

Chapter 6 :

Algorism, proportionality and the notation of the ars subtilior: Some observations on the dating of the works by Baude Cordier

In her monograph *Mensuration and Proportions Signs*, Anna Maria Busse Berger suggests that the advent of proportional signs using Indo-Arabic fractions in European musical notation was precipitated by the problem of ambiguity in notational devices consisting of novel coloration, note shapes and mensural signs as found especially in works in the late fourteenth century *ars subtilior* style.¹ While I, for the most part, agree with Busse Berger's assessment, there is a need to revise her statements concerning the broader context of the cultural movement towards a popular arithmetic, which includes proportionality. Busse Berger's assessment² that the presence of Indo-Arabic numerals is indicative of newly cultivated proportional concepts in music of the late fifteenth century does not accord with the intrinsic nature of musical notation of the *ars subtilior* in the late fourteenth century before the prevalence of those numerals. Indeed, it appears to conflate two essentially separate issues – the use of proportions and the role of Indo-Arabic numerals in musical notation. It is my thesis that the musical notation of the *ars subtilior* demonstrates a profound interest in proportionality at a level that already demonstrates the presence of new proportional mentality. This mentality was in turn instrumental in the adoption of Indo-Arabic numerals into music notation by a simple process of association, thereby marking the end of a long period of intense notational evolution seen in the fourteenth century as a whole.

This chapter's investigation of the rise of new proportionality in the music and notation of the *ars subtilior* is divided into two parts. In the first part, I outline the general history of proportions and Indo-Arabic arithmetic in the middle ages. In particular, I examine the relationship of Indo-Arabic numerals, when they first appear in musical notation in extant manuscripts, with proportional concepts. In this first part, I also discuss examples from the *ars subtilior* that demonstrate the presence of a new proportionality including those works that do not expressly use the new numerals. The second part of this

¹ Busse Berger, *Mensuration and Proportion Signs*, pp. 178-182.

² Busse Berger, *Mensuration and Proportion Signs*, p. 209.

chapter examines the notation of works ascribed to the composer Baude Cordier. By comparing the works of Baude Cordier, which demonstrate a fully developed system of proportionality relying on Indo-Arabic numerals, with the works of composers such as Jacob de Senleches and Johannes Suzoy, this chapter seeks to redress discrepancies in the currently received version of Cordier's biography which stands diametrically opposed to the history of arithmetic mentality and is at odds with the evidence of notational development provided by the sources of the *ars subtilior*.

6.1. *The rise of algorism in European culture*

According to Fabrizio della Seta, the rediscovery of the Euclid's *Elements* and its translation³ into Latin in the thirteenth century by Campanus de Novaria introduced the potential for a new concept of proportionality to medieval Latin culture.⁴ This potential lay in the definition of proportions in the *Elements* V, 3:

*Proportio est habitudo duarum quantaecunque sint eiusdem generis quantum, certa alterius ad alteram habitudo.*⁵

As will be seen below, the statement that proportions are descriptions of the relationship between two quantities of the same type has particular resonance in the early theory of proportions in music which stated that proportions were only ever reckoned in relation to the *minima*. Euclidean proportionality underwent several modifications during the fourteenth century when it was chiefly associated with continuous quantities by natural philosophers. The Merton Calculator, Thomas Bradwardine (c. 1290-1349), in particular, was instrumental in his synthesis of Boethian-Pythagorean proportionality into an expanded Euclidean system based on the distinction between discrete and continuous quantities respectively. Moreover, his use of the terminology “proportions of proportions” to describe geometric or proportional acceleration (or proportional differentia of velocity) influenced subsequent scholars in their considerations of continua composed of irrational, that is non-discrete or infinite, quantities. The concept of Bradwardinean proportions in

³ Adelard of Bath also translated some of the *Elements* in the 12th century, *vid.* H.L.L. Busard (ed.), *The First Latin Translation of Euclid's Elements commonly ascribed to Adelard of Bath : Books I-VIII and Books X.36-XV.2*, Toronto, 1983.

⁴ In this brief survey of proportionality, I have summarised the excellent assessment of its development by Fabrizio della Seta in his ‘Proportio: Vicende di un concetto tra scolastica e umanesimo’, in *In cantu et in sermone: A Nino Pirrotta nel suo 80° compleanno*, eds F. della Seta and F. Piperno, *Italian Medieval and Renaissance Studies* 2, Firenze, 1992, pp. 75-99.

⁵ “Proportion is the relationship of two quantities that are of the same what-so-ever sized type, a determined relationship of one to the other.”

relation to irrational quantities was further developed by Nicholas Oresme (c.1320-1382), especially in his considerations of proportional acceleration in his *De configurationibus qualitatum et motuum*,⁶ although his first treatment of Bradwardine's theorem occurs in the earlier *De proportionibus proportionum*. Further dissemination of these concepts is evidenced in the writings of Biagio da Parma, Prosdocimus de Beldemandis and Ugolino d'Orvieto.

Bradwardinean proportions distinguished themselves from Pythagorean-Boethian proportions. Pythagorean-Boethian proportions described the relation of discrete parts to a whole. Bradwardinean proportions described the relation of parts to each other. Bradwardinean proportions relied on multiplicative arithmetic, while Pythagorean-Boethian proportions were of a purely additive nature. A fundamental element of Bradwardinean multiplicative proportionality, was algorism. In his development of Bradwardine's theorems, Nicholas Oresme's *De proportionibus proportionum* explicitly states the relationship of the new proportionality with algorism:

*Si autem volueris per artem proportionum maioris inequalitatis alteri addere tunc oportet denominationem unius per denominationem alterius multiplicare. Et si volueris unam ab altera subtrahere hoc facies denominationem unius per denominationem alterius dividendo. Denominationum inventio postea docebit; quarum multiplicatio atque divisio habetur per algorismum.*⁷

While I will not concern myself here with greater inequalities, the last part of the previous passage leaves little doubt that the processes of algorism were associated with the multiplication and division of proportions.

An assessment of the reception of Indo-Arabic numerals into medieval Europe is problematic if one looks beyond generalities and argues that there exist distinct periods wherein Indo-Arabic numerals were adopted into specific parts of its culture. In his fascinating reading of medieval culture, *Reason and Society in the Middle Ages*, Alexander Murray writes concerning the history of the adoption of Indo-Arabic numerals in the middle ages:

The new numerals were available, complete with instructions to any educated persons who wanted them by 1200. It was only c. 1400 that they began an

⁶ Cf. Tanay, *Noting Music, Marking Culture*, pp. 239-244.

⁷ "If however you wish to add a greater inequality to another through the art of proportions, then it is appropriate to multiply one *denominatio* by another. And if you wish to subtract one from the other, do this by dividing one *denominatio* by the other. The finding of the *denominatio* will be taught below; their multiplication and division is gained by algorism."; Edward Grant, (ed.), *Nicole Oresme: De proportionibus proportionum and Ad pauca respicientes*, Madison and London, pp. 142-144. According to Grant, *op. cit.*, p. 12, the *De proportionibus proportionum* must have been written after 1361 in light of the dedication in Oresme's *Algorismus proportionum* to Philippe de Vitry, then Bishop of Meaux (1351-61).

effective conquest of all literate culture. This delay is our opportunity. The pattern of the numerals' adoption will reflect, not any foreign technological bombardment, but native aspirations and pressures.⁸

Murray's statement, admittedly, is only an approximation. Yet, his statement rings true in the context of extant musical sources where the period around 1400 does see the introduction of these numerals into musical notation. More importantly, I would like to pursue Murray's suggestion that an examination of the "pattern of the numeral's adoption", which I read as their adoption into musical notation, presents scholarship (*sc.* musicology) with the opportunity to discern those same "native aspirations and pressures" in its own unique, but not exclusive, context.

Algorism in the middle ages can be seen as a set of calculative processes employing Indo-Arabic numerals. This system was introduced into Europe sometime before 1143 through Latin translation(s) of the *Treatise on Calculation with Hindu Numerals* by the Arabic mathematician and astronomer Muhummad ibn Musa al-Khwarizimi.⁹ This treatise was frequently referred to by the Latin transliteration of the author's name as the *Algorismus*.¹⁰ Treatises by European authors based on the *De numero indorum* proliferated during the thirteenth century with notable works such as the *Carmen de algorismo* (c. 1225) by Alexander de Villa Dei and the *Algorismus vulgaris* (c. 1240) by John of Holywood or Halifax (otherwise known as Sacrobosco) becoming staples in the liberal arts programs of many universities.¹¹ Petrus Philomenus' enhancement of Sacrobosco's treatise in 1291 continued to be used in the university curriculum up to the sixteenth century.¹²

The calculative processes sent down in these treatises involve those familiar to persons today with basic numeracy. The *Carmen de Algorismo*, after setting out the forms of the Indo-Arabic numerals including the zero, which is strictly called the *cifra* (from the Arabic *al-cifr* = 'the vacant one'), and stressing the importance of reading numbers from

⁸ Oxford, 1978, pp. 167-8.

⁹ Some historians of mathematics attribute the introduction of Indo-Arabic numerals to Gerbert d'Aurillac (945-1003), later (999) Pope Sylvester II, although a sign for zero was not present and Gerbert "did not understand the full significance of the Indo-Arabic system.", *vid.* Victor J. Katz, *A History of Mathematics: An introduction*, 2nd edn, Reading (Massachusetts), 1998, pp. 289-90. The first translation of the *Algorismus* into Latin appears to have been made by Adelard of Bath (fl. 1116-1142).

¹⁰ *vid.* Michael S. Mahoney, 'Mathematics,' in *The Dictionary of the Middle Ages*, ed. J. R. Strayer, New York, 1987, vol. 8, pp. 208ff.

¹¹ Mahoney, *op.cit.*, p. 209. An edition of the *Carmen de Algorismo* may be found in Robert Steele, ed., *The Earliest Arithmetics in English*, London, 1922, pp. 72-80. Steele's edition also includes a middle English gloss on the *Carmen de Algorismo* and similarly a middle English translation of John of Holyword's *Algorismus* (or *De Arte Numerandi*).

¹² Mahoney, *op.cit.*, 209.

right to left (a concept presumably alien to the occidental medieval *litteratus*), proceeds to describe in order addition, subtraction, duplication, mediation or halving (including the notation of a half remainder in the case of uneven numbers), multiplication (including mental), division, square root and the cube root. The elements of algorism eventually contributing to its success lies in its conciseness of figures, its almost limitless rational bounds, its propensity for mental calculation without tools, and the establishment of central principles of calculation which could be expanded to other systems such as fractions.

Simultaneous with the advent of works concerning integer calculations, a series of new treatises on the manipulation of fractions, beginning with the *Tractatus minutiarum* and *Demonstratio de minutiis* of Jordanus Nemorarius (fl. 1220),¹³ demonstrated the inventiveness of European scholars by their adaptation of the algorithmic processes for new numeric systems. Perhaps the most widely read work on fractions was Jean de Lignères' *Algorismus de minutiis* (c. 1340), written specifically for astronomical applications, by which the familiar form of numerator/denominator was made popular.¹⁴ This last development was to have the most profound influence on music. While the principles of fractions in the form of Boethian-Pythagorean ratios had been a familiar aspect of medieval mathematics,¹⁵ the Indo-Arabic system played an important role in the expansion of arithmetic thought both in terms of conceptualisation and its influence beyond the university and cloister walls.

