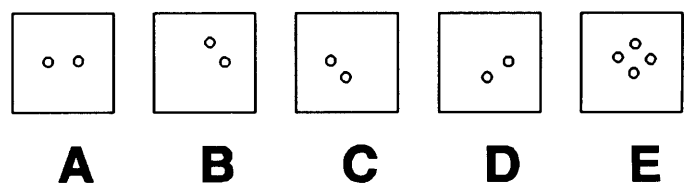
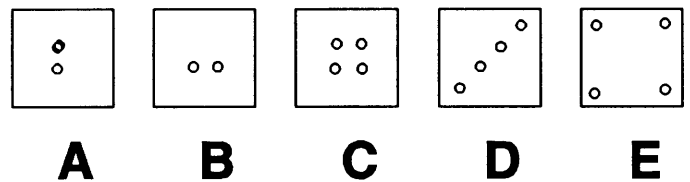
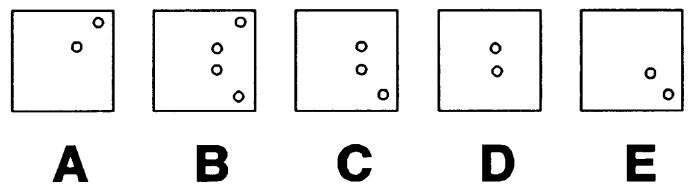
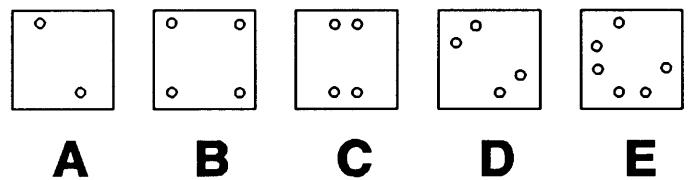
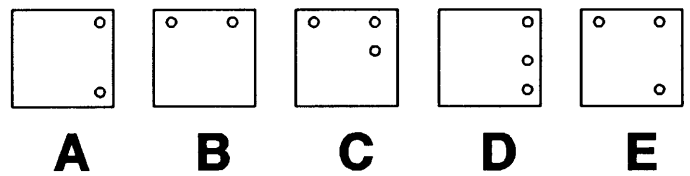
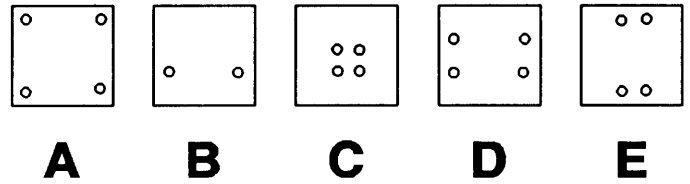
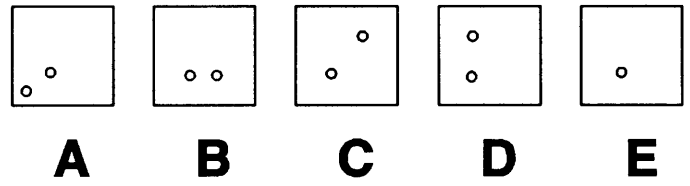
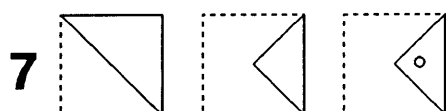
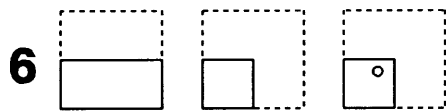
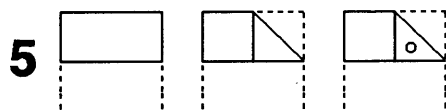
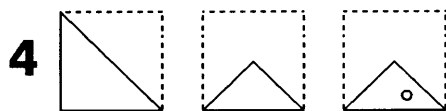
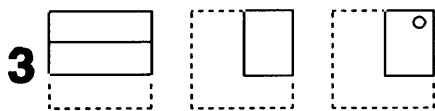
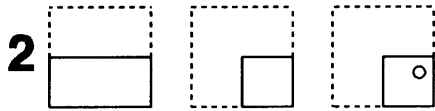
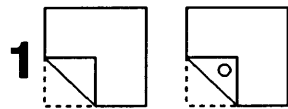


4



5

PAPER FOLDING TEST - TEST ITEMS



SETS TEST - ADMINISTRATIVE INSTRUCTIONS

If you look at the following Answer Sheet you will see that there are a number of boxes after each question number. For each question, you will be shown two groups, or sets, containing a number of members. As soon as the two sets are displayed an instruction will be given. Your task is to write in these boxes the members that will answer the question.

Members of each set, can be numbers, letters, words or symbols. In each set, characters are separated by commas. The sets are shown inside braces { }. Here are some examples:

{ A, B, C }	this set has members -	A, B, C
{ 1, 4, 67, 8 }	this set has members -	1, 4, 67, 8
{ girl, boy }	this set has members -	girl, boy

There are three types of instructions, one of which will be read out to you as soon as each question is displayed. They are:

- Write down all members in both sets, but do not repeat any member.
- or Write down members common to both sets.
- or Are the members in the second set also in the first? Answer Yes or No.

Example 1:	Write down all members in both sets, but do not repeat any member. <i>The answer you would write down is:</i>	{ 4, 7 } { 4, 8 } <b>4, 7, 8</b>
Example 2:	Write down members common to both sets: <i>The answer you would write down is:</i>	{ A, G } { C, G } <b>G</b>
Example 3:	Are all the members in the second set also in the first? Answer Yes or No. <i>The answer you would write down is:</i>	{ P, H } { F, T } <b>NO</b>

The teacher will show you each question, drawn on a large sheet of paper. Each sheet will be shown for five seconds. Look at it carefully while it is displayed. When the teacher takes it away, write down your answers. You will have ten seconds to do this before the next sheet is shown. We will begin with two practice exercises.

Remember:

- Each question will be shown for five seconds.
- When the question is taken away, you will have ten seconds to write your answer.

SETS TEST - TEST ITEMS

$\cup$  MEANS ALL MEMBERS IN BOTH SETS, but do not repeat any member.

$\cap$  MEANS MEMBERS COMMON TO BOTH SETS.

