PART IV - OVERVIEW AND CONCLUSION

CHAPTER FOURTEEN

CONCLUSION

14.1 Overview

In section 1.6, the six specific objectives of the thesis were listed. In this section, the results for each objective will be briefly summarised.

1. ***To demonstrate the need for alternative means of dealing with violence in PNG.***

After providing country background material in chapter 2, the nature, extent, causes and costs (broadly defined) of the major contemporary forms of violence were examined in chapter 3. The current policies to deal with violence are themselves essentially violent; they are also very costly and not very effective. There is therefore an opportunity to try alternative means of dealing with these conflicts.

Traditional PNG societies had and have well established nonviolent means of attempting to resolve conflict before it breaks out into violence; these are based on discussion; negotiation and consensus (chapter 4). These methods can be utilised, along with relevant Western conflict resolution procedures, to try to resolve contemporary conflicts.

2. ***To present the theoretical and practical cases for the peace studies at university level; this will include an evaluation of its impact in general and specifically in one Australian university.***

Building on the identified need for alternative ways of resolving conflict, the scope of peace studies and nonviolence was presented (chapter 5 & 6). The potential of university - level peace studies in changing students was then assessed. The few previous evaluative studies were discussed in section 8.2. Given the limitations in research designs (particularly the length of time over which evaluations typically occurred), the impact on students were modest and emphasised the importance of developing greater compassion rather than simply teaching facts.

Research was carried out to determine the impact of peace studies on undergraduates and postgraduates at the University of New England, Australia. A range of methods was employed and this study served to refine the methods used in the subsequent UPNG evaluation. Student responses concerning the impact of the courses was very positive (section 9.3). However, a specific test of impact based on four dimensions (section 9.2) showed only a small impact in the desired directions. Again, the time over which the study of peace occurred and was evaluated was limited.

3. ***To design a one semester peace studies course for teaching at the University of Papua New Guinea. The design will take account, among other things, of contemporary curriculum design, peace studies in developed countries and Papua New Guinea cultural practice in the resolution of conflicts.***

4. ***To teach this course in an environment where learning is enhanced by a range of co-operative learning practices.***

The curriculum of the course to be taught at UPNG was presented in detail in chapter 11 and was built on the foundations of earlier chapters, particularly chapters 3-6. Structural violence (section 5.2) was emphasised along with its counterpart, positive peace.

The importance of student learning taking place in a peaceful learning environment was discussed in chapter 12, and the processes and methods used were outlined. Emphasis was placed on active dialogue between students and between students and the lecturer.

The course was taught during second semester, 1996 to 23 second and third year undergraduates, mostly majoring in social work.

5. ***To evaluate this peace studies course in terms of its impact on the learners.***

The earlier experience of evaluating such courses was utilised to evaluate the UPNG course. A number of methods were employed, including before and after questionnaires (section 10.2), which were also applied to a control group of Politics students; interviews at the end of the course (section 10.3), and learning journals (section 10.4) The evaluation found that significant changes had occurred in respect to student optimism about being able to make positive change (section 13.4.4).

6. ***To consider the implications of the research findings for the teaching of peace studies at tertiary level in Papua New Guinea as a means of building a more peaceful society.***

This concluding chapter (section 14.2) reflects on the findings of this research for tertiary level in PNG. It suggest that there is a strong case for offering Peace Studies as a major within undergraduate degrees.

14.2 Reflections

The genesis of this research was the researcher's desire to search for effective solutions to violence in PNG. Violence has become so deeply embedded in the contemporary culture that many citizens have come to view violence as inevitable. It is not uncommon for the marginalised and disempowered to resort to violence in the hope of achieving some form of justice. The use of violence as peacemaking strategy has failed to provide any lasting solutions and has added to the portrayal of PNG as the most dangerous country in the South Pacific.

The fatalistic perception that violence is inevitable and beyond rectification, is a pessimistic view and needs to be changed. According to the researcher, this change involves empowering the majority through re-education. Melanesian approaches to conflict resolution demonstrate the power of nonviolent conflict resolution in dealing with disputes. Non-village people need to learn and re-learn such approaches as well to utilise relevant parts of Western CR approaches.

More importantly, people need to be conscientized concerning structural violence and injustice. This provides a powerful motive to work for peace and justice. It is vital that peace studies goes beyond the study of direct violence to uncover the root causes of violence, particularly structural violence and cultural violence (section 5.2), which is silent, slow and deadly.

The Peace Studies curriculum see (chapter 11) was therefore designed with the goals of addressing the root causes of violence and to searching for alternative ways of promoting peace and justice. The curriculum was designed to enable students to become sensitive to the dynamic and multiple linkages and connections between the many problems of peacelessness in PNG and at the global level. Greater personal and social peace is hard to achieve without promoting peace at all these interdependent levels.

While the choice of peace studies *content* is important, attention was also given to the *pedagogy* of peace studies i.e. the methods employed in classroom teaching. The methodology used was based on the ideas proposed by educators such as Freire which promotes classroom dialogue and participation amongst learners and between learners and the teacher. The practice was designed as a departure from the traditional method of teaching where the teacher dominates the classroom environment. Thus opportunities were given to students to contribute to knowledge by drawing on their own experience. This is an empowering approach and adds to the possibility that students will develop an internal motivation that recognises the need for change within society.

14. 3 Some issues concerning the evaluation.

In order to assess the practical efficacy of peace studies, this research study aimed to answer the crucial question: How might the study of peace lead to changes amongst students? Such changes were believed to arise in four dimensions (holistic understanding, conscientization, critical thinking, and actions for peace and justice)

In terms of evaluating the impact of such a course, there are three problem areas. First, a pure research methodology would randomly assign students to one or other of experimental and control groups. Otherwise, their conscious or unconscious attitudes towards peace are likely to determine their choice whether or not to study it. This is a difficult problem to overcome, given the freedom of choice typically available to students. Secondly, how long does it take for the study of peace to influence students' attitudes? Is it reasonable to expect such changes to occur during the four or five months of the course or will these emerge later, and if so how much later? Thirdly, the personality of the lecturer, their commitment to the content of the course and the learning methods used, may influence students as much as content. How can such variables be accounted for in research design?

For this research, the approach used was to employ a separate 'control' group of students doing other courses who were similar to the 'experimental' group doing peace studies. The UPNG results were impressive, given the relatively short time of the study. The results of the quantitative analysis was supported by the qualitative analysis based on interviews and learning journals.

