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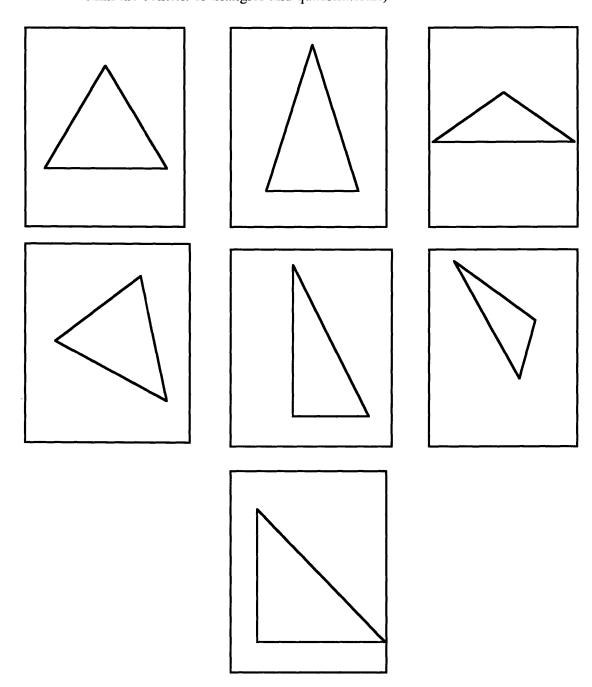
## **APPENDIX A**

### **PILOT STUDY**

## **Section 1: Relationships among figures**

### Part A: Relationships among triangles

Int: These cards have been placed into two groups. Can you tell me the way in which the cards have been sorted? (initial question to focus the student within the context of triangles and quadrilaterals)



Int: I would like you to sort them into smaller groups. As you sort them I

would like you to explain your reasons for sorting them the way you

have chosen.

Probes include: Why have you placed these triangles together?

Why is this triangle on its own?

Is there anything else that you can tell me about the groups?

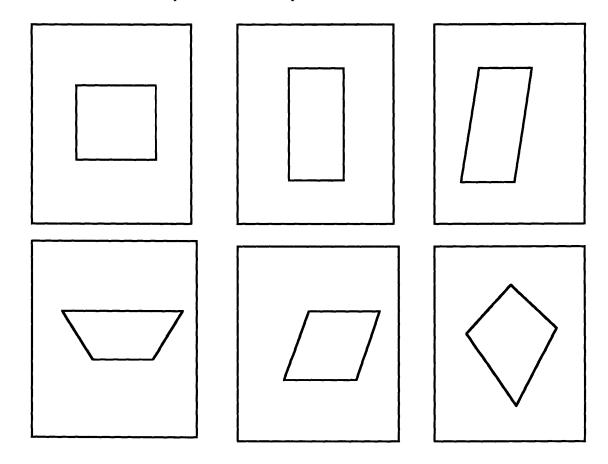
Prompts include: Are there any other ways that the triangles can be sorted?

What reasons do you think someone might have for placing the equilateral in with the isosceles? What would you think about doing

that?

## Part B: Relationships among quadrilaterals

The format above was repeated with the 6 quadrilateral cards below.



### Section 2: Relationships among properties

### Part A: Relationships among triangle properties

Int: I would like you to think about all that you know about the equilateral

triangle. Tell me all the properties that belong to that figure (these were

listed on cards by the interviewer).

Int: I want you to think very carefully now, as I would like you to come up

with a description or definition that accurately refers to that shape

with the least number of properties needed.

Int: Come up with as many combinations as you can.

Probes include: Why would your friend need that combination of cards?

Why is it possible to remove these cards?

Prompts include: What would happen if I removed this card? Would your friend still

recognise the triangle? Why?

Questions repeated for the right isosceles triangle.

### Part B: Relationships among quadrilateral properties

Format above repeated for the square, rhombus, and parallelogram.

### APPENDIX B

### STUDIES 1 AND 2 INTERVIEW PROFORMA

### **Study 1: Triangles**

### Phase 1: Relationships among triangle figures

- (i) Int: I would like you to write a list of all the triangle names you can think of. Begin with acute-angled scalene. Draw each triangle.
- (ii) Int: Design a tree diagram which links the different triangles. Draw a sketch to link each type.(discussion follows concerning the reasons for links and/or lack of links)(the following three points are addressed if required)
- (iii) Int: There are some triangles that we can add to this list. (provide triangles not recalled)

  Draw a sketch of each new triangle.
- (iv) Int: Design a second tree diagram incorporating all the triangles on the list.

  (discussion follows concerning the reasons for links and/or lack of links)
- (v) Int: Return now to your first map. I would like you to add the new triangles to your original tree. (discussion follows concerning the reasons for links and/or lack of links)

### **Study 2: Quadrilaterals**

### Phase 1: Relationships among quadrilateral figures

- (i) Int: I would like you to write a list of all the quadrilateral names you can think of.

  Draw each quadrilateral.
- (ii) Int: Design a tree diagram which links the different quadrilaterals. Draw a sketch to link each type.(discussion follows concerning the reasons for links and/or lack of links)(the following three points are addressed if required)
- (iii) Int: There are some quadrilaterals that we can add to this list. (provide quadrilaterals not recalled)Draw a sketch of each new quadrilateral.
- (iv) Int: Design a second tree diagram incorporating all the quadrilaterals on the list. (discussion follows concerning the reasons for links and lack of links)
- (v) Int: Return now to your first map. I would like you to add the new quadrilaterals to your original tree. (discussion follows concerning the reasons for links and/or lack of links)

### **Study 1: Triangles**

### Phase 2: Relationships among triangle properties

(i) Int: We are going to look closely at a few triangles.

I have placed some cards in front of you with triangle characteristics on them.

I would like you to begin by choosing the cards which belong to the equilateral triangle (selection made).

