Wavelet Filter Banks and Applications

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Abstract

Traditionally wavelet basis is an orthonormal basis of $L^2(R)$, which is formed by translations and dilations of a single wavelet function which is usually known as the mother wavelet. After Mallat and Meyer developed multiresolution analysis, the relationship of wavelet bases to filter bank theory was established. Multiresolution analysis introduced a new function called scaling function. It also enabled us to first design the scaling function and then complete the filter bank to obtain the wavelet function. Probably the most important notions wavelets added to the filter bank theory are vanishing moments and regularity.

The classical wavelet technique can be generalized in many ways most of it were motivated by the techniques existed in filter bank theory. The primary goal of the thesis is to develop new filter banks while borrowing the concepts of vanishing moments and regularity from wavelet theory. Thus our emphasis is on filter bank aspects rather than the resulting wavelet bases. We have investigated three types of filter banks, M-band bi-orthogonal filter banks, Double Density Filter banks and its generalizations, and Filter banks on the hexagonal lattice. I have been more successful in M-band bi-orthogonal filter banks and Double Density Filter banks compared to filter banks on the hexagonal lattice. I have run into difficulties of two dimension on the hexagonal lattice.

Our approach is same as the way classical wavelets were developed, in that we first design the scaling filter with regularity. Then we complete the filter bank to obtain wavelet filters. On M-band bi-orthogonal wavelets we have obtained the shortest analysis wavelet filters with linear phase symmetry and of equal size. In double density filter banks I have developed a new

factorization technique for a special case and generalized the technique to M-band Multiple density setting. On hexagonal filter banks, I was only able to provide an analysis of such filter banks.

Finally I have also investigated applications of wavelet filter banks. I was attracted to digital watermarking in wavelet domain. I have developed a new technique to embed binary watermark images in wavelet domain.

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