Chapter 45
Active Contour Texture Segmentation in Modulus Wavelet Feature Spaces

Ashoka Jayawardena and Paul Kwan

Abstract In this paper we discuss a model that is able to segment textures using active contours. Our technique is based on active contour techniques using curve evolution. We build our model on properties of human vision, in that we segment the textures in a certain feature space. We will show the advantages of using modulus feature spaces. Wavelet coefficients are shown to exhibit local features both in space and frequency domains. We will implement our model in modulus wavelet subbands.

45.1 Introduction

The idea behind the active contour image segmentation is that a contour evolves subject to constraints imposed by the image such as image gradient. The active contour image segmentation algorithms can be implemented using classical snakes [1, 2] or level sets. In both these implementations active contours are energy minimizing curves and hence are formulated as energy minimization problems.

The curve evolution models are particular interest to us. When the curve (or the front) evolves in the normal direction of the curve we arrive at the following evolution scheme [3]:

$$\frac{\partial \phi}{\partial t} + F|\nabla \phi| = 0 \tag{45.1}$$

A. Jayawardena (✉) · P. Kwan
School of Science and Technology, University of New England,
Armidale, NSW 2351, Australia
e-mail: ashoka@turing.une.edu.au

P. Kwan
e-mail: kwan@turing.une.edu.au

K. Elleithy and T. Sobh (eds.), *Innovations and Advances in Computer, Information, Systems Sciences, and Engineering*, Lecture Notes in Electrical Engineering 152,
DOI: 10.1007/978-1-4614-3535-8_45, © Springer Science+Business Media New York 2013

The speed F is normally modelled by mean curvature thus resulting in mean curvature motion. The mean curvature evolution equation is given by [3, 4]

$$\frac{\partial \phi}{\partial t} + |\nabla \phi| \mathrm{div}\left(\frac{\nabla \phi}{|\nabla \phi|}\right) = 0 \tag{45.2}$$

The mean curvature motion has been extensively used to model geometric flow. In the level sets implementation the evolving curve is normally embedded in the zeroth level set. They have shown to be able to undergo automatic topologic changes.

Active contour segmentation algorithms have been developed where object mean can be used to discreminate textures [4]. However mean fails to discreminate many textures in the presence og high variances. Julesz [5] has proposed a statistical description of texture that is consistent with human visual perception which is now recognized as the Julesz conjecture.

A texture is defined as a homogeneous random field (RF) $u(x,y)$ on a finite lattice $(x,y) \in L \subset \mathbb{Z}^2$. The Julesz conjecture quoted in [6] states that: there exists a set of functions $f_k(u)$ such that samples drawn from any two RFs that are equal in expectation over this set are visually indistinguishable under some fixed comparison conditions. Mathematically,

$$\begin{aligned} &\mathcal{E}(f_k(u)) = \mathcal{E}(f_k(v)), \\ &\quad \forall k \Rightarrow \text{samples of } u \text{ and } v \text{ are perceptually equivalent.} \end{aligned} \tag{45.3}$$

Thus as long as we find the right feature functions f we can use the expectation as the discrimanatory variable. However, we use feature functions which are evaluated at each pixel location to enable active contour segmentation.

45.2 Energy Minimization Model

Lets define $u : D_I \to R$ be the image which we want segment into two partitions. Lets define the feature function $\mathbf{f} : D_I \to R^N$, where N is the number of feature dimentions, be the feature space where the feature function is evaluated at each image location, i.e. pixels.

We use the following external energy model:

$$\begin{aligned} E_1(C,\mathbf{c}_1) + E_2(C,\mathbf{c}_2) = &\int_{inside(C)} (\mathbf{f}(x,y) - \mathbf{c}_1)^T \mathbf{D} (\mathbf{f}(x,y) - \mathbf{c}_1) dxdy \\ &+ \int_{outside(C)} (\mathbf{f}(x,y) - \mathbf{c}_2)^T \mathbf{D} (\mathbf{f}(x,y) - \mathbf{c}_2) dxdy \end{aligned} \tag{45.4}$$

where $\mathbf{c}_1$ and $\mathbf{c}_2$ are constants and $\mathbf{D}$ is a diagonal matrix with positive values.