Look carefully to make sure that you have included all the cards, which belong to that triangle.

(ii) Int: Suppose you wanted to leave some clues for a friend.

Do you think that your friend would need to see all these properties to know that you are thinking about an equilateral triangle?

What combination could you leave? (discussion follows concerning reasons for cards included in the combination and those that have been removed)

Do you think it could be made simpler? (discussion follows concerning reason for the simplification and inability to make simpler)

- (iii) Int: Let's put all the cards back. I would like you to make a different set of clues for your friend. (point (ii) repeated until student has provided all known combinations).
- (iv) First three steps repeated for the right isosceles triangle.

#### Triangle Characteristic Cards

3 SIDES
3 ANGLES
3 SIDES EQUAL
3 ANGLES EQUAL
HAS RIGHT ANGLE
1 AXIS OF SYMMETRY
NO AXES OF SYMMETRY
3 AXES OF SYMMETRY
HAS OBTUSE ANGLE
HAS ACUTE ANGLES
2 ANGLES EQUAL
2 SIDES EQUAL

### Study 1: Quadrilaterals

### Phase 2: Relationships among quadrilateral properties

(i) Int: We are going to look closely at a few quadrilaterals.

I have placed some cards in front of you with quadrilateral characteristics on them (see below).

I would like you to begin by choosing the cards which belong to the square (selection made).

Look carefully to make sure that you have included all the cards, which belong to the square.

(ii) Int: Suppose you wanted to leave some clues for a friend.

Do you think that your friend would need to see all these properties to know that you are thinking about a square?

What combination could you leave? (discussion follows concerning reasons for cards included in the combination and those that have been removed)

Do you think it could be made simpler? (discussion follows concerning reason for the simplification and inability to make simpler)

- (iii) Int: Lets put all the cards back. I would like you to make a different set of clues for your friend. (point (ii) repeated until student has provided all known combinations).
- (iv) First three steps repeated for parallelogram and rhombus.

Quadrilateral Characteristic Cards

4 SIDES
4 ANGLES

ALL SIDES ARE EQUAL

THERE ARE 4 RIGHT ANGLES

OPPOSITE SIDES ARE PARALLEL

OPPOSITE SIDES ARE EQUAL

DIAGONALS ARE EQUAL

DIAGONALS BISECT

DIAGONALS MEET AT RIGHT ANGLES

OPPOSITE ANGLES ARE EQUAL

2 PAIR OF EQUAL ADJACENT SIDES

1 PAIR OF OPPOSITE ANGLES EQUAL

**4AXES OF SYMMETRY** 

2AXES OF SYMMETRY

**1 AXIS OF SYMMETRY** 

1 PAIR OF PARALLEL SIDES

1 PAIR OF OPPOSITE SIDES EQUAL

## **APPENDIX C**

## **INTERVIEW RESOURCES**

## **Student Profile**

| Name:     |   | <br> | <br>        |         |
|-----------|---|------|-------------|---------|
| School: _ | · | <br> | <br><b></b> | . — — - |
| Year:     |   | <br> | <br>        |         |
| Age:      |   |      |             |         |

Study 1 Part 1: List of triangle names and sketches

Triangle tree diagram – 1 and 3

# Triangle tree diagram 2

Study 2 Part 1: List of quadrilateral names and sketches

Quadrilateral tree diagram - 1 and 3

# Quadrilateral tree diagram 2

Study 1 Part 2: Triangle property characteristic cards

| 3 SIDES             |
|---------------------|
| T1                  |
| 3 ANGLES            |
| T2                  |
| 3 SIDES EQUAL       |
| T3                  |
| 3 ANGLES EQUAL      |
| T4                  |
| HAS RIGHT ANGLE     |
| T5                  |
| 1 AXIS OF SYMMETRY  |
| T6                  |
| NO AXES OF SYMMETRY |
| T7                  |
| 3 AXES OF SYMMETRY  |
| T8                  |
| HAS OBTUSE ANGLE    |
| Т9                  |
| HAS ACUTE ANGLES    |
| T10                 |
| 2 ANGLES EQUAL      |
| T11                 |
| 2 SIDES EQUAL       |
| T12                 |