1:	{ G, A, D }	$\cup$	{ D, K }
2:	{ pig, cow, tap }	$\cup$	{ boy, pig }
3:	{ 1, 3, 5, axe }	$\cap$	{ 1, 5 }
4:	{ 4, 5 }	$\cup$	{ 5 }
5:	{ e, 8, f, 7 }	$\cap$	{ 15, 7, 87 }
6:	{ 2, 9, 45, 3 }	$\cap$	{ 9, 7, 4, 2 }
7:	{ hat, pen, lap, top }	$\cap$	{ pen, cow, hat }
8:	{ 1, 5, 3, 2, 7, 6 }	$\cap$	{ 6, 5, 1, 2 }
9:	{ F, A, D }	$\cup$	{ D, B, F }
10:	{ bed, bee, doe }	$\cup$	{ doe, bed }
11:	{ a, b, p }	$\cap$	{ f, p, c, d, a }
12:	{ 14, 21 }	$\cup$	{ 21 }

1:	{ A, P, F, T }		
	<b>YES or NO ?</b> (Are the bottom members in the top?)		
	{ T, P }		
2:	{ a, y, p, s, t, u }		
	<b>YES or NO ?</b>		
	{ y, p, a, s }		
3:	{ toy, A, 7, T, 11 }		
	<b>YES or NO ?</b>		
	{ T, A, toy, 71 }		
4:	{ 3, 33, 0.3, C, C3 }		
	<b>YES or NO ?</b>		
	{ 33, C3, 3.3, 3 }		

CORRECT ANSWERS - SHAPES TEST, PAPER FOLDING TEST AND SETS TESTSHAPES TEST

A	1, 5
B	2, 5
C	2, 5
D	1, 2, 4
E	1, 2, 3
F	1, 2, 5
G	1, 3, 4, 5
H	1, 2, 3, 5

PAPER FOLDING TEST

1	A
2	D
3	B
4	D
5	B
6	C
7	E

SETS TEST

1	G, A, D, K	9	F, A, D, B
2	pig, cow, tap, boy	10	bed, bee, doe
3	1, 5	11	a, p
4	4, 5	12	14, 21
5	7	S1	YES
6	2, 9	S2	YES
7	hat, pen	S3	NO
8	1, 5, 2, 6	S4	NO

**APPENDIX K****APTITUDE TEST RESULTS (Study 3)**

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**OBSERVED MEANS AND STANDARD DEVIATIONS OF THE SIX TEST VARIABLES USED IN STUDY 3 FOR COMBINED (N= 251), LORETTO (N=63), QUEENWOOD (N=49), ROSEBY (N=39), TARA (N=60) AND WENONA (N=40)**

TEST	$\bar{x}$	s
NUMBER SPAN (COMBINED)	33.64	7.65
Loretto	36.60	7.39
Queenwood	32.18	7.64
Roseby	30.64	6.53
Tara	34.83	8.37
Wenona	31.90	6.13
LETTER SPAN (COMBINED)	28.37	8.18
Loretto	30.94	7.52
Queenwood	26.63	7.08
Roseby	24.41	6.34
Tara	31.00	9.99
Wenona	26.35	6.47
WORD SPAN (COMBINED)	28.77	6.28
Loretto	31.59	5.72
Queenwood	28.04	5.22
Roseby	26.03	6.52
Tara	27.93	6.32
Wenona	29.18	6.56
SHAPES (COMBINED)	4.27	1.74
Loretto	4.32	1.83
Queenwood	4.31	1.64
Roseby	4.23	1.40
Tara	4.00	1.86
Wenona	4.58	1.84
PAPER FOLDING (COMBINED)	4.04	1.66
Loretto	3.98	1.51
Queenwood	4.29	1.81
Roseby	3.15	1.42
Tara	4.37	1.63
Wenona	4.23	1.70
SETS (COMBINED)	8.25	2.18
Loretto	9.02	2.37
Queenwood	8.16	1.77
Roseby	7.67	2.33
Tara	8.05	1.88
Wenona	8.00	2.34

**PEARSON'S CORRELATION COEFFICIENTS BETWEEN THE SIX TEST VARIABLES:  
NUMBER SPAN, LETTER SPAN, WORD SPAN, SHAPES, PAPER FOLDING AND SETS  
(N=251)**

	NUMBER SPAN	LETTER SPAN	WORD SPAN	SHAPES	PAPER FOLDING
LETTER SPAN	.70**				
WORD SPAN	.62**	.59**			
SHAPES	.02	.08	.11		
PAPER FOLDING	.08	.08	.16*	.41**	
SETS	.11	.14	.23**	.48**	.42**

1-tailed significance: \* <.01 \*\* <.001

VARIMAX COMPONENT LOADINGS OF THE FIVE VARIABLES  
NUMBER SPAN, LETTER SPAN, WORD SPAN, SHAPES, PAPER FOLDING  
ON THE TWO COMPONENTS OBTAINED FROM THE PRINCIPAL COMPONENT  
ANALYSIS OF RAW SCORE DATA (N=251)

	COMPONENT 1	COMPONENT 2
NUMBER SPAN	.90	-.01
LETTER SPAN	.88	.03
WORD SPAN	.83	.13
SHAPES	.02	.84
PAPER FOLDING	.08	.83

DISTRIBUTION OF SUBJECTS INTO NINE APTITUDE GROUPS  
BASED UPON SIMULTANEOUS AND SUCCESSIVE COMPONENT SCORES

GROUP	NO. OF SUBJECTS	% OF TOTAL
GROUP1-HH	23	9
GROUP2-HM	31	12
GROUP3-HL	24	10
GROUP4-MH	32	13
GROUP5-MM	30	12
GROUP6-ML	31	12
GROUP7-LH	25	10
GROUP8-LM	28	11
GROUP9-LL	27	11
TOTALS	251	100

DISTRIBUTION OF SUBJECTS COMPLETING INDIVIDUAL OBSERVATION  
AND CLINICAL INTERVIEW

SCHOOL	GROUP-HL	GROUP-HL	TOTAL
LORETTO	6	5	11
QUEENWOOD	5	5	10
ROSEBY	4	4	8
TARA	4	7	11
WENONA	5	4	9
TOTAL	24	25	49

**APPENDIX L****INSTRUCTIONAL MATERIALS and STRUCTURED PROBES (Study 3)**

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Correct Answers to Observed Exercises and Post Test	409



# SETS

We will work together through these notes on sets.

You will not find the material difficult. This is not a test.

The objective is to find out YOUR OPINION of where the notes make the learning very easy and clear, and where the notes do not help much at all, or perhaps even cause confusion.

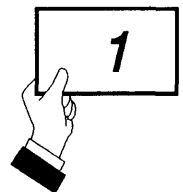
You are working with me to identify the kinds of lessons you find helpful and those you do not.

