CHAPTER 1 - INTRODUCTION

1.1 DEFINITION OF CHILDREN WITH LEARNING PROBLEMS

For decades attempts have been made to identify and define children with learning problems or special learning needs. In some cases the process was motivated by a search for explanations or understanding of their physical or intellectual conditions with a medical model in mind. In other cases the identification and definition was for the purpose of providing adequate services and making funding allocations for support of their special needs. Of most relevance and interest in this project, however, is definition and identification for the purpose of obtaining the closest possible match between children's difficulties and needs and the educational provisions made for them. Such terms as 'educable mentally retarded', 'slow learner', 'remedial','educationally subnormal', 'at risk', 'developmentally delayed', 'culturally deprived','special needs','learning disabled', and 'Attention Deficit Disorder' have all "been applied and enjoyed a temporary fashion" (Ward et.al.,1987,ix). Children with learning problems can clearly be defined and grouped in a myriad of ways depending upon one's focus and purpose. However, as far as a classroom teacher is concerned, such children have in common that they :

> are either out of touch, or becoming out of touch, with their peers in the educational mainstream... and may be experiencing major difficulties with their schoolwork, their emotional and social development and their participation in expressive, creative and recreational activities.
> (Ward et. al., 1987,ix).

A more general and less emotive definition of children with learning problems would be that they are "socio-educationally marginal" in that the major pointer of learning problems is generally failure to reach expected standards in academic achievement and/or personal adjustment (Ward et. al.,1987,ix). Frequently it is this failure to achieve or to 'keep up' that leads to multifaceted formal and informal testing in search of an individual profile of a student's needs.

One of the advantages of using such a broad definition is that the inherent dangers of long term labelling and its consequences, or of identification for its own sake and not for the sake of intervention or remediation , can be avoided to some degree. The definition is one of context and current behaviour and may be relevant to most learners at some stage without condemning them forever to a lowered set of expectations.

1.2 WHY REGULAR CLASSROOMS?

One of the most effective ways of increasing children's socio-educational marginality is to remove them from normal peer relationships and activities. Many studies have noted this effect and the development of institutionalised behaviours, lowered self-esteem and an increasingly widening gap in achievement between children with learning difficulties in special programs and their peers (Tawney & Gast, 1984, 52-3).

It is now generally accepted that in the vast majority of cases children with learning problems or special educational needs can and should be "maintained successfully within the regular school system, given that it provides an adequate level of resources, a supportive environment and skilled and sensitive teaching" (Ward et.al.,1987,ix). In fact one of the major roles of regular classroom teachers is to work towards reducing the proportion of children who are socio-educationally marginal at any one time. Special Education resource and support services should generally be directed to assisting classroom teachers to achieve this aim in the least invasive way and without removing responsibility for the child's progress either from the regular teacher or from the child.

Research studies have shown poor or conflicting results in academic gains made by children with mild learning problems in withdrawal or segregated situations, as well as poor transfer and maintenance of gains (Tawney & Gast, 1984, 56). Such findings, added to knowledge of the personal and social disruption of the learner, give strong support to the maintenance of socio-educationally marginal students in regular classrooms with appropriate teaching, resources and support as the preferred model.

In view of the previous discussion much of the following paper will consider the regular Year 3 classroom situation as the normal environment for children with learning difficulties in mathematics. In general the texts and manipulatives reviewed will be those used by primary teachers with all pupils, although the pace and intensity of instruction should differ with the fluctuating needs and skills of individuals within classes.

1.3 WHY UNDERSTANDING AND SKILLS?

Understanding can be defined in many ways. In terms of mathematical concepts and structures "a shared knowing of each other's meaning"; a familiarity with the "character or nature"; and "knowing the meaning of the words used" (Macquarie Dictionary) come close to encompassing the complexity of "understanding" as used in teaching. Skill could be described as "efficiency and excellence in performance"(Macquarie Dictionary). Thus, "skills" relates to performance of mathematical activities or computations with overtones of both accuracy and fluency.

From a currently popular, Constructivist viewpoint Hendrickson (1983,42) wrote:

> What we know of how children learn has not been applied to the curriculum, with only a few exceptions. Children are given little time to think and develop their own ways. The need to 'get inside the child's head', to understand the kind of thinking done by the child, and to recognise that the child constructs knowledge, rather than accepting it as someone else's construction, must be satisfied before teaching can be useful to the child.

That is, "no-one can teach mathematics. Effective teachers are those who can stimulate students to learn mathematics" (Clements & Battista,1990,34). In other words a shared knowing of each other's meaning is highly relevant since the child needs to acquire or construct understandings which are useful, and in line with the general view in order to operate effectively in mathematical situations in class and in real-life.

The aims of education may be broadly stated as "ensuring economic, vocational, and social independence; active participation as a citizen in the affairs of the community, state and nation; and educational competence" (Lowenbraun & Affleck,1976,3). For some exceptional children such aims are impractical or inappropriate and need modification. As educators, however, our responsibility is to ensure that as many as possible of our students move towards attaining these goals whilst in our care.

Narrowing our scope back to the teaching of mathematics to children with learning problems, if there is only focus on developing skills there is a limit being set for the student as an independent learner. The use of related understandings is necessary for skills to be applied in novel situations or in novel ways as the need arises. Conversely, an individual with a good grasp of concepts or understandings can also be limited in independence if the related skills are poorly learned, or lacking in fluency or accuracy. Thus, skills and understandings go hand in hand and need to be taught in an integrated way.

Outhred and Center (in Ward,1987,141) suggest that for both normal and exceptional learners a major cause of children's difficulty with number tasks has been :

> The emphasis that has been given to calculations expressed in symbolic forms in many traditional mathematics programs. The mechanical manipulation of symbols with no regard to meaning encourages the memorisation of rules rather than a consideration of the concepts involved.

Obviously this sort of learning situation may be made even more frustrating and fruitless in cases where the learner also experiences memory and perceptual problems.

Understanding refers not only to a grasp of the concepts relevant to the mathematical activity in hand but also to the interrelationships between mathematical concepts (eg. multiplication and repeated addition). Transfer of learning from one situation or field to another may occur spontaneously but more often must be overtly taught, especially with less able learners (Sweller & Low, 1992).

Various theories have been developed to explain levels or hierarchies of development of understanding (Cramer & Bezuk,1991). Bruner's enactive; iconic; and symbolic stages and Lesh's (i) real world situations
(ii) manipulatives (iii) pictures (iv) spoken symbols ,and (v) written symbols are readily absorbed into the concrete ----> abstract learning process generally recommended or prescribed. The N.S.W. Mathematics K-6 Syllabus (1989,5) states :

> As each new mathematical concept is encountered, learning should proceed, where possible, from the concrete to the abstract. Concepts should be continually developed and consolidated through a wide variety of learning experiences. The development of understanding should, as a general principle, precede a requirement for both automatic recall of factual information and speed and accuracy in performing mathematical computations. Skills should be maintained through meaningful practice and enjoyable drill.

Nevertheless, the Speedy report (1989 in Crawford,1990) revealed that in many Australian schools, classroom practice is "little different from what it was twenty years ago", and urgently recommended changes because:

> Mathematics on the whole, as a school subject, or as a higher education course, is not sufficiently related to the developing needs of our society which is being driven to respond to, and interact with, more technologically advanced countries and regions. The societal demands evolving from this situation, particularly in the context of the generation of knowledge, of information access and usage, and of human resource development, are extensive, urgent and very much the responsibility of mathematics education. Mathematics can no longer be treated as a 'chalk and talk' or 'paper and pencil' subject. To be skilled in the mechanics is no longer sufficient. To be skilled in applying mathematical knowledge across the whole of real life situations is imperative.

1.4 WHY TIME AND FRACTIONS?

Why teach Mathematics at all? The N.S.W. Mathematics K-6 Syllabus (1989,2) includes the following rationale for teaching Mathematics :

> Mathematics is useful.
>
> * Mathematics is essential for living. Some aspects of mathematics are required by individuals in order to function adequately as members of society. These aspects include strategies, skills and techniques involved in number facts, computation, mathematical problem solving and reasoning.
>
> * Mathematics is important and useful in many fields of endeavour. These fields include the sciences, medicine, economics, commerce, industry, engineering, business and the arts.
>
> * Mathematics provides a means of oral and written communication. Mathematics can be used to present and convey information in many ways. Some of these include explanations, figures, letters, tables, charts, diagrams, graphs and drawings.
>
> * Mathematics provides opportunities for development of reasoning abilities.