## Students triangle property choice

|   | <b>uilate</b><br>Choice                 |                               | angle                      |   |   |     |     |     |                |                  |                |
|---|---|-------------------------------|----------------------------|---|---|-----|-----|-----|----------------|------------------|----------------|
| 1   | 2                                       | 3                             | 4                          | 5   | 6 | 7   | 8   | 9   | 10             | 11               | 12             |
| Minir   | num Iı                                  | nforma                        | ation                      |   |   |     |     |     |                |                  |                |
| 1   | 2                                       | 3                             | 4                          | 5   | 6 | 7   | 8   | 9   | 10             | 11               | 12             |
| Made  | Simp                                    | ler                           |                            |   |   |     |     |     |                |                  |                |
| 1   | 2                                       | 3                             | 4                          | 5   | 6 | 7   | 8   | 9   | 10             | 11               | 12             |
| Other   | · Comb                                  | oinatio                       | n                          |   |   |     |     |     |                |                  |                |
| 1   | 2                                       | 3                             | 4                          | 5   | 6 | 7   | 8   | 9   | 10             | 11               | 12             |
| Other   | · Coml                                  | oinatio                       | n                          |   |   |     |     |     |                |                  |                |
| 1   | 2                                       | 3                             | 4                          | 5   | 6 | 7   | 8   | 9   | 10             | 11               | 12             |
| Other   | · Coml                                  | oinatio                       | n                          |   |   |     |     |     |                |                  |                |
| 1   | 2                                       | 3                             | 4                          | 5   | 6 | 7   | 8   | 9   | 10             | 11               | 12             |
| 2. Rig  | ght iso                                 | sceles                        | trian                      | gle   |   |     |     |     |                |                  |                |
|   | _                                       |                               |                            |   |   |     |     |     |                |                  |                |
| First   | Choice                                  | 9                             |                            |   |   |     |     |     |                |                  |                |
| First   | Choice<br>2                             | e<br>3                        | 4                          | 5   | 6 | 7   | 8   | 9   | 10             | 11               | 12             |
| 1   | 2                                       | 3                             |                            | _   | 6 | 7   | 8   | 9   | 10             | 11               | 12             |
| 1   |   | 3                             |                            | _   | 6 | 7   | 8   | 9   | 10<br>10       | 11<br>11         | 12<br>12       |
| 1<br>Minii<br>1                                     | 2<br>mum I<br>2                         | 3<br>nforma<br>3              | ation                      | 5   |   |     |     |     |                |                  |                |
| 1<br>Minii<br>1                                     | 2<br>mum I                              | 3<br>nforma<br>3              | ation                      | 5   |   |     |     |     |                |                  |                |
| Minin<br>1<br>Made<br>1                             | mum I<br>2<br>e Simp<br>2               | 3<br>nforma<br>3<br>ler<br>3  | ation<br>4                 | 5   | 6 | 7   | 8   | 9   | 10             | 11               | 12             |
| Minin<br>1<br>Made<br>1                             | 2<br>mum Ii<br>2<br>e Simp              | 3<br>nforma<br>3<br>ler<br>3  | ation<br>4                 | 5   | 6 | 7   | 8   | 9   | 10<br>10       | 11               | 12<br>12       |
| Mining Made  1 Other  1                             | 2 mum I 2 e Simp 2 r Coml               | 3 Informa 3 Ier 3 bination 3  | ation<br>4<br>4<br>on<br>4 | 5<br>5<br>5                                       | 6 | 7   | 8   | 9   | 10<br>10       | 11<br>11         | 12<br>12       |
| Mining Made  1 Other  1                             | mum II<br>2<br>Simp<br>2<br>r Coml<br>2 | 3 Informa 3 Ier 3 bination 3  | ation<br>4<br>4<br>on<br>4 | 5<br>5<br>5                                       | 6 | 7   | 8   | 9   | 10<br>10       | 11<br>11<br>11   | 12<br>12<br>12 |
| Minin<br>1<br>Made<br>1<br>Other<br>1<br>Other<br>1 | mum II 2 Simp 2 Coml 2 Coml             | 3 ler 3 bination 3 bination 3 | ation 4 4 on 4 on 4 on 4   | <ul><li>5</li><li>5</li><li>5</li><li>5</li></ul> | 6 | 7 7 | 8 8 | 9 9 | 10<br>10<br>10 | 11<br>11<br>. 11 | 12<br>12<br>12 |

Study 1 Part 2: Quadrilateral property characteristic cards

| 4 CIDEC                         |
|---------------------------------|
| 4 SIDES                         |
| Q1                              |
| 4 ANGLES                        |
| Q2                              |
| ALL SIDES ARE EQUAL             |
| Q3                              |
| THERE ARE 4 RIGHT ANGLES        |
| Q4                              |
| OPPOSITE SIDES ARE PARALLEL     |
| Q5                              |
| OPPOSITE SIDES ARE EQUAL        |
| Q6                              |
| DIAGONALS ARE EQUAL             |
| Q7                              |
| DIAGONALS BISECT                |
| Q8                              |
| DIAGONALS MEET AT RIGHT ANGLES  |
| Q9                              |
| OPPOSITE ANGLES ARE EQUAL       |
| Q10                             |
| 2 PAIR OF EQUAL ADJACENT SIDES  |
| Q11                             |
| 1 PAIR OF OPPOSITE ANGLES EQUAL |
| Q12                             |
| 4 AXES OF SYMMETRY              |
| Q13                             |
|                                 |

| 2 AXES OF SYMMETRY<br>O14      |
|--------------------------------|
| V17                            |
| 1 AXIS OF SYMMETRY             |
| Q15                            |
| 1 PAIR OF PARALLEL SIDES       |
| Q16                            |
| 1 PAIR OF OPPOSITE SIDES EQUAL |
| O17                            |

# **Students Quadrilateral Property Choice**

| 1. Sq | luare   |                |       |     |   |   |   |   |    |    |    |
|-------|---------|----------------|-------|-----|---|---|---|---|----|----|----|
| First | Choic   | e              |       |     |   |   |   |   |    |    |    |
| 1     | 2       | 3              | 4     | 5   | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 13    | 14      | 15             | 16    | 17  |   |   |   |   |    |    |    |
| Mini  | mum l   | Inform         | ation |     |   |   |   |   |    |    |    |
| 1     | 2       | 3              | 4     | 5   | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 13    | 14      | 15             | 16    | 17  |   |   |   |   |    |    |    |
| Made  | e Simp  | oler           |       |     |   |   |   |   |    |    |    |
| 1     | 2       | 3              | 4     | 5   | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 13    | 14      | 15             | 16    | 17  |   |   |   |   |    |    |    |
| Othe  | r Com   | binatio        | on    |     |   |   |   |   |    |    |    |
| 1     | 2       | 3              | 4     | 5   | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 13    | 14      | 15             | 16    | 17  |   |   |   |   |    |    |    |
| Othe  |         | binatio        | on    |     |   |   |   |   |    |    |    |
| 1     | 2       | 3              | 4     | 5   | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 13    | 14      | 15             | 16    | 17  |   |   |   |   |    |    |    |
|       |         | binatio        |       |     |   |   |   |   |    |    |    |
| 1     | 2       | 3              | 4     | 5   | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 13    | 14      | 15             | 16    | 17  | _ | • | _ |   |    |    |    |
|       |         |                |       |     |   |   |   |   |    |    |    |
| 2. Pa | arallel | ogram          | 1     |     |   |   |   |   |    |    |    |
| First | Choic   | e              |       |     |   |   |   |   |    |    |    |
| 1     | 2       | 3              | 4     | 5   | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 13    | 14      | 15             | 16    | 17  |   |   |   |   |    |    |    |
| Mini  | mum ]   | Inform         | ation |     |   |   |   |   |    |    |    |
| 1     | 2       | 3              | 4     | 5   | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 13    | 14      | 15             | 16    | 17  |   |   |   |   |    |    |    |
| Mad   | e Sim   | oler           |       |     |   |   |   |   |    |    |    |
| 1     | 2       | 3              | 4     | 5   | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 13    | 14      | 15             | 16    | 17  |   |   |   |   |    |    |    |
|       |         | binatio        |       |     |   |   |   |   |    |    |    |
| 1     | 2       | 3              | 4     | 5   | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 13    | 14      | 15             | 16    | 17  |   |   |   |   |    |    |    |
| Othe  | r Com   | binatio        | on    |     |   |   |   |   |    |    |    |
| 1     | 2       | 3              | 4     | 5   | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 12    |         |                | 1.    | 17  |   |   |   |   |    |    |    |
| 13    | 14      | 15             | 16    | 17  |   |   |   |   |    |    |    |
|       |         | 15<br>ibinatio |       | 1 / |   |   |   |   |    |    |    |
|       |         | binatio        |       | 5   | 6 | 7 | 8 | 9 | 10 | 11 | 12 |