The acceptance of the new arithmetic into universities was a gradual one, although some progress is witnessed in the fourteenth century.¹⁶ At Paris, one finds authors such as the aforementioned Petrus Philolomenus, Jean de Lignères, Johannes de Muris and Nicholas Oresme. At Oxford, the Calculators such as Richard Swineshead, William Heytesbury, John Dumbleton, and the *Doctor profundus* Thomas Bradwardine numbered among mathematical writers who detailed or utilised the new arithmetic in their tracts. Clearly, the lofty work on kinematics undertaken by the Oxford Calculators and Oresme was not for

¹³ Mahoney, *loc.cit.*

¹⁴ Concerning this treatise's wide readership, *vid.* Mahoney, *op.cit.*, p. 209. An edition of this work may be found in H. Busard, ed., *Het Rekenen met breuken in de Middeleeuwen, in het bijzonder bij Johannes de Lineriis*, Mededelingen van de koninklijke Vlaamse Academie voor Wetenschappen, Letteren en schone Kunsten van België 30, Brussels, 1968, pp. 21-36. The use of the horizontal bar in fractions appears to have been invented by Arab mathematicians, from whom Europeans adopted the practice, *vid.* David E. Smith, *History of Mathematics*, New York, 1958, vol. 2, pp. 215-216.

¹⁵ By ratio, I am not referring to the notational system of x:y, invented in the seventeenth century, but rather verbal descriptions (e.g. *quatuor ad tres*) or terms (e.g. *sesquitercia*).

¹⁶ The present assessment draws on Smith, *op.cit.*, vol. 1, pp. 230-240.

general consumption. A better indication of the mathematical diet at Oxford is given in the statutes of 1408 which specifies the bachelor's curriculum should include some knowledge of *algorismus integrorum* (integer arithmetic) and *computus ecclesiasticus* (calculation of the ecclesiastic calendar).¹⁷ In Italy, Andalò di Negro, Paolo Dagomari and Master Biagio of Parma wrote on arithmetic. Dagomari's *Trattato d'Abbaco*, which contained some commercial arithmetic, also hints at the teachings of the mid-century Italian *maesti d'abbaci*, such as Maestro Dardi di Pisa and Antonio de' Mazzinghi, particularly in schools oriented towards the education of the rising mercantile class, and their promotion of the Indo-Arabic system, algorism and algebra,¹⁸ despite some resistance to change.¹⁹

6.2. Algorism in theory and practice of mensural music

The preliminary survey of the use of Indo-Arabic numerals in mensural music treatises by Christopher Page reveals the adoption of the symbol set by music theorists from c. 1300 onwards.²⁰ Even Jacobus de Liège, who was an opponent to the new notational system of the *ars nova*, not only employs Indo-Arabic numerals in his *Speculum musicae* but also cites directly from a treatise on algorism by Jordanus Nemorarius.²¹ However, it is plain that algorism was not the only form of mathematics influencing the mensural music of the *ars nova*. Influence of Euclidean geometry, specifically algebra, is perhaps best exemplified by the case of the renowned Jewish astronomer and mathematician Leo Hebraeus (alias Gersonides) whom Philippe de Vitry, a foremost advocate and practitioner of the *ars nova*, asked to establish a proof that the *ars nova* system of divisions would yield discrete results

¹⁷ Smith, p. 245. On the importance of a knowledge of algorism in *computus* *vid.* Christopher Page, *Discarding Images : Reflections on Music and Culture in Medieval France*, Oxford, 1993, pp. 130-1.

¹⁸ V. J. Katz, *op. cit.*, p. 343.

¹⁹ Commerce (particularly that of Italy) and government were closely associated with the new numeracy, despite certain examples of conservatism and lack of preparedness to adopt Indo-Arabic numerals, *vid.* Murray, *op.cit.*, 188-203. The examples of vernacular glosses and translations of treatises on algorism found in Steele, *op.cit.*, may attest to the widening popularity of algorism in fourteenth- and fifteenth-century England. For a more recent discussion on this increasing awareness during the fourteenth century, *vid.* Paul Acker, 'The emergence of an arithmetic mentality in Middle English literature,' *The Chaucer Review*, 28, 1993, pp. 293-302. Similar evidence is found on the continent with the thirteenth century, Old French treatise by Philippe de Thaon which discusses the skill of *computus* in light of Indo-Arabic numerals and the importance of such knowledge for clerics, *vid.* Christopher Page, *op.cit.*, pp. 130-1. For another view on the influence of arithmetic thought on music, *vid.* Busse Berger, *Mensuration and Proportion Signs*, pp. 198-210 & *eadem*, 'Musical proportions and arithmetic in the late Middle Ages and the Renaissance,' *Musica Disciplina*, 44, 1990, pp. 89-118.

²⁰ Page, *op.cit.*, pp. 131-136.

²¹ Page, *op.cit.*, p. 136. Page also illustrates the dichotomy which exists in the music treatise *De musica* (c. 1300) by Johannes de Grocheio, a contemporary of Jacques de Liège, in that its author employs Roman numerals in his discussion of Boethian-Pythagorean arithmetic proportions and Indo-Arabic numerals in the sections on mensural notation.

without the possibility of the various divisions of the same quantity of smallest units.²² Beyond mathematical influences, Dorit Tanay has proposed that *ars nova* and *ars subtilior* music theory and the music itself is permeated by medieval logic not only through the adoption of its terminology and dialectic processes but more specifically in its reflection of philosophic sophistries also found in contemporary writings on natural philosophy by the Oxford Calculators, William of Ockham and Nicholas Oresme.²³

It is therefore not surprising to discern the influence of elements of the other six liberal arts upon music, especially when one considers what is known about the chief practitioners and theorists of the music of the fourteenth century. Among the generation of composers working in the *ars subtilior* style, there are several whose title or biographical data indicates that they belonged to a holy order,²⁴ a situation that, for the most part, would necessitate university training.²⁵ With an understanding of the education that these roles often required in the middle ages, one might safely assume that many composers and musicians were imbued to varying degrees with the diverse concepts embodied in the liberal arts. Notable musical theorists Johannes de Muris and Prosdocimus de Beldemandis are also known for their discussions on arithmetic, geometry and astronomy.²⁶ The learned, early

²² Eric Werner, 'The mathematical foundation of Philippe de Vitry's *Ars Nova*,' *Journal of the American Musicological Society*, 9, 1956, pp. 128-32.

²³ Tanay, *Noting Music, Marking Culture; q.v. eadem*, "Nos faysons contre Nature...": Fourteenth-century *Sophismata* and the musical Avant Garde', *Journal of the History of Ideas*, 59, 1998, pp. 29-51.

²⁴ Some examples include: Anthonellus 'Marot' de Caserta appears to have been an abbot, if the testimony of the fragment PAas 75 is accepted and if the composer of the Italian texted works is identical to the individual who composed the French-texted works in the *ars subtilior* style attributed to Anthonellus de Caserta. Master Egidius, composer of *Franchois sunt nobles* (MOe5.24, f. 11r) and *Courtois et sages* (MOe5.24, f. 35r; Pn 6771, f. 54r) was a member of the Augustinian order, as was Corradus de Pistoria (*ordo heremitarum*) and possibly Johannes de Janua (=Genoa) who are both referred to as *frater* ('Brother'). Bartholomeus de Bononia appears to have been a Benedictine. Antonius Zacharias is a notable exception by his being a married layman, although recent advances in his biography reveal that he was employed as a *scriptor* in the Roman court and member of the Bolognese papal chapel, *vid.* Chapter 3, pp. 137-140. Unfortunately, little is known of the leading composers Jacob de Senleches and Philipoctus de Caserta which might indicate their education, although both (the former indirectly) may have been associated with the Court of Giangaleazzo Visconti at Pavia, *vid.* Strohm, 'Filippotto da Caserta, ovvero i francesi in Lombardia', pp. 65-74. Several composers are referred to as *Magister* ('Master'), which may indicate that they had taken their university degree, although it was also a title accorded to respected persons.

²⁵ The successful study of theology, canon law, the *trivium* (grammar, rhetoric and dialectic) and plainchant were the usual requirements for those seeking to bear sacerdotal office, *vid.* Murray, *op.cit.*, p. 226. While the situation is complicated by the crisis in literacy c. 1400 (*ibid.*, 309ff), the works of the composers themselves act as testimony to their technical competence at a literary and musical level.

²⁶ Johannes de Muris is known to have written two treatises on music including his famous *Ars nove musicæ*, four treatises on arithmetic and two treatises on astronomy; *vid.* Lawrence Gushee, 'New sources for the biography of Johannes de Muris,' *Journal of the American Musicological Society*, 22, 1969, pp. 6-8. An anonymous fifteenth century copy of a music treatise also suggests De Muris may have also composed music (Page, *Discarding Images*, p. 113). Prosdocimus de Beldemandis wrote eight treatises on music, including one on speculative music, two treatises on arithmetic including a treatise on integer algorism, nine treatises on

fifteenth century theorist Ugolino of Orvieto was also a composer and a chapel singer for the Roman pope Gregory XII (r. 1406-1415).²⁷

The period encompassing the practice of the *ars subtilior* style (c. 1380-c. 1415) saw the appearance of Indo-Arabic numerals in musical sources. It will be recalled from previous chapters that, prior to this stage in the development of notation, the second decade of the fourteenth century had seen the expansion of notation by the addition of the *minima*. Further innovations witnessed on the two vying, quasi-national fronts include the use of the *quatre prolacions* in French notation and the use of the *divisiones* in Italian notation. In the previous chapter, I described the introduction and various uses of mensuration signs in the French and Italian notational systems. There, I concluded that the advent of this extrinsic mode of signification occurred in practice relatively late and primarily as a response to the demands of the *ars subtilior* style itself.

Close on the heels of mensuration signs being used in practice to indicate more complex relationships in music, Indo-Arabic numerals also began to appear in musical notation. Three regular²⁸ uses of these numerals in musical sources can be identified: i) the use of Indo-Arabic fractions to indicate mensuration, that is, as a substitute for mensural signs; ii) the use of single numerals (2, 3 and 4) with or without a mensuration sign to denote a simple proportion (sometimes in conjunction with a verbal canon); and iii) the use of complex proportions written as fractions (which is closely related to the second practice).

astronomy and one treatise on geometry; *vid.* Jan Herlinger (ed.), *Prosdocimo de' Beldemando: Contrapunctus*, Lincoln and London, 1984, pp. 2-4. While Johannes de Muris and Prosdocimus are extraordinary exponents of the *quadrivium*, it remains uncertain whether such a broad experience would be available to students of theology. However, a rudimentary knowledge of arithmetic was required for the *computus* (calculation of moveable events in the liturgical year) which during the thirteenth century had begun to use algorism, *vid.* Page, *op.cit.*, pp. 124-131.

²⁷ Di Bacco and Nádas, 'The papal chapels and Italian sources of polyphony during the Great Schism', p. 50, fn. 17; Ugolino is documented as a singer in the chapel of Gregory XII in May 1413. In 1417 it appears Ugolino was employed as one of two *cantores* at the Florentine cathedral of Santa Reparata (=Santa Maria del Fiore), *vid.* D'Accone, 'Music and musicians at Santa Maria del Fiore in the early Quattrocento', pp. 106 & 120. *Q.v.* Albert Seay, 'Ugolino of Orvieto: theorist and composer,' *Musica Disciplina*, 9, 1955, pp. 111-66.

²⁸ Some irregular uses of Indo-Arabic numerals, which nonetheless reflect the tightening grip of Indo-Arabic numerals upon musical thought in the early fifteenth century can also be observed in MOe5.24: the numerals 8 and 3 in the *tenor faciens contratenorem* in the *Gloria: spiritus et alme* on ff. 2v-3r indicate the number of *pause breves* before the derived *contratenor* enters after the *tenor*. The numerals 1-5 are found in the two uppermost voices of the next *Gloria* in MOe5.24 (ff. 4v-4r) and are used to enumerate successive *talee*. Transnotations and critical notes detailing these processes may be found in von Fischer and Gallo, *Italian Sacred and Ceremonial music*, #16 & 17 (transnotations), p. 271 (notes). Christopher Page provides a later fifteenth century example, wherein Indo-Arabic numerals are used to indicate the number of *minime* in each duration over which each is written, found in the MS Cambridge, Trinity College, R.14.26, f. 37r in *op.cit.*, p. 118.

