CHAPTER 5

EXTENSIONS TO THE BASIC DEA MODEL FOR THE SELECTION OF OPTIMAL PATHS OF ADJUSTMENT

5.1 Introduction

As previously stated, the main purpose of this thesis is the identification of optimal paths of adjustment under DEA for handling the observed production data of a set of comparable firms. Chapter 4 presents a basic DEA model for the selection of optimal paths of adjustment. The basic DEA model is derived from the fundamental dual DEA formulation.[1] This chapter presents extensions to the basic model in Chapter 4. The extensions are aimed to improve the basic model for practical applications.

The basic model optimises the present value of a profit function that includes the revenue along the time horizon, the period-to-period costs of inputs used for producing the outputs sold, and the period-to-period costs of adjusting the inputs.

[1] The fundamental dual DEA problem, defined by equation (3.5.1), consists of a profit maximisation objective function with the boundary of the technology described by $\boldsymbol{X\lambda} \leq \boldsymbol{x}_e$, $\boldsymbol{Y\lambda} \geq \boldsymbol{y}_e$ and $\boldsymbol{\lambda} \geq \boldsymbol{0}$.

The basic model considers symmetric costs of adjustment of inputs. This means that the costs of increasing and decreasing an input are the same. Section 5.2 presents the model of optimal paths of adjustment with asymmetric costs of adjustment. The basic model has implicit that the time of adjustment and the time horizon are the same. For cash flow evaluation purposes, Section 5.3 considers a time horizon longer than the time of adjustment. Section 5.4 presents an example that includes both asymmetric costs of adjustment and a time horizon longer than the time of adjustment. Section 5.5 presents the model of optimal paths of adjustment with dynamic (time-variable) outputs. Section 5.6 presents an example with dynamic outputs, asymmetric costs of adjustment, and period-to-period variable input prices, output prices, and costs of adjustment. Section 5.7 presents the incorporation of quasi-fixed (nondiscretionary) variables and an example. Section 5.8 presents the incorporation of a capital investment constraint. Section 5.9 presents the general model of optimal paths of adjustment with a dynamic (time-variable) boundary of technology. Without loss of generality, the improved models consider input-orientated systems and constant returns-to-scale technology. The output-orientated systems and variable returns-to-scale cases are trivial extensions. Section 5.10 presents conclusions.

5.2 Optimal Paths of Adjustment: Asymmetric Costs of Adjustment

This section considers asymmetric costs of adjustment, which is more realistic than symmetric costs of adjustment. Asymmetric costs of adjustment modify the cost of adjustment part of the objective function, the budget and the transition constraints. The modification makes explicit the character of increasing or decreasing inputs and of the corresponding price. Let x_{kt}^{+} and x_{kt}^{-} denote increases and decreases in inputs; w^{+} and w^{-} denote the costs of increasing and decreasing inputs, respectively. The modified mathematical expressions are:

$$\underset{x_{kt}^{+},x_{kt}^{-},\lambda_t}{maximise} \ \{\pi_k = \sum_{t=1}^{T} [s_t(y_k \ p - x_{kt} \ w) - s_{t-1} \ (x_{kt}^{+} \ w^{+} + x_{kt}^{-} w^{-})] \qquad (5.2.1)$$

subject to the budget constraint,

$$x_{kt}^{+} \ w^{+} + x_{kt}^{-} w^{-} \leq b_{kt};$$

and the transition equation,

$$x_{kt} = x_{kt-1} + x_{kt}^{+} - x_{kt}^{-}.$$

5.3 Optimal Paths of Adjustment: Time Horizon Longer than Adjustment Time

This section considers a time horizon that is greater than the adjustment time. The basic model considers that there is no difference between the time horizon and the adjustment time. However, it is more realistic that those times be different. Section 4.2 defines the concepts of time horizon and adjustment time (or time of adjustment).

If we view the adjustments of input quantities as an investment project, then the time of adjustment represents the number of periods during which investments are done. The investment is the period-to-period cost of adjustments. The number of periods that a firm effectively uses to perform the adjustment, t_a, is constrained by the maximum prefixed time of adjustment, t_a^*.

As stated in Chapter 4, the time horizon is the pre-fixed number of periods that management considers for the economic evaluation of each specific investment project. In most cases, the number of periods considered for economic evaluation of an investment project is larger than the number of periods the investment is done.

Because the savings derived from input adjustments are evaluated over the time horizon, the present value of savings may be larger than the present value of costs of adjustment that are evaluated only over the time of adjustment. Increasing the time horizon increases the present value of savings. In Appendix 7 we demonstrate that, with a constant time to adjust inputs, t_a, the present value of profit increases with increases in the time horizon.

This extension to the basic model modifies the budget constraint and transition equation, making explicit the period of time for which they are valid. We also include the constraint that the time taken to adjust inputs has to be less than or equal to the time of adjustment. The modified mathematical expressions are:

$$\boldsymbol{x}_{kt}^{+}\,\boldsymbol{w}^{+}+\boldsymbol{x}_{kt}^{-}\boldsymbol{w}^{-}\leq b_{kt},\ t=1,2,\ldots,t_a \qquad \text{(Budget constraint)} \qquad (5.3.1)$$

$$\boldsymbol{x}_{kt}=\boldsymbol{x}_{kt-1}+\boldsymbol{x}_{kt}^{+}-\boldsymbol{x}_{kt}^{-},\ t=1,2,\ldots,t_a \qquad \text{(Transition equation)}$$

$$t_a\leq t_a^{*}.$$

5.4 Example: Asymmetric Cost of Adjustment and Time Horizon Greater than Adjustment Time.

This section presents an example of the modified model for the selection of optimal paths of adjustment. The modified basic model includes asymmetric costs of adjustment and a time horizon greater than the adjustment time. Without loss of generality, we assume a constant returns-to-scale technology.

Consider the data of Table 3.1. We wish to determine the optimal path of adjustment that maximises the present value of profit over a five-period adjustment time and eight-period time horizon. The asymmetric adjustment costs (and the original data from Table 3.1) are presented in Table 5.1.

Table 5.1: Inputs, Outputs, Prices, and Costs of Adjustment for Five Firms

		Quantity at $t = 0$		
	Firm k	y_{1kt}	x_{1kt}	x_{2kt}
	1	100	100	100
	2	110	90	149
	3	120	150	85.9
	4	115	133	189
	5	103	152	61
	Price	8.0	2.0	3.0
Adjustment Cost	Increase		0.8	1.2
	Decrease		1.1	2.0

Additionally, consider a rate of discount of 9.0 per cent by period and assume that the budget constrains the expenditure of costs of adjustments up to \$20.0 for the first adjustment period, and up to \$30.0 for the second adjustment period. Under these conditions, the problem for firm 1 is:

$$\underset{x_{kt}^{+},x_{kt}^{-},\lambda_t}{maximise}\,\{\pi_1=\sum_{t=1}^{8}[\ s_t(100\times 8.0-x_{11t}\times 2.0-x_{21t}\ \times 3.0) \tag{5.4.1}$$

$$-s_{t-1}(x_{11t}^{-}\times 1.1+x_{21t}^{-}\times 2.0+x_{11t}^{+}\times 0.8+x_{21t}^{+}\times 1.2)]\}$$

subject to

Budget for period 1:

$x_{111}^{+}\times 0.8+x_{111}^{-}\times 1.1+x_{211}^{+}\times 1.2+x_{211}^{-}\times 2.0\leq 20.0;$

Budget for period 2:

$x_{112}^{+}\times 0.8+x_{112}^{-}\times 1.1+x_{212}^{+}\times 1.2+x_{212}^{-}\times 2.0\leq 30.0;$

Transition equations:

$$\left.\begin{array}{l} x_{11t}=x_{11t-1}+x_{11t}^{+}-x_{11t}^{-} \\ x_{21t}=x_{21t-1}+x_{21t}^{+}-x_{21t}^{-} \end{array}\right\}\quad t=1,2,3,4,5;$$

Time to adjust inputs is less than or equal to the adjustment time, $t_a\leq 5$;

boundary of technology at t = 1,2,3,4,5;

$100\,\lambda_{1t}+90\,\lambda_{2t}+150\,\lambda_{3t}+133\,\lambda_{4t}+152\,\lambda_{5t}\leq x_{11t}$

$100\,\lambda_{1t}+149\,\lambda_{2t}+85.9\,\lambda_{3t}+189\,\lambda_{4t}+61\,\lambda_{5t}\leq x_{21t}$

$100\,\lambda_{1t}+110\,\lambda_{2t}+120\,\lambda_{3t}+115\,\lambda_{4t}+103\,\lambda_{5t}\geq 100$

$EE_t=\dfrac{\mathbf{x}_{\mathbf{kT}}^{*}\mathbf{w}}{\mathbf{x}_{\mathbf{kt}}\mathbf{w}}$, t=1,2,3,4, 5 (Economic efficiency measurement); and

all variables are positive.

It is assumed that at the start of the adjustment period 5, or earlier, the firm has achieved the target input quantity vector and persists indefinitely with this optimal input quantity vector. The evaluation of profit is along the time horizon of eight periods.

As stated in the conclusions section of Chapter 4, the representation of the boundary of the technology in five adjustment periods may be understood as five basic DEA problems linked by the transition constraints and the objective function.

Similar LP problems must be written for firms 2, 3, 4 and 5. Table 5.2 shows the optimal paths of adjustment for the five firms. As in Table 4.2, we use the extended notation, λ_{ekt}, to display unambiguously the weights of peers. In the extended notation, e stands for the firm that is evaluated, whose profit is maximised, where $e = 1, 2, 3, 4, 5$; k stands for the peer of Firm e,, k= 1, 2, 3, 4, 5; and t stands for the period to adjust inputs, $t = 1, 2, 3, 4, 5$. Technical and economic efficiency measurements are included.

Comparison of Targets for Cobb-Douglas and DEA

The purpose of this subsection is to compare the target input quantity vectors determined by the Cobb-Douglas and the DEA approximations of the production function.

Consider that the data of Tabla 5.1 may be well represented by the Cobb-Douglas production function, $y = x_1^{0.6} x_2^{0.4}$, as we presented in the Section 2.5.

In this case, for each of the five firms, there are two scenarios that determine two different target input quantity vectors.

Table 5.2: Optimal Path of Adjustment, Asymmetric Costs of Adjustment, and the Time Horizon Greater than the Time of Adjustment

		Firm	1	2	3(1)	4	5
	Initial Values	Y	100	110	120	115	103
		x_1	100	90	150	133	152
		x_2	100	149	85.9	189	61
		TE	1.0	1.0	1.0	0.7567	1.0
		EE	0.9295	0.8154	1.0	0.6416	0.9829
	Target Values	x_1	125.00	137.50	150	143.75	128.75
		x_2	71.585	78.742	85.9	82.32	73.731
		TE	1.0	1.0	1.0	1.0	1.0
		EE	1.0	1.0	1.0	1.0	1.0
OPTIMAL PATH OF ADJUSTMENT	Adjustment Period 1	x_1	105.83	94.00	150	133.00	140.62
		x_2	93.261	141.2	85.9	179.00	67.233
		TE	1.0	1.0	1.0	1.0	1.0
		EE	0.9453	0.8359	1.0	0.6656	0.9912
		λ_{ek1}	λ_{111}=0.763 λ_{131}=0.197	λ_{211}=0.220 λ_{231}=0.800	λ_{331}=1.0	λ_{421}=0.508 λ_{451}=0.575	λ_{531}=0.420 λ_{551}=0.510
	Adjustment Period 2	x_1	114.82	100.00	150	133.00	128.75
		x_2	83.152	129.50	85.9	164.00	73.731
		TE	1.0	1.0	1.0	1.0	1.0
		EE	0.9700	0.8687	1.0	0.7051	1.0
		λ_{ek2}	λ_{112}=0.407 λ_{132}=0.494	λ_{212}=0.550 λ_{232}=0.500	λ_{332}=1.0	λ_{422}=1.048	λ_{532}=0.853
	Adjustment Periods 3 to 5	x_1	125.0	137.50	150	143.75	128.75
		x_2	71.583	78.742	85.9	82.32	73.731
		TE	1.0	1.0	1.0	1.0	1.0
		EE	1.0	1.0	1.0	1.0	1.0
		λ_{ekt}	λ_{13t}=0.833	λ_{23t}=0.917	λ_{33t}=1.0	λ_{43t}=1.0	λ_{532}=0.853

(1) Firm 3 is a peer for all firms, because it is the profit efficient firm.

The first scenario is the minimisation of the costs of inputs. The mathematical expression that defines the optimal quantity of the input 1 is $x_{11}^{*} = y\left(\frac{0.4}{0.6}\times\frac{w_1}{w_2}\right)^{-0.4}$ and of input 2 is $x_{21}^{*} = y\left(\frac{0.4}{0.6}\times\frac{w_1}{w_2}\right)^{0.6}$, where x_{11}^{*} is the optimal quantity of input 1, first scenario; x_{12}^{*} is the optimal quantity of input 2, first scenario; and w_1 and w_2 are the prices of one unit of inputs 1 and 2, respectively.

The second scenario is the minimisation of the sum of the present value of the costs of inputs and the present value of the costs of adjustment of inputs. The present value of the costs of inputs is evaluated over the prefixed horizon time, T. Assuming that input 1 increases from the initial quantity to the target, and that input 2 decreases, the mathematical expression for the inputs 1 and 2 that minimise the sum of the present value of inputs and the present value of the costs of adjustment of inputs is $x_{11}^{m} = y\left(\frac{0.4}{0.6}\times\frac{w_1\times f+WI1}{w_2\times f-WD2}\right)^{-0.4}$ for input 1 and $x_{21}^{m} = y\left(\frac{0.4}{0.6}\times\frac{w_1\times f+WI1}{w_2\times f-WD2}\right)^{0.6}$ for input 2, where x_{11}^{m} is the optimal quantity of input 1, second scenario; x_{12}^{m} is the optimal quantity of input 2, second scenario; f is the present worth factor of the cost of inputs over the prefixed time horizon; w_1 and w_2 are as previously defined; $WI1$ is the cost of increasing one unit of input 1; and $WD2$ is the cost of decreasing one unit of input 2.[2]

[2] From Table 5.2, firm 1 increases input 1 from 100 to 125.0 and decreases input 2 from 100 to 71.583 units.