| <b>3.</b> | Rhombus |
|-----------|---------|
|-----------|---------|

| First | Choic  | e       |       |    |   |   |   |   |    |    |    |
|-------|--------|---------|-------|----|---|---|---|---|----|----|----|
| 1     | 2      | 3       | 4     | 5  | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 13    | 14     | 15      | 16    | 17 |   |   |   |   |    |    |    |
| Mini  | mum l  | Inform  | ation |    |   |   |   |   |    |    |    |
| 1     | 2      | 3       | 4     | 5  | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 13    | 14     | 15      | 16    | 17 |   |   |   |   |    |    |    |
| Made  | e Simp | oler    |       |    |   |   |   |   |    |    |    |
| 1     | 2      | 3       | 4     | 5  | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 13    | 14     | 15      | 16    | 17 |   |   |   |   |    |    |    |
| Othe  | r Com  | binatio | on    |    |   |   |   |   |    |    |    |
| 1     | 2      | 3       | 4     | 5  | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 13    | 14     | 15      | 16    | 17 |   |   |   |   |    |    |    |
| Othe  | r Com  | binatio | on    |    |   |   |   |   |    |    |    |
| 1     | 2      | 3       | 4     | 5  | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 13    | 14     | 15      | 16    | 17 |   |   |   |   |    |    |    |
| Othe  | r Com  | binatio | on    |    |   |   |   |   |    |    |    |
| 1     | 2      | 3       | 4     | 5  | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
| 13    | 14     | 15      | 16    | 17 |   |   |   |   |    |    |    |

### APPENDIX D

#### INTRARATER AND INTERRATER RELIABILITY

To enable a measurement of the congruity of the system utilised to code students' responses to tasks concerning relationships among figures, and relationships among quadrilaterals, it was necessary to calculate both intrarater and interrater reliability. In an attempt to measure intrarater reliability the consistency the researcher's coding between responses is assessed. Interrater reliability requires a co-marker to utilise the described marking scheme and compare this coding against the principal researcher's coding. This assessment is discussed below.

Firstly, intrarater reliability was established through the random selection of one quarter of the students' responses to each of the seven tasks across Years 8–12. The percentage of responses, which were categorised into the same SOLO levels in both the initial codings and subsequent codings, was 96%.

Secondly, another researcher who has considerable experience working within the SOLO model over many years then coded the randomly selected sample of one quarter of the responses. For each of the seven tasks, the researcher worked within the described structure of levels for each particular task. The measure of agreement between the principal researcher and the co-marker was 92%.

Throughout the coding process, the consistency of the SOLO Model was also established via consultation between researcher and co-marker when rare difficulties occurred with categorisation of particular responses. This was particularly necessary in transitional cases. Overall, the following measures ascertained coding reliability for the SOLO model.

### APPENDIX E

#### PLAIN LANGUAGE STATEMENT / CONSENT FORM

Dear Parent / Guardian,

I am currently completing a Ph.D at the University of New England. As part of this program I am undertaking a study to investigate students' growth and understanding in Geometry. The focus of the study is to be on Years 8 to 12 students in Armidale High Schools. The purpose of this letter is to request your permission to include your son/daughter/ward as a participant in the study.

The study, which has the support of the Principal of the school, is designed in such a way that disruption to the normal school process will be minimal. It will consist of each student being interviewed on one occasion for approximately 40 minutes. At a later date each student will also be required to complete some pen and paper tasks. The interviews will be audiotaped for later analysis but there will be complete confidentiality for both students and schools with the use of pseudonyms where necessary. All records will be held within the Centre of Cognitive Research in Learning and Teaching (CRiLT).

The title of the project is "An investigation of students' understanding of class inclusion concepts in Geometry." Associate Professor John Pegg from the Department of Curriculum Studies, UNE, will also be involved in the study.

Participation by your son/daughter/ward is entirely voluntary and he/she will not be penalised for not wishing to be involved. It is also possible for the participant to withdraw consent and discontinue participation at any time.

If you have any concerns or enquiries you can contact me (Ph 73 5073), Assoc. Prof. John Pegg (Ph 73 5070) or the Principal for further information. If you are willing to allow your son/daughter/ward to participate could you please complete the attached consent form and return it to the school.

If your son / daugter / ward is selected at random as a participant, a letter will be sent to you before the commencement of interviews.