Please be frank and honest.

By the way,

whenever you see this symbol

stop and read out the number to me.



WHAT IS A SET ?

A SET is any collection of objects, such as .....

set of letters

set of words

set of animals

a set of vehicles

a set of sporting equipment.

CAPITAL LETTERS NAME SETS,

AND MEMBERS ARE ENCLOSED IN { } CALLED BRACES:

$$A = \{ 2, 4, 6, 8 \}$$

$$B = \{ \text{apples, pears} \}$$

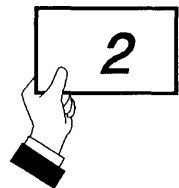
The set named SET A contains 2, 4, 6, 8 and SET B contains words apples and pears.

Here are some other sets:

$$\text{Set C} = \{ \text{Monday, Tuesday} \}$$

$$\text{Set T} = \{ \text{Lila, Lisa, Sarah} \}$$

$$\text{Set R} = \{ a, b, c \}$$



A SET CAN HAVE ITS ELEMENTS (ITS MEMBERS) LISTED IN ANY ORDER AND CAN HAVE ANY NUMBER OF ELEMENTS.

$$\text{In the set } P = \{ 7, 23, 12 \}$$

Set P has 3 elements.

$$\text{In the set } Q = \{ A, G, S \}$$

Set Q also has three elements.



The symbol  $\in$  is used to show "is a member of" or "is an element of":

In the set  $P = \{7, 12, 23\}$ , 7 is a member of the set P.

$$7 \in P$$

Note also:  $12 \in P$  and  $23 \in P$

**REMEMBER** - Capital letters are used to name sets:

$$A = \{2, 4, 6, 8\} \qquad B = \{\text{apples, pears}\} \qquad S = \{d, e, f\}$$

The objects or members of a set are called elements of the set. The elements are listed inside braces "{" and "}". The order of elements does not matter!

The elements of the set of letters in the word BABY are { B, A, Y }.

NOTE: Elements of a set are not repeated when defining a set's members.

### EXERCISE 1

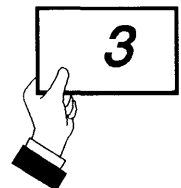
1. List the elements of each of the following sets **in braces**:

- (a) The set of letters in the word FROG  
 (b) The set of letters in the word BANANA

1(a):	1(b):
-------	-------

2. (a) Elements of a set can be listed in any order. (True or False?)  
 (b) How many elements in this set: { 4, 5, 6 } ?

2(a):	2(b):
-------	-------

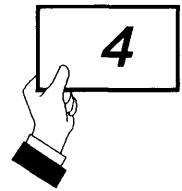


SUBSETS

If  $A = \{\text{Tom, Harry, Bill, Sarah, Cathy}\}$  and  $B = \{\text{Tom, Cathy}\}$

Set B is a subset of set A because every element of set B is also an element of set A.

To save time, this is written:  $B \subset A$       **The symbol  $\subset$  means "is a subset of".**



In the same way, if  $P = \{4, 23, 45, 56\}$  and  $Q = \{23\}$ , set Q is a subset of set P.

This is written:  $Q \subset P$

IF A SET HAS NO ELEMENTS AT ALL, IT IS CALLED THE EMPTY SET.

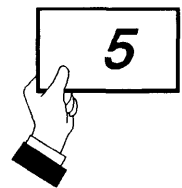
The empty set is written showing empty braces  $\{ \}$ .

Example: if set  $C = \{ \}$ , set C is an empty set, and contains no elements.

The empty set is a subset of any set! For example:

If  $A = \{3, 4\}$ , its subsets are  $\{3, 4\}$ ,  $\{3\}$ ,  $\{4\}$  and  $\{ \}$ .

Set  $A = \{ \text{pig, cow, hen} \}$



Remember:

**The symbol  $\subset$  means "is a subset of".**

EXERCISE 2

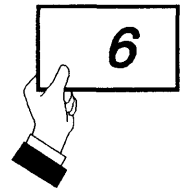
1. Write set P is a subset of set Q, using the symbol meaning "is a subset of".

1:
----

2. Let  $A = \{1, 2, 3, 4, 5\}$  and  $B = \{1, 3, 4\}$ . If the following statements are correct, write Yes, and if they are not correct write No.

- (a)  $A \subset B$       (b)  $B \subset A$

2(a):	2(b):
-------	-------

INTERSECTION OF SETS

Suppose  $A = \{\text{Lisa, Lila, Monique, Cathy, Kristen}\}$  is the set of girls in your class with blue eyes,

and  $B = \{\text{Lila, Cathy, Sheridan, Amanda}\}$  is the set of girls in your class with fair hair.

Then,  $\{\text{Lila, Cathy}\}$  is the set of all girls in your class with both blue eyes and fair hair.

This new set is called the intersection of A and B and is written using the  $\cap$  symbol, meaning "in intersection with":

$$A \cap B$$

So  $A \cap B = \{\text{Lila, Cathy}\}$

The intersection  $A \cap B$  of two sets  $A$  and  $B$  is the set of all elements which belong to both A and B. You can remember this by thinking of an intersection as containing the **ELEMENTS COMMON TO BOTH SETS**.

For example:

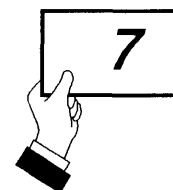
if  $A = \{1, 2, 3, 4\}$  and  $B = \{3, 4, 5, 6, 7\}$

then  $A \cap B = \{3, 4\}$ .

Another example:

if  $A = \{\text{black, brown, blue}\}$  and  $B = \{\text{orange, blue}\}$

what is  $A \cap B = ?$



### UNION OF SETS

Suppose  $A = \{\text{Lisa, Lila, Monique, Cathy, Kristen}\}$  is the set of girls in your class with blue eyes,

and  $B = \{\text{Lila, Cathy, Sheridan, Amanda}\}$  is the set of girls in your class with fair hair.

Then,  $\{\text{Lisa, Lila, Monique, Cathy, Kristen, Sheridan, Amanda}\}$  is the set of all girls in your class with blue eyes or fair hair.

The union of Sets  $A$  and  $B$  is **ALL ELEMENTS IN BOTH A AND B**.

This new set is called the union of  $A$  and  $B$  and is written using the  $\cup$  symbol, meaning "in union with":

$A \cup B$

So  $A \cup B = \{\text{Lisa, Lila, Monique, Cathy, Kristen, Sheridan, Amanda}\}$

The union  $A \cup B$  of two sets A and B is the set of all elements which belong to A and B (to both A and B). You can remember this by thinking of a union as a joining together.