The second scenario reduces to the first if the costs of adjustment of inputs are zero. Increasing the cost of adjustment increases the difference between the respective input targets. Increasing the rate of discount, i, and increasing the time horizon increases f. Increasing f decreases the difference between the respective input targets. Nonetheless, f has a finite value for an infinite time horizon. The largest value of f is the reciprocal of the rate of discount, r. The rate of discount is expressed in per unit terms.

The conclusion is that for a Cobb-Douglas production function, the target input quantity vector that minimises the cost of inputs is different from the target input quantity vector that minimises the sum of the present value of the costs of inputs and the present value of the costs of adjustment of inputs.

For firm 1, Table 5.3 presents the input targets under both scenarios for the Cobb-Douglas production function. The second scenario includes the input targets for the largest value of f, which corresponds to a time horizon of infinite periods, and the targets for a time horizon of eight periods. The values of Table 5.3 are determined with the data of Table 5.1.

Consider now that the data of Table 5.1 are well represented by a DEA production function, as we presented in the example of this Section. As in the Cobb-Douglas production function case, for each of the five firms there are two scenarios that determine two different target input quantity vectors.

The first scenario is the minimisation of the costs of inputs. Figure 4.1 presents the isocost line P'P" that is tangent to the DEA boundary of the technology at firm 3 with minimum costs of inputs. The slope of the isocost line is the ratio of the input prices. Although it is not possible to determine *a priori* the value of the optimal inputs, we know that they are on the lines that have slope ($-w_2/w_1$). From Figure 4.1, the minimum costs of inputs is at firm 3, as long as the slope of the isocost line is larger than the slope of the line defined by the firms 5 and 3, $-m_{53}$, and smaller than the slope of the line defined by the firms 3 and 1, $-m_{31}$.

The second scenario is the minimisation of the sum of the present value of the costs of inputs and the present value of the costs of adjustment of inputs. The present value of the costs of inputs is evaluated over the prefixed horizon time, T.

Again, assuming that input 1 increases from the initial quantity to the target, and that input 2 decreases, the sum of the present value of the costs of inputs and the present value of the costs of adjustment of inputs is

$$\mathrm{PV} = (w_1 x_{11}^m)f + (w_2 x_{21}^m)f + (x_{11}^m - x_{10})WI1 + (x_{20} - x_{21}^m)WD2,$$

where x_{10} and x_{20} are the initial values of inputs 1 and 2, and the other variables and parameters are as defined for the Cobb-Douglas case. Arranging terms, the expression becomes

$$\mathrm{PV} = (w_1 f + WI1)\, x_{11}^m + (w_2 f - WD2)\, x_{21}^m + x_{20} WD2 - x_{10} WI1,$$

and the values of the optimal inputs x_{11}^m and x_{21}^m are on the line which slope is

$-(w_2 f - WD2)/(w_1 f + WI1)$.

Then, so long as

$$-m_{53} < -w_2/w_1 < -m_{31}, \text{ and } -m_{53} < -(w_2 f - WD2)/(w_1 f + WI1) < -m_{31},$$

there is only one target for inputs 1 and 2. This is an important difference from the Cobb-Douglas case. While the Cobb-Douglas production function determines a continuous change of the target inputs for continuous changes of parameters, the DEA production function determines discrete changes of the target inputs for continuous changes of parameters.

For firm 1, Table 5.3 presents the input targets for the data of Table 5.1 with DEA production function. For this example, the input targets are independent of the scenario involved. Without loss of generality, for both production functions and both scenarios, the two target input quantity vectors are determined assuming that the adjustments are done completely in the first adjustment period; this implies that there are no budget constraints.

Table 5.3: Input Targets for Firm 1

	Cobb-Douglas			DEA
	First Scenario	Second Scenario		
		Time Horizon Infinite Periods	Time Horizon Eight Periods	Time Horizon Eight Periods
Input 1	138.32	133,09	127.37	125.00
Input 2	61.474	65.167	69.5655	71.583
Cost of Inputs	461.05	461.58	463.44	464.75

Conclusions from this Example

From the solution to this example, the main conclusions are the same as those derived from the solution to the basic model in Section 4.4. Although the conclusions are the same, the modified model is more realistic than the basic one. Nonetheless, it has to be pointed out that the modifications to the basic model do not modify the structure or the basic concepts on which the basic model was developed.

This example illustrates the effect of a time horizon greater than the time of adjustment. In this example, firm 5 performs adjustments to reach the target levels, but, in the example of Section 4.4, firm 5 did not perform adjustments because the present value of the costs of adjustment was larger than the present value of savings. In Example 5.4, firm 5 performs adjustments because the present value of savings is evaluated for three periods after the target input quantity vector is achieved. In this case, the present value of savings is larger than the present value of the costs of adjustment. In Appendix 7, we determine that increasing the time horizon over the time of adjustment improves the economic feasibility for making the adjustments.

In general terms, the targets determined with a Cobb-Douglas production function change with continuous changes of the parameters, the DEA production function determines discrete changes of the target inputs for continuous changes of the parameters. This is an important property of LP DEA, because some managers may feel more confident when pursuing input targets if these are independent among scenarios.

5.5 Optimal Paths of Adjustment: Dynamic Outputs

This section considers dynamic outputs. The basic model considers that outputs do not change along the time horizon. In this section, we present a model of optimal paths of adjustment with dynamic outputs. The term *dynamic output* stands for outputs that change its quantity from period to period, without modification of the technology. Future period-to-period target output quantity vectors may be estimated by experts or may be set by managers.

The existence of dynamic outputs implies that as long as the output quantity vector changes from period to period, the input quantity vector will also change from period to period. With dynamic output quantity vector, as long as outputs change over the time horizon there are period to period specific target input quantity vectors for each particular output quantity vector, and because of this there are no practical reasons for differentiating the time horizon and the time of adjustment.

This extension to the basic DEA model modifies the objective function and the output quantity vector on the right hand side of the boundaries of the technology. The modified mathematical expressions, including asymmetric costs of adjustment are presented in problem (5.5.1), where the subscript *t* indicates that a variable may change from period to period. These mathematical expressions allow for changes in input and output prices from one period to the next. The economic efficiency at time of adjustment *t*, EE_t is measured as the ratio of the cost of optimal input quantity vector at *t*, over the cost of input quantity vector at *t*.

$$\underset{x^+_{kt}, x^-_{kt}, \lambda_t}{maximise} \{ \pi_k = \sum_{t=1}^{T} [s_t(y_{kt}\ p_t - x_{kt}\ w_t) - s_{t-1}\ (x^+_{kt}\ w^+_t + x^-_{kt} w^-_t)] \qquad (5.5.1)$$

subject to the boundary of the technology,

$$\left. \begin{aligned} Y\lambda_t &= y_{kt} \\ X\lambda_t &= x_{kt} \end{aligned} \right\} \ 1 \le t \le T\,;$$

and the transition equation,

$$x_{kt} = x_{kt-1} + x^{+}_{kt} - x^{-}_{kt} \qquad t=1, 2, \ldots, T.$$

The significant difference between period to period constant output quantity vectors and period to period variable output quantity vectors is that at each period the corresponding expected or forecasted output quantity vector replaces the constant output quantity vector. This case is equivalent to solving as many basic static DEA models as time horizon periods are; each model has a specific output quantity vector.

5.6 Dynamic Outputs Example

This section presents another example of a modified model for the selection of optimal paths of adjustment. The modified basic model includes dynamic outputs and period-to-period variable asymmetric costs of adjustment, input prices, output prices and costs of adjustment variable with time. We continue to assume a constant returns-to-scale technology.

Consider the data of Table 3.1. We determine the optimal path of adjustment that maximises the present value of profit over an eight-period time horizon. The asymmetric adjustment costs (and the original data from Table 3.1) are presented in Table 5.1.

Data for estimating the expected future values of outputs, input prices and output prices, and of costs of adjustment are presented in Table 5.4. The values of the expansion factors in Table 5.4 are the ratios of the value of a variable of interest at time t over the value at time 0. The period-to-period variation of prices and costs of adjustment does not necessarily follow the assumption of ordinary differential equation for expectations of Luh and Stefanou (1996), which were presented as equations (2.8.5) to (2.8.8).

Let $\psi_{y_{jt}}$ denote the expansion factor for outputs; $\psi_{p_{jt}}$ the expansion factor for output prices; $\psi_{w_{1t}}$ the expansion factor for input 1 prices; $\psi_{w_{2t}}$ the expansion factor for input 2 prices; $\psi_{w_{it}^{+}}$ the expansion factor for cost of adjustment of input 1; and $\psi_{w_{it}^{-}}$ the expansion factor for cost of adjustment of input 2. Without loss of generality, we assume that the dynamic expansion factors of Table 5.4 are valid for all firms.

Table 5.4: Expansion Factors for Estimating Outputs, Prices and Costs of Adjustment at Period t

Period t	0	1	2	3	4	5	6	7	8
$\psi_{y_{jt}}$	1.000	1.025	1.045	1.070	1.095	1.120	1.150	1.175	1.200
$\psi_{p_{jt}}$	1.000	1.012	1.025	1.035	1.050	1.060	1.075	1.085	1.100
$\psi_{w_{1t}}$	1.000	1.010	1.020	1.030	1.040	1.050	1.060	1.070	1.080
$\psi_{w_{2t}}$	1.000	1.006	1.010	1.018	1.025	1.030	1.038	1.045	1.050
$\psi_{w_{it}^{+}}$	1.000	1.006	1.010	1.018	1.025	1.030	1.038	1.045	1.050
$\psi_{w_{it}^{-}}$	1.000	1.015	1.035	1.055	1.070	1.090	1.110	1.130	1.150

With these conditions, the problem for firm 1 is:

$$\underset{x_{kt}^{+},x_{kt}^{-},\lambda_t}{maximise}\ \{\pi_l=\sum_{t=1}^{8}[\ s_t(100\psi_{y_t}\times 8.0\psi_{p_t}-x_{11t}\times 2.0\psi_{w_{1t}}-x_{21t}\times 3.0\,\psi_{w_{2t}})$$

$$-s_{t-1}(x_{11t}^{-}\times 1.1\psi_{w_{1t}^{-}}+x_{21t}^{-}\times 2.0\psi_{w_{2t}^{-}}+x_{11t}^{+}\times 0.8\psi_{w_{1t}^{+}}+x_{21t}^{+}\times 1.2\psi_{w_{2t}^{+}})]\}\qquad (5.6.1)$$

subject to the transition equation,

$$\left.\begin{aligned} x_{11t}&=x_{11t-1}+x_{11t}^{+}-x_{11t}^{-}\\ x_{21t}&=x_{21t-1}+x_{21t}^{+}-x_{21t}^{-}\end{aligned}\right\}\quad t=1,2,\cdots,8;$$

the boundary of the technology for adjustment period t, $t=1,2,\cdots,8$:

$$100\lambda_{1t}+90\lambda_{2t}+150\lambda_{3t}+133\lambda_{4t}+152\lambda_{5t}\leq x_{11t}$$

$$100\lambda_{5t}+149\lambda_{5t}+85.9\lambda_{5t}+189\lambda_{5t}+61\lambda_{5t}\leq x_{21t}$$

$$100\lambda_{1t}+110\lambda_{2t}+120\lambda_{3t}+115\lambda_{4t}+103\lambda_{5t}\geq 100\,\psi_{y_{jt}};$$

and

all variables are positive.

The economic efficiency is measurement is $EE_t=\dfrac{x_{11t}^{*}\times 2.0\Psi_{w_{1t}}+x_{21t}^{*}\times 3.0\Psi_{w_{2t}}}{x_{11t}\times 2.0\Psi_{w_{1t}}+x_{21t}\times 3.0\Psi_{w_{2t}}}$,

$t=1,2,\cdots,8$

There are two important differences between this model and the model given by problem (5.4.1). The first difference is that instead of a constant output quantity vector at each of the eight periods in the time horizon, period-to-period output quantity vectors are considered. This situation is equivalent to considering that, at each period, a new firm is incorporated to the system. This difference is not real, because we may consider that, in each model and at each period, the pertinent output quantity vector is considered.

The second difference is that, in this model, the time of adjustment and the time horizon are the same. For this reason, in this model, we consider that the boundary of technology does not change over the whole time horizon, while, in the model to solve problem (5.4.1), the boundaries of the technology are considered only over the time of adjustment. This difference is only apparent because, in the model for solving problem (5.4.1), we may include $T\text{-}t_a$ times the boundary of the technology that is valid at t_a and assign to each firm the weight assigned at t_a.

The solution to problem (5.6.1) for the five firms is in Table 5.5.

Conclusions from this Example

From the solution to the example presented in Section 5.6, the main conclusions are the same as for the solution to the basic model. Although the conclusions are the same, the modified model is more realistic than the basic one. The modifications introduced to the basic model do not modify the structure or the basic concepts on which the basic model is developed. As mentioned above, in the dynamic output case, there is a specific period-to-period optimal input quantity vector.

Systems with dynamic outputs present a similar behaviour to static systems. The similarity is in the sense that, once a firm achieves the target input quantity vector, the peer is the same firm as the one that is its peer in the static output case. From this point of view, the dynamic outputs problem is similar to the static outputs problem, but with target quantities for inputs moving over the boundary of the technology. Nonetheless, if period-to-period prices are different from firm to firm, the economically efficient firm may change from period to period.