Should you have any complaints concerning the manner in which this research is conducted, please contact the Ethics Committee at the following address:

The Secretary
Human Research Ethics Committee
Research Services
University of New England
Armidale, NSW 2351
Telephone: (067) 73, 2352, Faccimile (

Telephone: (067) 73 2352 Facsimile (067) 73 3543

Yours Faithfully,

Penelope Serow Ph. D Student

Principal

| Student Consent                         |  |
|---|--|
| I,                                      | (the participant) have read the information concerning   |
| participate in this activity, realising | we asked have been answered to my satisfaction. I agree to ng that I may withdraw at any time. I agree that research |
| data gathered for this study may        | be published, provided my name is not used.  |
| Student Signature                       |  |
| Date                                    |  |
| Parent /Guardian Consent                |  |
| I,                                      | _ (parent/guardian) have read the information concerning   |
| the study and any questions I ha        | ave asked have been answered to my satisfaction. I give /ward to be a participant in this study, realising that my   |
| •                                       | I agree that research data gathered for this study may be  |
|   |  |
| Parent / Guardian Signature             |  |
|   |  |

Date

### APPENDIX F

#### RELATIONSHIPS AMONG TRIANGLES TASK ANALYSIS

A student's response concerning the relationships among triangles could include one of three sets of triangle relationships and the reasons for these links. The three sets of relationships, which could be identified and justified, are based on similar features, independent triangle-type classes, and triangle-type classes involving class inclusion.

### Set 1: Relationships based on similar features

There are three types of features upon which students could link triangles, namely, angle types (Figure F.1), side lengths and angle sizes (Figure F.2), and symmetry (Figure F.3).

### a) Angle Types

In Figure F.1 triangles are linked on one of three angle types. A student would select a feature and then link triangles that are seen to contain that feature.

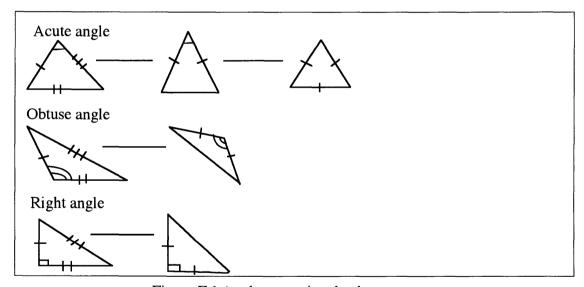


Figure F.1 Angle-type triangle classes

### b) Sides and/or Angles

In Figure F.2 the triangles are linked based on the properties associated with equality of sides and/or angles. The alternatives are three sides/angles equal, two sides/angles equal, and no sides/angles equal.

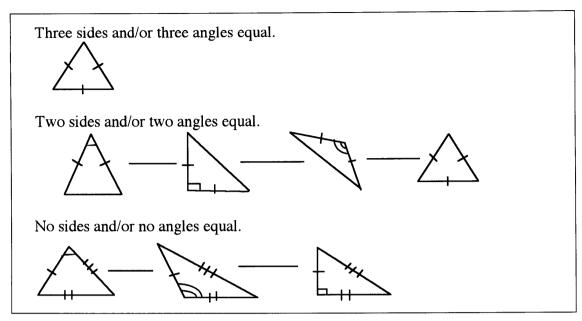


Figure F.2 Relationships based on equality sides and/or angles

### c) Symmetry

In Figure 4.3 the existence of symmetry is the defining feature. Here there are three possibilities, which are three axes of symmetry, one axis of symmetry, and no axes of symmetry.

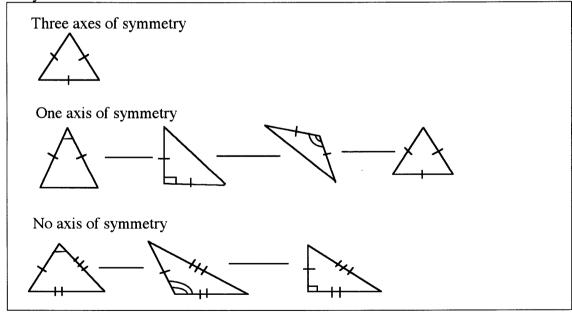


Figure F.3 Relationships based on symmetry

Set 2: The establishment of three triangle-type classes, namely, scalene, isosceles, and

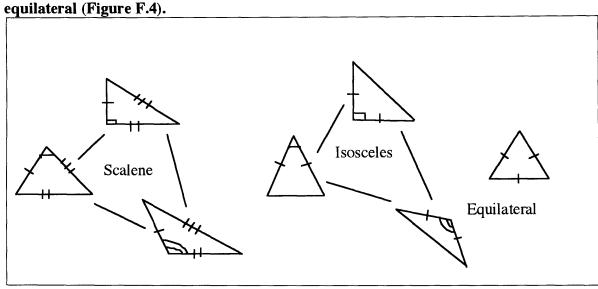


Figure F.4 Triangle type classes

In Figure F.4 the triangles are linked based on properties, such as equality of sides/angles, and/or the number of axes of symmetry. These combine to establish an identified and independent class of triangles.

Set 3: The relationships among the triangle types incorporate the notion of class inclusion (Figure F.5).

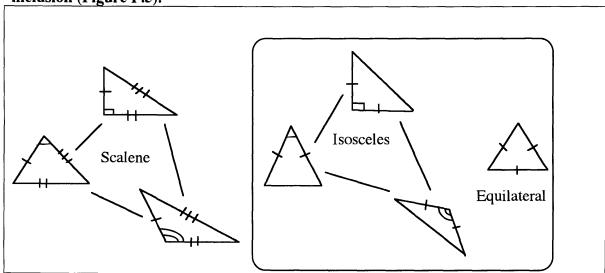


Figure F.5 Class inclusion incorporating triangle-type relationships

The equilateral triangle is a subset of the isosceles class of triangles. This relationship exists because the equilateral triangle is seen to include in its list of properties, the properties of an isosceles triangle, i.e., two sides/two angles equal and one axis of symmetry.

