## Appendix 1. Mathematical Derivation of Error Expressions For Results in Chapter 3

The mathematical tool 'order of magnitude' is used to examine the sizes of the errors of EDM estimates when the exogenous percentage shift $\lambda$ is near zero. We first introduce the notation of $\mathbf{O}$ (.) (Ledermann and Vajda 1978).

Definition: Let $f(x)$ and $g(x)$ be two functions. If

$$
\lim _{x \rightarrow 0}|\mathrm{f}(\mathrm{x}) / \mathrm{g}(\mathrm{x})|=\mathrm{c},
$$

where $0<c<+\infty$ is a constant, then we write:

$$
\mathrm{f}(\mathrm{x})=\mathbf{O}(\mathrm{g}(\mathrm{x}))
$$

Properties: As $\mathrm{x} \rightarrow 0$,
(i) $\mathbf{O}(1)=$ constant;
(ii) $\mathrm{c} \mathbf{O}(\mathrm{g}(\mathrm{x}))=\mathbf{O}(\mathrm{g}(\mathrm{x}))$ where $0<\mathrm{c}<+\infty$ is a constant;
(iii) If $f(x)=\mathbf{O}(g(x))$, then $[f(x)]^{k}=\mathbf{O}\left(\left[g\left(x^{\prime}\right)\right]^{k}\right), k:=1,2, \ldots$;
(iv) $\left[\mathbf{O}\left(\mathrm{x}^{\mathrm{m}}\right)\right]\left[\mathbf{O}\left(\mathrm{x}^{\mathrm{n}}\right)\right]=\mathbf{O}\left(\mathrm{x}^{\mathrm{m}+\mathrm{n}}\right), \mathrm{m}, \mathrm{n}=1,2 \ldots$;
(v) $\left[\mathbf{O}\left(\mathrm{x}^{\mathrm{m}}\right)\right] /\left[\mathbf{O}\left(\mathrm{x}^{\mathrm{n}}\right)\right]=\mathbf{O}\left(\mathrm{x}^{\mathrm{m}-\mathrm{n}}\right), \mathrm{m}, \mathrm{n}=1,2, \ldots$ and $\mathrm{m}>\mathrm{n}$;
(vi) $\mathbf{O}\left(\mathrm{x}^{\mathrm{m}}\right) \pm \mathbf{O}\left(\mathrm{x}^{\mathrm{n}}\right)=\mathbf{O}\left(\mathrm{x}^{\min (\mathrm{m}, \mathrm{n})}\right), \mathrm{m}, \mathrm{n}=1,2, \ldots$.

## A1.1 Parallel Shift and Linear Approxination

The derivations in this section relate to the definitions in Equations (3.7)-(3.9) and (3.18)(3.23) in Chapter 3 for the case of parallel shift and linear approximation using EDM.

Proposition 1: The percentage changes in price and quantity are of the same order of 'infinitesimal' as the percentage shift $\lambda$ when $\lambda \rightarrow 0$, that is:

$$
(E P)^{k}=\mathbf{O}\left(\lambda^{k}\right) \quad \text { and } \quad(E Q)^{k}=\mathbf{O}\left(\lambda^{\prime}\right) \quad(k=1,2, \ldots)
$$

Proof: Refer to Figure 3.1, connect points $\mathrm{E}_{1}$ and $\mathrm{E}_{2}$ with a straight line and consider the triangle $E_{1} E_{2} B$. Side $E_{1} B=K$, height $E_{2} D=\Delta Q$ and $E_{1} D=\Delta P$. Thus it is obvious using Property (ii) that $\Delta P=\mathbf{O}(K)$ and $\Delta Q=\mathbf{O}(K)$. Since $E P=\Delta P / P_{1}, E Q=\Delta Q / Q_{1}$ and $\lambda=K / P_{1}$, using Property (ii) again we have $\mathbf{O}(E P)=\mathbf{O}(E Q)=\mathbf{O}(\lambda)$. Therefore from Property (iii) $(E P)^{k}=\mathbf{O}\left(\lambda^{k}\right)$ and $(\mathrm{EQ})^{\mathrm{k}}=\mathbf{=} \mathbf{O}\left(\lambda^{\mathrm{k}}\right)(\mathrm{k}=1,2, \ldots) . \quad$ \#

Proposition 2: $E P-E P^{*}=\left[2 Q_{1}(\eta-\varepsilon)\right]^{-1} P_{1}^{2}\left[S^{(2)}\left(c_{2}\right)(E P-\lambda)^{2}-D^{(2)}\left(c_{1}\right)(E P)^{2}\right]=\mathbf{O}\left(\lambda^{2}\right)$
and

$$
\left.\mathrm{EQ}-\mathrm{EQ}^{*}=\left[2 \mathrm{Q}_{1}(\eta-\varepsilon)\right]^{-1} \mathrm{P}_{1}^{2}\left[\eta \mathrm{~S}^{(2)} \mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{2}-\varepsilon \mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{2}\right]=\mathbf{O}\left(\lambda^{2}\right)
$$

when $\lambda \rightarrow 0$. In other words, the errors in EDM estimation of price and quantity changes are of the infinitesimal order of $\mathbf{O}\left(\lambda^{2}\right)$ when $\lambda \rightarrow 0$.

Proof: Referring to equations (1)-(3) in the text, expanding the demand function $D_{1}$ at point $P_{1}$ using the Taylor Expansion formula with remainder and taking the value at point $\mathrm{P}_{2}$, we have:

$$
\mathrm{D}\left(\mathrm{P}_{2}\right)=\mathrm{D}\left(\mathrm{P}_{1}\right)+\mathrm{D}^{(1)}\left(\mathrm{P}_{1}\right)\left(\mathrm{P}_{2}-\mathrm{P}_{1}\right)+(1 / 2) \mathrm{D}^{(\cdot)}\left(\mathrm{c}_{1}\right)\left(\mathrm{P}_{2}-\mathrm{P}_{1}\right)^{2}
$$

that is: $\mathrm{Q}_{2}=\mathrm{Q}_{1}+\mathrm{D}^{(1)}\left(\mathrm{P}_{1}\right) \mathrm{P}_{1} \mathrm{EP}+(1 / 2) \mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right) \mathrm{P}_{1}{ }^{2} E \mathrm{P}^{2}$
or $\quad \mathrm{EQ}=\left(\mathrm{Q}_{2}-\mathrm{Q}_{1}\right) / \mathrm{Q}_{1}=\mathrm{D}^{(1)}\left(\mathrm{P}_{1}\right) \mathrm{P}_{1} / \mathrm{Q}_{1} \mathrm{EP}+\left(2 \mathrm{Q}_{1}\right)^{-1} \mathrm{P}_{1}{ }^{2} \mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{2}$
(A.1.1) $\quad=\eta \mathrm{EP}+\left(2 \mathrm{Q}_{1}\right)^{-1} \mathrm{P}_{1}{ }^{2} \mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{2}$
where $\mathrm{D}^{(\mathrm{i})}($.$) is the i$ th derivative of the demand lunction, $\eta$ is the demand elasticity at point $\mathrm{P}_{1}$ and $P_{2} \leq c_{1} \leq P_{1}$. Similarly, Taylor expanding he new supply function $S_{2}$ at point $P_{1}+K$ and evaluating at point $\mathrm{P}_{2}$, we have:

$$
\mathrm{Q}_{2}=\mathrm{S}\left(\mathrm{P}_{2}-\mathrm{K}\right)=\mathrm{S}\left(\mathrm{P}_{1}\right)+\mathrm{S}^{(1)}\left(\mathrm{P}_{1}\right)\left(\mathrm{P}_{2}-\mathrm{P}_{1}-\mathrm{K}\right)+(1 / 2) \mathrm{S}^{(2)}\left(\mathrm{c}_{2}\right)\left(\mathrm{P}_{2}-\mathrm{P}_{1}-\mathrm{K}\right)^{2}
$$

where $\mathrm{S}^{(\mathrm{i})}($.$) is the ith derivative of the supply function and \mathrm{P}_{2} \leq \mathrm{c}_{2} \leq \mathrm{P}_{1}$ is a constant. Thus:

$$
E Q=\left(\mathrm{Q}_{2}-\mathrm{Q}_{1}\right) / \mathrm{Q}_{1}=\mathrm{S}^{(1)}\left(\mathrm{P}_{1}\right) \mathrm{P}_{1} / \mathrm{Q}_{1}(\mathrm{EP}-\lambda)+\left(2 \mathrm{Q}_{1}\right)^{-1} \mathrm{P}_{1}{ }^{2} \mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{2}, \text { or }
$$

(A.1.2) $\mathrm{EQ}=\varepsilon(\mathrm{EP}-\lambda)+\left(2 \mathrm{Q}_{1}\right)^{-1} \mathrm{P}_{1}{ }^{2} \mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{2}$

Solving (A.1.1) and (A.1.2) jointly, we have:

$$
\begin{aligned}
& \mathrm{EP}=\lambda \varepsilon /(\varepsilon-\eta)+\left[2 \mathrm{Q}_{1}(\eta-\varepsilon)\right]^{-1}\left[S^{(2)}\left(\mathrm{c}_{2}\right)\left(\mathrm{P}_{2}-\mathrm{P}_{1}-\Delta \mathrm{S}\right)^{2}-\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)\left(\mathrm{P}_{2}-\mathrm{P}_{1}\right)^{2}\right] \\
& \mathrm{EQ}=\lambda \eta \varepsilon /(\varepsilon-\eta)+\left[2 \mathrm{Q}_{1}(\eta-\varepsilon)\right]^{-1} \mathrm{P}_{1}^{2}\left[\eta S^{(2)}\left(\mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{2}-\varepsilon D^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{2}\right]
\end{aligned}
$$

or
(A.1.3) EP-EP ${ }^{*}=\left[2 Q_{1}(\eta-\varepsilon)\right]^{-1} P_{1}^{2}\left[S^{(2)}\left(c_{2}\right)(E P-\lambda)^{-}-D^{(2)}\left(c_{1}\right)(E P)^{2}\right]$
(A.1.4)

$$
E Q-E Q^{*}=\left[2 Q_{1}(\eta-\varepsilon)\right]^{-1} P_{1}^{2}\left[\eta S^{(2)}\left(c_{2}\right)(E P \cdot \lambda)^{2}-\varepsilon D^{(2)}\left(c_{1}\right)(E P)^{2}\right]
$$

Since $|E P-\lambda| \leq|\lambda|$ and $|E P| \leq|\lambda|$ (from Figure 1), $\mid E P-\lambda)^{2}=\mathbf{O}\left(\lambda^{2}\right)$ and $(E P)^{2}=\mathbf{O}\left(\lambda^{2}\right)$ when $\lambda \rightarrow 0$. Thus:

$$
\begin{aligned}
& \left|E P-E P^{*}\right|=\mathbf{O}\left(\lambda^{2}\right) \\
& \left|E Q-E Q^{*}\right|=\mathbf{O}\left(\lambda^{2}\right) \quad(\lambda \rightarrow 0)
\end{aligned}
$$

Remark 1: $\mathrm{EP}=\mathrm{EP}^{*}$ and $\mathrm{EQ}=\mathrm{EQ}^{*}$ when the denand and supply curves are linear around the neighbourhood of the initial equilibrium point. This is obvious when $D^{(2)}\left(c_{1}\right)$ and $S^{(2)}\left(c_{2}\right)$ are zero in equations (A.1.3) and (A.1.4). \#

Remark 2: If we assume that the demand function is always increasing and concave and supply function is always decreasing and convex. that is:

$$
\begin{equation*}
\varepsilon>0, \quad \mathrm{~S}^{(2)}(\mathrm{P})<0 \tag{A.1.5}
\end{equation*}
$$

(A.1.6) $\quad \eta<0, \quad D^{(2)}(P)>0$
the upper bounds for the errors will be:
(A.1.7)|EP-EP $P^{*}\left|\leq\left|2 \mathrm{Q}_{1}(\varepsilon-\eta)\right|^{-1}\left(\mathrm{P}_{1}{ }^{2} \lambda^{2}\right)\right| \mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)-\mathrm{S}^{(2)}\left(\mathrm{c}_{2}\right) \mid \quad$ and
(A.1.8) $\left|E Q-E Q^{*}\right| \leq\left|2 Q_{1}(\varepsilon-\eta)\right|^{-1}\left(P_{1}{ }^{2} \lambda^{2}\right) \max \left(\left|\eta \mathrm{S}^{(2)}\left(\mathrm{c}_{2}\right)\right|,\left|\varepsilon D^{(2)}\left(\mathrm{c}_{1}\right)\right|\right)$.