Table 5.5: Optimal Paths of Adjustment:
Dynamic Outputs, Asymmetric Costs of Adjustment,
Dynamic Prices and Costs of Adjustment of Three Firms

		Firm	1		4		5	
	Initial Conditions	y	100		115		103	
		x_1	100		133		152	
		x_2	100		189		61	
		TE	1.0		0.7567		1.0	
		EE	0.9295		0.6415		0.9829	
			Optimal Path	Target	Optimal Path	Target	Optimal Path	Target
Optimal Path of Adjustment	Period 1	y	102.5	102.5	117.88	117.88	105.58	105.58
		x_1	123.72	128.13	142.27	147.34	151.28	131.97
		x_2	78.384	73.373	90.141	84.379	64.999	75.57
		EE	0.9872	1.0	0.9872	1.0	0.9859	1.0
	Period 2	y	104.50	104.50	120.18	120.18	107.64	107.64
		x_1	127.48	130.63	146.60	150.22	151.28	134.54
		x_2	78.384	74.805	90.1413	86.025	67.88	77.049
		EE	0.9911	1.0	0.9911	1.0	0.9876	1.0
	Period 3	y	107.00	107.00	123.05	123.05	110.21	110.21
		x_1	132.176	133.75	152.00	153.81	151.28	137.76
		x_2	78.384	76.594	90.141	88.083	71.490	78.892
		EE	0.9957	1.0	0.9957	1.0	0.9903	1.0
	Period 4	y	109.50	109.50	125.93	125.93	112.785	112.785
		x_1	136.88	136.88	157.41	157.41	151.28	140.98
		x_2	78.384	78.384	90.14	90.14	75.095	80.735
		EE	1.0	1.0	1.0	1.0	0.9925	1.0
	Period 5	y	112.00	112.00	128.80	128.80	115.36	115.36
		x_1	140.00	140.00	161.00	161.00	151.28	144.20
		x_2	80.173	80.173	92.199	92.199	78.701	82.579
		EE	1.0	1.0	1.0	1.0	0.9948	1.0
	Period 6	y	115.00	115.00	132.25	132.25	118.45	118.45
		x_1	143.75	143.75	165.31	165.31	151.28	148.06
		x_2	82.320	82.320	94.669	94.669	83.028	84.791
		EE	1.0	1.0	1.0	1.0	0.9977	1.0
	Period 7	y	117.50	117.50	135.125	135.125	121.025	121.025
		x_1	146.88	146.88	168.91	168.91	151.28	151.28
		x_2	84.11	84.11	96.727	96.727	86.634	86.634
		EE	1.0	1.0	1.0	1.0	1.0	1.0
	Period 8	y	120.00	120.00	138.00	138.00	123.6	123.6
		x_1	150.00	150.00	172.5	172.5	154.5	154.5
		x_2	85.90	85.90	98.785	98.785	88.477	88.477
		EE	1.0	1.0	1.0	1.0	1.0	1.0

In the example, firm 3 is the peer of all firms with the inputs and outputs at initial conditions, time of adjustment 0. The firms 1 and 4 achieve the dynamic targets (target input quantity vectors) at periods 4, 5, 6, 7 and 8.

In this example, firm 5 performs adjustments because the present value of savings is larger than the present value of the costs of adjustment, evaluated along the eight periods. Recall that firm 5 did not perform adjustments in the example of Section 4.4. Firm 5 achieves the dynamic targets at periods 7 and 8.

5.7 Incorporation of Quasi-fixed Variables

The variables considered in the preceding chapters and sections are discretionary, in the sense that they may be modified at management's discretion. Labour is an example of a discretionary variable. Quasi-fixed or non-discretionary variables are variables that may not be modified at management's discretion in a short time horizon. Only quasi-fixed variables that are relevant to productivity and efficiency measurement are considered in this thesis. The acres of land in a farm, the location of a firm, and the number of competitors close to a firm are examples of quasi-fixed variables. The terms quasi-fixed and non-discretionary are used interchangeably.

Staat (1999, p. 42) discusses DEA-techniques for modelling continuous and categorical non-discretionary variables. Staat (1999) presents the model developed by Golany and Roll (1993, p. 423f). For the purpose of this thesis, we adapt to profit maximisation the model of Banker and Morey (1986) that Cooper, Seiford and Tone

(2000, p. 63) present for technical efficiency measurement. The adapted model with discretionary outputs and discretionary and non-discretionary inputs is model (5.7.1):

$$\underset{x_{kt}^{+}, x_{kt}^{-}, \lambda_t}{maximise} \{ \pi_k = \sum_{t=1}^{T} [s_t (y_{kt}\ p_t - x_{kt}\ w_t) - s_{t-1} (x_{kt}^{+}\ w_t^{+} + x_{kt}^{-} w_t^{-})] \qquad (5.7.1)$$

subject to

$x_{kt}^{+}\ w^{+} + x_{kt}^{-} w^{-} \leq b_{kt} \quad t=1, 2, \ldots, t_a$ (budget constraint);

$x_{kt} = x_{kt-1} + x_{kt}^{+} - x_{kt}^{-} \quad t=1, 2, \ldots, t_a$ (transition equation);

$$\left. \begin{array}{l} Y\lambda_t \geq y_{kt} \\ X\lambda_t \leq x_{kt} \\ X_{nd}\lambda_t \leq x_{ndk} \end{array} \right\} \quad 1 \leq t \leq T.$$

(boundary of technology);

$t_a \leq t_a^{*}$ (Time to adjust inputs is lower than or equal to adjustment time);

and

all variables are non-negative;

where X_{nd} is the matrix of non-discretionary input quantity vectors ($I_{nd} \times K$); x_{ndk} is the non-discretionary input quantity vector ($1 \times I_{nd}$) of evaluated firm k; w_{nd} is the price vector of non-discretionary inputs; I_{nd} is the number of the non-discretionary inputs; and K is the number of observed firms.

This model, which considers constant returns to scale, is similar to previous models, but includes the non-discretionary variables that are forced to equality instead of to greater than or equal to the boundary of technology restriction.

We consider the data of Table 3.1 for an example. We determine the optimal path of adjustment that maximises the present value of profit over a three-period adjustment time and eight-period time horizon. For this example, input 2 is considered a non-discretionary input, the asymmetric adjustment costs (and the original data from Table 3.1) are presented in Table 5.6. For this example, the boundary of the technology is static and no budget constraints are considered.

Under these conditions, the problem for firm 4 is:

$$\underset{x_{kt}^{+},x_{kt}^{-},\lambda_t}{maximise}\{\pi_1=\sum_{t=1}^{8}[\ s_t(115\times 8.0-x_{14t}\times 2.0)-s_{t-1}(x_{14t}^{-}\times 1.1+x_{14t}^{+}\times 0.8)]\} \quad (5.7.2)$$

subject to the transition equations,

$$\left.\begin{array}{l} x_{14t}=x_{14t-1}+x_{14t}^{+}-x_{14t}^{-} \\ x_{nd24t}=x_{nd240} \end{array}\right\} \quad t=1,2,3;$$

the time to adjust inputs is less than or equal to the adjustment time, $t_a \leq 3$;

the boundary of the technology for t = 1,2,3:

$100\lambda_{1t}+90\lambda_{2t}+150\lambda_{3t}+133\lambda_{4t}+152\lambda_{5t}\leq x_{11t}$
$100\lambda_{1t}+149\lambda_{2t}+85.9\lambda_{3t}+189\lambda_{4t}+61\lambda_{5t}=189$

$100\lambda_{1t}+110\lambda_{2t}+120\lambda_{3t}+115\lambda_{4t}+103\lambda_{5t}\geq 115$

$x_{14t}=x_{143}$ t=4,5,6,7,8;

and

all variables are positive.

Table 5.6: Discretionary, Non-discretionary Inputs,
Outputs, Prices, and Costs of Adjustment

		Quantity at $t = 0$		
	Firm k	y_{1kt}	x_{1kt}	x_{nd2kt}
	1	100	100	100
	2	110	90	149
	3	120	150	85.9
	4	115	133	189
	5	103	152	75
	Price	8.0	2.0	3.0
Adjustment	Increase		0.8	
Cost	Decrease		1.1	

The solution to problem (5.7.2) for four values of quasi-fixed input 2 is in Table 5.7. The economic efficiency measurements presented in Table 5.7 are referred to as the costs of the target input quantity vector attainable with non-discretionary input 2 fixed at 189. The target input quantity vector is more expensive than the target input quantity vector attainable if both inputs are discretionary.

Table 5.7: Optimal Paths of Adjustment, Problem (5.7.2)

Period	x_{1t}	Economic Efficiency	λ_{2t}	λ_{4t}	Fixed Output	Quasi-fixed Input
0	133.0	0.9547	0.0168	0.9867	115	189
1	123.91	0.9761	0.6289	0.5042	115	189
2	114.16	1.0	1.2685	0	115	189
3	114.16	1.0	1.0455	0	115	189

Table 5.8 presents the target input quantity vector for firm 4, for discretionary and non-discretionary input 2; the target input quantity vector, when both inputs are discretionary, is from Table 5.2.

Figure 5.1 represents the data of Table 5.6. Bold numbers 2, 1, 3 and 5 represent the inputs by unit of output of firms 2, 1, 3 and 5, respectively. Assuming constant returns-to-scale technology, segments horizontal to 2, 2-1, 1-3, 3-5, and vertical from 5, define the boundary of the technology. Firm 4 is a non-efficient firm. Dotted line 0B represents a pseudo boundary of the technology.

The dotted arrow from firm 4 to dotted line 0B represents the optimal path of adjustment of this firm. It should be expected that the target of input 1 is on the horizontal part of the boundary of the technology, starting from firm 2, but the target is on the pseudo boundary of the technology, defined by the line 0B, that starts at 0 and passes by firm 2. For initial values of input 2 less than 114.16, there is no feasible solution to problem (5.7.2).

Table 5.8: Cost of Inputs of Initial and Target Input Quantity Vector[(1)]

	x_1	x_2	Cost
Initial	133	189	833.0
Target d-d	143.75	82.32	534.46
Target d-nd	114.16	189	795.32

[(1)] d-d stands for discretionary input 1 and discretionary input 2
d-nd stands for discretionary input 1 and non-discretionary input 2

Keeping input 1 constant, input 2 may be increased up to the frontier 0B. This point, not shown in Figure 5.1, is at 133 units for input 1 and 220.189 units for input 2. For initial values of input 2 larger than 220.189, there is no feasible solution to problem (5.7.2). The minimum value of input 1 is $(189 \times 90)/149$=114.16 and the maximum value of input 2 is $(133 \times 149)/90$ =220.189.

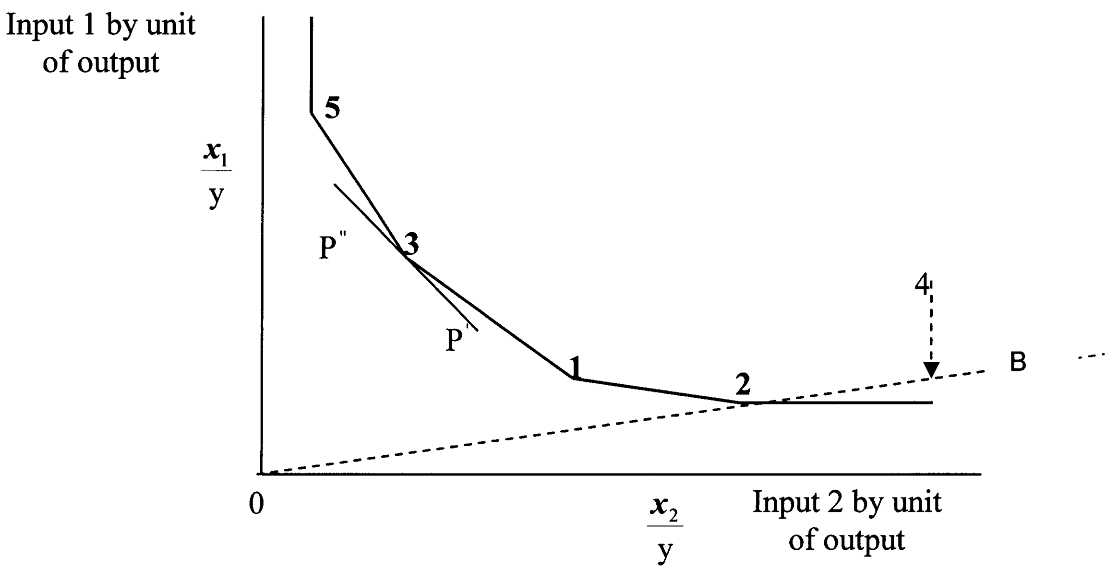

Figure 5.1: Optimal Path of Adjustment of Firm 4 Non-discretionary Input 2

5.8 Incorporation of Capital Investment Constraint

In addition to non-discretionary variables, we now present capital investment as a special input variable, which may be modified at management's discretion, but under prefixed constraints. We refer to such variables as constrained discretionary variables. The constraints to capital investment may come from a specific investment policy that is derived from a strategic planning process or from short-term capital allocation priorities. Two constraints are considered here: fixed assets, $K(t)$, and capital investment $I(t)$ at period t.

Rewriting equation (2.8.5) as, $\dot{\boldsymbol{K}}(t) = \boldsymbol{I(t)} - \delta \boldsymbol{K(t)}\ ;\ \boldsymbol{K(0)} = \boldsymbol{k}$, the discrete-time version of this first-order ordinary differential equation is:

$$\boldsymbol{K(t)} = \boldsymbol{K(t\text{-}1)} + \boldsymbol{I(t)} - \delta \boldsymbol{K(t\text{-}1)}\ ;\ \boldsymbol{K(0)} = \boldsymbol{k}, \qquad (5.8.1)$$

where $\boldsymbol{K}(t)$ is the fixed asset at the start of period t; $\boldsymbol{I}(t)$ is the capital investment in period t; δ is the rate of depreciation of assets; and $\boldsymbol{K}(0) = \boldsymbol{k}$ is the amount of capital at the time that the decisions are made.

Equation (5.8.1) is similar to the transition equations of the basic DEA model (4.3.3). The difference is that the adjustments to these transition equations are either increases or decreases, while the adjustments for the constrained discretionary variables may be both, increases and decreases; increase with investment and decrease with depreciation.

Under this perspective, the constrained discretionary variables may be considered as discretionary variables with a special transition equation. To be consistent with previous definitions of variables, let $\boldsymbol{x}_{cdk,t}$, instead of $\boldsymbol{K}(t)$, be the constrained discretionary input quantity vector of firm k in period t; $\boldsymbol{x}^{*}_{cdkt}$ be the maximum quantity of the constrained discretionary input quantity vector for firm k in period t; and $\boldsymbol{I}^{*}_{kt}$ be the maximum quantity of capital investment vector for firm k in period t. The cost per period of $\boldsymbol{x}_{cdk,t}$ is denoted $\boldsymbol{w}_{cdk,t}$.

