1. Introduction

1.1 Aims

Technological advances have made possible the collection of high definition potential field and radiometric data, which have proved quite useful in mapping geologic structures, particularly over large, often inhospitable areas. The use of such data has become common place in mineral exploration within Australia. Much of this data is interpreted after having only basic field corrections applied to the raw observations. While many interpretable features are typically present in these data sets, the relationships of the features to geologic structures can be deceptive, particularly for potential field data. The location, depth, trend and other aspects of structures can be enhanced considerable with a few basic processing steps that are generally absent from commercial software packages used in the mineral exploration industry and other geoscience applications. In addition, combining these geophysical data sets with geological data within a commercial image processing package permits direct overlay of different data sets, improved data manipulation capabilities, better geographic precision in individual data sets, and ultimately allows improved interpretations. The goals of this study are to;

1. obtain more information about subsurface features from existing data sets then is presently achieved.
2. produce easily interpreted images through applied Fourier methods.

investigate the application of the analytic signal to small scale, high resolution surveys.

3. investigate two-dimensional cross-correlation as a minerals exploration method.
4. illustrate the methodology developed here in a case study of a region in Queensland.

1.2 Case Study in Queensland

The ultimate goal of this study is to provide methods for improved interpretation of large geophysical data sets and then applying these methods to discover or improve the definition of subsurface resources (mineral deposits). The field area selected for this study was Goomeri-Biggenden area, in southeast Queensland. The methods investigated were the 3-dimensional analytic signal of the potential field data and the cross-correlation of the total magnetic intensities and the first vertical derivative of the total magnetic intensities with potassium-40 counts.

The analytic signal gave results that were similar to mapping the first vertical derivative of the total magnetic intensity. From this study, done at mid latitudes, the analytic signal gave no advantages. This may not be the case at low latitudes where anomalies are severely skewed and when reduction-to-the-pole is used high amplitude corrections are required.

Cross-correlation was computed in the Fourier plane and was implemented within ER Mapper. The cross-correlation of the total magnetic intensity and potassium-40 counts gave indications where hydrothermal activity occurred. It will be shown that as a preliminary exploration method, it can be used to identify areas most likely to contain mineralization. The images produced by this method are easy to interpret as they only show areas that correlate, anti-correlate or give no correlation with the model used in the hypothesis.

The cross-correlation of the first vertical derivative of the total magnetic intensity and potassium-40 will be shown to identify areas where structurally controlled, near surface mineralizations are most likely to occur. The verification of these results is by the location of past or present mining activity and for the Goonaloom Creek survey by drill hole locations.

1.3 North LTD. (Exploration Division)

1.3.1 Project Background and Support

The data for this project were supplied by North LTD, (Exploration Division), Queensland. The initial approach was to Derek Webb, then Supervising Geophysicist at Parkes, New South Wales. At this time, the Gunumbla Project was ending with the start of development of the North Parkes Mine. As Derek Webb was transferring to the Brisbane office, he, with the approval of Philip McInerny, Consulting Geophysicist, North LTD, Melbourne, supplied magnetic and radiometric data of the Goonaloom Creek and Biggenden aero-data surveys flown by Kevron in October, 1994.

On Derek Webb's departure from North LTD., Terry Hoschke transferred to Brisbane to take up the position of Supervising Geophysicist at the Brisbane office and assumed the role of supervisor for North LTD.

2 Fourier Series and Transformations

2.1 Fourier Series

Many mathematical methods rely on transforming data from a form that is cumbersome to handle to a form that is simple to handle, e.g., logarithms and Laplace transforms. In keeping with this, Fourier analysis involves transforming spatial or temporal data to the wavenumber or frequency domain where certain processes can be performed easily.

Fourier's theorem states that any periodic function can be analyzed into a sum of harmonic terms.

Fourier Series in trigonometric form

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left[a_n \cos\tfrac{n\pi x}{L} + b_n \sin\tfrac{n\pi x}{L}\right] \text{ for } -L \le x \le L$$

Equation 2—1

where

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$b_n = \frac{1}{L}\int_{-L}^{L} f(x)\sin\frac{n\pi x}{L}\,dx$$

For the Fourier expansion of $f(x)$ to exist, $f(x)$ must satisfy the Dirichlet conditions:-

The function $f(x)$, defined for $(-L \le x \le L)$ with $f(x+2L) = f(x)$ is bounded and has a finite number of finite step discontinuities. Then the Fourier expansion converges to

$$\frac{1}{2}\left[\lim_{x\to a^+} f(x) + \lim_{x\to a^-} f(x)\right] \text{ at every point } x = a$$

Equation 2—2

The Dirichlet conditions imply that the Fourier expansion is analytic and that the periodic function has existed for all time and will exist for all time. This condition is unrealistic under normal conditions. Generally, periodic functions are assumed to meet this requirement after the function has attained an unfluctuating level.

