THE FINITE SAMPLE PROPERTIES OF THE
ESTIMATORS OF THE TOBIT MODEL:
A MONTE CARLO STUDY

By

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Abstract

This study examines the small sample properties of some of the estimators of the tobit model. These estimators include, among others, the maximum likelihood estimator (MLE), Heckman’s 2-step (H2S) estimator and its weighted version, the weighted Heckman’s 2-step (WH2S) estimator, other Heckman-type 2-step estimators based on the unconditional expectation of the model, nonlinear and ordinary least squares estimators. Further, an improved Heckman-type estimator which is referred to as the three-step estimator (3SE) and its weighted version, the weighted three-step estimator (W3SE), are suggested and their properties investigated.

The study investigates the effects of sample size, degree (level) of censoring and error distribution on the properties of the estimators. Furthermore, it examines, among other things, the effects of correlation between the explanatory variables and the estimated inverse of Mill’s ratio on the performance of the (Heckman-type) estimators.

Under normally distributed error terms, the MLE estimator performed better than all estimators, followed by the 3SE estimator. The loss in efficiency of the 3SE, compared to the MLE, is quite marginal. However, both the MLE and 3SE estimators appear to be less efficient under the skewed (chi-square) distributed error terms. On the other hand, given a low level of censoring, the MLE estimator performs well under
the students'-t distribution. If the degree of censoring is high, the MLE estimates under the students'-t distribution can be less efficient than the 3SE estimator.

The H2S estimator, although less efficient compared to 3SE or MLE estimators, performs well in terms of bias. However, it can be highly inefficient depending on the level of censoring and/or the degree of correlation between the explanatory variables and the estimated inverse of Mill's ratio. More specifically, if the correlation between the explanatory variables and the estimated inverse of Mill's ratio is high, the H2S estimator can be even worst than the biased ordinary least squares estimator.

The nonlinear least squares estimators are generally inefficient and computationally very slow (unattractive) compared to the 3SE and MLE estimators. More importantly, they are very sensitive to the degree of censoring, and convergence is not always guaranteed.

When the model is estimated by the method of MLE, the variance-covariance matrix for the coefficients of the model can be estimated based on a number of asymptotically equivalent covariance matrix estimators, namely, the information matrix, the Hessian matrix, the outer product of the gradient vector and the robust (White-type) covariance matrix estimator. A detailed examination of these covariance matrix estimators in the estimation of variances and for hypothesis testing for the coefficients of the model reveals that the choice of one of the (four) alternative estimators appears to be neutral, provided that the model is correctly specified (i.e., under normality of the errors). However, if one assumes normality when in fact the errors are generated from the non-normal (i.e., the students'-t and chi-square) distributions, the robust (White-type) covariance matrix appears to be slightly better than others; followed by the covariance matrix estimator based on the outer product of the gradient vector.
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