5. Some Applications of CGE Models to Policy Analysis in Developing Countries

5.1 Introduction

The discussion in Chapter 4 has shown that within the general modelling framework of CGE models there is a lot of variety. In fact, in the last two decades, there has been a proliferation of CGE models applied to developed and developing countries. The reason for this, especially in the case of the latter, has been that CGE models, given their theoretical framework, are better suited to simulate economic policies than alternative forms of modelling.

In the literature, there are both one-country and multi-country CGE models. As pointed out by Bergman (1990), models in the former class are generally intended for analysis of resource allocation and income distribution issues. This is done within a national economy faced with exogenously given world market conditions, while those in the latter category are typically aimed at elucidating the corresponding issues in a regional or global perspective. Two widely used examples of one-country models are ORANI (Dixon et. al. 1982) and the World Bank model (Dervis et. al. 1982). A popular example of the multi-country CGE models is the GTAP model (see Hertel 1997).

Bergman (1990) continues by noting that CGE models differ significantly with respect to the level of aggregation as well as to the emphasis in the representation of the modelled economy. Thus, in some models the representation of production and technical change is rather elaborate, while a disaggregated household sector and a detailed treatment of the tax and transfer system are the main features of other CGE models. This is in addition to the fact that some CGE models can be characterised as large or even very large, multi-purpose models, while another category comprises CGE models that are especially designed and used for analysis of a particular problem in a particular context.

These differences between CGE models indicate that different models are intended for a number of different specific purposes even though they all tend to share the characteristic that they are intended for some kind of numerical comparative statics analysis of changes in exogenous conditions. Since the different CGE models that have been developed so
far are so many, it would be impossible to review each of them in this study and the reader is referred to bibliographic surveys by Bandara (1991), and Decaluwe and Martens (1988). This chapter only reviews some of the recent models that have been used to analyse the effects of external shocks and adjustment policies and also the CGE models that have been constructed for the Kenyan economy.

The chapter is organised as follows. CGE models applied in the analysis of effects of external shocks and of structural adjustment and macroeconomic stabilisation policies in developing countries are reviewed in Section 5.2. Section 5.3 presents a review of CGE models that have been formulated for the Kenyan economy. Section 5.4 provides concluding remarks.

5.2 Adjustment to Macroeconomic Crisis: CGE Models Applications

Most developing countries, especially those in Africa, faced an economic crisis in the early 1980s, characterised by worsening internal and external imbalances and stagnant economic growth. These imbalances, which were created mainly by external shocks, necessitated the introduction of macroeconomic and sectoral economic reforms under structural adjustment and macroeconomic stabilisation programs.

Cornia et al. (1987) heavily criticised these adjustment programs that were forced on developing countries by the IMF and the World Bank because of their lack of focus on the poor. For researchers, the question that has begged answers has been whether the macroeconomic and sectoral policy reforms have been appropriate and effective in remedying the imbalances that led to their introduction. Another more important question has been the issue raised by Cornia et al. (1987). That is, whether these economic reforms have impacted negatively on poverty and income distribution in the developing countries where they have been implemented. Therefore, issues in adjustment and stabilisation policies have occupied centre place in a majority of CGE model applications. In this section a review of some of these studies using CGE models addressing the effects of adjustment to economic imbalances is presented. The CGE models are divided into the two theoretical paradigms discussed in Chapter 4: neoclassical models and macro models incorporating a financial sub-model.
5.2.1 The neoclassical CGE models

Tokarick (1995) is a study which uses a CGE model of the economy of Trinidad and Tobago to assess the effects of trade liberalisation and terms of trade shocks on the real exchange rate and the overall fiscal position of the government. The model is also used to evaluate the implications of alternative tax policies designed to offset the increase in the budget deficit of the central government that results from both types of external shocks.

The study was necessitated by the response of the government of Trinidad and Tobago to consequences of Dutch disease\(^1\) which came from a major expansion in the petroleum sector in the late 1970s and early 1980s. The main objective of the study was to make clear the effects of trade liberalisation (an adjustment response to Dutch disease consequences) on the overall position of the public sector in the face of uncertainty surrounding the position. The thrust of the study is that no definite conclusions can be reached a priori concerning the direction of the change in the fiscal position of the government.

To achieve the stated objective, Tokarick (1995) constructs a static CGE model. The model is designed to compute the equilibrium exchange rate that results from changes in exogenous variables and to provide an estimate of the effects of alternative trade and tax policies on the volume of imports and exports and on the surplus or deficit of the central government. The model divides the economy into three sectors: exportable, importable and nontraded or home good. The domestic production process occurs through two levels of nesting. At the first level, labour and capital are combined to produce value added on a sectoral basis according to a CES aggregation function. At the second level of nesting, value added is combined with imported intermediate inputs in fixed proportions to produce gross output. The model includes a rich array of tax instruments too. The consumer demand functions for each type of good are obtained by maximising

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\(^1\) Benjamin (1994) defines Dutch disease as that condition describing the adjustment of a country to spending foreign earnings from an export boom. The new spending drives up prices of nontraded goods relative to traded goods, which are held to world prices by competition from imports. This real exchange rate appreciation leads to contraction of the nonbooming export sector as production of nontraded goods and windfall exports becomes relatively more attractive.
individual utility subject to a budget constraint.

The model is then used to perform three broad tasks: quantify the effects of trade liberalisation on the performance of the fiscal and external sectors under different assumptions concerning the flexibility of prices of nontraded goods; quantify the effects of an adverse shift in the terms of trade and evaluate the effects of alternative policy responses to the shock; and rank the effects on efficiency of alternative tax policies designed to prevent an increase in the budget deficit following both types of external shocks. But, like many other CGE model uses, the analysis is static and does not consider how the various external shocks affect the economy over time. Moreover, the study uses a parameter calibration technique to determine the values for the unknown parameters which means that the model replicates the base year data exactly.

The results showed that a policy of trade reform by itself raises economic welfare in aggregate. The results suggested in particular the need for policy makers to pursue policies that promote price flexibility in conjunction with trade reform. It also emerged that a nominal exchange rate depreciation would be a useful complement to a policy of trade liberalisation when the prices of nontradables are inflexible. The results did however indicate that trade reforms act to worsen the deficit of the central government. Nevertheless, the results show that it is possible to offset the increase in the deficit with increases in other taxes and still generate an aggregate welfare gain.

Benjamin (1996) is a study whose purpose was to analyse the implications for income distribution of alternative structural adjustment paths in a low-income, agricultural economy. This was done by performing simulations on a general equilibrium model of Cameroon. The study examines the different options that could have been used to bring back external balance to the Cameroon economy. Some imbalances had arisen in the second half of the 1980s as a result of three external shocks that Cameroon had been subjected to: oil prices decline; fall in agricultural prices; and substantial appreciation of Cameroon’s currency relative to the US dollar. Different implications for the distribution of income of the alternative tax and expenditure policies that could generate external balance were examined.

The specific nature of the model used included: a Cobb-Douglas production function
characterisation of production in each sector with four labour types and sector specific capital that can be augmented only between periods; domestic goods are imperfect substitutes for imports in the same sector; consumers purchase a CES aggregate of imported and domestic goods based on the relative price; domestic producers respond according to a CET function that determines the rate at which domestic production can be transferred to export markets; and consumer behaviour is governed by CES utility functions for each household.

The macroeconomic closure used in the model experiments involves a fixed net flow of foreign exchange between Cameroon and the rest of the world. The numeraire of the model is the world price level hence the price of foreign exchange. This numeraire effectively fixes the nominal exchange rate but the domestic price level is allowed to vary against the world price level, implying that the real exchange rate varies. The major finding of the study was that public spending to reduce transport and marketing costs are beneficial to the poor and for economic growth, even at the expense of competing public investments.

Dorosh (1996) undertook a study based on CGE model simulations of external shocks and policy changes to shed more light on the important linkages between macroeconomic adjustment and welfare of lower-income households in Madagascar. The key features of the model are that it is a variant of the neoclassical-structuralist model of Dervis et. al. (1982). The model specifies twenty-seven production activities, highlighting the importance of rice in Madagascar's economy with three separate technologies for paddy.

Three types of labour are modelled and capital is fixed in the short-run but updated with additions of new investment net of depreciation. Eleven institutions, eight of them being household groups are specified since the model is used for looking at welfare implications for these households.

In terms of production technology, value added is a CES production function and quantities of intermediate inputs are modelled as fixed shares of the quantity of output produced. Armington aggregation for internationally traded goods and domestically produced and consumed goods is employed. In a similar fashion, CET aggregation is used to define a composite production good of export goods and goods produced for
domestic consumption. The small country assumption is also invoked implying that Madagascar is a price taker for both imports and exports. Household consumption is specified as a function of prices and incomes, using an LES formulation. The income of the households is derived from their ownership of factors of production and access to rents.

In summary, the simulations carried out address the question of how stabilisation and structural adjustment policies of the Madagascar government during the 1980s affected income distribution and the welfare of the poor. Hence key aspects of stabilisation and structural adjustment policies were undertaken through four model simulations. The first simulation examined the impact of a large increase in foreign borrowing and investment and subsequent stabilisation where foreign debt is repaid. The simulation also illustrates the effects of the investment boom of 1978-1981 and the stabilisation of 1982-1984. The second simulation models the impact of the large increase in rice imports in the early 1980s and the results were used to deduce the effects of the subsequent reduction in rice imports as part of the stabilisation effort of 1982-1984.

Simulations three and four model trade policy reform, the centrepiece of the structural adjustment effort in the late 1980s. Simulation three shows the effects of a removal of import quotas with no change in foreign capital inflows. The fourth experiment, which includes a reduction in foreign capital inflows along with the elimination of quotas, more closely simulates the historical trade liberalisation in 1988.

Together, the four simulations of macroeconomic policy changes in the 1980s suggested that stabilisation and structural adjustment policies in Madagascar did not adversely affect most of the poor, that is, the rural poor. An inflow of foreign savings benefited all household groups to some extent but the investment boom benefited the urban households most. Stabilisation measures such as a decline in foreign savings and reduced rice imports had their largest negative impact on urban households and on rice deficit rural households. Trade liberalisation improved both efficiency and equity, redistributing income away from those who had captured quota rents and boosting incentives to produce tradeable goods which are an important income source in rural areas.

Subramanian (1996) is another study undertaken to address issues of equity as a result of
price shocks and economic policies put in place to address the shocks. The study uses a CGE model of Cameroon to examine several key issues. First, the model seeks to answer the question of what was the impact on income distribution of the different components of the price shocks and of the adjustment program? Second, what alternative adjustment policies could the Cameroon government have followed and how would they have affected growth and distribution? Third, would the economy have been less vulnerable to the adverse shock in the terms of trade had the government adopted a different set of trade and investment policies during the oil boom of the 1970s?

Like Dorosh's (1996) model of Madagascar, Subramanian's (1996) model of Cameroon is a standard neoclassical CGE model in the tradition of Dervis et al. (1982). In the model, each sector employs capital and a labour input which is an aggregation of four kinds. The model determines output by a CES function. Demand for the intermediates is obtained using input-output coefficients. Labour demand is determined by marginal conditions while the sectoral capital stocks are fixed in the short run. However, it appears that this fixing of capital stocks is not adhered to since both capital stock in each sector and aggregate capital stock are treated as endogenous variables in model closure.

One difference with the traditional Dervis et al. (1982) model is that Subramanian (1996) treats the petroleum sector differently so that output is exogenously specified with capacity utilisation allowed to adjust. This treatment helps capture the fact that petroleum production does not respond in the short run to prices which are above marginal costs but is determined largely by past investment and production capacity.

On the important issue of income distribution, labour income in the model accrues to the seven categories of households which also receive transfers from firms and government. Firms on the other hand receive capital income and transfers from government, and they pay taxes and save, transferring the residual to households. Government revenue is endogenously determined through indirect taxes, tariff revenues, and corporate tax.

The trade component of the model is similar to the Dervis et al. (1982) model. Trade

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2 The Cameroon economy was hit hard in the mid-1980s by the steep fall in world prices for oil and for coffee and cocoa, its principal exports.
flows are determined by making the assumption that domestic sales and exports are imperfect substitutes, as are domestic production and imports. The small country assumption is upheld, hence firms and consumers are price takers and respond to the prices of imports and exports relative to domestic prices. Import and export prices are determined by applying the exchange rate and the relevant tariff rates to world prices.

Before outlining the simulation results, the study highlights two important limitations in the model. Firstly, because the base year of the model is 1984/85 and the key issues examined pertained to the period after 1984/85, the model could not be calibrated using dynamic simulation forward in time. Instead, a calibration using simulations backward in time was employed to validate the model. This is a limitation because investment allocation decisions in dynamic models are typically made forward in time and cannot be run in reverse. The second limitation, as outlined in the study is that, since the CGE model is a neoclassical, Walrasian one, it is ill-suited for examining situations of macroeconomic disequilibrium, as when the economy is not at full employment. The study observes that a greater degree of realism could be achieved by incorporating non-Walrasian features such as mark-up pricing or unemployment. The acceptance of this view as a limitation is debatable since confidence in such a non-Walrasian treatment would require the labour market to include institutional details such as wage formation mechanisms and indexation which are not easy for a developing country like Cameroon where data on unemployment and underemployment are not reliable.

Simulations indicate that the sectoral and multisectoral impact of the oil price shock was small because of the unresponsiveness of output to price and the weak linkages between the petroleum sector and the rest of the economy. However, due to the close tie between government revenue and petroleum earnings, the major impact of the oil price shock was macroeconomic, operating through a worsening of the government budget and the balance of payments. On the other hand, the fall in world prices for Cameroon’s agricultural exports led directly to a substantial reduction in production in this sector and a decline in the real incomes of farm households. Reduced outputs also occurred in other sectors with linkages to agriculture.

The study concludes with regard to these terms of trade shocks that the inability of Cameroon to devalue its currency due to its membership in the Communaute
Francophone Africaine (CFA) zone prevented the government pursuing a better adjustment option. A devaluation which is expansionary would have resulted in increased government revenues and an improvement in the current account balance but the government could only depend on contractionary expenditure-reducing policies to bring about equilibrium. On the question of structural adjustment and poverty, the study notes that the measure used for expenditure reduction affected rural households indirectly through lowering the quality and supply of infrastructural and agricultural services and in public investment in agriculture. The effect was found to be small even though the relatively better-off non-agricultural and urban households were affected more significantly.

Dependence on primary commodity exports that characterise most economies in sub-Saharan Africa has been identified as the main source of balance of payments crises experienced in these economies in the 1980s. During most of the 1970s when commodity prices reached historic highs, real incomes in most of these countries rose with exports. But it is the subsequent reversals in the terms of trade contributed by the fall in commodity prices that compounded the crises in balance of payments which eventually led to adoption of stabilisation and structural adjustment measures. Dorosh et al. (1996) analyse the impact of changes in external conditions and government policies on real incomes of various household groups in Niger using a CGE model. The crises in Niger that led to the stabilisation and structural adjustment measures emanated from declines in uranium earnings in the early 1980s after sharp increases in both the volume and price of uranium exports in the 1970s. The crises were worsened by reduced foreign capital inflows, drought, and adverse effects of economic fluctuations in neighbouring Nigeria (Dorosh et al. 1996).

Once again Dorosh et al. (1996) CGE model for Niger is a standard neoclassical one following Dervis et al. (1982). However, a few minor differences are worth noting. First, for a number of industrial goods and various services, two separate production technologies (for formal and informal sectors) are modelled, each producing the same commodity output. The commodity output of the corresponding formal sector activity is treated as a perfect substitute for the production of that same commodity by the informal sector. Second, unlike in most models following the parent Dervis et al. (1982) model,
Dorosh et al. (1996) models consumption of each commodity as a fixed share of total expenditure for each household group. The model is highly disaggregated both in terms of activities and factors. Twenty activities producing fourteen commodities are modelled and eight primary factors of production are included.

In a similar fashion to other neoclassical models, in all commodity markets, prices adjust to equate supply and demand. Savings determine investment. In addition to this savings-investment closure, the nominal exchange rate and foreign savings are fixed exogenously. Therefore, changes in the aggregate price index bring about movements in the real exchange rate and equilibrium in the rest of the world accounts represented by components of balance of payments. The model is solved for a sequence of static solutions by updating capital stock according to previous period net sectoral investment. Labour supply, fixed in the static model, is increased exogenously by a constant population growth rate.

The simulation results for Niger showed that the fall in uranium prices led to declining investment and savings, reducing demand for labour and real incomes of all households. The ability to devalue (which was not possible due to membership of Niger to CFA zone) would have accelerated the adjustment process and raised real incomes. The simulation results also indicated that reducing government spending had adverse consequences primarily for urban households employed by the public administration.

5.2.2 Financial CGE models

In this sub-section, a brief review of applications of the models identified by Robinson (1991) as financial CGE models is presented. The studies reviewed concern the issue of effects of macroeconomic stabilisation and structural adjustment policies in developing countries.

Bourguignon et al. (1991) and Bourguignon et al. (1992) are among the few studies that have developed a working financial CGE framework for assessing the impacts of stabilisation and adjustment policies. Bourguignon et al. (1991) and Bourguignon et al. (1992) thus went a step further by constructing a CGE model framework that can be used to undertake analysis of the likely income distribution implications of alternative
adjustment strategies for the poor.

Their framework links short-run impact of macroeconomic policies that affect the distribution of income through inflation, interest rate and other price changes, with the more often emphasised medium-run impacts of adjustment policies that affect the distribution of income through relative commodity and factor price changes. However, the simulations were for a representative economy. All the same, their study is a significant contribution to the modelling of impact of adjustment programs as indicated by some of the studies reviewed in this section.

Morrisson (1991) uses the financial CGE model developed by Bourguignon et al. (1991) to investigate the short-term effects of a stabilisation program implemented in Morocco in 1983. The short-term stabilisation program instruments investigated were a devaluation, reductions in public investment and a slower growth in domestic credit and government employment. The study also simulates the medium-term effects of alternative policies under medium-term structural adjustment measures which included liberalising trade, agriculture and financial markets.

In applying the model to Morocco, the economically active population was divided into six groups. Labour and capital are the main factors of production. Adjustment in the goods and services market occurs through prices or quantities. The simulation results showed that most of the measures applied by the government were beneficial, although the program could have been more effective had Morocco followed a slightly different set of policy options.

An important outcome of this study was that even though Morocco waited before applying a stabilisation program to deal with growing external and internal imbalances which dated as far back as 1976, the timing of the 1983 program was found to be suited to the exogenous shocks Morocco faced. In terms of the high costs associated with adjustment, the timing of the application was found to have helped keep social costs low although additional macroeconomic measures and higher direct taxes on high-income households could have reduced them even further.

Lambert et al. (1991) adapt the maquette model developed by Bourguignon et al. (1991)
to look at the impact of two critical aspects of Cote d'Ivoire experience before assessing the effect on income distribution of three adjustment measures. The model has four economic agents: firms, households, financial intermediaries and government. The firms are divided into six categories and households into six socio-economic groups. The model also identifies three types of labour.

The model is used to analyse the impact of adjustment through simulations that examine the effects of various policy measures on macro-aggregates and income distribution. The specific alternative policy measures simulated are a reduction in public wages, increase in export tax and a 20 per cent nominal devaluation. The simulations of these alternative policies suggested that a cut in current expenditures (through a reduction in public wages) reduces income inequality but is ineffective for reducing poverty. Besides not being very effective in reducing the fiscal deficit, an increase in the export tax was found to be distributionally regressive. Only the 20 per cent nominal devaluation was found to reduce both income inequality and poverty.

Demery and Demery (1991) explore the effects on Malaysia's poor of the early 1980s' recession and the mid-1980s' adjustment. They also apply a financial CGE model which differs from the ones applied by Morrison (1991) and Lambert et al. (1991) discussed above in that they derive a stable money demand equation using available Malaysian time-series data. Their strategy resulted in a financial system with only two assets, money and bonds. The money demand function was then inverted in a general equilibrium model for Malaysia to provide an expression for the nominal interest rate which was assumed to adjust to ensure money market equilibrium given an exogenous money supply.

The resulting financial CGE model identified five commodity groups and productive activities and twenty labour markets (by levels of schooling, gender, and whether agriculture or non-agriculture). Due to the model focus on income distribution, factor income was distributed to fifteen household types defined by two levels of schooling and three ethnic groups (Malay, Chinese, and other races).

The applied general equilibrium model was then used to assess three policy packages against the government's preferred package. The first package assumed an earlier
preemptive adjustment to see whether delaying the decision to accept economic and political consequences of adjustment made matters worse for poor households. The second package assumed a milder fiscal restraint, timed with the actual restraint, but a bolder devaluation of the Malaysian ringgit, to see whether it might have been possible to speed recovery without increasing adverse effects on the poor. The third assumes higher taxes to raise revenue and correct public deficits that the milder restraint of the second package could not.

The results found that only the third package comes close to achieving the actual outcome, but at the cost of hurting the poor, as happened with the first two packages. The conclusion was that the government's chosen policies for cutting and switching expenditures and devaluing the exchange rate thus did much to protect the poor. The observed slowdown in poverty alleviation in the mid-1980s was the result of the recession, not of adjustment policies. The government had few alternatives to restoring macroeconomic balances without making matters worse for the poor.

Thorbecke (1991) is another study that integrates a real and a financial sector in the fashion of Bourguignon et al. (1991) for application to the Indonesian economy. The model was designed to explore the impact of adjustment on income distribution and many other macroeconomic and sectoral variables. As in the model of Bourguignon et al. (1991), the Indonesian model integrates a general equilibrium macroeconomic model describing the real side of the economy with a more micro-oriented financial model explaining financial asset choice. A particular strength in the Indonesian model which is not in most models is the detailed treatment of government expenditures (divided into thirteen categories), which is highly desirable for any assessment of the effects of policies on income distribution. In addition, most behavioural and technical coefficients were econometrically estimated.