The first practice will be described briefly before proceeding to the second and third uses of Indo-Arabic numerals used as indicators of proportionality.

The use of Indo-Arabic fractions as substitute mensuration signs is found in only two extant works from this period: Anthonellus de Caserta's *Dame d'onour en qui tout mon cuer maynt* (MOe5.24, f. 40v; *vid.* Vol. II, App. A, No. 61), which employs $\frac{3}{2}$, $\frac{2}{2}$ and $\frac{2}{3}$, and Goscalch's *En nul estat* (CH 564, f. 39v; *vid.* Vol. II, App. A, No. 5), which employs $\frac{3}{3}$, $\frac{3}{2}$, $\frac{2}{2}$ and $\frac{2}{3}$. The earliest theoretical explanation of how these signs should be interpreted is found in the *Tractatus Secundus* of the Berkeley Manuscript dated *circa* 1375:

*Item solent poni cifre numeri ternarii et binarii, una supra aliam directe. Inferior designat tempus, superior vero prolacionem.*²⁹

These signs do not function as proportion signs but as alternative mensuration signs with *minima* equivalence between one another except in the case of $\frac{2}{2}$ in *Dame d'onour en qui*. In this work alone, instead of indicating *tempus imperfectum minoris* with *minima* equivalence,³⁰ this sign indicates *tempus imperfectum minoris diminutum* at a *sesquitercia* proportion in relation to the regular *minima*. Based on the rareness of this device, and an ascription in one source, some scholars have suggested that the Berkeley treatise was written, and these signs invented, by Goscalch himself.³¹

The use of single numerals in the course of a composition to indicate proportional relationships is first found in works contained in the two principal sources of the *ars subtilior*, namely CH 564 and MOe5.24. Table 6.1 shows list from these manuscripts in addition to works of a similar style found in other sources which employ the numerals 2, 3 and 4 to indicate proportional relationships.³²

²⁹ 'Likewise ciphers of ternary and binary numbers are wont to be placed one directly over the other. The lower <number> indicates the tempus, the upper indeed the prolacion'; Ellsworth, *The Berkeley Manuscript, University of California Music Library, MS. 744*, p. 170. In Busse Berger, 'The origin and early history of proportion signs', p. 413, references to this and later authors discussing the signs may also be found.

³⁰ The use of these signs in *En nul estat* is complicated by the alternation of diminution between voices and sections. It may be purely coincidental that this sign ($\frac{2}{2}$) always occurs in passages whose note values must be diminished.

³¹ Ellsworth, *The Berkeley Manuscript*, pp. 13-15.

³² This use of numerals has no connection to the *modus cum tempore* signs found in sources compiled after c. 1440. On this device, *vid.* Busse Berger, *Mensuration and Proportion Signs*, pp. 149-163.

Table 6.1: Use of simple numerals to indicate proportions in notation of *ars subtilior*.³³

Work and Composer	Source(s)	<i>sesquialtera</i>	<i>sesquitercia</i>	other	Comments
<i>Amans, ames secretement</i> Baude Cordier	Ob 213, 123r.			2 = pr. dupla on [3,3]. 3 = pr. tripla on [3,2] and [3,2].	Other mensuration signs and proportions also employed. Numerals are cumulative but cancelled by mensuration signs.
<i>Angolorum psalat</i> Rodericus	CH 564, f. 48v.			2 = pr. subdupla (augmented) on [2,3]. 3 = pr. subtripla augmented on [2,3].	Proportional signs refer to <i>integer valor</i> only (non-cumulative).
<i>Belle, bonne, sage</i> Baude Cordier	CH 564, 11v			3 = pr. tripla on [2,3].	Other mensuration signs and proportions also employed. Numerals are cumulative but cancelled by mensuration signs.
<i>Credo</i> Anon.	Tn TIII2, 17v-18r.			2 = pr. dupla on [2,3]	Numerals cancelled by mensuration signs. T and Ct diminished on third iteration according to canon.
<i>Lorques Arthus</i> Jo. Cuvelier	CH 564, 40v	3 = pr. 3:2 on [2,3].	2 = pr 4:3 on [2,3]	4 = pr. dupla on [2,3].	Canon explains signs. Proportional signs refer to <i>integer valor</i> only (non-cumulative).
<i>Ma douce amour</i> Jo. Simonis de Haspre	MOe5.24, 28r; CH564, 34r; Ob 213, 123r.	3 = pr. 3:2 on [3,2].		2 = pr. dupla on [3,2]. 4 = pr. tripla on [3,2].	Canon explains signs. Proportional signs refer to <i>integer valor</i> only (non-cumulative).
<i>Medee fu en amer</i> Anon.	CH 564, 24v; Ob 213, 116v117r; Fn 26, 105v-106r.	3 = pr. 3:2 on [2,3] = [3,3] dim.	2 = pr. 4:3 on [2,3] = [2,2] dupla.	4 = pr. dupla on [2,3].	Canon explains signs. Proportional signs refer to <i>integer valor</i> only (non-cumulative).
<i>Ne Geneive</i> Jo. Cuvelier	CH 564, 41v.			2 = pr. dupla on [2,2]. 3 = tripla on [2,2]	Canon explains signs. Proportional signs refer to <i>integer valor</i> only (non-cumulative).

³³ Key to addition abbreviations and terms used in Table 6.1: [II, 2]= *modus imperfectus tempus imperfectum*, [III,3]=*modus perfectus tempus perfectum*, *dupla* = 2:1, *sesquialtera* = 3:2, *sesquitercia* = 4:3. The terms cumulative and non-cumulative are explained below.

<i>Plorans ploravi</i> Zacharias	Tn T.III.2, 1r (frag.); Las 184, LXIv - LXIir.		2 + void notes = 4:3 on [2,3] = [2,2] dupla.		Notated in void red in Las 184. (non- cumulative?)
<i>Que pena maior</i> Bartholomeus de Bolonia	MOe5.24, 37r.	3 = t. 3:2 on [3,2].		2 = pr. dupla on [3,2].	Canon explains signs. 3 operates at the Sbr level. Proportional signs refer to <i>integer valor</i> only (non- cumulative).
<i>S'aincy estoit</i> Solage	CH 564, f. 36r.			2 = [II,2]; 3 = [3,3].	If durations are diminished, numerals refer to <i>modus</i> ; if normal, numerals refer to <i>tempus</i> .
<i>Se doulz espour</i> Corradus de Pistoia	MOe5.24, 31v	3 = t. 3:2 on [2,2] = [3,2] dim.		2 = t. 2:3 on [3,2] dim; cancels 3 = [2,2]	Numerals are cumulative?
<i>Sus une fontayne</i> Johannes Ciconia	Ob 229, 56v; MOe5.24, 27r;		3 = pr. 4:3		3 only occurs in Ob 229; \circ used in MOe5.24. Proportional signs refer to <i>integer valor</i> only (non- cumulative).

The following observations can be drawn from the data in Table 6.1:

1. The most common meaning of the numeral 2 in works either with or without an explanatory canon is duple proportion with respect to the prolation (or at the *minima* level). Furthermore, in cases where 2 is used in conjunction with a canon or coloration (such as in *Lorques Arthus*, *Medee fu* and *Plorans ploravi*) and in the mensuration *tempus imperfectum prolationis maioris*, the *tempus* is diminished by a half with the result of a *sesquitercia* proportion at the *minima* level.
2. The use of the numeral 3 with or without a canon is by no means consistent, although three works without a canon use 3 to indicate triple proportion. Corradus de Pistoria's *Se doulz espour* (Vol. II, App. A, No. 62) and the Ob 229 version of Ciconia's *Sus une fontayne* (Vol. II, App. A, No. 30) use the numeral 3 to indicate two very different effects of *sesquialtera* at the *semibrevis* level (explained, however, by a canon in Bartholomeus de Bononia's *Que pena maior*) and *sesquitercia* at the *minima* level respectively. The work by Ciconia also occurs in MOe5.24, but here it employs the mensuration sign \circ to indicate *sesquitercia* according to widespread conventions observable in principal sources and theorists.
3. The numeral 4 is only ever used in conjunction with a canon in three works. The similarity of the use of 2, 3 and 4 in Johannes Cuvelier's *Lorques Arthus* (Vol. II, App. A, No. 63) and the anonymous *Medee fu* (Vol. II, App. A, No. 10) has prompted some

commentators to propose that both works are by the same composer.³⁴ This use of 4 in these two works, unlike Jehan Hasprois' *Ma douce amour* (Vol. 2, App. A, No. 65), can be seen to demonstrate rational relationships since, within a *tempus*, four *semibreves* are now sung in the space of two. This relationship, as will be seen, reappears in a work possibly composed at a later date and certainly in a different part of Europe, which instead demonstrates an early stage of fractional proportion signs: Anthonellus' de Caserta's *Amour m'a le cuer mis*.

It can be concluded from the previous observations that single numerals demonstrate proportional relationships, frequently causing a re-division of the prolation (*minima* level), which are often logical within a mensural context. However, some works remain cryptic in their use of Indo-Arabic numerals. The Ob 229 version of *Sus une fontayne* appears to have employed the numeral 3 solely for its unusual nature or perhaps in association with proportional nomenclature (*sesquitercia*). Solage's *S'aincy estoit* (Vol. II, App. A, No. 65) also represents an unusual practice where the numerals 3 and 2 appear to indicate *modus* in passages in augmented notation and *tempus* in normal notation.³⁵ However, an indication of the rudimentary state of the use of numerals in these sources is testified to by the frequency of canons being used to clarify the desired intent. While these lone numerals may alone be considered indications of multiple or superparticular proportions,³⁶ in several of the works in Table 6.1 they appear to be qualitative signifiers whose meaning depends on verbal instructions.³⁷

The earliest, surviving theoretical definition of the third use of Indo-Arabic numerals in the form of fractions indicating proportions occurs in 1404 in the *Expositiones* of Prosdocimus de Beldemandis.³⁸ In arguing that the notational device of void red *minime*

³⁴ Günther, 'Die Anwendung der Diminution in der Handschrift Chantilly 1047,' p. 19.

³⁵ Willi Apel maintained that the numerals in this work referred to *tempus*, Apel, *French Secular Music of the Late Fourteenth Century*, p. 9a. This, however, appears unlikely as the numerals would be incorrect in 2 out of 6 occurrences, whereas my solution only encounters difficulty in the refrain section of the Tenor voice where the *tempus* is ambiguous. On the structural use of augmented notation in this work *vid.* Günther, 'Der Gebrauch des Tempus perfectum diminutum in der Handschrift Chantilly', p. 280.

³⁶ The meaning of the names of various species of proportions used in this article, which are derived from medieval nomenclature, may be defined for a ratio of $x:y$, both of which are positive integers and $x \geq y$, and a is a positive integer, as such: a simple (*simplex*) proportion occurs when $x=y$ (e.g. 2:2), a multiple proportion when $x=ay$ (e.g. 3:1), superparticular when $x-y=1$ (e.g. 4:3), superpartient when $y > x-y > 1$ (e.g. 5:3), multiple superparticular when $x-ay=1$ (e.g. 5:2), multiple superpartient when $x-ay=1$, and multiple superpartient when $y > x-ay > 1$ (e.g. 8:3). The prefix sub- denotes the inversion of these relationships; *q.v.* Apel, *Notation of Polyphonic Music*, p. 146.