Finally, the inclusion of constrained discretionary variables in the DEA model for optimal paths of adjustment results in the following:

$\boldsymbol{x}_{kt}^{+}\, \boldsymbol{w}^{+} + \boldsymbol{x}_{kt}^{-} \boldsymbol{w}^{-} \le b_{kt}$, t=1, 2, …, t_a (budget constraint);

$\boldsymbol{x}_{cdk,t} \le \boldsymbol{x}_{cdkt}^{*}$

$\boldsymbol{I}_{kt} \le \boldsymbol{I}_{kt}^{*}$

$\boldsymbol{x}_{kt} = \boldsymbol{x}_{k,t\text{-}1} + \boldsymbol{x}_{kt}^{+} - \boldsymbol{x}_{kt}^{-}$ t=1, 2, …, t_a (transition equations)

$\boldsymbol{x}_{cdk,t} = \boldsymbol{x}_{cdk,t\text{-}1} + \boldsymbol{I}_{kt} - \delta \boldsymbol{x}_{cdk,t\text{-}1}$

$$\left.\begin{array}{l} Y\lambda_t \ge y_{kt} \\ X\lambda_t \le x_{kt} \\ X_{cd}\lambda_t \le x_{cdxt} \\ X_{nd}\lambda_t = x_{ndk0} \end{array}\right\},\ 1 \le t \le T$$ (boundary of the technology);

and

$t_a \le t_a^{*}$.

5.9 General DEA Model for Optimal Paths of Adjustment

This section presents a general DEA model for optimal paths of adjustment with dynamic technology boundaries. The basic DEA model of Chapter 4 considers that outputs, prices, costs of adjustment and the technology boundary do not change from period to period. In this section, we present a model of optimal paths of adjustment, with a dynamic technology boundary, dynamic outputs, and period-to-period variable prices and costs of adjustment.

As stated in Section 5.6, the term *dynamic output* means that outputs change from period to period. With dynamic technology boundaries, as long as outputs and inputs change over the horizon time, it does not make sense to differentiate between the time horizon and the time of adjustment (as in dynamic outputs). However, budget constraints may be included. Dynamic boundaries of technology modify the objective function, the output quantity vector at the right-hand side of the technology boundaries and the boundary of the technology. The modification to the boundary of the technology affects the matrix of inputs and outputs, $\boldsymbol{Y}_t$ and $\boldsymbol{X}_t$, respectively.

Problem (5.9.1) below presents a general DEA model for the design of optimal paths of adjustment. This general DEA model maximises the present value, over the horizon time, of period-to-period revenue less cost of inputs and less cost of adjustment of inputs, and allows for a dynamic technology boundary, dynamic outputs, and period-to-period variable prices and costs of adjustment. The problem is defined by:

$$\underset{x^+_{kt}, x^-_{kt}, \lambda_t}{maximise} \{ \pi_k = \sum_{t=1}^{T} [s_t(\boldsymbol{y}_{kt}\ \boldsymbol{p}_t - \boldsymbol{x}_{kt}\ \boldsymbol{w}_t - \boldsymbol{x}_{cdt}\boldsymbol{w}_{cdt}) - s_{t-1}\ (\boldsymbol{x}^+_{kt}\ \boldsymbol{w}^+_t + \boldsymbol{x}^-_{kt}\boldsymbol{w}^-_t)] \qquad (5.9.1)$$

subject to

$\boldsymbol{x}^+_{kt}\ \boldsymbol{w}^+ + \boldsymbol{x}^-_{kt}\boldsymbol{w}^- \leq b_{kt}$, t=1, 2, …, t_a (budget constraints);

$\boldsymbol{x}_{cdk,t} \leq \boldsymbol{x}^*_{cdkt}$

$\boldsymbol{I}_{kt} \leq \boldsymbol{I}^*_{kt}$

$\boldsymbol{x}_{kt} = \boldsymbol{x}_{k,t\text{-}1} + \boldsymbol{x}^+_{kt} - \boldsymbol{x}^-_{kt}$ t=1, 2, …, t_a (transition equations);

$\boldsymbol{x}_{cdk,t} = \boldsymbol{x}_{cdk,t\text{-}1} + \boldsymbol{I}_{kt} - \delta\ \boldsymbol{x}_{cdk,t\text{-}1}$

$$\left.\begin{array}{l} Y\lambda_t \geq y_{kt} \\ X\lambda_t \leq x_{kt} \\ X_{cd}\lambda_t \leq x_{cdxt} \\ X_{nd}\lambda_t = x_{ndk0} \end{array}\right\} \quad 1 \leq t \leq T. \qquad \text{(boundaries of the technology)}$$

$t_a \leq t_a^{*}$.

[$1'\lambda_t = 1$, variable returns to scale if applicable)]

$s_t = (1+r/100)^{-t}$, (present value factor);

and

all variables positive.

Solving this problem is equivalent to solving as many basic DEA models as there are time horizon periods. For each period, the specific basic DEA problem is defined by its specific technology boundary, input quantity vector, prices and costs of adjustment. The transition constraints and the objective function link the basic DEA models implicit in problem (5.9.1).

5.10 Conclusions

The first conclusion is that, starting from the basic DEA model, it is possible to formulate general DEA models that include period-to-period variable input and output quantity vectors. These general models reduce to the known special cases when fixing the vectors that do not change from period to period. For example, if the forecast is presented as in Table5.3, the static variables should have expansion factors valued at 1.0 for the periods they are static.

The second conclusion is that our general DEA model accounts for intertemporal optimisation. In Section 3.7, we present five of the limitations that one may encounter with DEA, as stated by Coelli, Rao and Battese (1998, pp. 180-181). The first four of these limitations remain in the basic and in the general DEA models. The limitation, that DEA does not account for intertemporal optimisation, deserves comment. If we understand *standard DEA* as the primal DEA LP, then it does not account for multi-period optimisation, because, to date, no one has developed an aggregate form of intertemporal total factor productivity. But if we understand *standard DEA* as the dual DEA LP that maximises the present value of period-to-period revenue, or minimises the present value of period-to-period cost, then it may account for multi-period optimisation.

Our basic DEA model derives from the dual DEA LP with an objective function that optimises an aggregative form of period-to-period present value of profits. Additionally, the period-to-period input variables are linked by the transition equations. Those two facts allow our general DEA model to account for intertemporal relationships of inputs and outputs. The key for dealing with intertemporal relationships of inputs and outputs is to consider that the peers and the weights of peers may change from period to period.

CHAPTER 6

DETERMINATION OF OPTIMAL PATHS FOR STORES OF THE CHILEAN RETAILING FIRM, DIJON

6.1 Introduction

This chapter presents a realistic application of the models of optimal path of adjustment, developed in Chapter 4 and extended in Chapter 5. The application uses relevant data of 35 comparable retail-level stores. The stores are branches of the Chilean retailing firm, Dijon.

Section 6.2 is a general overview of Dijon. Section 6.3 presents the relevant data to be considered in this application. Because of a confidentiality agreement with Dijon, the data are encrypted by a simple escalation procedure. Section 6.4 presents the optimal path of adjustment LP model for Dijon's stores. Section 6.5 presents the optimal paths of adjustment obtained from the LP model. Section 6.6 presents the main economic results of these optimal paths of adjustment. Section 6.7 presents the comparison of period-to-period changes of weights of the stores with respect to the constant weight model. Sections 6.8 and 6.9 present the main results derived from the application of optimal paths of adjustment to one specific store, considering the expected and optimal period-to-period variations of output, input prices and input costs of adjustment, respectively. Finally, Section 6.10 presents conclusions derived from the application of the optimal path of adjustment LP model to the 35 stores.

6.2 An Overview of Dijon

This overview includes a general description of the processes that Dijon has developed for managing its stores. Special attention is given to the features that indicate that Dijon is appropriate for the application of the model for optimal paths of adjustment.

By 1974, Chile moved from a tightly State-controlled economy to an open market economy. Chilean manufacturing industries had to update their production technology to one that could cope in a global market. Since 1974, prices have been determined in the markets instead of being fixed by the government. Importation taxes have been reduced to a flat 10 per cent and a value added tax (VAT) of 20 per cent is imposed on trade and service transactions, education excluded. Under this competitive scenario, Dijon developed its strategic policy.[1]

Dijon Limited is a Chilean family business that started in late 1978, after merging five non-competitive small textile manufacturing installations. By July 1979, the first business diversification took place, importing T-shirts and parkas made in Asia, for distribution to retailers. By April 1980, the second diversification started with the opening of the first three Dijon retailing stores that had a mix of products orientated to the clothing needs of medium and medium-low income people.

[1] The VAT was decreased from 20 to 16 per cent in 1988, but it was increased to 18 per cent in 1991 and then to 19 per cent on 9 July 2003.

From 1980 to 1986, Dijon grew to a national chain of retailing stores. By 1985, Dijon was vertically integrated in the clothing business, manufacturing its own brand products, while still importing goods from Asia. By 1995, the firm offered the Dijon credit card. In December 2001, Dijon opened its fifty-seventh store.

Of the 57 stores, 15 stores target children of medium-income families, seven stores are outlet-stores, and 35 stores target adults and children of medium and medium-low income families. Only these last 35 stores are involved in this thesis. Dijon estimates that it commands 25 per cent of the clothing market for medium- and low-income people, whose socio-economic classifications are C and D.

About 35 per cent of the population has socio-economic classification D, which had average monthly income of AU$410 in 2001. The percentages of the population in the socio-economic classifications C3, C2 and C1 are 25, 20 and 8 per cent, respectively; whose average monthly incomes in 2001 were AU$760, AU$1,640 and AU$5,300, respectively.

Dijon's managing process is tightly controlled from the Branches Manager's Office, at Dijon's headquarters, located in Huechuraba, which is 25 kilometres north-west from central Santiago. The Branches Manager (BMO) is responsible for the allocation of resources at each store. For each store, the BMO decides the assignment of the sales quantity target, the number of employee-hours, the number of cashier-hours, the sales and administration expenditures, and the marketing budget.

The BMO tries to apply, to the actual business, the same organization and operation policies that were developed by the owners many years ago. The 35 stores under study are organized according to the following pattern:

(a) Each store has a chief executive officer (CEO) who manages the store, carries out the policies from the BMO, to whom the CEO reports.

(b) The BMO assigns the number of employee-hours to each store: salespersons, cashiers, office white-collar workers, juniors and blue-collar workers.

(c) The BMO distributes the marketing budget for the whole system, assigning the corresponding amount to each store.

Since 1985, stores in Chile that sell clothes at the retail level are under aggressive competition from the fast-growing hypermarket industry. This competition has reduced the profits of Dijon because of the forced price reductions. To reverse this situation, Dijon is looking for cost reductions and for a more efficient resource allocation.

The above organizational pattern configures a system of partially indexed assignment of resources. In addition to this situation, and considering that the firm includes one non-discretionary input, in addition to four discretionary ones, Dijon is a valid study case to illustrate the application of the adjustment models that are developed in Chapters 4 and 5. The 35 stores to be considered are comparable in the sense indicated in Chapter 3. In this context, the model for optimal path of adjustment, as developed in previous chapters, is appropriate for application to Dijon.

6.3 Relevant Data for this Application

This section presents and discusses the data for the 35 stores in our study. Data were extracted from the accounting information for the years, 2000 and 2001. The BMO gave access to the data sources. The labour-related costs of adjustment were evaluated according to the Chilean labour regulations.

Under a confidentiality agreement with Dijon, data are encrypted by simple escalation transformations. The escalation factor is specific to each kind of variable. This simple encryption of data does not change the optimal value of the decision variables when transformed back to the original scale. The reason for this is that when using the dual LP model, at both sides of each constraint the same variable appears. There is no reason for expecting different optimal values if a variable is measured in a metric or any other system. Thus, the floor surface of the store may be measured in square metres or in square feet or sales expressed in Chilean pesos or in Australian dollars. Nonetheless, consistency is required, for easy interpretation of results.

After being encrypted, the monetary data are in "Dijon Monetary Units" (D$) and the store floor data are in "Dijon Surface Units" (DSU). The observed data are presented in Appendix 1. The monetary values are expressed in AU$, relative to December 29, 2001. For all purposes, *period* refers to a time period of six months.

Output

For performance evaluation of the stores, Dijon's senior managers consider that gross sales is the only tangible output. The gross sales data that are presented in Appendix 1 are encrypted average data for the six-month period, from January 1, 2000 to December 31, 2001. The original data are taken from the accounting records that the BMO uses for decision making.

Inputs

Dijon's senior managers consider that five inputs are relevant for branch performance evaluation. Four of these inputs are discretionary variables and a fifth is a non-discretionary variable. These five variables are:

1) *Salesperson-hours by period.* There is no legal constraint or contractual agreement with Dijon's Labour Union regulating salesperson-hours by period. The BMO tries to keep the ratio of salesperson hours to the total employee hours, around 0.50. For the purposes of this thesis, this variable is assumed to be continuous, although, in real terms, this is not true. Nonetheless, the assumption is based on the fact that a part-time employee counts as the fraction of time that the person works with respect to the time a full-time employee works. In Chile, a full-time employee works 48 hours per week. This discretionary variable is measured in thousands of hours by period (khour/period).
2) *Cashier-hours by period.* There is no legal or contractual agreement with Dijon's Labour Union regulating cashier-hours by period. The BMO tries to keep the ratio of cashier- hours to the total employee hours about 0.15. This discretionary variable is measured in thousand hours by period (khour/period).
3) *Sales and Administration Expenses.* This item includes the institutional administration cost, and the sales and administration expenses of the specific store. The BMO charges a fraction of the institutional administration cost to each store. The general criterion is to charge each store close to five per cent of

its gross sales. The CEO is responsible for the sales and administration expenses of the store. This discretionary variable is measured in D$ by period.

4) *Marketing*. This item includes the cost of institutional marketing and the cost of marketing that the specific branch does. The BMO charges a fraction of the institutional marketing cost to each store. The general criterion is to charge each store about seven per cent of its gross sales. This discretionary variable is measured in D$ by period.