It can be seen that when $\mathbf{c}_1$ is the mean of the feature space $\mathbf{f}$ inside the contour C and $\mathbf{c}_2$ is the mean of the feature space $\mathbf{f}$ outside the contour C, $E(C,\mathbf{c}_1,\mathbf{c}_2) = E_1(C,\mathbf{c}_1) + E_2(C,\mathbf{c}_2)$ achieves its minimum for the given contour C. To see this we calculate the partial derivatives of $E(C,\mathbf{c}_1,\mathbf{c}_2)$ with respect to $\mathbf{c}_1$ and $\mathbf{c}_2$.

$$\frac{\partial(E(C,\mathbf{c}_1,\mathbf{c}_2))}{\partial \mathbf{c}_1} = -2\mathbf{D} \int_{inside(C)} (\mathbf{f}(x,y) - \mathbf{c}_1) dxdy$$

$$\frac{\partial(E(C,\mathbf{c}_1,\mathbf{c}_2))}{\partial \mathbf{c}_2} = -2\mathbf{D} \int_{outside(C)} (\mathbf{f}(x,y) - \mathbf{c}_1) dxdy$$

When $\mathbf{c}_1$ and $\mathbf{c}_2$ are the mean of inside and outside regions of the contour C, the above two partial derivatives vanish. If we further assume that for each feature dimension the feature value of a texture object is approximately constant, it is clear that when C is the contour seperating the objects $E(C,c_1,c_2)$ achieves its global minimum. We have the following result.

Theorem 1 *Let $u : D_I \to R$ be an image function and $f : D_I \to R$ be a feature function of the image. Let the feature image f consists of two homogeneous random fields R_1 and R_2. Let D_1, D_2 be a disjoint partition of D_I resulted from the two random fields such that $D_1 \cup D_2 = D$. Let R_1 has μ_1 mean and R_2 has μ_2 mean. Then if $\mu_1 \neq \mu_2$,*

$$\begin{aligned} E(C,c_1,c_2) = &\int_{inside(C)} (f(x,y) - c_1)^2 dxdy \\ &+ \int_{outside(C)} (f(x,y) - c_2)^2 dxdy \end{aligned}$$

achieves its infimum at the object boundary.

Proof Let the D_I is arbitrarily partitioned into $D_A \cup D_B$ such that D_A and D_B are disjoint. Let R_A and R_B are the random fields corresponding of D_A and D_B. Let R_1 and R_2 has variences σ_1 and σ_2 and density functions $p_1(\mu_1,\sigma_1)$ and $p_2(\mu_2,\sigma_2)$ respectively. Since R_1 and R_2 are homogeneous random fields, it is clear that the densities of R_A and R_B are given by

$$p_A(\mu_A,\sigma_A) = k_A p_1(\mu_1,\sigma_1) + (1 - k_A) p_2(\mu_2,\sigma_2)$$

and

$$p_B(\mu_B,\sigma_B) = k_B p_1(\mu_1,\sigma_1) + (1 - k_B) p_2(\mu_2,\sigma_2)$$

respectively where k_A and k_B are given by

$$k_A = \frac{\int_{D_A \cap D_1} dxdy}{\int_{D_A} dxdy}$$

and

$$k_B = \frac{\int_{D_B \cap D_1} dxdy}{\int_{D_B} dxdy}.$$

Now it can be shown that

$$\begin{aligned}
\mu_A &= k_A \mu_1 + (1 - k_A)\mu_2 \\
\mu_B &= k_B \mu_1 + (1 - k_B)\mu_2 \\
\sigma_A &= k_A \sigma_1 + (1 - k_A)\sigma_2 + k_A(1 - k_A)(\mu_1 - \mu_2)^2 \\
\sigma_B &= k_B \sigma_1 + (1 - k_B)\sigma_2 + k_B(1 - k_B)(\mu_1 - \mu_2)^2.
\end{aligned}$$

