

**THE FINITE SAMPLE PROPERTIES OF THE
ESTIMATORS OF THE TOBIT MODEL:
A MONTE CARLO STUDY**

By

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Abstract

This study examines the small sample properties of some of the estimators of the tobit model. These estimators include, among others, the maximum likelihood estimator (MLE), Heckman's 2-step (H2S) estimator and its weighted version, the weighted Heckman's 2-step (WH2S) estimator, other Heckman-type 2-step estimators based on the unconditional expectation of the model, nonlinear and ordinary least squares estimators. Further, an improved Heckman-type estimator which is referred to as the three-step estimator (3SE) and its weighted version, the weighted three-step estimator (W3SE), are suggested and their properties investigated.

The study investigates the effects of sample size, degree (level) of censoring and error distribution on the properties of the estimators. Furthermore, it examines, among other things, the effects of correlation between the explanatory variables and the estimated inverse of Mill's ratio on the performance of the (Heckman-type) estimators.

Under normally distributed error terms, the MLE estimator performed better than all estimators, followed by the 3SE estimator. The loss in efficiency of the 3SE, compared to the MLE, is quite marginal. However, both the MLE and 3SE estimators appear to be less efficient under the skewed (chi-square) distributed error terms. On the other hand, given a low level of censoring, the MLE estimator performs well under

the students'-t distribution. If the degree of censoring is high, the MLE estimates under the students'-t distribution can be less efficient than the 3SE estimator.

The H2S estimator, although less efficient compared to 3SE or MLE estimators, performs well in terms of bias. However, it can be highly inefficient depending on the level of censoring and/or the degree of correlation between the explanatory variables and the estimated inverse of Mill's ratio. More specifically, if the correlation between the explanatory variables and the estimated inverse of Mill's ratio is high, the H2S estimator can be even worse than the biased ordinary least squares estimator.

The nonlinear least squares estimators are generally inefficient and computationally very slow (unattractive) compared to the 3SE and MLE estimators. More importantly, they are very sensitive to the degree of censoring, and convergence is not always guaranteed.

When the model is estimated by the method of MLE, the variance-covariance matrix for the coefficients of the model can be estimated based on a number of asymptotically equivalent covariance matrix estimators, namely, the information matrix, the Hessian matrix, the outer product of the gradient vector and the robust (White-type) covariance matrix estimator. A detailed examination of these covariance matrix estimators in the estimation of variances and for hypothesis testing for the coefficients of the model reveals that the choice of one of the (four) alternative estimators appears to be neutral, provided that the model is correctly specified (i.e., under normality of the errors). However, if one assumes normality when in fact the errors are generated from the non-normal (i.e., the students'-t and chi-square) distributions, the robust (White-type) covariance matrix appears to be slightly better than others; followed by the covariance matrix estimator based on the outer product of the gradient vector.

Contents

Acknowledgements	iii
Abstract	v
1 Introduction	1
1.1 Objectives of the Study	3
1.2 Outline of the Study	5
2 Review of Literature	9
2.1 Introduction	9
2.2 The Standard Tobit Model	11
2.3 Properties of Least Squares Estimators of the Tobit Model	13
2.4 The Maximum Likelihood Estimator (MLE)	16
2.5 Heckman's Two-step Estimator (H2S)	26
2.6 Weighted Heckman's two-step Estimator	30
2.7 Nonlinear Estimation based on Conditional Expectation	31
2.8 Two-step Estimators based on Unconditional Expectation	33

2.8.1	Heckman's two-step Estimator based on the Unconditional Expectation (H2SU) of the Model	35
2.8.2	Nonlinear Estimation based on the Unconditional Expectation	36
2.9	Highlights of Other Estimators	38
2.9.1	Semi-Parametric Estimators of the Model	39
2.9.2	Bounded Influence Estimators of the Model	41
2.9.3	Bayesian Estimation of the Tobit Model	43
2.10	Some Useful Results	44
2.11	Summary and Conclusions	45
3	An Improved Heckman Estimator and its Properties	48
3.1	Introduction	48
3.2	The Three-step Estimator (3SE)	49
3.3	Asymptotic Properties of the 3S estimator	53
3.3.1	Consistency	53
3.3.2	Asymptotic Distributions of $\hat{\sigma}_{3S}$ and $\hat{\beta}_{3S}$	57
3.4	Some Generalizations of the 3S Estimator	64
3.5	Summary and Conclusions	70
4	Small Sample Properties of Tobit Models: Relevant Monte Carlo/Simulation Studies	72
4.1	Introduction	72
4.2	Small Sample Studies of Tobit Models	73
4.3	Summary and Conclusions	82