2.2 Fourier Transform or Fourier Integral

Fourier expansion allows for the analysis of periodic functions, Fourier transforms allow for the analysis of non-periodic functions, e.g., impulses and random events.

Most texts reviewed, for the application of Fourier transforms use a heuristic approach to the Fourier Integral and heuristic or mathematical development are not covered herein and the Fourier transform is stated without proof.

The Fourier transform for one dimension in exponential form is defined as

$$F(s) = \int_{-\infty}^{\infty} f(x) e^{-i2\pi xs} dx \qquad \textit{Equation 2—3}$$

and the inverse transform as

$$f(x) = \int_{-\infty}^{\infty} F(s) e^{i2\pi xs} ds \qquad \textit{Equation 2—4}$$

2.3 Conditions and Requirements for Application of the Fourier Transform.

For the Fourier transform to exist a similar set of condition to the Dirichlet conditions for Fourier expansion should be met. First the function $f(x)$ must be bounded and have a finite number of finite step discontinuities.

These conditions exclude functions such as:-

$\sin(t)$	harmonic wave	$-\infty \prec t \prec \infty$
$H(t)$	Heavyside step function	$H(t) = 0 \; for -\infty \prec t \leq 0$ $H(t) = 1 \; for \; 0 \prec t \prec \infty$
$\delta(t)$	Impulse function	$\delta(0) = \infty$ $\delta(t) = 0 \quad t \neq 0$

However Fourier transforms for these and other useful functions can be approximated. In general, any accurately measured physical phenomenon will conform to the conditions of existence and have a transform. (Bracewell, 1965)

2.4 Two Dimensional Fourier

In geophysical analysis, one-dimensional Fourier transforms may be used to process line data. For area surveys a two-dimensional Fourier transform is required.

A two-dimensional function, $f(x,y)$ has a two-dimensional transform $F(u,v)$ and between them the following relationship exists :-

$$F(u,v) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(x,y)e^{-i2\pi(ux+vy)}dxdy \qquad \textit{Equation 2--5}$$

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)e^{i2\pi(ux+vy)}dudv \qquad \textit{Equation 2--6}$$

under the conditions that $f(x,y)$ is bounded and has an finite number of finite step discontinuities. These theorems apply to continuous functions. Data gathered electronically is discrete or sampled data and these theorems must be modified before they can be applied.

3. The Application of Fast Fourier Transforms to Discrete Sampled Data

3.1 Sampling and Nyquist Criteria

Bracewell (1965), states the sampling theorem as:

"A function whose Fourier transform is zero for $|s| \succ s_c$ is fully specified by values spaced at equal intervals not exceeding $\frac{1}{2}s_c^{-1}$ save for any harmonic term with zeros at the sampling points", with $s =$ frequency and $s_c =$ cut-off frequency (spatial or temporal). Simply put, the highest frequency present in the signal must be less than half the sampling frequency for an accurate reconstruction of the signal. If frequencies greater than half the sample frequency are present then aliasing occurs. This is also known as the Nyquist Sampling Criteria and $2s_c$ is referred to as the Nyquist frequency.

Blakely (1995), Bracewell (1965), Reynolds et al (1989), and many others from various fields of science, show that the spectrum of a sampled signal repeats itself with each occurrence centered on a integer multiple of the sampling frequency. If the signal is not band limited, i.e., frequencies above $\frac{1}{2}\omega_{sample}$ are present, there is an overlap in the spectra. Then the recovered signal will not be an accurate representation of the original signal as new spectral components will have been added. The new spectral components will be those frequencies above $\frac{1}{2}\omega_{sample}$ folded about $\frac{1}{2}\omega_{sample}$. Compromises made with regard to bandwidth, in data acquisition must be kept in mind, when analyzing discrete data.