Overall, the task analysis provides a number of expected outcomes that might be considered plausible possibilities. Of interest is whether students provide these possibilities, whether there is some sequencing of the responses in terms of development, and the nature of the thinking that accompanies the responses.

### APPENDIX G

### RELATIONSHIPS AMONG QUADRILATERALS TASK ANALYSIS

It is necessary to consider all possible elements of a response addressing the relationships between the six quadrilaterals focused upon in the interview, prior to coding into groups. A response concerning the relationships among quadrilaterals include two sets of relationships and the reasons for these links. These being:

### Set 1: Relationships based on similar properties

In Figures G.1, G.2, G.3, G.4, and G.5, the quadrilaterals are linked based on the properties associated with equality of sides, equality of angles, symmetry, diagonals, and parallelism.

### a) Sides

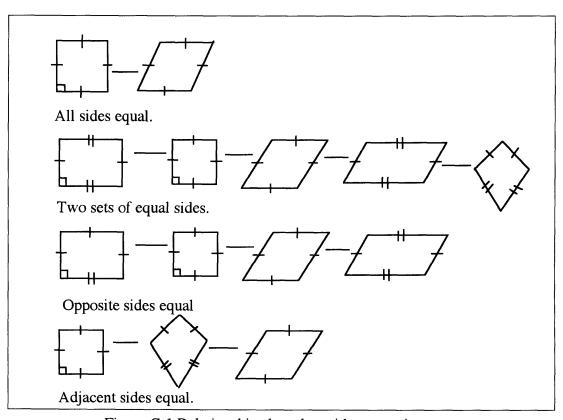


Figure G.1 Relationships based on side properties.

## b) Angles

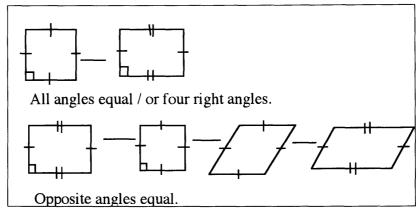


Figure G.2 Relationships based on angle properties

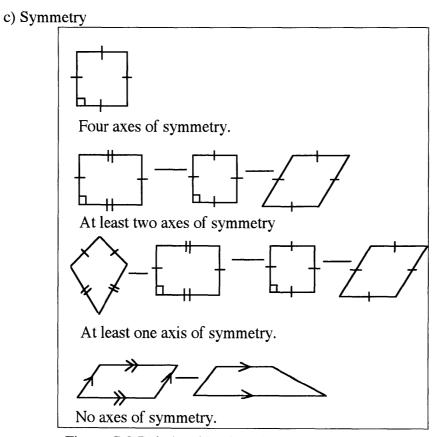


Figure G.3 Relationships based on symmetry properties

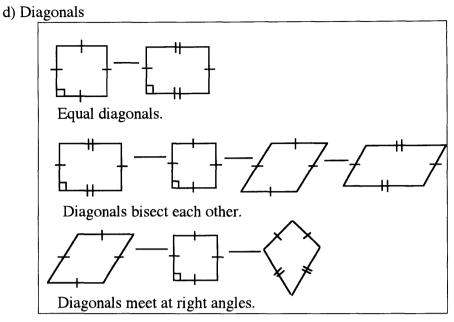


Figure G.4 Relationships based on diagonal properties

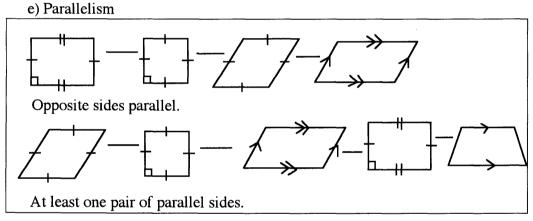


Figure G.5 Relationships based on parallelism.

Set 2: The establishment of three quadrilateral classes involving subsets with justification for each class based on properties such as sides, angles, symmetry, and diagonals (Figure 5.6).

The three classes include;

- a) Rectangle
- b) Rhombus
- c) Parallelogram

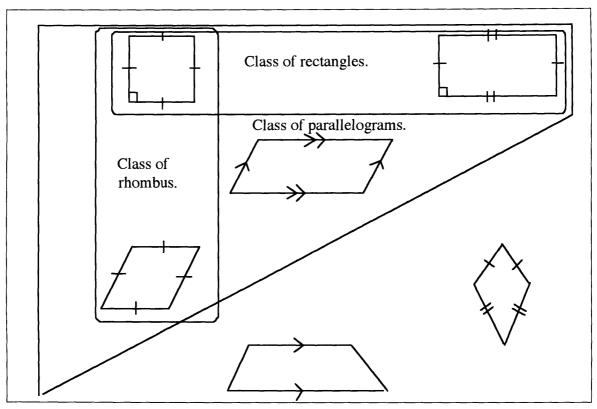


Figure G.6 Quadrilateral classes involving subsets

APPENDIX H

QUADRILATERAL PROPERTIES TASKS CONTEXTUAL GROUPINGS

| Student | Square | Parallelogram | Rhombus |
|---------|--------|---------------|---------|
| Scott   | 3      | 3             | 2       |
| Jason   | 2      | 2             | 3       |
| Brendan | 1      | 2             | 2       |
| Kathy   | 2      | 3             | 3       |
| Louise  | 3      | 2             | 3       |
| Narelle | 2      | 3             | 3       |
| Peter   | 2      | 3             | 2       |
| Andrew  | 2      | 2             | 3       |
| Arthur  | 3      | 2             | 3       |
| Alice   | 2      | 2             | 3       |
| Megan   | 3      | 2             | 2       |
| Ellen   | 2      | 3             | 2       |
| Nathan  | 1      | 2             | 2       |
| Adam    | 2      | 2             | 2       |
| Allan   | . 1    | 2             | 2       |
| Frances | 2      | 2             | 2       |
| Suzanne | 1      | 2             | 1       |
| Tracy   | 2      | 2             | 2       |
| Cameron | 2      | 3             | 3       |
| Michael | 2      | 2             | 3       |
| David   | 1      | 2             | 2       |
| Beth    | 1      | 3             | 2       |
| Dianne  | 2      | 3             | 2       |
| Jenny   | 2      | 3             | 3       |

