Chapter 7

Modelling Infrequency of Purchase and Equivalence Scale Estimation: A Bayesian Solution

In the last chapter, Bayesian techniques were employed to develop a procedure for estimating equivalence scales. Using household level micro-unit data, the procedure was applied to a model that aimed to measure the effect of demographic variables on household welfare as revealed by its consumption behaviour in the survey. Data based on household expenditure surveys, however, present a major estimation problem. Because of the short period for which expenditures are recorded, it is common to find a high proportion of surveyed households that report a non-purchase on one or more commodities. Demand models that do not give special treatment to the occurrence of zero observations in surveys yield biased results.

In this chapter, attention is focused on the treatment of observed zero expenditures that arise due to the "infrequency of purchase" of the consumer unit in survey data. A model that adjusts recorded expenditures by a "probability of purchase" factor is developed and Bayesian techniques are employed to derive an estimation procedure. Posterior densities for commodity-specific and general scales are derived using data from the 1988-89 Household Expenditure Survey. Note that this chapter is appearing as a book chapter in a forthcoming 1997 volume of *Advances*

in Econometrics, edited by T.B. Fomby and R.C. Hill.

7.1 The Zero Expenditure Problem

Household budget surveys commonly record expenditures over only a relatively short period of time. Data sources of this kind - which includes the Australian Household Expenditure Survey - are constructed from diary records of expenditures over a period of two weeks. Household weekly expenditures obtained from these surveys' microdata files are recorded as a per week average of expenditures data collected within that two-week recall period. Because of the short period of reference, zero expenditures indicating non-purchases on one or more commodities are recorded for a high proportion of households. This is clearly the case for the 1988-89 Household Expenditure Survey (HES) micro-unit data where zero expenditures were reported for the eleven broadly-defined commodity groupings as seen in Table 7.1 below. Zero expenditures are considered dubious indicators of consumption for such necessities as food, clothing and shelter. On the other hand, zero expenditures for Alcohol and Tobacco are acceptable because it is a fact that some households do not consume such commodities. Recorded zero expenditures in surveys can thus be misleading.

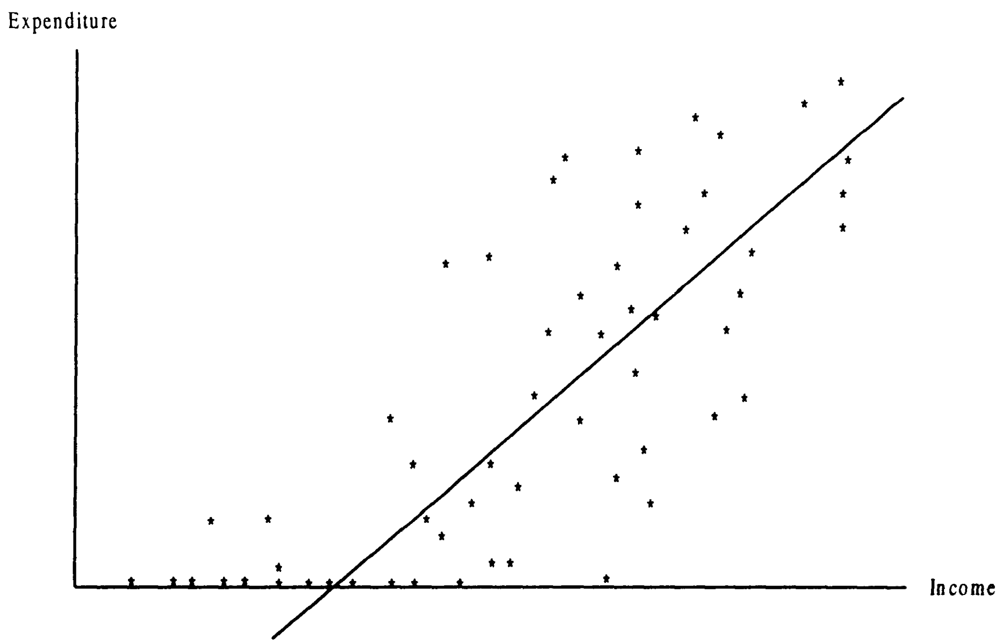
Table 7.1 Households with Zero Expenditures, 1988-89 HES

	Households	
Commodity Group	*Number*	*Proportion*
Total Expenditure	0	0.0%
Housing	140	2.5%
Fuel & Power	134	2.4%
Food	9	0.2%
Alcohol & Tobacco	1800	32.5%
Clothing & Footwear	1791	32.4%
Household Furnishings & Equipment	50	0.9%
Medical & Health Care	629	11.4%
Transport	199	3.6%
Recreation & Entertainment	190	3.4%
Personal Care	1186	21.4%
Others	463	8.4%

There are a number of possible causes that give rise to zero values in expenditure surveys. A recorded non-purchase can result from false reporting by either the respondent or the enumerator. It could also represent a "corner solution" case where a consumer chooses not to consume at the given price and income. The recording of a non-purchase may also reflect underlying taste differences across the sample – households may simply not consume some commodities at any given price or income. Finally, it is possible that zeros represent cases of "infrequency of purchase" in which households usually consume the items, but are not recorded as making expenditures for them within the timeframe of the survey period. Households who 'stock-up' on groceries and other food items will fall under this case.

The typical 'censored' sample looks something like the configuration illustrated in Figure 7.1. The model that has traditionally been used to account for such type of data common in commodity demand studies[1] is the Tobit model. The underlying

Figure 7.1 A censored sample



[1]It is also common to find such censoring in labour supply studies.

assumption in the Tobit model is that the same stochastic process determines both the value of continuous observations on the dependent variable and the discrete switch at zero. That is, a zero realisation for the dependent variable represents a corner solution or a negative value for the underlying latent dependent variable. This obviously restricts other quite reasonable determinants of zero observations such as infrequency of purchase or misreporting in commodity demand.

Such restrictions have been recognised in the past. To account for the possibility of misreporting, Deaton and Irish (1984) extended the Tobit model through the addition of a binary censor which aimed to explain how the zeroes were realised. The resulting 'p-tobit' model was applied to a single-equation analysis on alcohol and tobacco expenditure in survey data using maximum likelihood procedures in the estimation. Keen (1986) also proposed to model infrequency of purchase through a binary censor. He introduced an instrumental variable estimation procedure to obtain unbiased and consistent estimates of the model. Other bivariate alternatives to the Tobit model are suggested in Atkinson, Gomulka and Stern (1984), Blundell, Ham and Meghir (1986), and Blundell and Meghir (1987)[2].

The occurrence of zero expenditure observations presents a major econometric estimation problem. The problem lies in the fact that economic models of consumer behavior are formulated in terms of an agent's consumption, not an agent's expenditure. From the point of view of estimation, the case is that the information required for econometric modelling is household weekly consumption but the best information available is derived from survey microdata files which report household weekly expenditures. The occurrence of zeros, thus, creates a missing-values problem for some of the dependent-variable observations. It also means that recorded expenditures inflate consumption levels when they are positive. This chapter deals with the treatment of zero expenditures that arise due to the infrequent purchasing behaviour of households. Hence, we call the model developed here as the "infrequency of purchase" model. This is not to say that the other causes are not significant or are less important. Misreporting is, however, a serious concern only for comparatively few commodities, while preference variation

[2]Pudney (1989) presents an excellent summary of the economic literature concerning zeroes.

is likely to be especially important when dealing with relatively fine commodity classifications. Infrequent purchasing, however, is liable to be a problem even when dealing with broad commodity groups, as was seen in Table 7.1.

In the following sections, we develop a particular model for the infrequency of purchase problem where zero values for the dependent variable cannot be attributed to corner solutions. This is a consumer demand model where consumption is always positive but recorded expenditures are often zero. In line with this, the commodities used in the empirical application were further aggregated into just four groups as shown in Table 7.2. Here, it is clear that zero expenditures continue to be observed for the Food, Clothing and Housing commodities over the interview period. With such broadly aggregated commodity groupings, it seems reasonable to attribute the presence of zero observations to infrequent purchasing of households. Interestingly, in this type of model the latent dependent variable is never directly observed. This arises because a positive expenditure will represent a purchase of stock whose services will be consumed over future periods typically longer than the period of observation. The infrequency model can be viewed as a snapshot of the dynamic process determining stock accumulation and consumption of services.

Table 7.2 Total No. of Households with Zero Expenditures

Household Type	*Total No. of Households*	*No. of Households with Zero Expenditures*			
		Food	*Clothing*	*Housing*	*Others*
(2,0)	2074	0	540	34	0
(2,1)	532	0	80	7	0
(2,2)	889	0	122	12	0
(2,3)	388	0	44	2	0
(1,0)	1372	8	680	58	0
(1,1)	132	1	42	3	0
(1,2)	103	0	28	0	0
(1,3)	42	0	11	1	0
Total	5532	9	1547	117	0

7.2 Accounting for Zero Expenditures in a Demand Model

The model used in this chapter is the extended linear expenditure system (ELES) which was discussed previously in section 5.1.1 of Chapter 5. Recall that for the equivalence scale estimation problem, we first considered the following n-equation linear seemingly unrelated regression (SUR) system (first presented in Chapter 5 as equations (5.29) and (5.30)) and presented here again to set out notation:

$$\begin{bmatrix} v_{1h} \\ v_{2h} \\ \vdots \\ v_{nh} \end{bmatrix} = \begin{bmatrix} z_h & & & \\ & z_h & & \\ & & \ddots & \\ & & & z_h \end{bmatrix} \begin{bmatrix} \theta_{1h} \\ \theta_{2h} \\ \vdots \\ \theta_{nh} \end{bmatrix} + \begin{bmatrix} x_h & & & \\ & x_h & & \\ & & \ddots & \\ & & & x_h \end{bmatrix} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \vdots \\ \eta_n \end{bmatrix} + \begin{bmatrix} e_{1h} \\ e_{2h} \\ \vdots \\ e_{nh} \end{bmatrix}$$

or

$$\mathbf{V}_h = \mathbf{Z}_h \mathbf{\Theta}_h + \mathbf{X}_h \boldsymbol{\eta} + \mathbf{E}_h \tag{7.1}$$

where

h	$= 1, 2, ..., H$ refers to household composition type h;
n	refers to the number of commodity groups;
v_{ih}	is an $(M_h \times 1)$ vector of observations on expenditure for the i^{th} commodity and the h-type household;
z_h	is an $(M_h \times 1)$ vector of ones;
x_h	is an $(M_h \times 1)$ vector of observations on income for households of type h;
e_{ih}	is an $(M_h \times 1)$ vector of errors;
$\mathbf{V}_h$	is of dimension $(nM_h \times 1)$;
$\mathbf{Z}_h$	$= I_n \otimes z_h$ is an $(nM_h \times n)$ matrix of dummy variables;
$\mathbf{X}_h$	$= I_n \otimes x_h$ is an $(nM_h \times n)$ vector of household incomes;
$\mathbf{\Theta}_h, \boldsymbol{\eta}$	are $(n \times 1)$ vectors of unknown parameters;

$\mathbf{E}_h$ is an $(nM_h \times 1)$ vector of errors which is assumed to be distributed as

$$\mathbf{E}_h \sim N[0, \mathbf{\Omega}_h \otimes I_{M_h}] \tag{7.2}$$

where $\mathbf{\Omega}_h$ is a $(n \times 1)$ error covariance matrix. The per commodity expenditure equation is

$$v_{ih} = \theta_{ih} z_h + \eta_i x_h + e_{ih} \tag{7.3}$$

where z_h is an $(M_h \times 1)$ vector of ones and e_{ih} is an $(M_h \times 1)$ vector of errors. Now, as noted in the last section, recorded expenditure in a given week does not necessarily reflect weekly consumption. Some households may purchase a commodity less frequently than once a week, but still consume that commodity. How do we then account for this observation in the context of our model in (7.1)?

Suppose that v_{ihj} represents the j^{th} h-type household's average consumption of commodity i over a period of m weeks so that total consumption over this period is mv_{ihj}. It is worth stressing that v_{ihj} (a component of v_{ih} in equation (7.3)) now denotes consumption, not expenditures, as in the previous chapters. Suppose, also, that for a particular household, purchases of commodity i are made during n_{ihj} of those weeks ($n_{ihj} < m$). Let $\overline{y}_{ihj}$ be the average weekly expenditure for those weeks when an expenditure was made. Total expenditure over the m week period is $n_{ihj}\overline{y}_{ihj}$. It is reasonable to assume that average expenditure and average consumption over the period of m weeks are the same. Hence,

$$n_{ihj}\overline{y}_{ihj} = mv_{ihj} \tag{7.4}$$

from which we get

$$v_{ihj} = \frac{n_{ihj}}{m}\overline{y}_{ihj} \tag{7.5}$$

Let actual expenditure in a randomly chosen week in which an expenditure is made be

$$y_{ihj} = \overline{y}_{ihj} + \epsilon_{ihj} \tag{7.6}$$

Introduction of the error ϵ_{ihj} allows for the fact that a given household does not make exactly the same expenditure each time it incurs an expenditure. Also,

$$\frac{n_{ihj}}{m} = P_{ih} + w_{ihj} \tag{7.7}$$

where P_{ih} is the "average" probability of a household of type h making an expenditure on commodity i in a randomly selected week. In short, P_{ih} is called the probability of a purchase. Introduction of the error w_{ihj} allows for the fact that a given household will not necessarily purchase a commodity at completely regular intervals. Then, in a week when an expenditure is made, we can write

$$\begin{aligned} v_{ihj} &= (P_{ih} + w_{ihj})(y_{ihj} - \epsilon_{ihj}) \\ &= P_{ih}y_{ihj} + \epsilon^*_{ihj} \end{aligned} \tag{7.8}$$

where ϵ^*_{ihj} is a composite error reflecting contributions from w_{ihj} and ϵ_{ihj}. This results by substituting (7.6) and (7.7) into (7.5) above.

In the context of the commodity equation in (7.3), unobserved consumption in a survey week can be expressed as

$$\begin{aligned} v_{ihj} &= P_{ih}y_{ihj} + \epsilon^*_{ihj} \\ &= \theta_{ih} + \eta_i x_{hj} + e_{ihj} \end{aligned} \tag{7.9}$$

Thus,

$$P_{ih}y_{ihj} = \theta_{ih} + \eta_i x_{hj} + u_{ihj} \tag{7.10}$$

and $u_{ihj} = e_{ihj} - \epsilon^*_{ihj}$. For any commodity, the actual consumption of a household over some period can be equated with any observed expenditure on the item at that time multiplied by the household's corresponding probability of a purchase plus an error term. Equation (7.10) is the basic equation upon which the estimation procedure discussed in the next section is developed. Note that y_{ihj} is only observable if the j^{th} household (of type h) makes a purchase of commodity i during the survey period.

Once estimates for θ_{ij} and η_i are obtained, it is then possible to estimate a_{ih} and b_i from the relationships earlier defined in (5.17)-(5.20). These in turn lead to the estimation of equivalence scales s_{ih} and s_h from expressions (5.21) and (5.28) derived earlier.