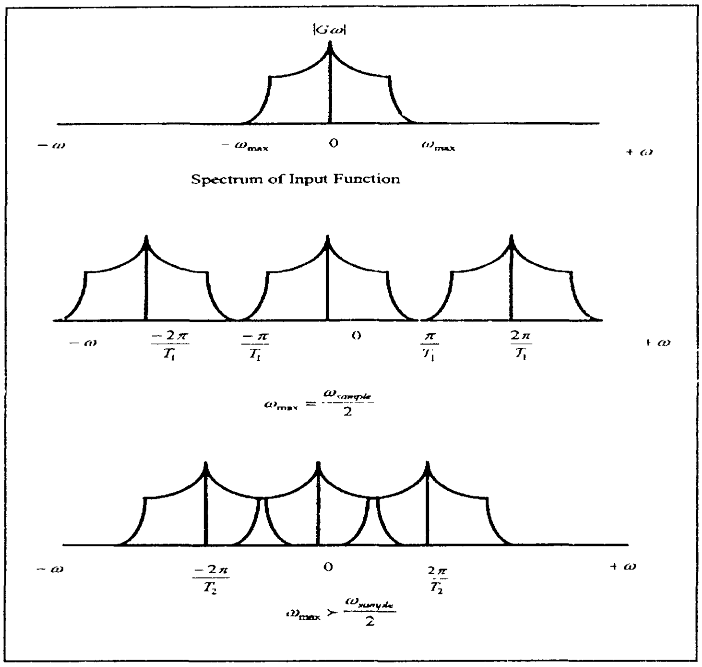

Figure 3—1: Aliasing, the overlap of spectra caused when the sampling frequency is less then twice maximum signal frequency present. TOP: input spectra. MIDDLE: output spectra at critical sampling. BOTTOM: output spectra when the input signal is under sampled.

3.2 DFT & FFT in One Dimension.

Fourier transforms as presented so far have been for continuous functions and their application to discrete data only hinted at by the sampling theorem. The discrete Fourier transform (DFT) and its more computer efficient refinements, the fast Fourier transform algorithms (FFT) were developed for the numerical analysis of sampled data.

Stearns and David, (1993), express the forward DFT as:

$$X_m = \sum_{k=0}^{N-1} x_k e^{-i(2\pi \frac{m}{N} k)} \quad for\ m = 0, 1, 2, 3, \ldots, \frac{N}{2}$$

Equation 3—1

where N is an even number of samples.

Or in trigonometric form as:

$$X_m = \sum_{k=0}^{N-1} x_k \cos(\frac{2\pi m}{N}k) - j\sum_{k=0}^{N-1} x_k \sin(\frac{2\pi m}{N}k)\ for\ m = 0, 1, 2, 3,\ldots,\frac{N}{2} \qquad \text{Equation 3–2}$$

The period of X_m is given as $\frac{N}{m}$ samples or $\frac{N\Delta x}{m}$ units of measure where Δx is the sample interval.

Lynn (1990), and others state the number of calculations of a DFT as being proportional to N^2 and for a FFT as being proportional to $N\log_2 N$. This improvement in computational efficiency was first pioneered by Cooley & Tukey (in Press et al., 1992) and achieved by recognizing redundant calculations and symmetry. By the use of symmetry, the number of calculations was reduced but an added requirement was that $N \in 2^n$ where n is a positive integer. The full consequences of this are discussed in Section 4-7, Edge Effects, Padding and Compromises

Stearns and David (1993), state the inverse DFT as:

$$x_k = \frac{1}{N}\sum_{m=0}^{N-1} X_m e^{i(2\pi k\frac{m}{N})} \quad for\ k = 0,1,2,3,\ldots,N-1 \qquad \text{Equation 3–3}$$

The inverse transform is similar to the forward transform with the only difference being the sign of the exponential, negative for forward transformations and positive for the inverse transformations and the scaling factor $\frac{1}{N}$ for the inverse transform. Because of this symmetry, the same computer algorithm is used for forward and inverse transformations. A direction flag (± 1) is used to control which transform is applied and the scaling factor, $\frac{1}{N}$ for the inverse transform, is often omitted from the algorithm. The programmer is expected to be aware of these practices (Press et al., 1992).

3.3 Two Dimensional DFT-FFT

As with the Fourier transform for continuous functions, the DFT or FFT can be extended to data of two or more dimensions. Press, et al. (1992), state for a complex function $h(k_1,k_2)$ defined over the two-dimensional grid, $0 \le k_1 \le N_1 - 1$, $0 \le k_2 \le N_2 - 1$, the two-dimensional discrete Fourier transform is a complex function $H(n_1,n_2)$, defined over the same grid,

$$H(n_1,n_2) \equiv \sum_{k_2=0}^{N_2-1}\sum_{k_1=0}^{N_1-1} h(k_1,k_2)\, e^{-i(2\pi k_2 \frac{n_2}{N_2})}\, e^{-i(2\pi k_1 \frac{n_1}{N_1})}$$

Equation3—4

which simplifies to the form usually stated for two dimensions,

$$H(n_1,n_2) \equiv \sum_{k_2=0}^{N_2-1}\sum_{k_1=0}^{N_1-1} h(k_1,k_2)\, e^{-i2\pi(k_2 \frac{n_2}{N_2} + k_1 \frac{n_1}{N_1})}$$

Equation 3—5

and for m dimensions as

$$H(n_1,n_2,\ldots,n_m) \equiv \sum_{k_m=0}^{N_m-1} \cdots \sum_{k_2=0}^{N_2-1}\sum_{k_1=0}^{N_1-1} h(k_1,k_2,\ldots,k_m) e^{-i(2\pi k_m \frac{n_m}{N_m})} \ldots e^{-i(2\pi k_2 \frac{n_2}{N_2})}\, e^{-i(2\pi k_1 \frac{n_1}{N_1})}$$

Equation 3—6

The sign of i used here is to conform to the definition for one dimension not the programming conventions of Press et al.