Mathematics is part of our culture.

* Mathematics has been part of human activity since the earliest times. It has made, and continues to make, a significant contribution to human culture. Mathematics allows children to appreciate their cultural heritage more fully by providing insights into many of the creative achievements of the human race.

Mathematics can be part of our leisure.

* Mathematics is a source of interesting and appealing puzzles and problems. When mathematics is enjoyable it encourages curiosity, exploration, discovery and invention.

So, why focus on the teaching of Time and Fractions? An understanding of and the ability to operate on Time and Fractions skilfully is :

(i) useful in everyday life

(ii) part of the language of everyday life

(iii) essential to full community participation and acceptance (eg. being on time, using community services)

(iv) frequently poorly learned or understood by less able students as a result of more emphasis being placed on the four basic operations of +,-, X and ÷ (Wyatt,1993,504; Cramer and Bezuk, 1991,34)

(v) ideal to establish links between mathematical understandings and skills and practical, real-life applications of mathematics.

The Cockcroft report (1982,10) places emphasis on telling the time as a necessary adult life skill, as well as the ability to estimate and apply mathematical understandings as can be seen below:

> 32. Therefore, whilst realising that there are some who will not achieve all of them, we would include among the mathematical needs of adult life the ability to read numbers and to count, to tell the time, to pay for purchases and to give change, to weigh and measure, to understand straightforward timetables and simple graphs and charts, and to carry out any necessary calculations associated with these...
>
> 33. We believe too that, as a necessary accompaniment to the list which we have given, it is important to have the feeling for number which permits sensible estimation and approximation...
>
> 34. Most important of all is the need to have sufficient confidence to make effective use of whatever mathematical skill and understanding is possessed, whether this be little or much.

Likewise, the Curriculum and Evaluation Standards for school mathematics (grades K-4) (NCTM, 1987) includes:

> STANDARD 1: MATHEMATICS AS PROBLEM SOLVING
> In grades K - 4, problem solving should permeate the mathematics curriculum so that students can:
> * use problem solving processes in their learning of all mathematical content;
> * use strategies in solving a wide variety of problems from many contexts;
> * discuss alternate solution strategies and relationships among problems;
> * formulate problems.
>
> STANDARD 4: ESTIMATION
> In grades K - 4, the curriculum should include estimation so students can:
> * explore estimation strategies;
> * recognise when an estimate is appropriate;
> * use estimation to determine reasonableness of results;
> * apply estimation in working with quantities, measurement, computation, and problem solving.

STANDARD 8: MEASUREMENT
In grades K - 4, the mathematics curriculum should include measurement so that students can:
* understand measurable attributes, the concept of a unit, and the process of measuring;
* make and use measurements in problem and everyday situations;
* make and use estimates in measurement situations.

STANDARD 11: FRACTIONS AND DECIMALS
In grades K - 4, the mathematics curriculum should include fractions and decimals so that students can:
* develop concepts of familiar fractions, mixed numbers, and decimals to tenths;
* extend number sense to include fractions and decimals;
* use models to relate fractions to decimals and to find equivalent fractions;
* use models to explore operations on fractions and decimals;
* apply fractions and decimals to problem situations.

Children enter school with a range of informal knowledge relating to Time and Fractions as they have impacted upon their everyday experiences. The language of Time and Fractions is probably used more widely in common speech than most other mathematical subject matter. Therefore, children are exposed to these real-life applications naturally and quite frequently. (Eg. "Come in now its tea-time"; "Good. You've half finished.")

The N.S.W. Mathematics K-6 Syllabus (1989) outlines a sequence of objectives for the teaching of Time and Fractions with recommendations as to which of them should be achieved by the end of Year 2 (Level 1), the end of Year 4 (Level 2) and the end of Year 6 (Level 3) (pp 51; 175; 262). These will be discussed in more detail in Chapter 4.

Research studies in Australia and around the world have revealed that while basic skills are often present, understandings and real-life applications relating to Time and Fractions are frequently poorly developed in both normal and disabled students (Strang,1990). This immediately alerts us to an examination of how Time and Fractions are generally taught. With regard to Fractions in Australia in many cases 'chalk and talk'; or symbolic demonstration without manipulatives is used by teachers. Also, children are moved rapidly from concrete to abstract experiences (Crawford,1990). Similarly in Europe, Strang reported (1990 in Steffe & Olive,1991):

> Researchers in Finland have warned for some years that children learn mathematics too mechanically in our comprehensive school. They learn rules and tricks, but not mathematical thinking. It is rote learning without meaning. I think this is what happens often with the fraction concept.

Examination of U.K.studies of Fraction learning revealed similar findings of poor understandings and limited skills based on rote memory techniques (Steffe & Olive,1991). Cramer and Bezuk (1991) reported similar findings in U.S.studies of children's competency in Fractions.

Time is probably better understood because it is so commonly used in real-life. However, Time is generally taught by relating visual cues to symbolic/ abstract skills. Wyatt (1993) found in the U.S. that whilst children could name times as displayed they had "no real understanding of the length of one minute". They were unable to estimate or mentally manipulate time concepts. Application of time knowledge to real-life problems - such as when to set out to walk to the bus stop in time to catch the 8:40 bus - requires estimation and mental arithmetic whilst operating on times.

If we are to teach Time well, this level of skill and understanding is necessary, not just rote recognition skills. Furthermore, the connection between Time and Fractions and real-life situations needs to be established for children's wider understanding, along with the inter-relationships or connections between Time and Fractions.

1.5 WHY USE CONCRETE MATERIALS ?

Concrete materials consist of "any objects used in a hands on, experiential, manipulative context to facilitate understanding of relationships and abstract concepts" (Lewis,1990). They can be everyday objects (eg. real containers and packages, money); readily available objects (eg. paddle pop sticks, pegs and string); or materials structured with special attributes designed to encourage the development of specific concepts (eg. Multi Attribute Blocks, rulers, centicubes).

In the initial stage of instruction "the use of concrete materials wherever possible is critical...Manipulative experiences are a critical part of the process of linking the concrete representations of the mathematical idea to its more abstract-symbolic representation" (Capps & Pickreign,1993,9).

The N.S.W. Mathematics K-6 Syllabus (1989,5) endorses this view, prescribing that "as each new mathematical concept is encountered, learning should proceed, where possible, from the concrete to the abstract. Concepts should be continually developed and consolidated through a wide variety of learning experiences".

Whatever theoretical stance teachers or researchers adopt - Cognitivist, Constructivist, Social Learning, Behaviourist, etc - there appears to be general agreement that concrete materials have a significant role to play in the development of mathematical understandings and skills (Stigler & Stevenson,1991). The amount of time given to or emphasis on the use of manipulatives may vary widely due to theoretical stance and the needs of learners, but all concede that manipulatives should be considered and utilised.

The use of the term "manipulatives" has become prominent of late. Not only does it convey the active nature of the materials more strongly, but it also breaks down the direct association back to Cognitivist, Developmental - Stage theories which can be limiting or offputting to the practitioner.

The advocacy of manipulatives is not new. Sophocles wrote "One must learn by doing the thing; for though he thinks he knows it, he has no certainty until he tries" (in Bauer & George,1976,11). Mathematics should be an active, performance (physical and mental) area of learning, not a dry and abstract pursuit.

A continuum of mathematical learning is a useful idea (Baur & George, 1976,38).

CONCRETE SEMICONCRETE ABSTRACT	Mathematical
<----------------------------------->	idea to be
	learned

Using this conceptualisation it can be seen that learning should proceed from concrete experiences to abstractions and that this can occur when any new learning is in

progress, whatever the age or stage of the learner.

CHAPTER 2 - THEORETICAL ISSUES AND BELIEFS

2.1 THEORETICAL OVERVIEW

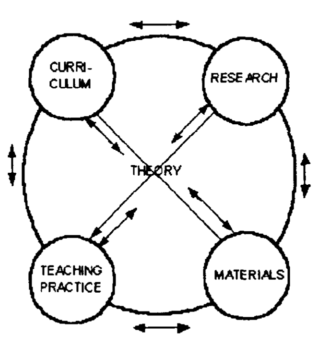
Theoretical constructs are relevant to and do impact on the teaching of mathematics, specifically Time and Fractions, to junior primary level students, including those with special needs or learning difficulties being educated in regular classrooms.