14.4 Peace Studies at UPNG

What can UPNG, as the highest learning institution in the country, do to address some of the root causes of violence? How is UPNG preparing its students to face the challenges to personally engage in the invitation to acquiesce in structural and cultural violence? When academic study is compartmentalised into specific subject disciplines, it restricts the holistic development of students. They need to be given the opportunity to understand how issues are interconnected, and involving both left and right brain. The left brain function recalls facts and information, in isolation from the more nebulous concepts of the emotions. The right brain involves the emotions, the artistic and the mystical and makes the connection with the whole person. Both are appropriate in university education.

In order for peace studies to gain academic credibility, it must break through the resistance by traditionalists to interdisciplinary innovations as well as administrative reticence to support such programs. The challenge is to encourage cooperative teaching across disciplinary boundaries and for administrative structures to be flexible enough to allow such a learning environment.

Peace studies does not want to be a 'gloss' on top of the already established disciplines instead it needs to be the foundation of the infrastructure from which all educational discursive strategies are interpreted

To date, peace studies has remained somewhat of an anomaly to the prescribed established areas of learning. A new interdisciplinary centre is needed, where academics from a range of disciplines, unified by a passion for peace and justice, can collectively offer courses leading to a major, along with a 'mainline discipline', within the Bachelor of Arts degree. Education students would also benefit for these courses, particularly if peace studies and allied subjects were taught at lower levels of education. According to a former Vice Chancellor:

> *Tomorrow's leaders require much more than technical, professional or administrative skills. They also require what used to be called a "well-rounded education" A civil engineer or a doctor, for example, needs to know a reasonable amount about the history, economics and politics of PNG, the region, and the world - not to help him in his engineering or medical practice in PNG, but because, by the very fact of his being an "educated" or "professional" Papua New Guinean, people will look to his leadership in fields outside those in which he is specialist. ...the question of ethics is of particular importance to a rapidly developing nation like PNG.*
>
> *Lynch, 1988: 178-179.*

This thesis suggests that the impact of individuals who have holistic understanding of conflict and violence, who are able and willing to criticise the mainstream and who are conscientized and motivated to work for peace will be a powerful one.

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APPENDIX A

ANALYSIS OF RESPONSES TO THE PHASE ONE STUDY

QUESTION BY QUESTION

QUESTION 1

Question 1 was the first question which asked students to consider what fractions were.

> ***How would you explain to someone, who didn't know, what a fraction was?***

Table A.1 indicates that there were nine categories identified for the responses to this question.

TABLE A.1

Analysis of adult learners' responses to Q1 on the Fraction Quiz

RESPONSE	NUMBER OF RESPONSES
No response	2
Uncodable	5
Couldn't say	2
Part of a whole	43
Drew a diagram or showed area or drew 1/2	8
One number over another	4
Part of a number	28
A percentage or decimal	6
Mathematical expression for division	5
Total	103

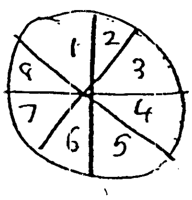
Typical Responses

Typical responses in the 'couldn't say' category included: "Actually, ... I don't really know what a fraction is" (DK) and 'I wouldn't be able to explain it to them" (LT).

Many of the forty-three students who identified fractions as 'part of a whole' did not elaborate on this statement. For example, one student (MD) wrote: "A part of a whole or something", while another student (MB) added: "when something is broken up into parts these parts are either small or large". Only three students wrote: "Not a whole number", possibly indicating that fractions were numbers, but could not be integers. It is difficult to determine how these students would have interpreted improper fractions.

However, some students who did elaborate on there answers would do so by spontaneously offering examples. For example, one student (ST) wrote: "I would probably use something as an example such as cutting something in to half then quarters etc". In these cases, 1/2 (and subsequent halving) was the preferred fraction. For example, one student (MY) wrote: "start with a watermelon and explain that if I cut it in half that will be two fractions and so on", while another student (MD) demonstrated the following:

Like a cake —



if you cut down the middle you are halving the cake now one cake ~~was~~ is in half 1/2.
if you cut across there is now 4 pieces 1/4
Another cut is in 6 pieces 1/6
& another one is 8 pieces 1/8

The final categories of answers differed qualitatively from the above responses. Prior to the 'one number over another' classification, responses indicated a preference to define a fraction in terms of a diagram or some concrete reference item. The fact that fractions were numbers was noticeably absent. In addition, there was a comparatively high non-response rate. As a consequence, this survey was followed up with an informal classroom discussion in which the students were asked if they thought fractions were numbers. While it is difficult to give exact numbers, some students refused to acknowledge that fractions were numbers. It became evident that many students thought that the term 'number' referred to **whole** numbers only. It is plausible that this widespread misconception may have influenced the results.

The category of 'one number over another' response appeared to indicate that some students clearly opted to describe the 'look' of a fraction. Only one student (PA) who gave this response, also acknowledged, for dubious reasons, that division may be related to fractions. The student wrote:

One whole number divided by another whole number with number being on the top of a ___ bar and the number being replaced by on the bottom of the bar. Or, I might say - replace the dot on top of the division sign with the number if front and replace the dot on the bottom of the division sign with the next number.

Students who wrote 'part of a number' usually added little else. When a student chose to elaborate on an answer, 1/2 was used as a referent. For example, one student (DH) concluded: "A fraction is a part of a whole number. It can also go toward making numbers whole e.g., 1/2 + 1/2 = 1".

Many adults avoided any reference to division as an equivalent fraction representation. When this possibility was mentioned, students usually qualified or restricted the fractions so that they were strictly less than one, e.g., one student (TA) stated: "The answer of one number divided by a larger number", while another student (PB) wrote: "A fraction has a purpose to divide wholes into smaller parts". Only one student (IS) acknowledged that fractions could not have a zero denominator.

QUESTION 2

Question 2 asked students to choose between a small selection of cards which demonstrated different aspects of fractions.

Which of the following cards would help someone to understand what the fraction 3/4 is? Explain why.

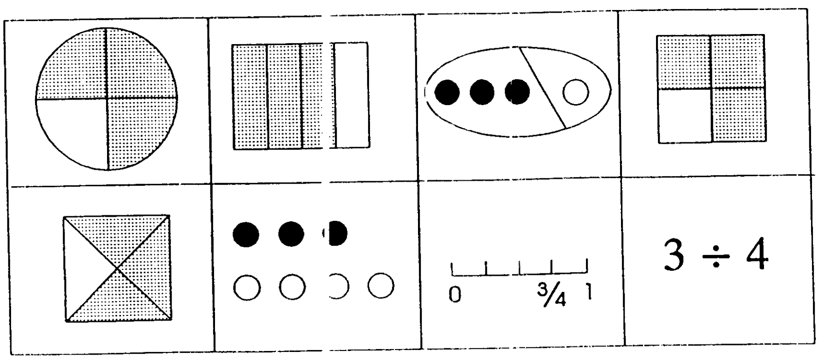


Table A.2 indicates the acceptance or rejection of particular models by the number of responses.