For two dimensions and higher, the inverse DFT is defined as: -

$$h(k_1,k_2,\ldots,k_m) \equiv \frac{1}{N_1 N_2 \ldots N_m} \sum_{k_m=0}^{N_m-1} \cdots \sum_{k_2=0}^{N_2-1}\sum_{k_1=0}^{N_1-1} H(n_1,n_2,\ldots,n_m) e^{i(2\pi k_m \frac{n_m}{N_m})} \ldots e^{i(2\pi k_2 \frac{n_2}{N_2})}\, e^{i(2\pi k_1 \frac{n_1}{N_1})}$$

Equation 3 7

3.3.1 Structure of Fourier data

As is obvious from the equations for Fourier transforms the function to be transformed may be real, imaginary or complex and the transformation may be real, imaginary or complex. Transformations of measured phenomenon are usually complex. Bracewell (1965) and others discuss the relationship between oddness and evenness and the transform being purely real, purely imaginary or complex. As this project deals with data gathered in "the real world", the following properties hold:

The input to a forward transformation is real only, with no assumptions or conditions for evenness or oddness.

1. The transformed data will be complex. If the input function is neither odd nor even then the output will contain real and imaginary components (complex).

2. The inverse transformation will be real only.

Blakely (1995), states that it can be shown that potential data is Hermitian and this is sufficient condition to ensure that the inverse transformation is real only.

If point 2 were not the case then Fourier transformations would be of little use in processing of geophysical data such as gravity or magnetics.

3.3.2 Examples of Spatial Data and their Fourier Transformation in Two-Dimensions

Figure 3—3 is the logarithm of the magnitude of the Fourier transform of the Goonaloom aeromagnetic data. As a Fourier transform is complex, it is normal to graph the magnitude only. The logarithm was taken to increase the range of values to display. Most log power spectrums will be similar in appearance to Figure 3—3.

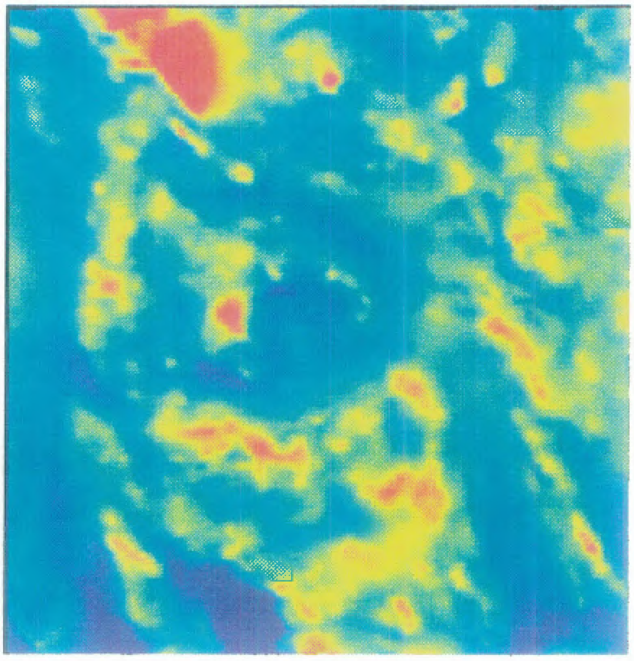

Figure 3—2 ER Mapper image of Goonaloom Creek Aeromagnetic Data. This data is displayed in the Fourier plane in Figure 3--3.

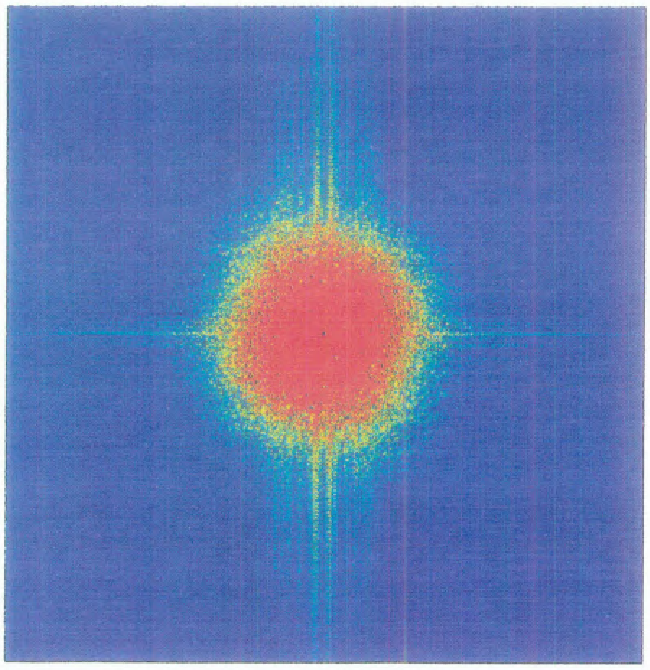

Figure 3—3: An example of a log power spectrum, Goonaloom Creek. aeromagnetic data (Figure 3-2).