5	The Design of the Monte Carlo Experiment	85
5.1	Introduction	85
5.2	The Specification of the Model	87
5.2.1	The Effects of Distributional Assumptions	89
5.2.2	The Effects of the Degree of Censoring	91
5.2.3	The Effects of Sample Size	92
5.3	Data Generation and Estimation Process	92
5.3.1	The Data Generation Process	93
5.3.2	The Generation of Random Variates	96
5.3.3	The Estimation (Monte Carlo) Process	102
5.3.4	Output Statistics	103
5.4	Summary and Conclusions	109
6	Discussion of Results	111
6.1	Introduction	111
6.2	Estimators using only N_1 observations	112
6.3	Estimators using all N observations	131
6.4	Further Analysis of Selected Estimators	142
6.4.1	Comparison of Variances and Hypothesis Testing	147
6.5	Summary and Conclusions	158
7	The MLE: Consistency, Variance Estimation and Hypothesis Test-	
	ing	161
7.1	Introduction	161
7.2	Consistency of the ML Estimator	165

7.2.1	Monte Carlo Comparisons for Consistency	169
7.3	Asymptotic Distribution of the MLE	175
7.3.1	Monte Carlo Comparison of Variance Estimators	181
7.3.2	Implications of the Variance Estimators for Hypothesis Testing	193
7.4	Summary and Conclusions	207
8	The 3SE Vs H2S Estimator: The Effects of Correlation	211
8.1	Introduction	211
8.2	An Overview of the H2S and 3S Estimators	212
8.3	The Design of the Experiment	215
8.3.1	The Model	215
8.3.2	The Data Generation Process	217
8.4	Comparison of Results	219
8.5	Summary and Conclusions	236
9	Selected Discussion and Comments	239
9.1	Introduction	239
9.2	Random Explanatory Variables	240
9.3	Left Vs Right Hand Censoring	245
9.4	The Effects of the Constant Term	251
9.5	Summary and Conclusions	258
10	Summary, Conclusions and Recommendations	261
	Appendix A: Output Tables	272
	References	281

List of Tables

3.1	Correlation between ξ and $\Phi(\xi)$	69
6.1	Results for Estimators using only N_1 observations given $N=100$ and 25% Degree of Censoring for the three Distributions.	114
6.2	Results for Estimators using only N_1 observations given $N=200$ and 25% Degree of Censoring for the three Distributions.	116
6.3	Results for estimators using only N_1 observations, given $N=400$ and 25% degree of censoring for the three distributions.	117
6.4	The Effects of the Degree of censoring for Estimators using only N_1 observations, given $N=100$ and normally distributed error terms. . . .	119
6.5	Results for Estimators using only N_1 observations given $N=100$ and 50% Degree of Censoring for the three Distributions.	120
6.6	Results for estimators using only N_1 observations, given $N=100$ and 75% degree of censoring for the three distributions.	122
6.7	The effects of the degree of censoring for estimators using only N_1 observations, given $N=400$ and normally distributed error terms. . . .	124
6.8	Comparisons of H2S and 3SE estimates, given a Sample Size of 100, 25% degree of Censoring and the three distributions.	127