This is because $|E P-\lambda| \leq|\lambda|,|E P| \leq|\lambda|$ and, from assumptions (A.1.5) and (A.1.6), $\mathrm{S}^{(2)}\left(\mathrm{c}_{2}\right)$ (EP$\lambda)^{2} \leq 0, D^{(2)}\left(c_{1}\right)(E P)^{2} \geq 0, \eta S^{(2)}\left(c_{2}\right)(E P-\lambda)^{2} \geq 0$ and $\varepsilon D^{(2)}\left(c_{1}\right)(E P)^{2} \geq 0$. \#

Remark 3: From assumptions (A.1.5) and (A.1.6), the error term in equation (A.1.3) is always nonnegative. Thus:

$$
\begin{equation*}
\mathrm{EP} \geq \mathrm{EP}^{*} \tag{A.1.9}
\end{equation*}
$$

The empirical implication of this result is given in the main text. \#

Remark 4: The sign in the error term in (A.1.4) is indeterminate depending on the relative sizes of $\eta S^{(2)}\left(c_{2}\right)(E P-\lambda)^{2}$ and $\varepsilon D^{(2)}\left(c_{1}\right)(E P)^{2}$. Henc』 EQ can be over or under estimated. \#

Proposition 3: $\quad \Delta C S-\Delta C S^{*}=-[2(\eta-\varepsilon)]^{-1} P_{1}{ }^{3}\left[S^{(2)}\left(c_{2}\right)(E P-\lambda)^{2}-D^{(2)}\left(c_{1}\right)(E P)^{2}\right]$

$$
\begin{aligned}
& -[2(\eta-\varepsilon)]^{-1} \eta P_{1}^{3}\left[S^{(2)}\left(c_{2}\right)(E P-\lambda)^{2} E P^{*}-D^{(2)}\left(c_{1}\right)(E P)^{2} E P^{*}\right]-(1 / 6) P_{1}^{3} D^{(2)}\left(c_{1}\right)(E P)^{3} \\
& -\left[8 Q_{1}(\eta-\varepsilon)^{2}\right]^{-1} \eta P_{1}^{5}\left[S^{(2)}\left(c_{2}\right)(E P-\lambda)^{2}-D^{(2)}\left(c_{1}\right)(E P)^{2}\right]^{2}-(1 / 6) P_{1}^{3} D^{(2)}\left(c_{1}\right)(E P)^{3} \\
& =\mathbf{O}\left(\lambda^{2}\right)
\end{aligned}
$$

that is, the estimate of consumer surplus change using Equation (3.18) involves error of the infinitesimal order $\mathbf{O}\left(\lambda^{2}\right)$ when $\lambda \rightarrow 0$.

Proof: Expanding $\mathrm{Q}=\mathrm{D}(\mathrm{P})$ at point $\mathrm{P}_{1}$

$$
D(P)=D\left(P_{1}\right)+D^{(1)}\left(P_{1}\right)\left(P-P_{1}\right)+(1 / 2) D^{(2)}\left(c_{1}\right)\left(P-P_{1}\right)^{2}
$$

Thus $\Delta \mathrm{CS}=\int_{P 2}^{P 1} D(P) d P$

$$
\begin{equation*}
=-\mathrm{P}_{1} \mathrm{Q}_{1} \mathrm{EP}-(1 / 2) \eta \mathrm{P}_{1} \mathrm{Q}_{1}(\mathrm{EP})^{2}-(1 / 6) \mathrm{P}_{1}^{3} \mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{3} \tag{A.1.10}
\end{equation*}
$$

From (A.1.3):

$$
\begin{equation*}
E P=E P^{*}+\left[2 Q_{1}(\eta-\varepsilon)\right]^{-1} P_{1}^{2}\left[S^{(2)}(c)(E P-\lambda)^{2}-D^{(2)}\left(c_{1}\right)(E P)^{2}\right]=E P^{*}+\Delta_{p} \tag{A.1.11}
\end{equation*}
$$

Substituting $E P^{*}+\Delta_{\mathrm{p}}$, for EP in (A.1.10) we have

$$
\begin{align*}
& \Delta C S=-\mathrm{P}_{1} \mathrm{Q}_{1}\left(\mathrm{EP}^{*}+\Delta_{\mathrm{p}}\right)-(1 / 2) \eta \mathrm{P}_{1} \mathrm{Q}_{1}\left(\mathrm{EP}^{*}+\Delta_{\mathrm{p}}\right)^{2}-(1 / 6) \mathrm{P}_{1}{ }^{3} \mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{3} \\
& =-\mathrm{P}_{1} \mathrm{Q}_{1} \mathrm{EP}^{*}\left(1+0.5 \mathrm{EQ}^{*}\right)-\mathrm{P}_{1} \mathrm{Q}_{1} \Delta_{\mathrm{p}}-\eta \mathrm{P}_{1} \mathrm{Q}_{1} \mathrm{Er}^{*} \Delta_{\mathrm{p}}-(1 / 6) \mathrm{P}_{1}^{3} \mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{3} \\
& \text { (using } E Q^{*}=\eta E P^{*} \text { from Equations (3.11) and (3.12)) } \\
& =\Delta C S^{*}-\mathrm{P}_{1} \mathrm{Q}_{1} \Delta_{\mathrm{p}}-\eta \mathrm{P}_{1} \mathrm{Q}_{1} E P^{*} \Delta_{\mathrm{p}}-(1 / 6) \mathrm{P}_{1}^{3} \mathrm{D}^{(3)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{3} \\
& \left.\Delta \mathrm{CS}=\Delta \mathrm{CS}^{*}-[2(\eta-\varepsilon)]^{-1} \mathrm{P}_{1}^{3}\left[\mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right) \mathrm{EP}-\lambda\right)^{2}-\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{2}\right]  \tag{A.1.12}\\
& -[2(\eta-\varepsilon)]^{-1} \eta P_{1}{ }^{3}\left[S^{(2)}\left(c_{2}\right)(E P-\lambda)^{2} E P^{*}-D^{(2)}\left(c_{1}\right)(E P)^{2} E P^{*}\right] \\
& -\left[8 \mathrm{Q}_{1}(\eta-\varepsilon)^{2}\right]^{-1} \eta \mathrm{P}_{1}^{5}\left[S^{(2)}\left(\mathrm{c}_{2}\right)(E P-\lambda)^{2}-\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(E P)^{2}\right]^{2} \\
& -(1 / 6) P_{1}{ }^{3} D^{(2)}\left(c_{1}\right)(E P)^{3}
\end{align*}
$$

Since $|E P-\lambda|=\mathbf{O}(\lambda),|E P|=\mathbf{O}(\lambda)$ and $\left|E P^{*}\right|=\mathbf{O}(\lambda)$ when $\lambda \rightarrow 0$, we have

$$
\Delta \operatorname{CS}-\Delta \mathrm{CS}^{*}=\mathbf{O}\left(\lambda^{2}\right)+\mathbf{O}\left(\lambda^{3}\right)+\mathbf{O}\left(\lambda^{4}\right)=\mathbf{O}\left(\lambda^{2}\right) \quad(\text { Property }(v i)) \quad \#
$$

Remark 1: The estimate of the consumer surplus change using (3.18) is exact when the demand and supply curves are strictly linear around the neighbourhood of the initial equilibrium point. It is obvious that $\Delta C S=\Delta C^{*}$ when $D^{(2)}\left(c_{1}\right)$ and $S^{(2)}\left(c_{2}\right)$ are zero in equation (A.1.12). \#

Remark 2: Considering the signs of the $\mathbf{O}\left(\lambda^{2}\right), \mathbf{O}\left(\lambda^{3}\right)$ and $\mathbf{O}\left(\lambda^{4}\right)$ terms in the error expression in (A.1.12) under the assumptions in (A.1.5) and (A.1.6), and noting that $|\mathrm{EP}-\lambda| \leq|\lambda|$ and $|\mathrm{EP}|$ $\leq|\lambda|$, the upper bound for the error in consumer surplus change is:

$$
\begin{align*}
& \text { 3) } \quad\left|\Delta \mathrm{CS}-\Delta \mathrm{CS}^{*}\right| \leq|2(\eta-\varepsilon)|^{-1} \mathrm{P}_{1}^{3}\left|D^{(2)}\left(c_{1}\right)-\mathrm{S}^{(2)}\left(\mathrm{c}_{2}\right)\right| \lambda^{2}  \tag{A.1.13}\\
& +\max \left(\left|[2(\eta-\varepsilon)]^{-1} \eta \mathrm{P}_{1}^{3}\left[\mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)-\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)\right]\right|,\left|(1 / 6) \mathrm{P}_{1}^{3} \mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)\right|\right) \lambda^{3} \\
& +\left|\left[8 \mathrm{Q}_{1}(\eta-\varepsilon)^{2}\right]^{-1} \eta \mathrm{P}_{1}^{5}\left[\mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)-\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)\right]^{2}\right| \lambda^{4}
\end{align*}
$$

When $\lambda$ is very small, the error in $\Delta \mathrm{CS}^{*}$ can be roughly estimated by the $\mathbf{O}\left(\lambda^{2}\right)$ term, that is:

$$
\begin{equation*}
\mid \Delta C S-\Delta \text { CS }\left.^{*}|\leq \approx| 2(\varepsilon-\eta)\right|^{-1} P_{1}^{3}\left|D^{(2}\left(c_{1}\right)-S^{(2)}\left(c_{2}\right)\right| \lambda^{2} \quad \# \tag{A.1.14}
\end{equation*}
$$