Special needs students have been variously described as "educationally disadvantaged, slow learning, minimally or mildly intellectually impaired, learning disabled or educationally marginal"(Ward et al,1987,xi). Numerical estimates of this population vary according to definitions and identification procedures, but a consensus of about 15% of children in regular schools falling into the special needs category at some time in their education seems realistic (Thorley & Mills,1986). Since the numbers are so great, withdrawal for special education or remediation by specialists is impractical and prohibitively expensive in all but the most severe cases; as well as being socially detrimental and of dubious long-term benefit (Tawney & Gast, 1984, 56). Hence, in the majority of situations, regular teachers in regular classrooms must deal with the special needs of these students by differentiating their curricula and applying appropriate theoretical constructs and teaching strategies.

This discussion will proceed from broad issues of general educational and psychological theory to a narrower review of the relationship of such theory to current mathematics curricula and teaching practice. Such a funnel-shaped review is necessitated by the very nature of the inter-relationships between theory, research, curriculum, materials, and teaching practice. In a perfect world, the relationship between these areas would be very close with little time-lag and much interaction between them. In general the reality is very different.

A step back in time to the set theory of the old "New Math" may help to illustrate this point. In a perfect world all research, curricula, teaching practice and materials would be entrenched in, yet active upon, theory. All areas would interact, relate, and develop in an osmotic manner.

Diagram 2A



The reality is that due to many insurmountable factors, this perfect world cannot exist. Some of the major hindrances to pure, theoretically based education are :-

(i) Theorists are frequently removed from practical experience (eg. creating abstractions; dealing with restricted populations).

(ii) Researchers must often alter theories or "reduce" them to gain valid and reliable results.

(iii) Reductionism in research can lead to "excellently constructed" research that is of very limited application or value to practitioners.

(iv) Practitioners, teachers, are concerned with specific individuals and very complex, interactive, personal situations. At this level, general theories or statistical truths across populations often break down.

(v) The time lag between the development of theories; publication; research studies; publication; and dissemination to practitioners is great and means that there is always a range of theories, research, curricula, materials, and practices being implemented at any one time.

(vi) The flow of information can be in any direction so that theories can develop from teaching practice; from research; or from some other area altogether (eg. from medicine, psychology, anthropology, or sociology).

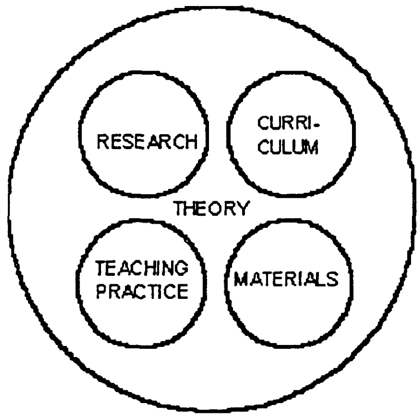
(vii) The financial cost of developing theories; researching and testing them; and then translating them into curricula, materials and practice is such that there is a reluctance to adopt new, potentially better, ways before old ones have been fully utilised.

(viii) The financial aspect can also influence which theories and research get developed and which fail for lack of funds. Politics, publicity, and general popularity (face validity) may also influence which theories flourish and which founder.

The following models of interaction are proposed as a way of illustrating the different perceptions of theoretical impact depending upon one's role in the educational situation.

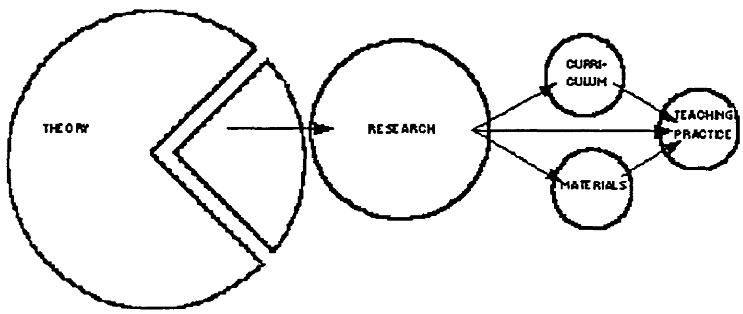
A theorist would consider the following an appropriate, if unlikely, scenario.

Diagram 2B



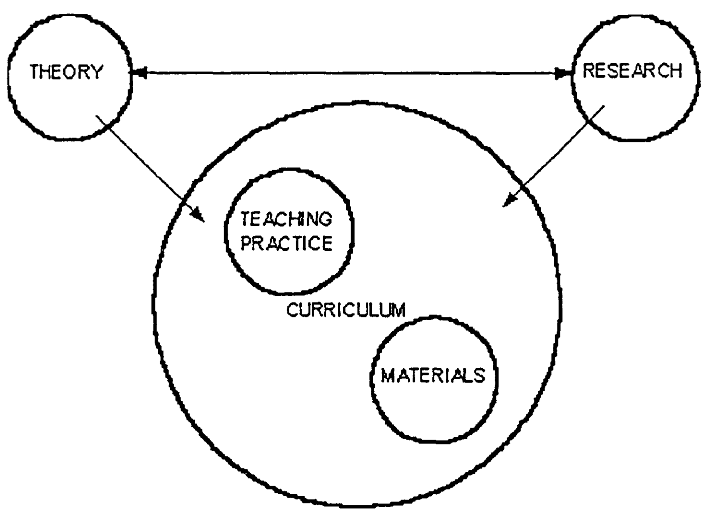
A researcher might see the situation more like this: -

Diagram 2C



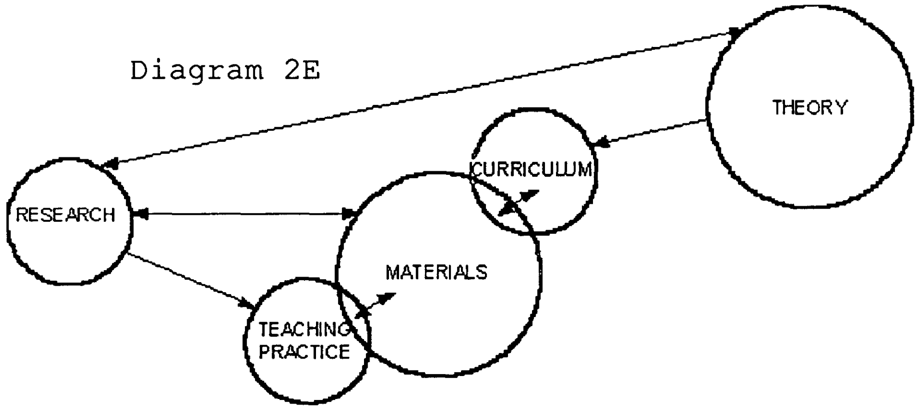
A curriculum planner might view the relationship thus: -

Diagram 2D



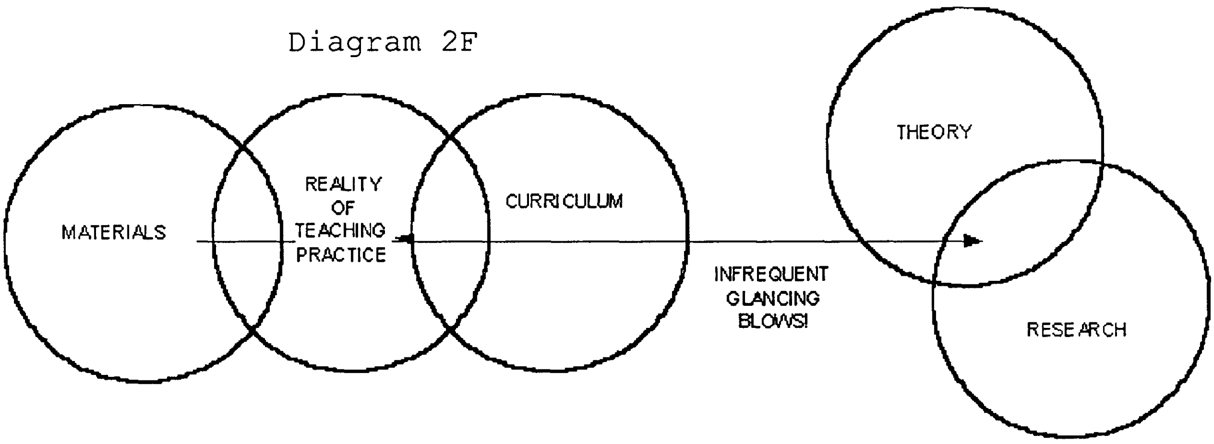
A teaching materials developer might see the situation more like this: -