The model was used to explore the impact of the stabilisation and structural adjustment policy package implemented in Indonesia during 1982-1988 following the drop in oil prices on growth, income distribution, internal and external equilibrium. The study argues that the adopted policy package that included widespread structural reforms with standard stabilisation policies was successful in restoring internal and external equilibrium. The argument is supported by simulating the impact of six alternative policy
scenarios on the whole socioeconomic system and comparing them in terms of short-run and long-run effects during the adjustment phase with the adopted adjustment package. The simulations revealed that, by and large, the selected adjustment package was successful in restoring equilibrium and improving income distribution.

The results from Thorbecke (1991) have far reaching implications for developing countries. The reason for this is that the most remarkable aspect of Indonesia's policy response to the oil-crisis is that it was undertaken voluntarily, quickly and in a balanced fashion. By following conservative fiscal and monetary policies during periods of expansion and recession, Indonesia managed to avoid accumulating large external and internal imbalances. Hence, it did not require emergency stabilisation or structural adjustment loans from the IMF or World Bank. The lesson to be learned by other developing countries, especially those in Africa, is that recognising the existence of an internal or external imbalance and taking steps to rectify it pays. It also avoids a situation in which a government has to be forced into undertaking policy reforms when economic conditions do not allow.

de Janvry et al. (1991) is yet another study that uses a financial CGE model to analyse effects of adjustment policies. Like similar studies, the thrust behind this study was the need to understand how social costs of adjustment in Ecuador could be reduced. The adjustment program being analysed was a necessity in Ecuador, just as in many other developing countries. Ecuador faced a severe foreign sector crisis during the 1980s as a result of deteriorating terms of trade and the debt crisis in Latin America of late 1982. The study explores alternative approaches to stabilisation that were made necessary by both a reduction in foreign borrowing and a fall in world oil prices. The approaches that the study considers included exchange rate devaluation, fiscal austerity directed at current or investment spending and reduced growth in the money supply. Each approach was assessed for its effects on GDP growth, on rural and urban poverty, and for its political sustainability assessed by changes in utility for politically dominant rural and urban interests.

Unlike in the cases of real neoclassical-structuralist CGE models, the study by de Janvry et al. (1991) argues that because inflation, interest rates, and capital outflight play important roles in stabilisation and adjustment, they decided to build a real-financial CGE
including a detailed financial sector. The model draws its strengths from the distinctive features that it includes. Firstly, the model recognises the role of inflation, thus it incorporates two mechanisms by which inflation affects macroeconomic equilibrium. Through the investment equation, the model captures switching effects of asset holding between domestic and foreign assets in an environment of volatile and high inflation. The same equation captures the indirect negative impact on investment by inflation since inflation tends to reduce domestic funds available for loans.

Secondly, the productivity effects of private and public investment are incorporated in the model. The study notes that capital accumulation affects growth not by increasing the stock of capital but through technological changes and productivity gains embedded in capital renewal. The model therefore specifies distinct and complementary productivity effects for private and public investment in sectoral production functions. By doing so, the study emphasises the importance of public investment for the growth of productivity and for the profitability of private investment.

Thirdly, the model recognises that nominal wages are sticky in the short run, with the eventual result that actual unemployment can differ from its “natural” rate. The study therefore included a lag in wage adjustment recognising that when nominal wages are renegotiated, they take account of past inflation leading to a loss in real wages in periods of increasing inflation but to an increase in real wages during periods of falling inflation.

Apart from the above three distinctive features, the rest of the model adopts the same framework by Bourguignon et al. (1991). The results of the de Janvry et al. (1991) study showed that cutting current spending was best for restoring growth and protecting the rural poor, but was politically untenable. Making the approach politically feasible required the following: complementary economic reforms to increase the flexibility of the economy and raise the share of public investment going to agriculture. The approach also required social programs allowing some leakages in antipoverty transfers toward politically essential nonpoor; and political campaigns to increase the relative autonomy of the state, lower the social rate of discount whereby the public assesses the outcome of policy, and induce the public to compare policy outcomes against the scenario of no intervention instead of pre-crisis living standards.
Another financial CGE model application is that of Wang et al. (1995) who constructed a short-run CGE model for the Argentinean economy and used it to simulate the impacts of alternative stabilisation policies in dealing with the high inflation that plagued the Argentine economy in the period between the mid-1970s to the early 1990s. Their model gives special attention to interrelationships between important macroeconomic variables, mainly the interaction between the general price level and the exchange rate. One important limitation of their study is that their model does not take explicitly into account the relationship between inflation and/or stabilisation policies on the one hand, and capital accumulation and growth on the other.

They used their model to carry out three policy simulations. In their first simulation, they analysed the expectations management program used in Argentina during the period 1976-81. In their simulation, they examined what might have happened if the expectations management policies had been implemented with all the budget deficits removed. They found out that policies of expectations management are not recommended even under the zero-deficit hypothesis.

The second simulation was an analysis of a stabilisation plan that was used between 1985 and 1989. They maintained the assumption of zero-deficit and then froze the exchange rate for the first nine months of the stabilisation period, gave up the price and wage freeze used by the government and then regulated the nominal interest rate at ten per cent. According to their second simulation results, the Argentine economy could have performed better in this period if the government used the policies from their simulation.

Their final simulation was antedating of a stabilisation program used by the government since 1991. They had a simulation that introduced the program in 1985. The results indicated that the introduction of the program in 1985 would have been very successful in arresting inflation and eliminating a budget deficit, though it would not have been free from side effects such as money supply shortages and high interest rates.

Another study that evaluates effects of macroeconomic policies on household welfare using a financial CGE model is that by Sarris (1996) in Tanzania. The study is an important contribution in a country where efforts to reform economic policies have been discouraged on the argument that such reforms adversely affect the poor. The study
presents results based on a dynamic macro-micro general equilibrium model that is explicitly designed to trace the links from macro policies to household activities. The model used by Sarris (1996), like that of Bourguignon et al. (1991) and Bourguignon et al. (1992), explicitly incorporates real as well as financial flows and gives nominal as well as real magnitudes.

On the real side of the model, there is not much difference from the Dervis et al. (1982) neoclassical model. One difference is that due to a parallel foreign exchange market in the model, there are also two sets of import prices and export prices that are respectively determined by official and parallel exchange rates. Another difference with the Dervis et al. (1982) model is the formation of a composite “leisure” good made from leisure of skilled and unskilled household labour that is consumed by households (which are divided according to whether they are rural or urban and whether they are poor or rich). The incorporation of financial aspects in the model is quite elaborate but is not very different from the “maquette” model mentioned earlier from Bourguignon et al. (1991) and Bourguignon et al. (1992).

However there is a major weakness with this model for Tanzania. That is, the model fails to recognise that unlike Tanzania’s neighbouring economies, that is Kenya and Uganda, the Tanzanian economy was a socialist one for part of the period covered by the study. The path adopted by President Nyerere for socialisation was accompanied by nationalisation and price controls and culminated in the early 1970s with the ‘Ujamaa’ campaign of forced villagisation. This makes it questionable whether this kind of model can be applicable to the Tanzania economy. The non-existence of a market economy up to early 1980s would make the results unconvincing, especially given that the database used for the model is from the period of an almost centrally-planned economy. The model might have better represented the economy by treating it more like an economy in transition rather than a market economy where market clearing is through endogenous price movements.

The results from the study suggest that structural adjustment policies will impose the major burden of adjustment on the non-poor. Short-term declines in real income for all household groups result from the major structural adjustment policies implemented. However, declines for the poor are minimal compared to those for the non-poor. The
explanation offered for this result is that the non-poor losses largely reflect the loss of benefits obtained from rent seeking in the formerly highly regulated economy.

5.3 CGE Models Developed for the Kenyan Economy

In this section, CGE models developed so far for the Kenyan economy are reviewed. The structure, policies analysed and results of these models are discussed in detail. The models are classified according to the authors and the studies in which a particular model has been applied are reviewed within the particular model classification.

5.3.1 Dick-Gupta-Mayer-Vincent (DGMV) model

This is the earliest CGE model for the Kenyan economy and is explained in Dick et al. (1982, 1983). The model is in the class of Johansen type of CGE models used in models such as ORANI (see Dixon et. al. 1982) and MONASH (see Peter et. al. 1996) for the Australian economy. In the model, the producers are assumed to minimise the cost of producing a given activity level subject to constant returns to scale (CRTS) production functions of a three level form. At the first level a Leontief assumption of no substitution between competing commodity categories or between them and an aggregate of primary factors and non-competing imports is made. At the second level, CES functions describing substitution possibilities between imported and domestic goods of the same type are used. At the same level, there are also CRESH functions describing substitution between the primary factors, aggregate labour, capital and land. At the third level, the model has CRESH functions describing substitution prospects between labour occupations within the aggregate labour category. The model assumes that the average household maximises its utility from a consumption bundle of commodity input categories subject to CES functions describing substitution prospects in consumption between domestically produced and imported sources of the commodity category and to an aggregate budget constraint.

As in other CGE models, the solution to the constrained optimisation problem yields a system of commodity demand equations (domestic commodities, competing imports), for current production, capital creation and household consumption and a system of factor demand equations (land, labour and capital). Government demands for commodities are
simply linked to aggregate domestic consumption. Commodity export demands are a function of shifts in the world demand curve facing a particular commodity together with the world export price. A special feature is the incorporation of multi-commodity output functions in agricultural industries. This implies that producers can alter their output mix according to relative product price changes. The ensuing system of commodity supply equations is derived assuming that producers in various agricultural industries choose their output mix to maximise their revenue subject to CRETH functions describing transformation prospects between products.

The model imposes competitive pricing behaviour with equations relating prices to costs for each of the activities' current production, capital creation, importing and exporting. A system of market clearing relationships equates demands with supplies for domestic commodities, occupational labour, fixed capital and land. The model also included miscellaneous equations which (a) describe capital accumulation, rates of return and investment allocation across industries, (b) aggregate variables (for example, aggregating occupational labour demands to the workforce level), (c) allow for various indexing relationships (for example the linking of macro aggregates such as real consumption and real investment), and (d) define useful summary variables (for example the GDP).

Since the model employs a comparative static framework of the Johansen type, it is solved by first converting to linear form by logarithmic differentiation of all the non-linear equations. The first application of the model is in Dick et al. (1982). The model is used to study resource allocation and income implications for Kenya of UNCTAD's proposed indexation of agricultural commodity prices. Two indexation methods are investigated, (a) buffer stock transactions, and (b) export quota entitlements. By definition, the terms of trade gains accruing to Kenya from commodity price indexation was supposed to be the same under the buffer stock and export quota alternatives. However, the two methods produce different implications for the composition of Kenya's industrial production and hence the efficiency with which its resources are used.

The following are the key features of the macroeconomic environment in which the price shocks are assumed in Dick et al. (1982). The first feature is that of mobile capital stocks between domestic industries but with the capital stock fixed in aggregate. This characterises the adjustment period as being medium run and is achieved by exogenising
relative rates of return to capital across industries. The second feature is the constant aggregate employment with an endogenous economy-wide wage. The relative wages between different labour occupations are held constant. The absolute wage level in the economy is forced to adjust to maintain employment. The third key feature is the balance of trade equilibrium with endogenous adjustment of real domestic absorption (household consumption expenditure, investment expenditure, government expenditure). This imposes a medium run balance of trade constraint. Kenya must alter its resource allocation between traded and non-traded sectors to achieve external balance rather than run up a balance of trade surplus. The final key feature is the fixed exchange rate. This fixes the numeraire and hence the model endogenises the real exchange rate.

Given the closure rules mentioned above, the model was used to compare projections of key economic variables in Kenya from the two methods of achieving the changes in commodity price relativities. The first experiment was through the buffer stock approach: that is, the exogenous shocks generating the results were shifts in commodity export demand curves. The second experiment referred to the export quotas approach in which case the exogenous shocks generating the results involved both changes in world export prices and changes in export quantities. The results suggested that there are efficiency losses associated with the buffer stock option which reduce GDP gains below pure terms of trade gains. However, efficiency gains associated with the export quota method resulted in GDP increases above those indicated by the pure terms of trade gains.

The DGMV model has also been used in Dick et al. (1983) to study the short-run impacts of fluctuating primary commodity prices on the economy of Kenya. It provides evidence of the effects of commodity price instability on economic activity in the country. The model imposes two types of model closures. In the first closure, the real domestic absorption (aggregate real consumption, investment and government expenditure) is set exogenously and the balance of trade is allowed to be endogenous. The idea behind this closure rule is that the short-run level of domestic absorption can be maintained (by a set of government policies which are not included in the model under review) in the face of an exogenous shock such as fluctuations in world commodity prices.

In the second closure type, aggregate real absorption is determined endogenously by
imposing a balance of trade constraint so that foreign currency export receipts are equal to foreign currency import expenditures. That is, the balance of trade is equal to zero. The composition of absorption between aggregate consumption, investment and government expenditure is assumed fixed. That is, Kenya must undertake equiproportional changes in its domestic absorption components to achieve balanced trade in the face of exogenous shocks.

In each of the closures, capital and land are exogenous in order to emphasise the short-run nature of the adjustment horizon. Hence rates of return to capital and land are determined endogenously. Next, the money price of labour is set exogenously with employment demand being endogenous. This reflects the short-run slack labour market with employers being able to absorb as much labour as they wish at the given money wage.

The main simulation involves analysing the effects of a ten per cent increase in coffee and tea prices. The results are interpreted in the light of the two macroeconomic adjustment options that were open to the government as specified in the closures. The results of the first simulation imposing the first closure indicates that the output effects of the commodity price increases are confined essentially to the agricultural sector. In other words, the results suggest that by pursuing a policy of fixed domestic absorption in the face of the price instability, the government can successfully isolate the effects of the price shock to the agricultural sector. However, the study notes that the main difficulty with this strategy is that foreign exchange reserves need to be large. With the second simulation using the alternative closure, the results showed that seeking to maintain balanced trade requires changes in the real exchange rate to facilitate a switch in the industrial composition of the economy between domestic oriented and the agricultural export sector. This allows the output effects of the price shock to be spread more evenly around the domestic economy. The second option implies the need for adjustment costs in shifting resources between sectors. Consequently, this option would be rational only when the price shock is considered as representing a shift in the price trend. The domestic absorption stabilisation would seem more appropriate in the case where world commodity price movements represent genuine fluctuations about trend rather than a shift in trend.
5.3.2 Blomqvist-McMahon model

Another CGE model for the Kenyan economy is Blomqvist and McMahon (1986). The model is a simple two-sector numerical general equilibrium model of a small, open economy where prices are given internationally and follow the standard Harris-Todaro formulation. Two factors of production, capital and labour, are combined through a CES technology. The assumptions of perfect competition and profit maximisation are invoked in deriving the rates of return to labour and capital. On the demand side, there is a linear expenditure system. The model includes neither transport costs nor non-traded goods and, with the exception of the market for urban labour, all markets are competitive. The model considers the wage in the urban formal sector (manufacturing) to be determined in a non-competitive fashion, reflecting some combination of government intervention and labour union market power. The manufacturing wage is assumed to change in response to changes in the prices of agricultural and manufacturing goods. The model also attempts to separate out the effects of the various distortions and policy measures on workers and capitalists.

The model was used to provide a series of numerical illustrations of the possible welfare effects of second-best tariff policy under a variety of assumptions concerning the nature of wage rigidity, the presence of domestic tax distortions in addition to labour market distortions, and the sectoral mobility or immobility of capital. The study also provides calculations which illustrate the potential gains from eliminating the wage rigidity itself and from implementing second-best commercial policies, respectively.

In the first set of simulations, Blomqvist and McMahon (1986) took the labour market imperfection as given and investigated the effectiveness of tariff policy as a means of partially offsetting the efficiency loss from this imperfection. A variety of model specifications, resulting from varying combinations of assumptions regarding inter-sectoral capital mobility, the presence or absence of tax on the domestic production of manufactured goods and the way the manufacturing sector wage depends on the price of the agricultural and manufactured goods respectively. In the second set of simulations, the study considered the hypothetical effects of not only using tariff policy but also eliminating the labour market imperfection as a way of improving resource allocation in the economy.
The simulation results suggested a number of general observations. As summarised by Blomqvist and McMahon (1986), their results showed that whether and to what extent an import tariff is beneficial to an economy with a distortion in the form of a non-competitive manufacturing wage and urban unemployment, depends on a number of factors. The first factor is whether the urban wage depends primarily on the price of the agricultural or the manufactured good. The more strongly it depends on the price of the agricultural good, the more likely it is that an import tariff (rather than an import subsidy) is the second best policy. However, the study found that the net efficiency gains in cases where the manufacturing wage was proportional to the price of food were typically quite small and the second-best policies in those cases tended to redistribute real income in favour of capital. The second factor is whether there are other tax distortions in the economy. For example, if there is a production tax on the manufactured good, the second-best commercial policy is more likely to involve an import tariff. And the third factor is whether or not capital is intersectorally mobile. In particular, if the capital intensity in manufacturing is higher than in agriculture, and the urban wage depends strongly on the price of the agricultural good, the second-best commercial policy may involve an import subsidy if capital is intersectorally mobile even though an import tariff is called for when capital is sector specific and immobile.

5.3.3 McMahon model

McMahon (1987) is another study that uses a CGE model for the Kenyan economy. The model identifies four producing sectors: coffee and non-coffee agriculture in the rural areas and secondary and tertiary activities in the urban areas. Standard CES functions are used with the factors of production being capital and labour in the secondary and tertiary sectors, labour and land in the coffee sector and labour, land and capital in the non-coffee agricultural sector. Capital and land are assumed fixed within each period, though they can change from year to year. Labour on the other hand can only move from rural to urban areas at the end of each year but is mobile within each area in each year. The model set up is such that the various factors of production are assumed to receive their marginal revenue products. There is no unemployment except in the urban labour market and the urban wage is assumed to be tied to prices of goods that urban workers consume.
There are four income groups: coffee labourers, agricultural labourers, urban labourers and capitalists. Each group receives its marginal revenue product minus taxes plus its redistributed portion of government tax revenue. The model also assumes that capitalists do all the saving, hence all the investment. The investment function is a composite commodity function consisting of domestically produced and imported investment. The consumption equations for the four income groups are linear expenditure systems. In a similar fashion to that of investment, secondary and tertiary goods are treated as composite commodities but are imperfect substitutes for foreign goods. In the foreign sector, in addition to the imports of secondary and tertiary goods that are determined from the composite commodity functions, all coffee output is exported as is any excess of agricultural supply over domestic demand. Exports of secondary and tertiary goods are modelled to depend on the relative prices of domestically produced and foreign goods, as well as the values of the export elasticities. The balance of payments is restricted to zero.

The McMahon (1987) model includes some dynamic adjustment equations. These are the rural-urban migration equation, the investment allocation equations, the labour force growth equations and the equation determining the amount of coffee hectarage. The model was used to estimate the actual costs and benefits that Kenya realised from the price-increasing ability of an International Coffee Agreement (ICA) to which it was a party. First, it is used to compare any gains that Kenya would have received if an ICA did not exist (1973-1979) with the losses that would have been suffered due to lower prices of coffee during the years the ICA was in operation (1964-1972). Second, it was used to investigate whether the use of planting requirements by the Kenyan government was the most efficient way of meeting its obligations to the ICA.

In the first set of simulations, it is assumed that the ICA did not exist, coffee hectarage was not fixed, and the price of coffee was 100, 90, 80 or 60 per cent of its historic price. The results suggested that, unless there had been a drastic reduction in the price of coffee, in the absence of ICA, Kenya benefited very little by being party to the agreement. They also suggest that unless the agreement was completely unsuccessful in its attempt to raise prices, Kenya was not hurt very much by being a member (except, perhaps, due to lower level of coffee tree stocks that is likely to have occurred under the
The second set of simulations involved using a production tax to control coffee production rather than controlling the number of new trees planted. The results indicated that there might have been more substantial gains for Kenya from being a member of the ICA if a production tax had been used to control coffee output rather than planting restrictions.

5.3.4 Tyler and Akinboade model

Tyler and Akinboade (1992a, 1992b) have also constructed a CGE model for the Kenyan economy. To date, this is the only model that has been used extensively to look at issues affecting the Kenyan agricultural economy (see Tyler and Akinboade 1992a, Tyler and Akinboade 1992b, Akinboade 1994, Akinboade 1996 and Tyler 1997). Unlike the other models discussed above, this model departs from presenting models conventionally as a set of equations showing the means by which prices and quantities are determined. Instead it models price and value flows, resulting in a set of equations expressing the model in the transaction value (TV) form. In effect, the model simply shows a set of equations describing the ways in which prices and transaction values are determined. The value flows and prices do in fact imply quantities, thus making it quite straightforward to translate a model expressed in TV form into a conventional format of prices and quantities, and vice versa (Akinboade 1994).

Essentially the CGE model by Tyler and Akinboade (1992a, 1992b) consists of a set of structural equations linking producing sectors, factor markets, households, government and the rest of the world, with market prices as well as production, employment, consumption, savings, trade etc., being endogenously determined. There are three production activities representing the three sectors of agriculture, industry and services. Each activity is assumed to combine labour of different categories, capital stock and "operating surplus" in a CRTS Cobb-Douglas production function to produce value added. Factors are combined optimally so that their marginal value products are equated to factor prices. Value added is then combined in fixed Leontief fashion with purchased intermediate inputs to produce gross output at producer prices.
For each production sector there are four commodity accounts, namely domestic, imported, exported and composite commodities. It is assumed that domestic and imported commodities are neither perfect substitutes nor perfect complements but are imperfectly substitutable. They are combined as a CES aggregate to produce composite commodities at minimum cost. These composite commodities are then made available to meet total demand for household consumption, government consumption, intermediate use and for investment.