³⁷ Cf. Busse Berger, *Mensuration and Proportion Signs*, p. 182.

³⁸ The only surviving version of the *Expositiones* is the revision of the lost 1404 version made by Prosdocimus in 1412, *vid.* Gallo, *Prosdocimi de Beldemandi Opera 1: Expositiones*, p. 7. It is arguable that the

representing *sesquitercia* or *sesquialtera* proportions is mathematically erroneous, Prosdocimus advocates the proportions $\frac{4}{3}$ and $\frac{3}{2}$ in their place, stating that:

...ista sunt signa comunissima, quoniam conveniunt omnibus figuris, scilicet maximis, longis, brevibus, semibrevibus, minimis et semiminimis atque omnibus earum pausis.³⁹

In this reading, *comunissima* does not indicate that the usage is widespread or commonly practised, but that the sign can accommodate proportions at every *gradus* of durations and their rests.⁴⁰ While this statement supports Busse Berger's assessment that proportions facilitated the scribe and composer with the simple means of notating all durations or pauses accurately without resorting to unusual, often ambiguous note shapes found at the height of the *ars subtilior*, in the broader context of the passage in which it occurs, the previous statement demonstrates that Prosdocimus saw fractions as a viable and simpler alternative to other notational devices indicating proportions already present in musical practice and notation, such as advanced coloration. The weight of this assessment will become evident in the subsequent paragraphs.

The limited number of examples of fractions used to represent proportions in the extant repertoire of the *ars subtilior* style operate in the manner whereby a number of durations after the sign, as indicated by the numerator (upper numeral), are equal to the number of durations before the sign, as indicated by the denominator (lower numeral). It

1404 version also contained the section on proportions discussed herein, since its context depends on the prior discussion of advanced notational devices typical of the late fourteenth century, such as *sesquitercia* coloration. The likelihood of its prior existence is strengthened by the fact that the fractions $\frac{2}{1}$, $\frac{3}{1}$, $\frac{3}{2}$ and $\frac{4}{3}$ are discussed by Prosdocimus in 1408 in his treatise largely based on the *Expositiones*, the *Tractatus practice de musica mensurabili* (edited in Coussemaker, *op.cit.*, vol. 3, pp. 200-228). The *Libellus* by De Muris contains a section discussing the diminution of tenors in motets by a half and a third which may be viewed as the earliest discussion of durational proportions.

³⁹ "...those signs are most accommodating since they suit all notes, namely *maxime, longe, breves, semibreves, minime* and *semiminime* as well as all rests of these (note values)."; Gallo, *Prosdocimi de Beldemandi Opera 1: Expositiones*, ch. LXI sent. 23. cf. Prosdocimus' *Tractatus practice de musica mensurabilis* where, after detailing several common proportions, the theorist suggests that the range of proportions which could theoretically be employed was infinite, in Coussemaker, *op.cit.*, pp. 218f.

⁴⁰ Ugolino d'Orvieto writes c. 1430: *Ideo quae accidunt mensuris possunt proportionatis convenire notis, ea enim sunt notarum perfectio et imperfectio, alteratio, puncti perfectio et divisio, ligaturae positio, ingeniosa syncopatio, vocum pausatio, augmentatio, diminutio, colorata notarum positio, talea, evacuatio, colorum transmutatio, et signorum positio, ut cum suis tractalis quadranguli, et cum suis puncti circuli vel semicirculi.* ("Therefore, perfection and imperfection of notes, alteration, points of perfection and division, the placing of ligatures, ingenious syncopation, rests, augmentation, diminution, the placing of coloured notes, talea, the hollowing out of notes, the changing of colour, and the placing of signs, such as the square with its strokes and the circle or semicircle with their dot, which occur in measures, are suitable for proportional notation"), Seay, *Vgolini de Vrbevetani Declaratio Musicae Disciplinae*, vol. 2, ch. VI-7, sent. 12.

remains, however, to be noted that although Prosdocimus and his successors referred to these signs as fractions, they are nothing more than proportions (which are identical to ratios) relating one part to another without adhering to strict mathematical principles of fractions. The use of the term *fractio* by musical theorists to describe one Indo-Arabic numeral superimposed over another in musical notation to indicate a proportional relationship belies a mathematical inexactitude wherein, for example, the fraction $\frac{3}{2}$ does not result in an increase of a note's temporal duration by a half, but an increase by one part of a unit formerly divided by two sub-units. If a strict mathematical relationship relative to the change in length of the subsequent unit in relation to the former, which occurs in the 3:2 proportion, were to be expressed mathematically, it would consist of the fraction $\frac{2}{3}$. This notation, mathematically speaking, would denote that the subsequent duration is (through multiplication) two-thirds the duration of the previous equivocal duration. Despite this (nonetheless conventional) contradiction, the following discussion will persist in the use of the term fraction to denote one Indo-Arabic numeral placed above another in mensural notation. At issue, however, is whether one can observe algorithmic processes at a date earlier than that previously accorded to music.

At this time (unlike in the subsequent periods), the unit of reference to which proportions are related is usually the *minima*.⁴¹ Anthonellus de Caserta's *Amour m'a le cuer mis*, found in MOe5.24, may be the earliest surviving example of fractions used to denote proportions,⁴² and, as with many other examples, numerical indications of proportionality operate at the *minima* level. A diplomatic copy of its opening and transnotation is found in Figure 6.1 (A complete transnotation can be found in Vol. II, App. A, No. 66).

⁴¹ Prosdocimus de Beldemandis writes: *Propter quod scire debes, quod quotienscumque proportionantur figure per talia signa, solum proportionantur in respectu ad minimas earum si minime non sunt et se minime sunt inter se proportionantur sine aliquo respectu.* ("Therefore you ought to know that whensoever notes are proportioned by such signs, they are only proportioned with respect to their minims: with or without minims among them they are proportioned with respect to nothing else."); Gallo, *Prosdocimi de Beldemandi Opera 1: Expositiones*, ch. LXI sent. 28.

⁴² On the basis that none of Anthonellus' works are found in CH 564, a manuscript whose textual references demonstrate a *terminus ante quem non* of 1395 (*vid.* Günther, 'Eine Ballade auf Mathieu de Foix', pp. 69-81), but that his works appear in MOe5.24 and Pn 6771 in sections dating from c. 1400-1410, it may be conjectured that Anthonellus was active around the beginning of the fifteenth century

Figure 6.1: Opening of Anthonellus de Caserta's *Amour m'a le cuer mis* (MOe5.24, f. 32v.)⁴³

The image displays a musical score for the opening of Anthonellus de Caserta's *Amour m'a le cuer mis* (MOe5.24, f. 32v.). The score is presented in a modern transcription format, featuring three systems of staves. Each system includes a vocal line (Tenor) and two instrumental lines (Cintrateneur and Teneur). The first system shows the vocal line with lyrics: "1 A 2.Ei mour sou m'a uent le fois cuer pense mis et en tel mar - ti sos - pi". The second system shows the vocal line with lyrics: "re re. Que sou". The third system shows the vocal line with lyrics: "que uent mayn me tes can". The score includes various musical notations such as clefs, time signatures, and rhythmic values.

This work demonstrates an early phase in the use of proportions in which they are non-cumulative. Unlike cumulative proportions, which refer to the immediately previous

⁴³ N.B. Ligature marks are not shown in transnotations, but they should be inferred by the reader from the original notation.

temporal division, non-cumulative proportions relate only to the original mensuration or *integer valor* of the work in which they occur. For example, in the third system of Figure 6.1, the non-cumulative proportion $\frac{4}{2}$ preceding S 13, a ratio which shows an affinity with the aforementioned use of the numeral 4 in *Lorques Arthus* and *Medee fu*, results in durations whose values are halved relative to the same note values in the original mensuration or at a proportion of 2:1. If proportions were cumulative in this work, the proportional relationship of durations in relation to the original mensuration would be 3:1 if we also take into consideration the earlier proportion of $\frac{9}{6}$ preceding B. 8, i.e. $\frac{4}{2} \times \frac{9}{6} = 3$.⁴⁴ This is clearly not the case in the context of the lower voices of this work. This non-cumulative practice is likely a residue of Boethian-Pythagorean thought,⁴⁵ most frequently witnessed in treatises from this period which deal with the division of the monochord, where each string division is expressed as a ratio in relation to the string's overall length.

In the works of Baude Cordier, proportions are always cumulative. Figure 6.2 shows the canonic upper voice of Cordier's *Tout par compas*,⁴⁶ the only version of which is found on one of the two inserted leaves beginning Codex Chantilly (A complete transnotation of this work can be found in Vol. II, App. A, No. 67).

⁴⁴ Rather than being considered unrefined, the use of the unreduced fraction $\frac{4}{2}$ in *Amour m'a le cuer mis* may be explained as an indication of temporal subdivisions of three groups of four rather than two groups of six relative to the *brevis* in the *integer valor*.

⁴⁵ *vid.* Busse Berger, *Mensuration and Proportion Signs*, p. 204.

⁴⁶ *Tout par compas* has long been examined and re-examined by modern musicology especially with regards to a transcribable solution, *vid.* Pierre Aubry, *Les plus anciens monuments la musique française*, Paris, 1905 [reprint: New York, 1969]; H. Reimann, *Handbuch der Musikgeschichte*, vol. 1, part 2, Leipzig, 1905, pp. 350; J. Handschin, 'The Summer Canon and its background,' *Musica Disciplina*, vol. 3, 1949, p. 84; Carl Parrish, *The Notation of Medieval Music*, New York, 1957, pp. 187-95; Ursula Günther, 'Der Gebrauch des *Tempus perfectum diminutum* in der Handschrift Chantilly,' pp. 277-297; John Bergsagel, 'Cordier's circular canon,' *The Musical Times*, 113, 1972, pp. 1175-77; R. Meylan, 'Reparation de la roue de Cordier,' *Musica Disciplina*, 26, 1972, pp. 69-71; Carol Williams, 'Two examples of Mannerist notation in the late fourteenth century,' *Miscellanea Musicologica*, 2, 1980, pp. 111-128.

Figure 6.2: Canonic upper voice of Baude Cordier's *Tout par compas*.

CH 564

1.4.7. Tout par
3. Re gar
5. Trois temps

5
com - pas suy com - po - ses,
de com suy dis - po - ses,
en tiers par toys po - ses

10
2.8 en ces te ro - de pro - pre - ment pour moy chan -
6. cha - cer me pues io - yeu - se - ment s'en chan - tant

15
ter plus seu - re - ment,
as vray sen - te - ment.

That proportions in this work are cumulative is clearly shown by the proportion preceding B. 12 in Figure 6.2, which effects a *sesquitercia* proportion within the major prolation. This in turn is cancelled by its opposite, 3:4, in B. 13, thereby restoring the original mensuration.

The distinction set out above between cumulative and non-cumulative proportions using Indo-Arabic fractions is considered by Busse Berger as an important indication of their chronological development, in that there was an historical progression towards cumulative proportionality.⁴⁷ However, the concept of cumulative or algorithmic proportionality was already employed by composers of the *ars subtilior* style. Rather than resorting to the extrinsic device of Indo-Arabic numerals, it can be generally said that many composers at first sought to express compound proportions by altering the intrinsic nature of note forms

⁴⁷ Busse Berger, *Mensuration and Proportion Signs*, p. 204 et passim.

through the modification of their colour and/or adding additional tails and/or flags. As such, this represented an extension of two different inventions witnessed with the birth of *ars nova* notation: the addition of the upward tail to form the *minima*, and red coloration. However, as seen in Chapter 4, the expansion of this system resulted in a complex array of note shapes that often existed on a piece-by-piece basis. This situation may have caused composers to look again to the foundations of *ars nova* notation where the prior existence of mensuration signs prompted the exploration of this extrinsic mode of signification by using unusual mensural signs or tying certain coloration or note shapes to verbal instructions to express proportional relationships. This eventual shift in the notational paradigm consequently paved the way for the introduction of the new numerals and a simplification of musical representation. However, the direct association of proportionality, especially of a cumulative nature, with Indo-Arabic numerals in the form of fractions occurred only at the very end of the cultivation of the *ars subtilior*.