5) *Store Floor Surface*. The total store floor surface is constant in a short time horizon. Nonetheless, the store is regularly remodelled and the floor surface redistributed, especially for end-of-season events. This is the non-discretionary variable, which is measured in DSU.

Analysis of Data

The five inputs described above are not highly correlated. Appendix 3 presents the correlation matrix generated with the software, STATISTICA. The highest sample correlation coefficient is 0.589 between Marketing and Cashier-hours.

Dyson et al. (2001, p. 248) indicate that for achieving a reasonable level of discrimination, the minimum number of units (stores) to be observed must be at least twice the product of the number of inputs and the number of outputs involved. For this application, the number of stores observed is 3.5 times this suggested minimum ($2 \times 5 \times 1 = 10$).

Costs of Adjustment

As previously presented, cost of adjustment is the cost of changing the operational conditions of a store. In general terms, cost of adjustment is the cost of changing the terms of a contract or of an agreement. For the purpose of this research, the cost of adjustment of an input is the cost of changing the quantity of that input. For example, the Chilean labour legislation grants to employees the property of their posts. For this reason, employers must indemnify any fired employee. This payment is part of the cost of adjustment of labour.

Only discretionary variables are susceptible to adjustment in a short- or medium-time horizon. In this application, the cost of adjustment is asymmetric, in the sense that the cost of increasing the quantity of an input is different from the cost of decreasing it. For example, in general terms, the cost of firing is higher than the cost of hiring an employee. Appendix 2 presents asymmetric costs of adjustment for the four discretionary inputs.

Time of Adjustment and Time Horizon for Economic Evaluation

Dijon's Vice-President defined up to four consecutive periods as the time for performing the adjustments. If and only if the adjustments are not feasible in four periods, up to two additional periods may be considered. Thus, the time of adjustment is four periods, extendable up to six.

Dijon's Vice-President defined that the time horizon or the number of periods for economic evaluation of the adjustment project is eight periods.

Rate of Discount

Dijon's Vice-President considers a rate of discount of 9.0 per cent per period as adequate for this project.[2] This rate of discount includes an estimated inflation of 1.5 per cent per period.

Budget Constraints

Dijon's Vice-President determined that, at the first and second adjustment periods, no store could spend in this project more than the one per cent of its gross profit in the period just before the adjustments start. This gross profit is evaluated as the residual of output and the cost of inputs.

Returns to Scale

The returns to scale of the boundary of the technology were investigated at the observed input-output data set of each of the technically efficient stores. The method used to determine the returns to scale is that proposed by Thanassoulis (2001, p. 125) and presented in Section 3.4 of this thesis.

For each of the technically efficient stores, a five per cent increase of all observed data of inputs returns a five per cent increase of output, at 100 per cent technical efficiency. For this reason, constant returns to scale are expected for the Dijon system

[2] For this project, Dijon expects that the rate of return of investment be no less than its weighted average cost of capital (WACC).

6. 4 The Optimal Path of Adjustment Program for Dijon's Stores

Based on the information provided in Sections 6.2 and 6.3, it is concluded that the optimal path of adjustment model, as developed in Chapter 5, is applicable to Dijon's 35 stores.

The optimal path of adjustment program determines the period-to-period adjustments that a specific store must undertake to perform at 100 per cent economic efficiency. The period-to-period adjustments are specified for each discretionary variable. The adjustments are to be performed at the start of each adjustment period. The sequence of period-to-period adjustments of the inputs is optimal in the sense that it minimises the total of the present values of period-to-period costs of the inputs and the period-to-period costs of adjustment of the inputs.

The general optimal path of adjustment program is presented in Appendix 4, with the specific data of Store 202. The general LP with the store-specific data is run store by store, independently of the other stores.

Program Description

The path of adjustment program is divided into five parts, as explained below:

1) Part One contains the identification and specific data of the store under study. The identification is the three-digit number assigned to each store. The specific data are the initial input quantity vector, the output, and the floor area of the

store, the price of inputs, the cost of increasing the inputs and the cost of decreasing them.

2) Part Two performs economic evaluations. This part determines the present worth factors for discounting the value of money to the present; the cost of inputs and its present value for each of the eight evaluation periods; the cost of adjustment of inputs and its present value at each of the four adjustment periods, and the economic efficiency at initial conditions and at each adjustment period.

3) Part Three is the objective function. The objective function is to minimise the value of two independent terms. The first term is the summation of the present value of the cost of inputs at each of the eight evaluation periods or time horizon and the summation of the present value of the cost of adjustment at each of the four adjustment periods. The second term is the cost of the input quantity vector that minimises the cost of inputs; this vector is the target input quantity vector. This term may be included because the optimisation variables of the second term are independent of the path of adjustment optimisation variables.

4) Part Four are the transition equations. The transition equations define the input quantity vector at each adjustment period, in terms of the cumulated adjustments. In real terms, the decision variables are the period-to-period input adjustment variables.

5) Part Five contains the production function and data envelopment analysis. This part is divided into three sections.

   i. The target input quantity vector is determined. The target input quantity vector minimises the cost of inputs. This target input quantity vector and the input quantity vector at each adjustment period are compared term by term. The purpose of the comparison is to determine if the path of adjustment of the inputs arrives to the target quantity vector. In this section, the weight of any store *k* is LT*k*.

ii. The path of adjustment is determined. The adjustments to the input quantity vector are to be done in four adjustment periods under budget constraints for the first two adjustment periods. As previously indicated, budget constraints indicate that, at the first and second adjustment periods, no more than the one per cent of the gross profit at the initial conditions may be spent. In this section, the weights of any store *k* are LA*k*, LB*k*, LC*k*, and LD*k*, respectively, for the first, second, third, and fourth adjustment periods. The period-to-period changes of weights of stores are for allowing period-to-period specific weights of peers and for allowing changes of peers from one period to the next.[3]

iii. If all elements of the error input vector are zero then the path of adjustment is optimal and is accepted. If one or more elements of the error input vector are not zero then the path of adjustment found under this condition is not the best and may be improved. The error input vector is the vector of the differences between the values of the inputs at the end of the path of adjustment and the target values of the inputs.

Two conditions may be considered for improving the non-optimal path of adjustment. The first condition to be considered is increasing the time horizon. The shortest time horizon, T, may be determined using equation (3.6) of Appendix 7. The second condition to be considered is relaxing the budget constraints. Relaxing the budget

[3] This dynamic aspect of DEA is presented in Section 4.3 of the basic model for the selection of optimal paths of adjustment.

constraints improves the quality of the path of adjustment. Increasing the time horizon and relaxing the budget constraints may be considered simultaneously.

For practical applications, it is advisable to re-design the optimal path of adjustment each period, with the new available data. In this case, the initial period is the period at which the re-design is started, including the new technical and economic data.

6. 5 Optimal Paths of Adjustment for Five Stores

As indicated above, for each of the 35 stores, the optimal path of adjustment program determines the period-to-period adjustments that must be undertaken for that store to perform at 100 per cent economic efficiency. The results indicate that for each of the 35 stores, the optimal path of adjustment program determines as the target the input quantity vector that minimises the cost of inputs. This means that the optimal path of adjustment program assigns, in a one-step procedure, the target input quantity vector that minimises the present value of the cost of period-to-period inputs and the period-to-period vector of inputs that minimise the total present value of both costs, the cost of period-to-period inputs and the cost of period-to-period adjustments of inputs.

Table 6.1 presents the optimal paths of adjustment and some relevant results for stores 202, 211, 232, 243 and 258. These firms were selected for the following reasons. Store 202 has one of the lowest economic and technical efficiencies of the 35 stores, when measured at initial input-output conditions. Store 202 has an optimal path of adjustment that is independent of whether constant or variable weights of firms are considered from period-to-period for the firms. For Store 202, the data and results are analysed at each of the four intended adjustment periods. For the other four stores, only relevant deviations from the behaviour of Store 202 are included.

Store 211 has a high economic efficiency and is fully technically efficient, when measured at initial input-output conditions. Store 211 has an optimal path of adjustment for variable period-to-period weights of firms that differs slightly from its optimal path of adjustment for constant weights of firms.

Store 232 has the lowest economic efficiency of the 35 stores, when measured at initial input-output conditions. Store 232 is technically efficient and has an optimal path of adjustment for variable period-to-period weights of firms that is different from the optimal path of adjustment for constant weights of firms.

Store 243 has a low economic efficiency, when measured at initial input-output conditions. Store 243 is technically efficient and has an optimal path of adjustment, considering variable period-to-period weights of firms that is also different from its optimal path of adjustment considering constant weights of firms.

Store 258 has full economic and technical efficiency, when measured with initial and any period-to-period input-output data. Store 258 is peer of 26 stores, itself included.

For each store in period 0, Table 6.1 gives the initial input quantity vector and the cost of inputs. From these data, the technical efficiency, the peers and their weights are calculated and presented.

Table 6.1: Optimal Paths of Adjustment for Five Stores.

Period	Store		202	211	232	243	258
	Output, D$		1,669.40	2,153.81	1,987.10	3,025.80	2,689.50
	Area, DS		10.90	7.55	7.31	17.86	17.76
0	Salesperson Hours, khours		5.539	6.120	4.287	5.959	6.699
	Cashier Hours, khours		1.790	1.532	2.143	1.190	1.340
	Sales & General Expenses, D$		18.350	14.534	41.013	25.765	15.135
	Marketing, D$		13.346	11.672	9.829	16.211	13.674
	Cost of Inputs, D$		407.51	419.92	1,172.96	941.78	459.25
	Technical Efficiency		0.6132	1.000	1.00	1.000	1.000
	Economic Efficiency		0.5801	0.9834	0.4039	0.7455	1.000
	Weight	Peer 204	0.0576				
		Peer 235	0.0365				
		Peer 240	0.0038				
		Peer 254	0.5235				
1	Salesperson Hours, khours		5.539	5.134	4.287	6.814	6.699
	Cashier Hours, khours		1.790	1.136	2.143	1.462	1.340
	Sales & General Expenses, D$		9.395	12.284	37.022	17.275	15.135
	Marketing, D$		11.778	13.912	10.137	15.399	13.674
	Cost of Inputs, D$		288.79	415.12	1,080.64	708.00	459.25
	Cost of Adjustment, D$		12.619	17.339	8.1414	20.840	0.00
	Economic Efficiency		0.8185	0.9948	0.4384	0.9917	1.000
	Weight	Peer 211		0.3121	0.0176		
		Peer 232			0.8580		
		Peer 236	0.0184	0.4431	0.0413		
		Peer 237				0.3418	
		Peer 246					
		Peer 247		0.0417		0.0240	
		Peer 251			0.0488		
		Peer 254					
		Peer 256					
		Peer 258	0.630			0.7217	1.000
2	Salesperson Hours, khours		5.182	4.687	4.287	7.4267	6.699
	Cashier Hours, khours		0.8361	0.9562	2.143	1.5826	1.340
	Sales & General Expenses, D$		9.395	11.264	33.031	17.043	15.135
	Marketing, D$		8.480	14.928	10.445	15.262	13.674
	Cost of Inputs, D$		240.88	412.94	988.32	702.11	459.25
	Cost of Adjustment, D$		12.619	7.867	8.1414	4.471	0.00
	Economic Efficiency		0.9814	1.000	0.4793	1.000	1.000
	Weight	Peer 211			0.03526		
		Peer 232			0.7159		
		Peer 236		0.6442	0.0826		
		Peer 251			0.0975		
		Peer 247	0.0184	0.0607		0.3144	
		Peer 258	0.6030			0.8214	1.000
3	Salesperson Hours, khours		4.152	4.687	4.448	7.4267	6.699
	Cashier Hours, kours		0.8361	0.9562	0.9827	1.5826	1.340
	Sales & General Expenses, D$		9.395	11.265	10.679	17.043	15.135
	Marketing, D$		8.480	14.928	12.401	15.262	13.674
	Cost of Inputs, D$		236.39	412.94	473.73	702.11	459.25
	Cost of Adjustment, D$		13.006	0.00	50.456	4.471	0.00
	Economic Efficiency		1.000	1.000	1.000	1.000	1.000
	Weight	Peer 236		0.6442	0.3888		
		Peer 247	0.0184	0.0607	0.3012	0.3144	
		Peer 258	0.6030			0.8214	1.000
4	Salesperson Hours, khours		4.152	4.687	4.448	7.4267	6.699
	Cashier Hours, khours		0.8361	0.9562	0.9827	1.5826	1.340
	Sales & General Expenses, D$		9.395	11.265	10.679	17.043	15.135
	Marketing, D$		8.480	14.928	12.401	15.262	13.674
	Cost of Inputs, D$		236.39	412.94	473.73	702.11	459.25
	Cost of Adjustment, D$		0.00	0.00	0.00	4.471	0.00
	Economic Efficiency		1.000	1.000	1.000	1.000	1.000
	Weight	Peer 236		0.6442	0.3888		
		Peer 247	0.0184	0.0607	0.3012	0.3144	
		Peer 258	0.6030			0.8214	1.000

Store 202

From Table 6.1, the cost of inputs for Store 202 at period 0 is D$407.51; the technical efficiency is 0.6132 and the economic efficiency is 0.5801. Radial reduction of inputs to the boundary of the technology determines that the peers of Store 202 are Stores 204, 235, 240, and 254 with weights 0.0576, 0.0365, 0.0038, and 0.5235, respectively. Under these conditions, the input cashier-hours has a surplus of 0.1504 thousand hours.[4]

The period-1 entries in Table 6.1 contain the input quantity vector for each store, as a result of the first adjustment at the start of the period. For this period, the input quantity vector, the peers and their weights, the economic efficiency, the cost of inputs and the cost of adjustment are presented. These variables are evaluated with the input quantity vector for period 0 and with the specific economic, floor surface and output data of each store. From Table 6.1, in period 1, the input quantity vector for Store 202 has 5.539 thousand salesperson-hours, 1.790 thousand cashier-hours, D$9.395 for sales and general expenses, and D$11.778 for marketing. These inputs cost D$288.79 and the cost of adjustment of inputs from the initial input quantity vector to the optimal of this period is D$12.619. This cost is the highest feasible cost of adjustment that the budget constraint permits for this period. The cost of adjustment is 10.6 per cent of the reduction of the cost of inputs.[5] In this period, the technical efficiency of Store 202 is 0.9981. For this measurement, Stores 236 and 258 are peers, with weights 0.0127 and 0.6060, respectively. Note that the peers of Store 202, when measuring technical efficiency and when defining the optimal path of adjustment are different.