Factor incomes are distributed to the institutions of households, companies, the government and the rest of the world. Incomes accruing to household groups are allocated to consumption, savings, direct taxes, transfers to companies and remittance transfers to other households in fixed value shares. Total consumption expenditures is then allocated to consumption of composite commodities in fixed proportions in quantity terms. The government consumption is modelled in a similar manner. As for capital accounts, total savings in the economy by households, companies, the government and foreign savings are distributed to investment in the three sectors (by destination) in fixed value shares. Within each sector, this investment is used to purchase the three composite commodities in fixed value shares (investment by sector of origin). Foreign saving is specified as a residual.

In the rest of the world account, imports are assumed to be available in perfectly elastic supply to Kenya at fixed world prices. Exports are not assumed to face perfectly elastic demand. But the demand for exports is assumed to depend on the price of exports relative to the world market price of comparable goods.

The Tyler and Akinboade (1992a, 1992b) model is closed by endogenising product and factor market prices. Only world prices of imports and exports in foreign currency and the exchange rate are exogenous. No restrictions are placed on the balance of payments. However, in the case of the labour markets, two different closures are specified alternately in some simulations. First, the Keynesian closure assuming unlimited supply of labour at a fixed nominal wage, so that the level of employment is determined endogenously by the demand for labour, is used. Second, the neoclassical closure that assumes fixed supply of each labour category so that the market wage is determined endogenously in a situation of full employment is also used. Investment is determined
endogenously and capital stock, which is sector specific, (unlike the labour which moves across sectors) is assumed fixed.

As mentioned previously, this model has been applied in a number of studies. Tyler and Akinboade (1992a, 1992b) simulate the effects of 10 per cent devaluation, 10 per cent increased investment and 10 per cent agricultural productivity improvements on the macro economy and real incomes of the poor. For each policy simulation two specifications for the labour markets are adopted as mentioned above in the explanation of the model closure. The first assumption is unlimited supply of labour at given nominal wages and the second is that of fixed labour supplies so that wages are endogenously determined.

A major conclusion is that the results of the policy simulations depend crucially on whether labour is assumed to be available in unlimited supplies or fixed within the time period under consideration. The results supported more strongly the assumption of a Keynesian closure of the labour market for Kenya. Under this assumption, devaluation provides a significant boost to real GDP and has highly favourable effects on agricultural production, exports, the current account deficit, employment and poverty. However, the devaluation was found to be inflationary under the second assumption but having only attenuated effects on real GDP and no effect on the current account deficit. With regard to the other two simulations, the agricultural productivity improvement was found to be less affected by the different specifications of the labour market and compares favourably with the devaluation except for its smaller impact on GDP. On the other hand the increased investment policy was found to be inferior on most counts. All in all, the three policies decrease poverty, though income distribution was found to be stable.

Akinboade (1994) uses the same model to analyse the implications of the changes urged on the Kenya government by World Bank to implement policies towards more intensive land use through irrigation schemes and use of ox-driven equipment and the like. The study looks at the allocative and distributional impact of these policy changes. The CGE model is used to assess the possible impact of technical change on employment, incomes and consumption of the poorer segment in the Kenyan population. In particular, the study examines allocative as well as distributional impacts of input-specific technical change in Kenya and compares this with the impact of general dis-embodied technical
change. A benchmark equilibrium data set is generated and compared with equilibria generated in the policy experiments. In the process, the significance of adopting one or other alternative hypothesis is assessed.

The simulations carried out included a 50 per cent technical change in intermediate inputs of agricultural, industrial and service origin and a 10 per cent technical change in the economy that is not input-specific. The intermediate input specific technical efficiency changes resulted in a small improvement in income and GDP. The impact was also small on consumption. The result of input-specific technical change was mixed on labour market specification. Under the Keynesian condition, technical change in inputs of agricultural origin resulted in a small increase in labour employment; if the technical change was from inputs of service origin, it imparted a small fall in non-agricultural employment whereas technical change in agriculture from inputs of industrial origin resulted in a smaller increase in the employment of professionals. Overall, improved input specific technical efficiency in agriculture marginally boosted investment in agriculture. The conclusions that were made from the input-specific technical change are: under the conditions of unlimited labour supply, agricultural technical change emanating from inputs from the industrial sector appeared to benefit the poor more than those originating from the services; the impact of technical efficiency of industrial and service origin appeared a little more beneficial to the poor than if technical efficiency were to originate from inputs of agricultural origin; when labour supply is fixed, technical change of service origin had in general the least preferred comparable impact on the employment and consumption of the poorer segment of the society; and agricultural technical change from inputs of industrial origin appeared to be most beneficial to the poor, irrespective of the labour market condition, and should be promoted.

The general dis-embodied technical change in agriculture was found to be moderately inflationary under the condition of fixed labour supply and appeared to have a slightly lower impact across the board as compared to general efficiency change under the condition of unlimited labour supply. The general efficiency change was found to be beneficial to all households and as a potent way of improving the employment, income and consumption of the poorer segment of the society.

The other study where the Tyler and Akinboade (1992a, 1992b) model has been applied
is Akinboade (1996). The study assesses the possible impact of some agricultural policy options on the employment, income and consumption of the poorer segment of the Kenyan society. Unlike in other applications of the model, the closure in this particular study is altered in such a way that it is able to reflect a monopsonistic situation for plantation agriculture. Thus, unskilled labour earnings are assumed to be fixed, implying a positively sloped labour-supply curve that permits effective modelling of market conditions a monopsonist is likely to face.

The options examined in Akinboade (1996) included those of increasing wages of the unskilled in plantation agriculture, removing indirect taxes on agricultural commodities and increasing world market prices of agricultural exports. The simulation of raising wages in estate agriculture to attract more labour into agriculture was found to be highly beneficial in that it boosted GDP, employment and household income. As for the simulation of indirect taxes, the Kenyan economy did not respond to elimination of indirect taxes on domestic agriculture. As for the improvements in the terms of trade, the model showed these would result in increased demand for Kenyan exports leading to increased prices and a significant boost to income and GDP.

Tyler (1997) is a recent study using the Tyler and Akinboade (1992a, 1992b) model. Tyler (1997) takes further the issue of devaluation considered in Tyler and Akinboade (1992a). This is done through extending the analysis to evaluate the implications of combining devaluation with other policy measures. Thus, the CGE model is used to simulate the effects of a 10 per cent devaluation combined with a more progressive tax regime and elimination of indirect industrial taxes. The elimination of indirect taxes would be expected to have beneficial effects to the economy in two ways (Tyler 1997). First, there would be a lower price of industrial commodities needed for capital investment, allowing the maintenance of the same level of real investment. Second, there would be a reduction in price distortions. An important point is that the negative effect of lower taxes on the government’s fiscal position are addressed by introducing a more progressive direct tax structure. As in the original Tyler and Akinboade (1992a) CGE model, for each policy simulation, two different macroeconomic closure rules are applied with respect to labour market specifications. The first closure assumes abundant supplies of labour at given nominal wages and the second one assumes fixed supplies of labour so
that wages are determined endogenously.

Results in Tyler (1997) show that elimination of domestic indirect taxes in the industrial sector, as would be expected, leads to a decline in industrial market prices. Since industrial commodities are intermediate inputs in all sectors, the decrease in industrial market prices provided a boost throughout the economy. However, the results present a better outcome when there is an assumption of unlimited supply of labour. An attempt to restore the state of government finances which worsen from lower indirect taxes through higher direct taxes led to a slight dampening effect on real GDP. This, as Tyler (1997) explains is a result of lower domestic demand. When the benefits of the cost-reducing effects of the lower indirect taxes (which are not wiped out by an increase in the direct tax incidence) are combined with the export-enhancing effects of a devaluation, the outcome is very encouraging under the assumption of unlimited supply of labour. But under the assumption of fixed labour supply, the expansionary effects that are expected from a devaluation are not realised as the move is highly inflationary. The important conclusion was that elimination of indirect industrial taxes and a shift to a more progressive direct tax structure combined with a devaluation could be beneficial for the Kenyan economy and for poverty alleviation (Tyler 1997). This means that with appropriate macroeconomic policy actions, there may be no conflict between improvements in the balance of payments position and the alleviation of poverty.

5.4 Concluding Remarks

In this chapter, some applications of CGE models in addressing issues related to external shocks and the adjustment policies to deal with them have been reviewed based on the two theoretical paradigms. Moreover, the various CGE models developed for the Kenyan economy have also been reviewed. No attempt will be made here to prescribe a particular paradigm for use in CGE modelling. However, for purposes of this study, the neoclassical assumptions will be used in formulating the Kenyan model. The justification for this is the strength of established traditional microeconomic theory. However, as with most other CGE models for developing countries, introduction of structuralist rigidities should not be ruled out.
As for the approach to be adopted, a marriage of the Johansen and the World Bank styles will be used. The Johansen approach will be employed mainly in the production part. The World Bank approach as documented in Dervis et. al. (1982) will be employed in capturing income distribution issues to the different institutions, allowing for different types of households to be modelled. Moreover, the investment and trade components of the model will follow the World Bank approach.
6. Theoretical Framework for Kenyan Economy
General Equilibrium Model (KEGEM)

6.1 Introduction

This chapter presents a step-by-step formulation of the Kenyan Economy General Equilibrium Model (KEGEM). The model emphasises and derives the core equations for the Kenyan economy. KEGEM is geared and aimed at presenting a theoretically consistent framework through which external shocks and impacts of policies introduced to counter the shocks can be captured. It is designed to capture effects on resource allocation, economic growth, structural change (economic structure), international trade and income distribution of terms of trade shocks and impacts of different fiscal, exchange rate and trade policies.

The chapter is organised as follows. Section 6.2 describes the building blocks of KEGEM which include the production sectors, factors of production and institutional income groups. This is followed in Section 6.3 by a description of how production in KEGEM takes place. In this section, the production technology is discussed and the demand functions for various production inputs are derived. Section 6.4 explains the way the different institutional groups make their income which leads to product demand by households and government in Section 6.5. Institutional savings and how these savings are allocated for investment in different sectors are discussed in Section 6.6. This is followed in Section 6.7 by discussion of Kenya's trade with the rest of the world. In this section, import demand and export supply functions are derived. The price system of KEGEM is explained in Section 6.8 and the equilibrium conditions for product and factor markets are in Section 6.9. Section 6.10 lists the full equation system and variables for KEGEM. The section also discusses the macroeconomic closure adopted in implementing KEGEM. Section 6.11 provides a conclusion.

6.2 The Building Blocks of KEGEM

The theoretical structure of KEGEM derives and explains the blocks of equations that determine prices endogenously. For this reason, the various productive activities, factors
of production and institutional income groups need to be identified. These are the main building blocks in the model presented in this section.

6.2.1 The production sectors

Three production sectors that are an aggregation of activities in the 1976 Kenyan input-output tables and SAM classification are identified for KEGEM. This definition and identification of the production sectors in a way that closely resembles input-output tables and the SAM provides an opening through which KEGEM can further be disaggregated to include all sectors classified in input-output tables in future versions of the model. The three sectors in KEGEM are: agriculture—which includes cereals, temporary industrial crops, permanent crops, livestock and livestock products, forestry and fishing; manufacturing—this includes mining and quarrying, electricity and water, building and construction, food and beverages, textiles and clothing, wood and wood products, paper and printing, petroleum refining, other chemicals, non-metallic minerals, metal products and other manufacturing; and services—this includes wholesale and retail trade, restaurants and hotels, transport and communication, finance, insurance, real estate and business services, ownership of dwellings, other private services (including export, import and domestic trade), public administration, defence, education, health, agricultural services and other public services and the traditional economy.

6.2.2 Factors of production

Producers in each of the above sectors demand factors of production necessary in undertaking their domestic production. The factors of production employed in different production activities fall into two main categories. These are: the domestically produced commodity and imported commodity for each sector used as intermediate inputs; and the primary factors—capital and labour of different categories. The various labour categories are: unskilled and semi-skilled workers; skilled workers; office workers and semi-professionals; professional workers and self-employed and family workers.

In total, there are six primary factors of production. That is, capital plus the five categories of labour and there are two intermediate inputs in each sector. For the
primary factors, capital is assumed to be sector specific. Labour on the other hand is thought of as being mobile across sectors. Hence, urban unskilled labour moves freely between the manufacturing and services sector. Skilled labour and self-employed labour also moves freely across all sectors even though it can best be taken as being specific to either rural or urban areas.

6.2.3 Institutional income groups

In addition to the production activities and the factors of production, institutional income groupings are specified for the Kenyan economy. There are four major categories into which income in the economy is distributed. These are: rural households; urban households; public institutions (the government); and private institutions (different sectors). The specification of the rural and urban households, which are further subdivided into various categories aims at identifying the socioeconomic status of the citizens. Household incomes, consumption and other expenditure patterns largely depend on the socioeconomic status of the household. Therefore, inclusion of different socioeconomic groups within the country establishes how different socioeconomic groups are affected by different policies.

The rural sector in Kenya is the most important for consideration of how economic policies impact on the agricultural sector and hence income distribution issues. This is mainly because most of the people involved in agricultural production are found in rural areas. Therefore, the classification of different households within the rural sector is important and must reflect each household's ability to generate income.

The households in the rural sector are classified depending on the amount of land they own and on whether they have other sources of income; that is, non-farm income. This is consistent with 1976 Kenyan SAM which classifies rural households according to the same criteria, viz. the size of land holdings in hectares and whether households derive income additional to that from farming activities. The rural institutional income groupings therefore are: rural households with less than half a hectare of land and little additional income; rural households with half a hectare and substantial additional incomes; rural households with between half and one hectare with little additional income; rural households with half to one hectare and substantial additional incomes;
rural households with one to eight hectares; rural households with greater than eight hectares (small farms only); and other rural households.

Three Kenyan urban institutional income groupings are identified and classified depending on their level of income. These groupings are: urban low income group (income less than Kshs 6000 per annum); urban middle income group (income between Kshs 6000 and 20000 per annum); and urban high income group (income greater than Kshs 20000 per annum). For the private institutional income groupings, there are three sectoral income groups one for each of the sectors included in the model. The central and local government are aggregated to form the public (government) institutional income group. In total, fourteen institutional categories are identified in KEGEM.

In the remaining sections of this chapter the explicit form of all behavioural relationships is described. The structural specification of these relationships are, as much as possible, consistent with economic theory. Hence, functional forms employed on the production and demand sides become manageable but, in making them manageable, care is taken to avoid imposing unrealistic assumptions through the functional forms.

6.3 Production in the Real Economy

KEGEM is based on the assumptions of the neoclassical paradigm. This is because producer and consumer behaviour in the neoclassical school of thought is assumed to be the way agents in Kenya behave.

There are different procedures that can be followed in deriving the core equations of a CGE model like KEGEM. In some models, the equations abstract from specific functional forms. In others, the equations, mainly the factor and commodity demands, are derived using the optimisation techniques applied in most economics work. In the development of KEGEM, effort is made to derive respective equations from first principles through optimisation techniques as documented in Appendix 6.1.

It is important to clarify the procedure followed in formulating KEGEM. To do so, it is worth recalling neoclassical CGE model assumptions. These models assume profit
maximisation behaviour by producers, utility maximisation by consumers and that markets clear because of flexible adjustment in wages and prices.

The modelling then begins with the derivation and presentation of the basic structure of the equations for the core CGE component for the real Kenyan economy. The structure of KEGEM follows the standard neoclassical specification of the CGE model in Dervis et. al. (1982), albeit with some modifications. The most important modification, discussed below, is the formulation of the production structure in the form used in ORANI (Dixon et. al. 1977, Dixon et. al. 1982) and MONASH-MRF (Peter et. al. 1996) models of the Australian economy allowing the intermediate inputs to be differentiated by source.

6.1.1 The Kenyan economy production technology structure

Each commodity identified in KEGEM is associated with a specific production sector in the Kenyan economy. Several inputs are used to produce a commodity in any sector in the Kenyan economy. These include domestically produced and imported intermediate inputs for each sector as well as primary factors. In order to accommodate various degrees of substitution and different elasticities among the different factors of production specified for Kenya, KEGEM proposes a tree structure technology from Dixon et al. (1977), Wong (1990) and Mitra (1994) that has been applied in models such as ORANI (Dixon et. al. 1977, Dixon et. al. 1997) and MONASH-MRF (Peter et. al. 1996) for Australia. That is, a nested (hierarchical) production function denoting the technology for the different sectors in the Kenyan economy. The production technology exhibits constant returns to scale and is in three level form.

More specifically, the production technology of different sectors (activities) in the Kenyan economy can be formalised through the separable and nested gross sectoral production function given in Eq. (6.1):

\[ X_i = A_i^X \min[CES(ID_{ij}, IM_{ij}); CES(K_i, CES(L_{ij}))] \]  

(6.1)

where
\[ A_i^X = \text{efficiency parameter capturing factor productivity}, \]
\[ X_i = \text{supply of gross sectoral production in sector } i, \]
\[ ID_{ji} = \text{domestically produced intermediate commodity in sector } j \text{ used as input in sector } i \text{ to form the sector's composite intermediate commodity}, \]
\[ IM_{ji} = \text{imported intermediate commodity in sector } j \text{ used as input in sector } i \text{ to form the sector's composite intermediate commodity}, \]
\[ K_i = \text{demand for capital in sector } i \text{ and} \]
\[ L_{li} = \text{demand for labour input of category } l \text{ used in sector } i. \]

The separable nature of the production function can be elaborated further. At the top level, a composite value added input and a composite intermediate input are used in fixed proportions to the output using a Leontief production function. Composite intermediate and composite value added are the effective inputs that are used in production of gross sectoral product at this level of the tree structure. The Leontief combination of the value-added input, \( VA_i \), and composite intermediate input, \( N_{ji} \), at the first level of gross sectoral production, \( X_i \), is given in Eq. (6.2):

\[
X_i = A_i^X \min \left( \alpha_{N_{ji}}, N_{ji}; \alpha_{VA_i} VA_i \right) \quad (6.2)
\]

where \( A_i^X \) is the efficiency parameter capturing factor productivity for gross sectoral output. \( \alpha_{N_{ji}} \) and \( \alpha_{VA_i} \) are the distribution (weighting) parameters for the composite intermediate input and composite value-added factor respectively. The activity or sectoral production buys raw materials domestically, from the rest of the world and hires factor services to produce commodities. No substitutability between domestic and imported intermediate inputs and primary inputs is introduced at the first level of production as a Leontief combination of the effective inputs is used. In other words, value added taken as a whole and composite intermediate, also taken as a whole, are combined in fixed proportions without allowing substitution.
For the other two levels, a CES assumption is invoked. The second level CES nest introduces substitutability between domestically produced intermediate commodity inputs and those from the rest of the world and among the different primary factors. In other words, substitution is allowed between imported and domestic intermediate inputs of the same type and among the primary factors.

Eq. (6.3) is the CES function describing substitution between domestic intermediate commodities and imported intermediate commodities in the creation of respective units of the composite intermediate commodity input that enter the first level. Domestically produced intermediate inputs are differentiated from imported intermediate inputs by a CES function that produces the composite intermediate input, $N_{ji}$:

$$N_{ji} = CES(ID_{ji}, IM_{ji}) = A^N_{ji} \left( \alpha_{ID_{ji}} ID_{ji}^{-\rho_{ii}} + \alpha_{IM_{ji}} IM_{ji}^{-\rho_{ii}} \right)^{-1/\rho_{ii}}$$  \hspace{1cm} (6.3)

Here, $A^N_{ji}$ is the efficiency parameter for the formation of the composite intermediate input and $\rho_{ii}$ is the substitution parameter for the two intermediate inputs from different sources for each sector. $\alpha_{IM_{ji}}$ and $\alpha_{ID_{ji}}$ are the distribution parameters\(^1\) for intermediate inputs by source for a given sector $i$.

Eq. (6.4) is the formation of effective value-added input. It describes the CES functions adopted to describe substitution possibilities between composite labour and capital in the creation of a unit of composite primary factor, the value-added factor. Capital and

---

\(^1\) If the functional form of the equation describing the formation of the composite intermediate input adopted a Cobb-Douglas production technology, the distribution parameters would represent cost shares of the intermediate inputs by source. However, due to the CES production technology used in KEGEM, these weights are no longer simply cost shares but what Dixon et. al. (1992, 1997) refer to as modified cost shares. Therefore, what is referred to here as distribution parameters for inputs forming the composite intermediate input are actually modified cost shares. The same case follows for distribution parameters of the inputs forming other composites within KEGEM where CES technology is invoked.
composite labour follow a CES function to produce value added, VA, allowing for smooth substitution between factor inputs of the same commodity specification:

\[ VA_i = CES(K_i, L_i) = A_i^{VA} \left( \alpha_k K_i^{-\rho_{va}} + \alpha_L L_i^{-\rho_{va}} \right)^{-1/\rho_{va}} \]  

(6.4)

Here, \( A_i^{VA} \) is the efficiency parameter for the composite value-added input formation, \( \rho_{va} \) the substitution parameter of the inputs forming the composite value-added input. \( \alpha_k \) and \( \alpha_L \) are the distribution parameters for capital and labour in the formation of the composite value-added input respectively.