3.3.3 Frequency Distribution of a Two-dimensional Fourier Transformation

Figure 3—4 is the two-dimensional log power spectrum of Figure 3—2. The *x*-axis of the spatial data has been mapped to the *u*-axis of the spectral data and the *y*-axis has been mapped to the *v*-axis.

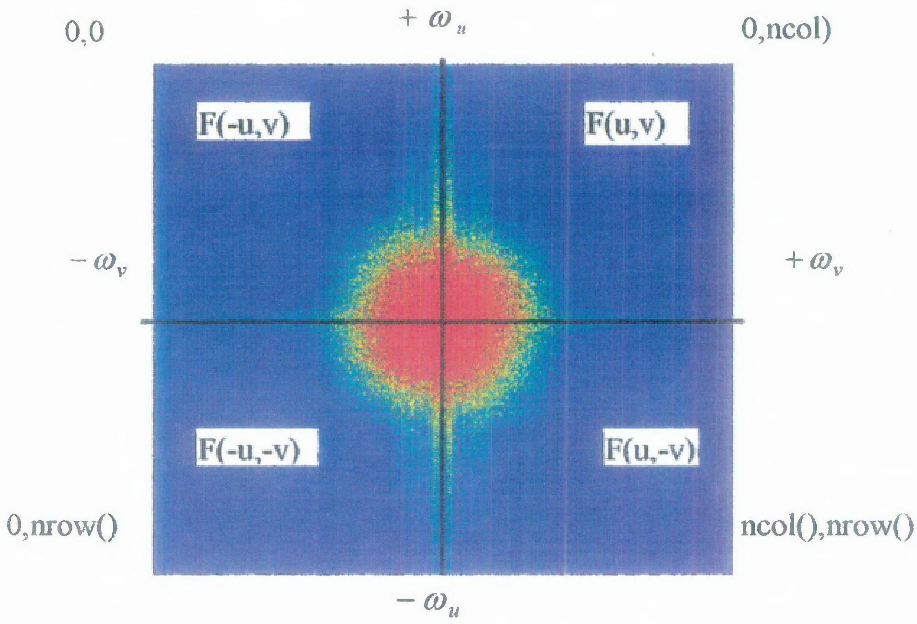

Figure 3–4 Log power spectrum with u and v, axes and quadrants labeled to show symmetry.

There appears to be an apparent rotation of the axes, with vertical (*y* direction) information displayed on the horizontal (*v*) axis, but if one visualizes a vertical corrugation, the wavelength or frequency is measured in the horizontal direction. A similar situation exits for latitude, the line on a map is horizontal but the measure is vertical.

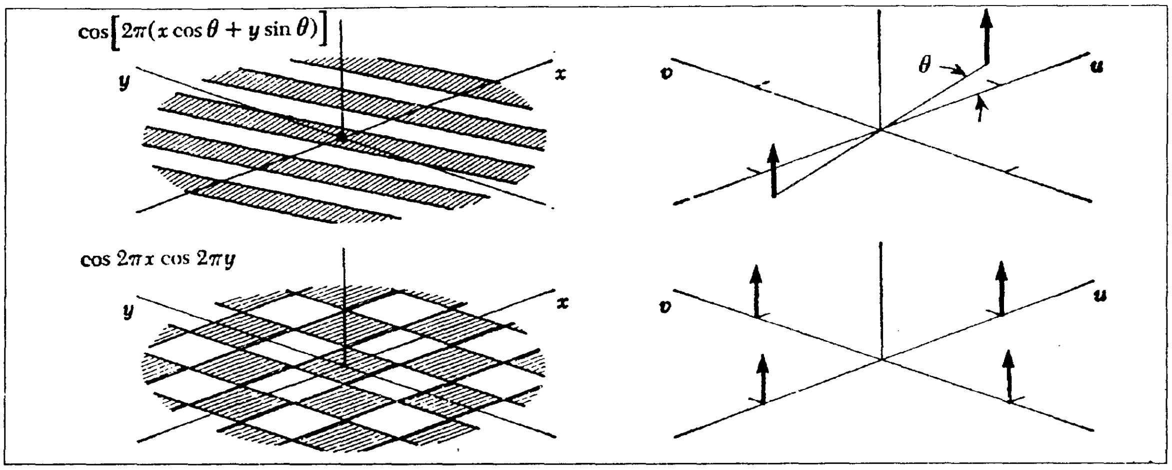

Figure 3–5 Fourier Transforms. a) Oblique corrugation (left) maps to 2 points in the Fourier plane (right); b) perpendicularly crossed corrugations (left) maps to 4 points in the Fourier plane (right) (Bracewell, 1965)

Now consider a vertical corrugation: If only one frequency of corrugation exists with no horizontal corrugation, the ω_u component is zero and the frequency of corrugation will appear as a single point on the v axis. If the corrugation is the sum of two frequencies then it will be fully represented by two points on the v axis.