Diagram 2E



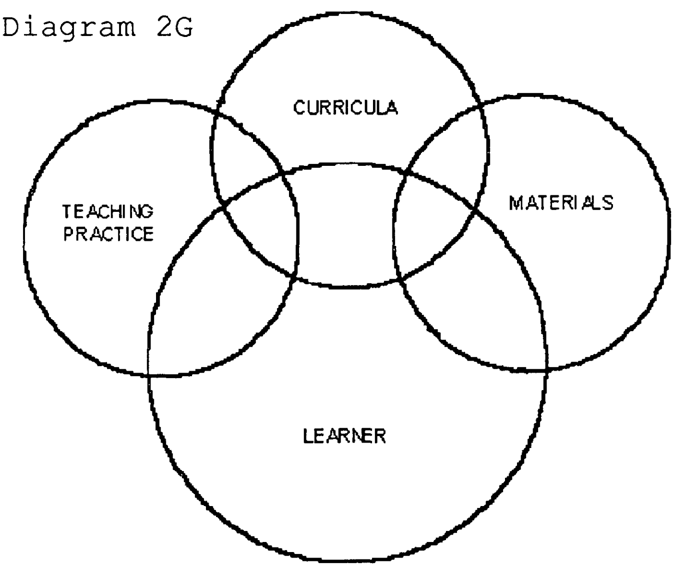
A teacher might view the situation more like this: -

Diagram 2F



A learner would most likely perceive the situation as follows: -

Diagram 2G



2.2 BELIEF SYSTEMS OF EDUCATORS

Most teachers are familiar mainly with the theoretical perspectives they were exposed to during their initial teacher training, along with their repertoire of "traditional beliefs" about teaching acquired during their own schooling. There is, as yet, no compulsion for teachers - unlike lawyers, doctors, - to keep abreast of theoretical and research developments directly. Their exposure to these generally comes indirectly through the impact of theory and research on curriculum documents and teaching materials. Teachers tend to be pragmatic and eclectic, rather than theoretical purists, in response to the "complex" and "uncontrollable" (Silver, 1990) nature of their work.

Nevertheless, traditional beliefs and educational theories have an influence on educational practice - however "bastardised" they may be.

The following table displays some widely held traditional beliefs held by educators identified by Lindgren (1972). Belief systems are complex and internalised yet can influence the way in which theories, curricula and materials are implemented by teachers. As Lindgren (1972, 162) put it: "Everyone has some kind of theory about learning". In many cases these beliefs become part of the theory "constructed" in the mind of the practitioner; or influence the theoretical standpoint adopted by a practitioner in order to accommodate their belief system.

TABLE 2.1: BELIEFS

*That learning occurs because the learner has been rewarded or punished.

*That learning is primarily a process of accumulating knowledge.

*That learning is primarily a process of accumulating skills.

*That things properly taught are retained indefinitely.

*That repetition is needed for learning.

*That learning results from being told.

*That learning should proceed deductively.

*That learning should proceed inductively.

*That learning transfers automatically.

*That learning should be stressful.

*That learning should be pleasant.

*That learning has intrinsic value.

*That learning must have practical value.

*That the intrinsic nature of the learner is of prime importance in educational situations.

*That extrinsic factors are of prime importance in educational situations.

(Adapted from Lindgren, 1972, 162)

It can be seen that some of these beliefs are in direct contrast to each other, whilst many would exist in the community on a continuum. Furthermore,many have been shown to be erroneous or of limited validity or reliability by research.

Nevertheless, there is a good case for teachers becoming aware of their underlying beliefs about learning since they are likely to influence which theoretical models they adopt and the way they translate theory into practice.

Elements of these beliefs can be readily identified in the various theories of Educational Psychology which have come to prominence and influenced educational research, planning and practice in general, and the teaching of Mathematics in particular.

2.3 OVERVIEW OF THEORIES OF EDUCATIONAL PSYCHOLOGY AND MATHEMATICS

An overview of the major theories of teaching/ learning is presented below in Summaries 1 to 4, grouped according to theoretical orientation, as well as loose historical development. The volume of literature relating to educational theories is enormous, dating back to ancient times. This overview is an attempt at analysing and synthesising some of this literature, whilst highlighting major points of interest and difference.

SUMMARY 1

MAJOR THEORIST: John Locke

ORIENTATION: External factors

MODEL: Tabula Rasa

IMPACT ON EDUCATIONAL PRACTICE: Concept of the learner, or child, as initially a "blank tablet".

IMPACT ON MATHEMATICS EDUCATION: 18th and 19th Century view of mathematics education as knowledge being installed.

****

MAJOR THEORIST: Thorndike

ORIENTATION: External focus

MODEL: Drill and practice

IMPACT ON EDUCATIONAL PRACTICE: Rote memorisation of facts; work broken down into small steps of increasing complexity or difficulty.

IMPACT ON MATHEMATICS EDUCATION: Stimulus- Response learning; memorisation of facts and algorithms; task analysis; facility with computation primary aim; deductive process; very influential in the 1920s - 30s.

****

MAJOR THEORISTS: Skinner, Popham

ORIENTATION: Behaviour Modification

MODEL: Operant Conditioning; Direct Instruction; Precision Teaching

IMPACT ON EDUCATIONAL PRACTICE: Determining educational objectives; developing, changing and maintaining behaviours.

IMPACT ON MATHEMATICS EDUCATION: Behavioural objectives; careful task analysis and sequencing; reinforcement schedules; skills / performance orientation. Still influential, especially in Special Education field.

REFERENCES: Gage & Berliner,1975,444-7.
Kroll, 1989, in NCTM Yearbook.
Rothstein,1990,53.

These models and theories share a common focus on the factors external to the learner, that is, factors that the teacher can change and control, with changes able to be monitored with regard to their effectiveness. A major criticism of such an approach is that all of the unseen internal factors within learners remain largely unseen and unused, which can lead to real-life problems for teachers and students and sometimes to reduced effectiveness.

SUMMARY 2

MAJOR THEORIST: Eric Erikson

ORIENTATION: The inner person

MODEL: Psychoanalytic

IMPACT ON EDUCATIONAL PRACTICE: 8- stage theory of psycho-social development; coined the phrase "identity crisis".

IMPACT ON MATHEMATICS EDUCATION: Focus on internal learning conditions, early to middle 20th Century.

****

MAJOR THEORIST:Kohlberg

ORIENTATION: The inner person

MODEL: Moral development

IMPACT ON EDUCATIONAL PRACTICE: Identified 6 stages of moral reasoning; stage theory.

IMPACT ON MATHEMATICS EDUCATION: Internal, learner oriented, "stage" theories becoming prevalent.

****

MAJOR THEORIST: Carl Rogers

ORIENTATION: The person

MODEL: Non-directive teaching

IMPACT ON EDUCATIONAL PRACTICE:Personal development through capacity for self instruction; self-understanding, self-discovery, self-concept.

IMPACT ON MATHEMATICS EDUCATION: Inductive learning process, discovery methods.

****

MAJOR THEORIST: William Glasser

ORIENTATION: The person, groups

MODEL: Classroom meeting.

IMPACT ON EDUCATIONAL PRACTICE: Personal understanding and responsibility; improved social functioning.

IMPACT ON MATHEMATICS EDUCATION: Concept development; learner responsibility; group learning.

****

MAJOR THEORIST: William Gordon

ORIENTATION: The person, problem-solving

MODEL: Synectics

IMPACT ON EDUCATIONAL PRACTICE: Personal creativity and creative problem solving.

IMPACT ON MATHEMATICS EDUCATION: Problem solving emphasis; creativity in mathematics.

MAJOR THEORIST: Brownell

ORIENTATION: Gestalt psychology

MODEL: Meaningful arithmetic

IMPACT ON EDUCATIONAL PRACTICE: Emphasis on relationships; incidental learning; activity - oriented approach.

IMPACT ON MATHEMATICS EDUCATION: Mathematics activities; understanding of arithmetic ideas and skills; application of mathematics skills to real-life problems; influential in 1930s - 50s, renewed interest in 1980s to the present.

REFERENCES: Gage & Berliner, 1975, 444-7.
Kroll, 1989, in NCTM Yearbook
Rothstein, 1990, 53.