The two sets of relationships given by Eq. (6.3) and Eq. (6.4), describe how the top level variables in the overall Leontief combination are determined. The third level applies only to the effective labour input required for value-added production. This effective labour is made from five types of labour categories. As in the second level, a CES function is specified. This allows for substitution between labour from different categories in the creation of a unit of composite labour, \( L_i \):

\[ L_i = CES(L_{il}) = A_i^{L} \left( \sum_l \alpha_{il} L_{il}^{-\rho_{li}} \right)^{-1/\rho_{li}} \]  

(6.5)

The effective labour input is a CES aggregation of labour of different categories with \( l \) being the number of the labour categories and \( A_i^{L} \) capturing the efficiency parameter for the formation of effective labour. \( \alpha_{il} \) represents the distribution parameter of category \( l \) in the effective labour of sector \( i \). \( \rho_{li} \) is the substitution parameter that determines the elasticity of substitution between different labour categories. The CES combination of the five types of labour assumes a single elasticity of substitution among all labour categories (that is, constancy of pair-wise substitution elasticities). The implication of equality between all pair-wise substitution elasticities in the CES is that the elasticity of substitution between labour inputs from category 1 and 2 is assumed to be the same as that between labour input from categories 2 and 3 and so on.

It is worthwhile noting what a composite (effective) output or input is at this point. Wong (1990) explains that:
Composite goods are imaginary goods and are defined for the sole purpose of conveniently handling several "like" variables simultaneously, e.g. capital and labour in value-added. There is no actual composite value-added good nor composite intermediate good. By a similar token, the price of a composite good appears only to facilitate calculations. A composite is always a function of its (real) component variables. (Wong 1990 in Quah and Chin (1990), pp 471)

Consequently, composite intermediate and value-added inputs specified in the tree structure are specifically for ease of calculations and attempt to make the model more realistic and tractable. Similar variables, capital and labour, are conveniently handled by the composite value-added input that they form. In the same vein, domestic intermediate and imported intermediate inputs are also conveniently handled through the composite intermediate input that they form. In the case of labour, the effective labour input is formed from a convenient combination of various labour categories.

Most of the interactions occurring in the Kenyan economy are assumed to be captured by CES production functions and substitution elasticities that are specified during implementation of the model. Since the Kenyan economy is aggregated to three sectors at the empirical stage, the CES production functions are representations of what is believed to be the technical production process.

Each production sector in KEGEM is represented using this tree production function. However, particular elasticities of substitution may differ from sector to sector. Furthermore, some inputs may be zero in particular sectors, where the output from the level in the tree is equal to the other (non-zero) input. For each level of the tree there is a specific production structure. With Leontief and CES assumptions the production function for the given level of the tree in a given sector can be defined.

Since KEGEM uses an assumption of CES to represent technology in the second and third nests, it is of a rather general form. It allows for imperfect substitution while allowing inclusion of the special cases of perfect substitution (when the elasticity of substitution is infinite), Cobb-Douglas technology (when the elasticity of substitution is one), and fixed coefficients, or Leontief technology (when the elasticity of substitution is zero). In other words, at all levels of tree production particular assumptions are made about the degree of substitutability between inputs entering that level. If the elasticity of
substitution is the same at two consecutive levels of the tree these two levels may be collapsed to one level. Since the structure can accommodate mixtures of these assumptions at various levels of the tree, it is very flexible.

6.3.2 Input demands for the gross sectoral output production

Having defined and elaborated on the separable production function that represents the sectoral gross output production the next step is to derive various input demand functions. The separable nature of the production technology reduces the parameters that require explicit evaluation. It also simplifies the representation of the systems of demand equations as indicated by Dixon et al. (1992).

Before deriving the various demand functions, it is important to recall that producers in this model act as profit maximisers in perfectly competitive markets. They take factor and output prices as given and express demands for factors so as to minimise unit costs of output. In addition to assuming that producers behave as profit maximisers, they are assumed to be small so that a producer takes market prices of inputs and outputs as given.

Like most neoclassical CGE models, KEGEM assumes producers are competitive and efficient. Dixon et al. (1977) explain that the competitiveness is due to their treatment of all input and output prices as exogenously given. The efficiency is implied by the fact that for any given level of output producers select the combination of inputs which minimises their costs subject to maintaining a given output level.

Given the separable and nested production structure, the profit-maximisation or cost-minimisation problem is separable. This means that if inputs are optimally chosen at every level of production technology, without reference to any other level, the overall input composition is efficient. In other words, substitutability is allowed only in the relevant component of each level. Therefore, inputs for the first level do not substitute for inputs from the second level. This is because inputs in the first level are outputs from the second level. Moreover, for the second level, inputs that go into production of composite intermediate inputs do not substitute with inputs that make the composite
value-added commodity. This implies that imported and domestic intermediate inputs do not substitute with primary factors directly.

Therefore, the easiest way to view producer behaviour is to assume that they minimise costs of producing each of the composites entering production. This leads to a sequence of optimisation problems. For every level of production technology, the producer chooses a combination of inputs which minimises costs subject to technological constraints. Each component of the production structure can be taken as an individual production function. In other words, the producer is assumed to be making decisions about how many inputs need to be used at each level to minimise costs.

Appendix 6.1 presents the derivations for the various demand functions for the different inputs at each level of the tree production technology structure. As is clear from the foregoing, there are inputs for each of the levels which may themselves be output for the following level of production. This kind of production structure allows the Kenyan economy to be modelled without the restricting assumptions of some CGE models where only one level production technology is specified. The next step is to briefly present the various demand functions whose derivations are carried out in Appendix 6.1, Sections A6.1.1, A6.1.2 and A6.1.3.

6.3.2.1 Demand for different labour categories

Starting at the lowest level and working upwards, the producer must first decide how much of each labour category should be used to produce one unit of effective aggregate labour input. In this instance, input demand functions for each labour category required in cost-minimisation production of one effective labour input (the output in this case) need to be derived. Therefore, five labour demand functions are needed for the formation of one aggregate effective labour input. These are: demand for unskilled and semi-skilled workers; demand for skilled labour; demand for office labourers and semi-professionals; demand for professional workers; and demand for self-employed and family workers in production. These demands are the ones that are required to form one effective labour input that is combined with capital to produce the value-added input.
This sub-section shows how the demands for specific labour categories that are required to optimally form effective labour input used in producing effective value added input are derived. These are inputs required in the third level of the production technology. More formally, and invoking the separability properties of the production technology, the optimisation problem can be formulated in a more explicit form to minimise Eq. (6.6),

$$TC_{L_i} = \sum_{l=1}^{5} w_{li} L_{li}$$  \hspace{1cm} (6.6)

subject to Eq. (6.7).

$$L_i = A_i^{L} \left( \sum_{l}^{5} \alpha_{li} L_{li}^{-\rho_{li}} \right)^{-1/\rho_{li}}$$  \hspace{1cm} (6.7)

where $\alpha_{li} + \alpha_{2i} + \alpha_{3i} + \alpha_{4i} + \alpha_{5i} = 1$. $TC_{L_i}$ is the total cost of producing effective labour input in sector $i$ from the five labour categories. $w_{li}$ is the wage rate for labour category $l$ in sector $i$.

If the costs that producers in sector $i$ incur in producing one effective labour input are to be minimised then as little as possible of the different labour categories input requirements must be used. Given this problem the following Lagrangian function can be formed:

$$\Lambda_{L_i} = \sum_{l=1}^{5} w_{li} L_{li} + \lambda_i \left( L_i^{-\rho_{li}} - \left( A_i^{L} \right)^{-\rho_{li}} \sum_{l=1}^{5} \alpha_{li} L_{li}^{-\rho_{li}} \right)$$  \hspace{1cm} (6.8)

The CES total cost function for a two labour category case is derived as shown in Appendix 6.1, Section A6.1.1:

$$TC_{L_i} = \left( \frac{L_i}{A_i^{L}} \right)^{1/(1+\rho_{li})} \left( w_{li}^{\rho_{li}/(1+\rho_{li})} + \alpha_{2i}^{1/(1+\rho_{li})} w_{2i}^{\rho_{2i}/(1+\rho_{2i})} \right)^{1/(1+\rho_{li})}$$  \hspace{1cm} (6.9)
It is possible using the envelope theorem\(^2\) as explained in Silberberg (1990) to derive the constant effective sectoral labour, specific labour category demands from the total cost function (Eq. (6.9)). The demand for labour category 1 in a two category case is:

\[
L_{ii} \left( \frac{L_i}{A_i} \right)^{1/(1+p_{4})} W_{ii}^{\rho_{ii}/(1+p_{4})} \left( \alpha_{li}^{1/(1+p_{4})} W_{ii}^{\rho_{ii}/(1+p_{4})} + \alpha_{li}^{1/(1+p_{4})} W_{ii}^{\rho_{ii}/(1+p_{4})} \right)^{1/(1+p_{4})} W_{ii}^{-1/(1+p_{4})}
\]

Generalising these results for the five labour categories case for Kenya and using the symmetry properties leads to Eq. (6.11), the different labour category demand functions:

\[
L_u = \left( \frac{L_i}{A_i} \right)^{1/(1+p_{4})} \left( \sum_{i=1}^{5} \alpha_{li}^{1/(1+p_{4})} W_{ii}^{\rho_{ii}/(1+p_{4})} \right)^{1/(1+p_{4})} W_{ii}^{-1/(1+p_{4})}
\]

\[\text{6.3.2.2 Demands for capital and aggregate sectoral labour}\]

Moving to the middle level of the production process, the producer must now decide how much of primary inputs, that is capital and effective labour, must be used in production of one value-added input. Consequently, demand functions for capital and for effective aggregate labour input must be derived.

In order to derive demands for capital and effective labour used in production of the effective value-added input, producers minimise the total cost of producing the effective value-added. In a more formal manner, producers seek to minimise:

\[
TC_{VA} = P_K K_i + w_i L_i
\]

subject to

\[
VA_i = A_i^{VA} \left( \alpha_{K_i} K_i^{-\rho_{ai}} + \alpha_{L_i} L_i^{-\rho_{ai}} \right)^{-1/\rho_{ai}}
\]

---

\(^2\) As noted by Silberberg (1990), the envelope theorem is often referred to as Shephard's lemma and is an important part of the duality theory of cost and production functions.
where $TC_{VA}$ is the total cost of producing effective value added input in sector $i$. $P_k$ is the price of capital and $w$ is the aggregate sectoral price of labour (wage rate). Producers' choice of primary inputs is a cost minimisation problem whose Lagrangian function is:

$$\Lambda_{VA} = P_k K_i + w_i L_i + \lambda_{VA} \left( VA_i^{1/(1+\rho_{wi})} - \left( A_{VA}^{1/(1+\rho_{wi})} \right) \right)$$

(6.14)

As shown in Appendix 6.1 Section A6.1.2, the following constant elasticity of substitution total cost function for the value-added input (which is the output at this level), is derived:

$$TC_{VA} = \left( \frac{VA_i}{A_{VA}} \right) \left[ \alpha_{K_i}^{1/(1+\rho_{wi})} P_k^{\rho_{wi}/(1+\rho_{wi})} + \alpha_{L_i}^{1/(1+\rho_{wi})} W_i^{\rho_{wi}/(1+\rho_{wi})} \right]^{1/(1+\rho_{wi})}$$

(6.15)

Using the envelope theorem from Silberberg (1990), it is possible to derive the constant value-added primary input demands from the CES total cost function. These derivatives are shown in Appendix 6.1, Section A6.1.2. The primary input demand functions are as follows. The demand for capital is given by:

$$K_i = \left( \frac{VA_i}{A_{VA}} \right) \left[ \alpha_{K_i}^{1/(1+\rho_{wi})} P_k^{\rho_{wi}/(1+\rho_{wi})} + \alpha_{L_i}^{1/(1+\rho_{wi})} W_i^{\rho_{wi}/(1+\rho_{wi})} \right]^{1/(1+\rho_{wi})} P_k^{1/(1+\rho_{wi})}$$

(6.16)

and demand for effective labour input is:

$$L_i = \left( \frac{VA_i}{A_{VA}} \right) \left[ \alpha_{K_i}^{1/(1+\rho_{wi})} P_k^{\rho_{wi}/(1+\rho_{wi})} + \alpha_{L_i}^{1/(1+\rho_{wi})} W_i^{\rho_{wi}/(1+\rho_{wi})} \right]^{1/(1+\rho_{wi})} W_i^{1/(1+\rho_{wi})}$$

(6.17)

### 6.3.2.3 Demand for the domestic and imported intermediate inputs

Still at the middle level, the producer must choose the cost minimising combination of imported and domestically produced intermediate inputs needed in the production of the composite intermediate commodity input. Therefore, two more demand functions must be derived for each sector's demand for the imported and domestically produced
intermediate inputs needed in production of the composite intermediate commodity input.

Demand functions for domestic and imported intermediate inputs are derived in a similar fashion to that employed for the primary factor inputs above. The derivation is given in Appendix 6.1, Section A6.1.3. The resulting demand functions are a function of the composite intermediate commodity and the respective prices of the intermediate inputs. The demand function for the domestic intermediate input is:

\[
ID_{ji} = \left( \frac{N_{ji}}{A_{ji}} \right) \left( \frac{P_{dm_j}^{p_n_i/(1+p_n_i)}}{\alpha_{IM_{ji}}^{1/(1+p_n_i)}} + \frac{P_{dm_j}^{p_n_i/(1+p_n_i)}}{\alpha_{ID_{ji}}^{1/(1+p_n_i)}} \right)^{1/(1+p_n_i)} P_{dm_j}^{-1/(1+p_n_i)} (6.18)
\]

and demand for imported intermediate inputs from sector \( j \) going to sector \( i \) production is:

\[
IM_{ji} = \left( \frac{N_{ji}}{A_{ji}} \right) \left( \frac{P_{dm_j}^{p_n_i/(1+p_n_i)}}{\alpha_{IM_{ji}}^{1/(1+p_n_i)}} + \frac{P_{dm_j}^{p_n_i/(1+p_n_i)}}{\alpha_{ID_{ji}}^{1/(1+p_n_i)}} \right)^{1/(1+p_n_i)} P_{dm_j}^{-1/(1+p_n_i)} (6.19)
\]

where in both demand functions, \( P_{dm_j} \) is the price the producer pays for one unit of imported intermediate input of type \( j \) used in sector \( i \). \( P_{id_j} \) is the price paid for one unit of domestic intermediate input of type \( j \) used in sector \( i \).

### 6.3.2.4 Demand for the composite intermediate and value-added inputs

Finally, the producer is left with deciding how much composite intermediate commodity and value-added inputs are required for production of the gross sectoral output. Thus, two demand functions, one for composite intermediate input and one for the value-added input are needed. This is at the top level of the production process where producers use the composite intermediate and value-added inputs in fixed proportion to gross sectoral output. The demand function for the composite intermediate commodity \( j \) used as an input in sector \( i \):

\[
N_{ji} = \alpha_{N_j} \left( \frac{X_j}{A_j} \right) (6.20)
\]
and for value-added input:

\[ VA_i = \alpha_{VA} \left( \frac{X_i}{A_i} \right) \]  \hspace{1cm} (6.21)

6.4 Institutional Incomes

Before considering the detail of the nature of demands for each institutional group, it is necessary to show where income that is spent on consumer goods, services and intermediate commodities comes from for each group.

6.4.1 Sectoral profits from production

It is assumed for simplicity that there is one firm representing each sector. The profits by sector, \( Y_i \), are given by the aggregate sales revenues of the relevant sector, net of all costs of production. The costs of production include two intermediate inputs by source and wage payments to different labour categories. The costs also include capital costs: depreciation, \( DEP_i \); interest payments, \( INT_i \); and indirect taxes, \( ITAX_i \). Any subsidies, \( SUBS_i \), received by firms are added to aggregate sales to give actual profitability for a particular sector. The subsidy payments to producing sectors are predetermined by the government and therefore treated as exogenous in the macroeconomic closure for KEGEM. Therefore, the equation determining sectoral profitability is:

\[ Y_i = P_x X_i - \sum_j P_{mx} IM_{ji} - \sum_j P_{ld} ID_{ji} - \sum_w w_i L_i \]
\[ = - DEP_i - INT_i - ITAX_i + SUBS_i \] \hspace{1cm} (6.22)

where \( P_x \) is the producer price, a composite of the domestic (basic price) and domestic currency export price in sector \( i \) including taxes. All other variables in Eq. (6.22) are as defined in the preceding equations. The cost components are described by particular relationships. Depreciation, which is consumption of fixed capital, is:

\[ DEP_i = DEPR_i P_a K_i \] \hspace{1cm} (6.23)
where $DEPR_{i}$ is the rate of depreciation in sector $i$ and $P_{a_{i}}$ is the asset price\(^3\) in the given sector which is the average price of its components weighted by the share of good $j$ in the capital good of sector $i$:

$$P_{a_{i}} = \sum_{j} \omega_{ji} P_{q_{j}}$$

(6.24)

$P_{q_{j}}$ defined later in Equation (6.51) in Section 6.8 dealing with KEGEM's price system is the composite price of the commodity formed through Armington's aggregation of the domestic and imported commodities. The interest payments are:

$$INT_{i} = r_{i} P_{k_{i}} K_{i}$$

(6.25)

where $r_{i}$ is an exogenous sector specific rate of interest. This allows for the situation that prevailed in Kenya whereby different rates of interest prevailed for the different sectors. Indirect taxes, exclusive of import duties, are:

$$ITAX_{i} = t_{d_{i}} P_{x_{i}} X_{i}$$

(6.26)

where $t_{d_{i}}$ is the indirect tax rate for sector $i$.

Profits in Eq. (6.22) represent the returns to capital suppliers who are the firms’ owners. The assumption inherent in this formulation is that KEGEM differentiates owners of capital from owners of labour. That is, capital used for production in firms of a given sector is owned by the operators who subsequently earn the profits. These firm operators also hire labour which is provided by different households. Thus the households receive wage income. The implication then is that the main source of income for households is wages as explained in the sub-section below.

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\(^3\) As explained in Dixon et. al. (1977), an asset price refers to the cost of buying a unit of capital for a given sector. This price is different to the rental price of capital which as the name implies is the cost of using or renting a unit of capital for a given sector.
6.4.2 Household income

As explained above, KEGEM assumes that owners of capital are different to owners of labour. That is, firm operators own the capital used in production but have to hire labour services provided by different households. It is tempting to simplify KEGEM by assuming that each household category is characterised by a single type of labour that it owns and supplies. However, each household category can own and supply, say, different labour categories, albeit in different proportions from the other household categories. Hence, Kenyan households are assumed to derive income from different labour categories depending on the amount they own in each category. Households are divided into socioeconomic groups as described in the discussion of the building blocks for the model.

Each household, $h$, receives income from labour of each category $l$ depending on its proportion of ownership given by $\beta_{hi}$. That is, the ownership share of household $h$ in labour category $l$. Therefore, household income is:

$$Y_h = \sum_l \sum_i \beta_{hi} w_{il} L_{il} + NTH_h$$

(6.27)

where $NTH_h$ is exogenous net households transfers. These transfers include payments to households by government and firm operators. Ideally, firms' transfers to households are in the form of dividend payments. Hence one would expect to see the transfer payments from the firms included in a way that they would be endogenous in the model. While this would remove the possibility of underestimating household incomes it does call for information on each household equity holdings in different sectors. This kind of information is not readily available in Kenya and any attempt to model transfers between domestic institutions where origin or destination of transfers is households would have to be arbitrary, involving little more than guesswork. Hence the decision to include the transfer payments as exogenous in the model closure, in deriving household incomes.
6.4.3 Government income

The government receives its income from taxation. The government receives direct taxes on the firms and households income, indirect taxes on sectoral production and import duties. Eq. (6.28) gives the determination of government income in KEGEM:

\[
YG = \sum_{h} HTAX_{h} + \sum_{i} CTAX_{i} + \sum_{i} ITAX_{i} + \sum_{i} DUTY_{i} - \sum_{i} SUBS_{i} + NTG
\]  

(6.28)

Here, \( YG \) is government income, \( HTAX_{h} \) is direct tax paid by household group \( h \) at rate of \( t_{h} \) (as shown in Eq. (6.29)), \( CTAX_{i} \) is direct tax paid by each sector at rate of \( t_{i} \) (as given in Eq. (6.30)), \( DUTY_{i} \) is the level of import duty paid by the respective sector at rate \( m_{i} \) (Eq. (6.31)) and \( NTG \) is exogenous net transfer payments by the government.

Direct taxes paid by households are a function of household income:

\[
HTAX_{h} = t_{h}Y_{h}
\]

(6.29)

Similarly, direct taxes paid by firms are a function of profits:

\[
CTAX_{i} = t_{i}Y_{i}
\]

(6.30)

Import duties in domestic currency are:

\[
DUTY_{i} = m_{i}P_{m_{i}}M_{i}
\]

(6.31)

where \( m_{i} \) is rate of import duty (tax), \( P_{m_{i}} \) is domestic currency price of imports and \( M_{i} \) is amount of imports in sector \( i \).

6.5 Domestic Product Demand in the Real Economy

With production in the Kenyan economic system defined through various sectoral supplies and associated input demand relations, attention now turns to product demand in the real system. The sources of income for each of the institutions, that is, the households, government and firms which are the key decision-making units in the Kenyan real economy, have been defined above. Each of these institutions makes
decisions on how to spend income generated hence they determine domestic demand for different goods and services produced by different sectors.

Various categories of households, defined by different socioeconomic ranks, demand consumer goods. The government also demands consumer goods and investment goods. Firms in different sectors demand intermediate commodities and capital goods for investment (investment demand). These demands are modelled in the following subsections.

6.5.1 Product demand for household consumption

Starting with households, it follows that consumption depends on disposable income, savings and prices of goods consumed. Therefore, prior to any consumption decisions, different households first make decisions on how much income to save. These savings are taken into consideration in deriving household consumption functions.