7.3 Stochastic Specification and Likelihood Function

First consider a single observation on a single commodity i for the j^{th} h-type household. Let

$$u_{ihj} \sim N(0, \omega_{ih}^2)$$

and

$$D_{ihj} = \begin{cases} 1 & \text{if } y_{ihj} > 0 \\ 0 & \text{if there is no corresponding expenditure} \end{cases} \tag{7.11}$$

$$\begin{aligned} \text{Prob}(D_{ihj} = 1) &= P_{ih} \\ \text{Prob}(D_{ihj} = 0) &= 1 - P_{ih} \end{aligned}$$

Given $D_{ihj} = 1$, the pdf for y_{ihj} is

$$\begin{aligned} f(y_{ihj}) &= \frac{P_{ih}}{\sqrt{2\pi}\omega_{ih}} \exp\left\{ -\frac{1}{2\omega_{ih}^2} (P_{ih} y_{ihj} - \theta_{ih} - \eta_i x_{hj})^2 \right\} \\ &= P_{ih} g_{ihj} \end{aligned}$$

Note that P_{ih} is the Jacobian of the transformation from $P_{ih}y_{ihj}$ to y_{ihj}. It is convenient to write the remainder of the pdf as g_{ihj}.

The joint pdf for D_{ihj} and y_{ihj} is

$$\begin{aligned} f(D_{ihj}, y_{ihj}) &= [P_{ih} f(y_{ihj})]^{D_{ihj}} [1 - P_{ih}]^{1-D_{ihj}} \\ &= \left(P_{ih}^2 g_{ihj}\right)^{D_{ihj}} (1 - P_{ihj})^{1-D_{ihj}} \end{aligned}$$

If independence is assumed over all commodities and households, then the likelihood function would be given by the product of the $f(D_{ihj}, y_{ihj})$ over all $i = 1, 2, ..., n$, $j = 1, 2, ..., M_h$ and $h = 1, 2, ..., H$. However, it is more conventional and in line with the stochastic specification in the previous chapters to allow for correlation between expenditures on different commodities in each household and independence across households.

To work towards a likelihood function for this case, consider a household with expenditures on all commodities during the survey week. Also, to facilitate exposition, we henceon set $n = 4$, in line with the 4 commodities considered later in the empirical work. Equation (7.10) can then be written as

$$\begin{bmatrix} P_{1h} & & & \\ & P_{2h} & & \\ & & P_{3h} & \\ & & & P_{4h} \end{bmatrix} \begin{bmatrix} y_{1hj} \\ y_{2hj} \\ y_{3hj} \\ y_{4hj} \end{bmatrix} = \begin{bmatrix} \theta_{1h} \\ \theta_{2h} \\ \theta_{3h} \\ \theta_{4h} \end{bmatrix} + x_{hj} \begin{bmatrix} \eta_1 \\ \eta_2 \\ \eta_3 \\ \eta_4 \end{bmatrix} + \begin{bmatrix} u_{1hj} \\ u_{2hj} \\ u_{3hj} \\ u_{4hj} \end{bmatrix}$$

or

$$\mathbf{P}_h \mathbf{y}_{hj} = \mathbf{\Theta}_h + x_{hj}\boldsymbol{\eta} + \mathbf{u}_{hj} \tag{7.12}$$

where $\mathbf{u}_{hj}$ is assumed to be distributed as

$$\mathbf{u}_{hj} \sim N(0, \mathbf{\Omega}_h)$$

Given all components of $\mathbf{y}_{hj}$ are positive, the pdf for $\mathbf{y}_{hj}$ can be written as

$$\begin{aligned} f_1(\mathbf{y}_{hj}) &= \mid \mathbf{P}_h \mid (2\pi)^{-2} |\mathbf{\Omega}_h|^{1/2} \exp\left\{-\frac{1}{2}(\mathbf{P}_h\mathbf{y}_{hj} - \mathbf{\Theta}_h - x_{hj}\boldsymbol{\eta})'\mathbf{\Omega}_h^{-1}(\mathbf{P}_h\mathbf{y}_{hj} - \mathbf{\Theta}_h - x_{hj}\boldsymbol{\eta})\right\} \\ &= P_{1h}P_{2h}P_{3h}P_{4h}g_{1hj} \end{aligned} \tag{7.13}$$

where

$$g_{1hj} = (2\pi)^{-2} \mid \mathbf{\Omega}_h \mid^{-1/2} \exp\left\{-\frac{1}{2}(\mathbf{P}_h\mathbf{y}_{hj} - \mathbf{\Theta}_h - x_{hj}\boldsymbol{\eta})'\mathbf{\Omega}_h^{-1}(\mathbf{P}_h\mathbf{y}_{hj} - \mathbf{\Theta}_h - x_{hj}\boldsymbol{\eta})\right\}$$

The reason for the subscript 1 in the notation $f_1(y_{hj})$ and g_{1hj} will become apparent shortly.

Observations where expenditures on some commodities are not observed are now considered. The appropriate pdf in each case will depend on which commodity expenditures are missing. The different possible combinations are given in Table 7.3 on the following page. A '+' indicates an observed positive expenditure; a '0' indicates an observed zero expenditure. Given type 2 observations (k=2) from households of type h, the pdf for observed expenditures is

$$f_2(\mathbf{y}_{hj}) = P_{2h}P_{3h}P_{4h}g_{2hj}$$

where g_{2hj} is similar to g_{1hj} except that the function becomes 3-dimensional, with matrix elements that correspond to the first commodity being deleted. Similarly, for type 3 observation, the pdf can be specified as

$$f_3(\mathbf{y}_{hj}) = P_{1h}P_{3h}P_{4h}g_{3hj}$$

where g_{3hj} is suitably defined. The pdfs for the other cases are obtained in similar fashion and the process is continued up to type 16.

Let

$$D_{khj} = \begin{cases} 1 & \text{if observation } j \text{ from household type } h \text{ is of type } k \\ 0 & \text{otherwise} \end{cases}$$

Table 7.3 Type of Household
According to Recorded Positive and Zero Expenditures

	Zero Expenditure Type k															
Commodity	*1*	*2*	*3*	*4*	*5*	*6*	*7*	*8*	*9*	*10*	*11*	*12*	*13*	*14*	*15*	*16*
Food	+	0	+	+	+	0	0	0	+	+	+	0	0	0	+	0
Clothng	+	+	0	+	+	0	+	+	0	+	0	0	0	+	0	0
Housing	+	+	+	0	+	+	0	+	0	0	+	0	+	0	0	0
Others	+	+	+	+	0	+	+	0	+	0	0	+	0	0	0	0

Note: (a) A '+' indicates an observed positive expenditure; a ' 0' indicates an observed zero expenditure.

So

$$\begin{aligned}
\text{Prob}(D_{1hj}=1) &= P_{1h}P_{2h}P_{3h}P_{4h} \\
\text{Prob}(D_{2hj}=1) &= (1-P_{1h})P_{2h}P_{3h}P_{4h} \\
\text{Prob}(D_{3hj}=1) &= P_{1h}(1-P_{2h})P_{3h}P_{4h} \\
&\vdots \\
\text{Prob}(D_{15,hj}=1) &= P_{1h}(1-P_{2h})(1-P_{3h})(1-P_{4h}) \\
\text{Prob}(D_{16,hj}=1) &= (1-P_{1h})(1-P_{2h})(1-P_{3h})(1-P_{4h})
\end{aligned}$$

This specification of the probabilities assumes that shopping for one commodity is independent of shopping for another one. For example, it says that if you are purchasing food at a given time, it does not mean that you will be more or less likely to shop for clothes, too.

To get the complete likelihood, we need to include the pdfs for all types of observations and their probabilities of occurring. The complete likelihood function can be written as

$$\begin{aligned}
L &= \prod_{h=1}^{H}\prod_{j=1}^{M_h}\Big\{\Big[P_{1h}P_{2h}P_{3h}P_{4h}f_1(\mathbf{y}_{hj})\Big]^{D_{1hj}}\Big[(1-P_{1h})P_{2h}P_{3h}P_{4h}f_2(\mathbf{y}_{hj})\Big]^{D_{2hj}} \\
&\quad \Big[P_{1h}(1-P_{2h})P_{3h}P_{4h}f_3(\mathbf{y}_{hj})\Big]^{D_{3hj}}\cdots \\
&\quad \Big[P_{1h}(1-P_{2h})(1-P_{3h})(1-P_{4h})f_{15}(\mathbf{y}_{hj})\Big]^{D_{15,hj}} \\
&\quad [(1-P_{1h})(1-P_{2h})(1-P_{3h})(1-P_{4h})]^{D_{16,hj}}\Big\} \\
&= \prod_{h=1}^{H}\prod_{j=1}^{M_h}\Big\{\Big[P_{1h}^2P_{2h}^2P_{3h}^2P_{4h}^2g_{1hj}\Big]^{D_{1hj}}\Big[(1-P_{1h})P_{2h}^2P_{3h}^2P_{4h}^2g_{2hj}\Big]^{D_{2hj}} \\
&\quad \Big[P_{1h}^2(1-P_{2h})P_{3h}^2P_{4h}^2g_{3hj}\Big]^{D_{3hj}}\cdots \\
&\quad \Big[P_{1h}^2(1-P_{2h})(1-P_{3h})(1-P_{4h})g_{15hj}\Big]^{D_{15,hj}} \\
&\quad [(1-P_{1h})(1-P_{2h})(1-P_{3h})(1-P_{4h})]^{D_{16,hj}}\Big\}
\end{aligned}$$

$$= \prod_{h=1}^{H} \{P_{1h}^{2n_{1h}} P_{2h}^{2n_{2h}} P_{3h}^{2n_{3h}} P_{4h}^{2n_{4h}} (1 - P_{1h})^{M_h - n_{1h}} (1 - P_{2h})^{M_h - n_{2h}}$$

$$(1 - P_{3h})^{M_h - n_{3h}} (1 - P_{4h})^{M_h - n_{4h}} \prod_{j=1}^{M_h} \prod_{k=1}^{15} g_{khj}^{D_{khj}}\} \tag{7.14}$$

where n_{ih} is the number of observations on households of type h for which expenditure on commodity i is positive.

The likelihood function in (7.14) has a very complicated structure with differing dimensions of the g_{khj} for each k and the presence of the P_{ih} within the g_{khj}. Such structure does not make maximum likelihood estimation an attractive proposition. As an alternative, this study investigates Bayesian estimation. At the outset, Bayesian techniques appear more feasible for the problem at hand. The use of Bayesian techniques in the treatment of zero expenditures in this context has not been done in the past so the exercise will provide a new alternative to existing estimation procedures used in this context. The results from the empirical application towards the end of the chapter will be of interest from both methodological and empirical points of views.

7.4 Bayesian Specification: Notation, Priors and Joint Posterior Pdf

In the light of the discussions above, the first question that needs to be addressed is whether "data augmentation" would be a productive direction. Data augmentation is a technique applied by Bayesians to handle a general class of problems involving missing data. Bayesian analysts handle augmented data problems by generating values of latent variables that facilitate the construction of posterior pdfs. Our problem here of observed zero expenditures falls under this general class and we, therefore, investigate whether data augmentation in the Bayesian sense is appropriate. For more details and other uses of data augmentation, see Tanner and Wong (1987) and Chib (1992).

From equation (7.10), observed positive expenditures y_{ihj} can be written as

$$y_{ihj} = (\theta_{ih} + \eta_i x_{hj} + u_{ihj})/P_{ij}$$

For observations where expenditures are observed to be zero, define the latent variable as

$$y^*_{ihj} = (\theta_{ih} + \eta_i x_{hj} + u_{ihj})/P_{ij}$$

The y^*_{ihj} will be useful for making the pdfs in an "augmented likelihood function" of the same dimension. It is difficult to attach any specific meaning to the y^*_{ihj}; they need to be viewed as latent variables as defined above, where u_{ihj} are random variables with properties consistent with those assumed earlier.

Let $\mathbf{y}^0_{hj}$ be a (4×1) vector that contains the y_{ihj} for commodities with an observed positive expenditure and y^*_{ihj} for those commodities with an observed zero expenditure. Generically,

$$\mathbf{y}^0_{hj} = \left\{y_{ihj}, y^*_{ihj}\right\}$$

This vector characterises the j^{th} h-type household as belonging to one of the k types defined in Table 7.3. The exact position of the y_{ihj} and the y^*_{ihj} in the vector $\mathbf{y}^0_{hj}$ will depend on the type of observation. Also, the following generic notation will be useful

$$\begin{aligned}
\mathbf{y}^* &= \{y^*_{ihj} \mid \text{for all } i, h, j\} \\
\mathbf{y} &= \{y_{ihj} \mid \text{for all } i, h, j\} \\
\mathbf{y}^0 &= \{\mathbf{y}^*, \mathbf{y}\}
\end{aligned}$$

Thus, $\mathbf{y}^*$ denotes the set of all latent observations, $\mathbf{y}$ denotes the set of all positive expenditures, and $\mathbf{y}^0$ is the union of the two sets. Other notation is set as follows:

$$\begin{aligned}
\mathbf{D} &= \{D_{khj} \mid \text{for all } k, h, j\} \\
\boldsymbol{\Omega} &= \{\boldsymbol{\Omega}_h \mid \text{for all } h\} \\
\mathbf{P} &= \{P_{ih} \mid \text{for all } i, h\}
\end{aligned}$$

$$\begin{aligned}\boldsymbol{\Theta} &= \{\theta_{ih} \mid \text{for all } i, h\} \\ \boldsymbol{\Gamma} &= \{\boldsymbol{\Omega}, \mathbf{P}, \boldsymbol{\Theta}, \boldsymbol{\eta}\}\end{aligned}$$

Now, recall Bayes theorem which states that the posterior pdf for a set of unknown parameters $\boldsymbol{\Gamma}$ given some relevant data can be expressed as

$$g(\boldsymbol{\Gamma} \mid \text{data}) = \frac{f(\text{data} \mid \boldsymbol{\Gamma})f(\boldsymbol{\Gamma})}{f(\text{data})} \tag{7.15}$$

where $f(\boldsymbol{\Gamma})$ is the prior density for $\boldsymbol{\Gamma}$. Equivalently, it can be written as

$$f(\boldsymbol{\Gamma} \mid \text{data}) \propto l(\boldsymbol{\Gamma} \mid \text{data})f(\boldsymbol{\Gamma}) \tag{7.16}$$

where $l(\boldsymbol{\Gamma} \mid \text{data})$ is the likelihood function. The use of Bayes theorem allows us to combine prior information with the likelihood function in (7.14) which then leads to the derivation of a form for the joint posterior density of the unknown parameters in our model. The following discussion works towards that direction.