[4] The technical efficiency is measured using a DEA technical efficiency measurement program.

[5] The budget constraint limits the costs of adjustment for periods 1 and 2 up to one per cent of the gross income at period 0; namely, $0.01 \times (1{,}669.4 - 407.51) =$ D$12.619

In Table 6.1, we omit the technical efficiency measurement from period 1 on, because the technical efficiency measurements of the five firms from this period on are 1.0. In period 1, the economic efficiency of Store 202 is 0.8185. This input quantity vector is not the target one because the economic efficiency is not one and two inputs have a surplus over their target values. Salesperson hours have a surplus of 1.404 thousand hours and cashier hours have a surplus of 0.9609 thousand hours.[6]

The adjustment to the input quantity vector is not a radial reduction from the initial to the optimal input quantity vector. The peers and slacks that define the optimal input quantity vector for period 1 are not necessarily the same when measuring technical and economic efficiency. At the next adjustment period, the adjustments are expected to be such that the optimal input quantity vector is on the frontier of the technology, towards the target input quantity vector. The extension of the adjustments depends on the budget constraint for the period, as previously indicated.

The period-2 entries in Table 6.1, contain the input quantity vector for each store, as a result of the second adjustment at the start of the period. For this period, the input quantity vector, the peers and their weights, the economic efficiency, the cost of inputs and the cost of adjustment are presented. When pertinent, these variables are evaluated with the input quantity vector for periods 0 and 1, and with the specific economic, floor surface and output data of each store. From Table 6.1, the period-2 input quantity vector of Store 202 has 5.182 thousand salesperson-hours, 0.8361 thousand cashier-hours, D$9.395 for sales and general expenses, and D$8.480 for marketing. These inputs cost

[6] The slack or surplus of input quantities is not shown in Table 6.1 or in Appendix 8.

D$240.88 and the cost of adjustments is D$12.619. As in period 1, this cost of adjustment is the highest feasible cost that the budget constraint permits for this period.

In period 2, the economic efficiency of Store 202 is 0.9814, the peers are stores 247 and 258 with weights of 0.0184 and 0.6030. These peers with these weights determine the target input quantity vector for Store 202. Nonetheless, the optimal input quantity vector for this period is not the target one because the economic efficiency is not unity. Salesperson hours have a surplus of 1.030 thousand hours over the target value. The inputs, cashier-hours, sales and general cost, and marketing, are at the target values. The peers of Store 202 for this period differ from the peers for period 1. The optimal input quantity vector is on the hyperplane of the target input quantity vector.

As for period 1, the adjustment to the input quantity vector is not a radial reduction from the initial to the optimal input quantity vector, which is on the frontier of the technology. In general terms, the peers and slacks that define the optimal input quantity vector for period 2 are not necessarily the same when measuring technical and economic efficiency. For the same input quantity vector, the peers and slacks are specific to the efficiency that is measured. Although technical efficiency is one, additional adjustments are required because the economic efficiency is not one. At the next adjustment period, the adjustments are expected to be such that the optimal input quantity vector remains on the frontier of the technology towards the target input quantity vector. The amounts of the adjustments depend on the budget constraint for the period, as previously indicated.

For period 3, Table 6.1 contains the input quantity vector for each store, as result of the third adjustment at the start of the period. For this period, the input quantity vector, the peers and their weights, the economic efficiency, the cost of inputs and the cost of adjustment are presented. When pertinent, these variables are evaluated with input quantity vector for periods 0, 1 and 2, and with the specific economic, floor surface and output data of each store. From Table 6.1, the period-3 input quantity vector of Store 202 has 4.152 thousand salesperson-hours; 0.8361 thousand cashier-hours; D$9.395 for sales and general expenses, and D$8.480 for marketing. These inputs cost a total of D$236.39 and the cost of adjustments is D$13.006. Because this period has no budget constraint, the adjustments are done as long as it is required for the optimal input quantity vector to be the target one. With the adjustments setting the input quantity vector at the target one, both the economic and technical efficiencies are one, and inputs do not have any surpluses.

For period 4, Table 6.1 contains the input quantity vector for each store. For this period, the input quantity vector, the peers and their weights, the economic efficiency, the cost of inputs and the cost of adjustment are presented. From Table 6.1, the period-4 input quantity vector for Store 202 is the same as in period 3 and, for this reason, the cost of adjustment is zero and the economic and the technical efficiencies are one.

The peers that define the period-to-period optimal input quantity vector for any store at any adjustment period are generally not the same peers that define the period-to-period measurement of technical efficiency. For any store, when the technical and economic efficiencies are one, and no input has slack, the peers and their weights that define the target input quantity vector and those that define the measurement of technical efficiency are the same. The peers of Store 202 and their weights do not change from period 2 to period 4.

Store 211

Basically, the analysis of the period-to-period information derived from the application of the optimal path of adjustment program to Store 211 is the same as for Store 202.

In period 1, the peers of Store 211 are itself and Stores 236 and 247, with weights 0.3121, 0.4431 and 0.0417, respectively. From period 2 to period 4, only Stores 236 and 247 are the peers, with weights 0.6442 and 0.0607, respectively. The peers with their respective weights determine the target input quantity vector. The weight of peer Store 236 in period 1 is different from its weight in the last three periods, where it is constant. The same is true for peer Store 247.

Store 211 has no slacks for any input in all of the four adjustment periods.

Store 232

Basically, the analysis of the period-to-period information derived from the application of the optimal path of adjustment program to Store 232 is the same as that derived from the application to Stores 202 and 211. Nonetheless, some differences exist.

In periods 1 and 2, the peers of Store 232 are Stores 211, 232, 236 and 251 with period-1 weights, 0.0176, 0.8580, 0.0413 and 0.0488, respectively, and period-2 weights, 0.03526, 0.7159, 0.0826 and 0.0975, respectively. Cashier-hours have surpluses of 0.1472 and 0.2945 thousand hours at periods 1 and 2, respectively. For periods 3 and 4, the peers of Store 232 are Stores 236 and 247, with weights, 0.3888 and 0.3012, respectively.

The weight of peer Store 211 in period 1 is different from its weight in period 2. For periods 3 and 4, this store is not a peer of Store 232. The weight of peer Store 232 in period 1 is different from its weight in period 2. For periods 3 and 4, this store is not the peer of itself.

The weights of peer Store 236 are different for periods 1 and 2 and different for periods 3 and 4. At the last two periods, the weight of this peer does not change. The same is true for peer Store 247. This means that the input quantity vector at periods 1 and 2 are not on the hyperplane of the target input quantity vector.

At the last two periods, the weights of the peer Stores 232 and 247 do not change. At the same time, the optimal input quantity vector is the target input quantity vector. This means that the optimal input quantity vector at periods 3 and 4 is on the hyperplane of the target input quantity vector.

Store 243

Basically, the analysis of the period-to-period information derived from the application of the optimal path of adjustment program to Store 243 is the same as those derived for Stores 202, 211 and 232. Nonetheless, we note one difference.

At period 1, only one of the three peers of Store 243 defines the target input quantity vector. This means that the input quantity vector at period 1 is not on the hyperplane of the target input quantity vector. In this period, Store 243 has a slack of 0.1155 thousand cashier-hours.

From period 2 to 4, the two peers of Store 243 are Stores 247 and 258. At these periods, the weights of the peers do not change and correspond to the peers that determine the target input quantity vector. This means that the optimal input quantity vector at periods 2, 3 and 4 are on the hyperplane of the target input quantity vector.

Store 258

For this store, we draw attention to a difference in the results from those for the stores discussed above. Store 258 is the single peer of itself at the four adjustment periods. As the single peer of itself, its weight is one. For this reason, this store does not perform adjustments. Store 258 is a peer of 25 other stores, 18 times with Store 247, three times with Store 266, twice with Store 262 and twice with two other stores. Similarly to Store 258, each of the Stores 236, 247 and 266 is the single peer of itself at the four adjustment periods with weights one, and does not perform adjustments.

6. 6 Economic Results

As indicated above, for all of the 35 stores the optimal path of adjustment program determines the period-to-period adjustments that they have to undertake, to perform at 100 per cent economic efficiency. The economic results indicate that for each of the 35 stores, the path of adjustment program determines as the target the input quantity vector that minimises the cost of inputs.

As previously indicated, the budgets for the first and second adjustment periods are constrained to be less than or equal to one per cent of the gross profit at the initial conditions. The gross profit at the initial conditions is evaluated as the residual of the output and cost of inputs at initial conditions.

Table 6.2 presents the economic results for the 35 stores if the adjustments of inputs are done as the optimal path of adjustment program determines. The cost of inputs at initial conditions is the present value of the cost of inputs at initial conditions for eight periods. The cost of inputs of the optimal input quantity vector is the total present value of the cost of inputs at initial conditions and at the start of the first to the eighth periods. From the start of the fourth period, the input quantity vector is the target one. The cost of adjustment is the present value of the cost of adjustment of the inputs. Net saving is the difference between the present value of the cost of inputs at initial conditions and the sum of the present value of the cost of inputs of the optimal path of adjustment and the present value of the cost of adjustment.

From Table 6.2, the present value of total net saving of the 35 stores is D$22,984.04; this saving is the 14.63 per cent of the cost of inputs at the initial conditions. The savings span from zero per cent for the initially fully economically efficient stores, 236, 247, 258 and 266, up to 36.82 per cent of the cost of inputs at initial conditions for Store 232.

Table 6.2: Economic Results of the Optimal Paths of Adjustment[7]

Store	Cost of Inputs, D$ Initial Conditions	Optimal Path of Adjustment	Cost of Adjustment D$	Net Saving D$	Net Saving per cent of Cost of Initial Inputs
202	2,662.99	1,767.74	35.14	860.11	32.30
204	1,306.07	1,132.61	38.30	135.16	10.35
211	2,744.12	2,707.48	25.21	11.43	0.42
224	2,732.12	2,151.81	34.25	546.06	19.99
225	4,961.25	3,931.99	19.07	1,010.19	20.36
226	3,665.98	2,861.05	20.63	784.30	21.39
232	7,665.05	4,784.88	58.02	2,822.15	36.82
234	8,267.93	7,092.56	32.32	1,143.05	13.83
235	3,389.13	3,119.53	14.05	255.55	7.54
236	4,041.65	4,041.65	-	-	-
237	3,754.99	3,689.43	13.37	52.19	1.39
238	3,294.03	2,528.80	29.39	735.84	22.34
239	4,456.64	4,060.88	15.52	380.24	8.53
240	7,655.98	5,374.79	26.00	2,255.19	29.46
242	4,871.91	3,528.45	20.60	1,322.86	27.15
243	5,125.60	3,888.79	30.16	1,206.65	23.54
244	8,677.54	8,004.83	32.72	639.99	7.38
245	5,315.90	4,841.88	11.30	462.72	8.70
246	3,877.13	3,722.85	15.41	138.87	3.58
247	2,799.34	2,799.34	-	-	-
248	6,154.35	4,833.24	25.03	1,296.08	21.06
250	4,072.16	3,682.69	12.15	377.32	9.27
251	2,440.61	2,176.33	16.32	247.96	10.16
252	4,364.36	4,003.33	10.05	350.98	8.04
254	3,902.10	3,689.25	15.97	196.88	5.05
256	6,452.96	4,564.97	31.60	1,856.39	28.77
257	4,506.53	3,665.55	16.53	824.45	18.29
258	3,001.09	3,001.09	-	-	-
259	5,539.69	4,774.90	17.88	746.91	13.48
260	5,788.37	5,065.92	30.09	692.36	11.96
262	1,739.35	1,676.93	22.71	39.71	2.28
263	5,635.05	4,836.84	31.20	767.01	13.61
264	4,712.89	4,503.52	16.52	192.85	4.09
265	5,297.11	4,632.30	32.22	632.59	11.94
266	2,238.79	2,238.79	-	-	-
TOTAL	157,110.76	133,376.99	749.73	22,984.04	14.63

[7] Costs and savings are present values. Cost of inputs is the present value of the cost of inputs for the time horizon of eight periods.

From Table 6.1, at initial conditions the economic efficiency of Store 202 is 58.01 per cent. The budget constraints determine that Store 202 requires three adjustment periods to transform the input quantity vector from the initial value to the target one. In other words, this store requires three adjustment periods to become 100 per cent economically efficient.

From the data of Table 6.1, for Store 202, the cost of adjustment of input quantity vector at each of periods 1 and 2 is 33 per cent of total cost of adjustment. The cost of adjustment of inputs at period 3 is the 34 per cent of the total cost of adjustment of this store. If there were no budget constraints, then the adjustment of inputs would be fully accomplished at the start of the first adjustment period. In this case, the present value of the cost of inputs from period 0 to period 8 would be D$1,715.89, and the present value of the cost of adjustment would be D$38.24. These costs give a net saving of D$908.86. This net saving is 1.057 times the corresponding net savings with budget constraints in effect. For this store, the budget constraints determine a small difference between the present value of savings of the constrained and unconstrained budget cases.

Store 232 behaves differently from that of Store 202. From Table 6.1, at initial conditions the economic efficiency of Store 232 is 40.39 per cent. The budget constraints determine that Store 232 requires, as does Store 202, three adjustment periods to transform the input quantity vector from initial conditions to the target one. In other words, this store requires three adjustment periods for becoming 100 per cent economically efficient.

From data of Table 6.1, the cost of adjustment of the input quantity vector at each of periods 1 and 2 is 12.2 per cent of the total cost of adjustment of this store. The cost of adjustment of inputs at period 3 is the 75.6 per cent of total cost of adjustment of this store. If there were no budget constraints, then the adjustment of inputs would be done at the start of the first adjustment period. In this case, the present value of the cost of inputs from period 0 to period 8 would be D$3,794.97, and the present value of the cost of adjustment would be D$66.74. These costs give a net saving of D$3,803.34. This net saving is the 49.62 per cent of the present value of the cost of inputs at initial conditions, evaluated for eight periods. This net saving is 1.35 times the corresponding percentage with the budget constraints in effect. For this store, the budget constraints make a significant difference between the savings of the constrained and unconstrained budget cases.