Example: if  $A = \{1, 2, 3, 4\}$  and  $B = \{3, 4, 5, 6, 7\}$   
 then  $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$ .

NOTE THAT THE SAME SET ELEMENT IS NOT REPEATED.

If  $A = \{\text{black, brown, blue}\}$  and  $B = \{\text{orange, blue}\}$

what is  $A \cup B = ?$

Summary:

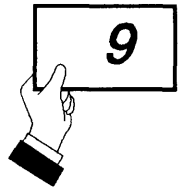
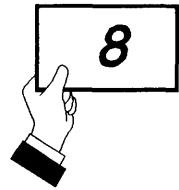
The symbol  $\in$  means "is a member of" or "is an element of".

The symbol  $\subset$  means "is a subset of".

The empty set is written showing empty braces  $\{ \}$ .

The  $\cap$  symbol means "in intersection with".

The  $\cup$  symbol means "in union with":



EXERCISE 3

1. If  $A = \{1, 3, 5\}$ ,  $B = \{2, 5, 7\}$  find:

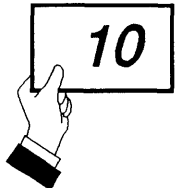
- (a)  $A \cup B$                       (b)  $A \cap B$

1(a):	1(b):
-------	-------

2. If  $A = \{a, b, c\}$  and  $B = \{a, b\}$  write True or False:

- (a)  $c \in A$                       (b)  $A \cup B = \{a, b, c\}$

2(a):	2(b):
-------	-------



STOP



# SETS

We will work together through these notes on sets.

You will not find the material difficult. This is not a test.

The objective is to find out YOUR OPINION of where the notes make the learning very easy and clear, and where the notes do not help much at all, or perhaps even cause confusion.

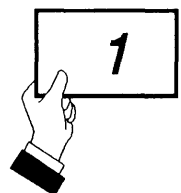
You are working with me to identify the kinds of lessons you find helpful and those you do not.

Please be frank and honest.

By the way,

whenever you see this symbol

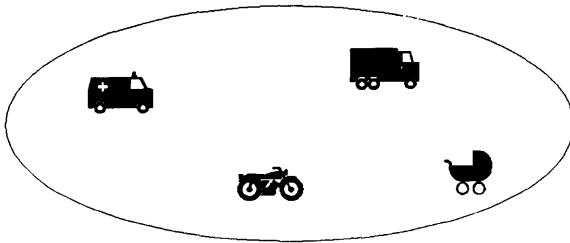
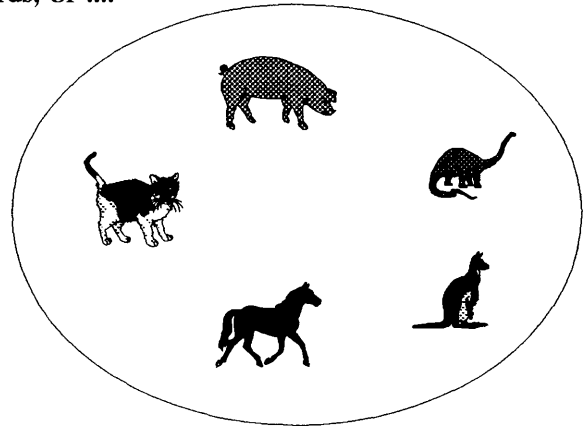
stop and read out the number to me.



WHAT IS A SET ?

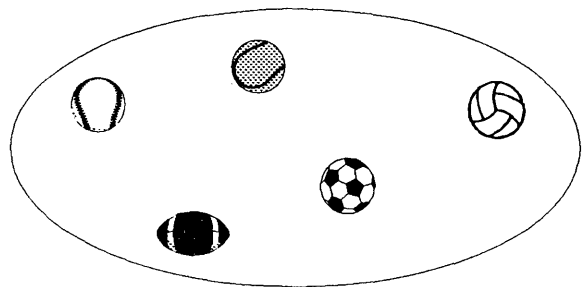
A SET is any collection of objects, or letters, or words, or ....

a set of animals:

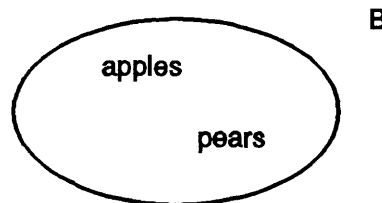
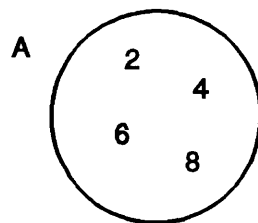


a set of vehicles:

a set of sporting equipment:



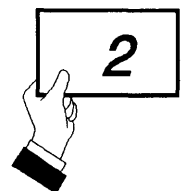
Here are some other sets, Set A and Set B:



The set named SET A contains 2, 4, 6, 8 and SET B contains words apples and pears.

Capital letters name sets, and its members are enclosed in { } called braces. Examples:

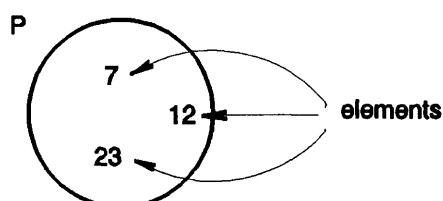
Set C = { Monday, Tuesday }      Set T = { Lila, Lisa, Sarah }      Set R = { a, b, c }



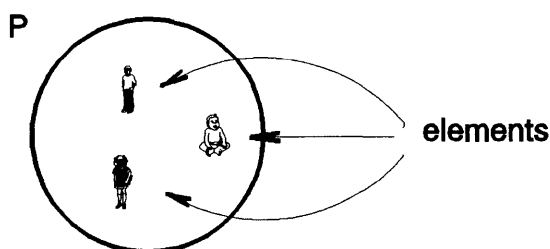
A SET CAN HAVE ITS ELEMENTS (ITS MEMBERS) LISTED IN ANY ORDER  
AND CAN HAVE ANY NUMBER OF ELEMENTS.

In the set  $P = \{7, 23, 12\}$

Set P has 3 elements.



This set P also has three elements.