As noted earlier, the likelihood function in (7.14) is inconvenient because of the differing dimensions of the g_{khj}. This problem can be overcome by treating the latent observations $\mathbf{y}^*$ as additional parameters. When we do so, and recognise that in this case data= $\{\mathbf{y}, \mathbf{D}\}$, Bayes' theorem in (7.16) can be written as

$$\begin{aligned}f(\mathbf{y}^*, \boldsymbol{\Gamma} \mid \mathbf{y}, \mathbf{D}) &\propto f(\mathbf{y}, \mathbf{D} \mid \mathbf{y}^*, \boldsymbol{\Gamma})f(\mathbf{y}^*, \boldsymbol{\Gamma}) \\ &= f(\mathbf{y}, \mathbf{D} \mid \mathbf{y}^*, \boldsymbol{\Gamma})f(\mathbf{y}^* \mid \boldsymbol{\Gamma})f(\boldsymbol{\Gamma}) \\ &= f(\mathbf{y}, \mathbf{y}^*, \mathbf{D} \mid \boldsymbol{\Gamma})f(\boldsymbol{\Gamma}) \\ &= f(\mathbf{y}^0, \mathbf{D} \mid \boldsymbol{\Gamma})f(\boldsymbol{\Gamma}) \qquad (7.17)\end{aligned}$$

The equalities are obtained using the basic definition of a conditional pdf for jointly continuous random variables (Larson, 1974). The advantage of the last line in (7.17) is that the "augmented likelihood function" $f(\mathbf{y}^0, \mathbf{D} \mid \boldsymbol{\Gamma})$ is relatively easy to specify. It is equivalent to the likelihood function specified in (7.14) except that now all the $f_k(\mathbf{y}_{hj})$ become "type 1" pdfs corresponding to type 1 observations where expenditures on all commodities are observed. That is, to obtain $f(\mathbf{y}^0, \mathbf{D} \mid \boldsymbol{\Gamma})$, the pdfs $f_2(\mathbf{y}_{hj}), \ldots, f_{15}(\mathbf{y}_{hj})$ are all replaced by $f_1(\mathbf{y}^0_{hj})$ as follows

$$
\begin{aligned}
f(\mathbf{y}^0, \mathbf{D} \mid \mathbf{\Gamma}) &= \prod_{h=1}^{H} \left\{ \left[\prod_{i=1}^{4} P_{ih}^{n_{ih}} (1 - P_{ih})^{M_h - n_{ih}} \right] \prod_{h=1}^{M_h} f_1(\mathbf{y}^0_{hj}) \right\} \\
&= \prod_{h=1}^{H} \left\{ \left[\prod_{i=1}^{4} P_{ih}^{M_h + n_{ih}} (1 - P_{ih})^{M_h - n_{ih}} \right] \prod_{h=1}^{M_h} g_{1hj} \right\} \qquad (7.18)
\end{aligned}
$$

where

$$
\begin{aligned}
\prod_{j=1}^{M_h} g_{1jh} = \prod_{j=1}^{M_h} (2\pi)^{-2} \, |\mathbf{\Omega}_h|^{-1/2} \; \exp\left\{ -\frac{1}{2} (\mathbf{P}_h \mathbf{y}^0_{hj} - \mathbf{\Theta}_h - x_{hj}\boldsymbol{\eta})' \mathbf{\Omega}_h^{-1} (\mathbf{P}_h \mathbf{y}^0_{hj} - \mathbf{\Theta}_h - x_{hj}\boldsymbol{\eta} \right\} \\
\propto | \, \mathbf{\Omega}_h \, |^{-M_h/2} \; \exp\left\{ -\frac{1}{2} \text{ tr } \left[(\mathbf{Y}^0_h \mathbf{P}_h - \mathbf{X}_h \mathbf{B}_h)' (\mathbf{Y}^0_h \mathbf{P}_h - \mathbf{X}_h \mathbf{B}_h) \mathbf{\Omega}_h^{-1} \right] \right\}
\end{aligned}
$$

(see Bayesian analysis of SURs in Judge, et.al. (1985) pp. 478-80) with

$$
\mathbf{Y}^0_h = \begin{bmatrix} \mathbf{y}^{0'}_{h1} \\ \mathbf{y}^{0'}_{h2} \\ \vdots \\ \mathbf{y}^{0'}_{hM_h} \end{bmatrix} \qquad \mathbf{X}_h = \begin{bmatrix} 1 & x_{h1} \\ 1 & x_{h2} \\ \vdots & \vdots \\ 1 & x_{hM_h} \end{bmatrix} \qquad \mathbf{B}_h = \begin{bmatrix} \mathbf{\Theta}'_h \\ \boldsymbol{\eta}' \end{bmatrix} \qquad (7.19)
$$

which are of dimensions $(M_h \times 4), (M_h \times 2)$ and (2×4), respectively.

For prior pdfs, all the $\mathbf{\Omega}_h, P_{ih}, \theta_{ih}$ and η_i are treated as *a priori* independent and, with the exception of $\mathbf{\Omega}_h$, non-informative uniform priors are used. For $\mathbf{\Omega}_h$, the following conventional non-informative prior for multivariate regression (see Judge, et.al. (1985), p.478) is used:

$$
f(\mathbf{\Omega}_h) \propto | \, \mathbf{\Omega}_h \, |^{-(n+1)/2}
$$

Thus, the joint prior pdf for all the parameters can be expressed as

$$
f(\mathbf{\Gamma}) = f(\boldsymbol{\eta}) \prod_{h=1}^{H} f(\mathbf{\Omega}_h) f(\mathbf{\Theta}_h) f(\mathbf{P}_h) \propto \prod_{h=1}^{H} | \, \mathbf{\Omega}_h \, |^{-5/2}
$$

Now, using (7.17) and (7.18), the joint posterior pdf for all parameters and the latent observations $\mathbf{y}^*$ can be written as

$$f(\mathbf{y}^*,\ \boldsymbol{\Omega}\ ,\boldsymbol{\eta},\boldsymbol{\Theta},\mathbf{P} \mid \mathbf{y},\mathbf{D}) \propto f(\mathbf{y}^0,\mathbf{D} \mid \boldsymbol{\Gamma}) \prod_{h=1}^{H} \mid \boldsymbol{\Omega}_h \mid^{-5/2}$$

$$= \prod_{h=1}^{H} \left\{ \mid \boldsymbol{\Omega}_h \mid^{-(M_h+5)/2} \exp\left\{ -\frac{1}{2} tr \left[(\mathbf{Y}_h^0 \mathbf{P}_h - \mathbf{X}_h \mathbf{B}_h)'(\mathbf{Y}_h^0 \mathbf{P}_h - \mathbf{X}_h \mathbf{B}_h) \boldsymbol{\Omega}_h^{-1} \right] \right\} \right.$$

$$\left. \prod_{h=1}^{4} P_{ih}^{M_h+n_{ih}} (1 - P_{ih})^{M_h - n_{ih}} \right\} \tag{7.20}$$

This pdf represents all our post-sample information on all the unknown parameters in our model $\mathbf{y}^*$, $\boldsymbol{\Theta}$, $\boldsymbol{\eta}$, $\boldsymbol{\Omega}$ and $\mathbf{P}_h$. Information from this pdf will lead to information about the equivalence scales s_{ih} and s_h. To be able to draw inferences about these parameters, it is necessary to derive or estimate their respective marginal posterior pdfs. However, given the form of the joint posterior in (7.20), the process of integrating out unwanted parameters to obtain the marginal posterior pdfs is not analytically feasible. So, following the direction taken in the last chapter, we investigate the feasibility of a numerical approach. Specifically, we explore the use of the Gibbs sampling and Metropolis-Hastings (M-H) algorithms to generate values from the marginal posterior densities of the parameters without the need to specify their particular forms[3]. The Gibbs sampling algorithm works by drawing observations from the conditional posterior pdfs of each of the unknown parameters. So, before we can describe in detail the algorithm that is employed here, we first need to specify the forms of the conditional posterior pdfs of each of the unknown parameters in (7.20). The next section shows how these conditional posterior pdfs are derived; it is followed by a section describing the "M-H Within Gibbs" sampling procedure.

[3] Appendix A gives a brief description of how these algorithms are applied in the general terms.

7.5 Conditional Posterior Pdfs

To derive the conditional posterior pdf for the latent variable $\mathbf{y}^*$ given the parameters $\mathbf{\Gamma}$ and the observed expenditures $\mathbf{y}$, it is first noted that

$$f(\mathbf{y}^* \mid \mathbf{\Gamma}, \mathbf{y}, \mathbf{D}) = \prod_{h=1}^{H} \prod_{j=1}^{M_h} f(\mathbf{y}^*_{hj} \mid \mathbf{\Gamma}, \mathbf{y}_{hj}, \mathbf{D}) \tag{7.21}$$

where $\mathbf{y}_{hj}$ and $\mathbf{y}^*_{hj}$ are vectors containing the observed positive expenditures and the latent observations, respectively, for the j^{th} observation for the h-type household. The dimensions of $\mathbf{y}_{hj}$ and $\mathbf{y}^*_{hj}$ and the positions of the elements in these vectors change for each observation; they depend on the observation type, where the possible types appear in Table 7.3. From (7.21) latent-variable observations can be generated by considering each household independently. Also, $f(\mathbf{y}^*_{hj} \mid \mathbf{\Gamma}, \mathbf{y}_{hj}, \mathbf{D})$ is a suitably chosen conditional normal pdf from the 4-dimensional joint pdf

$$\begin{aligned} f(\mathbf{y}^0_{hj} \mid \mathbf{\Gamma}, \mathbf{D}) &\propto \exp\left\{-\frac{1}{2}(\mathbf{P}_h \mathbf{y}^0_{hj} - \mathbf{\Theta}_h - x_{hj}\boldsymbol{\eta})' \mathbf{\Omega}_h^{-1} (\mathbf{P}_h \mathbf{y}^0_{hj} - \mathbf{\Theta}_h - x_{hj}\boldsymbol{\eta})\right\} \\ &\propto \exp\left\{-\frac{1}{2}(\mathbf{y}^0_{hj} - \mathbf{P}_h^{-1}\mathbf{\Theta}_h - x_{hj}\mathbf{P}_h^{-1}\boldsymbol{\eta})' \mathbf{P}_h \mathbf{\Omega}_h^{-1} \mathbf{P}_h (\mathbf{y}^0_{hj} - \mathbf{P}_h^{-1}\mathbf{\Theta}_h - x_{hj}\mathbf{P}_h^{-1}\boldsymbol{\eta})\right\} \end{aligned}$$

That is,

$$(\mathbf{y}^0_{hj} \mid \mathbf{\Gamma}, \mathbf{D}) \sim N\left[\mathbf{P}_h^{-1}(\mathbf{\Theta}_h + x_{hj}\boldsymbol{\eta}), \mathbf{P}_h \mathbf{\Omega}_h^{-1} \mathbf{P}_h\right] \tag{7.22}$$

It follows that $(\mathbf{y}^*_{hj} \mid \mathbf{y}_{hj}, \mathbf{\Gamma}, \mathbf{D})$ will have a normal distribution obtained by choosing the appropriately partitioned conditional distribution from (7.20). See, for example, Judge, et.al. (1988, p.50). The precise form of the conditional distribution for a given observation will again depend on the "type" of that observation where, for $n = 4$, the possible positions of the unobserved expenditures describe 16 types as in Table 7.3.

Consider next the conditional posterior pdf for $\mathbf{\Omega}_h$ given other parameters, and given the data. First note that the $\mathbf{\Omega}_h$ for different household types are conditionally independent. That is,

$$f(\mathbf{\Omega} \mid \mathbf{y}^*, \mathbf{y}, \boldsymbol{\eta}, \mathbf{\Theta}, \mathbf{P}) = \prod_{h=1}^{H} f(\mathbf{\Omega}_h \mid \mathbf{y}^*, \mathbf{y}, \mathbf{D}, \boldsymbol{\eta}, \mathbf{\Theta}, \mathbf{P})$$

where

$$f(\mathbf{\Omega}_h \mid \mathbf{y}^0, \mathbf{D}, \boldsymbol{\eta}, \mathbf{\Theta}, \mathbf{P}) \propto \mid \mathbf{\Omega}_h \mid^{-(M_h+5)/2} \exp\left\{-\frac{1}{2} \text{ tr } (\mathbf{A}_h \mathbf{\Omega}_h^{-1})\right\} \tag{7.23}$$

and

$$\mathbf{A}_h = \left((\mathbf{Y}_h^0 \mathbf{P}_h - \mathbf{X}_h \mathbf{B}_h)'(\mathbf{Y}_h^0 \mathbf{P}_h - \mathbf{X}_h \mathbf{B}_h)\right)$$

The pdf in (7.23) is an inverted Wishart with parameters $(4, M_h, \mathbf{A}_h)$. Random generation of observations from this pdf is straightforward (Anderson (1984), p.238).

To derive the conditional posterior pdf for the $\mathbf{\Theta}_h$, it is first noted that

$$\mathbf{X}_h \mathbf{B}_h = \mathbf{z}_h \mathbf{\Theta}_h' + \mathbf{x}_h \boldsymbol{\eta}'$$

where $\mathbf{z}_h = [1, 1, ..., 1]'$ and $\mathbf{x}_h = [x_{h1}, x_{h2}, ..., x_{hM_h}]'$ both with dimension $(M_h \times 1)$. Now, write

$$\begin{aligned} \mathbf{Y}_h^0 \mathbf{P}_h - \mathbf{X}_h \mathbf{B}_h &= \mathbf{Y}_h^0 \mathbf{P}_h - \mathbf{z}_h \mathbf{\Theta}_h' - \mathbf{x}_h \boldsymbol{\eta}' \\ &= \mathbf{Y}_h - \mathbf{z}_h \mathbf{\Theta}_h' \end{aligned} \tag{7.24}$$

where $\mathbf{Y}_h = \mathbf{Y}_h^0 \mathbf{P}_h - \mathbf{x}_h \boldsymbol{\eta}'$. Then, using *a posterior* conditional independence of the $\mathbf{\Theta}_h$ for the different households, and a result from Zellner (1971, p.225) on the traditional multivariate regression model, we have

$$\begin{aligned} f(\mathbf{\Theta}_h \mid \mathbf{y}^0, \mathbf{\Omega}, \boldsymbol{\eta}, \mathbf{P}, \mathbf{D}) &\propto \exp\left\{-\frac{1}{2} \text{ tr } [(\mathbf{Y}_h^0 \mathbf{P}_h - \mathbf{X}_h \mathbf{B}_h)'(\mathbf{Y}_h^0 \mathbf{P}_h - \mathbf{X}_h \mathbf{B}_h) \mathbf{\Omega}_h^{-1}]\right\} \\ &= \exp\left\{-\frac{1}{2} \text{ tr } [(\mathbf{Y}_h - \mathbf{z}_h \mathbf{\Theta}_h')'(\mathbf{Y}_h - \mathbf{z}_h \mathbf{\Theta}_h') \mathbf{\Omega}_h^{-1}]\right\} \\ &\propto \exp\left\{-\frac{1}{2} \text{ tr } \left[(\mathbf{\Theta}_h - \hat{\mathbf{\Theta}}_h) \mathbf{z}_h' \mathbf{z}_h (\mathbf{\Theta}_h - \hat{\mathbf{\Theta}}_h)' \mathbf{\Omega}_h^{-1}\right]\right\} \\ &= \exp\left\{-\frac{1}{2} (\mathbf{\Theta}_h - \hat{\mathbf{\Theta}}_h)' M_h \mathbf{\Omega}_h^{-1} (\mathbf{\Theta}_h - \hat{\mathbf{\Theta}}_h)\right\} \end{aligned} \tag{7.25}$$

where

$$\hat{\mathbf{\Theta}}_h' = (\mathbf{z}_h' \mathbf{z}_h)^{-1} \mathbf{z}_h' \mathbf{Y}_h$$

$$= \quad M_h^{-1}\mathbf{z}_h'\mathbf{Y}_h = (\overline{y}_{1h}, \overline{y}_{2h}, \overline{y}_{3h}, \overline{y}_{4h}) \tag{7.26}$$

and the $\overline{y}_{ih}$'s are the averages of the elements in the columns of the $\mathbf{Y}_h$. Thus, for generating the $\mathbf{\Theta}_h$ within the Gibbs' sampler, the following distribution for $\mathbf{\Theta}_h$ is used

$$f(\mathbf{\Theta}_h \mid \mathbf{y}^0, \mathbf{\Omega}, \boldsymbol{\eta}, \mathbf{P}, \mathbf{D}) \sim N(\hat{\mathbf{\Theta}}_h, M_h^{-1}\mathbf{\Omega}_h) \tag{7.27}$$