Remark 3: When $\lambda$ is very small,

$$
\begin{equation*}
\Delta \mathrm{CS} \leq \approx \Delta \mathrm{CS}^{*} \tag{A.1.15}
\end{equation*}
$$

since, from the above proof, $\Delta \mathrm{CS}-\Delta \mathrm{CS}^{*}=-\mathrm{P}_{1} \mathrm{Q}_{1} \Delta_{\mathrm{p}}+\mathbf{O}\left(\lambda^{3}\right)$ where $-\mathrm{P}_{1} \mathrm{Q}_{1} \Delta_{\mathrm{p}}=-[2(\eta-\varepsilon)]^{-1}$ $P_{1}{ }^{3}\left[S^{(2)}\left(c_{2}\right)(E P+\lambda)^{2}-D^{(2)}\left(c_{1}\right)(E P)^{2}\right] \leq 0 . \quad \#$

Proposition 4: $\left.\Delta \mathrm{PS}-\Delta \mathrm{PS}{ }^{*}=[2(\eta-\varepsilon)]^{-1} \mathrm{P}_{1}{ }^{3} \mid \mathrm{S}^{(2)}\left(\mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{2}-\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{2}\right]$

$$
\begin{aligned}
& +[2(\eta-\varepsilon)]^{-1} \varepsilon P_{1}^{3}\left(E P^{*}-\lambda\right)\left[S^{(2)}\left(c_{2}\right)(E P-\lambda)^{2}-D^{(21}\left(c_{1}\right)(E P)^{2}\right]+(1 / 6) P_{1}^{3} S^{(2)}\left(c_{2}\right)(E P-\lambda)^{3} \\
& +\left[8 \mathrm{Q}_{1}(\eta-\varepsilon)^{2}\right]^{-1} \varepsilon P_{1}^{5}\left[S^{(2)}\left(c_{2}\right)(E P-\lambda)^{2}-D^{(2)}\left(c_{1}\right)(E P)^{2}\right]^{2}=\mathbf{O}\left(\lambda^{2}\right)
\end{aligned}
$$

Proof: Expanding $S_{1}: Q=S(P-K)$ at point $P_{1}+K$ :

$$
S(P-K)=S\left(P_{1}\right)+S^{(1)}\left(P_{1}\right)\left(P-P_{1}-K\right)+(1 / 2) S^{(2)}\left(\mathrm{c}_{2}\right)\left(P-P_{1}-K\right)^{2}
$$

Thus $\quad \Delta \mathrm{PS}=\int_{P 1+\mathrm{K}}^{P_{2}} S(P-\mathrm{K}) d P$

$$
\begin{equation*}
=\mathrm{P}_{1} \mathrm{Q}_{1}(\mathrm{EP}-\lambda)+(1 / 2) \varepsilon \mathrm{P}_{1} \mathrm{Q}_{1}(\mathrm{EP}-\lambda)^{3}+(1 / 6) \mathrm{P}_{1}^{3} \mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{3} \tag{A.1.16}
\end{equation*}
$$

$=\mathrm{P}_{1} \mathrm{Q}_{1}\left(\mathrm{EP}^{*}-\lambda+\Delta_{\mathrm{p}}\right)+(1 / 2) \varepsilon \mathrm{P}_{1} \mathrm{Q}_{1}\left(\mathrm{EP}^{*}-\lambda+\Delta_{\mathrm{p}}\right)^{2}+(1 / 6) \mathrm{P}_{1}^{3} \mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{3}$ (using (A.1.11))
$=\mathrm{P}_{1} \mathrm{Q}_{1}\left(\mathrm{EP}^{*}-\lambda\right)\left(1+0.5 \mathrm{EQ}^{*}\right)+\mathrm{P}_{1} \mathrm{Q}_{1} \Delta_{\mathrm{p}}+(1 / 2) \varepsilon \mathrm{P}_{1} \mathrm{Q}_{1}\left[2\left(\mathrm{EP}^{*}-\lambda\right) \Delta_{\mathrm{p}}+\Delta_{\mathrm{p}}{ }^{2}\right]$
$+(1 / 6) \mathrm{P}_{1}{ }^{3} \mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{3} \quad$ (using $E Q^{*}=\varepsilon\left(E P^{*}-\lambda\right)$ from (4) \& (5))

$$
\begin{equation*}
=\Delta \mathrm{PS}^{*}+[2(\eta-\varepsilon)]^{-1} \mathrm{P}_{1}^{3}\left[\mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{2}-\mathrm{D}^{(2)}\left(\mathbf{c}_{1}\right)(\mathrm{EP})^{2}\right] \tag{A.1.17}
\end{equation*}
$$

$$
+[2(\eta-\varepsilon)]^{-1} \varepsilon P_{1}^{3}\left(E P^{*}-\lambda\right)\left[S^{(2)}\left(c_{2}\right)(E P-\lambda)^{2}-D^{(2)}\left(c_{1}\right)(E P)^{2}\right]+(1 / 6) P_{1}^{3} S^{(2)}\left(c_{2}\right)(E P-\lambda)^{3}
$$

$$
+\left[8 \mathrm{Q}_{1}(\eta-\varepsilon)^{2}\right]^{-1} \varepsilon \mathrm{P}_{1}^{5}\left[\mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)(E P-\lambda)^{2} \cdots D^{(2)}\left(c_{1}\right)(E P)^{2}\right]^{2}
$$

$$
=\Delta \mathrm{PS}^{*}+\mathbf{O}\left(\lambda^{2}\right)+\mathbf{O}\left(\lambda^{3}\right)+\mathbf{O}\left(\lambda^{4}\right)=\mathbf{O}\left(\lambda^{2}\right) \quad(\text { Property }(v i))
$$

Remark 1: The EDM estimate for the producer surplus change $\Delta \mathrm{PS}^{*}$ from (3.19) is exact when demand and supply are strictly linear around the neighbourhood of $\mathrm{E}_{1}$. It is obvious that $\Delta \mathrm{PS}=$ $\Delta \mathrm{PS}^{*}$ when $\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)$ and $\mathrm{S}^{(2)}\left(\mathrm{c}_{2}\right)$ are zero in the error expression (A.1.17). \#

Remark 2: Considering the signs of the $\mathbf{O}\left(\lambda^{2}\right), \mathbf{O}\left(\lambda^{3}\right)$ and $\mathbf{O}\left(\lambda^{4}\right)$ terms in the error expression in (A.1.17) under the assumptions in (A.1.5) and (A.1.6) and noting that $|\mathrm{EP}-\lambda| \leq|\lambda|$ and $|\mathrm{EP}| \leq$ $|\lambda|$, the upper bound for the error in producer surplas change is:

$$
\begin{align*}
& \text { 8) }\left|\Delta \mathrm{PS}-\Delta \mathrm{PS}^{*}\right| \leq|2(\eta-\varepsilon)|^{-1} \mathrm{P}_{1}^{3}\left|\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)-\mathrm{S}^{(2)}\left(\mathrm{c}_{2}\right)\right| \lambda^{2}  \tag{A.1.18}\\
& +\max \left(\left|[2(\eta-\varepsilon)]^{-1} \varepsilon \mathrm{P}_{1}^{3}\left[\mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)-\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)\right]\right|,\left|(1 / 6) \mathrm{P}_{1}^{3} \mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)\right|\right) \lambda^{3}
\end{align*}
$$

$$
+\mid\left[8 \mathrm{Q}_{1}(\eta-\varepsilon)^{2}\right]^{-1} \varepsilon P_{1}^{5}\left[S^{(2)}\left(\mathrm{c}_{2}\right)-\left.\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)\right|^{2} \mid \lambda^{4}\right.
$$

When $\lambda$ is very small, the $\mathbf{O}\left(\lambda^{3}\right)$ and $\mathbf{O}\left(\lambda^{4}\right)$ terms are much smaller than the $\mathbf{O}\left(\lambda^{2}\right)$ term. Thus the error in $\Delta \mathrm{PS}^{*}$ is approximately equal to the $\mathbf{O}\left(\lambda^{2}\right)$ term, that is:

$$
\begin{equation*}
\left|\Delta \mathrm{PS}-\Delta \mathrm{PS} \mathrm{~S}^{*}\right| \leq \approx|2(\eta-\varepsilon)|^{-1} \mathrm{P}_{1}^{3}\left|\mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)-\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)\right| \lambda^{2} \quad \# \tag{A.1.19}
\end{equation*}
$$

Remark 3: We almost always underestimate producer surplus gain under the assumptions (A.1.5) and (A.1.6). From (A.1.17), $\Delta \mathrm{PS}-\Delta \mathrm{PS}=\mathrm{P}_{1} \mathrm{Q}_{1} \Delta_{\mathrm{p}}+\mathbf{O}\left(\lambda^{3}\right)$ where the $\mathbf{O}\left(\lambda^{2}\right)$ term $P_{1} Q_{1} \Delta_{\mathrm{P}}=[2(\eta-\varepsilon)]^{-1} \mathrm{P}_{1}^{3}\left[\mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)(E P-\lambda)^{2}-\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(E P)^{2}\right] \geq 0$. Therefore when $\lambda$ is very small,

$$
\begin{equation*}
\Delta \mathrm{PS} \geq \approx \Delta \mathrm{PS}^{*} \tag{A.1.20}
\end{equation*}
$$

Proposition 5: $\quad \Delta \mathrm{TS}-\Delta \mathrm{TS}{ }^{*}=-[4(\eta-\varepsilon)]^{-1} \eta \mathrm{P}_{1}{ }^{3} \mathrm{EP}^{1}\left[\mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{2}-\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{2}\right]$

$$
\begin{aligned}
& +(1 / 6) \mathrm{P}_{1}^{3}\left[\mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{3}-\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{3}\right] \\
& +[4(\eta-\varepsilon)]^{-1} \varepsilon \mathrm{P}_{1}^{3}(\mathrm{EP}-\lambda)\left[\mathrm{S}^{(2)}\left(\mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{2}-\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{2}\right]=\mathbf{O}\left(\lambda^{3}\right)
\end{aligned}
$$

In other words, the EDM total surplus change measure from Equation (3.20) involves infinitesimal error of order $\mathbf{O}\left(\lambda^{3}\right)$, which is much smaller than the errors in the estimates of producer or consumer surplus change.