TABLE A.2

Analysis of adult learners' responses to Q2 on the Fraction Quiz

MODEL	NUMBER OF RESPONSES
No response	24*
Uncodable	4
Accepted only one model that was not an area model	3
Accepted only one area model, e.g., one of rectangle, circle or square	19
Accepted all the area models only	15
Accepted all the area models plus one or two others	16
Accepted all models	22
Total	103

* means that one class (n = 24) did not receive this question

Typical Responses

In general, and despite the prompt in the question, students did not provide details for their decisions to select (or reject) particular models. Clearly, there is overwhelming acceptance of the area models. Of those students who selected only one model, (22 in all), 19 students selected an area model. However, the type of area model selected (circle, rectangle) was apparently based on personal choice, rather than for any mathematical reason. For example, one student (MD) stated: "it would depend on the way the person thinks and relates. e.g., I relate to squares, my wife relates to pie charts etc". Another student (DH) wrote: "[The circle] would help best as the circle or 'pie' is easy to visualise and understand" (DH) while yet another student (NB) suggested: "This card could be a chocolate bar broken into 4 equal sections, 3 of these sections would represent 3/4".

The classification of 'accepted all the area models' appeared to be based on the assumption that a whole can be divided into equal parts as one student (DS) wrote: "The whole is divided into 4 equal parts so 3/4 is 3 of those 4 parts or 3/4 of the whole". Responses in this category exhibited considerable similarity, with respect to the choice of language used. For example: "The cards are all dived [sic] into parts and shaded in" (AD), "3 out of 4 sections are shaded so therefore is 3/4" (TB), and "3/4 is an object divided into 4 parts and you what [sic] 3 of them".

So far, all the examples presented have treated fractions as if they were objects. The next category, which consisted of 'accepted all the area models plus one or two others' was fundamentally different to the above. It was not until this category that students indicated that they were considering the possibility that fractions could be represented by other means than area diagrams. However, this did not necessarily imply that the students accepted fractions as numbers. For example, there were no consistent trends observed across the responses, i.e, all of the 'other' non-area models were noted in the scripts. Only two students chose $3 \div 4$ as a model of a fraction. One adult (RH) explained that this was "because the product is less than one". It is plausible that the student recognised the fact that $3 \div 4$ would yield an answer less than one and therefore selected this answer. Another student (PA) wrote: "3 divided by 4 is exactly what 3/4 is". It was only in the final category that all the different models of fractions were accepted, i.e., as one student (MW) wrote: "they explain what fraction 3/4 are [sic]".

QUESTION 3

Question 3 placed the division of three by five into a situation that should be familiar to many adults. It was seeking the connection between $3 \div 5$ and 3/5.

> ***You have three cakes. Could you share them equally between five people? Explain what you would do. (Use diagrams if necessary).***

The methods used by the students in obtaining the answer are summarised in Table A.3 below.

TABLE A.3

Analysis of adult learners' responses to Q3 on the Fractions Quiz

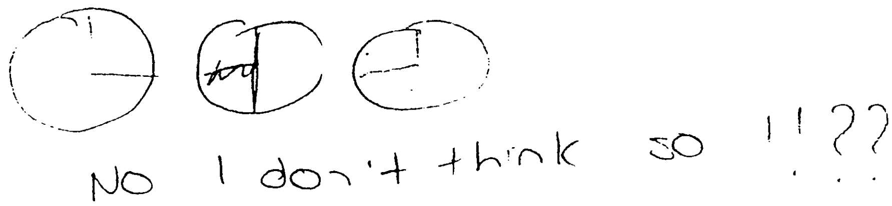
RESPONSE	NUMBER OF RESPONSES
No response	6
Uncodable	4
Incorrect answer and no diagram present	3
Incorrect answer and diagram present	11
Correct answer (3/5) obtained and a diagram was presented	43
Correct answer (3/5) obtained in the absence of a diagram	36
Total	103

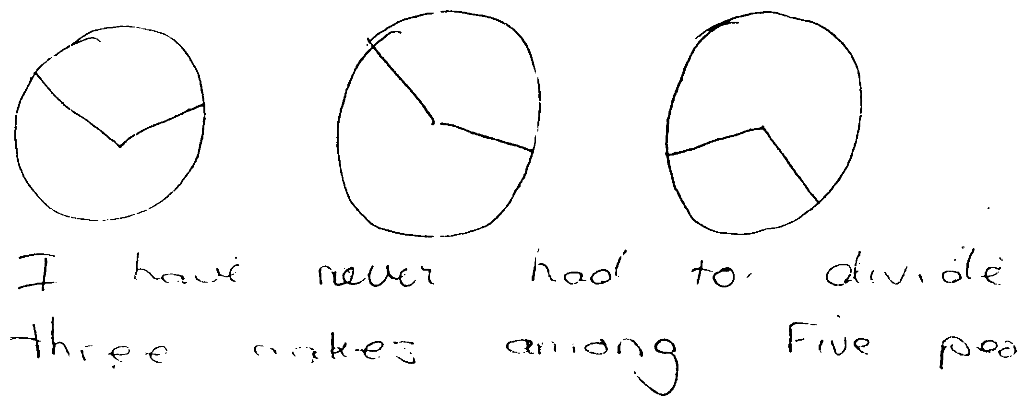
Typical Responses

It is worth noting that one of the students (MD), who was unable to attempt this question at all, acknowledged that he: "would have trouble doing this even with real cakes".

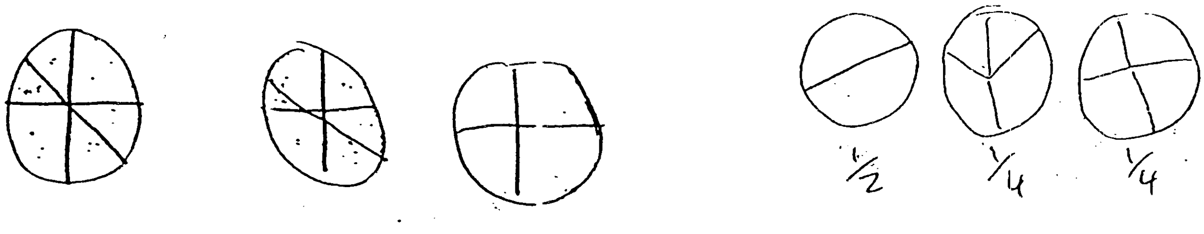
The 'incorrect' category consisted of responses which indicated that students had attempted to answer the problem, but were unable to complete it successfully. For example, one student wrote: "If the cakes are halved and 1/2 is given to each person one-half would be left over for anyone with a sweet tooth".