For the conditional posterior pdf for $\boldsymbol{\eta}$, equation (7.24) is again used but this time rewritten as

$$\begin{aligned} \mathbf{Y}_h^0\mathbf{P}_h - \mathbf{X}_h\mathbf{B}_h &= \mathbf{Y}_h^0\mathbf{P}_h - \mathbf{z}_h\mathbf{\Theta}_h' - \mathbf{x}_h\boldsymbol{\eta}' \\ &= \mathbf{Y}_h^* - \mathbf{x}_h\boldsymbol{\eta}' \end{aligned} \tag{7.28}$$

where $\mathbf{Y}_h^* = \mathbf{Y}_h^0\mathbf{P}_h - \mathbf{z}_h\mathbf{\Theta}_h'$. Also define $\hat{\boldsymbol{\eta}}_h = \mathbf{Y}_h^{*\prime}\mathbf{x}_h(\mathbf{x}_h'\mathbf{x}_h)^{-1}$ so that

$$\begin{aligned} f(\boldsymbol{\eta} \mid \mathbf{y}^0, \mathbf{\Omega}, \mathbf{\Theta}, \mathbf{P}, \mathbf{D}) &\propto \exp\left\{-\frac{1}{2}\sum_{h=1}^{H}\mathrm{tr}[(\mathbf{Y}_h^0\mathbf{P}_h - \mathbf{X}_h\mathbf{B}_h)'(\mathbf{Y}_h^0\mathbf{P}_h - \mathbf{X}_h\mathbf{B}_h)\mathbf{\Omega}_h^{-1}]\right\} \\ &= \exp\left\{-\frac{1}{2}\sum_{h=1}^{H}\mathrm{tr}[(\mathbf{Y}_h^* - \mathbf{x}_h\boldsymbol{\eta}')'(\mathbf{Y}_h^* - \mathbf{x}_h\boldsymbol{\eta}')\mathbf{\Omega}_h^{-1}]\right\} \\ &\propto \exp\left\{-\frac{1}{2}\sum_{h=1}^{H}\mathrm{tr}[(\boldsymbol{\eta} - \hat{\boldsymbol{\eta}}_h)\mathbf{x}_h'\mathbf{x}_h(\boldsymbol{\eta} - \hat{\boldsymbol{\eta}}_h)'\mathbf{\Omega}_h^{-1}]\right\} \\ &= \exp\left\{-\frac{1}{2}\sum_{h=1}^{H}(\boldsymbol{\eta} - \hat{\boldsymbol{\eta}}_h)'(\mathbf{x}_h'\mathbf{x}_h)\mathbf{\Omega}_h^{-1}(\boldsymbol{\eta} - \hat{\boldsymbol{\eta}}_h)\right\} \\ &\propto \exp\left\{-\frac{1}{2}\left[\boldsymbol{\eta}'\left(\sum_{h=1}^{H}\mathbf{x}_h'\mathbf{x}_h\mathbf{\Omega}_h^{-1}\right)\boldsymbol{\eta} - 2\boldsymbol{\eta}'\left(\sum_{h=1}^{H}\mathbf{x}_h'\mathbf{x}_h\mathbf{\Omega}_h^{-1}\hat{\boldsymbol{\eta}}_h\right)\right]\right\} \\ &\propto \exp\left\{-\frac{1}{2}(\boldsymbol{\eta} - \hat{\boldsymbol{\eta}})'\left(\sum_{h=1}^{H}\mathbf{x}_h'\mathbf{x}_h\mathbf{\Omega}_h^{-1}\right)(\boldsymbol{\eta} - \hat{\boldsymbol{\eta}})\right\} \end{aligned} \tag{7.29}$$

where

$$\hat{\boldsymbol{\eta}} = \left(\sum_{h=1}^{H}\mathbf{x}_h'\mathbf{x}_h\mathbf{\Omega}_h^{-1}\right)^{-1}\left(\sum_{h=1}^{H}\mathbf{x}_h'\mathbf{x}_h\mathbf{\Omega}_h^{-1}\hat{\boldsymbol{\eta}}_h\right) \tag{7.30}$$

Thus, for generating observations on $\boldsymbol{\eta}$ through the Gibbs' sampler, the following conditional distribution for $\boldsymbol{\eta}$ is used

$$(\boldsymbol{\eta} \mid \mathbf{y}^0, \boldsymbol{\Omega}, \boldsymbol{\Theta}, \mathbf{P}, \mathbf{D}) \sim N\left[\hat{\boldsymbol{\eta}}, \left(\sum_{h=1}^{H} \mathbf{x}_h' \mathbf{x}_h \boldsymbol{\Omega}_h^{-1}\right)^{-1}\right] \tag{7.31}$$

The remaining parameters for which we need a conditional posterior pdf are the P_{ih}'s. To derive this conditional pdf is less straightforward than those just derived because it has terms under the exponential as well as a part that is in the form of a beta pdf as is seen in equation (7.20). To write the exponential in a more convenient form, define $\mathbf{Y}_{ih}^0$ as the $(M_h \times 1)$ vector containing all observations (some positive expenditures, other latent variable observations) on the i^{th} commodity for the h-type household

$$\mathbf{Y}_{ih}^0 = (y_{ih1}^0, y_{ih2}^0, ..., y_{ihM_h}^0)'$$

and let $\mathbf{Y}_{Dh}$ be a $(4M_h \times 4)$ block diagonal matrix including these observations for all commodities,

$$\mathbf{Y}_{Dh} = \begin{bmatrix} \mathbf{Y}_{1h}^0 & & & \\ & \mathbf{Y}_{2h}^0 & & \\ & & \mathbf{Y}_{3h}^0 & \\ & & & \mathbf{Y}_{4h}^0 \end{bmatrix}$$

Also, let $\boldsymbol{\beta}_h =$ vec $(\mathbf{B}_h)$, $\mathbf{p}_h = [\, P_{1h}, P_{2h}, P_{3h}, P_{4h}\,]'$. Now

$$\begin{aligned} \mathrm{tr}[(\mathbf{Y}_h^0\mathbf{P}_h \;&-\; \mathbf{X}_h\mathbf{B}_h)'(\mathbf{Y}_h^0\mathbf{P}_h - \mathbf{X}_h\mathbf{B}_h)\boldsymbol{\Omega}_h^{-1}] \\ &= [\mathbf{Y}_{Dh}\mathbf{p}_h - (I \otimes \mathbf{X}_h)\boldsymbol{\beta}_h]'(\boldsymbol{\Omega}_h^{-1} \otimes I)[\mathbf{Y}_{Dh}\mathbf{p}_h - (I \otimes \mathbf{X}_h)\boldsymbol{\beta}_h] \\ &= \mathbf{p}_h'\mathbf{Y}_{Dh}'(\boldsymbol{\Omega}_h^{-1} \otimes I)\mathbf{Y}_{Dh}\mathbf{p}_h - 2\mathbf{p}_h'\mathbf{Y}_{Dh}'(\boldsymbol{\Omega}_h^{-1} \otimes I)(I \otimes \mathbf{X}_h)\boldsymbol{\beta}_h + K_1 \\ &= (\mathbf{p}_h - \hat{\mathbf{p}}_h)'\mathbf{Y}_{Dh}'(\boldsymbol{\Omega}_h^{-1} \otimes I)\mathbf{Y}_{Dh}(\mathbf{p}_h - \hat{\mathbf{p}}_h) + K_2 \end{aligned}$$

where K_1 and K_2 are quantities that do not involve $\mathbf{p}_h$ and where

$$\hat{\mathbf{p}}_h = [\mathbf{Y}'_{Dh}(\mathbf{\Omega}_h^{-1} \otimes I)\mathbf{Y}_{Dh}]^{-1}\mathbf{Y}'_{Dh}(\mathbf{\Omega}_h^{-1} \otimes \mathbf{X}_h)\boldsymbol{\beta}_h.$$

Note that the $\mathbf{p}_h$ for the different household types are conditionally independent, implying

$$f(\mathbf{P} \mid \mathbf{\Omega}, \mathbf{y}^*, \mathbf{y}, \boldsymbol{\eta}, \mathbf{D}, \mathbf{\Theta}) = \prod_{h=1}^{H} f(\mathbf{p}_h \mid \mathbf{\Omega}, \mathbf{y}^*, \mathbf{y}, \mathbf{D}, \boldsymbol{\eta}, \mathbf{\Theta}) \tag{7.32}$$

Thus the complete conditional posterior pdf for each $\mathbf{p}_h$ can be written as

$$f(\mathbf{p}_h \mid \mathbf{y}^0, \mathbf{\Omega}, \boldsymbol{\eta}, \mathbf{\Theta}, \mathbf{D}) \propto \left(\prod_{i=1}^{4} P_{ih}^{M_h+n_{ih}}(1-P_{ih})^{M_h-n_{ih}}\right)$$
$$\exp\left\{-\frac{1}{2}(\mathbf{p}_h - \hat{\mathbf{p}}_h)'\mathbf{Y}'_{Dh}(\mathbf{\Omega}_h^{-1} \otimes I)\mathbf{Y}_{Dh}(\mathbf{p}_h - \hat{\mathbf{p}}_h)\right\} \tag{7.33}$$

This pdf is in the form of the product of a beta kernel and a normal kernel and, as such, it is not in a recognisable form.

7.6 Numerical Sampling: The M-H Within Gibbs Sampling Procedure

In the previous section, the conditional posterior densities of each of the unknown parameters $\mathbf{y}^*, \mathbf{\Omega}, \boldsymbol{\eta}, \mathbf{\Theta}$, and $\mathbf{P}_h$ were derived. These are specified in (7.22), (7.23), (7.27), (7.31) and (7.33), respectively. Note that these conditional posterior pdfs are all in recognisable forms except for (7.32) for $\mathbf{P}_h$ which is, as pointed out, in the form of a product of a beta kernel and a normal kernel. Observations from the marginal pdfs for each of the parameters can be obtained using Gibbs sampling which samples iteratively from the conditional posterior pdfs. Such sampling is straightforward for the conditional densities for $\mathbf{y}^*$ in (7.22), for $\mathbf{\Omega}$ in (7.23), for $\boldsymbol{\eta}$ in (7.27) and for $\mathbf{\Theta}$ in (7.31). To draw observations from the conditional posterior pdf for $\mathbf{P}_h$, another MCMC algorithm had to be employed because the conditional pdf in (7.33) is not easily identifiable. For this case, the Metropolis-Hastings (M-H) sampling algorithm was chosen. The procedure developed and used here is thus a Gibbs sampling procedure involving an M-H sub-routine. Numerical sampling using this "M-H within Gibbs" sampling procedure proceeds as follows[4]:

1. Given some initial values for θ_{ih}, η_i, Ω_h and P_{ih}, generate, from the density in (7.22), replacements for the y^*_{ihj} for the observed zero expenditures. Note that the precise form of the conditional multivariate normal density for each household depends on the combination of commodities for which the observed positive and zero expenditures were recorded.

2. Using the y_{ihj}'s augmented by the y^*_{ihj}'s generated in step (1), compute for each household type h,
$$\mathbf{A}_h = (\mathbf{Y}_h^0\mathbf{P}_h - \mathbf{z}_h\mathbf{\Theta}_h' - \mathbf{x}_h\boldsymbol{\eta}')'(\mathbf{Y}_h^0\mathbf{P}_h - \mathbf{z}_h\mathbf{\Theta}_h' - \mathbf{x}_h\boldsymbol{\eta}')$$

3. Draw values $\mathbf{\Omega}_1, \mathbf{\Omega}_2, ..., \mathbf{\Omega}_H$ from respective inverted Wishart distributions with parameter matrices $\mathbf{A}_1, \mathbf{A}_2, ..., \mathbf{A}_H$ and degrees of freedom $M_1, M_2, ..., M_H$.

4. Compute $\hat{\mathbf{\Theta}}_1, \hat{\mathbf{\Theta}}_2, ..., \hat{\mathbf{\Theta}}_H$ as defined in (7.26) and, given the $\mathbf{\Omega}_h$ drawn in step (3), draw values $\mathbf{\Theta}_h$, $h = 1, 2, ..., H$ from $N(\hat{\mathbf{\Theta}}_h, M_h^{-1}\mathbf{\Omega}_h)$ distributions.

5. Using the values for $\mathbf{\Omega}_h$ and $\mathbf{\Theta}_h$ drawn in steps (3) and (4), respectively, compute $\hat{\boldsymbol{\eta}}$ as defined in (7.30).

6. Draw a value for $\boldsymbol{\eta}$ from a $\mathrm{N}(\hat{\boldsymbol{\eta}}, W^{-1})$ distribution where $W = \left(\sum_{h=1}^{H} \mathbf{x}_h'\mathbf{x}_h\mathbf{\Omega}_h^{-1}\right)$.

7. Generate P_{ih} values using M-H subroutine below.

8. Return to step (1) using the θ_{ih}, η_i, $\mathbf{\Omega}_h$ and P_{ih} drawn in steps (4), (6), (3) and (7), respectively, and continue to proceed iteratively through all the steps, until a large sample has been generated.

[4]This section describes a numerical procedure which combines the Gibbs and the Metropolis-Hastings (M-H) sampling algorithms. The discussion outlines the specific steps required to generate sample observations for our particular case. Brief descriptions of how to apply the Gibbs and M-H algorithms in general terms are found in Appendix A.

M-H Subroutine

This Metropolis-Hastings (M-H) algorithm is developed as a sub-routine of the Gibbs sampling procedure above (step 7). It is used to generate P_{ih} values because the conditional posterior pdf for the P_{ih}'s is drawn is not in a recognisable form. The following steps thus describes how the "sample" P_{ih} values are generated using this sub-routine.

7-a Using the generated values of $y^*_{ihj}, \theta_{ih}, \mathbf{\Omega}_h$ and η_i in steps (2), (3), (4) and (6) above, select initial values for the elements of $\mathbf{p}_h$, say $\mathbf{p}^o_h$. Set $s = 0$.

7-b Generate a candidate value $\mathbf{p}^*_h$ given by

$$\mathbf{p}^*_h = \mathbf{p}^s_h + \mathbf{d}$$

where $\mathbf{d}$ is drawn from a 4-dimensional normal distribution with mean zero and covariance matrix $c_h[\mathbf{Y}'_{Dh}(\mathbf{\Omega}_h^{-1} \otimes I)\mathbf{Y}_{Dh}]^{-1}$. The scalars c_h were set such that feasible potentially new draws were accepted about 50 percent of the time. This step is repeated if any of the elements of $\mathbf{p}^*_h$ do not lie between 0 and 1.