In the set  $P = \{7, 12, 23\}$ , 7 is a member of the set P. The symbol  $\in$  is used to show "is a member of" or "is an element of":

$$7 \in P$$

Note also:

$$12 \in P$$

and

$$23 \in P$$

**REMEMBER** - Capital letters are used to name sets:

$$A = \{2, 4, 6, 8\}$$

$$B = \{\text{apples, pears}\}$$

$$S = \{d, e, f\}$$

The objects or members of a set are called elements of the set. The elements are listed inside braces "{" and "}". The order of elements does not matter!



The elements of the set of letters in the word BABY are { B, A, Y }.

NOTE: Elements of a set are not repeated when defining a set's members.

**EXERCISE 1**

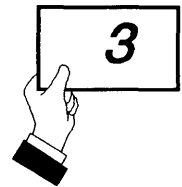
1. List the elements of each of the following sets **in braces**:

- (a) The set of letters in the word FROG  
 (b) The set of letters in the word BANANA

1(a):	1(b):
-------	-------

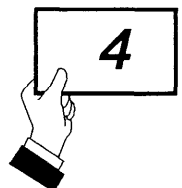
2. (a) Elements of a set can be listed in any order. (True or False?)  
 (b) How many elements in this set: { 4, 5, 6 } ?

2(a):	2(b):
-------	-------



**Remember:** The symbol  $\in$  means "is an element of".

$\in$  element  
 e for element



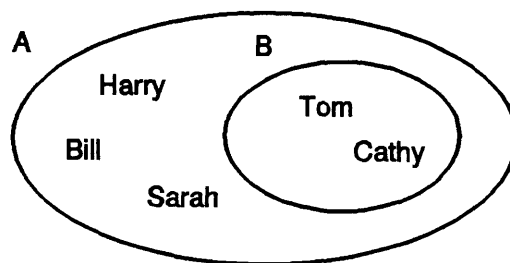
SUBSETS

If  $A = \{\text{Tom, Harry, Bill, Sarah, Cathy}\}$  and  $B = \{\text{Tom, Cathy}\}$

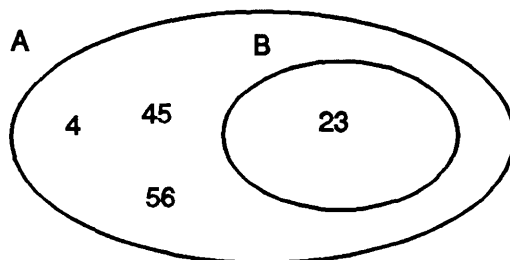
Set B is a subset of set A because every element of set B is also an element of set A.

To save time, this is written:  $B \subset A$       The symbol  $\subset$  means "is a subset of".

This diagram shows B is a subset of A:



In the same way, if  $A = \{4, 23, 45, 56\}$  and  $B = \{23\}$ , set B is a subset of set A.



This is written:  $B \subset A$

IF A SET HAS NO ELEMENTS AT ALL, IT IS CALLED THE EMPTY SET.

The empty set is written showing empty braces  $\{ \}$ .

Example: if set  $C = \{ \}$ , set C is an empty set, and contains no elements.

The empty set is a subset of any set! For example:

If  $A = \{3, 4\}$ , its subsets are  $\{3, 4\}$ ,  $\{3\}$ ,  $\{4\}$  and  $\{ \}$ .

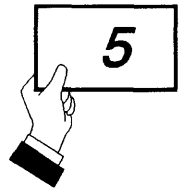
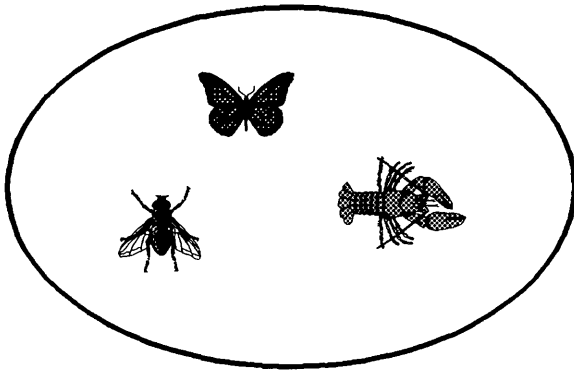
THIS MAY HELP YOU REMEMBER:



*means*

**subset**

(like jaws biting a piece of a set)



EXERCISE 2

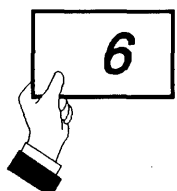
1. Write set P is a subset of set Q, using the symbol meaning "is a subset of".

1:
----

2. Let  $A = \{1, 2, 3, 4, 5\}$  and  $B = \{1, 3, 4\}$ . If the following statements are correct, write Yes, and if they are not correct write No.

(a)  $A \subset B$       (b)  $B \subset A$

2(a):	2(b):
-------	-------

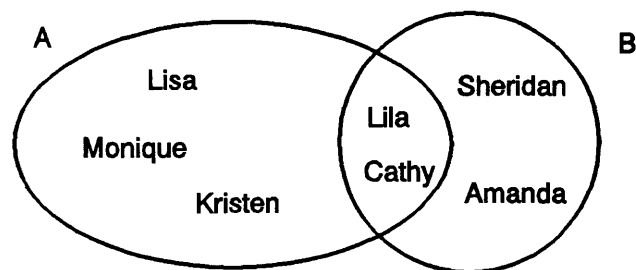


INTERSECTION OF SETS

Suppose  $A = \{\text{Lisa, Lila, Monique, Cathy, Kristen}\}$  is the set of girls in your class with blue eyes,

and  $B = \{\text{Lila, Cathy, Sheridan, Amanda}\}$  is the set of girls in your class with fair hair.

Then,  $\{\text{Lila, Cathy}\}$  is the set of all girls in your class with both blue eyes and fair hair.

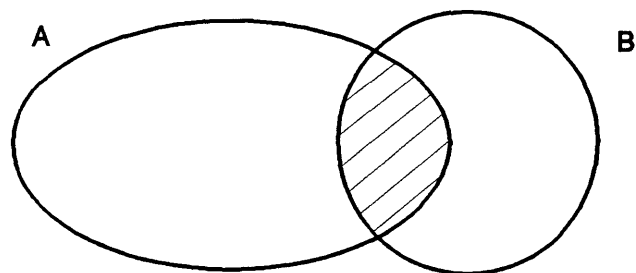


This new set is called the intersection of A and B and is written using the  $\cap$  symbol, meaning "in intersection with":

$$A \cap B$$

$$\text{So } A \cap B = \{\text{Lila, Cathy}\}$$

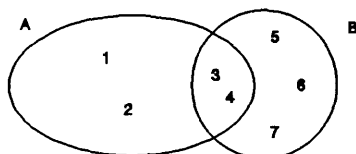
The intersection  $A \cap B$  of two sets A and B is the set of all elements which belong to both A and B (as shown in the hatched area below). You can remember this by thinking of an intersection as containing the **ELEMENTS COMMON TO BOTH SETS**.