Johannes Suzoy's *Pytagoras, Jobal et Orpheüs* is one such work in the *ars subtilior* style whose notation seeks to indicate cumulative proportional relationships with mensuration signs and verbal instructions. Little is known about this composer apart from the conjecture that he was the son of the Pierre de Susay who in 1332 was a clergyman in the French royal chapel,⁴⁸ and the possibility that he was the same person mentioned as a living contemporary by the anonymous author of the *Règles de la seconde rhétorique*,⁴⁹ a tract on French poetic forms and devices written between 1406 and 1408.⁵⁰ One transmission of *Pytagoras, Jobal et Orpheüs* is found in Codex Chantilly (f. 30v). A second transmission of this work occurs on ff. 4v-5r of the Boverio fragments (=Tn T.III.2).⁵¹ Tn T.III.2 was copied in northern Italy in the second decade of the fifteenth century and demonstrates connections to the Pisan papacy (1409-1415) at Bologna, and the northern Italian

⁴⁸ Reaney, 'The Manuscript Chantilly, Musée Conde, 1047', pp. 77-78

⁴⁹ M.E. Langlois, *op.cit.*, p. 14.

⁵⁰ The dating provided here is based upon the necessity that the *Règles* was completed after the deaths of the two *rhétoriques* Eustache Deschamps (c.1346-1406) and Jean Froissart (1333?-1400/1 or 1410?). The *Reglés* also mentions Tapissier as his contemporary (*de present*), which on the face of it would suggest that the tract was completed before 1408 if we take that date as Wright's presumed *obit* for the Burgundian musician Jean Tapissier, *vid.* Wright, 'Tapissier and Cordier: New documents and conjectures', p. 184. On Suzoy, *cf.* Ursula Günther, 'Susay [Suzoy], Jo(hannes),' in *The New Grove Dictionary of Music and Musicians*, 2nd edn, vol. 24, p. 732. Both Günther maintain that Suzoy's works were written in the 1380s. There is a Gloria attributed to a 'Suzay' in the MS Apt 16bis, f. 25v. For a summary of discussions on the attribution of this work *vid.* Giulio Cattin and Francesco Facchin (eds.), *French Sacred Music*, Monaco, 1989, p. 478.

⁵¹ A colour facsimile may be found in Ziino, *Il codice T. III. 2*.

cultivation of French music.⁵² The transmission of *Pythagoras, Jobal et Orpheüs* in Tn T.III.2 provides a crucial chronological framework that permits observations concerning current notational practices as typified by this manuscript's scribes.

Besides the notational variance between both versions, the transmission of *Pythagoras, Jobal et Orpheüs* in Tn T.III.2 is notable for its use of proportional mensuration signs.⁵³ Figure 6.3 below diplomatically reproduces the final part of the *S* of this work from both manuscripts with a transnotation given below. (A complete transnotation based on Tn T.III.2 can be found in Vol. II, App. A, No. 68).

Figure 6.3: Notational variance in the *S* of *Pythagoras, Jobal et Orpheüs* in Tn T.III.2 and CH 564.

In terms of variance between transmissions of musical works from this period, the notational devices are quite different, but they result in virtually identical readings. In other words, semantic integrity is maintained despite semiotic variation. Whereas Chantilly transmits the conventional mensural sign *O* to indicate *sesquialtera* at the *semibrevis* level, the Tn T.III.2 scribe uses the equally suitable device of red coloration sometimes in conjunction with the aforementioned mensuration sign. Such exchange of mensuration signs for coloration, and vice versa, is a common feature of this period, and in many respects explains the variants to be discussed below.

⁵² *vid.* Ziino, *Il codice T. III. 2*, pp. 102-111.

⁵³ Busse Berger discusses similar uses of mensuration signs to indicate proportional relationships in her *Mensuration and Proportions Signs*, pp. 164-178. The source discussed here was obviously not available at the time of her book's publication.

The next part of the phrase beginning on the second system of Figure 6.3 is marked in CH 564 with the subscription indicating that the duration of the written values must be halved (*hec cantetur per medium usque ad signum* = ‘this must be sung by halving <the durations> right up to the <mensuration> sign’). Equivalent meaning is conveyed in the Boverio fragments where the scribe instead employs the unusual mensuration sign \circ ⁵⁴ in conjunction with the canon found at the end of the tenor: *Canon Iste ballate. videlicet quod semicirculum et sursum in proportione dupla et alique prout Iacent tam in cantu quam in tenore* (‘This ballade’s canon: namely that the semicircle also facing upwards [indicates the passage is] in the proportion 2:1 and the rest just as they are written both in the *cantus* and in the *tenor*’).⁵⁵

After the *semibrevis* rest, the second system of Figure 6.3, the transmission found in CH 564 uses ambiguous full red coloration to denote a *proportio sesquitercia* organised as [2,2], resulting overall in a proportion of 8:3 (*proportio dupla superbipartiens tercias*). The transmission in Tn T.III.2 instead employs the mensuration sign \circ to indicate the same *sesquitercia* proportion in addition to the aforementioned upwards facing mensuration sign. The accumulation or multiplication of these signs in Boverio results in the same proportion found in Codex Chantilly of 8:3 at the *minima*.⁵⁶ However, the reiteration of the mensural sign \circ is unnecessary in this case. This is confirmed by the near identical end of the ballade’s *clos*, where \circ only is used in the equivalent position. The drawing of mensural signs in red ink in Boverio is inconsequential. The use of void red note forms after the

⁵⁴ A similar, but unrelated sign is mention in the *Regulae cantus mensurati secundum Johann Otteby* as signifying *proportio subsesquialtera*, *vid.* Reaney, *Opera omnia de musica mensurabili Johannes Hothby*, p. 54.

⁵⁵ The reading of this canon remains unclear due to the lacuna created by clasps perforating the paper at this point. There is some uncertainty whether the reading is *et sursum* or *reversum*. Ziino maintains the ambiguity in *Il codice T. III. 2*, pp. 57 & 113.

⁵⁶ Later examples of similar compound mensuration signs ($\text{\textcircled{d}}$, $\text{\textcircled{f}}$, $\text{\textcircled{e}}$) are found in a *Confiteor* from the *unicum* transmission of the anonymous *Missa L’ardant désir* found in Rome (Città del Vaticano), Biblioteca Apostolica Vaticana, Capella Sistina 51, ff. 98v-99r. While the work itself possibly dates from the 1460s, the choirbook itself was most likely copied sometime in the decade after 1470. A transnotation of the relative passage preserving the original mensuration signs can be found in Rex Eakins, (ed.), *An Editorial Transnotation of the Manuscript Capella Sistina 51, Biblioteca Apostolica Vaticana, Città del Vaticano: Liber Missarum*, Collected Works XVII/3, Ottawa, 2001, pp. 123-124. Eakins also notes the presence of additional esoteric elements in the transmission of this mass, *ibid.*, pp. xxxv-xxxvi. For an alternative interpretation of the *Confiteor*, *vid.* Rob Wegman, ‘Another Mass by Busnoys?’, *Music and Letters*, vol. 71, 1990, pp. 14-16. Busse Berger (*Mensuration and Proportion Signs*, 170-1) also notes a similar sign ($\text{\textcircled{e}}$) in the *Qui propter nos* of Reginald Liebert’s *Gloria* in the Trent, Museo Provinciale d’Arte, Castello del Buonconsiglio, MS 92, f. 59v, a source compiled c.1435-43. An example of a compound mensural sign not drawn in a vertical arrangement but horizontally, is found at the beginning the *contratenor* of Baude Cordier’s *Pour le deffault* ($\text{\textcircled{c}}$; Ob 213, f. 108v).

proportio dupla sign is initially puzzling, however, due to its redundancy. It is plausible that this coloration is a legacy from an earlier version of this work in which void red notation alone indicated *proportio dupla*. Alternatively, it may function to delimit the duration for which the proportion applies, with a return to the *integer valor* indicated by full black note forms. While both versions of this work contain ambiguities whose resolution is only made apparent - as is the case for most of *musica mensurabilis* - by their context, their identical outcomes, which result from the self-consistent application of coloration coupled with unusual mensuration signs and verbal instructions, testify to the semantic validity of each transmission.

The underlying proportional thought revealed by both transmissions of this work is complex and highly sophisticated in that it relies on the accumulation or multiplication of proportional relationships. This alone indicates the presence of an arithmetic mentality cultivated by algorism. Yet, the re-notation of at least one or more versions of this work suggests that proportional fractions and Indo-Arabic numerals in musical notation were not in wide use in northern Italy even in the second decade of the fifteenth century. It is possible but highly conjectural that both versions represent a reworking of the scribal exemplar. The CH 564-transmission of *Pythagoras, Jobal et Orpheüs* preserves the practice of ambiguous full red *sesquitercia* notation which may hark back to an earlier practice in the 1380-90s, while the void red notes in Tn T.III.2 are equivoques descended possibly from its exemplar. While it is difficult to state which version is the latest, both demonstrate a tradition of cumulative proportional notation employing mensuration signs and canons, but not Indo-Arabic numerals, that extends into the fifteenth century. As detailed above, scribes in both the older layer of Codex Chantilly (that is excluding the two Cordier inserts) and MOe5.24 employ Indo-Arabic numerals, although the level of complexity occurring in relation to this device does not extend to cumulative proportionality. Instead, as *Pythagoras, Jobal et Orpheüs* and other works in the repertoire demonstrate,⁵⁷ cumulative proportional signification beyond the use of special note shapes was achieved by the use of mensuration signs or verbal instructions often in conjunction with coloration.

⁵⁷ e.g. the anonymous *En Albion de fluns* (CH 564, f. 47v) notates several proportional relationships, including [3,2] *dim.* and [2,3] *dim.* by the combination of the sign \circ with full red and void red coloration respectively. Similar relationships are found in the next two works in CH 564, *De tous les moys* and Rodericus' *Angolorum psallat*. See Chapter 5, p. 274. Details of similar although slightly later devices used in English sources may be found in Andrew Hughes, 'Mensuration and proportion in early fifteenth century English music,' *Acta Musicologica*, 37, 1965, pp. 48-61.

Another pertinent facet in the history of musical notation in the early fifteenth century is provided by the Franco-Cypriot secular repertoire found in Tn J.II.9. The insular nature of this late source of music in the *ars subtilior* style, which was possibly compiled sometime between 1413 and 1422 in the Cypriot court of Janus de Luisignan and Charlotte de Bourbon,⁵⁸ prevents any direct comparison with works from the continent. However, the notation of this manuscript demonstrates several features in common with those found in MOe5.24 and CH 564 including special note shapes, advanced coloration and proportional mensuration signs.⁵⁹ While Richard Hoppin has wittingly generalised that "its notation is in fact considerably simpler <than MOe5.24 and CH 564>...straight forward and unequivocal...",⁶⁰ I would temper this statement with the observation that the notation of MOe5.24 and CH 564 is also contextually unequivocal in most cases despite the presence of a richer set of complex note shapes. However, the transitional nature of Tn J.II.9's notation is revealed by the admixture of devices used to denote proportional relationships. The only proportion written as an Indo-Arabic fraction is $\frac{3}{2}$. This always denotes the proportion of 3:2 at the minima level. Like in the continental repertoire, more complex proportional relationships in Tn J.II.9 are indicated by single numerals and unusual mensuration signs, which are usually explained by an accompanying set of verbal instructions.⁶¹ Tn J.II.9's proportions do represent daring departures from the usual superparticular and multiple superparticular proportions in the surviving mainland repertoire, to include superpartient and multiple superpartient proportions. Yet, proportions are never cumulative, but are still governed by Pythagorean-Boethian concepts.⁶² The scribe of Tn J.II.9 appears to be aware of developments in musical notation as described by Prosdocimus de Beldemandis, but he is likely tied to the older notational concepts that permeate this manuscript.