Some early work on CGE modelling did not assume explicit utility functions in deriving household demand function but simply ensured that demand functions were homogenous of degree zero in prices and incomes (Dervis et al. 1982). The justification was that in most empirical applications estimated demand was derived from additively separable utility functions which, while simplifying the problem of parameter estimation, are limiting. The limitation is that there is a lack of specific substitution effects and goods are non-inferior.

Nevertheless, KEGEM assumes different Kenyan households seek to maximise utility from goods they consume or demand that are produced in the Kenyan economy. Hence, an attempt is made to derive household demand functions from the behavioural postulate of maximising utility.

Given substitutability between different commodities allowed by the CES assumption, one could carry the CES framework over to price substitution in household consumer

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The commodity consumed by the households in a given sector is a composite of domestically produced and imported good. In other words, households consume a composite domestic commodity.
demand. However, economic theory shows that demand functions derived in this fashion are homothetic (Silberberg 1990). This is a major limitation in the sense that a restriction is automatically imposed which requires unitary elasticities of consumption of commodities with respect to income. Given Engel’s Law suggests that income elasticities of demand for most commodities differ from one a more general framework would seem to be more believable.

A linear expenditure system is used for KEGEM. While a two-stage budgeting process would be appealing, it is plausible to start with a LES given the level of aggregation of the model. The LES employed here has the property that theoretical restrictions of classical consumer demand theory are satisfied (Craven and Haidacher 1987). More specifically, the Stone-Geary LES obtained from the constrained maximisation of the Klein-Rubin utility function is applied.

The derivation of household demand for the different commodities is given in Appendix 6.1, Section A6.1.4. The general form of the household demand functions is:

\[
C_{jh} = \theta_{jh} + \frac{\beta_{jh} \left( Y_h - HTAX_h - SV_h - \sum_j P_{qj} \theta_{jh} \right)}{P_{qj}}
\]

(6.32)

where \( C_{jh} \) is household \( h \) demand for commodity \( j \) and \( SV_h \) is household saving. These demand functions are conveniently linear in prices, income and savings hence the name linear expenditure system. The \( \theta_{jh} \) parameters are the fixed levels of consumption of each commodity. That is, the subsistence requirements for commodity \( j \), are independent of income and prices. \( \theta_{jh} \) plays the key role of allowing expenditure elasticities of demand to differ from one (Dixon et al. 1980). \( \sum_j P_{qj} \theta_{jh} \) is the subsistence level of expenditure leading to the interpretation of \( \left( Y_h - HTAX_h - SV_h - \sum_j P_{qj} \theta_{jh} \right) \) as supernumerary expenditure, or the expenditure over and above subsistence requirements.
It follows that if all $\theta_{jh}$ are zero then all commodities would have constant budget shares implying unitary income elasticities of demand. The $\beta_{jh}$ parameters are defined as marginal budget shares which can be estimated:

$$
\beta_{jh} = \eta_{jh} \left( \frac{P_{qj} C_{jh}}{Y_h - HTAX_h - SV_h} \right) 
$$

(6.33)

where $\eta_{jh}$ is the income elasticity of demand for commodity $j$ by household $h$ and $P_{qj} C_{jh} / (Y_h - HTAX_h - SV_h)$ is budget share in household consumption from base year data. That is, additional expenditure on commodity $j$ associated with an additional shilling of total expenditure. This implies that the expenditure form of household demand can be interpreted as follows. There is the subsistence part, $P_{qj} \theta_{jh}$ and a supernumerary component,

$$
\beta_{jh} \left( Y_h - HTAX_h - SV_h - \sum_j P_{qj} \theta_{jh} \right) 
$$

with the supernumerary expenditure on commodity $j$ being proportional to the total supernumerary expenditure. Any excess of total spending $Y_h - HTAX_h - SV_h$ over subsistence expenditure is directed to commodities according to their marginal budget shares, $\beta_{jh}$.

### 6.5.2 Product demand for government consumption

Unlike households, government in KEGEM does not seek to maximise utility. However, it is worth emphasising that it would be more intuitively appealing if the government consumption behaviour was not modelled as a fixed proportion framework but in a way where government optimises, an objective function. While it is possible to implement an optimisation process for government subject to its budget constraint, it will be assumed that this is not the case for the Kenyan government. Instead, it will be assumed government keeps proportion of expenditure on each commodity fixed. Hence, government demand for any commodity $j$ is:
That is, government demand for consumption of commodity \( j \), \( CG_j \), is a fixed proportion of total consumption demand \( TGC \). \( \beta_j \) is the share of commodity \( j \) in the government's total consumption. This share is zero, as indicated in the input-output tables and the SAM, for all sectors except the producers of government services in public administration, defence, education, health, agricultural services and other services which are encompassed as the services sector of KEGEM. It is important to add that fiscal policy can be analysed by taking \( TGC \) as a policy variable. It would be interesting to see how an increase in government spending corresponding to a rise in \( TGC \) affects the agricultural sector and rest of the economy.

6.6 Savings and Investment

In this section, saving equations for various institutional groups and investment demand functions are formulated. This is important since in theory the level of investment is determined by domestic savings and available savings from the rest of the world.

6.6.1 Institutional savings

To begin with, saving for households, \( SV_h \), are given as a fixed proportion of a given household income. Total savings for a given group depend on the savings rate for the given group, \( s_h \). Therefore, the household savings function is given in Eq. (6.35) as:

\[
SV_h = s_h Y_h
\]

(6.35)

Sectoral savings, \( SV_i \), are given by net after tax income and net sectoral transfers:

\[
SV_i = Y_i - CTAX_i - NTF_i
\]

(6.36)

Government savings, \( SVG \), are given as net government income after accounting for government consumption of goods and services. The savings function for the government is:
\[ SVG = YG - \sum_j P_j CG_j \quad (6.37) \]

Savings of domestic institutions are supplemented by foreign savings, \( SVR \), converted to domestic currency by the exchange rate, \( ER \), to give the total savings, \( TSAV \):

\[ TSAV = \sum_h SV_h + \sum_i SV_i + SVG + SVR \cdot ER \quad (6.38) \]

### 6.6.2 Investment demand

Commodities produced in different sectors are used for investment purposes in addition to meeting government and household consumption demands. Total investment is determined by the level of available savings.

In order to model investment demand, it is necessary to make a distinction between investment demand by sector of origin and investment demand by sector of destination. Investment demand by sector of origin, \( Z_j \), is demand for commodity \( j \) for investment in other sectors. It captures total sales for capital formation by a given sector to other sectors.

The other definition is that of investment by sector of destination which is denoted here by \( I \). It is the share of investment by sector of destination accruing to the specified sector and hence the actual investment taking place in the given sector.

The purpose at this point is to show how investment demand by sector of origin is distributed to various destination sectors. This is done in KEGEM in a manner that is common to most CGE models which is borrowed from dynamic input-output models and is explained in Dervis et. al. (1982). Firstly, investment by sector of destination is a proportion of total savings:

\[ I = \kappa_i TSAV \quad (6.39) \]

where \( \kappa_i \) is exogenous sectoral investment share of each sector in total savings.

Second, investment demand by sector of origin can be modelled as:
where \( \sigma_{ji} \) is the capital composition coefficient matrix capturing the proportion of capital stock in sector \( i \) originating from sector \( j \). The condition \( \sum_i \sigma_{ji} = 1 \) for all \( i \) needs to be satisfied in the matrix.

6.7 Kenya's Trade with the Rest of the World

6.7.1 Kenyan demand for imports from the rest of the world

As in most CGE models, imperfect substitutability between domestic and imported commodities is assumed through the composite commodity, \( Q_j \). This implicit imposition of Armington's assumptions makes it possible for imported and domestic commodities to be handled as imperfect substitutes. This avoids a situation where imported and domestic products are regarded as a single commodity, implying perfect substitutability. Perfect substitutability would be inconsistent with the observation that relative prices of imported and domestic commodities can change without causing the exclusion of either of these commodities from the Kenyan market. Moreover, Armington assumptions remove the problem of having to treat imported and domestic products as non-substitutable. Non-substitutability of imported and domestic commodities would be inconsistent with the quantity share of imported commodities in Kenya responding to changes in the import to domestic price ratio.

The composite commodity, \( Q_j \), is assumed to be a CES combination of domestically produced commodity, \( D_j \), and the commodity produced abroad, import \( M_j \). The CES combination is:

\[
Q_j = A_j^q \left( \alpha_{D_j} D_j^{-\rho_n} + \alpha_{M_j} M_j^{-\rho_n} \right)^{-1/\rho_n}
\]  

(6.41)

where \( A_j^q, \alpha_{M_j}, \alpha_{D_j} \) and \( \rho_{q_i} \) are parameters and \( M_j \) and \( D_j \) are inputs producing aggregate commodity, \( Q_j \).
Through this CES aggregation, the Armington assumption becomes implicit in the model and from this function Kenyan demand for imports is determined. It is plausible that firms in each sector choose the cost-minimising combination of domestic and imported inputs compatible with $Q_j$. By virtue of the CES technology assumption, demand for the imported commodity is derived demand as with derived demands for various inputs in production.

Letting prices for imports and domestic commodities be $P_{m_j}$ and $P_{d_j}$ respectively, derived demand is determined in a procedure similar to the profit maximising problem of the firm. The firms minimise costs (see Eq. (6.42)) of obtaining the composite commodity:

$$P_{q_j}Q_j = P_{m_j}M_j + P_{d_j}D_j$$ (6.42)

subject to CES technology given by the aggregation of the composite commodity (Eq. (6.41)). This optimisation procedure leads to the demand function for imports. This function can easily be obtained by equating the marginal rate of substitution of the imported commodity to that for the domestic commodity. The derivation of sectoral derived import demands is given in Appendix 6.1, Section A6.1.5. The representative function for import demands as used in KEGEM is:

$$\frac{M_j}{D_j} = \left( \frac{P_{d_j}(1 + td_j)}{P_{m_j}} \right)^{\sigma_{q_j}} \left( \frac{\alpha_{D_j}}{\alpha_{M_j}} \right)^{\sigma_{q_j}}$$ (6.43)

The introduction of indirect taxes assumes that the purchasers of the composite commodity pay these taxes. The parameter $\sigma_{q_j} = (1/1 + \rho_{q_j})$ is the trade-substitution elasticity, whose magnitude determines the responsiveness of demand for Kenyan imports to changes in relative price of imported commodities brought about by trade and exchange rate policy changes or other exogenous events. In other words, $\sigma_{q_j}$ explains and determines the impact of trade and exchange rate policies.

One implicit implication of the small-country assumption for Kenya is that domestic tradeable goods prices are determined largely by import prices as modelled in Eq. (6.52).
Recognition of aggregative problems and product differentiation through Armington assumptions makes it possible to drop, as has already been done, the assumption of perfect substitutability between Kenyan imports and domestically produced commodities. Armington assumptions lead to a domestic price system that is no longer rigidly linked to world prices.

Unfortunately, Armington assumptions violate the form of the small-country assumption that requires there to be equality between domestic and world prices of tradeables adjusted for tariffs. Consequently, this rigid form of the small-country assumption is 'softened' in this model. A weaker form of the assumption is taken on the import side where the implicit implication of the small-country assumption is that Kenya's terms of trade are fixed due to fixed, and hence, exogenous prices of imports and exports. In this softer form, the product differentiation introduced through Armington assumptions does not violate the small-country assumption.

Kenya is small from the import side since it imports a very small percentage of the commodities produced in the rest of the world. Invoking Armington assumptions to allow for product differentiation does not affect the small nature of Kenya as an import market and hence it is plausible to assume the exogeneity of the import prices through the assumption of an infinite elastic foreign supply curve of Kenyan imports.

6.7.2 Kenya's export supply function to the rest of the world

Since Kenya does not control any large shares of world export markets, the small-country assumption is valid. However, where a commodity is sold in the world market as a differentiated product, Kenya may not be 'small' in the market for those products. It would be unreasonable in such situations to consider export prices as being exogenous because fixed terms of trade no longer hold for this form of the small-country assumption. In such cases, the small-country assumption breaks down for Kenya and, export demand from Kenya to the rest of the world can be less than infinitely elastic as explained in Dervis et. al. (1982).

For example, this can occur if the agricultural sector is considered as producing one agricultural commodity with different sub-sector's products taken as differentiated
commodities. If agricultural exports are viewed in terms of various sub-sectoral products, Kenya would not be considered small in export markets such as tea, pyrethrum, horticultural products and coffee.

However, due to the lack of disaggregated data for specific sub-sectors, it is necessary to consider each of the sectors as producing one undifferentiated commodity destined for export. Consequently, the small-country assumption is assumed to hold for Kenya. As a result, Kenyan export prices are fixed in the world market independently of quantities exported. This means that Kenya is faced with an infinitely elastic export demand function for its exports. Unfortunately, treating Kenyan exports in this way fails to recognise product differentiation by country of origin and take them as imperfect substitutes for one another. But if Armington assumptions were to be invoked on the export side then less than infinitely elastic demands for Kenya's exports as in Taylor et al. (1980) for Brazil, Benjamin and Devarajan (1984) for Cameroon, Dick et al. (1984) for Chile, Mayer (1983) for Colombia, Taylor et al. (1984) for India, McMahon (1986) for Kenya, and Dervis et al. (1982) for Turkey would have to be conceived.

Since the products in a given sector are assumed to be homogenous and since the market shares of these sectors' products are perceived as very small, it is not unreasonable to assume that export prices faced by Kenyan producers are fixed independently of the quantities they export. Therefore, there is no problem in not recognising the existence of downward-sloping demand curves for Kenya's exports. In this regard, exports in KEGEM can be determined through an export supply function alone without a formal incorporation of an export demand function. This is a framework that has been extensively applied in CGE models adopting the World Bank model documented in Dervis et. al. (1982). In this framework, exports for a particular country are determined through an export supply function. This means that there is a ready demand for the exports supplied in the rest of the world through the infinitely elastic export demand.

To model export supply in KEGEM, it is important to begin at the point where one recognises that a commodity produced in sector $j$ can either be absorbed domestically or exported. In this model, Kenyan export supply functions are derived from an explicit optimising framework that explains the proportions in which the domestic consumption and exports of the commodity appear in sector $j$'s output. This framework entails
assuming a constant elasticity of transformation between quantity domestically sold and quantity exported as proposed in Powell and Gruen (1968). The optimising framework consists of a revenue-maximising framework that has the advantage of greater visibility of the theoretical underpinnings as explained in Dixon et. al. 1982. Thus, the export supply function is introduced by assuming that for any given level of production activity, \(X_j\), and commodity prices \(P_d\) and \(P_e\), sector \(j\) chooses outputs, \(D_j\) and \(E_j\), to maximise the revenue:

\[
P_j X_j = P_d D_j + P_e E_j
\]

subject to the CET function:

\[
X_j = B_j^X \left( \gamma_d D_j^{\phi_d} + \gamma_e E_j^{\phi_e} \right)^{1/\psi_x}
\]

Export supply is determined by a constant elasticity of transformation function between domestically consumed \(D_j\) and exported commodities \(E_j\). Export supply functions are derived in Appendix 6.1, Section A6.1.6 and are of the general form:

\[
\frac{E_i}{D_i} = \left( \frac{\gamma_d}{\gamma_e} \right)^{\psi_x} \left( \frac{P_e}{P_d (1 + td_i)} \right)^{\psi_x}
\]

where \(\gamma_d\) is the distribution parameter for the domestically consumed commodity compared to that exported. Again the indirect tax variable is introduced to capture the notion that domestic buyers of a given output pay a tax which influences the level of domestic demand. \(\psi_x\) is the elasticity of transformation between domestically consumed and exported components of domestic product and is given by \(\psi_x = 1/(\phi_x - 1)\) where \(\phi_x\) is an elasticity parameter in the CET function. With undifferentiated export products, the supply of exports from any sector is equal to total domestic production minus domestic use. Therefore, an increase in domestic price, \(P_d\), raises exports from sector \(j\) through domestic supply expansion and contracting domestic use. In this respect, it is theoretically consistent for the price elasticity of total domestic
production and price elasticity of domestic demand to determine the export supply elasticity.

6.1 The Price System in the Model

KEGEM, like most other CGE models uses several sets of prices which are outlined in this sub-section. The price system is a combination of the framework in models such as ORANI (Dixon et. al. 1982) and the one in Dervis et. al. (1982). The domestic price of a commodity produced in sector \(i\) is determined in a fashion similar to ORANI. However, KEGEM differentiates the basic (domestic) price with producer and consumer prices. The producer and consumer prices are determined as composites of the domestic price and the export price and import price respectively as in models that use the Dervis et. al. (1982) style. The prices of effective inputs formed in the nested production structure are also determined in a similar fashion to those in ORANI.

6.1.1 The domestic or basic price

The domestic or basic price of commodities, \(P_{d_i}\), is determined in the same way as in ORANI given that KEGEM follows a similar production technology structure. The key assumption in determining the domestic or basic price as in ORANI-style models is that, in equilibrium all firms make zero profit. As explained in Siriwardana (1996), the assumption of constant returns to scale in production and competitive pricing behaviour in each of the economic activities ensures that zero pure profits are earned in equilibrium. Zero pure profits in production are imposed by setting unit domestic price (i.e. the commodities basic value) equal to unit costs. The price of the domestic commodity is the sum of payments for intermediate inputs, labour services and capital payments. Therefore, domestic price for the commodities is an average price of the costs of inputs. Thus, domestic price \(P_{d_i}\) is determined through the Eq. (6.47) ensuring zero profits in production in a given sector \(i\):

\[
P_{d_i}X_i = \sum_l w_l L_{il} + \sum_j P_{m_j} IM_{ij} + \sum_j P_{d_j} ID_{ij} + P_k K_i
\]  

(6.47)
The left side of Eq. (6.47) is the basic value of the output of sector $i$ and right side is the total payment for inputs. The equality is implied by the assumption of no pure profits as explained in Dixon et. al. (1977, 1982). The implication of the basic price equation is that the basic value of good $i$ is a weighted average of the prices of the inputs to the production of $i$, the weights being the cost shares of each input. Given the domestic price of a given sector’s commodity, the purchasing price for the domestic intermediate input produced in sector $j$ used in sector $i$ can be deduced:

$$P_{sd_i} = P_{d_j} (1 + td_j)$$

(6.48)

### 6.1.2 The composite producer price

In ORANI-style models the basic values of commodities are the prices received by producers. KEGEM however differentiates the basic price from the producer price as in models that follow Dervis et. al. (1982). The producer price in KEGEM is a combination of the basic (domestic) price with the domestic currency export price. That is, the composite producer price is a weighted average of the domestic commodity and export commodity price. This relationship is obtained from Eq. (6.49).

$$P_{P_i} X_i = P_{d_i} (1 + td_i)D_i + P_{e_i} E_i$$

(6.49)

$P_{e_i}$ is the domestic currency price of Kenyan exports. The aggregate world price for commodities in category $i$ is exogenously fixed due to the small-country assumption. Therefore, $P_{e_i}$, the Kenyan shilling price of a unit of Kenyan exports from sector $i$ is:

$$P_{e_i} = \bar{P}_{e_i} \cdot ER$$

(6.50)

where $\bar{P}_{e_i}$ is the dollar price of the same category of exports in the world market. Unlike in most developing countries, for the period covered by this study, export taxes or subsidies were not an important policy instrument for the Kenyan government. Hence, Eq. (6.50) differs with most CGE models for developing countries by not incorporating an export subsidy (tax) variable.
6.8.3 The composite consumer price

A commodity sold in the domestic market is combined with an imported commodity within a constant elasticity of substitution aggregation function to determine the total supply of the composite commodity, $Q_i$. In the same way that the producer prices are determined, the domestic price is combined with the domestic currency import price to determine the composite consumer price. It follows that the price of this commodity, $P_{qi}$, is like $P_i$, a weighted average of the import price and the price of commodities sold in Kenya which can be written in the form:

$$P_{qi} = P_d_i (1 + td_i)D_i + P_m M_i$$

(6.51)

Import prices, $P_m$, are equal to world price of imports, $P_m^w$, converted into Kenyan shillings at exchange rate, $ER$, and adjusted for import taxes, $tm_i$. Hence, the price for imports in Kenya for each of the sectors is:

$$P_m = P_m^w \cdot ER(1 + tm_i)$$

(6.52)

The purchasers’ price for the imported intermediate commodity from sector $j$ used as an input in sector $i$ is given by a similar equation to that of imports, adjusted for indirect taxes, $td_j$:

$$P_{jm} = P_m^w \cdot ER(1 + tm_i + td_j)$$

(6.53)

The introduction of exchange rate and exogenous world import prices in the imports price equation allows world prices to have a determining influence on Kenyan domestic prices in Eq. (6.51).

6.8.4 Prices of effective inputs

Due to the nested and separable nature of KEGEM, there are other important prices whose origins need to be shown. These are the prices of the composites that are formed in the assumed production technology. They are the composite prices for the effective intermediate input, effective primary input and effective labour input. It is worth
remembering that these composite commodities have been assumed to involve independent decisions, that is, the separability assumption. Therefore, it is plausible that for each of these composites there is an independent price equation. The prices for these effective units are formed in a manner similar to the ORANI-style models, in particular the MONASH-MRF model (see Peter et. al. 1996). Eq. (6.54) explains the formation for the price of the composite intermediate commodity \( j \) used in the production process of sector \( i \):

\[
P_{\pi_j} N_{ji} = P_{im_j} IM_{ji} + P_{d_j} ID_{ji}
\]  
(6.54)

Eq. (6.54) implies that the price of the composite intermediate commodity \( j \) used in the production process of sector \( i \) is a cost-weighted Divisia index of individual prices of intermediate inputs by source.