7-c Compute the values of the conditional density in (7.33) evaluated at the points $\mathbf{p}^s_h$ and $\mathbf{p}^*_h$ i.e. compute $f(\mathbf{p}^s_h \,|\, \mathbf{y}^0, \mathbf{\Omega}, \boldsymbol{\eta}, \Theta, \mathbf{D})$ and $f(\mathbf{p}^*_h \,|\, \mathbf{y}^0, \mathbf{\Omega}, \boldsymbol{\eta}, \Theta, \mathbf{D})$. Define $\mathbf{r}$ as the ratio

$$\mathbf{r} = \frac{f(\mathbf{p}^*_h \mid \mathbf{y}^0, \mathbf{\Omega}, \boldsymbol{\eta}, \Theta, \mathbf{D})}{f(\mathbf{p}^s_h \mid \mathbf{y}^0, \mathbf{\Omega}, \boldsymbol{\eta}, \Theta, \mathbf{D})}$$

7-d If $\mathbf{r} \geq 1$, set $\mathbf{p}^{(s+1)}_h = \mathbf{p}^*_h$; set $s = s + 1$, go to step (7-b).
If $\mathbf{r} < 1$, generate an independent uniform random variable, say u, from the interval [0,1]. Set $\mathbf{p}^{(s+1)}_h = \mathbf{p}^*_h$ if $u < \mathbf{r}$. Then, set $s = s + 1$ and go to step (7-b). Otherwise, set $\mathbf{p}^{(s+1)}_h = \mathbf{p}^s_h$ and $s = s + 1$, and go to step (7-b).

This process is repeated until $s = 35$. The observation chosen for each iteration of the Gibbs' sampler was the last accepted observation after 35 feasible Metropolis-Hastings iterations.

Markov chain theory guarantees that, after a particular point, the observations from this large sample represent observations from the marginal (or joint) posterior pdfs (Geman and Geman, 1984). The point at which they represent points from the marginal pdfs is the point at which the Markov chain (created by the "M-H within Gibbs" sampling procedure) has converged. Because observations at the beginning of the iterative procedure will not necessarily be from the marginal posterior pdfs, it is conventional to drop a number of these, treating them as initial observations in a "burn-in" period.

Once estimates for θ_{ij} and η_i are obtained, estimates for the parameters a_{ih} and b_i can then be calculated from equations (5.17)-(5.20). These in turn lead to the estimation of equivalence scales s_{ih} and s_h from expressions (5.21) and (5.28) derived earlier.

7.7 Empirical Application

The Bayesian procedure described above was applied to data from the 1988-89 Australian Household Expenditure Survey. This data set is described in detail in Chapter 3. To reduce unweildiness in the computational work, only four commodity groupings were used (unlike the previous chapters which used all the eleven commodity aggregates described on pages 35-36). The four commodity groups considered are (i) Food, (ii) Clothing, (iii) Housing, and (iv) Others. Commodities (i)-(iii) maintain identical definitions as before. Commodity (iv) is an aggregate of all the other remaining commodities in the list. The breakdown of the data over the different household types and the number of those with zero expenditures by type were earlier provided in Table 7.2. The initial estimates of the P_{ih} can be taken as n_{ih}/M_h and these values are given in Table 7.4. Note that clothing is the only commodity for which these probabilities are very different from 1.0. Table 7.5 gives the number of households exhibiting each particular type of zero expenditure. Only 6 of the 16 possible types were actually observed, one of which is where there are observed expenditures on all 4 commodities and for which generation of $\mathbf{y}^*_{hj}$ is not necessary. These numbers are important for the choice of the conditional pdfs when generating the observations on the latent variables. As expected, most correspond to one missing value which pertains to Clothing, but there are a number with Housing as the only missing value and a few others where both Housing and Clothing are missing.

For Bayesian estimation, 20,000 observations on each parameter element in $\mathbf{P}_h$, $\boldsymbol{\Omega}_h, \boldsymbol{\Theta}_h$ and $\boldsymbol{\eta}$ were generated for the eight household types. The first 3000 observations provided the 'burn-in' period of the Gibbs sampler and hence were discarded, leaving 17,000 observations in the final estimation sample. These observations were used to estimate the posterior means and standard deviations of all the commodity-specific and general scales as well as to provide information for graphing the marginal posterior pdfs for some selected scales. All calculations were carried out using the econometric package SHAZAM. Convergence of the series generated were checked through diagrams showing the path of observations.

Table 7.6 shows that posterior means and standard deviations of the P_{ih} from Bayesian estimation. The mean estimates are not far off from the initial estimates tabulated in Table 7.4. Standard deviations are generally small. Across commodities, the P_{ih}'s for Clothing exhibited the largest variances. Across households meanwhile, single adult households exhibited larger variances compared to the two-adult households and this could be attributed to the smaller numbers of households of these types included in the sample.

Table 7.7 shows the posterior means and standard deviations of the scales from Bayesian estimation derived from the estimated θ_{ih} and η_i values through the relationships in (5.21)-(5.20), (5.21) and (5.28)[5]. As expected, for most commodities, there is an increase in the per household equivalent expenditure as household size increases. The foodscale 1.225 for the (2,1) household indicates that to maintain the same standard of living, a couple will need to meet an extra 22 percent increase in their food expenditure with the addition of one child. A second child will impose a further increase in the food requirement by 18 percent while the third child a further 12 percent. For a single adult, the increase in the requirements per

[5]It was not possible to store the series generated for all parameters in the model as that required an enormous amount of computer memory. The selected series that were stored and used in the discussion of results are those of the P_{ih}'s and both the commodity-specific and general scales.

additional child is slightly less. The increases generally occur at a decreasing rate indicating economies of scale for additional children. The scale values for Clothing are comparable to those for Food and also exhibit a diminishing rate of increase with additional children.

The magnitudes of housing scales indicate larger scale relativities compared to those with Food or Clothing. The housing scale of 1.428 for the (2,1) household type shows that a first additional child to a two-adult household will increase housing expenditure requirements by 43 percent. The scales of 1.478 and 1.526 for household types (2,2) and (2,3), respectively, show that the second and third additional children will further increase housing requirements, but only marginally. The scale estimates indicate the same amount of marginal increases in the expenditure requirements of one-adult households due to the addition of children. These observations make economic sense because the addition of a first child to a new family usually requires parents to provide a separate room for the child. It is, however, common practice for families to let the second and/or third child share a room with the first child, hence, resulting in lower marginal cost for children after the first. For the Others commodity, the increase in the requirements is relatively low compared to that of other commodities with increases in the scale values also exhibiting diminishing rates of increase.

With the general scales, the value of 1.212 for the (2,1) household type implies that the addition of a first child to a two-adult household will increase the total expenditure requirement by 21 percent; the second child by a further 9 percent and the third child by a further 7 percent. For the one-adult household meanwhile, a first child will impose an 8 percent increase in the expenditure requirements; and the 2nd or 3rd child about a 4 percent increase in total expenditures. Standard deviations are invariably low. The standard deviations for the clothing scales are much larger compared to the three other commodity types. Across households, the scales for (1,3) type households exhibited the largest variances.

The posterior pdfs of the P_{ih} and the commodity-specific and general scales are presented in Figures 7.2-7.5. Figure 7.2 allows a comparison of the posterior distributions of P_{ih} across commodity type. It is shown that the posterior pdfs for Food and Others are concentrated heavily towards 1.0, while those for Housing exhibit some deviation from 1.0. In stark contrast, the Bayesian posteriors for the P_{ih}'s for clothing are centered away from 1.0 with pdfs for each household type showing large differences in the means and the variances.

Figure 7.3 allows for the comparison of posterior distribution by household type. The posterior pdfs of the P_{ih}'s for Food, Housing and Others are concentrated towards the value of 1.0 whereas those for clothing exhibit large movements across the x-axis as one moves from one household type to another. It is also apparent from Figure 7.3 that the posterior pdfs become flatter as we move towards household types where a smaller proportion of the sample is used – that is (1,3), (1,2) and (1,1) household types in particular. This indicates that households with small samples tend to exhibit larger posterior variances for the P_{ih}. It is interesting to note that the Bayesian mean is not 1.0 when all households in that category purchase each week. This is in contrast with what presumably would be with the ML estimate. How far away from 1.0 the Bayesian estimate can be depends on the number of households in that category.

Finally, the posterior pdfs for the scales are shown in Figure 7.4. With the foodscales, it is immediately obvious that the posterior pdf for the (1,0) household type has a relatively small variance while that of the household type (1,3) has a relatively large variance. In general, the pdfs shift to the right with the addition of children in the household. For example, with food, the effect of increasing the number of adults and children in the household has a clear and distinct effect - overlapping of the pdfs is minimal. With clothing, the location of the pdfs are as expected but considerable overlapping of the pdfs mean that we are more uncertain about the relative magnitudes of the scales. Across commodities for which the posterior pdfs have been plotted, the posterior pdfs of the food scales are observed to have less variability compared to those of the clothing scales or housing scales. The posteriors for food also indicate lesser gains in economies of scale compared to housing or clothing. This observation conforms with our expectation.

The posterior pdfs for the general scales in Figure 7.5 yield familiar patterns of movement across the x-axis consistent with observations for the commodity-specific scales. Pdfs shift to the right at a diminishing rate with the addition of children in the household. The least variance was observed for household type

(1,0) while the largest variance was observed for household type (1,3). It is also noted the standard errors for the general scales are mostly smaller than those for the commodity-specific scales.

Table 7.8 gives a comparison of Bayesian posterior means and standard deviations estimated using a model that did not account for zero expenditures (called model *m1*) and a model that provided for the occurrence of zero expenditures (called model *m2*). The former was used in Chapter 6 and the latter is the model developed in this chapter. Recall that in Chapter 6, commodity-specific scales were estimated for 11 commodity groupings while in this Chapter, commodity specific scales were obtained for only 4 aggregated commodities. The comparison in Table 7.8 is still possible because apart from the fourth commodity Others in Chapter 7, the commodity groups of Food, Clothing and Housing have identical definitions. The general scales are directly comparable.

From the table, estimates from *m1* are consistently higher than those from *m2*. Standard deviations are also larger. Estimated standard deviations for the posterior means were also observed to be smaller compared to those obtained in Chapter 6, indicating some gain in precision.

Significant differences were observed for all household types, for all the three commodity-specific scales as well as for the general scales. Estimates from *m1* are thought to reflect an upward bias which arose out of taking recorded zero and positive expenditures as direct replacements for consumption. Observed positive expenditures would be overestimates of actual consumption while zero expenditures do not actually mean non-consumption because households demonstrate not-so-regular shopping habits and surveys do not normally have lengthy periods of recall to be able to account for the "infrequency of purchase". The model developed in this chapter recognise this deficiency of survey expenditure data. Expenditure data was therefore adjusted accordingly so that they are able to reflect consumption patterns better. Estimates from this model are therefore believed to be more plausible.

It is important to point out that the model developed here would be too difficult to estimate within the sampling theory framework. In was demonstrated in this chapter that the Bayesian approach to estimation is a viable alternative

– estimation becomes feasible and the resulting estimates are more reliable and plausible.

Table 7.4 Initial P_{ih} estimates for each household type

Household Type	*Total No. of Households*	*Initial P_{ih} estimates* Food	Clothing	Housing	Others
(2,0)	2074	1.00000	0.73963	0.98361	1.00000
(2,1)	532	1.00000	0.84962	0.98684	1.00000
(2,2)	889	1.00000	0.86277	0.98650	1.00000
(2,3)	388	1.00000	0.88660	0.99485	1.00000
(1,0)	1372	0.99417	0.50437	0.95773	1.00000
(1,1)	132	0.99242	0.68182	0.97727	1.00000
(1,2)	103	1.00000	0.72816	1.00000	1.00000
(1,3)	42	1.00000	0.73810	0.97619	1.00000

Note: P_{ih} is the probability of purchase for commodity i for the h-type household

Table 7.5 Number of Sample Households by Expenditure Type
(Combination of Observed Positive & Zero Expenditures)

Expenditure Type k	*Household Type* (1,0)	*(1,1)*	*(1,2)*	*(1,3)*	*(2,0)*	*(2,1)*	*(2,2)*	*(2,3)*
1 - (+ + + +)	663	87	74	30	1511	448	760	343
3 - (+ 0 + +)	644	41	29	11	529	77	117	43
4 - (+ + 0 +)	29	2	0	1	23	5	7	0
6 - (0 0 + +)	7	1	0	0	0	0	0	0
9 - (+ 0 0 +)	28	1	0	0	11	2	5	2
12 - (0 0 0 +)	1	0	0	0	0	0	0	0
Total	1372	132	103	42	2074	532	889	388

Notes: (a) Expenditure type as classified in Table 7.3. A '+' indicates observed expenditure; a '0' indicates zero expenditure.
(b) The '+'s and '0's refer to expenditure on Food, Clothing, Housing and Others, respectively.
(c) No observations were found for types 2, 5, 7, 8, 10, 11, 13, 14 15 or 16 (all zeroes).

Table 7.6 Posterior Means and Standard Deviations for the P_{ih}

Household Type	*Total No of Households*	*Food*	*Clothing*	*Housing*	*Others*
(2,0)	2074	0.99944	0.74039	0.98339	0.99943
		(0.00051)	(0.00950)	(0.00279)	(0.00053)
(2,1)	532	0.99787	0.84833	0.98491	0.99781
		(0.00194)	(0.01560)	(0.00526)	(0.00201)
(2,2)	889	0.99868	0.85929	0.98508	0.99867
		(0.00120)	(0.01180)	(0.00413)	(0.00124)
(2,3)	388	0.99700	0.88406	0.99202	0.99695
		(0.00275)	(0.01609)	(0.00437)	(0.00276)
(1,0)	1372	0.99325	0.50957	0.95709	0.99910
		(0.00226)	(0.01318)	(0.00557)	(0.00084)
(1,1)	132	0.98438	0.68149	0.96963	0.99093
		(0.01035)	(0.03980)	(0.01463)	(0.00813)
(1,2)	103	0.98945	0.71780	0.98863	0.98872
		(0.00956)	(0.04445)	(0.01024)	(0.01012)
(1,3)	42	0.97520	0.73328	0.95307	0.97426
		(0.02175)	(0.06473)	(0.03071)	(0.02339)

Notes: (a) P_{ih} is the probability of purchase for commodity i for the h-type household

(b) The posterior standard deviations are in parentheses.

Table 7.7 Bayesian posterior means and standard deviations of commodity-specific and general scales

Household Type	*Commodity-Specific Scales*				*General Scales*
	Food	*Clothing*	*Housing*	*Others*	
(2,0)	1.00000	1.00000	1.00000	1.00000	1.00000
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
(2,1)	1.22490	1.24060	1.42790	1.15880	1.21230
	(0.02881)	(0.10346)	(0.07327)	(0.04872)	(0.04048)
(2,2)	1.40990	1.38890	1.47760	1.23740	1.30600
	(0.02841)	(0.09859)	(0.06412)	(0.04586)	(0.03836)
(2,3)	1.52070	1.53590	1.52610	1.30040	1.37830
	(0.03820)	(0.12662)	(0.07925)	(0.05892)	(0.05011)
(1,0)	0.51590	0.50169	0.75462	0.53211	0.55917
	(0.01183)	(0.04104)	(0.03363)	(0.02062)	(0.01738)
(1,1)	0.67719	0.74000	0.88405	0.56732	0.63898
	(0.03517)	(0.12877)	(0.06671)	(0.04521)	(0.04009)
(1,2)	0.88582	0.80157	1.00960	0.59880	0.71125
	(0.04429)	(0.13330)	(0.07659)	(0.04803)	(0.04246)
(1,3)	0.95947	1.09480	1.06130	0.63315	0.76674
	(0.09656)	(0.24572)	(0.15209)	(0.10345)	(0.09353)

Note: The posterior standard deviations are in parentheses.