The main difference between the economic behaviours of Store 202 and Store 232 is that the measurement of economic efficiency for the first store is 0.5801, at initial input conditions, and for the second the same measurement is 0.4039. This means that Store 202 must decrease the cost of its inputs by 41.59 per cent of the cost of inputs at initial conditions, while Store 232 must decrease the cost of its inputs by 59.61 per cent. The higher the economic efficiency, the lower is the reduction of the cost of inputs for becoming 100 per cent economically efficient. In general terms, the higher the economic efficiency, the higher is the gross profit at initial conditions. Finally, the higher the gross profit at initial conditions, the weaker are the budget constraints and the adjustments are done earlier than if the budget constraints are effective.

6. 7 Period-to-Period Changes of Weights of Stores

The DEA based formulation of the optimal path of adjustment program considers that the weight of each store may change from period-to-period. With this consideration, the optimisation problem has more degrees of freedom than imposing that the weight of each store is constant for all adjustment periods. This restriction reduces the weights of each store from four to one. In general terms, for a given system, it is reasonable to expect that the optimum improves with increasing the degrees of freedom.[8]

Three cases are identified when comparing the optimal paths of adjustment and the target input quantity vectors, determined assuming period-to-period variable weights of stores, with the optimal paths of adjustment and target input quantity vectors determined by imposing period-to-period constant weights of stores. Each case identifies different degrees of equality of these optimal paths of adjustment and target input quantity vectors.

Equal Case

The *equal case* occurs when the period-to-period input quantity vectors determined by assuming period-to-period variable weights of stores are the same as those determined by imposing period-to-period constant weights of stores. The initial input quantity vector is the same for both situations. Twenty-four stores are found to be in this category. For example, consider Store 202. From Table 6.1, at adjustment period 1, the peers are Stores 236 and 258, with respective weights of 0.01530 and 0.60446; at adjustment periods 2, 3 and 4 the peers are Stores 247 and 258, with respective weights

[8] For any store, the optimal weights at each adjustment period may be equal, although initially they are assumed to be different.

of 0.01835 and 0.60298. When imposing on Store 202 period-to-period constant weights, the resulting peers and their weights correspond to the stores and weights that define the technically feasible input quantity vector that minimises the cost of inputs for the given output. These peers and weights are the same for periods 2, 3 and 4 under period-to-period variable weights of stores, because at those periods the input quantity vector of Store 202 is the target one.

Although the optimal path of adjustment for Store 202 is the same in both weight options, the slacks are different. This means that the period-to-period optimal input quantity vector is on the hyperplane of the target input quantity vector. The input quantity vector exceeds the target in the slacks. Table 6.3 presents the slacks for both weights options.

Table 6.3: Slacks for Constant and Variable Weights of Peers for Store 202

	Sales		Salesperson-hours		Cashier-hours		Marketing	
	D $		khours		khours		D $	
Period	Variable	Constant	Variable	Constant	Variable	Constant	Variable	Constant
1	3.7890	0.0	1.3870	1.3872	0.95946	0.95386	3.1758	3.2972
2	0.0	0.0	1.0298	1.0298	0.0	0.0	0.0	0.0
3	0.0	0.0	0.0	0.00345	0.0	0.0	0.0	0.0
4	0.0	0.0	0.0	0.00345	0.0	0.0	0.0	0.0

In the discussion on Store 258 in Section 6.5, we note that Stores 236, 247, 258 and 266 are single peers of themselves at the four adjustment periods. As single peer of itself, its weight is one. This expected singular behaviour includes these four stores in this equal case.

Partially Equal Case

The *partially equal case* is said to occur when the period-to-period input quantity vectors that are determined by assuming period-to-period variable weights of stores are not equal to those determined by imposing period-to-period constant weights of stores, but the target input quantity vectors are equal. Stores 211, 232 and 256 are in this category. The initial input quantity vector is the same for both situations. Table 6.4 presents the factors that define this second case.

Table 6.4: Different Paths of Adjustment and Same Target Input Quantity Vector

Store		211		232		256	
Weight Status		Variable	Constant	Variable	Constant	Variable	Constant
Objective Function D$		2,732.04	2,732.16	4,842.96	4,924.42	4,596.57	4,605.61
Cost of Inputs D$	Initial	419.92	419.92	1,172.96	1,172.96	987.47	987.47
	Period 1	415.12	415.25	1,080.64	1,129.28	726.43	736.28
	Period 2	412.94	412.94	988.32	1,032.10	630.65	630.64
	Period 3	412.94	412.94	473.73	473.73	630.41	630.41
	Target	412.94	412.94	473.73	473.73	630.41	630.41

From Table 6.4, for practical purposes, Store 211 may be included in the set of firms that are in the equal case.

From Table 6.4, Store 232 is a good example of the partially equal case. The optimal path of adjustment defined with period-to-period variable weights of stores is better than the optimal path defined by imposing period-to-period constant weights of stores. The paths of adjustment differ at periods 1 and 2. The input quantity vectors at periods 3 and 4 are equal to the target input quantity vector. Store 256 behaves similarly, with the difference that it requires only two periods to adjust the input quantity vector to the target one, instead of the three periods for Store 232.

As expected, the immediate conclusion is that the optimal value of the objective function of the period-to-period variable weights of stores is equal to or better than the objective function of the period-to-period constant weights of each store.

Different Case

The *different case* is said to occur when there is no equality between the input quantity vectors that are determined by assuming period-to-period variable weights of stores with those determined by imposing period-to-period constant weights of stores. Of course, the target input quantity vectors are different in this case, although the initial input quantity vectors are the same for both situations.

Table 6.5 presents the results for the eight stores whose paths of adjustment are different if determined assuming period-to-period variable weights of stores or imposing period-to-period variable weights of stores.

For weight status being variable, the cost of inputs at period 4 is the cost of the target input quantity vector.

Table 6.5: Different Paths of Adjustment and Target Input Quantity Vector

Store	Weight Status	Objective Function	Cost of Inputs Period 1	Cost of Inputs Periods 2 & 3	Cost of Inputs Period 4	Cost of Adjustment Period 1	Cost of Adjustment Period 2	Cost of Adjustment Period 3	Cost of Adjustment Period 4
234	Variable	7,124.9	1,060.24	1,051.38	1,051.38	27,51	5.24	0.0	0.0
	Constant	7,170.5	1,065.16	1,058.96	1,061.35	27,51	0.365	0.0	4.44
240	Variable	5,400.8	876.02	736.25	736.25	16.54	10.31	0.0	0.0
	Constant	5,428.9	888.61	739.29	740.26	16.54	8.35	0.0	1.79
242	Variable	3,549.0	533.14	496.77	496.77	15.42	5.65	0.0	0.0
	Constant	3,576.4	556.62	497.84	498.21	15.42	4.82	0.0	0.753
243	Variable	3,918.9	589.14	553.62	553.62	17.07	14.07	0.0	0.0
	Constant	4,056.5	717.94	557.13	558.41	17.07	10.58	0.0	1.90
248	Variable	4,858.3	708.00	702.11	702.11	20.84	4.57	0.0	0.0
	Constant	4,891.3	708.00	708.00	710.27	20.84	0.0	0.0	4.15
260	Variable	5,096.01	763.73	753.56	753.56	24.52	6.07	0.0	0.0
	Constant	5,136.32	790.65	756.34	757.42	24.52	4.15	0.0	1.94
263	Variable	4,868.04	726.20	716.48	716.48	23.11	8.81	0.0	0.0
	Constant	4,916.18	726.20	725.02	728.41	23.11	2.80	0.0	6.00
265	Variable	4,664.52	700.55	688.48	688.48	23.05	10.0	0.0	0.0
	Constant	4,725.10	709.65	697.63	701.50	23.05	2.55	0.0	6.73

From Table 6.5, corresponding to the weight status being constant for each of the eight stores, the period-to-period costs of inputs of the optimal paths of adjustment are no smaller than the corresponding costs of inputs for the optimal paths of adjustment when the weight status is variable.

The immediate conclusion is that, as expected, the optimal value of the objective function for the period-to-period variable weights of stores is equal to or better than the value of the objective function of the period-to-period constant weights of each store.

With four adjustment periods, to behave as an equal case, Store 265 requires 16 or more periods as a time horizon for economic evaluation. If the number of intended adjustment periods is five, Store 265 requires at least 21 periods as the time horizon to behave as an equal case. If the number of intended adjustment periods is six, Store 265 requires at least 21 periods as the time horizon to behave as an equal case. Nonetheless, behaving as an equal case, Store 265 adjusts the input quantity vector to the target one in only two periods. For 11 intended adjustment periods, Store 265 requires at least 22 periods as the time horizon to behave as an equal case.

Imposing period-to-period constant weights to each store may determine that the input quantity vector corresponding to the last adjustment period is not technically efficient. Table 6.6 presents the input quantity vectors and the technical and economic efficiencies for Store 265 with six intended adjustment periods and 20 periods as the time horizon.

From Table 6.6, it is noted that the economic efficiency increases along the optimal path of adjustment and that the technical efficiency may increase or decrease. If five adjustment periods are involved, then the optimal path of adjustment is presented in Table 6.6, from period 0 to period 5.

Table 6.6: Input Quantity Vector, Technical and Economic Efficiencies for Store 265

	Salesperson-hours	Cashier-hours	General Expenses	Marketing	Cost of Inputs	Efficiency	
Period	khours	khours	D $	D $	D $	Technical	Economic
Initial	4.287	2.144	23.304	15.361	810.6	1.0	0.8494
1	6.544	2.144	18.423	15.883	709.6	0.9916	0.9702
2	6.544	1.670	17.953	15.883	697.6	1.0	0.9869
3	6.544	1.670	17.953	15.883	697.6	1.0	0.9869
4	7.588	1.670	17.953	15.883	701.5	0.9851	0.9814
5	7.588	1.670	17.953	15.883	701.5	0.9851	0.9814
6	6.544	1.465	17.953	15.883	697.3	1.00	0.9873

For 11 intended adjustment periods, the optimal input quantity vectors for periods 1, 2, and 3 are the input quantity vectors for these three periods presented in Table 6.6; for periods 4, 5, 6, and 7 is the input quantity vector presented for period 4 in Table 6.6, and for periods 8 to 11 are the input quantity vectors presented for periods 5 and 6 in Table 6.6.

For Store 265, the input quantity vectors at the last adjustment period for five or more intended adjustment periods are the input quantity vectors presented for periods 5 and 6 in Table 6.6. This input quantity vector has an economic efficiency of 0.9873, which is the highest obtainable by imposing period-to-period constant weights to each store.

6. 8 Expected Outputs with Variable Prices and Costs of Adjustment for Store 202

The versatility of the optimal paths of adjustment program allows for extensions. This section presents the optimal path of adjustment for Store 202, considering expected period-to-period variations of output, with variable input prices, and input costs of adjustment. Although the data are expected values, for the purpose of this study, these values are considered as observed, in the sense that they are taken as true values. This extension of the optimal paths of adjustment program considers that stores use the same technology in the four intended adjustment periods.

Experts and senior managers may assign expected period-to-period sales or outputs. Also, based on contracts with suppliers and on economic agreements with labour unions, costs of inputs and costs of adjustment may be determined at each period. For Store 202, Table 6.7 presents expected outputs and estimated costs of inputs and costs of adjustment for the four intended adjustment periods.

Table 6.7: Variable Outputs, Prices and Costs of Adjustment for Store 202

	Period	Initial	1	2	3	4
	Output	1,669.40	1,686.09	1,702.79	1,719.48	1,736.18
Salesperson_hours D$/khours	Price	4.36	4.45	4.45	4.53	4.53
	Cost of Increase	7.12	7.26	7.26	7.40	7.40
	Cost of Decrease	12.63	12.88	12.88	13.14	13.14
Cashier_hours D$/khours	Price	2.07	2.11	2.11	2.15	2.15
	Cost of Increase	2.07	2.11	2.11	2.15	2.15
	Cost of Decrease	3.38	3.45	3.45	3.52	3.52
Sales & Gen. Expen. D$	Cost	10.90	11.23	11.23	11.66	11.66
	Cost of Decrease	1.15	1.18	1.18	1.23	1.23
Marketing D$	Cost	13.46	13.80	13.80	13.80	13.80
	Cost of Decrease	1.48	1.52	1.52	1.52	1.52

Keeping eight periods as the time horizon, four as the number of intended adjustment periods, and budget constraints for periods 1 and 2, Table 6.8 presents the optimal path of adjustment for Store 202. For comparison purposes, Table 6.8 includes constant outputs, constant prices, and constant costs of adjustment of inputs.

Table 6.8: Optimal Path of Adjustment

Constant and Variable Outputs, Prices and Costs of Adjustment for Store 202

	Period	Output D$	Salesperson hours khours	Cashier hours khours	Sales & General Exp., D$	Marketing D$	Cost of Inputs D$	Cost of Adjustment D$
	Initial	1,669.4	5.539	1.79	18.35	13.346	407.5	0.0
Constant output, prices and cost of adjustment	1	1,669.4	5.539	1.79	9.395	11.778	288.79	12.619
	2	1,669.4	5.182	0.8361	9.395	8.480	240.88	12.619
	3	1,669.4	4.152	0.8361	9.395	8.480	236.39	13.006
	4	1,669.4	4.152	0.8361	9.395	8.480	236.39	0.0
Variable output, prices and cost of adjustment	1	1,686.1	5.539	1.79	18.35	13.346	418.56	0.0
	2	1,702.8	5.539	1.79	18.35	13.346	418.56	0.0
	3	1,719.5	4.295	0.885	9.680	8.716	254.53	37.21
	4	1,736.2	4.295	0.885	9.774	8.795	256.71	0.0

From Table 6.8, Store 202 does not perform adjustments in period 1 nor in period 2. The most significant adjustment is done in period 3, with a cost of D$37.21. In period 4, Sales and General Expenses increases D$0.094 and Marketing increases D$0.079; both increments are without cost of adjustment. Because the output and unit prices are different in both the cases presented in Table 6.8, the optimal paths of adjustment are also different.