For example:

if  $A = \{1, 2, 3, 4\}$  and  $B = \{3, 4, 5, 6, 7\}$

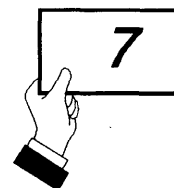
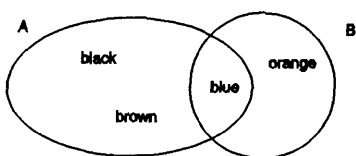
then  $A \cap B = \{3, 4\}$ .



Look at this example:

if  $A = \{\text{black, brown, blue}\}$  and  $B = \{\text{orange, blue}\}$

what is  $A \cap B = ?$

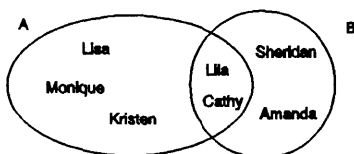


### UNION OF SETS

Suppose  $A = \{\text{Lisa, Lila, Monique, Cathy, Kristen}\}$  is the set of girls in your class with blue eyes,

and  $B = \{\text{Lila, Cathy, Sheridan, Amanda}\}$  is the set of girls in your class with fair hair.

Then,  $\{\text{Lisa, Lila, Monique, Cathy, Kristen, Sheridan, Amanda}\}$  is the set of all girls in your class with blue eyes or fair hair.



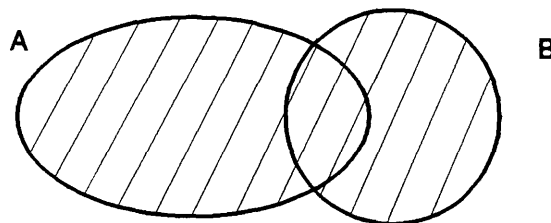
The union of Sets A and B is ALL ELEMENTS IN BOTH A AND B.

This new set is called the union of A and B and is written using the  $\cup$  symbol, meaning "in union with":

$$A \cup B$$

$$\text{So } A \cup B = \{\text{Lisa, Lila, Monique, Cathy, Kristen, Sheridan, Amanda}\}$$

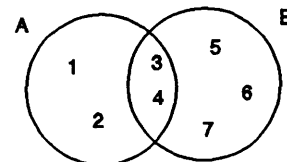
The union  $A \cup B$  of two sets A and B is the set of all elements which belong to A and B (to both A and B). You can remember this by thinking of a union as a joining together. This can be pictured diagrammatically:



For example:

if  $A = \{1, 2, 3, 4\}$  and  $B = \{3, 4, 5, 6, 7\}$

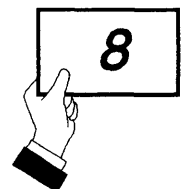
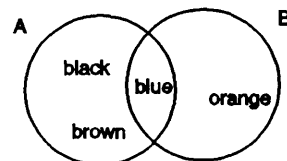
then  $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$ .




NOTE THAT THE SAME SET ELEMENT IS NOT REPEATED.


If  $A = \{\text{black, brown, blue}\}$  and  $B = \{\text{orange, blue}\}$


what is  $A \cup B = ?$



## HINTS FOR REMEMBERING SYMBOLS

$\in$   **e**lement  
e for element

$\cup$   **u**nion  
u for union

$\cap$   **i**ntersection  
n in intersection

$\subset$  **s**ubset  
(like jaws biting a piece of a set)

Summary:

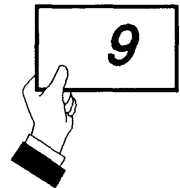
The symbol  $\in$  means "is a member of" or "is an element of".

The symbol  $\subset$  means "is a subset of".

The empty set is written showing empty braces  $\{ \}$ .

The  $\cap$  symbol means "in intersection with".

The  $\cup$  symbol means "in union with":

EXERCISE 3

1. If  $A = \{1, 3, 5\}$  and  $B = \{2, 5, 7\}$  :

(a)  $A \cup B$

(b)  $A \cap B$

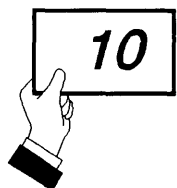
1(a):	1(b):
-------	-------

2. If  $A = \{a, b, c\}$  and  $C = \{c\}$ , write Yes if the following sentences are correct, and write No if they are not.

(a)  $c \in A$

(b)  $A \cup B = \{a, b, c\}$

2(a):	2(b):
-------	-------





STRUCTURED PROBES

2A. How do you rate the lessons so far? Clear or unclear? Helpful or not?



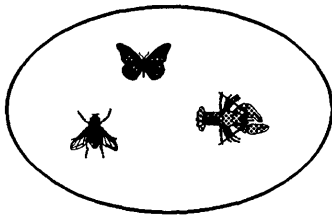
COMMENT?:.....  
 .....

2B. How do you rate the understanding of SETS being shown? Show your preferences.

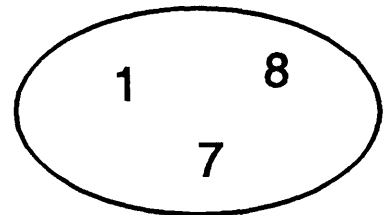
PICTORIAL  
 additions

TEXT  
 only

DIAGRAMMATIC  
 additions



{ 3, 7, 9 }



3. How would rate the lessons so far? Comment on how you did the last exercises.



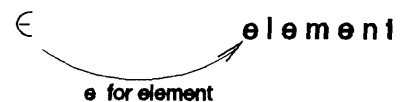
COMMENT?:.....  
 .....

4. Show by rating, which way would help you remember that  $\in$  means "is an element of":

TEXT

DIAGRAMMATIC

$\in$  means "is an element of"



5. What are the SUBSETS of the set of animals shown ?.....

How did you go about doing this? .....

.....

6. Look again at Exercise 2, Question 2. Did you work out the answer "with an overview" or "by dealing with one number at a time" ?

COMMENT ? :.....  
 .....

7A. What is the intersection of Set A and Set B shown in the notes? .....

How did you do about doing this? .....