⁵⁸ Hoppin, 'The Cypriot-French repertory of the Manuscript Torino, Biblioteca Nazionale, J.II.9', pp. 92-93.

⁵⁹ Cf. Hoppin, 'The Cypriot-French repertory of the Manuscript Torino, Biblioteca Nazionale, J.II.9', pp. 104-106.

⁶⁰ Richard H. Hoppin, 'The Manuscript J.II.9. in the Biblioteca Nazionale of Torino,' in *L'Arts Nova Italiana del Trecento I*, Certando, 1963, pp. 81-82.

⁶¹ *Vid.* Table 5.5, p. 280.

⁶² It is true that in Tn J.II.9, as Busse Berger puts it, "...the introduction of the fraction resulted in the use of proportions not naturally inherent in the mensural system, that is, numbers not divisible by two or by three" (*Mensuration and Proportion Signs*, p. 181). However, this statement, in light of the complex proportions indicated by mensuration signs in Tn J.II.9, strengthens my argument concerning the influence of algorism before the introduction of Indo-Arabic fractions into musical notation.

6.3. Baude Cordier reconsidered

I now turn my attention to the composer Baude Cordier and his works, which demonstrate some of the earliest uses of cumulative proportionality. The problem with Baude Cordier's works and their dating revolves around the issue of his identity. Earlier scholars maintained that Baude Cordier was active in the first or second decades of the fifteenth century.⁶³ In an article published in 1973, Craig Wright challenged this view by arguing that Cordier could be identified with Baude Fresnel, who served Philip the Bold, Duke of Burgundy, as a valet and harpist from 1384, and that Cordier is merely a sobriquet in a similar vein to Fresnel's colleague Jean de Noyers who was referred to frequently in documents as "Jehan de Noyers, dit Tapissier".⁶⁴

However, the evidence is at best circumstantial for Wright's duly cautious conjecture. It rests on (1) the coincidence of the composer's *Tout par compas* stating that Cordier was from Rheims, as was Fresnel, (2) that a *Gloria*, ascribed elsewhere to Cordier,⁶⁵ is found in the manuscript Apt 16bis, whose contents are largely representative of composers documented at the court of Burgundy,⁶⁶ and (3) that the *Amen* of Cordier's *Gloria* is "almost

⁶³ An example of a past author struggling with the Fresnel hypothesis may be found in Gilbert Reaney, 'Cordier, Baude,' in S. Sadie (ed.), *The New Grove Dictionary of Music and Musicians*, London, 1980, vol. 4, pp. 767-8. Reaney firstly gives an approximate date of c. 1400 for the composition of *Tout par compas* and then discusses Wright's conjecture which he assumes results in a dating up to a decade or more earlier. The same entry was reprinted with little change in *The New Grove Dictionary of Music and Musicians*, 2nd ed., London, 2001, vol. 6, pp. 455-456, although additional literature after 1980 is cited to support Reaney's previous objection to Wright's hypothesis.

⁶⁴ Wright, 'Tapissier and Cordier: New documents and conjectures,' pp. 177-98. *q.v. idem*, *Music at the Court of Burgundy 1364-1419: A Documentary History*, pp. 124-134, 166-8, *et passim*.

⁶⁵ A Credo lacking an ascription is found on f. 26v-27r of Apt 16bis which corresponds to a work found in Bc 15 ascribed to "baudet cordier".

⁶⁶ While ascriptions in Apt 16bis demonstrate connections to Burgundy, Andrew Tomasello's hypothesis concerning its connection to Avignon can no longer be proven on account of the recent re-reading of Vatican *Introitus et Exitus* 372, 83r. Instead of the original reading *Jo. de Bosco P<ellison>. servit*, which was used by Tomasello to argue that the composer Pelison named in Apt was identical to Johannes de Bosco, Ursula Günther and her colleagues find the correct reading to consist of two names: *Jo. de Bosco Jo. Frewit, vid.* Ursula Günther, 'Composers at the court of the antipopes in Avignon: Research in the Vatican Archives,' in *Musicology and Archival Research*, ed. Barbara Hagg, Frank Daelemans, and André Vanrie, Bruxelles, 1994, pp. 328-337. Similarly, Tomasello's conjecture that the manuscript was planned and executed mostly by Richardus de Bozonvilla (=Scribe C), a member of the chapel of Benedict (XIII) between 1395 and 1405, and it was completed before the departure of Benedict and his retinue from Avignon in 1403, must also be questioned. There is significant amplitude in the dating of watermarks in the fifth and sixth paper fascicles of Apt 16bis to argue for its completion as late as the second decade of the fifteenth century. *Vid.* Tomasello, *Music and Ritual at Papal Avignon*, pp. 123-50. It is also possible that Cordier's *Gloria* (assuming the Bc 15 ascription is correct) is the work of a younger Cordier. Additionally, the likelihood that Tapissier's *Credo* was entered in Apt 16bis after Cordier's *Gloria* is not an absolute determinant in the chronological priority of one work over the other, if one considers other factors such as availability of exemplar, geographic proximity of composers and scribal organisation. The last category is a feature of Apt 16bis.

identical, note for note" to an *Amen* in a *Credo* by Tapissier (Apt 16bis, ff. 34v-35r), thereby constituting a pairing.⁶⁷ Shortly after its publication, Richard Hoppin dismissed Wright's hypothesis by arguing that, unlike Tapissier, nowhere is the sobriquet "Cordier" found in documents from the court of Burgundy.⁶⁸ Despite the superficial merit of Wright's conjecture, there remains no concrete evidence to suggest that Cordier and Fresnel are one and the same. Yet, even the most recent writers on this period have not hesitated in accepting this hypothesis as a historical fact.⁶⁹ The central problem with Wright's conjecture is Baude Fresnel's death in 1397/98: the corollary that Cordier's works were composed prior to this date paints a picture of an explosive development of notational devices which, upon the background of the greater part of the *ars subtilior* repertoire, appear anachronistic or extraordinarily visionary.⁷⁰

As can be seen in the third measure of Figure 6.2 (pg. 300), *Tout par compas* also uses the mensuration sign \ominus to indicate *tempus perfectum diminutum*. In Cordier's works found in CH 564 and Ob 213,⁷¹ this sign always signifies that the duration of written note values must be diminished by one half. This meaning relies on its simultaneous use with other undiminished integral mensurations. The sign \ominus appears to have been an alternative or predecessor to the more familiar form of ϕ in use from *circa* 1420 onwards.⁷² Similar

⁶⁷ While "note for note" is perhaps an exaggeration, both *Amens* contain significant melodic quotations of one another. For a transnotation of these works *vid.* Cattin and Facchin, *French Sacred Music*, 1989, #33 & #53. For a recent discussion of the pairing of Tapissier's *Credo* with *Glorias* by both Cordier and Thomas Fabri *vid.* Robert E. Palmer, 'Squaring the triangle: Interrelations and their meanings in some early fifteenth century mass pairs,' *Journal of Musicology* 16, 1998, pp. 494-518. Palmer has also discussed Wright's hypothesis and argues in its favour on the basis of these relationships.

⁶⁸ Hoppin, *Medieval Music*, p. 486.

⁶⁹ Cf. Strohm, *The Rise of European Music, 1380-1500*, pp. 144 *et passim*.

⁷⁰ Margaret Bent states that "a date late in the 1410s or even 1420s would accord better with their [*sc.* Cordier's composition's] style and usage" in her 'The early use of the sign ϕ ', p. 223, endnote 2. Likewise, based on stylistic considerations, Ursula Günther suggests "that Cordier must be regarded as a composer of the early fifteenth century" in her 'Polymetric rondeaux from Machaut to Dufay: Some style-analytical observations', in *Studies in Sources and Style: Essays in Honor of Jan LaRue*, eds E. K. Wolf and E. H. Roesner, Madison, 1990, p. 102.

⁷¹ Oxford, Bodleian Library, ms Canonici Misc. 213 is a source written for the greater part in white notation containing works either bearing or alluding to dates from 1422 to May 1436 (in Venice), *vid.* David Fallows (ed.), *Oxford, Bodleian Library, MS. Canon. Misc. 213*, pp. 18-19; Gilbert Reaney, 'The Manuscript Oxford, Bodleian library, canonici misc. 213,' *Musica Disciplina*, 9, 1956, pp. 73-104; A facsimile of the complete MS may be found in Fallows, *Oxford, Bodleian Library, MS. Canon. Misc. 213*.

⁷² I ask that the reader refer to studies concerning the subsequent use of 'cut' signs in the fifteenth century. The problem of whether such signs, when appearing simultaneously in all parts, indicate an exact doubling of time, or a slightly faster tempo than the sign without the stroke, has received extensive treatment in recent literature: *vid.* Eunice Schroeder, 'The stroke comes full circle: \emptyset and ¢ in writings on music, ca. 1450-1540', *Musica Disciplina*, vol. 36, 1982, pp. 133-137; Rob Wegman, 'What is *acceleratio mensurae*?', *Music and Letters*, vol. 73, 1992, pp. 522-23; Anna Maria Busse Berger, 'Cut signs in fifteenth-century musical practice', in *Music in Renaissance Cities and Courts: Studies in Honor of Lewis Lockwood*, eds J. A. Owens and A. M. Cummings,

notational practices are also found in Bc 15, a source contemporary to Ob 213.⁷³ That the stroke through a mensural sign is always used in Cordier's works to indicate *proportio dupla* (or *diminutum per medium*) is confirmed by the examination of another work found ascribed to him in Ob 213. In this manuscript, the rondeau *Amans, ames secretement* (f. 123r; *vid.* Vol. II, App. A, No. 69)⁷⁴ was perhaps suitably entered below Hasprois' *Ma douce amour, je me doy bien complaindre*, a work already listed above as one containing Indo-Arabic numerals attached to a verbal canon. Although *Amans, ames secretement* does not employ proportional fractions, the following set of mensuration signs are used without any other explanation in this very compact, late *ars subtilior* essay:

1. ○, ⊙, ⊕, ⊗ Used in regular capacity, with *minima* equivalence
2. ∅, ℓ and ℓ̄ Indicate regular mensuration with a 2:1 proportion at the *minima* level.
3. ⊙3 and ○3 Indicate regular mensuration with a 3:1 proportion at the *minima* level.

In the second category, the diagonal stroke is equivalent in meaning to the horizontal or vertical stroke and is the result of a simple scribal trait. The use of the compound proportion/mensuration signs in the third category to indicate *proportio tripla* is analogous to the use of the numeral 3 in another of Cordier's works in Codex Chantilly, *Belle, bonne, sage*.