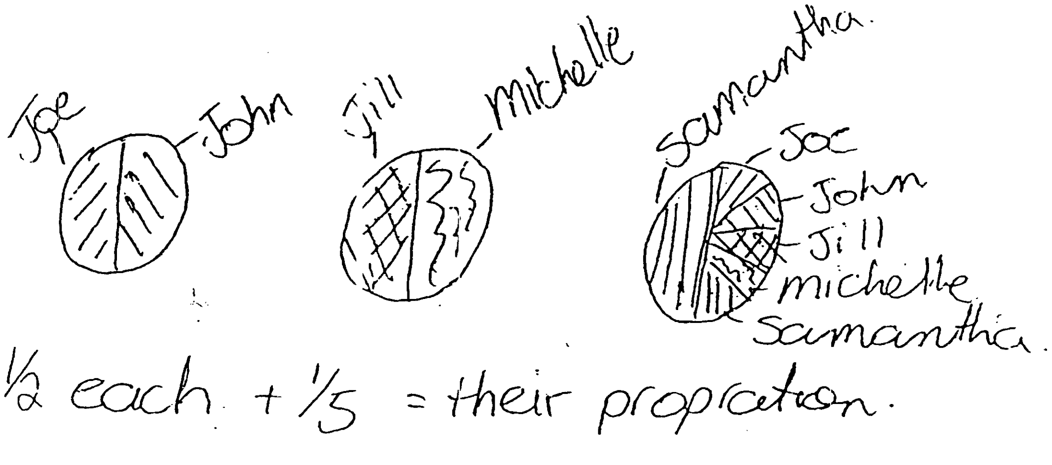
Many responses in the 'incorrect and a diagram present' section indicated that students had attempted to provide a diagram, but were unable to successfully use their own diagrams to answer the problem. The following two examples indicate the reliance on diagrams and the lack of 'connectedness' between the correct answer and the diagram.





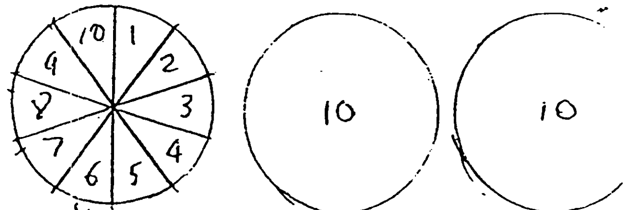
A slightly more sophisticated version of the above consisted of drawing three circles and then dividing them into (usually) 1/2. The halves would then be distributed to the each of the five people. The remaining half would then be dealt with in a variety of ways, depending on the expertise of the student. A common approach was to treat the remaining half as a new 'whole', and then repeat the procedure of dividing the half. One student decided that this could be achieved by halving again, apparently unaware of the unequal sized pieces this produced. Some students could eventually obtain a correct solution by subdividing the remaining 1/2 into fifths. When a correct answer was obtained via this method, it formed the basis of the 'correct and a diagram present' classification. The following examples are presented to show the increasing complexity of the 'halving' strategy.





Other typical responses included in the second classification included variations which appeared to rely on the preference of certain fractions, such as tenths. As one student wrote:

You should divide the cakes into 10 equal parts. like this:
you would end up with.
30 equal parts of cake



You can now divide equally 30 small pieces of cake amont 5 people. they should each have 6 small pieces of cakes.

In general, the 'correct and a diagram present' category consisted of responses, which although correct, included diagrams which appeared to form a major part of the students' problem-solving strategy. In these cases, students not only drew diagrams, but appeared to actually need to use them to answer the question. Without recourse to an interview, it is difficult to differentiate between those students who needed the diagrams and those who simply preferred to add diagrams to embellish their solutions. The obvious prompt in the question may have caused some students to draw diagrams, even though they did not utilise them to solve the problem.

Responses in the final category were fundamentally different to all previous classifications. There were no diagrams associated with this level and the answer was usually expressed simply as '3/5'. Some students did not, or could not, provide a reasons for there answers. For example, student (AD) wrote: "Every one would get 3/5 of a cake but I can't explain it".

QUESTION 4

Question 4 required students to treat fractions as numbers and asked students to determine how many numbers there were between two consecutive whole numbers.

> ***How many numbers are there between 2 and 3? And between 0 and 1?***

Eight categories were identified for the responses to this question, and are presented in Table A.4.

TABLE A.4

Analysis of adult learners' responses to Q4 on the Fraction Quiz

RESPONSE	NUMBER OF RESPONSES
No response	17
Uncodable	3
None	17
No whole numbers, only fractions or decimals	19
One or one whole number	3
A small number, e.g., 4, 9 or 10	11
A large number, e.g., 100 or 1000	7
Infinite	26
Total	103

Typical Responses

Results coded 'uncodable' for this question were ambiguous. For example, one student (CW) wrote "I have no idea.". This response was included in the 'uncodable' classification since it was impossible to determine if this student literally had no idea, or was simply unable to 'count' the number of fractions, recognising that there were so many.

The 'none' category consisted of responses which stated that there were no numbers between 2 and 3 (or 0 and 1). As one student wrote: "There is nothing between the numbers as [each] follows each other". This response indicated that the student had been confused by the term 'number', and had focused on only whole numbers.

The association of the term 'number' with whole numbers continued throughout the next classification, but was not confined to it. For example, one student (TA) stated that there were "None ... only numerous fractions", while another student wrote: "I don't think you could actually say there were any whole numbers but there are fractions". Of surprise, was the admission by one student (MD) in the 'infinite' classification, who wrote: "if you want to call fractions numbers ... you cannot put a figure to it". Again, without recourse to an interview, it is impossible to ascertain whether these students thought fractions were or were not numbers.

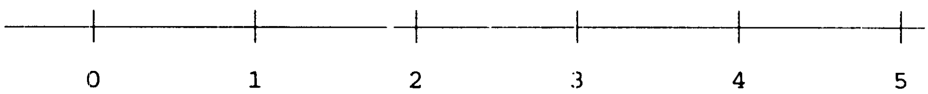
The next three categories consisted of responses which indicated that students attempted to 'count' the number of numbers between 2 and 3 (or 0 and 1). For example, three students stated that there was only one number, but did not state what it was. Some students replied by listing the numbers (2.1 2.2 2.3 2.4 2.5 2.6 2.7 2.8 2.9) or simply stated '9' or '10' or '11' if they had trouble counting. Finally, some students realised that there must be 'a very large number' such as 100 or 1000.

It was only in the last category that responses indicated the 'infinite' nature of fractions, as one student (UK) wrote: "How long is a piece of string?"

QUESTION 5

Question 5 asked students to plot three numbers on a typical number line. One of the numbers was expressed as a whole number (4), one as fraction less than one (3/5) and one as a fraction greater than one (1 1/5).