All of these models and theories emphasise the internal features of learners - their creativity, personal complexity, and the effects of personality and social relationships on learning. In many ways this forms a truer view of learning and learners yet adds a great many internal, uncontrollable and ethereal factors to the teacher's role. Many teachers and theorists, whilst acknowledging the existence of such factors, feel that they cannot be acted upon in an effective and coherent way.

SUMMARY 3

MAJOR THEORIST: Hilda Taba

ORIENTATION: Information processing

MODEL: Inductive teaching

IMPACT ON EDUCATIONAL PRACTICE: Emphasis on inductive theory-building and reasoning processes.

IMPACT ON MATHEMATICS EDUCATION: Spiral curriculum development, inductive teaching, emphasis on reasoning not rote learning.

****

MAJOR THEORIST: Jerome Bruner

ORIENTATION: Information processing, concepts.

MODEL:Concept attainment.

IMPACT ON EDUCATIONAL PRACTICE: Inductive reasoning; learning through discovery.

IMPACT ON MATHEMATICS EDUCATION: Activity-based inductive learning situations, emphasis on development of concepts rather than skills.

****

MAJOR THEORIST: Robert Gagne

ORIENTATION: Information processing

MODEL: Principle learning

IMPACT ON EDUCATIONAL PRACTICE: Problem-solving and the learning of principles inductively.

IMPACT ON MATHEMATICS EDUCATION: Subordinate and superordinate skills required in problem-solving and principle learning.Still influential especially in the area of Talented and Gifted Students.

****

MAJOR THEORIST: Arnold Gesell

ORIENTATION: Developmental stages

MODEL: Developmental sequence

IMPACT ON EDUCATIONAL PRACTICE: Influential in child-rearing practices, medical model, and early childhood education.

IMPACT ON MATHEMATICS EDUCATION: Specific sequential

stages of development; learners' mental structures and capacity in focus.
****

MAJOR THEORISTS: Jean Piaget, Irving Sigel, Lawrence Kohlberg

ORIENTATION: General intellectual development.

MODEL: Cognitive development.

IMPACT ON EDUCATIONAL PRACTICE: Learner's inherent developmental stages in logical reasoning; social and moral growth related to instruction, planning and curricula.

IMPACT ON MATHEMATICS EDUCATION: Change of expectations of learner's age appropriate development of logical reasoning; big impact on materials and curricula and maths teaching practice. Structure of the discipline becoming important, inductive learning with cognitive processing became common model used widely in 1960s-70s, especially in "New Math" movement.

****

MAJOR THEORIST: David Ausubel

ORIENTATION: Information processing

MODEL: Advance organiser

IMPACT ON EDUCATIONAL PRACTICE: Increased efficiency of information processing, manipulation of cognitive development and concept formation rather than dependence upon intrinsic developmental stages.

IMPACT ON MATHEMATICS EDUCATION: Increasing capacity for meaningful reception and relating of bodies of related learning.

****

MAJOR THEORIST: Benjamin Bloom

ORIENTATION: Levels of complexity of cognitive processing.

MODEL: Bloom's Taxonomy

IMPACT ON EDUCATIONAL PRACTICE: Different levels of information processing from simple to complex identified; differentiation of teaching/learning activities indicated; significantly, learners able to move between levels as appropriate.

IMPACT ON MATHEMATICS EDUCATION: Especially relevant to problem-solving activities and differentiation of curricula to cater to individual needs - eg. Special education of children with learning difficulties and Talented and Gifted Students.

****

MAJOR THEORISTS:Conway;Biggs & Collis; Munari & Butler; Kamii.

ORIENTATION: Cognitive development, thinking about thinking.

MODELS: Metacognition; S.O.L.O.Taxonomy; Neo-Piagetian; Constructivism

IMPACT ON EDUCATIONAL PRACTICE: Strong influences relating to cognitive development and information processing continuing to influence teaching practice, curriculum development and assessment procedures up until present.

IMPACT ON MATHEMATICS EDUCATION: Continued influence in teaching materials and practice; emphasis on the internal cognitions of learners, stages of development, and reasoning and problem solving activities. Existing side by side with behaviourally oriented outcomes-based education and cooperative learning model relating to sociological learning research.

REFERENCES: Gage & Berliner,1975,444-7
Kroll, 1989 in NCTM Yearbook
Rothstein,1990,53.

Information processing and cognitivist theories became popular and remain influential in educational circles up until the present. They would seem in some ways to represent the other side of the coin to the "tabula rasa" mentality apparently adopted by the behaviourist school of thought which has held sway concurrently. Both significantly impact upon education in general and Mathematics education in particular.

SUMMARY 4

MAJOR THEORISTS:Donald Oliver; John Dewey; Byron Massialas; Benjamin Cox; National Training Laboratory; Davidson.

ORIENTATION: Social interaction

MODEL: Social learning; Group investigation; Co-operative learning.

IMPACT ON EDUCATIONAL PRACTICE: Processing information and thinking about social issues; emphasis on social (group) skills and academic inquiry; emphasis on personal development; personal awareness and flexibility.

IMPACT ON MATHEMATICS EDUCATION: Popular movement in the 1980s and 90s; focus on language use in mathematics; role of female learners and minority groups; reinforced emphasis on problem solving, group interaction and cognitive structures.

REFERENCES: Gage & Berliner,1975,444-7.

It is clear that the orientation of these groups of theories differs substantially, as does their view of the direction in which learning should take place to achieve optimal retention and understanding.

2.4 THEORETICAL BASES OF POLICIES AND CURRICULA

One way of investigating the impact of theories upon mathematics instruction is to examine mathematics policies and curricula for evidence of theoretical influences. The highly influential U.K. Report of the Committee of Inquiry into the Teaching of Mathematics in Schools under the chairmanship of Dr W.H.Cockcroft "Mathematics Counts" (1982,10) reveals an emphasis on fluency and accuracy in basic numeracy skills and real-life problem solving; a constructivist slant towards overall concepts and principles; as well as a light dusting of concern for personal development and positive self-concept in sections such as 32 and 34 which have been quoted previously in Chapter 1.

Paragraph 39 further stresses this dual concern for skills with understanding stating "Our concern is that those who set out to make their pupils 'numerate' should pay attention to the wider aspects of numeracy and not be content merely to develop the skills of computation" (Cockcroft, 1989, 11).

The Cockcroft report's implications for teachers in "Why teach Mathematics?" (1989, 4) follows:

> In our view the mathematics teacher has the task

* of enabling each pupil to develop, within his capabilities, the mathematical skills and understanding required for adult life, for employment and for further study and training, while remaining aware of the difficulties which some pupils will experience in trying to gain such an appropriate understanding;
* of providing each pupil with such mathematics as may be needed for his study of other subjects;
* of helping each pupil to develop so far as is possible his appreciation and enjoyment of mathematics itself and his realisation of the role which it has played and will continue to play both in the development of science and technology and of our civilisation;
* above all, of making each pupil aware that mathematics provides him with a powerful means of communication.

Several theoretical issues are raised in these implications, such as: mathematics being useful and related to real situations; children's intrinsic capabilities (relating also to learning stage theories); interrelationships between maths and other subject areas; the 'learning should be fun' aspect discussed in the beliefs section; and the social aspects of learning mathematics including the language focus. These ideas and theoretical stances are further developed in sections 287,289 and 290 (Cockcroft, 1989, 84-5) in statements such as:

287 The primary mathematics curriculum should enrich children's aesthetic and linguistic experience, provide them with the means of exploring their environment and developing their powers of logical thought, in addition to equipping them with the numerical skills which will be a powerful tool for later work and study.

289 Practical work is essential throughout the primary years if the mathematics curriculum is to be developed in the way which we have advocated in

paragraph 297.

290 Children vary greatly in the amount of time which they take to move through these stages. It is as harmful to insist that one child should continue to use practical materials for a process which he understands and can carry out by using symbols as to insist that another should proceed to diagrammatic or symbolic representation before he is able to carry out the process by using practical materials. It is therefore a mistake to suppose that there is any particular age at which children no longer need to use practical materials or that such materials are needed only by those whose attainment is low. It is not 'babyish' to work with practical materials while the need exists and we believe that many children would derive benefit from a much greater use of these materials in the later primary years than occurs in many classrooms.