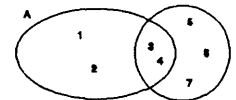
.....

7B. Show by rating, which way would help you understand "intersection of sets"?:

TEXT  
only

DIAGRAMMATIC  
additions

if  $A = \{1,2,3,4\}$  and  $B = \{3,4,5,6,7\}$   
 then  $A \cap B = \{3,4\}$



8A. What is the union of Set A and Set B shown in the notes? .....

How did you do about doing this? .....

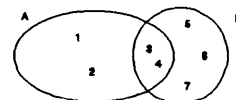
.....

8B. Show by rating, which way would help you understand "union of sets"?:

TEXT  
only

DIAGRAMMATIC  
additions

if  $A = \{1,2,3,4\}$  and  $B = \{3,4,5,6,7\}$   
 then  $A \cup B = \{1,2,3,4,5,6,7\}$



9. Show by rating, which way would help you remember that  $\subset$  means "is a subset of":

TEXT

$\subset$  means "is a subset of"



PICTURE IMAGE

$\subset$  means subset  
(like jaws biting a piece of a set)



10A. Look again at Exercise 3, Question 1. Did you work out the answer "with an overview" or "by dealing with one number at a time" ?

COMMENT ? : .....

.....

10B. Look again at Exercise 3, Question 2. Did you work out the answer "with an overview" or "by dealing with one number at a time" ?

COMMENT ? : .....

.....

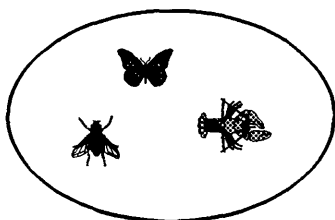
10C. Can you comment on the lessons you have just completed?

COMMENT ? : .....

.....

10D. How do you think you would learn best? Rate: pictorial additions to text, text only, and diagrammatic additions.

PICTORIAL

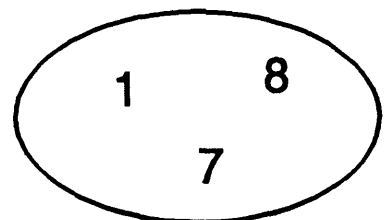


TEXT

{ 3, 7, 9 }



DIAGRAMMATIC



STUDY 3 POST TEST

Attempt all questions. Circle the letter of your answer. If you change your mind, cross out the circle you no longer want, and circle the new answer. Write the time after each question.

1. Which one of the following statements is **not** correct?

- A:** A set is a collection of elements.  
**B:** An element can belong to more than one set.  
**C:** A set must have at least one element.  
**D:** A set may have its elements listed in any order.

Your answer: **A B C D**

mins:	secs:
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2. Which one of the following is a correct statement?

- A:** A set may have no elements.  
**B:** The empty set must have one letter as an element.  
**C:** Two sets are equal if they have an equal numbers of elements.  
**D:** Elements can be letters or numbers, but not words.

Your answer: **A B C D**

mins:	secs:
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3. Which one of the following is a correct statement?

- A:** All sets labelled A in every textbook have the same number of elements.  
**B:** The symbol  $\subset$  is used for the words "is a subset of".  
**C:** Every Set A must have A as one of its elements.  
**D:** The symbol  $\in$  is used for the words "is a subset of".

Your answer: **A B C D**

mins:	secs:
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4. Which one of the following statements is correct?

- A:**  $P \subset Q$  means that the elements in set Q are a subset of the elements in set P.  
**B:**  $\{\} \subset P$  means that set P is a subset of the empty set.  
**C:**  $P \subset Q$  means that P is a subset of set Q.  
**D:**  $P = \{Q, R, S\}$  means that P has 4 elements.

Your answer: **A B C D**

mins:	secs:
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5. Which one of the following statements is correct?

- A: A set S cannot be defined as a set of sporting equipment.  
 B: A set can have any number of members.  
 C: A set must have at least one member.  
 D: A set cannot be empty.

Your answer: A B C D

mins:	secs:
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6. If  $A = \{\text{dog, cat, cow}\}$  and  $B = \{\text{cat, horse}\}$ , what are the elements of  $A \cap B$  ?

- A: {dog, cat, cow, horse}  
 B: {dog, cat, cow, cat, horse}  
 C: {dog, cat}  
 D: {cat}

Your answer: A B C D

mins:	secs:
-------	-------

7. If  $A = \{\text{dog, cat, cow}\}$  and  $B = \{\text{cow, horse}\}$ , what are the elements of  $A \cup B$  ?

- A: {dog, cat, cow, horse}  
 B: {horse}  
 C: {dog, cat}  
 D: {cat}

Your answer: A B C D

mins:	secs:
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8. If  $B = \{1, 2, 3, 4, 5\}$  and  $C = \{1, 3, 5\}$ , which sentence is a correct statement?

- A: Set  $C = \{\text{all numbers less than } 20\}$   
 B:  $C \subset B$   
 C:  $B \subset C$   
 D:  $B = C$

Your answer: A B C D

mins:	secs:
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9. Which set shows the elements in the word ASSESS ?

- A: {A, 4 Ss, E}  
 B: {A, four Ss, E}  
 C: {A, S, S, E, S, S}  
 D: {A, S, E}

Your answer: A B C D

mins:	secs:
-------	-------

10. If  $P = \{5, 10, 15, 20\}$  and  $Q = \{5, 20\}$  which of the following statements is correct?

- A:  $\{15\} \subset Q$   
 B:  $\{15\} \subset P$   
 C:  $\{5, 15\} \subset Q$   
 D:  $\{5, 25\} \subset P$

Your answer: A B C D

mins:	secs:
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11. Given the sets  $S = \{a, b, c, d, e\}$  and  $A = \{a, c, e\}$ , which one of the following statements is correct?

- A: The set A is a subset of set S.  
 B: The set  $B = \{f\}$  is a subset of the empty set.  
 C: The set  $C = \{d\}$  is a subset of set A.  
 D: The set  $D = \{e, d, c\}$  has the same number of elements as set S.

Your answer: A B C D

mins:	secs:
-------	-------

12. If  $A = \{1, 2, \text{apple}\}$  and  $B = \{3, \text{apple}\}$ , which of the following statements is correct?

- A:  $A \cap B = \{1, 2, 3\}$   
 B:  $A \cap B = \{3\}$   
 C:  $A \cap B = \{\text{apple}\}$   
 D:  $A \cap B = \{1, 2\}$

Your answer: A B C D

mins:	secs:
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13. If  $A = \{4, 5, 6\}$ ,  $B = \{5, 6, 7\}$  and  $C = \{6, 7, 8\}$ , which of the following statements is correct?