Where would the number 4 go on this line? And the number 3/5? And the number 1 1/5?



Since this question had more than one part to it, a comparison between each part is presented in Table A.5 below. None of the students had difficulty correctly plotting 4 at the correct position, so this number has been omitted from the analysis.

TABLE A.5

Analysis of adult learners' responses to Q5 in the Fractions Quiz

RESPONSE FOR FRACTION 3/5	NUMBER OF RESPONSES	RESPONSE FOR 1 1/5	NUMBER OF RESPONSES
No response	13	No response	20
Uncodable	10	Uncodable	6
Placed at 3/5 of length of line	5	Placed at 1 1/5 of length of line	1
Correctly placed at 3/5	75	Correctly placed at 1 1/5	76
Total	103	Total	103

Typical Responses

Although two types of responses were classified as 'uncodable', they are worth noting. In the first case, one student (SJ) identified 3/5 left of 0. Although the student had completed a Year 12 Mathematics course, the student appeared to confuse fractions with negative numbers. It is plausible that the student confused fractions with the definition of negative indices, which results in a fractional answer. In the second case, one student plotted 3/5 at 3.5 and correctly wrote 3.5 on the number line. This response indicates a misunderstanding of the relationship between fractions and decimals, i.e., confusion exists as to the role of the 'fraction bar (vinculum)' with that of the decimal point.

Students who plotted 3/5 at 3 on the number line, apparently interpreted the fractions 3/5 to mean 3/5 of the length of the line. However, this approach did not carry on the 1 1/5 fraction. It is also worth noting that the number of uncodable or non responses increased for the second part of the question. Since there were approximately the same proportion of students who could successfully plot both fractions, responses in this category indicated that students found plotting mixed numbers on a number line was not any more difficult than plotting proper fractions, or a whole number. In addition, of those students who could successfully plot both fractions, three students completed the task by first converting the fractions to their equivalent decimal representation. All three labelled the points as 0.6 (and not 3/5) and 1.2 (and not 1 1/5), respectively.

QUESTION 6

Question 6 required students to compare two equivalent fractions, but placed the fractions into a familiar situation, such as comparing two equal portions of a cake.

> ***Would you rather have 2/3 or 10/15 of a cake you particularly liked? Explain why?***

Table A.6 indicates 5 categories of students' responses to Question 6.

TABLE A.6

Analysis of adult learners' responses to Q6 on the Fraction Quiz

RESPONSE	NUMBER OF RESPONSES
No response	6
Uncodable	5
10/15 is larger	3
2/3 is larger	6
Fractions are equal/equivalent	83
Total	103

Typical Responses

In general, responses in the '10/15 is larger' or '2/3 is larger' classifications did not elaborate their answers. For example, "I would like 10/15 of the cake because its larger than 2/3" (TP) or "I would rather have 2/3 because it is a bigger number" (JB). Despite the familiar context, both of these answers indicated that some students did not understand equivalent fractions, even at an elementary or intuitive level.

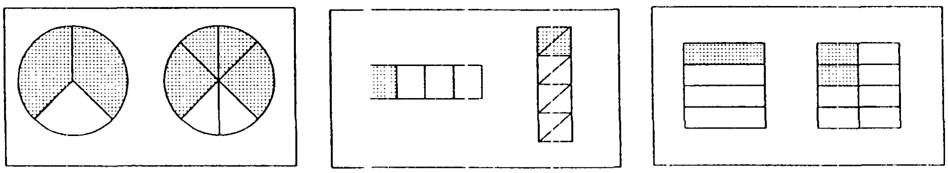
The final category consisted of responses which could be further divided into three main subgroups. The first group although acknowledging that the fractions were equal, then selected either 2/3 or 10/15 for reasons which indicated that they clearly treated the situation as if dealing with real cakes, i.e., they related their answers back to the original question. For example, one respondent wrote: "I would rather have 10/15 of the cake although they are both equal. I have a dainty mouth." or "2/3 of the cake even though 10/15 is the same amount. 2/3 would be much easier to hold. One piece for each hand".

The next subgroup of responses consisted of students who 'proved' that 2/3 was equivalent to 10/15. Typically this 'proof' included diagrams or cancelling 10/15 to 2/3. However, at least one student (AD) made a minor calculational error which lead to an incorrect conclusion, i.e., "2/5 = 6/15 therefore 10/15 is the greater portion". It is unknown if the student misread the question or if the student was trying to manipulate fraction symbols, and could not successfully complete the problem. The remaining responses in this category consisted of simple statements, such as 'they are the same or equal'.

QUESTION 7

Question 7 asked students to select equivalent fractions from a variety of models of fractions.

> ***Suppose you saw these diagrams in a textbook. What could you tell from them?***



The were four categories identified for the responses to this question. These are presented in Table A.7. It is worth noting that one class (n = 24) did not receive this question. These responses have been included in the 'No response' category.

TABLE A.7

Analysis of adult learners' responses to Q7 on the Fraction Quiz

RESPONSE	NUMBER OF RESPONSES
No response	29*
Uncodable	8
Diagrams represent fractions	16
Diagrams represent equivalent fractions	50
Total	103

* means that one class (n = 24) did not receive this question

Typical Responses

Some of the 'uncodable' responses were ambiguous. For example, one student wrote: "They all had the same shaded in area even though the fractions are different".

Clearly, a majority of the remaining responses indicated that students related the diagrams to fractions. The only major difference between the last two classifications was 'detail'. For example, the responses which identified the diagrams as 'they represent fractions', did not indicate if they meant 'equivalent' or not. At least one student (TM) listed the fractions, but did not connect them in any way, e.g., "2/3 4/6 1/4 2/8 1/4 2/8".

A majority of responses which indicated that the diagrams represented 'equivalent fractions', did not elaborate on this statement. One student (DM) wrote: "You'd be showing the same amount shaded in but in different fractions".

APPENDIX B

FRACTION QUIZ

QUESTION 1

Imagine you are writing a dictionary of Mathematical terms. Explain, giving as many details as possible, how you would describe what a fraction is.

QUESTION 2

Compare and contrast what is meant by:

a. $\frac{5}{7}$ and $\frac{7}{5}$?

b. $\frac{2}{3}$ and $\frac{3}{5}$?

QUESTION 3

Place in order from smallest to biggest:

a. $\frac{1}{3}$, $\frac{1}{2}$, $\frac{1}{4}$

b. $\frac{2}{3}$, $\frac{5}{7}$, $\frac{3}{4}$

QUESTION 4

You have 9 pizzas to be shared between 15 people. Describe how you would do this.