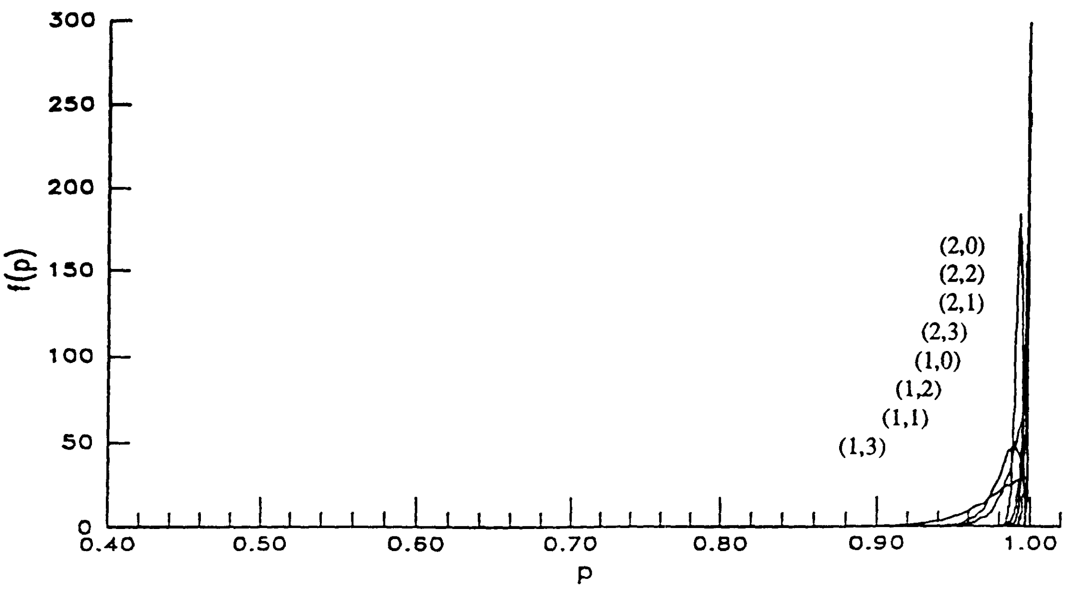
Table 7.8 Bayesian posterior means and standard deviations of selected scales from models 1 and 2 (*m1* and *m2*)

Household Type	*Commodity-Specific Scales*						*General Scales*	
	Food		*Clothing*		*Housing*			
	m1	*m2*	*m1*	*m2*	*m1*	*m2*	*m1*	*m2*
(2,0)	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)	(0.00000)
(2,1)	1.23641	1.22490	1.27952	1.24060	1.48807	1.42790	1.23380	1.21230
	(0.03288)	(0.02881)	(0.11570)	(0.10346)	(0.09238)	(0.07327)	(0.05705)	(0.04048)
(2,2)	1.42314	1.40990	1.40232	1.38890	1.51560	1.47760	1.33410	1.30600
	(0.03215)	(0.02841)	(0.11224)	(0.09859)	(0.07938)	(0.06412)	(0.05458)	(0.03836)
(2,3)	1.57516	1.52070	1.64687	1.53590	1.64963	1.52610	1.47000	1.37830
	(0.04369)	(0.03820)	(0.15340)	(0.12662)	(0.09952)	(0.07925)	(0.07253)	(0.05011)
(1,0)	0.52986	0.51590	0.53373	0.50169	0.82269	0.75462	0.58189	0.55917
	(0.01302)	(0.01183)	(0.04929)	(0.04104)	(0.04040)	(0.03363)	(0.02198)	(0.01738)
(1,1)	0.72651	0.67719	0.90953	0.74000	1.02559	0.88405	0.72262	0.63898
	(0.04096)	(0.03517)	(0.15584)	(0.12877)	(0.08350)	(0.06671)	(0.05323)	(0.04009)
(1,2)	0.94421	0.88582	0.92086	0.80157	1.14785	1.00960	0.78616	0.71125
	(0.04761)	(0.04429)	(0.15551)	(0.13330)	(0.09622)	(0.07659)	(0.05633)	(0.04246)
(1,3)	1.05883	0.95947	1.41036	1.09480	1.27995	1.06130	0.89943	0.76674
	(0.12146)	(0.09656)	(0.35590)	(0.24572)	(0.22548)	(0.15209)	(0.15416)	(0.09353)

Note: Model *m1* refers to the model used in the Bayesian estimation in Chapter 6 while model *m2* is the model used in this chapter. The former did not account for zero expenditures while the latter did.

Figure 7.2
Posterior Distributions of the P_{ih} by commodity type and by household type *(no. of adults, no. of children)*

Commodity: Food



Commodity: Clothing

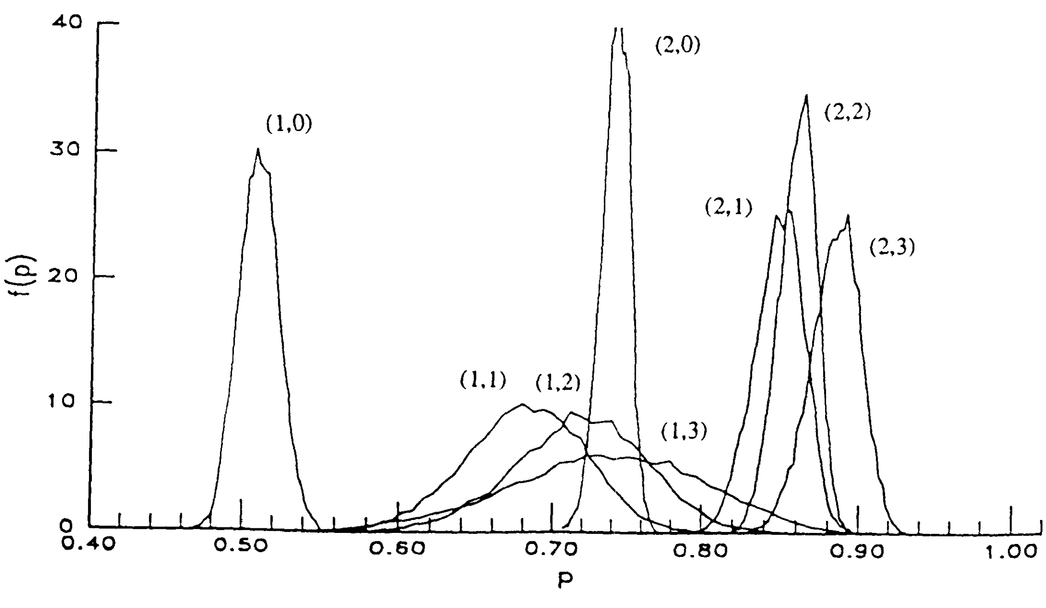
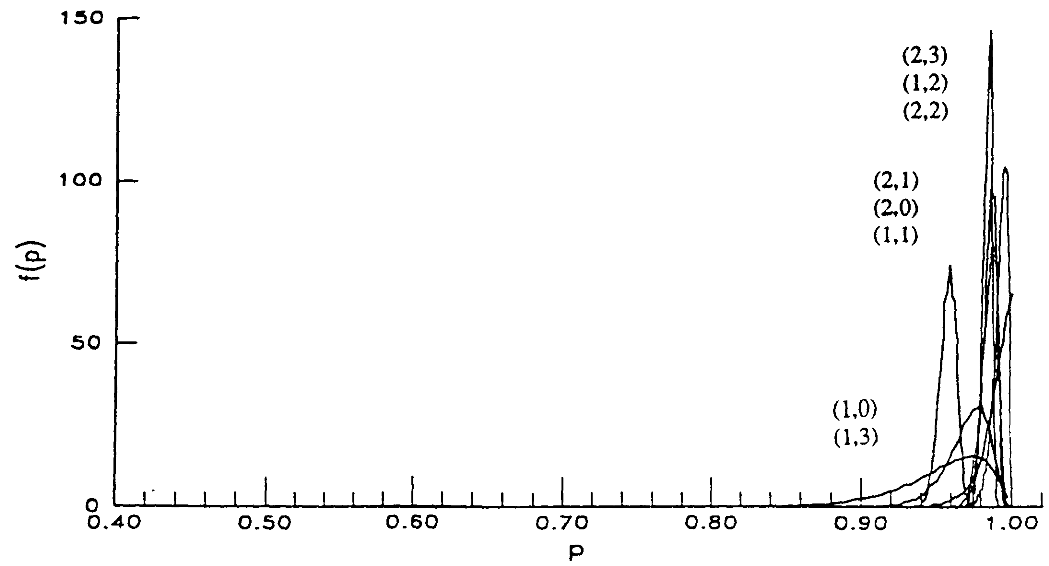


Figure 7.2 (cont.)
Posterior Distributions of the P_{ih} by commodity type and by household type *(no. of adults, no. of children)*

Commodity: Housing



Commodity: Others

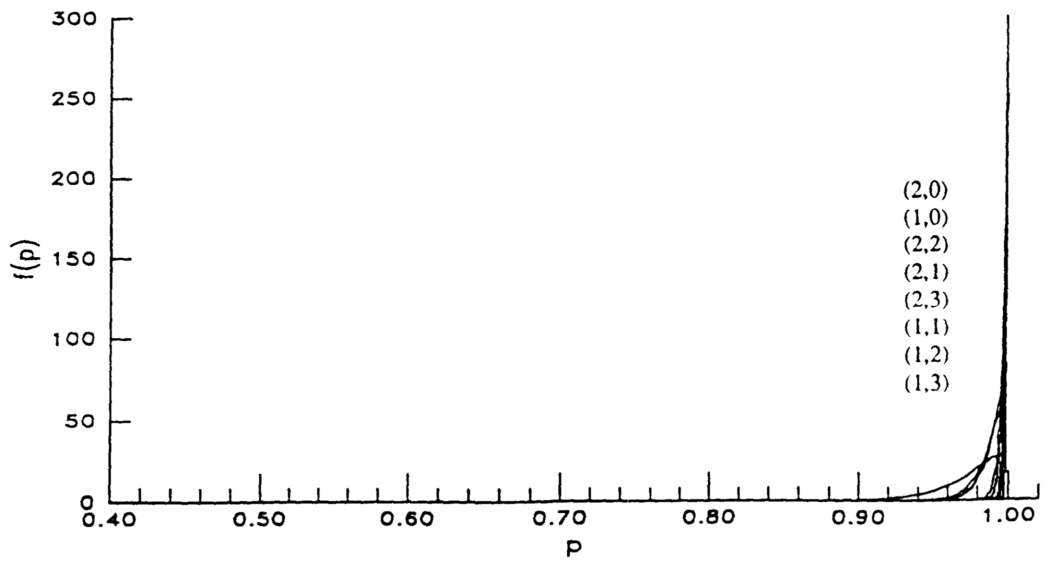


Figure 7.3
Posterior Distributions of the P_{ih} by household type
***(no. of adults, no. of children)* and by commodity type**

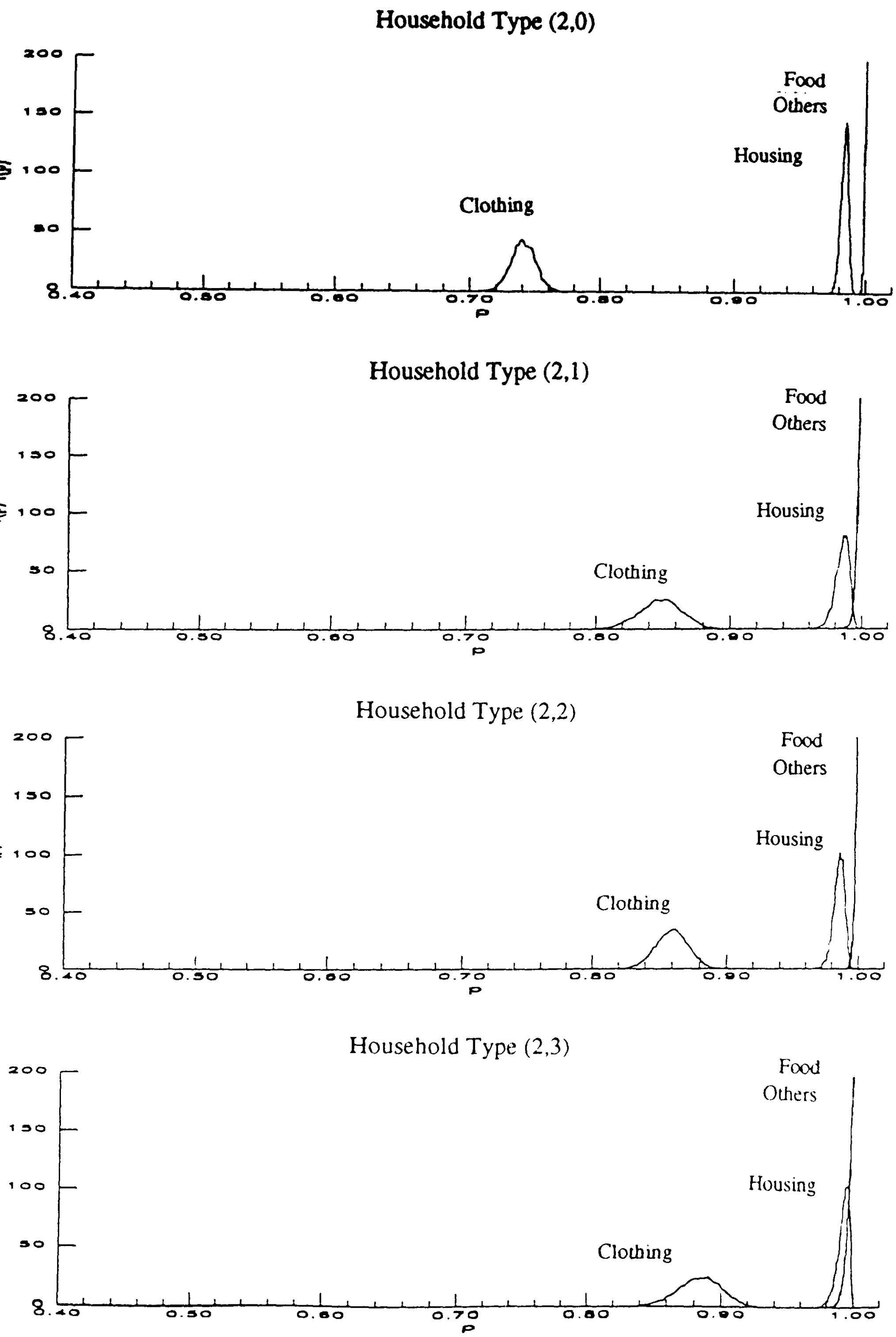


Figure 7.3 (cont.)
Posterior Distributions of the P_{ih} by household type
(no. of adults, no. of children) **and by commodity type**