Michigan, 1997, pp. 101-112; Alexander Blanchy, 'Reading Tinctoris for guidance on tempo', in *Antoine Busnoys: Method, Meaning, and Context in Late Medieval Music*, ed. P. Higgins, Oxford, 1999, pp. 399-427. For a thorough survey of late fifteenth and early sixteenth theoretical attitudes on cut-signs, *vid.* Busse Berger, *Mensuration and Proportion Signs*, pp. 125-148. Margaret Bent's series of studies concerning the interpretation of these signs is a work in progress which questions several conclusions drawn by previous scholarship and readings of contemporary theory, *vid.* Margaret Bent, 'The early use of the sign φ', pp. 199-225; *eadem*, 'The use of Cut Signatures in sacred music by Ockeghem and his contemporaries', in *Johannes Ockeghem: Actes du XLe Colloque internationale d'études humansites*, ed. P. Vendrix, Paris, 1998, pp. 641-680; *eadem*, 'The use of Cut Signatures in sacred music by Binchois', in *Binchois Studies*, eds A. Kirkman and D. Slavin, Oxford, 2000, pp. 277-312. For one reaction to Bent's scholarship, *vid.* Rob Wegman, 'Different strokes for different folks? On tempo and diminution in fifteenth-century music', *Journal of the American Musicological Society*, vol. 53, no. 3, 2000, pp. 461-505, and Bent's response in 'On the interpretation of φ in the fifteenth century: A response to Rob Wegman', *Journal of the American Musicological Society*, vol. 53, no. 3, 2000, pp. 598-612. The use of a cut sign in one voice only of a polyphonic composition, which indicates a simple proportional relationship at the *minima* (2:1), should not be confused with the former issue. In addition to the articles given above, the following published discussions on the related issue of *tempus perfectum diminutum* should be noted: Heinrich Bessler, *Bourdon und Fauxbourdon: Studien zum Ursprung der Niederländischen Musik*, Leipzig, 1950, pp. 121-138; Günther, 'Der Gebrauch des *Tempus perfectum diminutum* in der Handschrift Chantilly', pp. 277-297; Reynolds, *op.cit.*, pp. 350-398. Reynolds addresses the issue of re-notation of ⊙ into φ (*ibid.*, pp. 387-392), which has been recently rekindled in relation to the motets in Bc 15 by Julie E. Cumming in her *The Motet in the Age of Du Fay*, Cambridge, 1999, pp. 99-146.

⁷³ *vid.* Bobby Wayne Cox, 'Pseudo Augmentation' in the Manuscript Bologna, Civico Museo Bibliografico Musicale, Q 15 (BL)', *Journal of Musicology*, vol. 1, no. 4, 1982, pp. 419-448. The term 'pseudo-augmentation' used by Cox and others, describes the *dupla* relationship between *minime* in integral major prolation and cut minor prolation.

⁷⁴ A facsimile may be found in Apel, *Notation of Polyphonic Music*, p. 175.

In general, many unusual mensuration signs found in especially the older repertoire of the seventh and eighth gatherings of Ob 213 rely on the same processes used to alter mensural signs in *Amans, ames secretement*. Another work in Ob 213, whose use of mensuration signs is very similar to Cordier's usage, is Gillet Velut's *Laissies ester vestres chans* (f. 100r).⁷⁵ There is some evidence to suggest that Velut was present at Cambrai in 1409, and may have been among the retinue of Charlotte of Bourbon that travelled to the household of her new husband, Janus de Luisignan, in Cyprus.⁷⁶ This, and similar biographical data of composers of the early fifteenth century,⁷⁷ may account for the broad geographical distribution of the older works in Ob 213 and their notational devices.

As a means of further gauging the differences between Cordier's works and works dating from the late fourteenth or very early fifteenth centuries, and if the Fresnel hypothesis is to be reconsidered, a comparison of two works demonstrating similar musical outcomes is in order. A comparison between Cordier's *Tout par compas* and Jacob de Senleches' *La harpe de melodie*⁷⁸ is appropriate since both works utilise a pair of canonic

⁷⁵ The anonymous *Tant plus je voy* also found in Ob 213 (f. 124r) bears superficial similarities with Cordier's apparent notational style. However, the meaning of mensuration signs in this work is significantly different in that cut signs govern *modus* and *tempus*, and the numeral 2 indicates prolation is imperfect. On the basis of these advanced devices, it seems unlikely that this work might be attributed to Cordier, as Reaney tentatively proposed in his edition of this work (in *Early Fifteenth Century Music*, vol. 4, *Corpus Mensurabilis Musicae* 11, [Rome], 1955, p. xiv), unless Cordier was still active in the 1420s. It should also be noted that the resulting musical rendering shows few complexities and is clearly linked to the new style.

⁷⁶ Strohm, *The Rise of European Music*, p. 145. On the listing of a "Gillet Velliout" among the priests in Charlotte de Bourbon's retinue and the possible identification of this person with Gillet Velut, *vid.* Hoppin, 'The Cypriot-French repertory of the Manuscript Torino, Biblioteca Nazionale, J.II.9,' p. 89. The composers Guillaume Legrant, Nicholas Grenon, Pierre Fontaine, Jean Cesaris, Jean/Jacques Charité and Mahieu Paullet whose works occur in the older repertoire of gatherings 5-8 of Ob 213 are documented to have been present at various times between 1405-16 at the Sainte-Chapelle of Bourges, an institute founded and maintained by Jean I, Duke of Berry (1340-1416), *vid.* Paula Higgins, 'Music and musicians at the Sainte-Chapelle of the Bourges Palace, 1405-1515', pp. 689-701. For a summary concerning the associations of these and other composers in Ob 213 with the courts of Burgundy, and the cathedrals of Cambrai and Laon during this period, *vid.* Strohm, *The Rise of European Music*, pp. 145-150.

⁷⁷ The early fifteenth century sees the continuation of a trend witnessed in the late fourteenth century of French and Flemish musicians and composers in the employ of courts and institutes in Italy and Spain. The early career of Guillaume Du Fay (native of Cambrai) is exemplary for the many years he spent in Italy. On Du Fay's early career and possible connections to the court of Carlo Malatesta during the period 1420-23, *vid.* Alejandro Enrique Planchart, 'The early career of Guillaume Du Fay,' *Journal of the American Musicological Society*, vol. 46, 1993, pp. 361-62. On his residency at Bologna 1426-28 and subsequent appointment to the papal chapel at Rome 1429-1434, *vid. idem*, 'Guillaume du Fay's benefices and his relationship to the Court of Burgundy,' *Early Music History*, vol. 8, 1988, pp. 125-131.

⁷⁸ *La harpe de melodie* has also retained a large portion of scholarly interest, *vid.* Josephson, 'Die Konkordanz zu "en nul estat" und "La harpe de melodie"', pp. 292-300; *idem*, 'Vier Beispiele der ars subtilior,' *Archiv für Musikwissenschaft*, 27, 1970, pp. 41-58; Willi Apel, 'La harpe de melodie,' in *Scritti in onore di Luigi Ronga*, Milan and Naples, 1973; Tilman Seebass, 'The visualisation of music through pictorial imagery and notation in late medieval France', in *Studies in the Performance of Late Medieval Music*, pp. 19-33; Reinhard Strohm, 'La harpe de melodie, oder Das Kunstwerk als Akt de Zueignung', in *Das musikalische*

upper voices, are notated in a pictorial fashion, and can be discerned to employ similar proportional thought, albeit transmitted in a very different manner. Although a version of *La harpe de melodie* is also found in CH 564, the source used for this comparison is the manuscript Chicago, Newberry Library, ms. 54.1 (US-Cn 54.1) where Senleches' work is notated in an irregular fashion on the strings of an illustration of a harp, each of its strings representing a pitch.⁷⁹ The Newberry version also represents the earliest and most faithful extant copy by an English scribe, Brother William, made in Pavia in Lombardy sometime around 1391.⁸⁰ The date of composition for this work can reasonably conjectured to be the late 1380s or very early nineties. Hence, under Wright's hypothesis, this work and *Tout par compas* would be contemporaneous.

The pictorial aspect of both works is not merely *Augenmusik*, but in both cases is integral to the understanding of the text. The refrain of *La harpe de melodie* reads:

*La harpe de melodie
faite sanz mirancholie
par plasir.
doit bien cescun resjoir
pour l'armonie oïr, sonner,
et veïr.⁸¹*

The concept of visual harmony alluded to in the text of *La harpe de melodie* is only fully understood upon seeing its pictorial representation as found in Cn 54.1. Similarly, the first four lines the text of *Tout par compas* hint at the work being a musical round simply because the music is notated on staves inscribed by a compass:⁸²

*Tout par compas suy composes
en ceste rode proprement.
Pour moy chanter plus seurement
Regarde com suy disposes.⁸³*

Kunstwerke. Festschrift Carl Dahlhaus zum 60. Geburtstag, ed. H. Danuser, H. d. L. Motte, S. Leopold and N. Miller, Köln, 1988.

⁷⁹ Facsimiles of this version of *La harpe de melodie* appear in Seebass, 'The visualisation of music through pictorial imagery and notation in late medieval France', plate V, and Richard Hoppin, ed., *Anthology of Medieval Music*, New York, 1978, front cover.

⁸⁰ A critical assessment of variants, both musical and textual may be found in Josephson, 'Die Konkordanz zu "en nul estat" und "La harpe de melodie"', pp. 195-300.

⁸¹ "The melodious harp, made without melancholy to please. Well might all rejoice in hearing, playing and seeing its harmony."

⁸² Numerous facsimiles of this work have been published in the twentieth century, including Pierre Aubry, *Les plus anciens monuments*, pl. XXII (black & white), Freidrich Gennrich (ed.), *Übertragungsmaterial zum Abriß der Mensuralnotation des XIV und ersten häfte des XV Jahrhunderts*, Langen bei Frankfurt, 1965, Tab. XX (black and red); Bent, 'The early use of the sign ϕ ', p. 225.

⁸³ "All by a compass I am properly composed in this round. To sing me most accurately, note how I am written down."

The difference between these verbal instructions lies in the audience to which they are directed. In *La harpe de melodie*, it is plain that the text addresses both the performer and the listener, while in *Tout par compas* the text is solely directed to the performer on the composer's behalf. While one could imagine the scenario suggested by Reinhard Strohm where the illustration of the harp was presented beforehand to the composer's patron for contemplation during the performance of the work,⁸⁴ the text of Cordier's work appears to be addressed to musicians only, suggesting that the work may have been created as a diversion for skilled performers.⁸⁵ However, the visual aspect of the notated score remains an integral part of the experience of both of these works.

Both *La harpe de melodie* and *Tout par compas* begin with the same *integer valor* ([2,3]) and both utilise diminution by a half, although upon different mensurations. Table 6.2 compares techniques used to notate various durations in both works.

Table 6.2: Notational devices in Senleches' *La harpe de melodie* and Cordier's *Tout par compas* compared.

Proportion (with respect to <i>integer valor</i>)	<i>La harpe de melodie</i>	<i>Tout par compas</i>
a. 2:1 (<i>dupla</i>) within <i>prolatio</i>	$[\textcircled{C}] \downarrow \downarrow$ (or $\downarrow \downarrow$) = $[\textcircled{C}] \overset{\text{r}}{\downarrow} \overset{\text{r}}{\downarrow}$ (with canon)	$\ominus \downarrow \downarrow = \textcircled{C} \downarrow$
b. 3:2 within <i>tempus</i>	$[\textcircled{C}] \square \square \square = [\textcircled{C}] \blacksquare \blacksquare$ (dim.)	$\textcircled{C} \overset{\text{r}}{\blacklozenge} \overset{\text{r}}{\blacklozenge} \overset{\text{r}}{\blacklozenge} = \textcircled{C} \blacklozenge \blacklozenge$
c. 3:2 within <i>tempus diminutum</i>	$[\textcircled{C}] \downarrow \downarrow \downarrow = [\textcircled{C}] \blacklozenge \blacklozenge = [\textcircled{C}] \overset{\text{r}}{\blacklozenge} \overset{\text{r}}{\blacklozenge}$	$\ominus \blacklozenge \blacklozenge \blacklozenge = \textcircled{C} \blacklozenge$
d. 3:2 within <i>prolatio diminuta</i>	$[\textcircled{C}] \downarrow \downarrow \downarrow = [\textcircled{C}] \downarrow \downarrow$	$\ominus \frac{3}{2} \downarrow \downarrow \downarrow = \ominus \downarrow \downarrow$
e. 2:3 within <i>prolatio</i>	$[\textcircled{C}] \downarrow \downarrow = [\textcircled{C}] \downarrow \downarrow \downarrow$ (diminished by half)	$\textcircled{C} \frac{4}{3} \blacklozenge \blacklozenge = \textcircled{C} \downarrow \downarrow \downarrow$ (2 x <i>La harpe</i>)
f. 3:4 within <i>tempus imperfectum diminutum</i>	$[\textcircled{C}] \downarrow \downarrow \downarrow = [\textcircled{C}] \square = [\textcircled{C}] \downarrow \downarrow \downarrow \downarrow$	$[\ominus \frac{3}{2} \overset{\text{r}}{\blacklozenge} \overset{\text{r}}{\blacklozenge} \overset{\text{r}}{\blacklozenge}]$ (Not in actual piece, but my conjecture.)