6.9	Summary Notes on the Relative Performance of the Various Estimators.	130
6.10	Results for Estimators using all observations given N=100 and 25% Degree of Censoring for the three Distributions.	133
6.11	Results for Estimators using all observations, given N=400 and Degree of Censoring of 25% for the three Distributions.	134
6.12	The Effects of Sample Size on the Estimators using all observations, given 25% Degree of Censoring and Normally Distributed error terms.	135
6.13	The Effects of Degree of Censoring for Estimators using all observa- tions, given N=200 and Normally distributed error terms.	137
6.14	The Effects of Degree of Censoring for Estimators using all observa- tions, given N=200 and Chi-Square Distributed error terms.	139
6.15	Summary Notes on the Relative Performance of the Various Estimators.	141
6.16	Relative Root Mean Square Errors (RMSE) for all Sample Sizes and Distributions, given 25% Degree of Censoring.	144
6.17	Relative Root Mean Square Errors (RMSE) for all Sample Sizes and Distributions, given 50% Degree of Censoring.	145
6.18	Comparison of Variances of Estimators for all Sample Sizes and Dis- tributions, given 25% Degree of Censoring.	149
6.19	Comparison of Variances of Estimators for all Sample Sizes and Dis- tributions, given 50% Degree of Censoring.	150
6.20	95% Confidence Intervals of Estimators for all Sample Sizes and Dis- tributions, given 25% Degree of Censoring.	156
6.21	95% Confidence Intervals of Estimators for all Sample Sizes and Dis- tributions, given 50% Degree of Censoring.	157

7.1	Empirical Inconsistency ($I_N(\theta)$) of the MLE for the three Distributions and Sample sizes.	170
7.2	Comparison of Variance Estimators of the MLE Estimator for all Sample Sizes and Distributions, Given 25% Degree of Censoring.	183
7.3	Comparison of Variance Estimators of the MLE Estimator for all Sample Sizes and Distributions, Given 50% Degree of Censoring.	187
7.4	Comparison of Variance Estimators of the MLE Estimator for all Sample Sizes and Distributions, given 75% Degree of Censoring.	189
7.5	95% Confidence Intervals using the four Variance Estimators of the MLE for all Sample Sizes and Distributions, Given 25% Degree of Censoring.	196
7.6	95% Confidence Intervals using the four Variance Estimators of the MLE for all Sample Sizes and Distributions, Given 50% Degree of Censoring.	198
7.7	95% Confidence Intervals using the four Variance Estimators of the MLE for all Sample Sizes and Distributions, Given 75% Degree of Censoring.	200
7.8	Ranking on the Robustness of the Estimators.	202
7.9	Further Ranking of Robustness of the Estimators.	204
8.1	Comparison of Estimators under Various levels of Correlation, Given N=100 and 25% Degree of Censoring.	221
8.2	Comparison of Estimators under Various levels of Correlation, Given N=200 and 25% Degree of Censoring.	224

8.3	Comparison of Estimators under Various levels of Correlation, Given N=400 and 25% Degree of Censoring.	227
8.4	Comparison of Estimators under Various levels of Correlation, Given N=200 and 50% Degree of Censoring.	230
8.5	Finite Sample Root Mean Square Errors Relative to those of the MLE estimator.	232
8.6	Finite Sample Standard Errors Relative to Asymptotic Standard Errors.	235
9.1	Results for Estimators using Random Explanatory variables, given a 25% Degree of Censoring for the three Distributions.	243
9.2	Results for Estimators based on a 25% Left-hand Censoring for the Chi-square Distribution.	249
9.3	The likely Effects of Changes in the Constant term (β_0) on Responses associated with the Tobit Model for N=200 and 25% Degree of Censoring	256
9.4	The likely Effects of Changes in the Constant term (β_0) on Responses associated with the Tobit Model for N=200 and 50% Degree of Censoring.	257
A.1	The effects of Sample size for estimators using only N_1 observations, given 25% degree of censoring and normally distributed error terms. .	273
A.2	The effects of Sample size for estimators using only N_1 observations, given 25% degree of censoring and chi-square distributed error terms.	274
A.3	Results for estimators using only N_1 observations, given N=200 and 50% degree of censoring for the three distributions.	275
A.4	Results for estimators using only N_1 observations, given N=400 and 50% degree of censoring for the three distributions.	276

A.5	Results for estimators using only N_1 observations, given $N=400$ and 75% degree of censoring for the three distributions.	277
A.6	Estimated Results for the MLE estimators for all Distributions and degrees of Censoring, Given a Sample size of 100.	278
A.7	Comparison of Estimators under Various levels of Correlation, Given $N=100$ and 50% Degree of Censoring.	279
A.8	Comparison of Estimators under Various levels of Correlation, Given $N=400$ and 50% Degree of Censoring.	280