QUESTION 5

Complete the following:

$$\frac{14}{16} = \frac{\_}{24}$$

QUESTION 6

a. You have 3 pizzas to be shared between 5 people. Describe how you would do this.

b. You have 2 pizzas to be shared between 5 people. Describe how you would do this. (You may use diagrams if you wish).

QUESTION 7

You have two recipes to choose from to make a drink of punch for a party. One recipe calls for 3 bottles of sherry and 6 bottles of soda water. The other calls for 2 bottles of sherry and 5 bottles of soda water. Which is the stronger drink. Why?

QUESTION 8

You have 5 pizzas to be shared between 3 people. Describe how you would do this.

QUESTION 9

Two people, who have different occupations, save a certain part of their salaries each week. The first person saves 1/5 of their salary. The second person saves 1/3. Is it possible for each to save the same amount? Give details.

QUESTION 10

Complete the following:

a. $\frac{1}{2} + \frac{1}{4}$

b. $\frac{3}{5} + \frac{2}{7}$

c. $\frac{3}{4} - \frac{2}{3}$

d. $\frac{5}{9} \times \frac{3}{5}$

e. $\frac{1}{2} \div \frac{1}{4}$

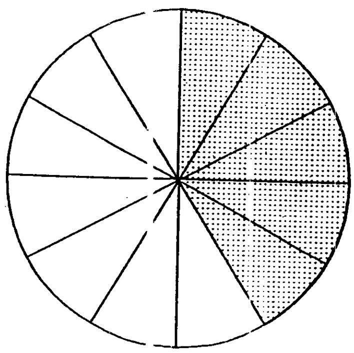
f. $\frac{2}{3} \div \frac{5}{9}$

QUESTION 11

At a recent function, the punch bowl was 2/5 full. At the end of the function, the bowl was 3/8 full. How much punch was consumed during the evening?

QUESTION 12

A student was asked to add 1/6 to 1/4. She drew:



and then concluded that: $\frac{1}{6} + \frac{1}{4} = \frac{1}{10}$

Discuss her conclusion.

QUESTION 13

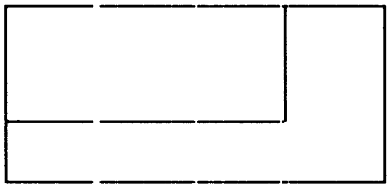
If I add 2 to both the top number (numerator) and bottom number (denominator) of 1/5, describe, in detail, what wil happen. [I'm looking for more than just the answer].

QUESTION 14

If I have the fraction 2/3 and I double the numerator (top number) and the denominator (bottom number), desc ibe the effect this will have on the 2/3. Why?

QUESTION 15

A carpet piece is placed in the corner of a room as shown. The carpet is found to go along 4/5 on one wall, and 2/3 of the other wall. What fraction of the floor does the carpet cover?



QUESTION 16

What does $\frac{3/4}{2/3}$ mean?

APPENDIX C

STUDENT INFORMATION SHEET

Please circle the most appropriate answer to the following questions.

1. What is your gender? F M

2. What is your (approximate) age?

   under 21
   21-30
   31-40
   41-50
   51-60
   over 60

3. Please indicate the highest level of mathematics you have passed or studied (please state) <u>before</u> starting this course. Approximately how many years ago was this? _______

   under year 8
   year 8
   year 9
   year 10
   year 11 - Maths 1
   - Maths 2
   - Maths in Society (MIS)
   year 12 - Maths 1
   - Maths 2
   - Maths in Society (MIS)
   other (please state) ________ __________

APPENDIX D

CONDENSED SUMMARY TABLES OF TABLES 5.2 TO 5.8

EQUIVALENCE (Q5, Q13 and Q14) AND SHARING (Q4, Q6a, Q6b and Q8)

Because of a constraint involved with the application of the Quest package, it has been necessary to re-group the data into only four major classifications. This number was chosen to enable statistically significant calculations to be performed and because this was a suitable number of categories to select for a majority of questions on the quiz. The new groupings for each of the seven questions described are shown in the next seven tables. The context-free questions are presented first, followed by the in-context questions.

TABLE D.1 (5.2)

Summary of adult learners' responses (re-grouped) to Q5 on the Fraction Quiz

Step Number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview for clarification	12	17
1	22/24 . added 8 . no working shown	11	5
2	21/24 . by patterns (e g., add '1/2') . by multiplying by 1.5/1.5	17	17
3	21/24 . by cancelling to 7/8 first . algebra . no working shown	15	13

TABLE D.2 (5.3)

Summary of adult learners' responses (re-grouped) to Q13 on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview for clarification	8	15
1	The fraction stays the same	1	4
2	3/8 or 2/7 Added 1/5 + 2/2 . unsuccesful . = 6/5	12	7
3	3/7 . the fraction decreases . the fraction changes (but does not describe how) . the fraction is larger . used diagrams . concluded that the fraction gets closer to 1	34	26

TABLE D.3 (5.4)

Summary of adult learners' responses (re-grouped) to Q14 on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview for clarification	5	9
1	The fraction changes/increases or decreases	11	3
2	2/3 = 4/9 Multiplied by 2 to get 4/3 = 1 1/3	3	5
3	4/6 . the fractions are the same . drew diagrams	36	35

TABLE D.4 (5.5)

Summary of adult learners' responses (re-grouped) to Q4 on the Fraction Quiz

Step Number	RESPONSES	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview for clarification	3	5
1	Cut into pieces and distribute until none left Drew 9 pizzas . unsuccessful solution (½ or ¼'s) . successful solution (1/5's) Cut each pizza into: . 1/2's . 1/4's then distributed evenly	10	11
2	Stated that they would need to multiply the number of pieces by 9 and divide by 15 Cut each pizza into: . 1/10's . 1/15's . 1/5's then distribute evenly	15	13
3	15/9 or 5/3 or 1 2/3 or 9/15 or 3/5	27	23

TABLE D.5 (5.6)

Summary of adult learners' responses (re-grouped) to Q6a on the Fraction Quiz

Step number	RESPONSES	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview for clarification	4	5
1	Drew 3 cakes . wrote 3 ÷ 5 . divided into 1/2's to distribute . divided into 1/4's to distribute . divided into 1/5's to distribute Diagram independent . cut into 1/2's to distribute Cut the cakes into even portions and gave each person an even amount	8	6
2	Diagram independent . cut into 1/10's to distribute . cut into 1/15's to distribute . cut into 1/5's to distribute	15	19
3	5/3 or 1 2/3 or 6/10 or 0.6 or 3/5	28	22

TABLE D.6 (5.7)