## APPENDIX I

## SOLO RESPONSE CODINGS FOR STUDIES 1, 2, AND 3

|         | ID  | 1. Tri        | 2. Quad    | 3. Equ    | 4. Isos   | 5. Squ  | 6. Para  | 7. Rhom |
|---------|-----|---------------|------------|-----------|-----------|---------|----------|---------|
| Scott   | 101 | M2/R2 (CS)    | M2 (CS)    | U1 (F)    | U1 (F)    | U1 (F)  | R2 (CS)  | U2 (CS) |
| Jason   | 102 | R2 cat 1 (CS) | M2 (CS)    | U1 (F)    | U1 (F)    | M1 (F)  | R1 (F)   | M2 (CS) |
| Brendan | 103 | M2 (CS)       | U2 (F)     | R2 (CS)   | M2 (CS)   | R2 (CS) | M1 (F)   | M1 (F)  |
| Kathy   | 104 | R2 cat 2 (CS) | M2 (CS)    | M2 (CS)   | R2 (CS)   | U2 (CS) | M1 (F)   | M2 (CS) |
| Louise  | 105 | M2/R2 (CS)    | M2 (CS)    | R2 (CS)   | R2 (CS)   | U1 (F)  | U1 (F)   | M1 (F)  |
| Narelle | 106 | R1 (CS)       | R1 (CS)    | M2 (CS)   | M2 (CS)   | R2 (CS) | R2 (CS)  | U2 (CS) |
| Peter   | 107 | U2 cat 1 (CS) | M2 (CS)    | U1 (F)    | U2 (CS)   | U1 (F)  | R2 (CS)_ | U1 (F)  |
| Andrew  | 108 | M2 (CS)       | M2 (CS)    | M2 (CS)   | U1 (F)    | R2 (CS) | M1 (F)   | M2 (CS) |
| Arthur  | 109 | U2 cat 1(CS)  | M2 (CS)    | M2 (CS)   | U1 (F)    | R2 (CS) | U1 (F)   | M2 (CS) |
| Alice   | 110 | M2 (CS)       | M2/R2 (CS) | R1 (CS)   | M2 (CS)   | U1 (F)  | M2 (CS)  | M1 (F)  |
| Megan   | 111 | M2 (CS)       | U2 (CS)    | M1 (F)    | U1 (F)    | R2 (CS) | R2 (CS)  | U1 (F)_ |
| Ellen   | 112 | U2 cat 2 (CS) | M2 (CS)    | R1 (CS)   | U2 (CS)   | M2 (CS) | R2 (CS)  | U1 (F)_ |
| Nathan  | 113 | U1 (F)        | R1 (F)     | M2 (CS)   | M2 (CS)   | M1 (F)  | R1 (F)   | U2 (F)  |
| Adam    | 114 | U2 (F)        | R1 (F)     | U1 (F)    | R2 (CS)   | M1 (F)  | M1 (F)   | M1 (F)  |
| Allan   | 115 | R2 cat 2 (CS) | R2 (CS)    | M1 (F)    | U1/M1 (F) | M1 (F)  | M1 (F)   | R2 (CS) |
| Frances | 116 | R2 cat 3 (CS) | R1 (CS)    | M2 (CS)   | U2 (F)    | U1 (F)  | M2 (CS)  | M2 (CS) |
| Suzanne | 117 | M2/R2 (CS)    | M2/R2 (CS) | M2 (CS)   | M2 (CS)   | U1 (F)  | M2 (CS)  | U1 (F)  |
| Tracy   | 118 | U2 cat 2 (CS) | R1 (CS)    | U1 (F)    | U1 (F)    | M1 (F)  | U1 (F)   | R2 (CS) |
| Cameron | 119 | R2 cat 1 (CS) | U2 (CS)    | U1 (F)    | U1 (F)    | M1 (F)  | M1 (F)   | U1 (F)  |
| Michael | 120 | R2 cat 2 (CS) | R2 (CS)    | U1 (F)    | M2 (CS)   | M1 (F)  | R2 (CS)  | U1 (F)  |
| David   | 121 | R1 (F)        | R1 (F)     | U1 (F)    | U1 (F)    | U2 (CS) | U2 (F)   | R2 (CS) |
| Beth    | 122 | R2 cat 4 (CS) | M2/R2 (CS) | U1 (F)    | R2 (CS)   | U1 (F)  | R1 (F)   | U2 (F)  |
| Dianne  | 123 | M1 (F)        | R2 (CS)    | M2 (CS)   | R2 (CS)   | R2 (CS) | U1 (F)_  | R2 (CS) |
| Jenny   | 124 | R2 cat 1 (CS) | R2 (CS)    | R1 (CS)   | U1/M1 (F) | M2 (CS) | U1 (F)   | U1 (F)  |
| Scott   | 201 | R2 (CS)       | R2 (CS)    | U2 (F)    | M1 (F)    | R1 (F)  | U2 (F)   | U1 (F)  |
| Jason   | 202 | U2 (F)        | U2 (F)     | U2 (F)    | U2 (F)    | M1 (F)  | U2 (F)   | U2 (F)  |
| Brendan | 203 | M2 (CS)       | U2 (F)     | U2 (F)    | U1 (F)    | U2 (F)  | U2 (F)   | M1 (F)  |
| Kathy   | 204 | M2/R2 (CS)    | M2/R2 (CS) | U1/M1 (F) | U2 (F)    | U1 (F)  | U1 (F)_  | U1 (F)  |
| Louise  | 205 | M2/R2 (CS)    | R2 (CS)    | M1 (F)    | M1 (F)    | M1 (F)  | M1 (F)   | U1 (F)  |
| Narelle | 206 | R1 (F)        | R2 (CS)    | M1 (F)    | U2 (F)    | M1 (F)  | U1 (F)   | M1 (F)  |
| Peter   | 207 | R2 (CS)       | M2 (CS)    | U1/M1 (F) | M2 (CS)   | M1 (F)  | U1 (F)   | M1 (F)  |
| Andrew  | 208 | M2 (CS)       | R2 (CS)    | R1 (F)    | U1 (F)    | M1 (F)  | M1 (F)   | M1 (F)  |
| Arthur  | 209 | R2 (CS)       | M2/R2 (CS) | U1 (F)    | U1 (F)    | M1 (F)  | R1 (CS)  | M1 (F)  |
| Alice   | 210 | M2 (CS)       | M2 (CS)    | U2 (F)    | U2 (F)    | M1 (F)  | M1 (F)   | R2 (CS) |
| Megan   | 211 | R2 (CS)       | R2 (CS)    | M2 (CS)   | U1 (F)    | R2 (CS) | M2 (CS)  | U1 (F)  |
| Ellen   | 212 | M2 (CS)       | R2 (CS)    | U1/M1 (F) | U2 (CS)   | R2 (CS) | U2 (CS)  | R2 (CS) |