Proof: $\Delta \mathrm{TS}=\Delta \mathrm{CS}+\Delta \mathrm{PS}$

$$
\begin{aligned}
& =-\mathrm{P}_{1} \mathrm{Q}_{1} \mathrm{EP}-(1 / 2) \eta \mathrm{P}_{1} \mathrm{Q}_{1}(\mathrm{EP})^{2}-(1 / 6) \mathrm{P}_{1}^{3} \mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{3} \\
& +\mathrm{P}_{1} \mathrm{Q}_{1}(\mathrm{EP}-\lambda)+(1 / 2) \varepsilon \mathrm{P}_{1} \mathrm{Q}_{1}(\mathrm{EP}-\lambda)^{2}+(1 / 6) \mathrm{P}_{1}^{3} \mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{3} \\
& =\lambda \mathrm{P}_{1} \mathrm{Q}_{1}-(1 / 2) \eta \mathrm{P}_{1} \mathrm{Q}_{1}\left(\mathrm{EP}^{*}+\Delta_{\mathrm{P}}\right)^{2}+(1 / 2) \varepsilon \mathrm{P}_{1} \mathrm{Q}_{1}\left(\mathrm{EP}^{*}+\Delta_{\mathrm{p}}-\lambda\right)^{2} \\
& +(1 / 6) \mathrm{P}_{1}^{3}\left[\mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{3}-\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{3}\right]
\end{aligned}
$$

Using $E Q^{*}=\eta E P^{*}$ and $E Q^{*}=\varepsilon\left(E P^{*}-\lambda\right)$ from (4) \& (5), we have:

$$
\begin{aligned}
& \Delta \mathrm{TS}=\lambda \mathrm{P}_{1} \mathrm{Q}_{1}+(1 / 2) \lambda \mathrm{P}_{1} \mathrm{Q}_{1} E \mathrm{Q}^{*}-(1 / 2) \eta \mathrm{P}_{1} \mathrm{Q}_{1} \mathrm{EP} \Delta_{\mathrm{p}} \\
& \quad+(1 / 6) \mathrm{P}_{1}^{3}\left[\mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{3}-\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)\left(E \mathrm{EP}^{3}\right]+(1 / 2) \varepsilon \mathrm{P}_{1} \mathrm{Q}_{1} \Delta_{\mathrm{p}}(\mathrm{EP}-\lambda)\right.
\end{aligned}
$$

$$
(\mathrm{A} .1 .21)=\Delta \mathrm{TS}^{*}-[4(\eta-\varepsilon)]^{-1} \eta \mathrm{P}_{1}^{3} \mathrm{EP}\left[\mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)\left(\mathrm{EP}-\lambda 1^{2}-\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{2}\right]\right.
$$

$$
+(1 / 6) \mathrm{P}_{1}^{3}\left[\mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{3}-\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{3}\right]
$$

$$
+[4(\eta-\varepsilon)]^{-1} \varepsilon P_{1}^{3}(E P-\lambda)\left[S^{(2)}\left(c_{2}\right)(E P-\lambda)^{2}-D^{(2)}\left(c_{1}\right)(E P)^{2}\right]
$$

$$
=\mathbf{O}\left(\lambda^{3}\right) \quad \#
$$

Remark 1: The EDM measure of total surplus change $\Delta \mathrm{TS}^{*}$ is exact for local linear demand and supply functions. This is obvious when $D^{(2)}\left(c_{1}\right)$ and $S^{(2)}\left(c_{2}\right)$ are zero in the error expression (A.1.21). \#

Remark 2: The upper bound for the error in total surplus change is:

$$
\left|\Delta \mathrm{TS}-\Delta \mathrm{TS}^{*}\right| \leq \max (|(5 \eta-2 \varepsilon)|,|\beta \varepsilon|)|12(\eta-\varepsilon)|^{-1} \mathrm{P}_{1}{ }^{3}\left|\mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)-\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)\right| \lambda^{3}
$$

Denote the three terms in the error expression as $\Delta \mathrm{TS}-\Delta \mathrm{TS}^{*}=\mathrm{T}_{1}+\mathrm{T}_{2}+\mathrm{T}_{3}$. Under the assumptions in (A.1.5) and (A.1.6), $T_{1}<0, T_{2}<0$ and $T_{3}>0$, and also $|E P-\lambda| \leq|\lambda|$ and $|E P| \leq|\lambda|$. Therefore, $\left|\Delta \mathrm{TS}-\Delta \mathrm{TS}^{*}\right| \leq \max \left(\left|\mathrm{T}_{1}+\mathrm{T}_{2}\right|,\left|\mathrm{T}_{3}\right|\right) \leq \max (|(5 \eta-2 \varepsilon)|,|3 \varepsilon|)|12(\eta-\varepsilon)|^{-1} \mathrm{P}_{1}^{3} \mid \mathrm{S}^{(2)}\left(\mathrm{c}_{2}\right)-$ $\mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right) \mid \lambda^{3}$

Remark 3: $\Delta \mathrm{TS}$ can be over or under estimated depending on the relative sizes of $\left|\mathrm{T}_{1}+\mathrm{T}_{2}\right|$ and $\left|\mathrm{T}_{3}\right|$. \#

Proposition 6: If the price and quantity changes were known, that is, $E P=E P^{*}$ and $E Q=E Q^{*}$, the errors in the surplus measures would only be of the order of $\mathbf{O}\left(\lambda^{3}\right)$ when $\lambda \rightarrow 0$, that is, if we define
(A.1.22) $\quad \Delta \mathrm{CS}^{\sim}=-\mathrm{P}_{1} \mathrm{Q}_{1} \mathrm{EP}(1+0.5 \mathrm{EQ})$
(A.1.23)

$$
\Delta \mathrm{PS}^{\sim}=\mathrm{P}_{1} \mathrm{Q}_{1}(\mathrm{EP}-\lambda)(1+0.5 \mathrm{EQ})
$$

(A.1.24)

$$
\Delta \mathrm{TS}^{\sim}=-\lambda \mathrm{P}_{1} \mathrm{Q}_{1}(1+0.5 \mathrm{EQ})
$$

then
(A.1.25) $\left|\Delta \mathrm{CS}-\Delta \mathrm{CS}^{-}\right|=\mathbf{O}\left(\lambda^{3}\right)$
(A.1.26) $\quad \mid \Delta \mathrm{PS}-\Delta \mathrm{PS} \rightleftharpoons=\mathbf{O}\left(\lambda^{3}\right)$
(A.1.27) $\quad \mid \Delta \mathrm{TS}-\Delta \mathrm{TS} \sim=\mathbf{O}\left(\lambda^{3}\right)$

Proof: From equation (A.1.10):

$$
\begin{aligned}
\Delta \mathrm{CS} & =-\mathrm{P}_{1} \mathrm{Q}_{1} \mathrm{EP}-(1 / 2) \eta \mathrm{P}_{1} \mathrm{Q}_{1}(E P)^{2}-(1 / 6) \mathrm{P}_{1}^{3} \mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{3} \\
& =-\mathrm{P}_{1} \mathrm{Q}_{1} \mathrm{EP}-(1 / 2) \eta \mathrm{P}_{1} \mathrm{Q}_{1}(\mathrm{EP})^{2}+\mathbf{O}\left(\lambda^{\hat{j}}\right)
\end{aligned}
$$

Also, from equation (A.1.1):

$$
\begin{aligned}
& \quad \mathrm{EQ}=\eta \mathrm{EP}+\left(2 \mathrm{Q}_{1}\right)^{-1} \mathrm{P}_{1}{ }^{2} \mathrm{D}^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{2}=\eta \mathrm{EP}+\mathbf{O}\left(\lambda^{2}\right) \\
& \therefore \quad \Delta \mathrm{CS}=-\mathrm{P}_{1} \mathrm{Q}_{1} \mathrm{EP}-(1 / 2) \mathrm{P}_{1} \mathrm{Q}_{1} \mathrm{EP}\left(\mathrm{EQ}+\mathbf{O}\left(\lambda^{2}\right)\right)+\mathbf{O}\left(\lambda^{3}\right) \\
& \\
& =-\mathrm{P}_{1} \mathrm{Q}_{1} \mathrm{EP}(1+0.5 \mathrm{EQ})-(1 / 2) \mathrm{P}_{1} \mathrm{Q}_{1} \mathrm{EP} \mathbf{O}\left(\lambda^{2} 1+\mathbf{O}\left(\lambda^{3}\right) \quad\right. \text { (Properties (ii) \& (vi)) } \\
& \\
& =\Delta \mathrm{CS}^{\sim}+\mathbf{O}\left(\lambda^{3}\right) \quad \text { and }(\mathrm{A} .1 .25) \text { is thus proven. }
\end{aligned}
$$

Similarly, from (A.1.16):

$$
\Delta \mathrm{PS}=\mathrm{P}_{1} \mathrm{Q}_{1}(\mathrm{EP}-\lambda)+(1 / 2) \varepsilon \mathrm{P}_{1} \mathrm{Q}_{1}(\mathrm{EP}-\lambda)^{2}+(1 / 6) \mathrm{P}_{1}^{3} \mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{3}
$$

$$
=\mathrm{P}_{1} \mathrm{Q}_{1}(\mathrm{EP}-\lambda)+(1 / 2) \varepsilon \mathrm{P}_{1} \mathrm{Q}_{1}(\mathrm{EP}-\lambda)^{2}+\mathrm{O}\left(\lambda^{3}\right)
$$

$$
\begin{aligned}
& \text { From (A.1.2): } \mathrm{EQ}=\varepsilon(\mathrm{EP}-\lambda)+\left(2 \mathrm{Q}_{1}\right)^{-1} \mathrm{P}_{1}{ }^{2} \mathrm{~S}^{(2)}\left(\mathrm{c}_{2}\right)(\mathrm{EP}-\lambda)^{2}=\varepsilon(\mathrm{EP}-\lambda)+\mathbf{O}\left(\lambda^{2}\right) \\
& \begin{aligned}
\therefore \quad \Delta \mathrm{PS} & \left.=\mathrm{P}_{1} \mathrm{Q}_{1}(\mathrm{EP}-\lambda)(1+0.5 \mathrm{EQ})+(1 / 2) \mathrm{P}_{1} \mathrm{Q}_{1} \mathrm{EP}-\lambda\right) \mathbf{O}\left(\lambda^{2}\right)+\mathbf{O}\left(\lambda^{3}\right) \\
& \left.=\Delta \mathrm{PS}^{\sim}+\mathbf{O}\left(\lambda^{3}\right) \quad \text { (Properties (ii) } \&(v i)\right)
\end{aligned}
\end{aligned}
$$

This proved (A.1.26). (A.1.27) is obvious since $\Delta \Gamma S=\Delta \mathrm{CS}+\Delta \mathrm{PS}$ and $\Delta \mathrm{TS}^{\sim}=\Delta \mathrm{CS}^{\sim}+\Delta \mathrm{PS}^{\sim}$. \#

## A1.2 Proportional Shift and Log-Linear Approximation

The derivations in this section relate to the specifications in Equations (3.13)-(3.15) and (3.29)(3.34) in Chapter 3 for the case of proportional shift and log-linear approximation using EDM.

