AN INVESTIGATION INTO STUDENTS' UNDERSTANDINGS OF CLASS INCLUSION CONCEPTS IN GEOMETRY

PENELOPE ANNE SEROW

Dip. in Teaching (Catholic College of Education Sydney, NSW)
B.Ed. (University of New England, NSW)

Thesis submitted for the degree of
Doctor of Philosophy
of the
University of New England

October 2002

CERTIFICATE

I certify that the substance of this thesis has not already been submitted for any degree and is not being currently submitted for any other degree.

I certify that any help received in preparing this thesis, and all sources used, have been acknowledged in this thesis.

Signature ___[redacted]___

ACKNOWLEDGEMENTS

One of the most rewarding elements of completing a project such as this is stepping back and acknowledging the people whose support, encouragement, and assistance has made it all possible. I wish to express sincerely my gratitude to those people.

My thanks go to the support of many staff and students within the School of Education at the University of New England. My supervisor, Professor John Pegg played a pivotal role in the establishment of the project, and subsequent writing of the thesis. I would also like to thank Dr Ted Redden for his professional support in the final stages of writing, Dr Ken Vine and Dr Scott Dickson for the expert advice in quantitative aspects of the study. In another dimension, I acknowledge the powerful support provided by those who could always be relied upon to keep one's self-esteem in tact. In this light, the friendship and assistance provided by Kathy Jenkins, Linley Lloyd, Michaela Inglis, Dr Judy Miller, and Cathy Coleman had a significant role to play. I thank Professor Mike Shaughnessy for the inspiration and fellowship he provided despite the distance involved.

On reflection, I recognise the essential gifts provided by my parents, that of instilling the importance of the development of knowledge and creating an environment of belief that you can reach your goals. In addition, my Dad's own example of excellence in teaching provided the foundation to continue in this field. I would like to thank all my family for their tireless support in many different aspects of the thesis journey.

To my husband Chris, whose faith in me, constant encouragement, love, support and patience was evident throughout the many hurdles involved in writing this thesis. To my children, Elise, Michael, and Charles, whose understanding of 'study time' and cooperation greatly assisted the completion of this thesis, thank you.

To enable the work involved in this study, I acknowledge the financial assistance in the form of an Australian Postgraduate Award provided by the University of New England.

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

ABSTRACT

This study investigated students' understandings of class inclusion concepts in Geometry. The purpose was to identify a developmental pathway leading to an understanding of the interrelationships among two-dimensional figures and their properties. The design involved a tightly focused investigation of the manner in which geometrical class inclusion concepts evolve, in particular, relationships among triangle and quadrilateral figures, and relationships among their properties. Empirical evidence is provided to explain the difficulties students face in understanding of class inclusion notions. This evidence has theoretical as well as practical implications.

The theoretical base for this study is the van Hiele Theory, which comprises five levels of development in Geometry. Numerous studies have involved a focus upon the holistic aspects of the first four van Hiele levels and this has resulted in supportive empirical evidence of the existence and nature of the levels. Pertinent to this study, the level associated with a student who accepts and utilises notions of class inclusion is described as Level 3. This aspect of Level 3 is regarded as both a difficult concept to acquire and a prerequisite for formal deductive reasoning. This study extends research into the van Hiele Theory by narrowing the microscopic lens and providing a focused analysis on the understanding and development of class inclusion concepts in Geometry. In an attempt to refine the characteristics of the development of this concept, this study utilised the SOLO model to provide deeper insights into the van Hiele levels.

The investigation comprised three studies. The first of these, Study 1, explored the context of triangles, and included two main components. These components were relationships among triangle figures, and relationships among triangle properties. Study 2 extended the baseline data of Study 1 via the investigation of students' understanding of relationships among quadrilateral figures and relationships among quadrilateral properties. Each of these studies involved in-depth interviews with 24 students of higher mathematical ability, purposely selected, within Years 8–12 (ages 13–18 years) in two secondary schools. Study 3 also consisted of two parts. The first of these, a quantitative synthesis, based upon the application of ACER's QUEST analysis program, utilised Rasch measurement theory. This part of Study 3 also considered developmental changes from a longitudinal perspective. The second part of Study 3 considered developmental changes in the form of four case studies.

A central finding of this study was the identification of a broad generic framework which describes the developmental pathway leading to an understanding of class

inclusion notions. This pathway characterises student growth in understanding of relationships among figures, and relationships among properties. The pathway was characterised by two cycles of responses of the concrete symbolic mode (SOLO), and two cycles of responses of the formal mode (SOLO). The existence of this pathway has challenged accepted characterisations of van Hiele's Level 3. Behaviours previously described as requiring Level 3 thinking have been found by this study to include Level 3, Transitional Level 3/4, and Level 4.

This study identified student difficulties associated with attaining Transitional Level 3/4. Here, students need to focus upon relationships that are not supported by visual cues. This is identified as formal thinking. The characterisation of transitional groups, evident at Level 3/4, provides guidance concerning teaching activities and implications, to assist students' in their progression from Level 3 to Level 4.

In general, the known property relationships assisted students in the formation of sub-class relationships. In addition, property relationships did not emerge as an identifiable sequence; instead they appeared dependent upon student familiarity with individual properties. However, developmental patterns were evident in terms of language-use where property descriptions appeared to hinder the formation of relationships at Level 2, and property descriptions were conducive to the utilisation of relationships at Level 3, Transitional Level 3/4, and Level 4. Of surprise were the similarity of results for two different contexts of quadrilaterals and triangles. This finding providing support for the notion that thinking at a particular level in one context assists the progression to the same level in other contexts.

The quantitative synthesis across contexts validated the chosen instrument and the developmental trends highlighted by the application of the SOLO model. There was consistency across the triangle and quadrilateral contexts concerning relationships among figures and relationships among properties. The longitudinal student responses, over the two-year period, were interpreted along the previously identified developmental path. Evidence presented in the case studies indicated that individual student responses to similar tasks within different contexts were not always at a consistent SOLO level, dependent upon individual familiarity of triangles or quadrilaterals. It was also evident that some students responded at a higher SOLO level concerning either relationships among properties or relationships among figures.

The research highlights the reasons students find class inclusion concepts in Geometry difficult to grasp. Secondary–school (ages 12–18 years) curriculum content concerning

such notions have been identified as requiring thinking at van Hiele's Level 3, Level 3/4 and Level 4. Thus the hurdles encountered by many students are detailed through the characterisation of the development of relationships among figures and relationships among properties. In addition, this study highlights the use of the SOLO model as an interpretive tool for research in Mathematics education.