⁸⁴ Reinhard Strohm, *The Rise of European Music*, pp. 57-8

⁸⁵ The additional three strophes accompanying this work are in a different rhyme and syllable number scheme (although still in rondeau form) and are addressed to certain 'seigneurs', who are asked 'Pries pour celi qui m'a fait' ("pray ye for him who composed me"). The question of whether these lines should be sung to the music has vexed scholarship for most of the 20th century, and continues to do so this century, *vid. supra*, fn. 46. Granted the difficulty of setting these strophes to the music, these lines may have been recited before the performance. The voice, the personification of the round itself, remains current in all sections of the text.

As shown by row *a* of Table 6.2, Senleches' composition employs red coloration in conjunction with a canon found in an accompanying rondeau which specifies: *Par blanc et noir per mi sans oblier / lay le tonant ou tu li feras tort* ('Let the black and white notes sound by half without forgetting or you will do them wrong').⁸⁶ Observing this instruction in the upper voice results in black and void notes whose duration at the *minima* level is halved relative to red notes. However, red notes in the upper voice are equivalent in duration to black notes in the *tenor*. White notes in the upper voice behave in the normal manner by providing a *sesquialtera* proportion at the *semibrevis* level with black *minima* equivalence. As also shown by rows *c* to *f* in Table 6.2, further re-division of time in *La harpe de melodie* relative to void and black durations is achieved by complex note shapes employing stems and flags.⁸⁷

Tout par compas, on the other hand, achieves the same proportions mainly through fractions, mensural signs and some coloration, avoiding the more complex note shapes found in *La harpe de melodie*. As shown in rows *a*, *c*, *d* & *f* of Table 6.2, diminution by a half is indicated by the mensuration sign \ominus , whose nature has been described above. As shown in rows *c* to *f* of Table 6.2, *Tout par compas* employs simple note shapes within the proportions $\frac{3}{2}$ and $\frac{4}{3}$ as well as proportional mensurations and thus it can be stated that the composer's mind was firmly entrenched in the arithmetic mentality and that he used Indo-Arabic numerals as an unambiguous expressions of his intent. Yet, it cannot be denied that Jacob de Senleches' *La harpe de melodie* contains evidence of arithmetic, proportional thought processes. The note shape \downarrow , for example, relies on cumulative proportions. The voiding of a note shape indicates a *sesquialtera* proportion at the *semibrevis* level (with *minima* equivalence) and the addition of the downward flagged stem to the void *minima* indicates a further *sesquialtera* proportion at the *minima*.

The issue, however, is complicated by Cordier's flamboyant complexity and fondness for notational equivoques in contrast to Senleches' precise economy in notation. In her article *Gebrauch des tempus perfectum diminutum*,⁸⁸ Ursula Günther saw these notational

⁸⁶ Translated in Hoppin, *Anthology of Medieval Music*, p. 171. Nors Josephson correctly interpreted the canon in 'Die Konkordanz zu "en nul estat" und "La harpe de melodie"', p. 299, fn. 13.

⁸⁷ A full discussion of these note shapes can be found on page 223 of this present study.

⁸⁸ Günther, 'Der Gebrauch des *Tempus perfectum diminutum* in der Handschrift Chantilly', p. 282.

practices in the first half of *Tout par compas* as an imitation of an earlier style, but in a manner which suggests complexity for its own sake. The beginning of *Tout par compas* best clarifies this statement (See Figure 6.2, p. 300). In the upper voice after the initial *minima* up-beat, a new mensuration is indicated by a compound sign of $\overset{\circ}{\underset{1}{3}}$ requiring [3,3] in *proportio tripla*. Yet, only *breves* and *semibreves* are employed after this sign. Granted that this notation is not indicative of a lost performance practice, to a composer or musical scribe of the *ars subtilior*, it would most likely have been far simpler to notate this passage in *semibreves* and *minime* in regular [3,3]. Instead, Cordier's compositional *ethos* appears to emulate the *ars subtilior* style, a style which is no longer central to musical experience but has been misconstrued as notational complexity rather than the notational representation of complex musical relationships. While Cordier's notation may be seen as a parody on his contemporaries' musical style, the presence of Indo-Arabic fractions and cut mensuration signs in the notation of his compositions suggests that Cordier was active after the peak of the *ars subtilior* style which appears to have occurred in the last years of the fourteenth century.

Setting aside for a moment concerns regarding the influence of geography and culture on the scribes of manuscripts, it is patently clear that, between the completion of Tn J.II.9 and the Boverio fragments, and the completion of Ob 213, there were significant shifts in notational process paralleling the shifts in musical styles between these sources. The *ars subtilior* repertoire in Tn J.II.9 and Tn T.III.2 represents the end of a tradition, whereas Ob 213 contains the works of a new generation of oftentimes Italian based composers which includes the young Guillaume Du Fay. Yet, the scribe of Ob 213 was also interested in collecting the music of Du Fay's predecessors, and the presence of Cordier in this document by no means assures him of a later dating. However, as the previous paragraphs have argued, the most telling indications of Cordier's chronology lies in his notation.

I would like to conclude by examining three distinct, although by no means exclusive, possibilities which might resolve the conflict between notational and cultural data supplied in the paragraphs above and Wright's Cordier hypothesis. Firstly, it might be argued that, based upon the likelihood or fact that Codex Chantilly, the Modena manuscript and Boverio fragments were compiled outside France, Baude Fresnel was an innovator living in Burgundy whose notational techniques were unknown to his contemporaries in Italy. This situation, however, appears improbable if the early discussion of fractions used to indicate

proportions by Prosdocimus de Beldemandis and the prevalence of algorism in northern Italian culture is recalled. Indeed, on this basis it does not seem too bold to propose that musicians in northern Italy played a significant role in the development of fractional proportions in musical notation. Furthermore, religious (especially the ecumenical councils) and political circumstances, and the general francophile tendencies of this period in many courts outside the kingdom of France suggests that it was unlikely that current French thought was foreign to northern Italy.⁸⁹ Indeed, the Venetian manuscript Ob 213, which contains the greatest number of works ascribed to Cordier, can be seen as extant testimony of his reputation in northern Italy.

On the other hand, one might conjecture that Cordier's works were all re-notated by fifteenth century scribes from exemplars employing notational devices closer to those found in the works of Suzoy or perhaps Senleches. While re-notation at a minor level is a feature of some works in the *ars subtilior* repertoire, there is no surviving evidence to suggest that complex notation, as in the works of composers such as Senleches and Suzoy, was rewritten using Indo-Arabic proportions and cut sign notation. The temporal gap between the peak of the *ars subtilior* style and the introduction of Indo-Arabic proportion signs is simply too great in terms of notational and musical chronology to warrant this situation. Furthermore, the similarity of notational devices in his Chantilly inserts and works in Ob 213 tend to indicate that the notation found in these sources reflects Cordier's idiom.

Finally, there is the undeniable possibility that Baude Cordier was not Baude Fresnel, but a composer who was active in the first two decades of the fifteenth century. As detailed at the beginning of this section, the currently received version of Baude Cordier's biography demonstrates several weaknesses by linking him to the renowned Burgundian harpist, Baude Fresnel, not only in its lack of irrefutable evidence but in the corollary that Cordier's works would have been composed before 1397/98. The surviving scribal evidence argues against Craig Wright's hypothesis on several fronts. Firstly, the association of Indo-Arabic fractions with cumulative, proportional relationships in music was a late development that is only generally witnessed in the third decade of the fifteenth century by Ob 213. When fractions are first encountered in Anthonellus de Caserta's *Amour m'a le cuer mis*, they show no relation

⁸⁹ Early fifteenth century catalogues from libraries of the great houses of northern Italy, such as the Gonzaga, Visconti and d'Este, show that a substantial part of the collections (between 10 and 20 per cent) consisted of French manuscripts, *vid.* David Fallows, 'French as a courtly language in fifteenth century Italy: the musical evidence,' *Renaissance Studies*, 3, 1989, pp. 429-441. Michael Long also demonstrates the

to the cumulative proportionality especially typical of French composers, such as Senleches and Suzoy, but are still couched in the age-old concepts of Boethian-Pythagorean ratios. This aspect and the use of cut mensurations signs suggests that Cordier's works belong to innovations which occurred towards the end of the first quarter of the fifteenth century. Next, there is the absence of Cordier's secular works in the older layer (Layer I) of Codex Chantilly and any other compilations of French music before the third decade of the fifteenth century. While chance or scribal taste obviously mitigate this observation to a lower status, it is strange that a composer who is accorded so much space in Ob 213 should not have at least one work transmitted in the sources of the *ars subtilior* repertoire proper if he was active before their compilation. Finally, there is the nature of Cordier's notational and musical style which can be perceived as a mannered imitation of older *ars subtilior* techniques, often employing ostensibly redundant equivoques and complexity for its own sake.

6.4. *Conclusions*

Throughout the preceding discussion, I have demonstrated that musical notation of the *ars subtilior* contains elements connected to wider cultural, especially intellectual, movements. The years around the beginning of the fifteenth century witness the emergence of a new popular arithmetic mentality that was nurtured by a growing interest in Indo-Arabic numerals and, more importantly, the methods of quick calculation facilitated by them. However, that the notational devices of composers such as Senleches and Suzoy were clearly influenced by advanced concepts of this new arithmetic mentality, but without the tell-tale presence of the Indo-Arabic numerals themselves, requires reconsideration of the view that the presence of Indo-Arabic numerals alone equates to a new arithmetic mentality. This conclusion challenges currently held wisdom. Yet the step of using the actual symbols that formed the basis of algorism indicates that, while the notational devices of the *ars subtilior* are by no means deficient in their nature, they lack a permanence and wide spread adoption which tends to indicate ongoing experimentation under the influence of the Indo-Arabic fractions and the growing dominance of this symbol-system in medieval culture. With the arithmetic concepts already in place, it was only a matter of time before the musical scribes in approximately the second decade of the fifteenth century took the

influence of French thought on the intellectual élite in *trecento* Florence in Long, 'Francesco Landini and the Florentine cultural élite', pp. 83-99.

logical step of using Indo-Arabic numerals/fractions themselves as precise indications of rhythmic proportions. This evaluation is ironic in that with the simple, uniform system of notation achieved, the interest or necessity of the style which brought it about appears to have declined to handful of dedicatory or exhibition pieces in the fifteenth century.⁹⁰

⁹⁰ On the extended but relatively rare cultivation of *ars subtilior* techniques into the fifteenth and sixteenth centuries, *vid.* David Fallows, "The end of the *ars subtilior*", *Basler Jahrbuch für Historische Musikpraxis*, 20, 1996, pp. 21-40.