For convinience of mathematical manipulation, assume that the demand and supply functions in Equations (3.1) and (3.2) can be expressed in logarithmic relationships as
(A.1.28) $\quad \mathrm{S}_{\mathrm{L} 1}: \mathrm{U}=\mathrm{S}_{\mathrm{L}}(\mathrm{V}) \quad$ initicl supply curve
(A.1.29) $\quad \mathrm{D}_{\mathrm{L} 1}: \mathrm{U}=\mathrm{D}_{\mathrm{L}}(\mathrm{V}) \quad$ initicl demand curve
where
(A.1.30) $U=\ln Q \quad$ and $\quad V=\ln P$

Define percentage change as
(A.1.31)

$$
\mathrm{E}(.)=\Delta \ln (.)
$$

Assume that a new technology will cause a proportional supply shift by a constant percentage $\lambda$ along the price direction, which, according to the definition in (A.1.31), is equilvalent to a
parallel shift of $\mathrm{S}_{\mathrm{L}}$ on the ( $\mathrm{U}, \mathrm{V}$ ), or ( $\operatorname{lnQ}, \ln \mathrm{P}$ ), plane by a constant $\lambda$ along V direction ${ }^{1}$. The new supply curve will be
(A.1.32) $\quad \mathrm{S}_{\mathrm{L} 2}: \quad \mathrm{U}=\mathrm{S}_{\mathrm{L}}(\mathrm{V}-\lambda) \quad$ new supply curve
where $\lambda<0$ representing a downward supply shift. On the $(\mathrm{U}, \mathrm{V})$ plane, the initial equilibrium point is $E_{1}\left(U_{1}, V_{1}\right)$ and the new equilibrium point is denoted as $E_{2}\left(U_{2}, V_{2}\right)$, where $U_{i}=\ln Q_{i}$ and $\mathrm{V}_{\mathrm{i}}=\ln \mathrm{P}_{\mathrm{i}}(\mathrm{i}=1,2)$. From Equation (3.14),

$$
\begin{equation*}
E P=V_{2}-V_{1} \text { and } E Q=U_{2}-U_{1} \tag{A.1.33}
\end{equation*}
$$

Similar results as for the linear case can be shown by Taylor expanding $\mathrm{S}_{\mathrm{L}}($.$) and \mathrm{D}_{\mathrm{L}}($.$) on the$ $(\mathrm{U}, \mathrm{V})$ plane, instead of $\mathrm{S}($.$) and \mathrm{D}($.$) on the (\mathrm{Q}, \mathrm{P})$ plane.

Proposition 7: Under the specifications in Equations (3.13)-(3.15), when $\lambda \rightarrow 0$,

$$
\begin{equation*}
E P-E P^{*}=[2(\eta-\varepsilon)]^{-1}\left[S_{L}^{(2)}\left(k_{2}\right)(E P-\lambda)^{2}-D_{L}^{(2)}\left(k_{1}\right)(E P)^{2}\right]=\mathbf{O}\left(\lambda^{2}\right) \tag{A.1.34}
\end{equation*}
$$

$$
\begin{equation*}
\text { EQ - EQ }{ }^{*}=[2(\eta-\varepsilon)]^{-1}\left[\eta S_{L}^{(2)}\left(\mathrm{k}_{2}\right)(E P-\lambda)^{2}-\varepsilon \mathrm{D}_{\mathrm{L}}^{(2)}\left(\mathrm{k}_{1}\right)(\mathrm{EP})^{2}\right]=\mathbf{O}\left(\lambda^{2}\right) \tag{A.1.35}
\end{equation*}
$$

Proof. Taylor expanding demand function $\mathrm{D}_{\mathrm{LI}}$ at point $\mathrm{V}_{1}=\ln \mathrm{P}_{1}$ using Taylor Expansion formula with remainder and taking the value at point $\mathrm{V}_{2}=\ln \mathrm{P}_{2}$, we have

$$
D_{L}\left(V_{2}\right)=D_{L}\left(V_{1}\right)+D_{L}^{(1)}\left(V_{1}\right)\left(V_{2}-V_{1}\right)+(1 / 2) D_{L}^{(2)}\left(k_{1}\right)\left(V_{2}-V_{1}\right)^{2}
$$

ie. $\quad U_{2}=U_{1}+D_{L}{ }^{(1)}\left(\mathrm{V}_{1}\right) E P+(1 / 2) D_{L}^{(2)}\left(k_{1}\right)(E P)^{2}$
or

$$
\mathrm{EQ}=\mathrm{U}_{2}-\mathrm{U}_{1}=\mathrm{D}_{\mathrm{L}}^{(1)}\left(\mathrm{V}_{1}\right) \mathrm{EP}+(1 / 2) \mathrm{D}_{\mathrm{L}}^{(2)}\left(\mathrm{k}_{1}\right)(\mathrm{EP})^{2}
$$

$$
\begin{equation*}
=\eta E P+(1 / 2) D_{L}^{(2)}\left(k_{1}\right)(E P)^{2} \tag{A.1.36}
\end{equation*}
$$

[^0]where $D_{L}{ }^{(i)}($.$) is the ith derivative of D_{L}(),. \eta$ is the demand elasticity at point $P_{1}$ and $V_{2} \leq k_{1} \leq$ $\mathrm{V}_{1}$. Similarly, Taylor expanding new supply function $\mathrm{S}_{\mathrm{L} 2}$ in equation (A.1.32) at point $\mathrm{V}_{1}+\lambda$ and evaluating at point $V_{2}$, we have
$$
\mathrm{U}_{2}=\mathrm{S}_{\mathrm{L}}\left(\mathrm{~V}_{2}-\lambda\right)=\mathrm{S}_{\mathrm{L}}\left(\mathrm{~V}_{1}\right)+\mathrm{S}_{\mathrm{L}}{ }^{(1)}\left(\mathrm{V}_{1}\right)\left(\mathrm{V}_{2}-\mathrm{V}_{1} \cdot \lambda\right)+(1 / 2) \mathrm{S}_{\mathrm{L}}^{(2)}\left(\mathrm{k}_{2}\right)\left(\mathrm{V}_{2}-\mathrm{V}_{1}-\lambda\right)^{2}
$$
where $\mathrm{S}_{\mathrm{L}}{ }^{(\mathrm{i})}($.$) is the ith derivative of \mathrm{S}_{\mathrm{L}}($.$) and \mathrm{V}_{2} \leq \mathrm{k}_{2} \leq \mathrm{V}_{1}$ is a constant. Thus
\[

$$
\begin{equation*}
\mathrm{EQ}=\mathrm{U}_{2}-\mathrm{U}_{1}=\varepsilon(\mathrm{EP}-\lambda)+(1 / 2) \mathrm{S}_{\mathrm{L}}^{(3)}\left(\mathrm{k}_{2}\right)(\mathrm{EP}-\lambda)^{2} \text {, or } \tag{A.1.37}
\end{equation*}
$$

\]

Solving (A.1.36) and (A.1.37) jointly, we have

$$
\begin{aligned}
& \mathrm{EP}=\lambda \varepsilon /(\varepsilon-\eta)+[2(\eta-\varepsilon)]^{-1}\left[\mathrm{~S}_{\mathrm{L}}^{(2)}\left(\mathrm{k}_{2}\right)(\mathrm{EP}-\lambda)^{2}-\mathrm{D}_{\mathrm{L}}^{(2)}\left(\mathrm{k}_{1}\right)(\mathrm{EP})^{2}\right] \\
& \mathrm{EQ}=\lambda \eta \varepsilon /(\varepsilon-\eta)+\left[2 \mathrm{Q}_{1}(\eta-\varepsilon)\right]^{-1}\left[\eta \mathrm{~S}_{\mathrm{L}}{ }^{(2)}\left(\mathrm{c}_{2}\right)(E P-\lambda)^{2}-\varepsilon D_{\mathrm{L}}{ }^{(2)}\left(\mathrm{c}_{1}\right)(\mathrm{EP})^{2}\right]
\end{aligned}
$$

or

$$
\begin{aligned}
& E P-E P^{*}=[2(\eta-\varepsilon)]^{-1}\left[S_{L}^{(2)}\left(k_{2}\right)(E P-\lambda)^{2}-D_{\mathrm{L}}^{(2)}\left(k_{1}\right)(E P)^{2}\right] \\
& E Q-E Q^{*}=[2(\eta-\varepsilon)]^{-1}\left[\eta S_{L}^{(2)}\left(k_{2}\right)(E P-\lambda)^{2}-\varepsilon D_{L}^{(2)}\left(k_{1}\right)(E P)^{2}\right]
\end{aligned}
$$

Since $|\mathrm{EP}-\lambda| \leq|\lambda|$ and $|\mathrm{EP}| \leq|\lambda|$ (obvious from Figure 3.1), $(\mathrm{EP}-\lambda)^{2}=\mathbf{O}\left(\lambda^{2}\right)$ and $(E P)^{2}=\mathbf{O}\left(\lambda^{2}\right)$ when $\lambda \rightarrow 0$. Thus

$$
\begin{aligned}
& \left|E P-E P^{*}\right|=\mathbf{O}\left(\lambda^{2}\right) \\
& \left|E Q-E Q^{*}\right|=\mathbf{O}\left(\lambda^{2}\right) \quad(\lambda \rightarrow 0)
\end{aligned}
$$

Remark 1: $E P=E P^{*}$ and $E Q=E Q^{*}$ when the demand and supply are log-linear around the neighbourhood of the initial equilibrium point. This is obvious when $D_{L}^{(2)}\left(k_{1}\right)$ and $S_{L}^{(2)}\left(k_{2}\right)$ are zero in equations (A.1.34) and (A.1.35). \#

Remark 2: The upper bounds for the errors will the

$$
\begin{equation*}
\left|E P-E P^{*}\right| \leq|2(\varepsilon-\eta)|^{-1} \lambda^{2}\left(\left|D_{L}^{(2)}\left(k_{1}\right)\right|+\left|S_{L}{ }^{(2)}\left(\mathrm{k}_{2}\right)\right|\right) \tag{A.1.38}
\end{equation*}
$$

$$
\begin{equation*}
\left|E Q-E Q^{*}\right| \leq|2(\varepsilon-\eta)|^{-1} \lambda^{2}\left(\left|\varepsilon D_{L}^{(2)}\left(k_{1}\right)\right|+\left|\eta S_{L}^{(2)}\left(\mathrm{k}_{2}\right)\right|\right) \tag{A.1.39}
\end{equation*}
$$

This is because $|E P-\lambda| \leq|\lambda|$ and $|E P| \leq|\lambda| . \quad \#$

Proposition 8: When the exogenous supply shift is of a proportional nature, the economic surplus changes associated with the log-linear curves, as shown by $S_{l}{ }^{*}, S_{2}{ }^{*}$ and $D_{l}{ }^{*}$ in Figure 3.2, are given by

$$
\begin{equation*}
\Delta \mathrm{CS}^{* *}=\operatorname{Area}\left(\mathrm{P}_{1} \mathrm{E}_{1} \mathrm{E}_{2}{ }^{*} \mathrm{P}_{2}{ }^{*}\right)=\int_{P \cdot *}^{P_{1}} \mathrm{D}_{1}{ }^{*}(\mathrm{P}) \mathrm{dP}=\mathrm{P}_{1} \mathrm{Q}_{1}(\eta+1)^{-1}\left(1-\mathrm{e}^{(\eta+1) \mathrm{EP} *}\right) \tag{A.1.40}
\end{equation*}
$$

$$
\begin{align*}
& \Delta \mathrm{PS}^{* *}=\operatorname{Area}\left(\mathrm{OE}_{2}{ }^{*} \mathrm{P}_{2}{ }^{*}\right)-\operatorname{Area}\left(\mathrm{OE}_{1} \mathrm{P}_{1}\right)=\int_{0}^{P_{2}{ }^{*}} \mathrm{~S}_{2}{ }^{*}(\mathrm{P}) \mathrm{dP}-\int_{0}^{P_{1}} \mathrm{~S}_{1}{ }^{*}(\mathrm{P}) \mathrm{dP}  \tag{A.1.41}\\
& =\mathrm{P}_{1} \mathrm{Q}_{1}(\varepsilon+1)^{-1}\left(\mathrm{e}^{(\eta+1) \mathrm{EP}}-1\right) \quad \text { and }
\end{align*}
$$

$$
\begin{equation*}
\Delta \mathrm{TS}^{* *}=\mathrm{P}_{1} \mathrm{Q}_{1}\left((\eta+1)^{-1}-(\varepsilon+1)^{-1}\right)\left(1-\mathrm{c}^{(\eta+1) \mathrm{EP} *}\right) . \tag{A.1.42}
\end{equation*}
$$