- A:  $A \cap B = \{5, 6, 7\}$   
 B:  $A \cup B = \{5, 6, 7\}$   
 C:  $B \cap C = \{6, 7\}$   
 D:  $B \cup C = \{5, 6, 7\}$

Your answer: A B C D

mins:	secs:
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14. Which set shows the elements in the union of the set of letters in the word ASSESS and the set of letters in the word SEAS ?

- A:  $\{2 \text{ As, } 6 \text{ Ss, } 2 \text{ Es}\}$   
 B:  $\{A, S, E\}$   
 C:  $\{A, S, S, E, S, S, S, E, A, S\}$   
 D:  $\{\text{two As, six Ss, two Es}\}$

Your answer: A B C D

mins:	secs:
-------	-------

CORRECT ANSWERS TO OBSERVED EXERCISES AND POST TEST

EXERCISE 1	ANSWER	POST TEST	ANSWER
1(a)	{F,R,O,G}	1	C
1(b)	{B,A,N}	2	A
2(a)	TRUE	3	B
2(b)	3	4	C
EXERCISE 2		5	B
1	$P \subset Q$	6	D
2(a)	NO	7	A
2(b)	YES	8	B
EXERCISE 3		9	D
1(a)	{1,2,3,5,7}	10	B
1(b)	{5}	11	A
2(a)	TRUE	12	C
2(b)	TRUE	13	C
		14	B

**APPENDIX M****STUDY 3 RESULTS**

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OBSERVED EXERCISES AND POST TEST OBSERVED MEANS  
AND STANDARD DEVIATIONS

	OBSERVED EXERCISES	POST TEST
<b>ALL SUBJECTS</b>		
$\bar{x}$	8.86	8.43
s	2.19	3.18
<b>GROUP HL</b>		
$\bar{x}$	10.17	10.00
s	1.17	2.84
<b>GROUP LH</b>		
$\bar{x}$	7.60	6.92
s	2.21	2.77
<b>SUBJECTS USING "SPATIAL" TREATMENT</b>		
<b>GROUP HL</b>		
$\bar{x}$	10.00	10.00
s	1.18	1.84
<b>GROUP LH</b>		
$\bar{x}$	7.71	6.86
s	2.09	3.30
<b>SUBJECTS USING "VERBAL" TREATMENT</b>		
<b>GROUP HL</b>		
$\bar{x}$	10.31	10.00
s	1.18	3.56
<b>GROUP LH</b>		
$\bar{x}$	7.46	7.00
s	2.46	2.05

[GROUP HL - high simultaneous - low successive  
GROUP LH - low simultaneous - high successive]

NUMBER OF SUBJECTS ADOPTING SIMULTANEOUS AND SUCCESSIVE SOLUTION STRATEGIES FOR SET MANIPULATION TASKS

PROBE QUESTION NUMBER	APTITUDE GROUP	SIMULTANEOUS SOLUTION STRATEGY	SUCCESSIVE SOLUTION STRATEGY
6*	HL	22	2
	LH	22	3
10A	HL	17	7
	LH	21	4
10B	HL	19	5
	LH	22	3

(\* probe numbers from student materials Appendix L, p. 417)

RESPONSES TO STRUCTURED PROBES ON PREFERRED PRESENTATION

PROBE		"T"	"D"	"P"	"E"
1	2B*	16	8	21	4
2	4	26	23	-	-
3	7B	30	17	-	2
4	8B	36	12	-	1
5	9	21	-	25	3
6	10D - OVERALL	24	3	18	4

"T": Text only instruction, "D": Diagrammatic adjuncts to text  
 "P": Picture adjuncts to text, "E": Equal preference

\* probe numbers from student materials Appendix L, p. 417.

POST TEST - MEAN TIMES (in seconds)

QUESTION	GROUP HL		GROUP LH		COMBINED
	"S"	"V"	"S"	"V"	
1	36.5	46.4	37.3	41.3	40.4
2	42.7	47.6	42.9	49.5	45.6
3	36.9	46.8	52.7	55.6	48.3
4	53.3	55.7	56.3	41.6	52.2
5	41.0	59.8	41.5	37.0	45.2
6	29.0	39.2	35.4	39.2	35.8
7	40.5	34.6	37.2	33.1	36.3
8	46.5	46.8	40.4	28.5	40.8
9	31.5	28.6	36.7	39.9	34.1
10	50.0	73.6	56.2	35.9	54.9
11	40.0	38.9	42.6	29.7	38.1
12	30.4	48.0	35.9	32.5	37.1
13	56.8	51.1	50.4	31.5	47.8
14	37.3	29.2	51.1	34.6	38.5
TOTAL	572.3	646.5	616.5	530.0	595.1

GROUP HL: high simultaneous - low successive  
 GROUP LH: low simultaneous - high successive

"S": "Spatial" treatment  
 "V": "Verbal" treatment

ADDITIONAL ANECDOTAL EVIDENCE ON SET MANIPULATION TASKS

The following observations are quoted as typical of the responses gained when subjects were queried on their procedure for solving Set manipulation tasks:

- "I looked at both sets at once."
- "Because the last one is blue in both sets and I saw that straight away."
- "I could see in my head that blue came up twice."
- "Looked at it once and saw the answer straight away."
- "I looked at the two sets in one glance."
- "I probably looked at it all together but then back to see the letters separately."
- "You knew when you'd read the question."
- "I could see the two blues quickly."

More detailed responses of interest were provided by the following students:

- Amy: "I first looked at Set B because it is smaller, then looked at Set A and checked if all the numbers were there, but I not need to read them to myself one at a time because it was easy to see the answer was blue."
- Sarah: "I spanned the line at once and saw the answer, but went over it again to check [my answer]."
- Anne: "I looked at each number one at a time, but just to make sure, to check my answer I already knew."
- Tina: "I looked at both of the sets first, then looked at Set A, then Set B, then looked for the missing numbers, but it was very quick. I don't think it took more than two seconds."
- Sally: "The Set A had more numbers so I could see that it was not a subset of B right away. It only took one look."
- Jenny: "I looked at Set A all at once and saw that there was more letters in A than B. That was one answer. A could not be a subset of B. Then I looked at B and checked if all the numbers were in A, and they were, so I go the second answer."
- Sascha: "I looked at set B first because it was shorter, and then saw blue and blue, and that was the answer."