

NONLINEAR MODE COUPLING IN VIBRATING
MECHANICAL SYSTEMS.

By
Katherine Ann Legge

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DECLARATION

I certify that the substance of this thesis has not already been submitted for any degree and is not currently being submitted for any other degree.

I certify that to the best of my knowledge any help received in preparing this thesis, and all sources used, have been acknowledged in this thesis.



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Motivation

Those who have heard the mysterious sound of a gong, whether it be the gliding pitch of a chinese opera gong or the rich crescendo of a large tam-tam, would not be surprised to learn of the “powers” often associated with these instruments. Such were its “powers” that even to be touched by a gong could create happiness and strength. The sound of a gong was believed to heal sickness and chase away evil spirits. Indeed the Chinese used a gong to terrify the dragon attempting to devour the moon during an eclipse.

To the ear of a present-day acoustician, the sound of a gong may be no less evocative than it was to the ancient civilizations, though the images of spirits and dragons have been replaced by thoughts of complex nonlinearities and high frequency masking. While nonlinearities quite obviously cause the pitch glide in the above mentioned opera gongs, it is perhaps not so obvious to the ear whether the delayed shimmer of the tam-tam is due to low to high mode couplings or just simply the masking of the high frequencies by more swiftly damped lower frequencies.

The answer to this fundamental question came unexpectedly while we tinkered with an old, and rather temperamental Sonograph. During an attempt to produce a reasonable recording, the first 3 seconds of the sound of a struck gong was used as a sample. Fig. 0.1 shows that, while we didnt manage to get the Sonograph to completely behave itself, we did manage to show that the shimmer of a gong must be due to mode coupling.

Thus we were provoked into investigating the phenonemon. The scarcity of information on coupling of vibrational modes with differing frequencies, however, meant that we needed to consider simpler systems before embarking on the complex three-dimensional problem of the gong.

It is for this reason that our investigations contain three fairly distinct parts. The first is the analysis of a flexible string passing over a nonrigid bridge, for which the nonlinearity has already been established but never considered as a source of mode coupling. The

second analysis examines related behaviour in symmetrically kinked bars, the kink being found to be essential to the process if simple clamped boundary conditions are assumed. In this case the nonlinearity is due to the geometry of the bar and has to be established. A symmetrically kinked bar was chosen partly for simplicity and partly because it represented an appropriate progression between the string and a gong where the gong is in the form of a plate bounded by a conical flange. Finally we investigate mode coupling on a large Chinese gong.

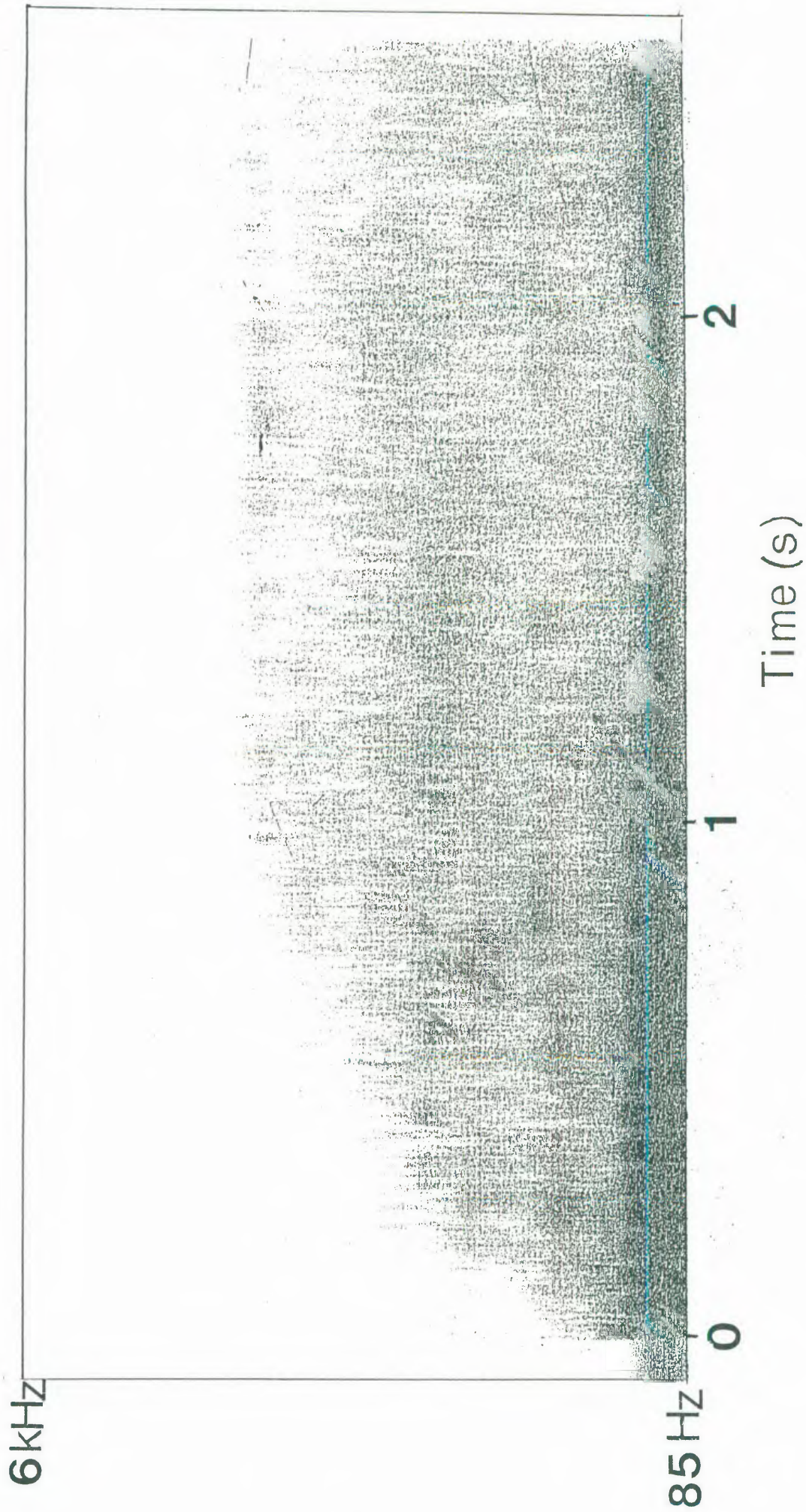


Figure 0.1: Frequency composition vs. time for a struck gong.

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