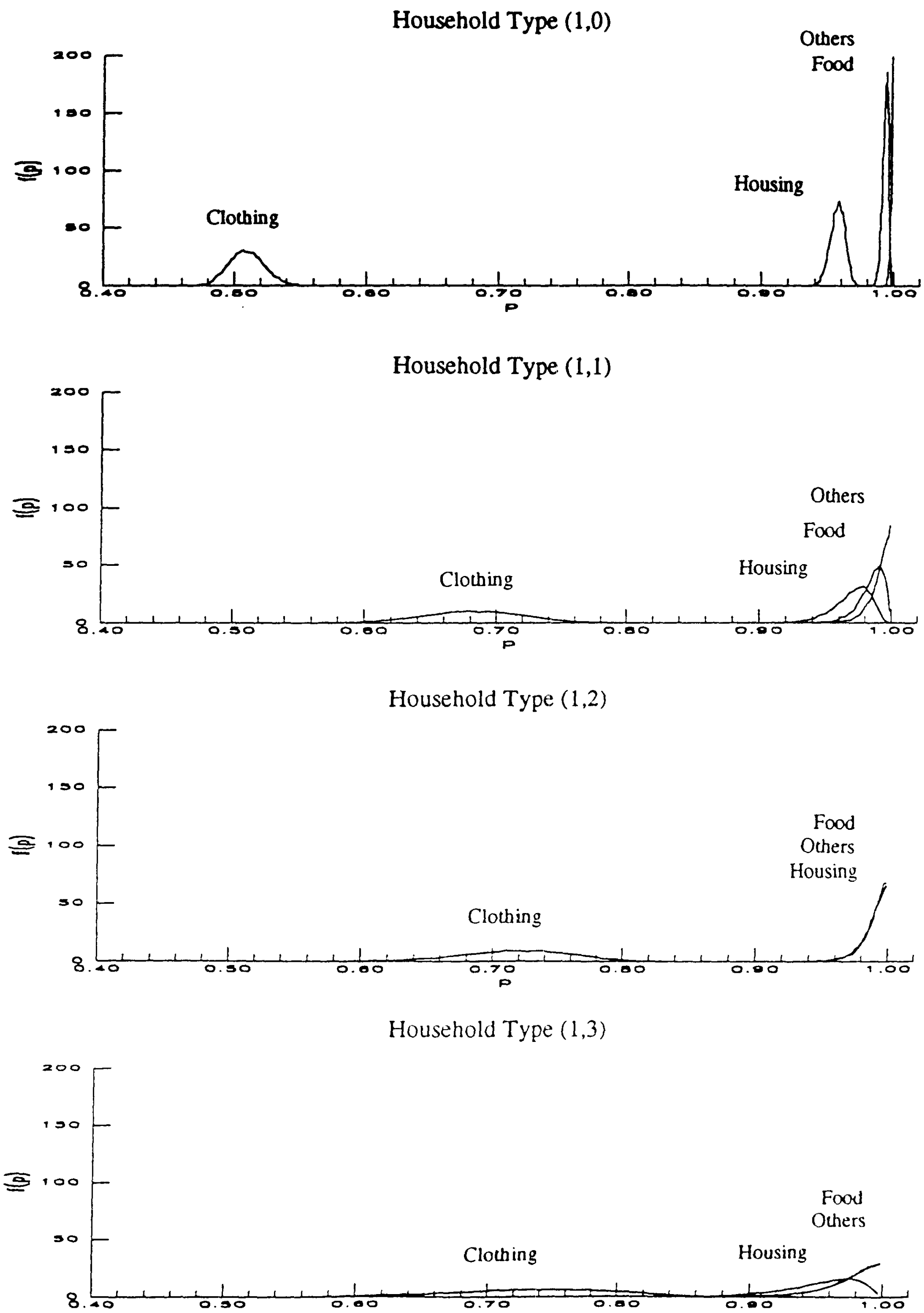
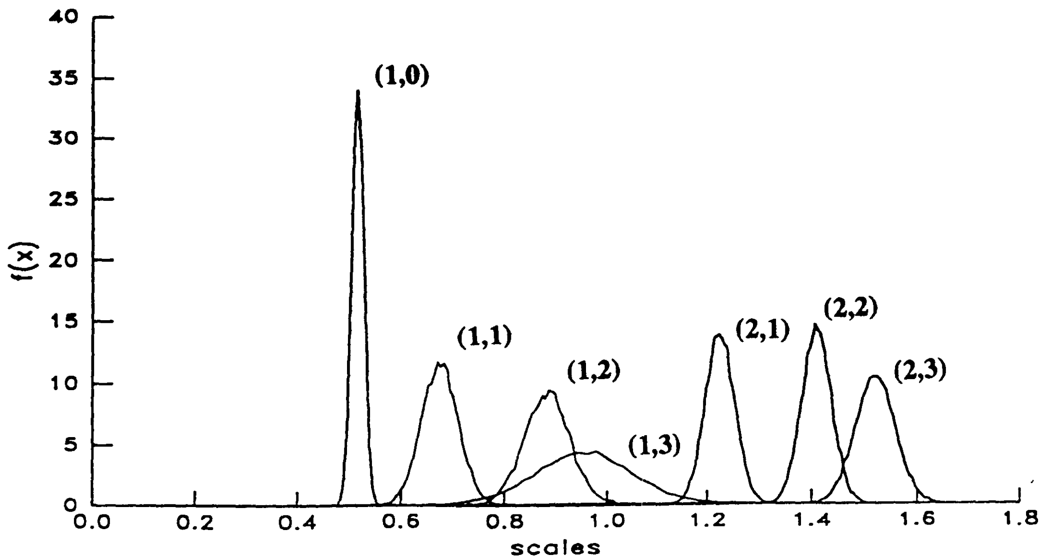


Figure 7.4

Posterior Distributions of Food Scales for each Household Type *(no. of adults, no. of children)*



Posterior Distributions of Clothing Scales for each Household Type *(no. of adults, no. of children)*

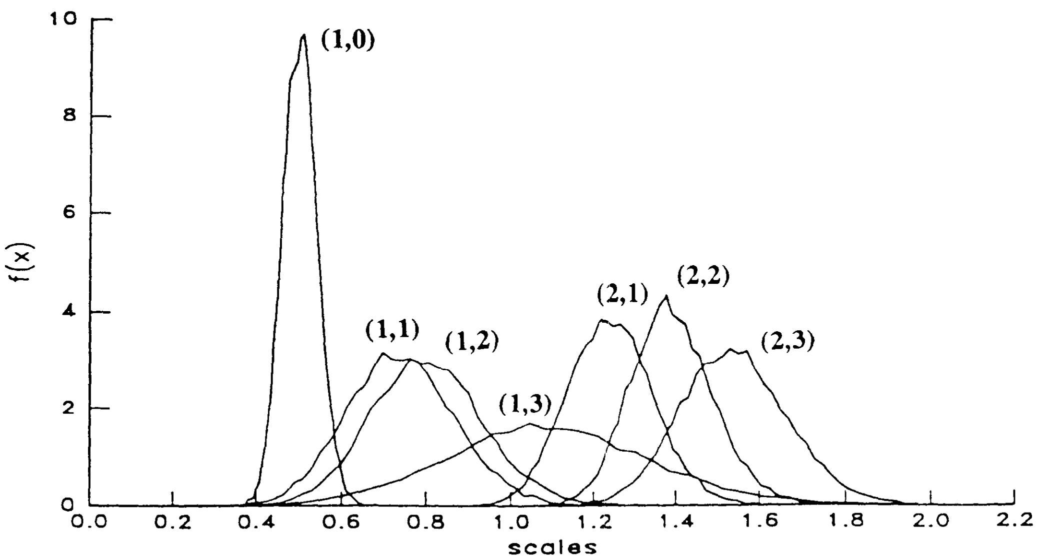
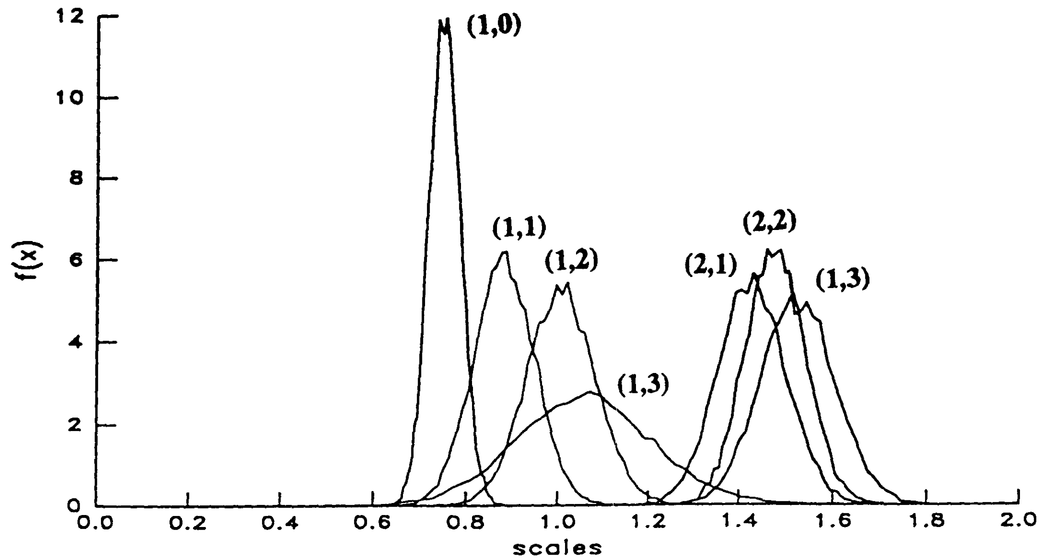


Figure 7.4 (cont.)

Posterior Distributions of Housing Scales for each Household Type *(no. of adults, no. of children)*



Posterior Distributions of Others Scales for each Household Type *(no. of adults, no. of children)*

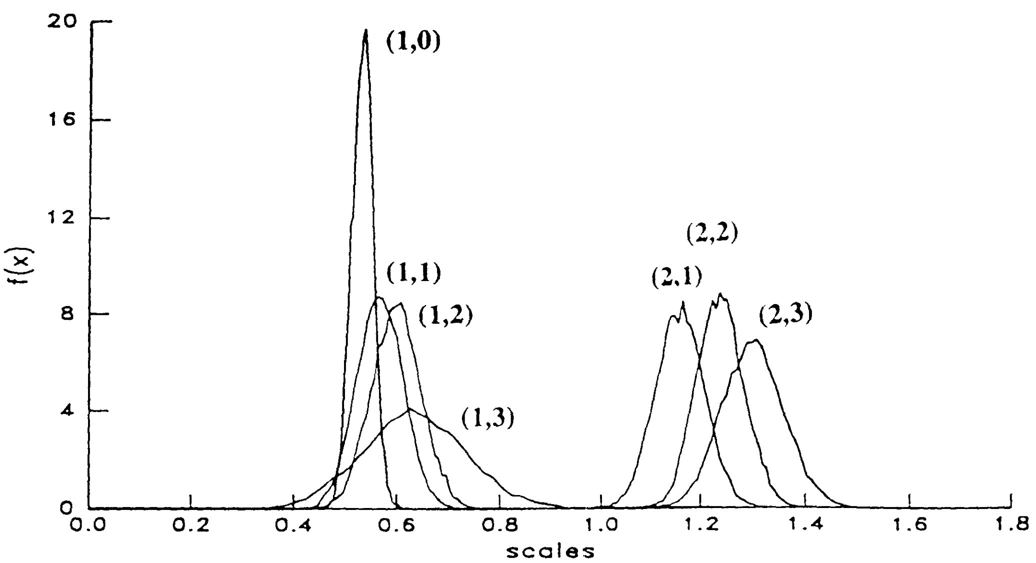
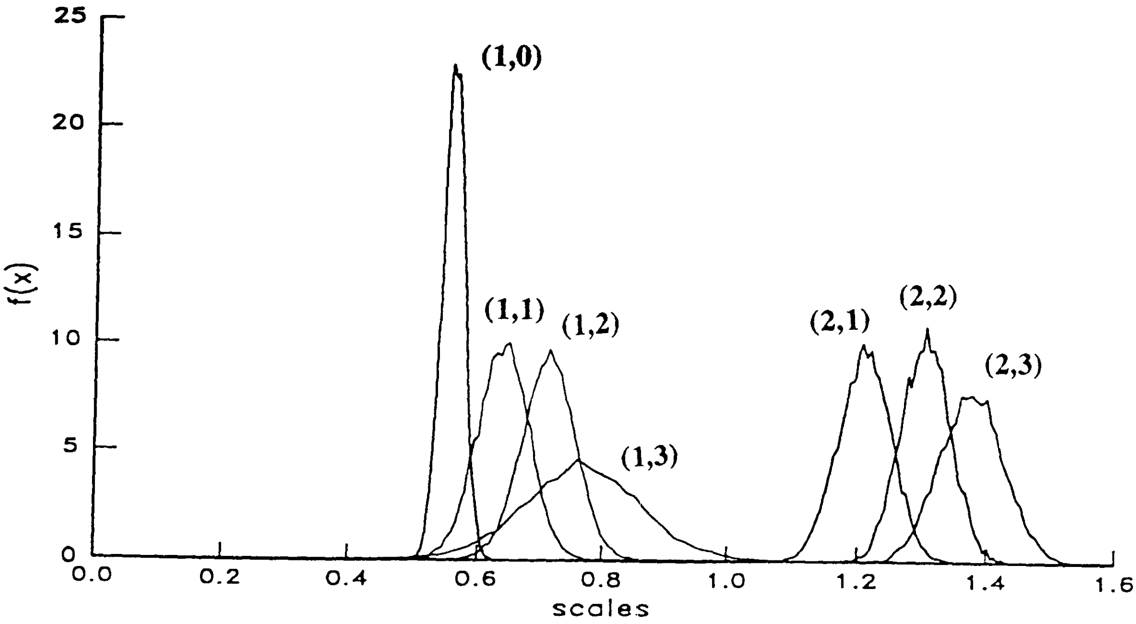


Figure 7.5

Posterior Distributions of General Scales for each Household Type *(no. of adults, no. of children)*



Chapter 8

Summary and Conclusion

The equivalence scale is a concept with considerable policy significance. As an instrument for comparing welfare levels of households differing in size and composition, it seeks to answer questions such as, "How much income does a household with two adults and one child need, in relation to a couple without children, to enjoy the same level of welfare as the latter?" Notwithstanding conceptual and methodological problems in interpersonal welfare comparisons (see Sen (1987)), and additional complications from using the household rather than the individual as the unit of decision making, such comparisons are inevitable in major policy exercises such as the measurement of inequality and poverty, studying the effects of a set of tax changes on welfare levels of different households and calculating the compensation that a household with a child requires for the additional cost of that child.

The main interest of this study is the estimation of household equivalence scales. A household equivalence scale shows the relative cost of maintaining household h with composition $\boldsymbol{\delta}_h$ at the same utility level $u = u_r$ enjoyed by the reference household r with composition $\boldsymbol{\delta}_r$. In terms of cost functions, an equivalence scale for household h is expressed as

$$s_h = \frac{C(u, \mathbf{p}, \boldsymbol{\delta}_h)}{C(u, \mathbf{p}, \boldsymbol{\delta}_r)}.$$

where $\mathbf{p}$ is the price vector for the expenditure goods. Deaton and Muellbauer

(1980) state that equivalence scales are to welfare comparisons between households of different characteristics what cost-of-living indices are to welfare comparisons for a given household facing different prices.

Research into equivalence scales estimation for welfare comparisons has a long and chequered history originating from the pioneering work of Engel (1895) on Belgian working class expenditure data. The first estimates of equivalence scales, due to Engel, were based on the assumption that the expenditure share of food is a correct indicator of a household's level of welfare. This assumption did not prove appealing to demand theorists because of its ad hoc nature. Notwithstanding, the Engel approach continue to be popularly used today because of its minimum data requirements and simple calculation techniques. It is certainly the most widely used method in empirical studies on equivalence scales.