## APPENDIX J

### RASCH ANALYSIS CODING

|         | ID  | 1.  | 2.   | 3.  | 4.   | 5.  | 6.   | 7.   |
|---------|-----|-----|------|-----|------|-----|------|------|
|         |     | Tri | Quad | Equ | Isos | Squ | Para | Rhom |
| Scott   | 101 | 3   | 3    | 5   | 5    | 5   | 4    | 2    |
| Jason   | 102 | 4   | 3    | 5   | 5    | 6   | 7    | 3    |
| Brendan | 103 | 3   | 8    | 4   | 3    | 4   | 6    | 6    |
| Kathy   | 104 | 4   | 3    | 3   | 4    | 2   | 6    | 3    |
| Louise  | 105 | 3   | 3    | 4   | 4    | 5   | 5    | 6    |
| Narelle | 106 | 1   | 1    | 3   | 3    | 4   | 4    | 2    |
| Peter   | 107 | 2   | 3    | 5   | 2    | 5   | 4    | 5    |
| Andrew  | 108 | 3   | 3    | 3   | 5    | 4   | 6    | 3    |
| Arthur  | 109 | 2   | 3    | 3   | 5    | 4   | 5    | 3    |
| Alice   | 110 | 3   | 3    | 1   | 3    | 5   | 3    | 6    |
| Megan   | 111 | 3   | 2    | 6   | 5    | 4   | 4    | 5    |
| Ellen   | 112 | 2   | 3    | 1   | 2    | 3   | 4    | 5    |
| Nathan  | 113 | 5   | 7    | 3   | 3    | 6   | 7    | 8    |
| Adam    | 114 | 8   | 7    | 5   | 4    | 6   | 6    | 6    |
| Allan   | 115 | 4   | 4    | 6   | 5    | 6   | 6    | 4    |
| Frances | 116 | 4   | 1    | 3   | 8    | 5   | 3    | 3    |
| Suzanne | 117 | 3   | 3    | 3   | 3    | 5   | 3    | 5    |
| Tracy   | 118 | 2   | 1    | 5   | 5    | 6   | 5    | 4    |
| Cameron | 119 | 4   | 2    | 5   | 5    | 6   | 6    | 5    |
| Michael | 120 | 4   | 4    | 5   | 3    | 6   | 4    | 5    |
| David   | 121 | 7   | 7    | 5   | 5    | 2   | 8    | 4    |
| Beth    | 122 | 4   | 3    | 5   | 4    | 5   | 7    | 8    |
| Dianne  | 123 | 6   | 4    | 3   | 4    | 4   | 5    | 4    |
| Jenny   | 124 | 4   | 4    | 1   | 5    | 3   | 5    | 5    |
| Scott   | 201 | 4   | 4    | 8   | 6    | 7   | 8    | 5    |
| Jason   | 202 | 8   | 8    | 8   | 8    | 6   | 8    | 8    |
| Brendan | 203 | 3   | 8    | 8   | 5    | 8   | 8    | 6    |
| Kathy   | 204 | 3   | 3    | 5   | 8    | 5   | 5    | 5    |
| Louise  | 205 | 3   | 4    | 6   | 6    | 6   | 6    | 5    |
| Narelle | 206 | 7   | 4    | 6   | 8    | 6   | 5    | 6    |
| Peter   | 207 | 4   | 3    | 5   | 3    | 6   | 5    | 6    |
| Andrew  | 208 | 3   | 4    | 7   | 5    | 6   | 6    | 6    |
| Arthur  | 209 | 4   | 3    | 5   | 5    | 6   | 1    | 6    |
| Alice   | 210 | 3   | 3    | 8   | 8    | 6   | 6    | 4    |
| Megan   | 211 | 4   | 4    | 3   | 5    | 4   | 3    | 5    |
| Ellen   | 212 | 3   | 4    | 5   | 2    | 4   | 2    | 4    |

Key:  $1=R_1(CS), \quad 2=U_2(CS), \quad 3=M_2(CS), \quad 4=R_2(CS), \\ 5=U_1(F), \quad 6=M_1(F), \quad 7=R_1(F), \quad 8=U_2(F)$