The value-added price, \( P_{VA_i} \), is defined through Eq. (6.55) that captures the prices of the primary inputs:

\[
P_{VA_i} = P_{K_i} K_i + w_i L_i
\]  
(6.55)

Again, the effective primary factor price is a cost-weighted Divisia index of the rental price of capital and the average sectoral wage. The rental (unit) price of capital is endogenous and changes accordingly to clear the market for the fixed capital and is different to the price of capital goods or the price of assets used for investment in sector \( i \) defined previously in Eq. (6.24). The wage mechanism adopted for the Kenyan labour markets affect the values of equilibrium quantities for output and endogenous prices. However, in order to uphold consistency with price formation for the other composites in the production structure, the sectoral wage, \( w_i \), is determined by a weighted average of wages of different labour categories:

\[
w_i L_i = \sum_l w_l L_{li}
\]  
(6.56)
That is, the average wage given in Eq. (6.56) is a Divisia index of the wages of different labour categories. The wage rate of different occupations is derived through an indexation to the consumer price index, \( P_\pi \):

\[
w_{ri} = \varphi_i (P_\pi)^\nu
\]  

(6.57)

where \( \varphi_i \) is the wage shift parameter for category \( l \) and \( \nu \) is the wage indexation parameter which ranges between zero and one. The consumer price index is computed through a weighting formula from the price of the composite domestic commodity, \( P_{\text{di}} \), capturing both domestic and import prices:

\[
P_\pi = \sum_i \mu_i P_{\text{di}}
\]

(6.58)

where \( \mu_i \) is the share of sector \( i \) in the national economy. The consumer price index captures the differing influences that domestic and imported inputs in the different sectors have in the Kenyan economy. The producer price index is computed in KEGEM in a manner similar to the CPI as shown in Eq. (6.59):

\[
P_\theta = \sum_i \mu_i P_{\text{pi}}
\]

(6.59)

The producer price index captures both domestic and export prices.

\section*{6.9 Equilibrium in the Products and Factor Markets}

\subsection*{6.9.1 Supply demand equilibrium of commodities produced}

Being a general equilibrium model, it is important that various product markets are in equilibrium. Hence, to complete the model, equilibrium is imposed in product markets between demand and supply of domestic products. Eq. (6.60) ensures that demand equals supply for domestically produced commodities. Hence, the domestic output level for commodity \( j \) satisfy the following output equation:

\[
X_j = DUR_j \left( \sum_k C_{jk} + CG_j + Z_j \right) + \sum_i ID_{ji} + E_j
\]

(6.60)
where $DUR_j$ is the domestic use ratio as defined in Dervis et. al. (1982, pp. 253) and applied in Sadoulet and de Janvry (1995, pp. 350). This ratio represents the domestic commodity component of the composite final demands in KEGEM. Therefore, the domestic use ratio as used in KEGEM is defined in Eq. (6.61):

$$DUR_j = \frac{D_j}{M_j + D_j}$$  \hspace{1cm} (6.61)

The left hand side of Eq. (6.60) is the supply from a given sector. The right hand side captures the total demand which is made up of domestic demand of domestic output for intermediate inputs to current production; demands for households consumption; demands for government purchases; demand for inputs to capital formation; and export supply. Note that unlike in ORANI which treats imports of commodity $j$ as a distinct product from domestically produced $j$ both in intermediate and final demands, the final demands in KEGEM are composites. This explains the presence of $DUR_j$ in Eq. (6.60) defined in Eq. (6.61) which in effect captures the domestic component of the final demand composites. Domestic output in Eq. (6.60) can be simplified to domestic absorption and export supply. The domestic absorption, $D_j$, as used in the Armington aggregation and CET equations is the residual after exporting of the domestic output. Hence:

$$D_j = X_j - E_j$$ \hspace{1cm} (6.61)

which can be expressed simply as in Eq. (6.62)

$$D_j = DUR_j \left( \sum_h C_{jh} + CG_j + Z_j \right) + \sum_i ID_{ji}$$ \hspace{1cm} (6.62)

The domestic commodity that is not exported is combined with the imported commodity to form the composite commodity. The product supply of the composite domestic commodity is determined by total demand for the commodity. Total demand is the sum of all demands by households, government and other sectors for intermediate and investment use. Therefore, the market-clearing equation, ensuring equilibrium in every composite commodity's market is the usual condition:
\[ Q_j = \sum_i N_{ji} + \sum_h C_{jh} + CG_j + Z_j \] (6.63)

6.9.2 Balance of trade

The condition for a balance of trade equilibrium is formulated as part of the market equilibrium conditions. This is captured through the following domestic currency expression:

\[ BOT = \sum_i P_{e_i} E_i - \sum_i P_{m_i} M_i \] (6.64)

6.9.3 Equilibrium in the labour and capital markets

Tyler and Akinboade (1992a) found that in spite of uncertainties surrounding Kenyan labour markets, Keynesian closure can be applied to the labour market. This assumes unlimited supply of labour at a fixed nominal wage so that the level of employment is determined endogenously by demand for labour. The equilibrium condition for labour in category \( l \) is:

\[ \sum_i L_i = L_i \] (6.65)

It is assumed that capital is fixed and is sector specific such that the equilibrium condition for capital stock in each sector is:

\[ K_i = \bar{K}_i \] (6.66)

6.10 The Complete Model and Macroeconomic Closure

Tables 6.1 and 6.2 list the full equations system and variables for KEGEM. The coefficients in the model are listed in Table 6.3. Since in KEGEM the number of variables exceeds the number of equations, it is necessary to have some predetermined variables to solve the equations. This involves determining the closure of the model, or as Harrison and Pearson (1994) call it, the economic environment in which the simulations are carried out. A closure of a particular model is chosen when the partition
(split) of the set of variables into exogenous and endogenous is determined. For the closure to be valid, the number of endogenous variables must equal the number of equations and the exogenous variables must be independent. Table 6.4 lists the variables treated as exogenous in the macroeconomic closure.

In closing KEGEM, product and factor prices are endogenously determined and world prices of exports and imports are exogenous. There is no restriction on the balance of trade which is also endogenous in the model. The different labour types are assumed to be available in unlimited supplies with wages for different labour categories indexed to the consumer price index with indexation ranging between zero and one. For example, when the wage indexation parameter is one an unlimited supply of labour is available at fixed real wages. As noted in Chapter 8, 30 per cent wage indexation was employed in the simulation results reported in Chapters 8, 9 and 10.

Total savings (from domestic and foreign sources) are distributed in fixed value shares to investment in the three production sectors. Investment is endogenously determined and adjusts to the level of savings. Foreign savings, unlike domestic savings, are exogenously determined. There are no direct connections between investment and capital stock and capital stock is exogenous and sector specific unlike for the labour categories where labour shifts between categories. In other words, sectoral investment is endogenous, and does not augment capital stocks. This means that all the results from the simulations using the model are short run. The exchange rate is also exogenous. All the tax parameters for direct, indirect and import taxes are exogenous. The tax parameters are the fiscal and trade policy instruments available to government. Government total consumption of goods and services is also exogenously determined and is treated as a fiscal policy instrument. The level of subsidies in each of the producing sectors is also exogenously determined. This closure captures the assumptions underlying all simulations undertaken using KEGEM.
Table 6.1: Equation System for Real KEGEM

<table>
<thead>
<tr>
<th>Identifier</th>
<th>Subscript Range</th>
<th>No. Description</th>
</tr>
</thead>
<tbody>
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<td>Inputs demand</td>
<td></td>
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</tr>
<tr>
<td>(6.11) $L_{ii} = \left( \frac{L_i}{A_i} \right) \left( \sum_{i=1}^{5} \alpha_{ii}^{\frac{1}{1+p_{ii}}} w_{ii}^{\frac{1}{1+p_{ii}}} \right)^{-1} \alpha_{ii}^{\frac{1}{1+p_{ii}}} w_{ii}^{\frac{1}{1+p_{ii}}} \right)^{-1} \frac{1}{1+p_{ii}}$</td>
<td>$i = 1,\ldots,3; l = 1,\ldots,5$</td>
<td>15 Demand for labour categories</td>
</tr>
<tr>
<td>(6.16) $K_i = \left( \frac{VA_i}{A_i} \right) \left( \sum_{i=1}^{3} \alpha_{ii}^{\frac{1}{1+p_{ii}}} k_{ii}^{\frac{1}{1+p_{ii}}} + \alpha_{ii}^{\frac{1}{1+p_{ii}}} w_{ii}^{\frac{1}{1+p_{ii}}} \right)^{-1} \alpha_{ii}^{\frac{1}{1+p_{ii}}} k_{ii}^{\frac{1}{1+p_{ii}}} + \alpha_{ii}^{\frac{1}{1+p_{ii}}} w_{ii}^{\frac{1}{1+p_{ii}}} \right)^{-1} \frac{1}{1+p_{ii}}$</td>
<td>$i = 1,\ldots,3$</td>
<td>3 Demand for capital</td>
</tr>
<tr>
<td>(6.17) $L_i = \left( \frac{VA_i}{A_i} \right) \left( \sum_{i=1}^{3} \alpha_{ii}^{\frac{1}{1+p_{ii}}} k_{ii}^{\frac{1}{1+p_{ii}}} + \alpha_{ii}^{\frac{1}{1+p_{ii}}} w_{ii}^{\frac{1}{1+p_{ii}}} \right)^{-1} \alpha_{ii}^{\frac{1}{1+p_{ii}}} k_{ii}^{\frac{1}{1+p_{ii}}} + \alpha_{ii}^{\frac{1}{1+p_{ii}}} w_{ii}^{\frac{1}{1+p_{ii}}} \right)^{-1} \frac{1}{1+p_{ii}}$</td>
<td>$i = 1,\ldots,3$</td>
<td>3 Demand for composite labour</td>
</tr>
<tr>
<td>(6.18) $ID_{ji} = \left( \frac{N_{ji}}{A_{ji}} \right) \left( \sum_{i=1}^{3} \alpha_{ji}^{\frac{1}{1+p_{ji}}} k_{ji}^{\frac{1}{1+p_{ji}}} + \alpha_{ji}^{\frac{1}{1+p_{ji}}} w_{ji}^{\frac{1}{1+p_{ji}}} \right)^{-1} \alpha_{ji}^{\frac{1}{1+p_{ji}}} k_{ji}^{\frac{1}{1+p_{ji}}} + \alpha_{ji}^{\frac{1}{1+p_{ji}}} w_{ji}^{\frac{1}{1+p_{ji}}} \right)^{-1} \frac{1}{1+p_{ji}}$</td>
<td>$i = 1,\ldots,3; j = 1,\ldots,3$</td>
<td>9 Domestic intermediate demand</td>
</tr>
<tr>
<td>(6.19) $IM_{ji} = \left( \frac{N_{ji}}{A_{ji}} \right) \left( \sum_{i=1}^{3} \alpha_{ji}^{\frac{1}{1+p_{ji}}} k_{ji}^{\frac{1}{1+p_{ji}}} + \alpha_{ji}^{\frac{1}{1+p_{ji}}} w_{ji}^{\frac{1}{1+p_{ji}}} \right)^{-1} \alpha_{ji}^{\frac{1}{1+p_{ji}}} k_{ji}^{\frac{1}{1+p_{ji}}} + \alpha_{ji}^{\frac{1}{1+p_{ji}}} w_{ji}^{\frac{1}{1+p_{ji}}} \right)^{-1} \frac{1}{1+p_{ji}}$</td>
<td>$i = 1,\ldots,3; j = 1,\ldots,3$</td>
<td>9 Imported intermediate demand</td>
</tr>
<tr>
<td>(6.20) $N_{ji} = \alpha_{N_{ji}} \left( \frac{X_i}{A_i} \right)$</td>
<td>$i = 1,\ldots,3; j = 1,\ldots,3$</td>
<td>9 Composite intermediate demand</td>
</tr>
<tr>
<td>(6.21) $VA_i = \alpha_{VA_i} \left( \frac{X_i}{A_i} \right)$</td>
<td>$i = 1,\ldots,3$</td>
<td>3 Composite value-added demand</td>
</tr>
<tr>
<td>Institutions income</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6.22) $Y_i = P_{x_i} X_i - \sum_j P_{im_{ji}} IM_{ji} - \sum_j P_{id_{ji}} ID_{ji} - \sum_l \omega_{lL_{ii}} -$</td>
<td>$i = 1,\ldots,3$</td>
<td>3 Sectoral profits</td>
</tr>
<tr>
<td>(6.23) $DEP_i = DEPR_i P_{a_i} K_i$</td>
<td>$i = 1,\ldots,3$</td>
<td>3 Consumption of fixed capital</td>
</tr>
<tr>
<td>(6.24) $P_{a_i} = \sum_j \sigma_{ji} P_{q_j}$</td>
<td>$i = 1,\ldots,3; j = 1,\ldots,3$</td>
<td>3 Capital assets price</td>
</tr>
</tbody>
</table>
Table 6.1 (continued)

<table>
<thead>
<tr>
<th>Identifier</th>
<th>Subscript Range</th>
<th>No.</th>
<th>Description</th>
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<tbody>
<tr>
<td>(6.25) INT&lt;sub&gt;i&lt;/sub&gt; = ( r_i ) ( P_{h} K_i )</td>
<td>( i = 1,\ldots,3 )</td>
<td>3</td>
<td>Interest payments by sector</td>
</tr>
<tr>
<td>(6.26) ITAX&lt;sub&gt;i&lt;/sub&gt; = ( t_d ) ( P_{h} X_i )</td>
<td>( i = 1,\ldots,3 )</td>
<td>3</td>
<td>Sectoral indirect taxes</td>
</tr>
<tr>
<td>(6.27) ( Y_h = \sum_i \sum_w \beta_{hi} w_i I_{w} + NTH_h )</td>
<td>( h = 1,\ldots,10 )</td>
<td>10</td>
<td>Households income by category</td>
</tr>
<tr>
<td>(6.28) ( YG = \sum_h HTAX_h + \sum_i CTAX_i + \sum_i ITAX_i + \sum_i DUTY_i - \sum_i SUBS_i + NTG )</td>
<td></td>
<td>1</td>
<td>Government's income</td>
</tr>
<tr>
<td>(6.29) ( HTAX_h = t_h Y_h )</td>
<td>( h = 1,\ldots,10 )</td>
<td>10</td>
<td>Households direct taxes</td>
</tr>
<tr>
<td>(6.30) ( CTAX_i = t_i Y_i )</td>
<td>( i = 1,\ldots,3 )</td>
<td>3</td>
<td>Sectoral direct taxes</td>
</tr>
<tr>
<td>(6.31) ( DUTY_i = t_m P_{m} M_i )</td>
<td>( i = 1,\ldots,3 )</td>
<td>3</td>
<td>Import duty</td>
</tr>
</tbody>
</table>

**Product demand for consumption**

\[
C_{ih} = \theta_{ih} + \frac{\beta_{ih} \left( Y_h - HTAX_h - SV_h - \sum_i P_{hi} \theta_{ih} \right)}{P_{hi}}
\]

(6.32) \( h = 1,\ldots,10; i = 1,\ldots,3 \) | 30 | Household consumption |

(6.34) \( C_{ig} = \beta_i TGC \) | \( i = 1,\ldots,3 \) | 3 | Government consumption |

**Institutional savings and investment demand**

(6.35) \( SV_h = s_h Y_h \) | \( h = 1,\ldots,10 \) | 10 | Households savings by category |
| (6.36) \( SV_i = Y_i - CTAX_i - NTF_i \) | \( i = 1,\ldots,3 \) | 3 | Sectoral savings |
| (6.37) \( SVG = YG - TGC \) | | 1 | Government savings |
| (6.38) \( TSAV = \sum_h SV_h + \sum_i SV_i + SVG + SVR \cdot ER \) | | 1 | Total savings |
| (6.39) \( I_i = \kappa_i TSAV \) | \( i = 1,\ldots,3 \) | 3 | Investment by destination sector |
| (6.40) \( Z_j = \sum_i \sigma_{ji} I_i \) | \( j = 1,\ldots,3 \) | 3 | Investment by sector of origin |
Table 6.1 (continued)

<table>
<thead>
<tr>
<th>Identifier</th>
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<th>No.</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>Kenya’s trade with the rest of the world</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6.43) $M_i = \left( \frac{P_{di}}{P_{m_i}} \right)^{\gamma_{M_i}} \left( \frac{\alpha_{D_i}}{\alpha_{M_i}} \right)^{\gamma_{\alpha_{D_i}}} D_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Import demand</td>
</tr>
<tr>
<td>(6.46) $E_i = \left( \frac{\gamma_{d_i}}{\gamma_{e_i}} \right)^{\psi_{e_i}} \left( \frac{P_{e_i}}{P_d (1+t_{d_i})} \right)^{\psi_{d_i}} D_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Export supply</td>
</tr>
<tr>
<td>Price system</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(6.47) $P_{di} X_i = \sum_j w_{ij} L_{ji} + \sum_j P_{im_j} IM_j i + \sum_j P_{id_j} ID_j i + P_{ki} K_i$</td>
<td>$i=1,...,3$</td>
<td>3</td>
<td>Zero profit for production</td>
</tr>
<tr>
<td>(6.48) $P_{id_j} = P_{d_j} (1+t_{d_j})$</td>
<td>$i=1,...,3; j=1,...3$</td>
<td>9</td>
<td>Purchasers price for dom. inter.</td>
</tr>
<tr>
<td>(6.49) $P_{xi} X_i = P_{d_i} (1+t_{d_i}) D_i + P_e E_i$</td>
<td>$i=1,...,3$</td>
<td>3</td>
<td>Composite producer price</td>
</tr>
<tr>
<td>(6.50) $P_{e} = \text{ER}$</td>
<td>$i=1,...,3$</td>
<td>3</td>
<td>Export price</td>
</tr>
<tr>
<td>(6.51) $P_{q_j} Q_i = P_{d_j} (1+t_{d_j}) D_i + P_{m_j} M_j$</td>
<td>$i=1,...,3$</td>
<td>3</td>
<td>Composite consumer price</td>
</tr>
<tr>
<td>(6.52) $P_{m_i} = \text{ER} \left( 1 + t_{m_i} \right)$</td>
<td>$i=1,...,3$</td>
<td>3</td>
<td>Import price</td>
</tr>
<tr>
<td>(6.53) $P_{m_j} = \text{ER} \left( 1 + t_{m_j} + t_{d_j} \right)$</td>
<td>$i=1,...,3; j=1,...3$</td>
<td>9</td>
<td>Purchasers price for imp. inter.</td>
</tr>
<tr>
<td>(6.54) $P_{N_j} N_j = P_{m_j} IM_j i + P_{id_j} ID_j i$</td>
<td>$i=1,...,3; j=1,...3$</td>
<td>9</td>
<td>Composite intermediate price</td>
</tr>
<tr>
<td>(6.55) $PVA_i VA_i = P_k K_i + w_i L_i$</td>
<td>$i=1,...,3$</td>
<td>3</td>
<td>Composite value added price</td>
</tr>
<tr>
<td>(6.56) $w_i L_i = \sum_{j} w_{ij} L_{ji}$</td>
<td>$i=1,...,3$</td>
<td>3</td>
<td>Composite sectoral wage</td>
</tr>
<tr>
<td>(6.57) $w_{ji} = \varphi_{i} (P_{x})^{\psi_{x}}$</td>
<td>$i=1,...,3; l=1,...5$</td>
<td>15</td>
<td>Wage indexation</td>
</tr>
<tr>
<td>(6.58) $P_{x} = \sum_{i} \mu_i P_{xi}$</td>
<td></td>
<td>1</td>
<td>Consumer price index</td>
</tr>
<tr>
<td>(6.59) $P_{\theta} = \sum_{i} \mu_i P_{xi}$</td>
<td></td>
<td>1</td>
<td>Producer price index</td>
</tr>
</tbody>
</table>
### Market clearing conditions

<table>
<thead>
<tr>
<th>Identifier</th>
<th>Subscript Range</th>
<th>No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(6.60) $X_j = DUR_j \left( \sum_h C_{jh} + CG_j + Z_j \right) + \sum_i ID_{ji} + E_j$</td>
<td>$j = 1, \ldots, 3$</td>
<td>3</td>
<td>Equilibrium for domestic output</td>
</tr>
<tr>
<td>(6.61) $DUR_j = \frac{D_j}{M_j + D_j}$</td>
<td>$j = 1, \ldots, 3$</td>
<td>3</td>
<td>Domestic use ratio</td>
</tr>
<tr>
<td>(6.62) $D_j = DUR_j \left( \sum_h C_{jh} + CG_j + Z_j \right) + \sum_i ID_{ji}$</td>
<td>$j = 1, \ldots, 3$</td>
<td>3</td>
<td>Domestic absorption of prodn.</td>
</tr>
<tr>
<td>(6.63) $Q_j = \sum_i N_{ji} + \sum_h C_{jh} + CG_j + Z_j$</td>
<td>$j = 1, \ldots, 3$</td>
<td>3</td>
<td>Dem. for composite commodity</td>
</tr>
<tr>
<td>(6.64) $BOT = \sum_i P_i E_i - \sum_i P_i M_i$</td>
<td></td>
<td>1</td>
<td>Trade balance in dom. currency</td>
</tr>
<tr>
<td>(6.65) $\sum L_{hl} = L_l$</td>
<td>$l = 1, \ldots, 5$</td>
<td>5</td>
<td>Labour demand = labour supply</td>
</tr>
<tr>
<td>(6.66) $K_i = K_i$</td>
<td>$i = 1, \ldots, 3$</td>
<td>3</td>
<td>Capital demand = capital stock</td>
</tr>
</tbody>
</table>
## Table 6.2: Variables of KEDEM