Paragraphs 300 and 301 (Cockcroft, 1989, 88) deal specifically with the areas of mathematics under investigation - Time and Fractions. A combination of inductive and deductive approaches is implied, with both concept development and skills introduction and practice being advocated. The real-life or functional basis of mathematics is also reiterated, as is the language of teaching/learning aspect.

With regard to children whose mathematics attainment is low the Cockcroft Report (1989) reveals theoretical underpinnings relating strongly to cognitive processing models and concept attainment or the construction of mental structures of related learnings and principles. Certainly the Behaviourist model is not being strongly or exclusively endorsed. An emphasis on language, meaningful practice and the use of technology is evident throughout the document.

The Cockcroft Committee was convinced that children's needs and performance in mathematics were much more similar than different when a world-wide perspective was adopted. The Committee concluded:

> It is clear that the differences in attainment which exist between children of the same age in any one country are very much greater than the small variations which exist between the performance of an 'average' pupil from each country (1989,99).

It may be extrapolated, therefore, that many of these recommendations are also relevant to Australian Mathematics education, and certainly the Cockcroft Report has had a substantial international impact since its release.

Similarly, in the United States the publication of the Standards Report of the National Council of Teachers of Mathematics (or NCTM) in 1987 brought into focus "the importance of considering not only what Mathematics itself is and why various mathematical concepts are important to know but also, perhaps most important, how students learn Mathematics"(Kroll, 1989,210 in NCTM Yearbook). Constructivist theory, an outgrowth of Piagetian and other cognitive theories, currently holds sway. This can be seen in statements such as "individuals

approach each task with prior knowledge, assimilate new information, and construct their own meanings."
(NCTM,1987,8)

An examination of the NCTM Curriculum Standards for Grades K-4 (NCTM,1987) reveals a range of theoretical underpinnings. The inductive, problem-solving approach is revealed in recommendations such as Standard 1 which advocates that problem-solving should be pivotal within the K-4 mathematics curriculum so that students can use problem-solving processes in learning of all mathematical content.

The importance of language in mathematics and the construction of meaning is clearly shown in Standard 2 where it is recommended that children use mathematics as a form of communication developing a language, with symbols, to extend and communicate mathematical ideas (NCTM, 1987).

Mathematics as useful, real-life numeracy is advocated in Standard 4 (NCTM, 1987) where students are expected to be able to demonstrate when the use of an estimate is appropriate and use estimation when working with quantities, measurement, computation and problem solving.

Furthermore the use of concrete materials or manipulatives and a real-life orientation is suggested in Standard 5 (NCTM, 1987).

The importance of language and the influence of cognitive processing models is evident in Standard 6 (NCTM,1987) with recommendations that students discuss a variety of problem situations using informal language and materials before explicit instruction on the four operations; and relate problem-solving situations to mathematical language.

Standard 7 (NCTM,1987) contains an emphasis on accuracy and fluency and the necessity for drill and practice with the suggestion that students should be able to model and explain basic facts and algorithms and develop operational proficiency.

Inductive problem-solving and conceptualisation is stressed in Standards 9 and 10 (NCTM, 1987) with recommendations that students be able to use skills such as investigating, predicting, collecting, organising, using data, experimenting and applying statistics to solve problems.

Standard 11: Fractions and decimals (NCTM, 1987) emphasises concept development through both inductive and deductive processes. It is suggested that students in Grades K-4 need to build concepts of fractions, develop number sense to incorporate fractions and decimals, relate fractions to decimals and recognise equivalent fractions and use fractions and decimals in problem situations.

Examination of the current N.S.W. Mathematics K-6 Syllabus (1989,8) reveals a complex combination of theories and underlying principles in the teaching and learning of Mathematics in the primary school. It is stated that:

> The aims of Mathematics education will be achieved in different levels...according to the stage of development of the students at these levels. These aims are to develop in students confidence and enjoyment in doing mathematical activities, knowledge, skills and understanding in certain specified areas, and awareness of the place of Mathematics in solving problems of everyday life and in contributing to the development of our society.

This statement, and the detailed discussion which follows it in the Mathematics K-6 Syllabus (1989), incorporates the following theoretical stances and beliefs about learning: that learning should be pleasant;that language is vital to the learning of Mathematics;that active problem-solving is necessary to develop understanding; that mathematics is and should be useful; that concrete materials and technology should be used to enhance mathematics teaching and learning: as well as elements of each of the Cognitive Processing, Constructivist and Behaviourist models of instruction.

Examination of the content summary relating to Time in the N.S.W. Mathematics K-6 Syllabus (1989, 12) reveals a now familiar eclecticism with regard to theoretical underpinnings in TABLE 2.2 below:

UNDERLYING THEORY	CONTENT SUMMARY : TIME
Constructivism	Awareness of concepts related to time
Realism	Passage of time related to routine event
Deductive approach/drill	Names of days, special days
Reasoning	Comparison of time
Stages of cognitive development	Passage of time using informal units
Deductive - drill & practice	Seasons, months, weeks and days
	Hours, minutes and seconds
Experience/ concept development	Time - o'clock, half hour
Reasoning/problem-solving	Reading digital clocks
Drill & practice	Time - 1 minute intervals
Metacognition	Comparison and ordering of time intervals
Concepts/ constructs	a.m. and p.m. notation
	Relationships between time units
Extension	Timetables,timelines and 24 hour time
Skills	Use of a stopwatch
Principle development	Speed
Extension	Geographical and astronomical time

In this and other content ar as the Syllabus selects theories as appropriate to pupils' and society's needs as they are currently perceived. Theories as widely different as Behaviourism and Constructivism are treated as though they exist on a continuum rather than in opposition to each other. This echoes the common pragmatic, eclectic approach of many teachers. The curriculum also provides a range of options so that all learners and teachers should be able to be accommodated with reasonable success. There is scope for both acceleration and extension of talented and gifted students (T.A.G.S.) in the range of content and methods available; as well as provision for task analysis, inductive and deductive learning situations, and drill and practice for learners with special needs or learning difficulties.

The summaries of objectives for the teaching of Time and Fractions in the Maths K-6 Syllabus (1989,175 & 262) further expand the recommendations for children's understanding and skilled use of time, as can be seen in detail in Chapter 4.

These objectives reveal a strong emphasis on sequential concept development from level to level which is reminiscent of Behaviourist task analysis teaching strategies. A more inductive, constructivist approach would necessarily result in a less ordered, more spontaneous and individualised development of Fraction and Time concepts. Obviously, there is room for both orientations as appropriate to particular pupils and their needs. Many practical, real-life, problem -solving situations are prescribed as relevant and useful objectives, with accompanying teaching strategies. However, an emphasis on recall, accuracy, fluency and skill attainment is definitely present, especially in the higher levels. This is both an acceptance of the important role in mathematics learning and teaching played by drill and practice, as well as a reflection of the cognitivist, stage theory orientation towards children's cognitive maturation from concrete, hands-on learning to the more abstract, formal operations stage usually by upper primary age. A combination of concept acquisition and skills attainment is an ongoing and recurrent theme throughout this document.

It is evident that Piagetian, Neo-Piagetian, and Constructivist orientations currently have a great influence on teaching in general, and the teaching of mathematics in particular. At the same time drill and practice are still recognised as important features of successful mathematics programs. Influences from other disciplines have also led to an emphasis on the language used in teaching and learning mathematics and on the use of social, interactive situations to enhance students' learning. The consistent use of concrete materials in lower levels in all areas of mathematics, and continued use for concept development in higher levels is also in line with Information Processing, and Cognitive Developmental models of Educational Psychology. The pragmatism of the classroom teacher seems to have overcome, ignored or adapted to the long-standing competition and disputation between Cognitivists and Behaviourists, and to have added a large pinch of social interaction and personal development for good measure. Purists would argue that such a position is untenable and doomed to be counterproductive. Realists would suggest that progress comes out of adaptation and compromise and that in the long run teaching has as much impact on theory as theory does on teaching.

CHAPTER 3 - RESEARCH REVIEW

3.1 IMPORTANT FACTORS IN TEACHING CHILDREN WITH LEARNING PROBLEMS: DIFFERENCES.

Slowness and inefficiency in the acquisition of knowledge, concepts and skills are primary factors which are consistently used to identify or describe children with learning problems. Much research and experimentation regarding the best ways to enhance learning for such children has been carried out over the last four decades. Important factors which have been identified in teaching children with learning problems and recommendations for dealing with them are reviewed below:-

* Deficiencies in short-term memory are a major problem. The use of rehearsal strategies, verbal cues, imagery and concrete materials is recommended.
* Difficulty in organising input material can be overcome by deliberate and overt grouping of material; external cuing; simultaneous rather than serial stimulus presentations; and by incorporating redundancy.
* Attention deficits can be mediated by the use of cues; the use of reinforcement; removal of distractors and by facilitating attention to the relevant dimensions of the stimuli.