Now the error E is given by

$$\begin{aligned}
E =& \sigma_A \int_{D_A} dxdy + \sigma_B \int_{D_B} dxdy \\
=& \sigma_1 \int_{D_A \cap D_1} dxdy + \sigma_2 \int_{D_A \cap D_2} dxdy \\
&+ \frac{\int_{D_A \cap D_1} dxdy \int_{D_A \cap D_2} dxdy}{\int_{D_A \cap D_1} dxdy + \int_{D_A \cap D_2} dxdy} (\mu_1 - \mu_2)^2 \\
&+ \sigma_1 \int_{D_B \cap D_1} dxdy + \sigma_2 \int_{D_B \cap D_2} dxdy \\
&+ \frac{\int_{D_B \cap D_1} dxdy \int_{D_B \cap D_2} dxdy}{\int_{D_B \cap D_1} dxdy + \int_{D_B \cap D_2} dxdy} (\mu_1 - \mu_2)^2 \\
=& \sigma_1 \int_{D_1} dxdy + \sigma_2 \int_{D_2} dxdy \\
&+ \frac{\int_{D_A \cap D_1} dxdy \int_{D_A \cap D_2} dxdy}{\int_{D_A \cap D_1} dxdy + \int_{D_A \cap D_2} dxdy} (\mu_1 - \mu_2)^2 \\
&+ \frac{\int_{D_B \cap D_1} dxdy \int_{D_B \cap D_2} dxdy}{\int_{D_B \cap D_1} dxdy + \int_{D_B \cap D_2} dxdy} (\mu_1 - \mu_2)^2
\end{aligned}$$

Thus it is clear that the error of any other partition is larger than the error of object partition $\sigma_1 \int_{D_1} dxdy + \sigma_2 \int_{D_2} dxdy$. □

Let the step function H and the dirac impulse function δ are given by

$$H(t) = \begin{cases} 1 & \text{if} \quad t \geq 0 \\ 0 & \text{if} \quad t < 0 \end{cases}$$

and

$$\delta(t) = \frac{d}{dt}(H(t)) \text{ respectively.}$$

Then we can write the external energy Eq. 45.4 as follows:

$$\begin{aligned}
&E(C, \mathbf{c}_1, \mathbf{c}_2) = \\
&\int_{D_I} (\mathbf{f}(x,y) - \mathbf{c}_1)^T \mathbf{D}(\mathbf{f}(x,y) - \mathbf{c}_1) H(\phi(x,y)) dxdy + \\
&\int_{D_I} (\mathbf{f}(x,y) - \mathbf{c}_2)^T \mathbf{D}(\mathbf{f}(x,y) - \mathbf{c}_2)(1 - H(\phi(x,y))) dxdy
\end{aligned} \tag{45.6}$$

In order to perform the gradient decent of the energy equation, we can decompose the evolution equation of $\phi_t(x,y)$ as:

$$\frac{\partial \phi}{\partial t} = \frac{\partial \phi}{\partial t}_{external} + \frac{\partial \phi}{\partial t}_{internal}$$

Now external energy component of the evolution equation is given by

$$\begin{aligned}
\frac{\partial \phi}{\partial t}_{external} &\stackrel{def}{=} \frac{d(E(C))}{d\phi} \\
&= \delta(\phi)\big(-(\mathbf{f}(x,y) - \mathbf{c}_1)^T \mathbf{D}(\mathbf{f}(x,y) - \mathbf{c}_1) \\
&\quad + (\mathbf{f}(x,y) - \mathbf{c}_2)^T \mathbf{D}(\mathbf{f}(x,y) - \mathbf{c}_2)\big)
\end{aligned}$$

It is clear from the above equation that if the image domain consists only a single texture object then external energy contribution to the evolution equation is zero. Therefore, only locally discriminative features contribute to the evolution equation.