For an oblique corrugation of a single frequency, ω_c, measured perpendicular to strike. The corrugation will contain frequency components in both the u and v directions and will be fully represented by the point ($\omega_c \cos\vartheta$, $\omega_c \sin\vartheta$) where ϑ is the angle of the corrugation to the horizontal or the frequency of corrugation is $\omega_c = \sqrt{\omega_u^{\,2} + \omega_v^{\,2}}$

Now consider a point feature in the spatial data (figure 4-6), which can be represented as an impulse. The Fourier transform of the impulse, $\delta(x - x_0, y - y_0)$ at x, y, or anywhere, is defined as '1' everywhere, meaning all frequencies are represented equally. This is the limiting case for small spatial features. The position of the impulse in the inverse transformed data will be fixed by the phase (imaginary) component in the forward transform.

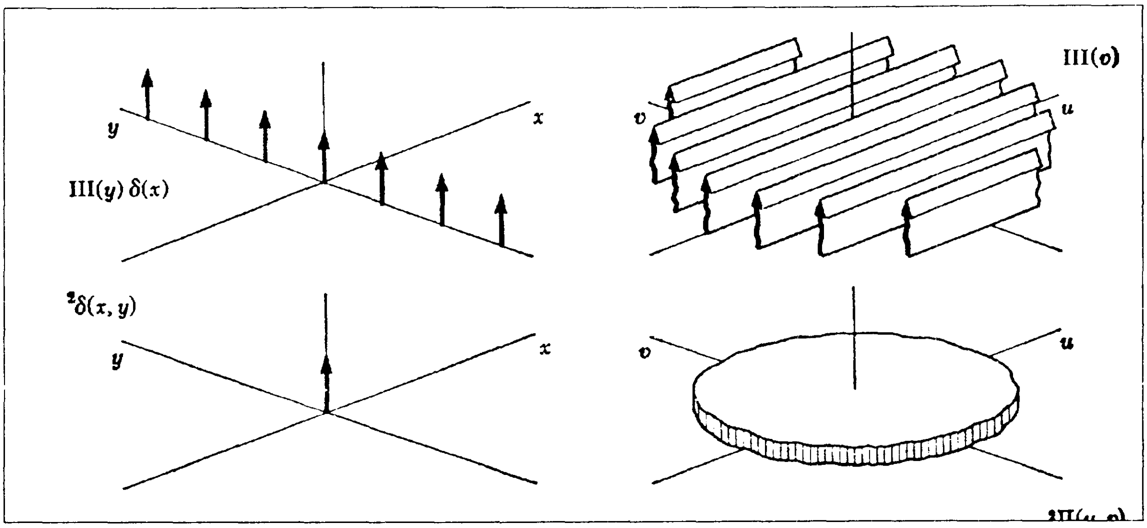

Figure 3–6 Fourier Transform. a) Row of points⇒ constant values. b) Impulse at x=0, y=0 ⇔ 1 everywhere. (Bracewell 1965)

Now consider a vertical line in the spatial data. In the horizontal direction, the line appears as an impulse and in the vertical direction as a Heavyside step. The impulse component contributes, '1' everywhere, to the transform and the Heavyside step confines the '1 everywhere' to a line of constant ω_u. The two-dimensional Fourier transform may be viewed as the convolution ot two, one dimensional, perpendicular transforms.

These examples may be generalized as follows:

features of large spatial extent are represented in the Fourier plane by limited extents

spatial features of limited extent occupy large frequency extents in the Fourier plane.

features of mixed extent such as lineations (length>breadth) show mixed frequency extents in inverse proportions to their respective spatial extents.

The zero frequency term is the sum of all samples for that vector and is equivalent to the average or $A_{(0)}$ term of a Fourier transform of a continuous function. The coordinates for the transform matrix conform to ER Mapper's coordinate system for spatial data, top left corner is 0,0 and the bottom right is ncol(), nrow(). Where ncol()

and nrow() are ER Mapper functions that return the maximum number of columns (cells per line) or rows (number of lines), respectively, in the data. Understanding the coordinate system definition is critical for the use of filtering and wavenumber transformation operations. The filter masks are defined using these coordinates and ***k*** for wavenumber transformations is calculated from these coordinates.