Summary of adult learners' responses (re-grouped) to Q6b on the Fraction Quiz

Step number	RESPONSES	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview for clarification	9	7
1	Drew 2 cakes . divided into 1/2's to distribute . divided into 1/4's to distribute . divided into 1/8's to distribute . divided into 1/3's to distribute . divided into 1/10's to distribute . divided into 1/5's to distribute and incorrect solution . divided into 1/5's to distribute and correct solution	15	10
2	Diagram independent . cut into 1/10's to distribute . cut into 1/5's to distribute	18	15
3	5/2 or 2 1/2 or 4/10 or 0.4 or 2/5	13	20

TABLE D.7 (5.8)

Summary of adult learners' responses (re-grouped) to Q8 on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview	4	5
1	Cut into pieces, divided by the number of people Gave each person one watermelon. Leaves 2 between 3, 1/2 both then 1/2 again	3	4
2	Gave each person one watermelon. Leaves 2 between 3, which is then divided into 1/3's . with diagrams Cut each watermelon into 3, then divided the remaining 1/3's	27	19
3	1 2/3 or 5 ÷ 3 or 5/3 or 1.6	21	24

APPENDIX E

THRESHOLD VALUES FOR UNDERSTANDING OF FRACTIONS THEME

Q5, Q13 and Q14 (context-free) and Q4, Q6a, Q6b, Q8 (in-context)

Figures shown in brackets indicate approximate numbers of students who attained this level.

Question number	Context-free questions 1	2	3	Question number	In-context questions 1	2	3
5	-.16 (87)	.35 (54)	1.13 (29)	4	-1.22 (100)	-.12 (85)	.74 (44)
13	-.25 (87)	-.06 (85)	.42 (54)	6a	-1.03 (98)	-.31 (87)	.73 (44)
14	-.63 (96)	-.09 (85)	.14 (75)	6b	-.72 (96)	.23 (67)	1.33 (16)
				8	-.95 (98)	-.55 (96)	.92 (29)

APPENDIX F

CONDENSED SUMMARY TABLES OF TABLES 6.2 TO 6.8

CONTEXT-FREE QUESTIONS (Q2a, Q2b, Q3a, Q3b) AND IN-CONTEXT QUESTIONS (Q7 and Q9)

Because of a constraint involved with the application of the Quest package, it has been necessary to re-group the data into only four major classifications. This number was chosen to enable statistically significant calculations to be performed and because this was a suitable number of categories to select for a majority of questions on the quiz. The new groupings for each of the six questions described are shown in the next six tables. The context-free questions are presented first, followed by the in-context questions.

TABLE F.1 (6.2)

Summary of adult learners' responses (re-grouped) to Q2a on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview for clarification	7	15
1	Wrote seven over five or seven fifths Drew diagrams as illustrations of fractions	17	18
2	Compared each fraction to the number 1	15	10
3	Converted before comparing . used percentages . used common denominators 5/7 is smaller or 7/5 is larger (no working)	16	9

TABLE F.2 (6.3)

Summary of adult learners' responses (re-grouped) to Q2b on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview for clarification	12	18
1	Wrote two parts out of three Drew a diagram to represent 2/3 and 3/5	16	16
2	Compared two fractions to a whole and noted that both are less than a whole Compared both fractions to a half . unsuccessful conclusion . successful conclusion	5	5
3	Converted to a common denominator or to a percentage 2/3 is larger than 3/5 (no working)	22	13

TABLE F.3 (6.4)

Summary of adult learners' responses (re-grouped) to Q3a on the Fraction Quiz

Step number	RESPONSES	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview for clarification	15	2
1	Wrote one part out of etc. Drew diagrams . stated fractions were different . and ranked in correct order	6	0
2	Focused on one fraction only e.g., 1/2 is the largest	5	0
3	Ranked in correct order . converted to percentages, decimals or common denominators . no reason given	29	50

TABLE F.4 (6.5)

Summary of adult learners' responses (re-grouped) to Q3b on the Fraction Quiz

Step number	RESPONSES	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview for clarification	26	7
1	Wrote two out of three Drew diagrams . and stated fractions were different . and ranked in correct order	5	1
2	Focused on only one fraction, e.g., 3/4 is the largest Stated an unusual order, e.g., 2/3, 3/4, 5/7	10	24
3	Ranked in correct order, e.g., 2/3, 5/7, 3/4 . converted two of the three fractions and then compared the third to one of the other two . converted to percentages/decimals or common denominators . no reason given	14	20

TABLE F.5 (6.6)

Summary of adult learners' responses (re-groped) to Q7 on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview for clarification	2	7
1	Stated: . 2 bottles stronger . they are equal . first recipe (or 3 bottles, no reason given)	17	11
2	Concluded that three bottles are stronger: . compared 1:2 and 1:2.5 . compared 3/6 = 1/2 to 2/5 which is < 1/2	22	21
3	Concluded that three bottles are stronger . compared 3/6 to 2/5 by using common denominators . compared 3/6 to 2/7 . compared 3/9 to 2/7 by using common denominators or percentages	14	13

TABLE F.6 (6.7)

Summary of adult learners' responses (re-grouped) to Q9 on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview for clarification	7	16
1	Answered 'No' and assumed 1/3 > 1/5 always Answered 'No' and assumed the wages were the same, but calculated (using LCD's = 15's) that first wage earner needs to increase contribution by 2/15	12	8
2	Stated it was possible, but did not provide reasons Stated it was possible, if the wages were different, but did not provide reasons Stated it was possible, if first wage earner > second wage earner	19	10
3	Stated the above, but also provided an example to indicate the two respective wages, i.e., to indicate that the student had an overview of the problem Stated it was possible if 1/5 of the first person's salary = 1/3 of the second person's salary Stated it was possible if the second's salary is 5/3 times the first's salary	17	18

APPENDIX G

THRESHOLD VALUES FOR COMPARISON OF FRACTIONS THEME

Q2a, Q2b, Q3a, Q3b (context-free) and Q7 and Q9 (in-context)

Figures shown in brackets indicate approximate numbers of students who attained this level.

Question number	Context-free questions 1	2	3	Question number	In-context questions 1	2	3
2a	-.84 (98)	.33 (39)	1.20 (19)	7	-1.53 (103)	-.23 (73)	1.25 (12)
2b	-.50 (94)	.45 (39)	.77 (28)	9	-.69 (98)	-.03 (68)	.85 (28)
3a	-.70 (98)	-.47 (85)	-.32 (73)				
3b	-.33 (73)	-.12 (68)	.87 (28)				

APPENDIX H

CONTEXT-FREE QUESTIONS (Q10a, Q10b, Q10c, Q10d, Q10e, Q10f) AND IN-CONTEXT QUESTIONS (Q11, Q12, Q15, Q16)

Because of a constraint involved with the application of the Quest package, it has been necessary to re-group the data into only four major classifications. This number was chosen to enable statistically significant calculations to be performed and because this was a suitable number of categories to select for a majority of questions on the quiz. The new groupings for each of the ten questions described are shown in the next ten tables. The context-free questions are presented first, followed by the in-context questions.