Using similar steps as in [4] we arrive at the following level set formulation of the curve evolution:

$$\begin{aligned}
\frac{\partial \phi}{\partial t} =& \delta_\epsilon(\phi)\Big[\mu \operatorname{div}\left(\frac{\nabla \phi}{|\nabla \phi|}\right) - v \\
&- \lambda_1 (\mathbf{f}(x,y) - \mathbf{c}_1)^T \mathbf{D}(\mathbf{f}(x,y) - \mathbf{c}_1) \\
&+ \lambda_2 (\mathbf{f}(x,y) - \mathbf{c}_2)^T \mathbf{D}(\mathbf{f}(x,y) - \mathbf{c}_2)\Big]
\end{aligned}$$

45.3 Modulus Wavelet Feature Space

Since $\int \psi(x,y)dxdy = 0$, the $E(w)$, i.e. the expectation of a wavelet coefficient, is zero. However variance of the wavelet coefficients depends on the texture. Since coefficient mean is the discriminating criteria of our active contour segmentation model, we need to transform the wavelet coefficients to a certain feature space where variance energy is transferred into mean. The modulus feature space does exactly that. To quantify, to what extent variance energy gets transferred into the mean, lets assume that wavelet coefficients are zero mean gaussian process with variance σ. It can be shown that [7]

$$E(|x|) = \sigma\sqrt{\frac{2}{\pi}}$$
$$E(|x|^2) = \sigma^2$$

Therefore,

$$E((|x| - E(|x|))^2) = E(|x|^2) - (E(|x|))^2 = \sigma\left(\frac{\pi - 2}{\pi}\right)$$

Thus the variance is reduced and transferred into mean.

45.3.1 Wavelet Subband Pre-Processing

Since our classification algorithm is based on the deviation from mean, each subband may respond to a particular object differently, i.e. may respond with a higher object mean than the global mean or lower object mean than the global mean. When using more than one feature space, the expectation of feature values for each texture objects is critical since even though each feature space may discriminate the texture objects when combined they may cancel out discriminatory features in terms of expectation and variance of feature values. This is indeed the case for horizontal and vertical modulus wavelet subbands since those subbands are orthogonal. When this occurs we need transform some subbands into negative images to have the same classification response for the same object. We apply the following transform:

$$-abs(f(x,y)) + k \tag{45.7}$$

where $abs(.)$ is the absolute value function and k is some constant. This pre-processing step improves the feature space correlation resulting in improved homogeneity of the feature space for a particular object.