4. Wavenumber Filtering

After transforming spatial data to the frequency domain, filtering would appear to be the simple process of setting to zero the value of the unwanted complex frequency components in the transform. This is only half-true. Filtering is performed by setting to zero or zeroing, the unwanted frequencies but there are side effects. These are dependant on how the zeroing is performed, or more precisely, the shape of the cut-off. Features in the spatial data are a summation of complex frequencies, amplitude and phase, in the Fourier plane. By zeroing the unwanted frequencies, complex phase relationships are affected and out of band responses occur. This phenomenon also occurs with analog filtering. Filtering in *N*-dimensions can be viewed as the convolution of the *N*-dimensional data with a suitable window function. The inverse transform is defined over the whole Fourier plane, so complex frequency components of the window function will also be transformed, leading to out-of-band responses, which may appear in the inverse transformed data as processing artifacts. ER Mapper uses a cosine taper for high-pass and low-pass filtering to reduce these out of band responses to a minimum.

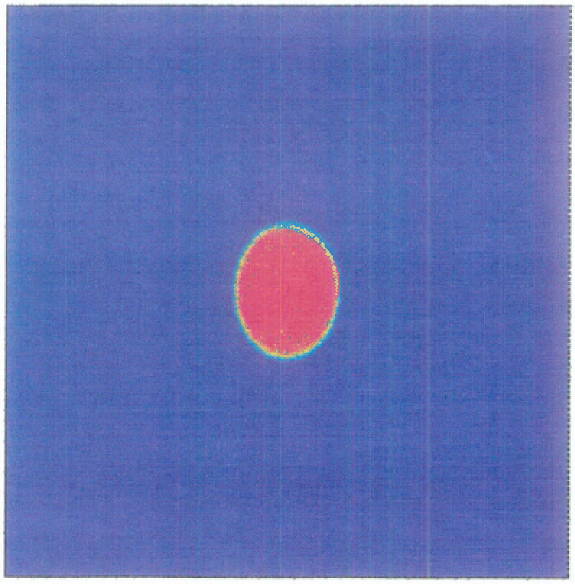

Figure 4—1: Log power spectrum of low-pass filtered data. The blue area represents the high frequency components that have had their values set to zero, the red area is the passed frequencies and the band between the red and blue is the cosine taper.

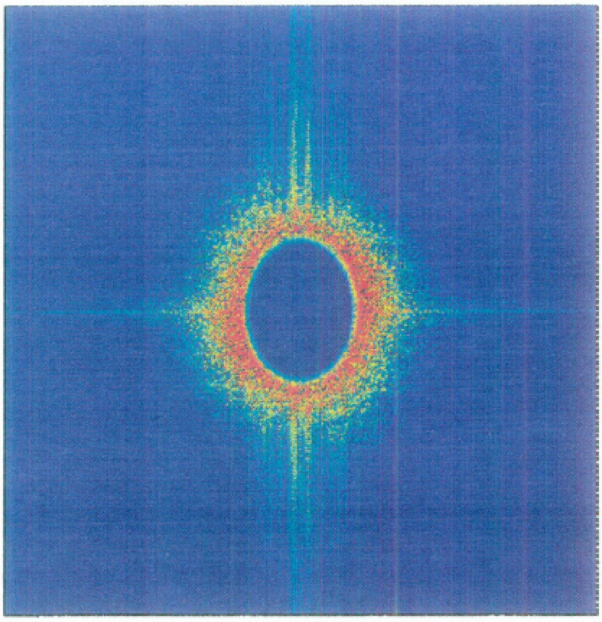

Figure 4—2: Log power spectrum of high-pass filtered data. The blue area in the centre represents the low frequency components that have had their values set to zero, the remaining area is the passed frequencies and the band between these is the cosine taper. Cut-off frequencies for Figure 5-1 and Figure 5-2 are identical.

4.1 High-pass Filters and Low-pass Filters

High-pass and low-pass filters are commonly used in geophysical signal processing with one-dimensional data. Here, the data sets have areal coverage and two-dimensional filtering is necessary.

As discussed before, the data are gridded with evenly spaced values along the x and y axes, comprising the spatial domain. For two-dimensional high-pass and low-pass filtering, the cut-off frequencies for the u and v axes (the wavenumber domain) are required and an additional frequency width is given for the interval over which the taper or roll-off function is applied.

As ER Mapper does not define filtering in terms of frequencies but in terms of lines (v axis) and pixels (u axis). Wavelength in sampled data is defined as:-

$$\lambda = \frac{N\Delta x}{m} \qquad \textit{Equation 4—1}$$

Where

N = number of samples

Δx = sample spacing

m = row or column number

Equation 4—1 transposed for m is required to evaluate the values for line 1 and line 2 as well as the pixel value for high-pass and low-pass filtering. The information for Equation 4--1 can be obtained from the file header.