Proof: When the parameters for the constant elasticity demand and supply curves, $D_{l}{ }^{*}, S_{l}{ }^{*}$ and $S_{2}{ }^{*}$, are assumed, the economic surplus changes can be calculated through integration. Assume that the constant elasticity demand and supply curves are:

$$
\begin{aligned}
& \mathrm{D}_{1}^{*}: \quad \ln \mathrm{Q}=\alpha_{\mathrm{D}}+\eta \ln \mathrm{P} \quad(\mathrm{P}>0, \mathrm{Q}>0 \text { and } \eta<0) \\
& \\
& \text { or } \quad \mathrm{Q}=\mathrm{e}^{\alpha \mathrm{D}} \mathrm{P}^{\eta}=\mathrm{Q}_{1}\left(\mathrm{P} / \mathrm{P}_{1}\right)^{\eta}
\end{aligned}
$$

where $\eta<0$ is the constant demand elasticity and the scale parameter is identified by the base equilibrium point as $\alpha_{D}=\ln Q_{1}-\eta \ln P_{1}$, and

$$
\begin{aligned}
& \mathrm{S}_{1}^{*}: \quad \ln \mathrm{Q}=\alpha_{\mathrm{s}}+\varepsilon \ln \mathrm{P} \quad(\mathrm{P}>0, \mathrm{Q}>0 \text { and } \varepsilon>0) \\
& \quad \text { or } \quad \mathrm{Q}=\mathrm{e}^{\alpha s} \mathrm{P}^{\varepsilon}=\mathrm{Q}_{1}\left(\mathrm{P} / \mathrm{P}_{1}\right)^{\varepsilon}
\end{aligned}
$$

where $\varepsilon>0$ is the constant supply elasticity and $\alpha=\ln Q_{1}-\varepsilon \ln P_{1}$. The new supply curve $S_{2}{ }^{*}$ is the proportional shift of $\mathrm{S}_{1}{ }^{*}$, which is a parallel shift in the $(\ln \mathrm{P}, \ln \mathrm{Q})$ relationship,

$$
\begin{aligned}
& \mathrm{S}_{2}^{*}: \quad \ln \mathrm{Q}=\alpha_{s}+\varepsilon(\ln \mathrm{P}-\lambda) \quad(\mathrm{P}>0, \mathrm{Q}>0 \text { and } \varepsilon>0) \\
& \\
& \text { or } \quad \mathrm{Q}=\mathrm{e}^{\alpha s-\lambda} \mathrm{P}^{\varepsilon}=\mathrm{Q}_{1}\left(\mathrm{e}^{-\lambda} \mathrm{P} / \mathrm{P}_{1}\right)^{\varepsilon}
\end{aligned}
$$

Thus

$$
\begin{equation*}
\Delta \mathrm{CS}^{* *}=\int_{P_{2^{*}}}^{P_{1}} \mathrm{D}_{1}{ }^{*} \mathrm{dP}=\int_{P_{2^{*}}}^{P_{1}} \mathrm{Q}_{1}\left(\mathrm{P} / \mathrm{P}_{1}\right)^{\eta} \mathrm{dP}=\left(\mathrm{Q}_{1} / \mathrm{P}_{1}{ }^{\eta}\right)(\eta+1)^{-1}\left(\mathrm{P}_{1}{ }^{\eta+1}-\mathrm{P}_{2}^{* \eta+1}\right) \tag{A.1.43}
\end{equation*}
$$

Note that from Equation (3.15),

$$
\begin{equation*}
\mathrm{EP}^{*}=\ln \mathrm{P}_{2}^{*}-\ln \mathrm{P}_{1}, \quad \mathrm{EQ}^{*}:=\ln \mathrm{Q}_{2}^{*}-\ln \mathrm{Q}_{1} \tag{A.1.44}
\end{equation*}
$$

which imply that

$$
\begin{equation*}
\mathrm{P}_{2}{ }^{*}=\exp \left(\ln \mathrm{P}_{1}+E P^{*}\right)=\mathrm{P}_{1} \mathrm{e}^{\mathrm{EP} *} \tag{A.1.45}
\end{equation*}
$$

$$
\begin{equation*}
\mathrm{Q}_{2}^{*}=\exp \left(\ln \mathrm{Q}_{1}+\mathrm{EQ}^{*}\right)=\mathrm{Q}_{1} \mathrm{e}^{\mathrm{EQ}^{*}} \tag{A.1.46}
\end{equation*}
$$

Substituting equation (A.1.45) for $\mathrm{P}_{2}{ }^{*}$ in (A.1.43) gives:

$$
\begin{aligned}
\Delta C S^{* *} & =\left(Q_{1} / P_{1}^{\eta}\right)(\eta+1)^{-1}\left(P_{1}^{\eta+1}-\left(P_{1} e^{E P^{*}}\right)^{\eta+1}\right) \\
& =P_{1} Q_{1}(\eta+1)^{-1}\left(1-e^{(\eta+1) E P^{*}}\right)
\end{aligned}
$$

Similarly,

$$
\begin{aligned}
\Delta \mathrm{PS}^{* *} & \left.=\int_{0}^{P_{2}{ }^{*}} \mathrm{~S}_{2}{ }^{*} \mathrm{dP}-\int_{0}^{P_{1}} \mathrm{~S}_{1}{ }^{*} \mathrm{dP}=\int_{0}^{P_{2} *} \mathrm{Q}_{1} \mathrm{e}^{-\lambda} \mathrm{P} / \mathrm{P}_{1}\right)^{\varepsilon} \mathrm{dP}-\int_{0}^{P_{1}} \mathrm{Q}_{1}\left(\mathrm{P} / \mathrm{P}_{1}\right)^{\varepsilon} \mathrm{dP} \\
& =\left(\mathrm{Q}_{1} / \mathrm{P}_{1}{ }^{\varepsilon}\right)(\varepsilon+1)^{-1}\left(\mathrm{e}^{-\varepsilon \lambda} \mathrm{P}_{2}{ }^{{ }^{\varepsilon}+1}-\mathrm{P}_{1}{ }^{\varepsilon+1} ;\right. \\
& \left.=\mathrm{P}_{1} \mathrm{Q}_{1}(\varepsilon+1)^{-1}\left(\mathrm{e}^{(\varepsilon+1) E \mathrm{P}^{*}-\varepsilon \lambda}-1\right) \quad \text { (using (A.1.45) for } \mathrm{P}_{2}{ }^{*}\right)
\end{aligned}
$$

$$
=P_{1} Q_{1}(\varepsilon+1)^{-1}\left(e^{(\eta+1) E P^{*}}-1\right) \quad\left(\text { using } E P^{*}=\lambda \varepsilon /(\varepsilon-\eta) \text { from }(3.15)\right)
$$

and

$$
\Delta \mathrm{TS}^{* *}=\Delta \mathrm{CS}^{* *}+\Delta \mathrm{PS}^{* *}=\mathrm{P}_{1} \mathrm{Q}_{1}\left((\eta+1)^{-1}-(\xi+1)^{-1}\right)\left(1-\mathrm{e}^{(\eta+1) E \mathrm{EP}^{*}}\right) \#
$$

## Appendix 2. Derivation of Integrability Conditions

In this appendix, the properties of cost, revenue, profit and utility functions are discussed in turn and used to derive the required properties for the demand and supply functions and their implications for the market parameters in the EDM model specification.

## A2.1 Cost Functions and Output-Constrained Input Demand

Consider first the properties of the cost function $C(w, y)$ of any multi-output technology as defined in Equation (4.3.3). To be a cost function, $C(w, y)$ needs to be positive for $y>0$, nondecreasing in $w$, concave and continuous in $w$, linearly homogenous in $w$, equal to zero when $y=0$ (as a $n \times 1$ vector) and nondecreasing in $y$ (Chambers 1991, p262). When $C(w, y)$ is twice-continuously differentiable, the comparative static properties of the derived outputconstrained input demands in Equation (4.3.6) are characterised by the requirements that (i) the derived demands $x(w, y)=\nabla_{w} C(w, y)$ be homogenous of degree zero in $\mathbf{w}$; (ii) the Hessian matrix $\nabla_{w w} C(w, y)=\nabla_{w} x(w, y)$ be symmetric and negative semidefinite; (iii) the gradient of marginal costs $\nabla_{y} C(w, y)$ be homogenous of degree 1 in $w$; and (iv) $\left(\partial^{2} C / \partial w_{i} \partial y_{j}\right)=\left(\partial^{2} C / \partial y_{j} \partial w_{i}\right)$ $(i=1, \ldots, k ; j=1, \ldots, n)$ (Chambers 1991, p262. The definition of gradient $\nabla$ is obvious from the discussion). These are the four conditions that input demands for the six industry sectors in Equations (4.4.5)-(4.4.8), (4.4.19)-(4.4.22), (4.428)-(4.4.32) and (4.4.41)-(4.4.46) in the model need to satisfy in order to be integrable to recover the "proper" cost functions in Equations (4.3.16)-(4.3.21). As market elasticity values are required to solve the displacement model in Equations (4.4.1)'-(4.4.58)', the implications of the above integrability conditions for the elasticities are examined below.