* Poor incidental learning and inefficient responses can be improved by eliciting correct responses; providing immediate differential feedback; teaching in a variety of contexts; keeping relevant cues constant; providing repetition; use of reinforcement; proceeding in small steps; and building on what children have already learned.

* The tendency towards having an external locus of control is common to children with learning difficulties. The development of an internal locus of control can be assisted by aiming for realistic successes; setting specific goals; providing consistent consequences; and reinforcing statements which reflect an internal locus of control.

* A history of failure at school is common to these children. To assist in developing self-esteem realistic goals and experiences of success should be devised.

* High expectancy of failure is associated with a history of failure. Again realistic goal setting is recommended. Ambiguous feedback should be avoided and rewards can be paired with desirable events.

* Outer directedness in problem-solving is a common feature of children with learning difficulties. Modelling can therefore be used as a major teaching strategy. Grouping is best with similar peers to avoid copying of work or frustration and lowered self-esteem.

* Slow unlearning of incorrect skills is a significant

problem. Incorrect responses need to be replaced by incompatible correct responses to extinguish the undesirable responses.

* Confusion in instruction, language and materials is to be avoided.

* Poor transfer of skills mastered in one area to other areas or real-life situations is a major problem for children with learning difficulties. In order to overcome this, overt teaching of connections into other areas or between areas should occur. Focus on such connections is recommended for all learners but will often occur spontaneously in more able students.

* Problems in reading lead to increased difficulties in all other subject areas. In mathematics the readability levels of material can be altered; readers can be assigned or the teacher can read and scribe for the less able child; alternative texts can be assigned; or the child with learning difficulties can be provided with regular direct instruction on a one-to-one or small group basis to minimise lack of progress due to insufficient reading skills.

* Developmental delays and difficulties with abstractions are common amongst children with learning difficulties. In mathematics especially an emphasis on concrete operations, manipulatives, visual cues and real-life applications is necessary. Realistic goal setting is advisable.

* A slow rate of progress is both a sign and an effect of learning difficulties. Systematic and explicit instruction is recommended, as well as the provision of appropriate content, teaching to mastery and regular monitoring of progress and provision of feedback.
References: Baur & George, 1976,362-6; Mercer & Snell, 1977; Outhred & Center, in Ward,1989; Mercer & Miller,1992)

Christenson, Ysseldyke and Thurlow (1989) undertook an exhaustive research review into instructional effectiveness for students with mild handicaps and came up with the following further aspects which have shown a high correlation with improved student achievement:-

* The degree to which classroom management is effective and efficient.
* The degree to which there is a positive atmosphere.
* The degree to which there is an appropriate instructional match.
* The degree to which students understand teaching goals and expectations for student's success.
* The degree to which lessons are presented clearly and follow specific instructional procedures.
* The degree to which instructional support is

maintained throughout acquisition, guided practice and independent practice sessions.

* Allocation of sufficient time and efficient use of time.
* High rates of student response opportunities.
* Active monitoring of student progress and understanding.
* Appropriate and frequent evaluation of student performance.

3.2 SIMILARITIES OF CHILDREN WITH LEARNING PROBLEMS TO "NORMAL" LEARNERS.

Christenson, Ysseldyke and Thurow's (1989) recommendations for instructional effectiveness hold just as true for normal learners as for those with learning problems, although they may not be as critical because of the ability of normal learners to learn incidentally and generalise and transfer spontaneously more often. Educational goals can be modified and instruction differentiated for children with learning difficulties. However, the aim is to integrate and challenge all students (Lowenbraun & Affleck,1976,34). It is a matter for concern that as Engelmann, Carnine and Steely (1991) reported:

> Poor performance in mathematics extends beyond the realms of students with learning disabilities and students from impoverished backgrounds. In the

> National Assessment of Educational Progress (Carpenter, Coburn, Reyes, & Wilson, 1976), only 25% of the fourth graders and 62% of the eighth graders could solve five story problems...Their performance dropped even further during the next 5 years (Carpenter, Corbitt, Kepner, Lindquist & Reyes, 1981). At that time, only one third of seventh graders could add fractions such as 1/3 and 1/2... International comparisons reveal similar problems.

Some reasons cited for this generally poor mathematics performance included:-

* The inordinate amount of time spent teaching computational skills, at the expense of concept understanding and problem solving.

* At least 70% of topics covered were taught for exposure and received less than 30 minutes of instructional time.

* A great variance among teachers in the actual amount of time spent teaching mathematics.

* The "low intensity", spiral curriculum in which content and goals linger from year to year with class programs attempting to build upon previous content that was poorly mastered.

* Failure to ensure that students have relevant, prior knowledge.

* Too rapid a rate for introducing new concepts.

* Incoherent presentation of mathematics strategies.

* Unclear, inconcise instructional activities.

* Inadequate transition between initial teaching and independent work.

* Sparse or absent review of skills and understandings. (Engelmann, Carnine & Steely,1991, 292-3).

Two major theoretical positions are widely held by mathematics educators - the Developmental and the Behaviourist positions. Whichever view is held, is of as much relevance to normal learners as it is to children with learning difficulties since it has been shown that (i) behavioural strategies are effective in increasing desired behaviours and (ii) cognitive development can be accelerated by instructional intervention (Pasnak,1987).

It has been shown that for all learners a major source of failure to enjoy and understand mathematics is a teaching bias towards "completing computational problems without first ensuring that the children understand the concept underlying the procedure they are applying" (Outhred &

Center in Ward,1989,142). Another common cause of failure is the failure of students to relate school mathematics to real-life, everyday situations, that is, the failure to realise the relevance of mathematics. The deliberate inclusion of real-life examples in mathematics teaching, and explicit development of connections between mathematics and life and mathematics and other subjects is necessary to overcome this difficulty (Outhred & Center, in Ward, 1989,142-3).

3.3 FACTORS IN THE EFFECTIVE USE OF MANIPULATIVES

Lesh's Translation Model (1979, in Cramer & Bezuk,1991) relates manipulatives to other forms of mathematics learning - pictures, spoken symbols, written symbols and real-world situations - and suggests that teachers need to organise learning activities to involve all these areas so that connections can be made between multiple experiences to result in strong concept and skill development. The use of manipulatives per se will not necessarily result in better learning.

Stigler and Stevenson (1991,20) reported that "teachers rely primarily on two powerful tools for representing mathematics: language and the manipulation of concrete objects. How effectively teachers use these forms of representation plays a critical role in determining how well children will understand mathematics". In a comparison of American and Asian teachers endeavouring to discover reasons for Asian students' general mathematical superiority they found that "Asian teachers generally are more likely than American teachers to engage their students, even very young ones, in the discussion of mathematical concepts" (Stigler & Stevenson,1991,20). It would appear that the linking of appropriate language to manipulative experiences is vital so that children are actively forming mathematical understandings rather than just enjoying an isolated concrete experience (Yackel,Cobb, Wood & Merkel,1990). The movement from concrete experiences to symbolic abstract understandings must be encouraged by teachers. The formation of "connections" is not spontaneous for most learners and even less so for children with learning problems whose memory and transfer skills are frequently poor.

While there is a growing body of evidence that concrete materials are an important teaching/learning aid a warning note must also be sounded. Raphael and Wahlstrom (1989) reported that of 40 studies using concrete materials, 24 showed positive results, 12 showed no differences and 4 were negative. They concluded that other factors such as the amount of time on task or opportunity to learn, the experience of the teacher, and the appropriateness of the concrete materials chosen were

all significant variables in the effectiveness of the mathematics teaching, as shown by pupil achievement.
Calvin Iron's (1982) study showed that the use of concrete materials per se could be both ineffective and inappropriate. Careful choice of the type of material, the language used and the manner of representing findings are all necessary for the most favourable learning outcomes. For example, Multi Attribute Blocks in conjunction with a place value chart using linear representation - thereby directly linking physical experiences, language and thought processes - is a more powerful, effective and appropriate use of concrete materials than just Multi Attribute Blocks or a place value chart alone.

