Title: **Mathematical models of chemical reactions illustrating Hopf bifurcations and oscillatory behaviour:**

A. The Belousov-Zhabotinski reaction.

B. An enzyme-catalysed reaction known as the S-A system.

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ABSTRACT

This thesis gives a detailed and much expanded account of the following two papers, filling in details of the proofs, and using *Maple* for numerical work:

Hastings, S.P. and Murray, J.D. 1975. 'The Existence of Oscillatory Solutions in the Field-Noyes Model for the Belousov-Zhabotinski Reactions', *SIAM Journal of Applied Mathematics*, **28**(3), pp. 678-688.

Hassard, B. and Jiang, K. 1992. 'Unfolding a Point of Degenerate Hopf Bifurcation in an Enzyme-catalyzed Reaction Model', *SIAM Journal of Mathematical Analysis*, **23**(5), pp. 1291-1304.

Two different chemical reactions, each of which are modelled by systems of ordinary differential equations, are examined to show that Hopf bifurcations occur at certain parameter values, giving rise to periodic solutions.

A. When citric acid is oxidised by bromate in aqueous sulphuric acid with the cerium ion as catalyst, and the mixture is stirred, temporal oscillations occur in the concentrations of the cerium ions *Ce(III)* and *Ce(IV)*.

Starting with the quantitative model of Field and Noyes, the analysis of Hastings and Murray is examined to show that the system of differential equations yields oscillatory solutions, of finite amplitude, at least one of which is periodic. The existence of a Hopf bifurcation giving rise to these oscillatory solutions is also investigated.

B. The change in concentrations of two chemical species in the presence of an enzyme-catalyzed reaction inside a compartment, with transport from an outside reservoir, is modelled by a system of two ordinary differential equations, known as an S-A system (substrate-activator).

An analysis of this system by Hassard and Jiang is examined. It is shown that a point of degenerate Hopf bifurcation exists. Using singularity theory, universal unfoldings of the degeneracy are determined to describe the families of periodic solutions that occur for parameter values near that at which the degeneracy occurs.

TABLE OF CONTENTS

Page

LIST OF FIGURES