<table>
<thead>
<tr>
<th>Variable</th>
<th>Subscript range</th>
<th>No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Gross output for a given sector $i$.</td>
</tr>
<tr>
<td>$N_{ji}$</td>
<td>$i = 1,...,3; j = 1,...,3$</td>
<td>9</td>
<td>Composite intermediate commodity from sector $j$ to sector $i$.</td>
</tr>
<tr>
<td>$VA_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Value-added composite in sector $i$.</td>
</tr>
<tr>
<td>$L_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Composite labour in sector $i$.</td>
</tr>
<tr>
<td>$ID_{ji}$</td>
<td>$i = 1,...,3; j = 1,...,3$</td>
<td>9</td>
<td>Domestic intermediate input from sector $j$ to sector $i$.</td>
</tr>
<tr>
<td>$IM_{ji}$</td>
<td>$i = 1,...,3; j = 1,...,3$</td>
<td>9</td>
<td>Imported intermediate input from sector $j$ to sector $i$.</td>
</tr>
<tr>
<td>$K_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Demand for capital by sector $i$.</td>
</tr>
<tr>
<td>$L_{li}$</td>
<td>$l = 1,...,5; i = 1,...,3$</td>
<td>15</td>
<td>Labour of category $l$ in sector $i$.</td>
</tr>
<tr>
<td>$L_l$</td>
<td>$l = 1,...,5$</td>
<td>5</td>
<td>Supply of labour by category $l$.</td>
</tr>
<tr>
<td>$Y_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Sectoral net income for sector $i$.</td>
</tr>
<tr>
<td>$Y_h$</td>
<td>$h = 1,...,10$</td>
<td>10</td>
<td>Income of household $h$.</td>
</tr>
<tr>
<td>$CG_j$</td>
<td>$j = 1,...,3$</td>
<td>3</td>
<td>Government’s demand for consumption of commodity from sector $i$.</td>
</tr>
<tr>
<td>$Q_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Composite commodity for use in the domestic economy sector $i$.</td>
</tr>
<tr>
<td>$M_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Imports by sector $i$.</td>
</tr>
<tr>
<td>$E_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Exports supply by sector $i$.</td>
</tr>
<tr>
<td>$P_{di}$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Price of domestic commodity in sector $i$.</td>
</tr>
<tr>
<td>$P_{mi}$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Import price in Kenyan shillings in sector $i$.</td>
</tr>
<tr>
<td>$P_{di}$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Price of composite domestic commodity in sector $i$.</td>
</tr>
<tr>
<td>$PN_{ji}$</td>
<td>$i = 1,...,3; j = 1,...,3$</td>
<td>9</td>
<td>Price of composite intermediate input type $j$ used in sector $i$.</td>
</tr>
<tr>
<td>$P_{di}$</td>
<td>$i = 1,...,3; j = 1,...,3$</td>
<td>9</td>
<td>Price of intermediate domestic input type $j$ used in sector $i$.</td>
</tr>
<tr>
<td>$P_{imj}$</td>
<td>$i = 1,...,3; j = 1,...,3$</td>
<td>9</td>
<td>Price of intermediate imported input type $j$ used in sector $i$.</td>
</tr>
<tr>
<td>$PVA_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Price of composite value added input in sector $i$.</td>
</tr>
<tr>
<td>$P_a$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Price of assets in sector $i$.</td>
</tr>
<tr>
<td>$P_c$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Rental price of capital in sector $i$.</td>
</tr>
<tr>
<td>$w_l$</td>
<td>$l = 1,...,5; i = 1,...,3$</td>
<td>15</td>
<td>Wage rate of labour category $l$ in sector $i$.</td>
</tr>
<tr>
<td>$P_e$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Export price in Kenyan shillings in sector $i$.</td>
</tr>
<tr>
<td>$P_s$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Producer price in sector $i$.</td>
</tr>
<tr>
<td>$P_n$</td>
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<td>Consumer price index.</td>
</tr>
<tr>
<td>$P_p$</td>
<td>$i = 1,...,3$</td>
<td>1</td>
<td>Producer price index.</td>
</tr>
<tr>
<td>$SV_h$</td>
<td>$h = 1,...,10$</td>
<td>10</td>
<td>Savings by households of group $h$.</td>
</tr>
<tr>
<td>$SV_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Savings by firms in sector $i$.</td>
</tr>
<tr>
<td>$SVG$</td>
<td>$i = 1,...,3$</td>
<td>1</td>
<td>Government savings.</td>
</tr>
<tr>
<td>$TSAV$</td>
<td>$i = 1,...,3$</td>
<td>1</td>
<td>Total savings</td>
</tr>
<tr>
<td>$I_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Investment demand by sector of destination.</td>
</tr>
<tr>
<td>$Z_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Investment demand by sector of origin.</td>
</tr>
<tr>
<td>Variable</td>
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<td>No.</td>
<td>Description</td>
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<tr>
<td>------------</td>
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<td>------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>$D_i$</td>
<td>$i = 1,\ldots,3$</td>
<td>3</td>
<td>Domestic commodity from sector $i$.</td>
</tr>
<tr>
<td>$DUR_j$</td>
<td>$j = 1,\ldots,3$</td>
<td>3</td>
<td>Domestic use ratio</td>
</tr>
<tr>
<td>$BOT$</td>
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<td>Balance of trade.</td>
</tr>
<tr>
<td>$CTAX_i$</td>
<td>$i = 1,\ldots,3$</td>
<td>3</td>
<td>Direct taxes from sector $i$.</td>
</tr>
<tr>
<td>$DEP_i$</td>
<td>$i = 1,\ldots,3$</td>
<td>3</td>
<td>Depreciation or consumption of fixed capital in sector $i$.</td>
</tr>
<tr>
<td>$INT_i$</td>
<td>$i = 1,\ldots,3$</td>
<td>3</td>
<td>Interest payments by sector $i$.</td>
</tr>
<tr>
<td>$ITAX_i$</td>
<td>$i = 1,\ldots,3$</td>
<td>3</td>
<td>Indirect taxes from sector $i$ excluding import tariffs.</td>
</tr>
<tr>
<td>$DUTY_i$</td>
<td>$i = 1,\ldots,3$</td>
<td>3</td>
<td>Import duties from sector $i$.</td>
</tr>
<tr>
<td>$HTAX_{h}$</td>
<td>$h = 1,\ldots,10$</td>
<td>10</td>
<td>Direct taxes from household $h$.</td>
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<td>$ER$</td>
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<td>The exchange rate for the KenyaN shilling to the US dollar.</td>
</tr>
<tr>
<td>$P_{m_i}$</td>
<td>$i = 1,\ldots,3$</td>
<td>3</td>
<td>World import prices in sector $i$.</td>
</tr>
<tr>
<td>$P_{e_i}$</td>
<td>$i = 1,\ldots,3$</td>
<td>3</td>
<td>World export prices in sector $i$.</td>
</tr>
<tr>
<td>$TGC$</td>
<td></td>
<td>1</td>
<td>Government’s total consumption demand.</td>
</tr>
<tr>
<td>$SVR$</td>
<td></td>
<td>1</td>
<td>Foreign capital inflow (savings from the rest of the world).</td>
</tr>
<tr>
<td>$t_i$</td>
<td>$i = 1,\ldots,3$</td>
<td>3</td>
<td>Tax rate on profits from sector $i$.</td>
</tr>
<tr>
<td>$t_h$</td>
<td>$h = 1,\ldots,10$</td>
<td>10</td>
<td>Direct average tax rate for households of category $h$.</td>
</tr>
<tr>
<td>$td_i$</td>
<td>$i = 1,\ldots,3$</td>
<td>3</td>
<td>Indirect tax rate for commodities in sector $i$.</td>
</tr>
<tr>
<td>$tm_i$</td>
<td>$i = 1,\ldots,3$</td>
<td>3</td>
<td>Import duty applied in sector $i$.</td>
</tr>
<tr>
<td>$\overline{K}_i$</td>
<td>$i = 1,\ldots,3$</td>
<td>3</td>
<td>Fixed amount of sector specific capital stock.</td>
</tr>
<tr>
<td>$r_i$</td>
<td>$i = 1,\ldots,3$</td>
<td>3</td>
<td>Rate of interest for sector $i$.</td>
</tr>
<tr>
<td>$DEPR_i$</td>
<td>$i = 1,\ldots,3$</td>
<td>3</td>
<td>Rate of depreciation in sector $i$.</td>
</tr>
<tr>
<td>$NTR_i$</td>
<td>$i = 1,\ldots,3$</td>
<td>3</td>
<td>Net transfers by sector $i$.</td>
</tr>
<tr>
<td>$SUBS_i$</td>
<td>$i = 1,\ldots,3$</td>
<td>3</td>
<td>Subsidy payments to sector $i$.</td>
</tr>
<tr>
<td>$NTH_{h}$</td>
<td>$h = 1,\ldots,10$</td>
<td>10</td>
<td>Net transfers to household $h$.</td>
</tr>
<tr>
<td>$NTG$</td>
<td></td>
<td>1</td>
<td>Net transfers to government.</td>
</tr>
<tr>
<td>$\varphi_l$</td>
<td>$l = 1,\ldots,5$</td>
<td>5</td>
<td>Wage shifter by category $l$.</td>
</tr>
</tbody>
</table>

Total Variables 298
Table 6.3: Coefficients of KEDEM

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_i^X$</td>
<td>Efficiency parameter in sector $i$'s gross sectoral production.</td>
</tr>
<tr>
<td>$A_j^N$</td>
<td>Efficiency parameter in the formation of composite intermediate input.</td>
</tr>
<tr>
<td>$A_j^V$</td>
<td>Efficiency parameter in forming the value-added composite in sector $i$.</td>
</tr>
<tr>
<td>$A_j^L$</td>
<td>Efficiency parameter in the formation of composite labour in sector $i$.</td>
</tr>
<tr>
<td>$\alpha_{VA_j}$</td>
<td>Distribution parameter for the composite value-added input in sector $i$.</td>
</tr>
<tr>
<td>$\alpha_{VA_i}$</td>
<td>Distribution parameter for composite intermediate input from sector $j$ used in sector $i$.</td>
</tr>
<tr>
<td>$\alpha_{ID_{ji}}$</td>
<td>Distribution parameter for intermediate domestic input type $j$ used in sector $i$.</td>
</tr>
<tr>
<td>$\alpha_{IM_{ji}}$</td>
<td>Distribution parameter for the intermediate imported input type $j$ used in sector $i$.</td>
</tr>
<tr>
<td>$\alpha_{K_i}$</td>
<td>Distribution parameter for capital in sector $i$.</td>
</tr>
<tr>
<td>$\alpha_{L_i}$</td>
<td>Distribution parameter for composite labour in sector $i$.</td>
</tr>
<tr>
<td>$\alpha_{L_{ii}}$</td>
<td>Distribution parameter for labour of category $l$ in sector $i$'s composite labour.</td>
</tr>
<tr>
<td>$\beta_{sh}$</td>
<td>Marginal budget share of sector $i$'s commodity in household consumption.</td>
</tr>
<tr>
<td>$\theta_{si}$</td>
<td>The fixed level subsistence consumption by households of commodity $i$.</td>
</tr>
<tr>
<td>$\alpha_{Di}$</td>
<td>Distribution parameter for domestic commodity in the formation of the composite domestic commodity in sector $i$.</td>
</tr>
<tr>
<td>$\alpha_{Mi}$</td>
<td>Distribution parameter for imported commodity in the formation of the composite domestic commodity in sector $i$.</td>
</tr>
<tr>
<td>$\gamma_{di}$</td>
<td>Distribution parameter for the domestic commodity in the CET function of sector $i$'s gross production.</td>
</tr>
<tr>
<td>$\gamma_{si}$</td>
<td>Distribution parameter for the exported commodity in the CET function of sector $i$'s gross production.</td>
</tr>
<tr>
<td>$\rho_{si}$</td>
<td>Substitution parameter of the composite value-added input and composite intermediate input in the gross sectoral production in sector $i$.</td>
</tr>
<tr>
<td>$\rho_{nh}$</td>
<td>Substitution parameter between domestic and imported intermediate inputs.</td>
</tr>
<tr>
<td>$\rho_{va_{ji}}$</td>
<td>Substitution parameter of the primary factors in the value-added composite formation in sector $i$.</td>
</tr>
<tr>
<td>$\rho_{Li}$</td>
<td>Substitution parameter of the different labour categories in the formation of composite labour input in sector $i$.</td>
</tr>
<tr>
<td>$\phi_{si}$</td>
<td>Substitution parameter between domestic and export commodities in the transformation of gross product from sector $i$.</td>
</tr>
<tr>
<td>$\sigma_{\varphi_{si}}$</td>
<td>Elasticity of substitution between domestic and imported goods in sector $i$.</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Wage indexation parameter.</td>
</tr>
<tr>
<td>$\psi_{\varphi_{si}}$</td>
<td>Elasticity of transformation between the domestically consumed and the exported components of gross sectoral product in sector $i$.</td>
</tr>
<tr>
<td>$\mu_i$</td>
<td>Weights of different sectors in the aggregate price index.</td>
</tr>
<tr>
<td>$\beta_{hl}$</td>
<td>The proportion of household group $h$ ownership of labour of category $l$.</td>
</tr>
<tr>
<td>$\sigma_{ji}$</td>
<td>The share of commodity $j$ in the capital good of sector $i$.</td>
</tr>
<tr>
<td>$\beta_{i}$</td>
<td>Proportion of commodity from sector $i$ in government's total consumption.</td>
</tr>
<tr>
<td>$\kappa_i$</td>
<td>Exogenous sectoral investment share of sector $i$ in total investments.</td>
</tr>
<tr>
<td>$s_h$</td>
<td>Savings rate of a given household group $h$.</td>
</tr>
</tbody>
</table>
Table 6.4: List of Variables Treated as Exogenous in KEGEM Closure

<table>
<thead>
<tr>
<th>Variable</th>
<th>Subscript range</th>
<th>No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ER</td>
<td></td>
<td>1</td>
<td>The exchange rate for the Kenyan shilling to the US dollar.</td>
</tr>
<tr>
<td>$P_{m_i}$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>World import prices in sector $i$.</td>
</tr>
<tr>
<td>$P_{w_i}$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>World export prices in sector $i$.</td>
</tr>
<tr>
<td>TGC</td>
<td></td>
<td>1</td>
<td>Government’s total consumption demand.</td>
</tr>
<tr>
<td>SVR</td>
<td></td>
<td>1</td>
<td>Foreign capital inflow (savings from the rest of the world).</td>
</tr>
<tr>
<td>$t_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Tax rate on profits from sector $i$.</td>
</tr>
<tr>
<td>$t_h$</td>
<td>$h = 1,...,10$</td>
<td>10</td>
<td>Direct average tax rate for households of category $h$.</td>
</tr>
<tr>
<td>$td_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Indirect tax rate for commodities in sector $i$.</td>
</tr>
<tr>
<td>$tm_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Import duty applied in sector $i$.</td>
</tr>
<tr>
<td>$\overline{K}_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Fixed amount of sector specific capital stock.</td>
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<td>$r_i$</td>
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<td>3</td>
<td>Rate of interest for sector $i$.</td>
</tr>
<tr>
<td>DEPR$_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Rate of depreciation in sector $i$.</td>
</tr>
<tr>
<td>NTF$_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Net transfers by sector $i$.</td>
</tr>
<tr>
<td>SUBS$_i$</td>
<td>$i = 1,...,3$</td>
<td>3</td>
<td>Subsidy payments to sector $i$.</td>
</tr>
<tr>
<td>NTH$_i$</td>
<td>$h = 1,...,10$</td>
<td>10</td>
<td>Net transfers to household $h$.</td>
</tr>
<tr>
<td>NTG</td>
<td></td>
<td>1</td>
<td>Net transfers to government.</td>
</tr>
<tr>
<td>$\varphi_l$</td>
<td>$l = 1,...,5$</td>
<td>5</td>
<td>Wage shifter by category $l$.</td>
</tr>
<tr>
<td><strong>Total Variables</strong></td>
<td></td>
<td>59</td>
<td></td>
</tr>
</tbody>
</table>

6.11 Concluding Remarks

This chapter has discussed the theoretical framework for KEGEM. While three sectors have been identified for the purposes of this study, KEGEM is a model that can be applied to any number of sectors beyond these three. All that is required is for data to be compiled for the relevant number of sectors chosen. With the theoretical model complete, it can now be used to analyse issues ranging from effects of external shocks to the various policy options available to the Kenyan government. This is the next task in this study. The remaining chapters deal with the application of KEGEM to address the objectives of the study dealing with terms of trade shocks and effects of adjustment policies.
Appendix 6.1 The Algebraic Derivations

This appendix presents the derivations of the input demand functions for each level of the tree production technology structures using a CES aggregation, the household consumption demand functions, the import demand functions, and the export supply functions.

A6.1.1 Demands for the Different Labour Categories

This section presents the derivation for the demands for the specific labour categories that are required to optimally form the effective labour input used in the production of the effective value added input. These are the inputs that are required in the third level of the production technology. More formally, and invoking the separability properties of the production technology, the optimisation problem can be formulated in a more explicit form as minimise:

\[ TC_L = \sum_{i=1}^{5} w_i L_{ii} \]

subject to

\[ L_i = A_i^L \left( \sum_{i=1}^{5} \alpha_{ii} L_{ii}^{\rho_{ii}} \right)^{-1/\rho_{ii}} \]

where \( \alpha_{1i} + \alpha_{2i} + \alpha_{3i} + \alpha_{4i} + \alpha_{5i} = 1 \).

If the costs that producers in sector \( i \) incur in producing effective labour inputs are to be minimised, then as little as possible of the different labour categories input requirements must be spent. Given this problem then, the following Lagrangian function can be formed:

\[ \Lambda_{L_i} = \sum_{i=1}^{5} w_{ii} L_{ii} + \lambda_i \left( L_i^{-\rho_{ii}} - (A_i^L)^{-\rho_{ii}} \sum_{i=1}^{5} \alpha_{ii} L_{ii}^{-\rho_{ii}} \right) \]

Differentiating with respect to \( L_{ii} \) gives the following first-order conditions:
To solve for \( L_{ni} \) in these first order conditions, one can simplify the problem by taking advantage of the symmetry properties that they present. Therefore, taking first-order conditions given by the first two, which is synonymous to solving for the labour demands in the case where there are only two categories, and eliminating \( \lambda _L \) yields:

\[
\frac{\partial \lambda _L}{\partial L_{ni}} = w_{li} - (A_i^T)\alpha _{ii} (-\rho _{L_i}) \lambda _L L_{ni}^{-\rho _L} = 0
\]

Multiplying through by \( L_{ni} / L_{2i} \),

\[
\frac{w_{li} L_{ni}}{w_{2i} L_{2i}} = \frac{\alpha _{ii} L_{ni}^{-\rho _L}}{\alpha _{2i} L_{2i}^{-\rho _L}}
\]

Now, adding 1 to both sides of the above equation gives

\[
\frac{w_{li} L_{ni} + w_{2i} L_{2i}}{w_{2i} L_{2i}} = \frac{\alpha _{ii} L_{ni}^{-\rho _L} + \alpha _{2i} L_{2i}^{-\rho _L}}{\alpha _{2i} L_{2i}^{-\rho _L}}
\]

The numerator on the LHS is the total cost for a two labour category case while the numerator of the RHS is the effective aggregate labour input from the two labour categories. Thus, the problem can be simplified to a two labour category case as,

\[
\frac{TC_{ni}}{w_{2i} L_{2i}} = \frac{(L_i \lambda _L)^{\rho _L}}{\alpha _{2i} L_{2i}^{-\rho _L}}
\]

Solving for \( L_{2i} \) gives,

\[
L_{2i} = TC_{ni}^{1/(1+\rho _L)} \left( \frac{L_i}{\lambda _L} \right)^{\rho _L/(1+\rho _L)} w_{2i}^{-\rho _L/(1+\rho _L)} \alpha _{2i}^{1/(1+\rho _L)}
\]

and by symmetry
\[ L_{ii} = TC_L^{1/(1+\rho_u)} \left( \frac{L_i}{A_i} \right)^{\rho_u/(1+\rho_u)} w_{1i}^{1/(1+\rho_u)} \alpha_{1i}^{1/(1+\rho_u)} \]

Therefore,

\[ w_{2i}L_{2i} = TC_L^{1/(1+\rho_u)} \left( \frac{L_i}{A_i} \right)^{\rho_u/(1+\rho_u)} w_{2i}^{\rho_u/(1+\rho_u)} \alpha_{2i}^{1/(1+\rho_u)} \]

and

\[ w_{ii}L_{ii} = TC_L^{1/(1+\rho_u)} \left( \frac{L_i}{A_i} \right)^{\rho_u/(1+\rho_u)} w_{ii}^{\rho_u/(1+\rho_u)} \alpha_{ii}^{1/(1+\rho_u)} \]

Adding the two produces the CES total cost function for a two labour category case:

\[ TC_L = \left( \frac{L_i}{A_i} \right) \left[ \alpha_{1i}^{1/(1+\rho_u)} w_{1i}^{\rho_u/(1+\rho_u)} + \alpha_{2i}^{1/(1+\rho_u)} w_{2i}^{\rho_u/(1+\rho_u)} \right]^{1/(1+\rho_u)} \]