Other models of equivalence scales have been advanced since. Rothbarth (1943) proposed to use expenditures on 'adult goods' to indicate welfare. Prais and Houthakker (1955), on the other hand, formulated an equivalence scale model for the calculation of commodity-specific scales in recognition of the fact that a change in the demographic composition of a household will affect the consumption of different commodities in different magnitudes. More recently, it has become more conventional to derive equivalence scales by first specifying a utility function. This approach was started off by Barten (1964) when he generalised Engel's work by explicitly considering a "collective utility function" of the form

$$u_h = U\left(\frac{q_1}{s_1(\boldsymbol{\delta}_h)}, \frac{q_2}{s_2(\boldsymbol{\delta}_h)}, \ldots, \frac{q_n}{s_n(\boldsymbol{\delta}_h)}\right)$$

where $q_i/s_i(\boldsymbol{\delta}_h)$ refers to the 'Barten-scaled' consumption of commodity i and the $s_i(\boldsymbol{\delta}_h)$'s are the commodity-specific scales. A large value of $s_i(\boldsymbol{\delta}_h)$ implies that the household needs a relatively large amount of that commodity (compared to the reference household) in order for that commodity to have the same input into the utility function. Barten's scaling model and other utility-based approaches such as the translation, the Barten-Gorman and the reverse-Gorman models, are much more preferred by economists than the "proxy" approaches of Engel and Rothbarth because they allow for a more systematic method of incorporating demographic variables in a demand system.

Following Pollak and Wales (1979), recent literature has generated considerable controversy on the interpretation and use of equivalence scales as conventionally calculated, in welfare comparisons across households. Pollak and Wales distinguish between "conditional" and "unconditional" scales. They argue that welfare comparisons across households require "unconditional" equivalence scales which need budget data and a theoretical framework that is based on the treatment of a household's decision on having children as endogenous similar to its expenditure decisions on various items. In reality, however, traditional budget data only allow the calculation of what are called "conditional" scales in view of the assumed exogeneity of children. Conditional equivalence scales, according to Pollak and Wales, only have behavioural applications, e.g. in studying the impact of household composition, like prices, on expenditure pattern, but have no policy significance. Deaton and Muellbauer (1986), Binh and Whiteford (1990), Pashardes (1991) and Nelson (1993), among others, disagree with the Pollak and Wales view and see virtue in conventional calculation of equivalence scales. Deaton and Muellbauer (1986) write: "how parents choose to have children" is not relevant to the problem of measuring child costs because "that parents choose to have children means that the benefits of having them are greater than the costs, but it does not mean that the costs are zero". Nelson (1993) further adds that "as questions of the distribution of pure subjective happiness (welfare) are rarely raised in practical application, equivalence scales in the older, more materialistic and more objective sense remain of great practical concern". Blundell and Lewbel (1991) take an intermediate position and argue "that demand equations alone provide no information about equivalence scales in any one price regime, but if equivalence scales in any one price regime were known, then demand data can identify the unique true equivalence scales in all other price regimes."

This dissertation makes no attempt to resolve this "crucial identification problem" in the equivalence scale literature. The position taken here is that the household utility or welfare that unconditional and conditional equivalence scales purport to measure are not the same and, in fact, lead to rather different models of preferences. Unconditional equivalence scales equates utility with the feeling of happiness or satisfaction that may be gained from the consumption of goods and services; in particular, this includes the happiness gained from the the presence of children in the household. On the other hand, conditional equivalence scales are concerned with welfare in the material standard of living sense. Specifically, this refers to the "capability" of each household member to be well-clothed, well-fed, well-rested, etc. In this work, conditional equivalence scales are the object of estimation. The underlying premise is that conditional equivalence scales are of great practical importance and play significant roles in a host of policy applications. It does not help that unconditional equivalence scales are not, at the present time, estimable and hence, not available. Therefore, unless a better alternative to conditional scales exists, the use and derivation of such scales will continue to occupy an important place in the economic and social policy research arena.

In practice, estimating equivalence scales is generally a difficult exercise. For this reason and inspite of its theoretical inadequacies, the Engel model remains the most widely used approach for estimating equivalence scales because it is easy to apply and has minimal data requirements. Chapter 4 of this work updates Engel scale estimates for Australia using the 1988-89 Australian Household Expenditure Survey (HES) unit record data file. The calculated Engel scales based on budget foodshares show that to maintain the same level of welfare or utility as a childless couple, a couple with one child needs 24 percent more income; a second child will push income requirements up by a further 12 percent. The estimated scales are also shown to increase when the "proxy for welfare" basket is generalised from the budget share of food only to that of food plus other necessities. Estimated scales based on the budget share of a composite basket consisting of food, clothing, housing and/or medical care show higher relativities compared to those based on the budget share of food alone.

A section in Chapter 4 is devoted to an international comparison of Engel equivalence scales obtained for Australia, the Philippines and Thailand. This cross country comparison is undertaken because it is often not clear to public policy researchers, particular to those who focus on international comparisons, which equivalence scale should be used. There is often contradicting evidence in the economic literature in this regard and when the interest is comparing Australia with its Asian neighbours, no formal comparison of equivalence scales has been attempted. This was until the recent publication of Valenzuela (1996) which is based on Chapter 4 of this dissertation. The comparison of Australian equivalence scales with those of the Philippines and Thailand is facilitated by the availability of the unit record data files for the year 1988 from the two latter countries i.e. the 1988 Family Income and Expenditure Survey (FIES) public-use files from the Philippines' National Statistics Office and the 1988 Socio Economic Survey (SES) from the National Statistics Office of Thailand.

This cross-country comparison has a number of interesting results. If the budget share of food is used as the indicator of welfare or utility, the results show that an additional child costs about 44 percent of a couple in the Philippines, and 41 percent of the same in Thailand, but only 24 percent of the same in Australia. These observed differentials between Australia, on the one hand, and the Philippines and Thailand, on the other, is attributed to two factors. First, the budget shares of food in the Philippines and Thailand are higher than that in Australia. Therefore, the addition of a child (who is largely food consuming) to the family will impose greater demand for Philippine and Thai households than it will for Australian households. A second reason is the availability of government "allowances" for most Australian children which softens the impact of the additional demand on the family's resources due to the presence of an additional child. This type of assistance is not available in the two other countries, implying that the need to meet the increase in demand is the sole burden of the household, hence, the higher relative costs. Another important finding in this section is the observation that when the expenditure share basket is redefined from that including foodshares alone to that of a combination of the shares of food and other necessities, the calculated scales for the three countries converge and become comparable to each other. This observed regularity is surprising but economically defensible.

Commodity specific scales are also of interest because they show the effect of a change in demographic composition of a household on a range of different consumption items. Chapter 5 dealt with the estimation of commodity-specific equivalence scales using the scaling procedure of Barten (1943) to incorporate demographic variables into the chosen demand system. The model considered in this study is the extended linear expenditure system (ELES), a demand system that has proven popular among researchers using Australian data. The main contribution of this chapter is the development of an iterative maximum likelihood (ML) estimation procedure to estimate both commodity-specific and general scales. The new procedure is demonstrated and is an improvement over the two-step procedure (due to Kakwani (1977)) conventionally used for such model. The following chapter Chapter 6, meanwhile, applies Bayesian econometrics to arrive at a new procedure for the estimation of ELES-based equivalence scales. Using non-informative priors, a Bayesian procedure is presented with a detailed derivation of expressions for posterior and conditional probability density functions. The chapter provides a step-by-step description of how a numerical sampling algorithm called the Gibbs sampler is used to operationalise the Bayesian procedure.

The ML estimates were compared with the Bayesian posterior means and standard deviations. As it turns out, there was not much difference between the estimated scale values from each procedure, presumably because of the large sample size used in the estimation. It was nonetheless shown that the Bayesian approach can be a viable alternative to conventional methods of equivalence scale estimation. Chapter 6 was also useful in providing a first step towards the development of a Bayesian methodology for handling observed zero expenditures from survey data developed in the next chapter.

This dissertation uses household level micro-unit level data from the 1988-89 Australian HES. It is well known that such expenditure survey-based data sets present a major estimation problem in that zero expenditures are observed, when, for some commodities, such as food, clothing and housing, zero consumption is not probable. Estimation of models that do not account for the occurrence of these misleading zeros are known to yield biased results. In this context, an econometric model for the occurrence of zero observations due to consumers' infrequency of purchase is proposed in Chapter 7. Here, a Bayesian estimation procedure based on this model is developed and applied. The estimated scales show lower relativities compared to the Bayesian estimates in Chapter 6, where these latter estimates were based on a model that did not account for the occurrence of the misleading zeros. Numerical sampling techniques were again required to operationalise the

Bayesian procedure derived for the model in Chapter 7. This chapter thus develops a numerical sampling process which incorporates the Metropolis-Hastings sampling algorithm within a Gibbs sampling procedure. A step-by-step description for this "M-H within Gibbs" procedure is outlined for the case under investigation.

Two new contributions to the literature emanate from this chapter: (i) it presents a new zero expenditures or "infrequency-of-purchase" model to account for the presence of misleading zero expenditure from survey data; and (ii) it develops and demonstrates a corresponding new estimation procedure based on Bayesian methods. Such a model and estimation procedure would have been too difficult to handle within the conventional sampling theory framework.

In general, it was demonstrated in this dissertation that the Bayesian approach can be a viable and reliable alternative to traditional sampling theory approach for the equivalence scale estimation problem. The Bayesian procedures proposed are also shown to facilitate statistical inference through the convenient estimation of posterior densities and associated posterior means and variances. It was shown, in a number of instances, that Bayesian estimation can be less cumbersome and less difficult compared to using conventional sampling theory methods for equivalence scale estimation. The presentation of empirical results proved to be another major advantage of the Bayesian procedure over the sampling theory-based methods. Bayesian estimates of parameters are presented in the form of density functions. Such a presentation is highly desirable because the Bayesian posterior probability density function carries more information and provides for a multidimensional characterisation of the estimated parameters, relative to the more traditional point estimate.

Some areas for future research based on extensions of this study could involve work towards the following directions: (i) exploring the imposition of prior information in the form of inequality restrictions on the scales, and on subsistence parameters; (ii) application of the proposed estimation techniques to a more flexible demand system; and, (iii) extending the procedure to accommodate price information on time-series budget data.

Appendix A

The Gibbs and Metropolis-Hastings Sampling Procedures

This appendix contains introductory descriptions of how the Gibbs sampler and the Metropolis-Hastings sampling procedures are applied. This exposition is meant to supplement the work in Chapters 6 and 7 which developed Bayesian procedures involving these two algorithms. The descriptions found in this appendix are in very general terms and are drawn mostly from Casella and George (1992) and Chib and Greenberg (1995a). Albert and Chib (1996) and Gilks, Richardson and Spiegelhalter (1996) also provide introductory expositions on the subjects.

A.1 Introduction

The Gibbs and Metropolis-Hastings (M-H) sampling procedures are two very popular numerical sampling algorithms based on Markov Chain Monte Carlo (MCMC) methodology. Given a posterior density function for some unknown parameters θ, $g(\theta \mid y)$, a MCMC method produces a (correlated) simulated sample $\{\theta^{(1)}, \theta^{(2)},..., \theta^{(T)}\}$, called a Markov chain, from $g(\theta \mid y)$. This simulated sample is then a basis for computing a variety of inferential summaries. The Gibbs sampler and the M-H algorithms are among the many ways of constructing these Markov chains. MCMC methods have proved useful in the application of Bayesian statistics because they can be applied to the simulation of Bayesian posterior density functions.

A.2 The Gibbs Sampler

The Gibbs sampler is an algorithm for generating random variables from a (marginal) distribution indirectly without having to calculate the density itself. Its main applications have been to that of Bayesian models but it has been shown to be extremely useful for classical models as well as in other practical problems (Tanner 1991).

The Gibbs sampler is based on elementary properties of Markov chains and its application can be described in a straightforward manner. Suppose we are given a joint density $f(\boldsymbol{\theta}) = f(\theta_1, \theta_2, ..., \theta_k)$ and are interested in obtaining characteristics of the marginal densities $f(\theta_s)$, $s = 1, 2, ...k$. All that is required is the complete set of conditional distributions $f(\theta_s \mid \boldsymbol{\theta}_{(s)})$, where $\boldsymbol{\theta}_{(s)}$ denotes the random vector of $k-1$ random variables with the s^{th} random variable being deleted. The Gibbs sampling algorithm proceeds as follows:

1. Given the initial values $(\theta_1^{(o)}, \theta_2^{(o)}, ..., \theta_k^{(o)})$, generate a value $\theta_1^{(1)}$ from the conditional density
$$f(\theta_1 \mid \theta_2^{(o)}, \theta_3^{(o)}, ..., \theta_k^{(o)}).$$
2. Generate a value $\theta_2^{(1)}$ from the conditional density
$$f(\theta_2 \mid \theta_1^{(1)}, \theta_3^{(o)}, ..., \theta_k^{(o)}).$$
3. Continue the process for all s, up to the value $\theta_k^{(1)}$ from the conditional density
$$f(\theta_k \mid \theta_1^{(1)}, \theta_2^{(1)}, ..., \theta_{k-1}^{(1)}).$$
4. With the new realisation $\boldsymbol{\theta}^{(1)} = (\theta_1^{(1)}, \theta_2^{(1)}, ..., \theta_k^{(1)})$, replace the initial values in step 1 above.

5. Using this new realisation, go through steps 1 through 4 to obtain $\boldsymbol{\theta}^{(2)} = (\theta_1^{(2)}, \theta_2^{(2)}, ..., \theta_k^{(2)})$.

6. Iterate the above process T times, producing $\boldsymbol{\theta}^{(T)} = (\theta_1^{(T)}, \theta_2^{(T)}, ..., \theta_k^{(T)})$.

The process allows the generation of a series of random values

$$\boldsymbol{\theta}^{(1)}, \boldsymbol{\theta}^{(2)}, ..., \boldsymbol{\theta}^{(T)}$$

called the "Gibbs sequence". Geman and Geman (1984) show that the Gibbs sequence above converges in distribution to the joint distribution $f(\theta_1, \theta_2, ..., \theta_k)$, "if T is sufficiently large". It is conventional to treat the initial m samples as observations from a "burn-in period" and thus, in practice, the first m samples in the generated series are dropped. The sample $\theta_s^{(t)}, t = T - m, ..., T$ can be regarded as a simulated observation from the marginal distribution of $f(\theta_s)$. Therefore, corresponding characteristics such as mean and variances of the random variable in the Gibbs sequence pertain to the marginal distribution $f(\theta_s)$.

A.3 The Metropolis-Hastings Sampling Algorithm

The Metropolis-Hastings (M-H) algorithm is another general MCMC method that can be used to sample intractable distributions that arise in Bayesian econometrics. It can be used in conjunction with the Gibbs sampler to sample from intractable conditional distributions.

Consider the general problem of simulating a sample from a probability density function $f(\theta)$. The M-H algorithm proceeds as follows:

1. Select an initial value for θ.

2. Given a candidate-generating density $g(\theta, \theta')$, generate a candidate value θ'.

3. Given $g(\theta, \theta')$ is symmetric, compute an acceptance probability $\alpha(\theta, \theta')$ given by
$$\alpha(\theta, \theta') = \min \left\{ \frac{f(\theta')}{f(\theta)}, 1 \right\}$$

4. Accept candidate θ' with probability $\alpha(\theta, \theta')$. If rejected, the new value for θ is taken to be the current one.

This procedure is iterated several times to produce a large sample from $f(\theta)$. An important feature of this algorithm is that the calculation of $\alpha(\theta, \theta')$ does not require knowledge of the normalising constant of $f(\theta)$ which makes it useful for Bayesian applications.

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