Lowebraun and Affleck (1976,65-7) show how careful matching of objectives to concrete materials, as well as examination of underlying skill and concept requirements in the selection of manipulatives and other teaching materials, is particularly important when dealing with students with learning difficulties. These students may not have sufficient entry skills or reading ability to successfully learn from inappropriately selected materials. They recommend that teachers should be "sensitive to individual strengths and provide materials that will best enable the child to make correct responses" and thereby develop the desired concepts and skills in a given area. That is, teachers should aim to provide a learning situation in which successful learning is ensured.

Lowenbraun and Affleck (1976,66-7) synthesised much research and teaching experience to identify important variables in the choice and effective use of teaching materials, including manipulatives, with mildly handicapped children in regular classrooms.
Their recommendations included examination of: the objectives of the materials, the entry skills required, the actual content, the sequencing of the content, provision for assessment and ongoing feedback, adaptability to individualisation, physical characteristics and teacher information and resources.

Appropriate selection of manipulatives is a vital part of ensuring their effective use. However, teaching strategies and organisation are equally significant factors. Great materials used poorly are not a formula for success. Joyner (1990) identified the following teaching factors in the effective use of manipulatives in teaching primary mathematics:

*free exploration time

*pre-packaging and prior organisation

*clear expectations

*clear guidelines for acceptable use

*modelling the use of the materials
*verbalisation or "thinking aloud"
*management guidelines
*assistance (eg. classroom helpers familiar with materials).

It can be seen that concrete materials used in the context of careful selection for appropriateness and true representation of the concept being examined, and in combination with deliberate fostering of relevant language and abstract representations should be an effective tool in mathematics teaching.

3.4 IS THERE A PLACE FOR OPEN-ENDED CONCRETE MATERIALS ?

Within any regular classroom of students there is a range of ages, stages, abilities, skills, learning style preferences, and so on... The range can be very wide or quite narrow according to streaming policies, numbers of children and many other factors.

It has been discussed previously that current research indicates that children with learning problems are at least not worse off and are probably better off integrated into regular classes. It has also been shown that the effective use of manipulatives in combination with skilful language development is most likely to improve the mathematics understandings and skills of children with learning difficulties. Careful choice of manipulatives and teaching strategies is necessary for optimising such students' progress.

However, do we service a whole regular class best by selecting materials and strategies to suit the bottom end of the range? Normal learners - whilst equally well requiring careful selection, language development and "connections" being made overt - may need to spend less supervised time using concrete materials; more time using them independently; and should move towards iconic and symbolic mathematical systems more rapidly.

Gifted and talented students within regular classrooms have been repeatedly found to spontaneously employ "effective learning strategies similar to older students" (Scruggs et al,1985). In relation to these students concrete materials need to stimulate independent thought and to be useful for hypothesis testing. Manipulatives which are open-ended and can be combined with other mathematical systems or applied to diverse areas of mathematics are of more benefit and effectiveness for such students (Scruggs et al,1985).

The catch-cry of education for many years has been the fulfilment of each child's potential. Different materials and/or different strategies and/or effective use of grouping/helpers/individualisation are necessary to move towards achieving this goal. It would seem well developed, open ended manipulatives would be of most use in teaching primary mathematics across the whole range of abilities.

Piaget commented (in Baur & George,1976,32):

> The principal goal of education is to create men who are capable of doing things, not simply of repeating what other generations have done - men who are creative, inventive, and discover. The second goal of education is to form minds which can be critical, can verify, and not accept anything they are offered.

3.5 RESEARCH REVIEW OF THE USE OF CONCRETE MATERIALS OR MANIPULATIVES TO ENHANCE MATHEMATICAL UNDERSTANDINGS AND SKILLS IN TIME AND FRACTIONS FOR CHILDREN WITH LEARNING PROBLEMS IN THE REGULAR CLASSROOM.

The Journal for Research in Mathematics Education which is the official journal of the National Council of Teachers of Mathematics, conducts an exhaustive annual review of research on mathematics education. More than 60 journals are searched each year, as well as the Current Index to Journals in Education and Psychological Abstracts. In addition dissertations are reviewed.

Examination of these reviews from 1987 to 1993 resulted in the following findings:

TABLE 3.1

MATHEMATICAL TOPIC	NUMBER OF RELATED RESEARCH ARTICLES
Fractions	47
Time	2
Fractions & Time	0

The total number of articles, reviews and dissertation searched was in excess of 2110. Of the articles relating to fractions none directly related to the use of concrete materials to enhance mathematical understandings and skills for children with learning problems in the regular classroom, nor did the two articles dealing with Time concepts. The connection between Fractions and Time was not explored at all. Noticeably few "connections" between areas of mathematics, or mathematics and other areas, have been researched to date.

It would not be too strong to say that there is a dearth of previous research in the area of inquiry under investigation here. The few directly relevant pieces of research so far unearthed suggest that:- "understanding is vital, and thus children need experiences with concrete materials for the various interpretations of rational numbers" (Driscoll,1984). McBride and Lamb (1986) support this emphasis on concrete experiences in fraction learning as does Davis (1983) in his longitudinal study.

Internationally, Steffe and Olive (1991) report:

> Strang's findings are based on the results of testing nearly three thousand students, aged nine to twelve years, in the Finnish comprehensive school. But are the implications of the findings restricted to students living in Finland? Kerslake (1986) found that English students of thirteen to fourteen years relied on rote memory of previously learned techniques when working with fractions. She believes the underlying problem is that 'with the exception of certain simple examples such as 1/2 and 1/4, fractions do not form a normal part of a child's environment, and the operations on them are abstractly defined'. Students of similar ages in the United States also learn rules and tricks for fractions and rely on rote memory of these rules and tricks (Hunting,1980; Kieren,1988; Nik Pa,1987; Payne,1976).

Mack (1990) found that Sixth Grade students possessed "a rich store of informal knowledge of fractions". Instruction was related to this informal knowledge and children developed refined fraction concepts. However, knowledge of rote procedures "frequently interfered with students' attempts to build on their own informal knowledge". This study advocates the teaching of concepts prior to procedures. Hunting and Sharpley (1988) support this study in their examination of preschoolers' fraction knowledge.

Harrison,Brindley and Bye (1989) compared regular instruction to the use of a concrete, process-oriented approach to teaching fractions and found that: "the concrete, process oriented approach resulted in significantly improved achievement in and attitude towards, fractions and ratios", along with the development of general mathematical strategies and computational facility. They also discovered the broad range of cognitive levels and abilities within a regular classroom which require consideration and adaptation of materials.

Resnick, Nesher, Leonard, Magone, Omanson and Peled (1989) studied the conceptual sources of children's errors in fraction learning and suggested that "errors are a natural concomitant of student's attempts to integrate new material...with already established knowledge". They reminded teachers that errors are a major source of diagnostic information. Combined with appropriate language new learning can spring from children's errors.

Streefland's (1991) three year research study in the Netherlands lends further support to the gradual movement from first "constructive handling of meaningful concrete material, second generating the relations between fraction materials, and third developing the relations on a formal mathematical level".

With regard to Time concepts and skills, Harris (1991) raised the interesting prospect that children bring not only informal knowledge to any subject in mathematics but also a cultural perspective. The Aboriginal perspective on Time reported was based on a present-past, Dreaming view with little scope for industrialised, Western society's emphasis on being "on time". Similar cultural perspectives for various other national groups were suggested as relevant to appropriate Time use and connections being made.

Andrade (1992) similarly employed concrete materials effectively to teach concepts of Time and the analog clock. Wyatt (1993, 504), however, found that children who had learned to "tell the time" had "no real understanding of the length of one minute". Their knowledge of this practical, real-life mathematical skill was purely abstract. The connections to real-life had not been made. Active participation and estimation exercises overcame this deficiency.

Since research studies and findings are relatively scarce further investigations into the use of concrete materials or manipulatives to enhance mathematical understanding and skills in Time and Fractions for children with learning problems in the regular classroom will need, of necessity, to proceed along less traditional lines.

In the following chapters a survey of general teaching practices will be discussed, as well as an examination of methods currently used in and advocated by mathematics textbooks and teacher reference materials. In addition, a review of concrete materials currently available in schools and through educational suppliers will be presented to provide further insight into current practice. These avenues of investigation will culminate in a discussion of the development and testing of a set of concrete materials designed specifically for the purpose of teaching Time and Fractions to children with learning problems in the regular classroom.