For practical application of this extension, it is advisable to re-design the optimal path of adjustment each period that new data are available. In this case, the initial period is the period at which the re-design is started when the technical and economic data are available.

6. 9 Optimal Outputs with Variable Prices and Costs of Adjustment for Store 202

Section 6.8 presents the optimal path of adjustment for expected period-to-period output variations. This section presents the optimal path of adjustment for Store 202, considering optimal period-to-period variations of output with variable input prices and input costs of adjustment. This extension of the optimal paths of adjustment program considers that, in the five intended adjustment periods, the stores do not increase nor decrease the current floor surface areas and that there are no changes in technology.

Senior managers may seek to generate the largest technically feasible output for the given non-discretionary variables. Although the system has constant returns to scale, the non-discretionary variables limit the technically feasible increase of outputs, not allowing outputs to be unbounded.[9]

In this case, we consider that any output increase has a specific cost of adjustment. This cost of adjustment refers to the cost of extra goods and the cost of installation of additional services needed to facilitate increases of output. This cost of adjustment does not include the costs of adjustment of corresponding inputs. It is assumed that, for Store

[9] In Section 2.4, we indicate that only the systems with decreasing returns to scale may maximise profit because the rate of increasing the cost of inputs is higher than the rate of increasing the value of inputs.

202, the cost of increasing (adjusting) the output is D$2.0. The period-to-period variable costs of inputs and costs of adjustment of inputs, presented in Table 6.7, are considered in this section.

In this case, the objective is to maximise the present value of net profit from the initial period to the evaluation time horizon. For any period, the net profit is the residual of the corresponding inputs and three costs: the cost of inputs, the cost of adjustment of inputs and the cost of adjustment of output. For the first, second and third adjustment periods, the budget constrains the sum of the cost of adjustment of inputs and the cost of adjustment of output to five per cent of the net initial profit. For the fourth adjustment period, the budget constrains the cost of adjustment of output to be half the difference between the target output and the output at period 4.

Table 6.9 presents the period-to-period outputs and the optimal path of adjustment for Store 202, with twelve periods as the time horizon, five intended adjustment periods, and budget constraints for periods 1, 2, 3 and 4.

From Table 6.9, Store 202 may increase output from actual D$1,669.4 to the target output D$3,145.0. This target is the largest output that may be generated by any Dijon store with 10.90 DSU as the floor surface of the store.

Table 6.9: Optimal Path of Adjustment

Optimal Output, Variable Price and Cost of Adjustment

Store 202		Period					
		Initial	1	2	3	4	5
Output	D$	1,669.4	1,691.6	1,723.2	1,754.7	2,218.1	3,145.0
Salesperson hours	khrs	5.539	5.539	5.539	5.539	5.539	6.799
Cashier hours	khrs	1.790	1.360	1.360	1.360	1.360	1.360
Sales & General Expenses	D$	18.35	9.521	9.701	9.880	12.515	16.345
Marketing	D$	13.346	8.585	8.733	8.882	11.061	22.293
Cost of Inputs	D$	407.51	246.30	250.25	254.20	312.22	510.68
Cost of Adjust Inputs	D$	0.0	18.652	0.00	0.00	0.00	8.970
Cost of Adjust Outputs	D$	0.0	44.442	63.095	63.095	926.83	1,853.7
Gross Profit	D$	1,261.9	1,445.3	1,482.0	1,500.5	1,905.9	2,634.3
Net Profit	D$	1,261.9	1,382.2	1,409.9	1,437.4	979.05	771.6

In period 1, the output is increased from D$1,669.4 to D$1,691.6 with a cost of increasing output of D$44.443, and the cost of inputs decreases from D$407.51 to D$246.30 with a cost of adjustment of D$18.652. The gross profit is D$1,445.3 and the net profit is D$1,382.2.

In period 2, the output is increased from D$1,691.6 to D$1,723.2 with a cost of increasing output of D$63.095, and the cost of inputs increases from D$246.30 to D$250.25 without cost of adjustment. There is no cost of adjustment of inputs because the two inputs that are adjusted are Sales & General Expenses and Marketing; increasing both inputs involve no costs. The gross profit is D$1,482.0 and the net profit is D$1,409.9.

In period 3, the output is increased from D$1,723.2 to D$1,754.7 with a cost of increasing output of D$63.095, and the cost of inputs increases from D$250.25 to D$254.20 without cost of adjustment. The gross profit is D$1,500.5 and the net profit is D$1,437.4

In period 4, the output is increased from D$1,754.7 to D$2,218.1 with a cost of increasing output of D$926.83, and the cost of inputs increases from D$254.20 to D$312.22 without cost of adjustment. The output increase is limited by its cost of increasing. As previously indicated, the budget for this period assigns to this item up to half of the difference of the target output and the output at period 4. The gross profit is D$1,905.9 and the net profit is D$979.05. The net profit decreases with respect to previous periods because the significant output increase has a large cost of increase.

In period 5, the output is increased from D$2,218.1 to the target D$3,145.0 with a cost of increasing the output of D$1,853.7. The cost of inputs increases from D$312.22 to D$510.68 with a cost of adjustment D$8.97. This period has no budget constraint. The gross profit is D$2,634.3 and the net profit is D$771.6. The net profit decreases with respect to previous periods because the significant output increase has a large cost of increase. From this period on, because there are no adjustments, the net profit corresponds to the gross profit D$2,634.3.

Table 6.10 presents the weights of peer stores and the slacks of inputs for the optimal path of adjustment for Store 202, with twelve periods as the time horizon, five intended adjustment periods, and budget constraints for periods 1, 2, 3 and 4.

Table 6.10: Optimal Path of Adjustment
Period-to-Period Weights of Peers and Slacks

	Store 202		Period 1	2	3	4	5
Weight of Peers	Store 247		0.0401	0.0710	0.1019	0.5555	
	Store 258		0.5902	0.5721	0.5540	0.2882	
	Store 236						1.0149
Slacks	Salesperson hrs	khrs	1.340	1.271	1.204	0.2084	0.0
	Cashier hrs	khrs	0.5076	0.4845	0.4614	0.1221	0.0

Stores 247 and 258 are the peers of Store 202 at adjustment periods 1, 2, 3 and 4. At target conditions, Store 236 is the peer, with weight 1.0149. This weight is the ratio of the floor surface of Store 202, at initial conditions, to the floor surface of peer Store 236; this is 10.9/10.74 = 1.0149

Salesperson hours and cashier hours are the only two inputs that present slacks at adjustment periods 1, 2, 3 and 4. Slacks decrease while the inputs and outputs adjust. At target conditions, no input presents slacks.

6.10 Conclusions

The basic DEA model considers that peers may change from one period to the next, because firms that perform adjustments may crossover the firms that are on the boundary of the technology while improving their cost efficiency. Changes include weights of peers, deletion of one or more peers and the incorporation of one or more peers. This is a relevant aspect of this DEA model, because the optimal paths of adjustment determined with period-to-period variable weights of stores are economically better than the optimal paths of adjustment determined with period-to-period constant weights of stores.

The extension of the concept of period-to-period variable weights of stores, developed in this thesis, to period-variable outputs for input-orientated systems and to period-variable technology is trivial. The extension of the concept of period-to-period variable weights of stores to productive systems that involve the concurrence of two or more technologies is also trivial. Färe and Grosskopf (1996, pp. 110-116) use a similar approach "to introduce a vintage nonparametric DEA model that explicitly recognizes that technical change may have its origin in new technology introduced over vintages through investment".

CHAPTER 7

CONCLUSIONS

7.1 Summary

This chapter presents the main conclusions derived from the application of the optimal path of adjustment program developed in Chapter 4 and extended in Chapter 5. The application uses relevant data of 35 comparable retail-level stores, which are branches of the Chilean retailing firm, Dijon.

This section presents a summary of the conclusions derived from the application of the optimal path of adjustment program. Section 7.2 presents opportunities for further research.

For input-orientated systems, the standard DEA model assigns target input quantity vectors that minimise the cost of inputs. The targets are technically and economically fully efficient. Nonetheless, this target assignment has the following three main limitations:

i. The outputs and their prices, the price of inputs and their costs of adjustment, and the production technology are assumed to be constant over time.

ii. The target assignment does not consider the costs of adjustment of inputs nor budget constraints.

iii. The target assignment does not give information on how the adjustments are to be accomplished.

These limitations are removed in the research of this thesis. The main contribution of this research is a generalised dynamic DEA model. Our dynamic DEA model extends the standard DEA methodology, allowing for period-to-period variable output quantities, period-to-period variable input and output prices, period-to-period variable costs of adjustments of inputs, period-to-period budget constraints to the costs of adjustments, period-to-period variable production technology, and the determination of the period-to period optimal input quantity vectors that minimise the sum of the present value of the costs of inputs and the present value of the costs of adjustments of the inputs. The generalised dynamic DEA model reduces to the standard forms, while setting constant the parameters that are time-independent. The extension of the generalised dynamic DEA model to output-orientated systems is trivial. For the empirical application, the extended dynamic DEA methodology is included in the optimal path of adjustment program.

Each firm has its specific optimal input quantity vector that minimises the sum of the present value of the costs of inputs and the present value of the costs of adjustments. This optimal vector is called the target input quantity vector, and depends on the prices of the inputs and the costs of adjustments of inputs. Input prices may or may not be the same for the firms involved. For example, labour costs may differ from firm to firm. Equal prices from firm to firm are not a necessary condition because the minimisation of the cost of inputs is done separately for each firm. The same is true for the cost of adjustments of inputs.

For a Cobb-Douglas production function, the target input quantity vector that minimises the costs of inputs is different from the target input quantity vector that minimises the sum of the present value of the costs of inputs and the present value of the costs of adjustments of inputs.

For DEA, the target input quantity vector that minimises the costs of inputs may be the same target input quantity vector that minimises the sum of the present value of the costs of inputs and the present value of the costs of adjustments of inputs. In Section 5.4, under the subsection, *Comparison of Targets for Cobb-Douglas and DEA,* we analyse the conditions for both vectors to be the same.

In Section 5.4, we stated that, in general terms, the targets determined with a Cobb-Douglas production function change continuously with continuous changes of the parameters, and that the DEA production function determines discrete changes of the target inputs for continuous changes of the parameters. This is an important property of DEA because some managers may feel more confident when pursuing input targets that equally optimise the costs of inputs and the sum of the present value of the costs of inputs and the present value of the costs of adjustments of inputs.

The path of adjustment is the period-to-period adjustment that a firm must perform, for transforming the initial input quantity vector into the target one. The optimal path of adjustment minimises the present value of inputs and the present value of the costs of adjustments. The present value of inputs is evaluated along a predefined time horizon. The present value of costs of adjustments is evaluated along a predefined adjustment time. For the application to 35 stores of Dijon, the time horizon is eight periods and the adjustment time is four periods, where a period is a six-month interval. To determine the optimal path of adjustment requires knowing the costs of adjustments of the firm under optimisation.

With the data from Dijon, the optimal paths of adjustment for the different stores are one-step paths, unless budget constraints are included. The incorporation of budget constraints gives practical use to the optimal path of adjustment program.

The DEA optimal path of adjustment model considers that peers may change from one adjustment period to the next. The reason for considering that peers may change from one adjustment period to the next is that firms performing adjustments crossover the firms that are on the boundary of the technology while improving their economic efficiency. The changes include the weight values of peers, the deletion of one or more peers and the incorporation of one or more peers. This is a relevant aspect of this extended DEA model, because the optimal paths of adjustment determined with period-to-period variable weights of stores are economically better than the optimal paths of adjustment determined assuming period-to-period constant weights of stores. Assuming that, while performing adjustments, the weights of peers are constant implies assuming that the peers are unique and are the peers that define the target input quantity vector.

The dynamic DEA model allows period-to-period variable output. Considering that peers may change from one adjustment period to the next allows the output quantity vector to change from one adjustment period to the next. This is the dynamic output case of input-orientated systems.

The dynamic DEA model allows period-to-period variable output and input prices and input costs of adjustment. Considering that peers may change from one adjustment period to the next allows the input and output prices and the costs of adjustments to change from one adjustment period to the next.

The dynamic DEA model allows period-to-period variable technology. Considering that peers may change from one adjustment period to the next allows the technology to change from one adjustment period to the next. This is the dynamic technology case. This extension configures a basic model to a dynamic DEA model.

Assigning a period-specific weight to each firm allows considering period-specific sets of observed input-output data. This extension includes the concurrence of two or more technologies. In this last case, a specific weight by period and technology is assigned to each store.[1]

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[1] In Section 6.8, we mentioned that Färe and Grosskopf (1996, pp. 110-116) use a similar approach "to introduce a vintage nonparametric DEA model that explicitly recognizes that technical change may have its origin in new technology introduced over vintages through investment".

7.2 Opportunities for Further Research

In this research, it is assumed that the boundary of the technology is valid along the adjustment periods, as defined by the best-performing firms at some specific time. Depending on how long is each adjustment period, more adjustment periods than the initially assigned may take place until the static technological boundary is reached.

If changes in technology may be foreseen, it may be preferred to consider some expected period-to-period variation of technology, instead of constant technology. To deal with *expected* period-to-period data instead of observed period-to-period data requires the extension of DEA fundamentals and may demand the generation of new concepts.

In this research, it is assumed that, once an input quantity vector is established, the store immediately performs as expected. This assumption means that there is no learning time; the firm does not spend a behavioural-adjustment time. The estimation of the transient behaviour of firms to changes of inputs may be an interesting field of research.

In this research, the future prices and costs of adjustments of inputs are handled as observed instead of estimated values. The incorporation of forecast concepts to prices and the evaluation of the risk of the investments involved in the adjustments may lead to more realistic applications for medium- and long-time horizons for business planning.

In this research, the optimal path of adjustment is determined for known or for given outputs. The presence of non-discretionary variables allows the consideration of the case of simultaneous change of outputs and inputs, as presented in Section 6.8. Change of technology and considerations relative to the cost of increasing the market share may be considered as opportunities for further research.

In general terms, these opportunities for further research may need a deeper insight of firm theory and firm economics.