First, $x=x(w, y)=\left(x_{l}(w, y), \ldots, x_{k}(w, y)\right)^{\prime}$ homogenous of degree zero $(\mathrm{HD}(0))$ in $w$ implies that for any $\lambda>0$,

$$
\begin{equation*}
\left.\mathrm{x}_{\mathrm{i}}(\lambda \mathrm{w}, \mathrm{y})=\mathrm{x}_{\mathrm{i}}(\mathrm{w}, \mathrm{y}) \quad(i=1, \ldots, \mathrm{k}) \quad \text { (homogeneity }\right) . \tag{A.2.1}
\end{equation*}
$$

The necessary and sufficient condition for a function $f(z)=f\left(z_{l}, \ldots, z_{k}\right)$ to be $\operatorname{HD}(\mathrm{m})$ is that $\sum_{j=1}^{k} z_{j} \frac{\partial f(z)}{\partial z_{j}}=m f(z)$ (Euler Theorem. Berck and Sydseter 1992, p15). Thus, $x(w, y)$ is $\operatorname{HD}(0)$ in $w$ if and only if $\sum_{j=1}^{k} \frac{\partial x_{i}(w, y)}{\partial w_{j}} w_{j}=0$, or $\sum_{j=.}^{k} \frac{\partial x_{i}(w, y)}{\partial w_{i}} \frac{w_{j}}{x_{i}(w, y)}=0(i=1, \ldots, k)$. That is,

$$
\begin{equation*}
\sum_{\mathrm{j}=1}^{\mathrm{k}} \widetilde{\eta}_{\mathrm{ij}}(\mathrm{w}, \mathrm{y})=0 \quad(i=1, \ldots, \mathrm{k}) \quad \text { (homogeneity), } \tag{A.2.1}
\end{equation*}
$$

where $\tilde{\eta}_{\mathrm{ij}}(\mathrm{w}, \mathrm{y})$ is the constant-output input demand elasticity of $x_{i}$ with respect to a change in the input price $w_{j}(i, j=1, \ldots, k)$. Using Allen-Uzawa's definition of elasticity of input substitution (McFadden 1978, p79-80)

$$
\begin{equation*}
\tilde{\eta}_{\mathrm{ij}}(\mathrm{w}, \mathrm{y})=\mathrm{s}_{\mathrm{j}}(\mathrm{w}, \mathrm{y}) \sigma_{\mathrm{ij}}(\mathrm{w}, \mathrm{y}) \quad(i, j=1, \ldots, \mathrm{k}), \tag{A.2.2}
\end{equation*}
$$

Equation (A.2.1)' can be written as
(A.2.1)"

$$
\sum_{j=1}^{k} s_{j}(w, y) \sigma_{i j}(w, y)=0 \quad(i=1, \ldots, \mathrm{k}) \quad \text { (homogeneity) }
$$

where $s_{j}():.=\left(w_{j} x_{j} / C\right)$ is the share of the $j$ th input in total cost and $\sigma_{i j}(w, y)$ is the Allen-Uzawa elasticity of substitution between the $i$ th and $j$ th inputs $(i, j=1, \ldots, k)$.

Second, by definition the Hessian matrix can be written as

$$
\mathrm{H}=\nabla_{\mathrm{ww}} \mathrm{C}(\mathrm{w}, \mathrm{y})=\nabla_{\mathrm{w}} \mathrm{x}(\mathrm{w}, \mathrm{y})=\left(\frac{\partial x_{i}(w, y)}{\partial w_{j}}\right)_{k \times k}=\left(\eta_{i j}(w, y) \frac{x_{i}(w, y)}{w_{j}}\right)_{k \times k} .
$$

When the homogeneity condition is satisfied, the columns of H are linearly correlated satisfying $\sum_{j=1}^{k} \frac{\partial x_{i}(w, y)}{\partial w_{j}} w_{j}=0(i=1, \ldots, \mathrm{k})$. This implies that H is singular, or

$$
|\mathrm{H}|=0 .
$$

H is symmetric implies that

$$
\begin{equation*}
\frac{\partial x_{i}(w, y)}{\partial w_{j}}=\frac{\partial x_{j}(w, y)}{\partial w_{i}} \quad(\mathrm{i}, \mathrm{j}:=1, \ldots, \mathrm{k}) \text { (symmetry), or } \tag{A.2.3}
\end{equation*}
$$

$$
\tilde{\mathrm{\eta}}_{\mathrm{ij}}(\mathrm{w}, \mathrm{y}) \frac{\mathrm{x}_{\mathrm{i}}}{\mathrm{w}_{\mathrm{j}}}=\tilde{\eta}_{\mathrm{ji}}(\mathrm{w}, \mathrm{y}) \frac{\mathrm{x}_{\mathrm{j}}}{\mathrm{w}_{\mathrm{i}}} \quad(i, j=1, \ldots, \mathrm{k}) \quad \text { (symmetry), or }
$$

(A.2.3)'

$$
\mathrm{s}_{\mathrm{i}}(\mathrm{w}, \mathrm{y}) \tilde{\eta}_{\mathrm{ij}}(\mathrm{w}, \mathrm{y})=\mathrm{s}_{\mathrm{j}}(\mathrm{w}, \mathrm{y}) \tilde{\eta}_{\mathrm{ji}}(\mathrm{w}, \mathrm{y})
$$

(i,j=1, .., k) (symmetry).

Using Equation (A.2.2), the above symmetry cond lition becomes

$$
\sigma_{i j}(w, y) \frac{w_{j} x_{j}}{C} \frac{x_{i}}{w_{j}}=\sigma_{j i}(w, y) \frac{w_{i} x_{i}}{C} \frac{x_{j}}{w_{i}}, \quad \text { or }
$$

(A.2.3)"
$\sigma_{i j}(w, y)=\sigma_{j i}(w, y)$
$(i, j=1, \ldots, \mathrm{k})$
(symmetry).

In other words, in terms of input substitution, the symmetry condition simply means that the Allen-Uzawa substitution elasticity is symmetric.
$H$ is negative semidefinite (NSD) if and only if all eigenvalues of $H=\left(b_{i j}\right)_{k \times k}$ are nonpositive, or if and only if $(-l)^{m} H_{m} \geq 0$ where $H_{m}$ is the $m$ th principal minor of $H(m=l, \ldots, k)$. That is, the principal minors of $H$ alternate between nonpositive (when $k$ is odd) and nonnegative (when $k$ is even). As is already shown in Equation (4.4.\&), the $k$ th principal minor $H_{k}=|H|=0 ; H$ is NSD if and only if

$$
(-1)^{\mathrm{m}} \mathrm{H}_{\mathrm{m}}=(-1)^{\mathrm{m}}\left|\begin{array}{cccc}
b_{11} & b_{12} & \cdots & b_{1 m}  \tag{A.2.4}\\
b_{21} & b_{22} & \cdots & b_{2 m} \\
\vdots & \vdots & & \vdots \\
b_{m 1} & b_{m 2} & \cdots & b_{m n}
\end{array}\right| \geq 0 \quad(m=1, \ldots, \mathrm{k}-1) \quad \text { (concavity) }
$$

where $\mathrm{b}_{\mathrm{ij}}=\frac{\partial \mathrm{x}_{\mathrm{i}}(\mathrm{w}, \mathrm{y})}{\partial \mathrm{w}_{\mathrm{j}}}(i, j=1, \ldots, k)$.

In terms of demand elasticities, for $m=1, \ldots, k$,

$$
H_{m}=\left|\left(\eta_{i j} \frac{x_{i}}{w_{j}}\right)_{m \times m}\right|=\left(\prod_{i=1}^{m} \frac{x_{i}}{w_{i}}\right)\left|\left(\eta_{i j}\right)_{m \times m}\right| .
$$

Thus, as $\prod_{i=1}^{m} \frac{x_{i}}{w_{i}} \geq 0, H$ is NSD iff $H_{\eta}=\left(\eta_{i j}\right)_{k \times k}$ is NSD, or, in terms of principal minors of $H_{\eta}$ ( as it can be shown that $H_{\eta}$ is also singular), $H$ is NSD iff

$$
(-1)^{\mathrm{m}} \mathrm{H}_{\mathrm{nm}}=(-1)^{\mathrm{m}}\left|\begin{array}{llll}
\tilde{\eta}_{11} & \tilde{\eta}_{12} & \cdots & \tilde{\eta}_{1 \mathrm{~m}}  \tag{A.2.4}\\
\tilde{\eta}_{21} & \tilde{\eta}_{22} & \cdots & \tilde{\eta}_{2 \mathrm{~m}} \\
\vdots & \vdots & & \vdots \\
\tilde{\eta}_{\mathrm{m} 1} & \tilde{\eta}_{\mathrm{m} 2} & \cdots & \tilde{\eta}_{\mathrm{n}}
\end{array}\right| \geq 0 \quad(m=1, \ldots, \mathrm{k}-1) \text { (concavity). }
$$

Similarly, as $H_{n m}=\left|\left(\tilde{\eta}_{i j}\right)_{m \times m}\right|=\left|\left(s_{j} \sigma_{i j}\right)_{m \times m}\right|=\left(\prod_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{s}_{\mathrm{j}}\right)\left|\left(\sigma_{\mathrm{ij}}\right)_{\mathrm{m} \times \mathrm{m}}\right|$ and $\left(\prod_{\mathrm{j}=1}^{\mathrm{n}} \mathrm{s}_{\mathrm{j}}\right) \geq 0, H$ is NSD if and only if $H_{\sigma}=\left(\sigma_{i j}\right)_{k x k}$ is NSD, or, because $H_{\sigma}$ is also singular under homogeneity,
(A.2.4)" $\quad(-1)^{\mathrm{m}} \mathrm{H}_{\sigma \mathrm{m}}=(-1)^{\mathrm{m}}\left|\begin{array}{llll}\sigma_{11} & \sigma_{12} & \cdots & \sigma_{1 m} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2 m} \\ \vdots & \vdots & \vdots \\ \sigma_{m 1} & \sigma_{m 2} & \cdots & \sigma_{m m}\end{array}\right| \geq 0 \quad(m=1, \ldots, \mathrm{k}-1)$ (concavity).

Now consider Conditions (iii) and (iv). Under the assumptions of separable inputs and outputs and constant returns to scale, the cost function can be written as $C(w, y)=g(y) \hat{c}(w)$ as in Equations (4.3.4) and (4.3.5). Thus, $\frac{\partial \mathrm{C}(\mathrm{w}, \mathrm{y})}{\partial y_{j}}=g_{j}^{\prime}(y) \hat{c}(w)(j=l, \ldots, n)$. This implies that Condition (iii) that $\nabla_{y} C(w, y)=\left(\frac{\partial \mathrm{C}}{\partial y_{1}}, \frac{\partial \mathrm{C}}{\partial y_{2}}, \ldots, \frac{C}{c} \frac{\mathrm{C}}{y_{n}}\right)$ are $\mathrm{HD}(1)$ in $w$ is equivalent to that unit cost function $\hat{c}(w)$ is $\operatorname{HD}(1)$ in $w$. This, given the separable cost function, is equivalent to a cost function $C(w, y)$ with $\mathrm{HD}(1)$ in w. As in gencral $f(z)$ is $H D(m)$ in $z$ implies $\nabla_{z} f(z)$ is $H D(m$ 1) in $z$ (Berck and Sydseter 1992, p15), a $\operatorname{HD}(1) C(w, y)$ means that $x(w, y)=\nabla_{w} C(w, y)$ are $\mathrm{HD}(0)$ in $w$. In other words, under the three assumptions given at the beginning of Section 4.3, Condition (iii) implies condition (i) in terms of integrability requirements in input demands. Also, when the cost function is assumed twice-continuously-differentiable, Condition (iv) is always satisfied. Thus, the integrability conditions for the input demands in the model are reduced to Conditions (i) and (ii), or, specifically, homogeneity, symmetry and concavity conditions in Equations (A.2.1), (A.2.3) and (A.2.4), or their two equivalent forms in Equations (A.2.i)' and (A.2.i)''( $\mathrm{i}=1,3,4$ ).