TABLE H.1 (7.3)

Summary of adult learners' responses (re-grouped) to Q10a on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response	1	2
1	2/6 = 1/3 or 1/2	3	6
2	Provided unusual answers	0	5
3	3/4 . converted to 6/8 then 3/4 . used LCD's . no working	51	39

TABLE H.2 (7.4)

Summary of adult learners' responses (re-grouped) to Q10b on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response	1	6
1	5/12	1	4
2	Provided unusual answers	4	2
3	31/35 (correct) . used LCD's . no working	49	40

TABLE H.3 (7.5)

Summary of adult learners' responses (re-grouped) to Q10c on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response	1	9
1	1/1 = 1	0	2
2	Provided unusual answers	1	1
3	1/12 . uses LCD's . no working	53	40

TABLE H.4 (7.6)

Summary of adult learners' responses (re-grouped) to Q10d on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response	4	6
1	Provided unusual answers	5	6
2	Employed common denominators	9	3
3	Did not cancel fractions and obtained answers, such as 3/9, 15/45 1/3 . cancelled only one common factor first (i.e., intermediate step of 3/9's etc) . cancelled both common factors first . no working	37	37

TABLE H.5 (7.7)

Summary of adult learners' responses (re-grouped) to Q10e on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response	5	9
1	Provided unusual answers	15	8
2	Employed common denominators	9	2
3	2 . 1/2 × 4/1 = 2 . no working	26	33

TABLE H.6 (7.8)

Summary of adult learners' responses (re-grouped) to Q10f on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response	8	12
1	Provided unusual answers	8	7
2	Employed common denominators	12	2
3	6/5 or 1 1/5 . did not cancel, i.e., 18/15 . cancelled 2/3 x 9/5 . no working	27	31

TABLE H.7 (7.9)

Summary of adult learners' responses (re-grouped) to Q12 on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview for clarification	13	20
1	Focused on 'picturing' 1/6 and/or 1/4 and argued that the answer of 1/10 was wrong because it was < 1/4 (or 1/6)	6	0
2	Focused on 1/6 + 1/4, but did not obtain 5/12 and became confused by the diagram Focused on the use of common denominators to solve 1/6 + 1/4 and ignored the diagram	33	25
3	Focused on the correctness of both the diagram and the written answer.	3	7

TABLE H.8 (7.9)

Summary of adult learners' responses (re-grouped) to Q11 on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview for clarification	2	14
1	Incorrect application (used + or x)	3	4
2	Correct choice of process (subtraction)/incorrect application, e.g., obtained 1/3	7	2
3	Correct process (subtraction) and correct manipulation of fractions to obtain the correct answer (1/40)	43	32

TABLE H.9 (7.11)

Summary of adult learners' responses (re-grouped) to Q15 on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview for clarification	16	17
1	Wrote 2/3 and 4/5 on diagram (correctly)	4	2
2	Added fractions . incorrectly . correctly (22/ 5 = 1 7/15) Added 1/5 to 1/3 to obtain 8/15	11	13
3	Correct process 4/5 × 2/3 . incorrect solution . obtained 7/15 (i.e., area uncovered) . = 8/15 (correct)	24	20

TABLE H.10 (7.12)

Summary of adult learners' responses (re-grouped) to Q16 on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview for clarification	15	15
1	1½/(5/3) 3/2 ÷ 5/3	14	22
2	3/2 × 3/5 . no further working . unsuccessful solution	4	0
3	9/10 . manipulated 3/2 x 3/5 . no working	20	15

APPENDIX I

THRESHOLD VALUES FOR OPERATIONS ON FRACTIONS THEME

Q10a, Q10b, Q10c, Q10d, Q10e, Q10f (context-free) and Q11, Q12, Q15, Q16 (in-context)

Figures shown in brackets indicate approximate numbers of students who attained this level.

Question number	Context-free questions 1	2	3	Question number	In-context questions 1	2	3
10a	-2.63 (all)	-1.38 (90)	-.76 (88)				
10b	-1.27 (90)	-.69 (88)	-.42 (86)	12 (+)	.17 (88)	.36 (80)	2.80 (2)
10c	-.77 (88)	-.58 (87)	-.49 (86)	11 (-)	-.41 (97)	-.06 (91)	.17 (88)
10d	-.98 (88)	-.18 (82)	.24 (75)	15 (x)	.22 (84)	.40 (80)	.99 (42)
10e	-.78 (88)	.42 (73)	.79 (56)				
10f	-.27 (83)	.36 (73)	.81 (48)	16 (÷)	-.03 (91)	1.11 (35)	1.23 (35)

APPENDIX J

CONDENSED SUMMARY TABLE OF TABLE 8.1

The new groupings for Question 1 are shown in Table J.1.

TABLE J.1

Summary of adult learners' responses (re-grouped) to Q1 on the Fraction Quiz

Step number	RESPONSE	NUMBER OF STUDENTS AD	TP
0	No response Responses that require an interview for clarification	8	9
1	Described the look of a fraction as 'one number over another' Focused on a fraction as an object . part of a whole . part of a whole and defines a number in terms of objects such as cakes, pies, etc	15	27
2	Described a fraction as part of a whole or number Focused on a fraction as a number . part of a number . part of a number and gives an example to support this definition, e.g., 1/2	12	8
3	Listed several aspects to do with fractions such as decimals or percentages. Usually responses also stated 'not a whole number'. Fractions are numbers and related several aspects to do with fractions, e.g., $1 \div 2 = 1/2$	20	8

APPENDIX K

THRESHOLD VALUES FOR OVERVIEW OF FRACTION UNDERSTANDING

ALL QUESTIONS

Question number	Thresholds 1	2	3
1	-.84	.62	1.20
2a	-.53	.57	1.29
2b	-.16	.67	.93
3a	-.39	-.19	-.10
3b	.02	.17	1.03
4	-1.22	-.17	.61
5	-.13	.29	.95
6a	-1.03	-.36	.59
6b	-.72	.17	1.08
7	-1.28	.01	1.38
8	-.94	-.61	.74
9	-.36	.23	.99
10a	-1.38	-.97	-.67
10b	-.91	-.61	-.46
10c	-.63	-.55	-.50
10d	-.81	-.33	-.04
10e	-.75	.10	.38
10f	-.39	.07	.39
11	-.45	-.21	-.02
12	-.03	.14	2.54
13	-.23	-.07	.30
14	-.59	-.12	.05
15	.03	.17	.69
16	-.19	.80	.92