4.2 Wavenumber Filtering and Transform Pairs

An understanding of filtering in the Fourier domain is assisted by an understanding of Fourier transform pair. A transform pair is a spatial function and a Fourier transform function with the property of each being the Fourier transform of the other, not only the first being the inverse of the second.

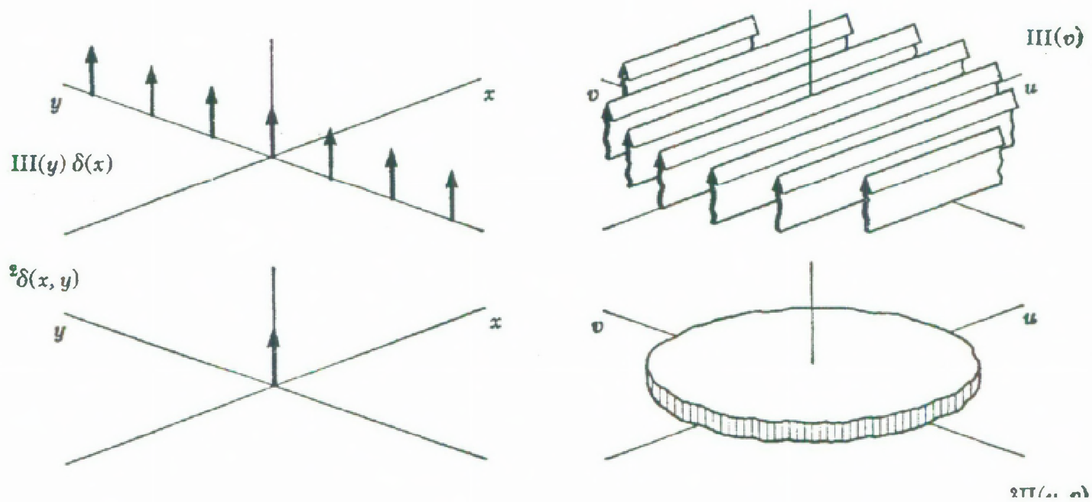

Figure 4—3 Fourier Transform Pairs (Bracewell 1965)

As can be seen from the upper pair of Figure 4-3, a row of points in the spatial domain transforms into a row of lines in the Fourier domain and a row of lines in the spatial domain will be represented in the Fourier domain by a row of points. From the lower example of Figure 4-3, a noise spike in the two dimensional spatial domain will be represented across the entire Fourier transform plane making it very difficult to remove. The converse is that a feature such as periodic speckle noise in the spatial data will be a single feature in the transform and can be easily removed. These properties allow one to see what undesirable features of a spatial data set can be removed by filtering in the Fourier domain and those that are best removed in the spatial plane.

4.3 Notch Filtering

Notch filtering in ER Mapper is the setting to zero, cells or groups of cells in the Fourier transform. In notch filtering no roll-off function is used. This form of filtering can be used for removing grid lines and for removing periodic noise such as speckle noise.

5. Convolution and Cross-Correlation in the Two-Dimensional Fourier Plane

5.1 Convolution in the Two –Dimensional Fourier Plane

Convolution in the Fourier plane is defined as:

$$F(k_x, k_y)G(k_x, k_y) \qquad \textit{Equation 5—1}$$

where F and G are Fourier transformations of $f(x,y)$ and $g(x,y)$. The result of a convolution in the Fourier plane will be complex and Hermitian so the inverse transformation will be real and the phase information preserved.

5.2 Cross-Correlation in the Two-Dimensional Fourier Plane

Cross correlation in the Fourier domain is defined as:

$$F(k_x, k_y)G^*(k_x, k_y) \qquad \textit{Equation 5--2}$$

and auto-correlation is defined as:

$$F(k_x, k_y)F^*(k_x, k_y) \qquad \textit{Equation 5--3}$$

The result of a cross correlation in the Fourier plane will be complex Hermitian so the inverse transformation will be real preserving the location information.

The result of auto-correlation is real only and all phase or location information is lost.

The term Fourier matrix has been used to describe transformed data stored in a two-dimensional matrix format. The operations of convolution and cross correlation in the Fourier plane are not operations as defined for linear systems where multiplication is a cross product or scalar product. Convolution and correlation are point by point elemental multiplication of two Fourier transform data sets, that is,

c [i, j] = a [i, j] b [i, j] for i = 1 to n, j = 1 to m,

where * denotes the complex conjugate and all terms are defined as for convolution.

Cross correlation is the comparison of frequency components in two Fourier transformed data sets. Point by point multiplication of the corresponding frequencies ($\omega_{x,y}$) will amplify or attenuate that particular frequency depending on the relative amplitudes of each. Inversion of the resultant data set will be the cross-correlated image that is not normalized. As can be seen from the cross-correlation formula, $F(k_x, k_y)G^*(k_x, k_y)$ and the description of elemental multiplication, F and G must be dimensionally identical.