Using the envelope theorem (or in other words applying the Shephard's lemma), it is possible to derive the constant effective sectoral labour, specific labour categories demands from the above total cost function. The demand for labour category 1 in a two category case is given as:

\[ \frac{\partial TC_L}{\partial w_{1i}} = L_{1i} = \left( \frac{L_i}{A_i} \right) \left[ \alpha_{1i}^{1/(1+\rho_u)} w_{1i}^{\rho_u/(1+\rho_u)} + \alpha_{2i}^{1/(1+\rho_u)} w_{2i}^{\rho_u/(1+\rho_u)} \right]^{1/(1+\rho_u)} \]

which can be written as:

\[ L_{ii} = \left( \frac{L_i}{A_i} \right) \left[ \alpha_{1i}^{1/(1+\rho_u)} w_{1i}^{\rho_u/(1+\rho_u)} + \alpha_{2i}^{1/(1+\rho_u)} w_{2i}^{\rho_u/(1+\rho_u)} \right]^{1/(1+\rho_u)} \alpha_{ii}^{1/(1+\rho_u)} w_{ii}^{1/(1+\rho_u)} \]

Generalising these results for the five labour categories case for Kenya, and making use of the symmetry properties lead to the following labour demand functions for the five labour categories:
\[ L_{1i} = \left( \frac{L_i}{A_{iL}} \right) \left( \sum_{l=1}^{5} \alpha_{li}^{1/(1+p_{li})} \mu_{il}^{p_{li}/(1+p_{li})} \right)^{1/p_{li}} \alpha_{li}^{1/(1+p_{li})} \mu_{il}^{-1/(1+p_{li})} \]

\[ L_{2i} = \left( \frac{L_i}{A_{iL}} \right) \left( \sum_{l=1}^{5} \alpha_{li}^{1/(1+p_{li})} \mu_{il}^{p_{li}/(1+p_{li})} \right)^{1/p_{li}} \alpha_{li}^{1/(1+p_{li})} \mu_{il}^{-1/(1+p_{li})} \]

\[ L_{3i} = \left( \frac{L_i}{A_{iL}} \right) \left( \sum_{l=1}^{5} \alpha_{li}^{1/(1+p_{li})} \mu_{il}^{p_{li}/(1+p_{li})} \right)^{1/p_{li}} \alpha_{li}^{1/(1+p_{li})} \mu_{il}^{-1/(1+p_{li})} \]

\[ L_{4i} = \left( \frac{L_i}{A_{iL}} \right) \left( \sum_{l=1}^{5} \alpha_{li}^{1/(1+p_{li})} \mu_{il}^{p_{li}/(1+p_{li})} \right)^{1/p_{li}} \alpha_{li}^{1/(1+p_{li})} \mu_{il}^{-1/(1+p_{li})} \]

\[ L_{5i} = \left( \frac{L_i}{A_{iL}} \right) \left( \sum_{l=1}^{5} \alpha_{li}^{1/(1+p_{li})} \mu_{il}^{p_{li}/(1+p_{li})} \right)^{1/p_{li}} \alpha_{li}^{1/(1+p_{li})} \mu_{il}^{-1/(1+p_{li})} \]

**A6.1.2 Demands for Capital and Aggregate Labour**

In order to derive the demand for capital and labour needed in the production of the effective value-added input, \( VA_i \), it is assumed that the producers seek to minimise:

\[ TC_{VA} = P_k K_i + w_i L_i \]

subject to

\[ VA_i = A_{iVA} \left( \alpha_{ki} K_i^{-\rho_{wi}} + \alpha_{li} L_i^{-\rho_{wi}} \right)^{-\rho_{wi}} \]

The producers' choice of the primary inputs can be formally represented as a cost minimisation problem, thus,

\[ \Lambda_{VA} = P_k K_i + w_i L_i + \lambda_{VA} \left( VA_i^{-\rho_{wi}} - \left( A_{iVA}^{\rho_{wi}} - \rho_{wi} \left( \alpha_{ki} K_i^{-\rho_{wi}} + \alpha_{li} L_i^{-\rho_{wi}} \right) \right) \right) \]

Differentiating the Lagrangian function with respect to capital and labour gives,

\[ \frac{\partial \Lambda_{VA}}{\partial K_i} = P_k + \lambda_{VA} \left( -A_{iVA}^{\rho_{wi}} \right)^{-\rho_{wi}} \alpha_{ki} (-\rho_{wi}) K_i^{-\rho_{wi}-1} = 0 \]
To solve for $K_i$ and $L_i$, it is possible to eliminate the Lagrangian multiplier in a similar way as shown in Silberberg (1990) which yields:

$$
\frac{P_k}{w_i} = \frac{\alpha K_i}{\alpha L_i} \frac{\rho_{w_i}}{L_i^{\rho_{w_i} - 1}}
$$

Multiplying through by $(K_i / L_i)$ gives

$$
\frac{P_k K_i}{w_i L_i} = \frac{\alpha K_i}{\alpha L_i} \frac{\rho_{w_i}}{L_i^{\rho_{w_i} - 1}}
$$

Adding 1 to both sides of the above expression leads to:

$$
\frac{P_k K_i}{w_i L_i} + w_i L_i = \frac{\alpha K_i}{\alpha L_i} \frac{\rho_{w_i}}{L_i^{\rho_{w_i} - 1}} + \frac{\alpha L_i}{\alpha L_i} \frac{1}{L_i^{\rho_{w_i} - 1}}
$$

The above expression can be written for a two-primary factor case value-added input formation as:

$$
TC_{VA} = \left( \frac{VA_i}{A_i} \right)^{\rho_{w_i}} \alpha L_i \frac{1}{L_i^{\rho_{w_i} - 1}}
$$

Solving for $L_i$ leads to:

$$
L_i = TC_{VA_i}^{1/(1+\rho_{w_i})} \left( \frac{VA_i}{A_i} \right)^{1/(1+\rho_{w_i})} \alpha L_i \left( \frac{1}{L_i^{\rho_{w_i} - 1}} \frac{1}{w_i^{\rho_{w_i}}} \right)^{1/(1+\rho_{w_i})}
$$

Due to symmetry, a similar expression for the capital input is given as:

$$
K_i = TC_{VA_i}^{1/(1+\rho_{w_i})} \left( \frac{VA_i}{A_i} \right)^{1/(1+\rho_{w_i})} \alpha K_i \left( \frac{1}{K_i^{\rho_{w_i} - 1}} \frac{1}{P_{ki}^{\rho_{w_i}}} \right)^{1/(1+\rho_{w_i})}
$$
Therefore,

\[ w_i L_i = TC_{VA}^{1/(1+\rho_{m_i})} \left( \frac{VA_i}{A_i^{VA}} \right)^{\rho_{m_i}/(1+\rho_{m_i})} \alpha_{L_i}^{1/(1+\rho_{m_i})} W_i^{\rho_{m_i}/(1+\rho_{m_i})} \]

and

\[ P_k K_i = TC_{VA}^{1/(1+\rho_{m_k})} \left( \frac{VA_i}{A_i^{VA}} \right)^{\rho_{m_k}/(1+\rho_{m_k})} \alpha_{K_i}^{1/(1+\rho_{m_k})} P_{k_i}^{\rho_{m_k}/(1+\rho_{m_k})} \]

Adding the two expressions together gives the constant elasticity of substitution total cost function which is given as:

\[ TC_{VA} = \left( \frac{VA_i}{A_i^{VA}} \right)^{\rho_{m_i}/(1+\rho_{m_i})} \alpha_{K_i}^{1/(1+\rho_{m_k})} P_{k_i}^{\rho_{m_k}/(1+\rho_{m_k})} + \alpha_{L_i}^{1/(1+\rho_{m_i})} W_i^{\rho_{m_i}/(1+\rho_{m_i})} \]

Using the envelope theorem, it is possible to derive the constant effective value-added primary input demands from the above CES total cost function. This is done by again following the procedure enunciated in Silberberg (1990) which requires just the derivation of the first-order condition with respect to the variable whose demand function is sought by applying the Sheophard's lemma. Therefore, the primary input demand functions are as follows. The demand for the effective labour input is given by:

\[ L_i = \left( \frac{VA_i}{A_i^{VA}} \right)^{\rho_{m_i}/(1+\rho_{m_i})} \alpha_{K_i}^{1/(1+\rho_{m_k})} P_{k_i}^{\rho_{m_k}/(1+\rho_{m_k})} + \alpha_{L_i}^{1/(1+\rho_{m_i})} W_i^{\rho_{m_i}/(1+\rho_{m_i})} \]

Similarly, the demand for capital stock input can be found by taking its first derivative. This results in the demand for capital stock:

\[ K_i = \left( \frac{VA_i}{A_i^{VA}} \right)^{\rho_{m_k}/(1+\rho_{m_k})} \alpha_{K_i}^{1/(1+\rho_{m_k})} P_{k_i}^{\rho_{m_k}/(1+\rho_{m_k})} + \alpha_{L_i}^{1/(1+\rho_{m_i})} W_i^{\rho_{m_i}/(1+\rho_{m_i})} \]
A6.1.3 Demands for the Domestic and Imported Intermediate Inputs

Applying the same procedure like the one used in the case of the primary inputs demand functions, the demand functions for the domestic and imported intermediate inputs can be derived. Thus, the assumption again is that the producers seek to minimise:

\[ TC_{N_j} = P_{im_j} IM_{ji} + P_{id_j} ID_{ji} \]

subject to

\[ N_j := A_j \left( \alpha_{ID_j} ID_{ji}^{-\rho_n} + \alpha_{IM_j} IM_{ji}^{-\rho_n} \right)^{-1/\rho_n} \]

The producer’s choice of the intermediate inputs can be formally represented as a cost minimisation problem through the following Lagrangian function:

\[ \Lambda_{N_j} = P_{im_j} IM_{ji} + P_{id_j} ID_{ji} + \lambda_{N_j} \left( N_j^{-\rho_n} - (A_j^{-1})^{-\rho_n} \left( \alpha_{IM_j} IM_{ji}^{-\rho_n} + \alpha_{ID_j} ID_{ji}^{-\rho_n} \right) \right) \]

Differentiating the Lagrangian function with respect to the imported and domestic intermediate inputs, we get:

\[ \frac{\partial \Lambda_{N_j}}{\partial IM_{ji}} = P_{im_j} + \lambda_{N_j} \left( -A_j^{-1} \right)^{-\rho_n} \alpha_{IM_j} (\rho_n) IM_{ji}^{-\rho_n -1} = 0 \]

\[ \frac{\partial \Lambda_{N_j}}{\partial ID_{ji}} = P_{id_j} + \lambda_{N_j} \left( -A_j^{-1} \right)^{-\rho_n} \alpha_{ID_j} (\rho_n) ID_{ji}^{-\rho_n -1} = 0 \]

To solve for the demand functions of the two intermediate inputs, the same steps as in the previous section above are followed. The demand function for the domestic intermediate input that is arrived at is given by the function:

\[ ID_{ji} = \left( \frac{N_j}{A_j} \right) \left[ P_{im_j}^{-\rho_n/(1+\rho_n)} \alpha_{IM_j}^{1/(1+\rho_n)} + P_{id_j}^{-\rho_n/(1+\rho_n)} \alpha_{ID_j}^{1/(1+\rho_n)} \right]^{1/\rho_n} \left[ P_{im_j}^{-\rho_n/(1+\rho_n)} \alpha_{IM_j}^{1/(1+\rho_n)} + P_{id_j}^{-\rho_n/(1+\rho_n)} \alpha_{ID_j}^{1/(1+\rho_n)} \right]^{-1/\rho_n} \]

and the demand function for the imported intermediate input of sector j going to sector i production is given by:
A6.1.4 Consumer Demand

The Stone-Geary LES obtained from the constrained maximisation of the Klein-Rubin utility function is applied in this model. The LES is derived in the usual way by maximising a Stone-Geary utility function given by,

\[ U_h = \sum_j \beta_{jh} \log(C_{jh} - \theta_{jh}) \]

subject to the household budget constraint

\[ SV_h + \sum_j P_{qj} C_{jh} = Y_h - HTAX_h \]

where \( C_{jh} \) is the quantity demanded of commodity \( j \) by household of category \( h \), \( SV_h \) is the amount of income that goes to savings from this household group and the other variables are as defined in the main text of Chapter 6.

From this utility maximisation problem, it is possible to write the problem as a Lagrangian function,

\[ \Lambda(C_{jh}, \lambda) = \sum_j \beta_{jh} \log(C_{jh} - \theta_{jh}) - \lambda \left( SV_h + \sum_j P_{qj} C_{jh} - (Y_h - HTAX_h) \right) \]

The first-order conditions for this Lagrangian function are

\[ \frac{\beta_{jh}}{C_{jh} - \theta_{jh}} - \lambda P_{qj} = 0 \]

and

\[ SV_h + \sum_j P_{qj} C_{jh} - (Y_h - HTAX_h) = 0 \]
From the first expression of the first-order conditions, it follows that

\[ P_{qj} C_{jh} = P_{qj} \theta_{jh} + \beta_{jh} / \lambda \]

thus.

\[ \beta_{jh} = \lambda(P_{qj} C_{jh} - P_{qj} \theta_{jh}) \]

Hence from the second expression of the first-order conditions and the expression for \( \beta_{jh} \) above we get

\[ \sum_j \beta_{jh} = \lambda \left( Y_h - HTAX_h - SV_h - \sum_j P_{qj} \theta_{jh} \right) = 1 \]

which can be re-written as

\[ \lambda = 1 / \left( Y_h - HTAX_h - SV_h - \sum_j P_{qj} \theta_{jh} \right) \]

Substituting this value of \( \lambda \) into the first first-order expression and rearranging gives

\[ \beta_{jh} \left( Y_h - HTAX_h - SV_h - \sum_j P_{qj} \theta_{jh} \right) = P_{qj} C_{jh} - P_{qj} \theta_{jh} \]

which can be rewritten as

\[ C_{jh} = \theta_{jh} + \frac{\beta_{jh} \left( Y_h - HTAX_h - SV_h - \sum_j P_{qj} \theta_{jh} \right)}{P_{qj}} \]

or if the demand function is written in expenditure form:

\[ P_{qj} C_{jh} = P_{qj} \theta_{jh} + \beta_{jh} \left( Y_h - HTAX_h - SV_h - \sum_j P_{qj} \theta_{jh} \right) \]
A6.1.5 The Import Demand Functions

The supply of composite commodity, $Q_j$, is assumed to be a CES combination of domestically produced commodity, $D_j$, and commodity produced abroad, import $M_j$. The CES combination can be given more explicitly as:

$$Q_j = A_j^Q \left( \alpha_{D_j} D_j^{-\rho_{Q_j}} + \alpha_{M_j} M_j^{-\rho_{Q_j}} \right)^{-1/\rho_{Q_j}}$$

where $A^Q_j, \alpha_{M_j}, \alpha_{D_j}$ and $\rho_{Q_j}$ are parameters and $M_j$ and $D_j$ are the inputs producing the aggregate commodity, $Q_j$. $\sigma_{Q_j}$, the elasticity of substitution, is given by $\sigma_{Q_j} = 1/(1 + \rho_{Q_j})$.

From the Armington aggregate function, the Kenyan demand for imports is determined. It is plausible to assume that firms in each sector choose the cost-minimising combination of domestic and imported inputs that are compatible with $Q_j$. By virtue of the CES technology assumption, the demand for the imported commodity is regarded as derived demand, just like the derived demands for the various inputs in the production process.

Letting the prices of imported and the domestic commodities to be $P_{m_j}$ and $P_{d_j}$ respectively, the derived demand can be determined in a procedure similar to the profit maximising problem of the firm. The firms minimise the cost of obtaining the composite commodity:

$$TC_{Q_j} = P_{m_j} M_j + P_{d_j} D_j$$

subject to the production technology given by the CES aggregation of the composite commodity. By letting $\lambda_{Q_j}$ be the Lagrangian multiplier, the following Lagrangian function can be formed:

$$\Lambda_{Q_j} = P_{m_j} M_j + P_{d_j} D_j - \lambda_{Q_j} \left( Q_j - A_j^Q \left( \alpha_{M_j} M_j^{-\rho_{Q_j}} + \alpha_{D_j} D_j^{-\rho_{Q_j}} \right)^{-1/\rho_{Q_j}} \right)$$
Following the normal optimisation process, the first-order conditions from the Lagrangian function are:

\[
\lambda_{q_j} \left( \frac{1}{\rho_{q_j}} \right) A_j^Q \left( \alpha_{M_j} M_j^{\rho_{q_j}} + \alpha_{D_j} D_j^{\rho_{q_j}} \right)^{-1/\rho_{q_j}} A_{M_j} = P_{m_j}
\]

\[
\lambda_{q_j} \left( \frac{1}{\rho_{q_j}} \right) A_j^Q \left( \alpha_{M_j} M_j^{\rho_{q_j}} + \alpha_{D_j} D_j^{\rho_{q_j}} \right)^{-1/\rho_{q_j}} A_{D_j} = P_{d_j}
\]

\[
Q_i^j - A_j^Q \left( \alpha_{M_j} M_j^{\rho_{q_j}} + \alpha_{D_j} D_j^{\rho_{q_j}} \right)^{-1/\rho_{q_j}} = 0
\]

Elimination of \( \lambda_{q_j} \) from the first two first-order condition equations which is tantamount to getting their ratio leads to:

\[
\frac{\alpha_{M_j} M_j^{\rho_{q_j}}}{\alpha_{D_j} D_j^{\rho_{q_j}}} = \frac{P_{m_j}}{P_{d_j}}
\]

which upon rearrangement leads to,

\[
\left( \frac{M_j}{D_j} \right)^{-\rho_{q_j}} = \left( \frac{P_{m_j}}{P_{d_j}} \right) \left( \frac{\alpha_{D_j}}{\alpha_{M_j}} \right)
\]

which can further be rearranged to give,

\[
\frac{M_j}{D_j} = \left( \frac{\alpha_{D_j}}{\alpha_{M_j}} \right)^{-1/(1+\rho_{q_j})} \left( \frac{P_{m_j}}{P_{d_j}} \right)^{-1/(1+\rho_{q_j})}
\]

But it is known that \( \sigma_{q_j} = 1 / (1 + \rho_{q_j}) \), therefore the derived demand can be specified as:

\[
\frac{M_j}{D_j} = \left( \frac{P_{d_j}}{P_{m_j}} \right)^\sigma_{q_j} \left( \frac{\alpha_{M_j}}{\alpha_{D_j}} \right)^\sigma_{q_j}
\]
A6.1.6 The Export Supply Function

The firms in their respective sectors maximise revenue from a given level of sectoral gross output. In other words, the revenue maximising problem is to maximise

\[ P_x X_j = P_e E_j + P_d D_j \]

subject to

\[ X_j = B_j^X \left( \gamma_{e_j} E_j^{\rho_{e_i}} + \gamma_{d_j} D_j^{\rho_{d_j}} \right)^{1/\rho_{e_j}} \]

where \( X_j \) is gross output and \( B_j^X \) and \( \gamma \)'s are constants. The elasticity of transformation \( \psi_{X_j} \) is given by

\[ \psi_{X_j} = 1 / (\phi_{x_j} - 1) \]

Note that the sign convention of the CET production frontier requires that the elasticity of transformation be positive for the transformation frontier to be concave. The solution to the above revenue maximisation problem yields the required export supply function that represents the allocation between domestic sales, \( D_j \) and exports, \( E_j \). The export supply for the Kenyan exports is yielded from the above problem in the normal optimisation technique. If \( \lambda_{X_j} \) is the Lagrangian multiplier, then one can form the following Lagrangian function:

\[ \Lambda_{X_j} = P_e E_j + P_d D_j + \lambda_{X_j} \left( X_j - B_j^X \left( \gamma_{e_j} E_j^{\rho_{e_i}} + \gamma_{d_j} D_j^{\rho_{d_j}} \right)^{1/\rho_{e_j}} \right) \]

The first order conditions that follow from this expression are:

\[ \lambda_{X_j} \left( 1 / \phi_{x_j} \right) B_j^X \gamma_{e_j} \phi_{x_j} E_j^{\rho_{e_i}-1} \left( \gamma_{e_j} E_j^{\rho_{e_i}} + \gamma_{d_j} D_j^{\rho_{d_j}} \right)^{1/\rho_{e_j}-1} = P_e \]

\[ \lambda_{X_j} \left( 1 / \phi_{x_j} \right) B_j^X \gamma_{d_j} \phi_{x_j} D_j^{\rho_{d_j}-1} \left( \gamma_{e_j} E_j^{\rho_{e_i}} + \gamma_{d_j} D_j^{\rho_{d_j}} \right)^{1/\rho_{d_j}-1} = P_d \]

\[ X_j - B_j^X \left( \gamma_{e_j} E_j^{\rho_{e_i}} + \gamma_{d_j} D_j^{\rho_{d_j}} \right)^{1/\rho_{e_j}} = 0 \]
Elimination of $\lambda_{x_j}$ from first-order conditions in the above system leads to:

$$\frac{\gamma_{e_j} E_j^{\phi_{x_j} - 1}}{\gamma_{d_j} D_j^{\phi_{x_j} - 1}} = \frac{P_{e_j}}{P_{d_j}}$$

which can be rearranged leading to:

$$\left(\frac{E_j}{D_j}\right)^{\phi_{x_j} - 1} = \left(\frac{\gamma_{d_j}}{\gamma_{e_j}}\right) \left(\frac{P_{e_j}}{P_{d_j}}\right)$$

which gives the export supply function:

$$\frac{E_j}{D_j} = \left(\frac{\gamma_{d_j}}{\gamma_{e_j}}\right)^{\phi_{x_j}} \left(\frac{P_{e_j}}{P_{d_j}}\right)^{\phi_{x_j}}$$

recalling that $\psi_{x_j} = 1 / (\phi_{x_j} - 1)$. 