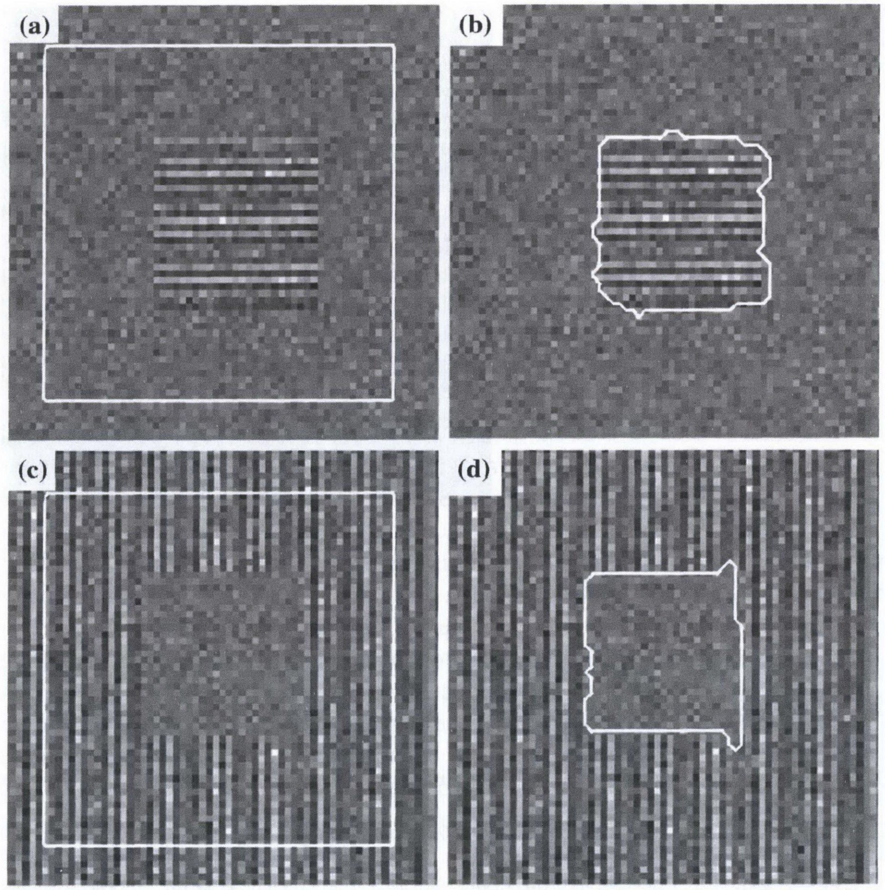

Fig. 45.1 **a** is the horizontal wavelet subband of the original texture image. **b** is the segmented image of the image (**a**). **c** is the vertical wavelet subband of the original texture image. **d** is the segmented image of the image (**c**)

45.4 Results and Discussion

The active contour algorithm of Chan and Vese [4] fails to segment the texture images in wavelet domain since the mean of the wavelet coefficients zero.

We have only used horizontal and vertical subbands of the wavelet transform. The diagonal subband has been omitted since it contains mixture of both horizontal and vertical directional features. The low pass subband has been omitted since our choice of texture images has the same mean for both texture objects.

We have used $max_{(x,y)}(abs(f(x,y)))$ for the value of k in 7. Separable wavelet subbands are capable of extracting horizontal and vertical features of the image. When texture objects can be discriminated using horizontal and vertical features individual subbands are sufficient for active contour segmentation as illustrated in

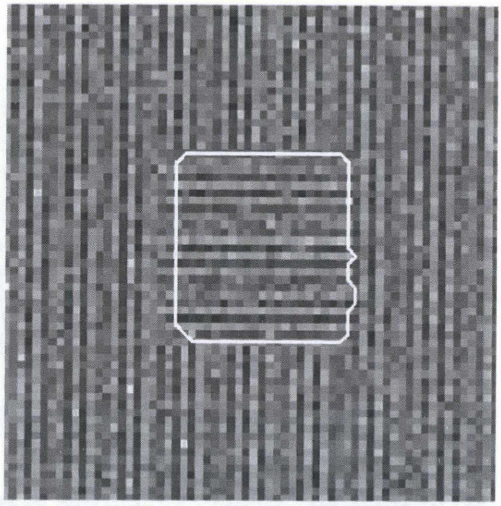

Fig. 45.2 The segmentation in the combined horizontal and vertical subbands

Fig. 45.1. The Fig. 45.2 illustrates the active contour segmentation with both horizontal and vertical subbands.

References

1. Kass M, Witkin A, Terzopoulos D (1988) Snakes: active contour models. Int J Comput Vis 1:321–331
2. Caselles V, Catte F, Coll T, Dibos F (1993) A geometric model for active contour models in image processing. Numer Math 66:1–31
3. Osher S, Sethian J (1988) Front propagating with curvature dependent speed: algorithms based on hamilton-jacobi formulation. J Comput Phys 79:12–49
4. Chan TF, Vese LA (2001) Active contours without edges. IEEE Trans Image Process 10: 266–277
5. Julesz B (1963) Visual pattern discrimination. IRE Trans Inf Theory, IT-8:84–92
6. Portilla J, Simoncelli EP (2000) A parametric texture model based on joint statistics of complex wavelet coefficients. Int J Comput Vis 40(1):49–71
7. Papoulis A (1991) Probability, random variables, and stochastic processes, 3rd edn. WCB/ McGraw-Hill, New York