## A2.2 Revenue Functions and Input-Constrained Output Supply

To be a multi-output revenue function for a given input bundle $x, R(p, x)$ needs to be nonnegative, nondecreasing in output price $p, \operatorname{HD}(1)$ in $p$, convex and continuous in $p$, and nondecreasing in $x$ (Chambers 1991, p263). Also, if $R(p, x)$ is differentiable in $p$, the inputconstrained output supply can be derived (Chambers 1991, p264) as

$$
y_{j}(p, x)=\frac{\partial R(p, x)}{\partial p_{j}} \quad(j=1, \ldots, n) .
$$

Based on the above properties, the comparative static properties for a twice-continuously differentiable $R(p, x)$ and the derived output supplies are that (i) $y(p, x)$ be $\operatorname{HD}(0)$ in $p$; (ii) the Hessian matrix $\nabla_{p p} R(p, x)=\nabla_{p} y(p, x)$ be symmetric and positive semidefinite (PSD); (iii) $\nabla_{x} R(p, x)$ be $\mathrm{HD}(1)$ in $p$; and (iv) $\left(\partial^{2} R(p, x) / \partial x_{i} \partial p_{j}\right)=\left(\partial^{2} R(p, x) / \partial p_{j} \partial x_{i}\right)(i=1, \ldots, k ; j=1, \ldots, n)$ (Chambers 1991, p265).

Similar to the analysis of cost function (thus derivation is not repeated here), under the three assumptions made at the beginning of Section 4.3 (ie. profit maximization, input and output separability and constant returns to scale), the above comparative static properties are exhausted by the following homogeneity, synmetry and convexity restrictions, or their equivalent forms.

The homogeneity condition is given by

$$
\begin{equation*}
y_{j}(\lambda p, x)=\lambda y_{j}(p, x) \quad(\forall \lambda>0 ; j=1, \ldots, n) \quad \text { (homogeneity), or } \tag{A.2.5}
\end{equation*}
$$

$$
\begin{equation*}
\sum_{\mathrm{j}=1}^{\mathrm{n}} \widetilde{\mathrm{E}}_{\mathrm{ij}}(\mathrm{p}, \mathrm{x})=0 \quad(\mathrm{i}=1, \ldots, \mathrm{n}) \quad \text { (homogeneity), } \tag{A.2.5}
\end{equation*}
$$

where $\varepsilon_{i j}(p, x)$ is the input-constrained output supply elasticity of $y_{i}$ with respect to a change in output price $p_{j}(i, j=1, \ldots, k)$. Using Allen-Uzawa's definition of elasticity of product transformation (McFadden 1978, p79-80), ie.

$$
\begin{equation*}
\tilde{\varepsilon}_{\mathrm{ij}}(\mathrm{p}, \mathrm{x})=\gamma_{\mathrm{j}}(\mathrm{p}, \mathrm{x}) \tau_{\mathrm{ij}}(\mathrm{p}, \mathrm{x}) \tag{A.2.6}
\end{equation*}
$$

where $\gamma_{j}()=.\left(p_{j} y_{j} / R\right)$ is the share of the $j$ th out ${ }^{\prime}$ ut in total revenue and $\tau_{i j}(p, x)$ is the AllenUzawa elasticity of product transformation between the $i$ th and $j$ th outputs ( $i, j=1, \ldots, n$ ), homogeneity can also be written as
(A.2.5)"

$$
\sum_{j=1}^{n} \gamma_{j}(p, x) \tau_{i j}(p, x)=0 \quad(\mathrm{i}=1, \ldots, \mathrm{n}) \quad \text { (homogeneity). }
$$

The symmetry condition is given by

$$
\begin{equation*}
\frac{\partial y_{i}(w, y)}{\partial p_{j}}=\frac{\partial y_{j}(w, y)}{\partial p_{i}} \tag{A.2.7}
\end{equation*}
$$

( $\mathrm{i}, \mathrm{j}=1, \ldots, \mathrm{n}$ ) (symmetry), or
(A.2.7)'

$$
\gamma_{\mathrm{i}}(\mathrm{p}, \mathrm{x}) \tilde{\varepsilon}_{\mathrm{ij}}(\mathrm{p}, \mathrm{x})=\gamma_{\mathrm{j}}(\mathrm{p}, \mathrm{x}) \tilde{\varepsilon}_{\mathrm{ji}}(\mathrm{p}, \mathrm{x}) \quad(i, j=1, \ldots, \mathrm{n}) \quad \text { (symmetry). }
$$

Using Allen-Uzawa's elasticity of substitution, the symmetry condition becomes

$$
\tau_{i j}(p, x) \frac{p_{j} y_{j}}{R} \frac{y_{i}}{p_{j}}=\tau_{j i}(p, x) \frac{p_{i} y_{i}}{R} \frac{y_{j}}{p_{i}}, \quad \text { or }
$$

(A.2.7)"

$$
\tau_{i j}(w, y)=\tau_{j i}(w, y) \quad(1, j=1, \ldots, \mathrm{n}) \quad \text { (symmetry). }
$$

In other words, the symmetry condition simply implies symmetry of Allen-Uzawa product transformation elasticities.

The convexity condition requires that the Hessialı matrix

$$
\mathrm{H}=\nabla_{\mathrm{pp}} \mathrm{R}(\mathrm{p}, \mathrm{x})=\nabla_{\mathrm{p}} \mathrm{y}(\mathrm{p}, \mathrm{x})=\left(\frac{\partial y_{i}(p, x)}{\partial p_{j}}\right)_{n \times n}
$$

is PSD. It can be shown that $H$ is PSD iff. $\mathrm{H}_{\varepsilon}=\left(\tilde{\varepsilon}_{\mathrm{ij}}\right)_{\mathrm{n} \times \mathrm{n}}$ is PSD, or $H_{\tau}=\left(\tau_{i j}\right)_{n \times n}$ is PSD. Similar to the case of cost function, it can also be shown that under the homogeneity condition, all three matrices $H, H_{\varepsilon}$ and $H_{\tau}$ are singular. Thus, in terms of the principal minors of these matrices, the convexity condition is equivalent $t$ )
(A.2.8) $\quad \mathrm{H}_{\mathrm{m}}=\left|\begin{array}{cccc}b_{11} & b_{12} & \cdots & b_{1 m} \\ b_{21} & b_{22} & \cdots & b_{2 m} \\ \vdots & \vdots & & \vdots \\ b_{m 1} & b_{m 2} & \cdots & b_{m m}\end{array}\right| \geq 0 \quad(m=1, \ldots, \mathrm{n}-1) \quad$ (convexity)
where $\mathrm{b}_{\mathrm{ij}}=\frac{\partial \mathrm{y}_{\mathrm{i}}(\mathrm{p}, \mathrm{x})}{\partial \mathrm{p}_{\mathrm{j}}}(i, j=l, \ldots, n)$, or
(A.2.8)' $\quad \mathrm{H}_{\mathrm{\varepsilon m}}=\left|\begin{array}{llll}\widetilde{\varepsilon}_{11} & \tilde{\varepsilon}_{12} & \cdots & \widetilde{\varepsilon}_{1 \mathrm{~m}} \\ \tilde{\varepsilon}_{21} & \tilde{\varepsilon}_{22} & \cdots & \widetilde{\varepsilon}_{2 \mathrm{~m}} \\ \vdots & \vdots & & \vdots \\ \tilde{\varepsilon}_{\mathrm{m} 1} & \tilde{\varepsilon}_{\mathrm{m} 2} & \cdots & \tilde{\varepsilon}_{\mathrm{mm}}\end{array}\right| \geq 0 \quad(m=1, \ldots, \mathrm{n}-1)$ (convexity), or
(A.2.8)" $\quad \mathrm{H}_{\mathrm{tm}}=\left|\begin{array}{cccc}\tau_{11} & \tau_{12} & \cdots & \tau_{1 m} \\ \tau_{21} & \tau_{22} & \cdots & \tau_{2 m} \\ \vdots & \vdots & & \vdots \\ \tau_{m 1} & \tau_{m 2} & \cdots & \tau_{m m}\end{array}\right| \geq 0 \quad(m=1, \ldots, \mathrm{n}-1) \quad$ (convexity).

Thus, the integrability conditions for the output supplies in Equations (4.4.13)-(4.4.16), (4.4.25)-(4.4.26), (4.4.35)-(4.4.38) and (4.4.51)-(4.4.54) in the model are satisfied if the homogeneity, symmetry and convexity conditions in Equations (A.2.5), (A.2.7) and (A.2.8), or their two equivalent forms in Equation (A.2.i,' and (A.2.i)" $(i=5,7,8)$, hold. 'These conditions will ensure the recovery of the underlying revenue functions in Equations (4.3.22)(4.3.27).

## A2.3 Profit Functions and Exogenous Factor Supplies

A multioutput (including single output as a special case) profit function $\Pi(p, w)$ needs to be nonnegative, nondecreasing in output prices $p$. nonincreasing in input prices $w$, convex, continuous and $\mathrm{HD}(1)$ in all arguments (Chambers, 1991, p269). When $\Pi(p, w)$ is differentiable, using Hotelling's Lemma, a unique set of profit-maximizing output supplies and input demands can be derived as
(A.2.9) $y_{j}(p, w)=\frac{\partial \Pi(p, w)}{\partial p_{j}}$ and $-x_{i}(p, w):: \frac{\partial \Pi(p, w)}{\partial w_{i}}(i=1, \ldots, k ; j=1, \ldots, n)$.

The comparative static properties for these input demands and output supplies are that (i) z(p, $w)=\left(y_{j}(p, w),-x_{i}(p, w)\right)$ are $\mathrm{HD}(0)$ in $q=(p, w)$; and (ii) the Hessian matrix $\nabla_{q} z(q)=\nabla_{q q} \Gamma(q)$ is symmetric and PSD.

Similar to the cases of cost and revenue functions, the demand and supply functions in Equation (A.2.9) need to satisfy homogeneity, symmetry and convexity conditions. These conditions can be expressed in terms of market clasticities. And the exogenous input supplies of $X_{1}, X_{n 2}, X_{s 2}, F_{n 2}, F_{n 3}, Y_{p}, Z_{m e}$ and $Z_{m d}$ in Equations (4.4.1), (4.4.3)-(4.4.4), (4.4.17)-(4.4.18), (4.4.27) and (4.4.39)-(4.4.40) need to satisfy these conditons.

However, if let $x$ represent any one of the exogenous inputs to the model $\left(x=X_{1}, X_{n 2}, X_{s 2}, F_{n 2}\right.$, $F_{n 3}, Y_{p}, Z_{m e}$ and $Z_{m d}$ ), the supply of $x$ is the only equation in the model that is derived from its supplier's profit function. Other variables influencing the factor supply are assumed exogenous and kept constant. Thus, the supply of each of these factors needs to satisfy the economic restrictions associated with the demand and supply of other variables in its supplier's profit function, but not with any demand or supply specifications within the model. As a result, homogeneity and symmetry conditions do not impose restrictions on the exogenous factor supply functions that involve any other equations in the model. The only restriction implied by a PSD Hessian is that the own-price supply elasticities are non-negative, ie.

$$
\varepsilon_{\mathrm{x}} \geq 0 \quad\left(x=X_{1}, X_{n 2}, X_{s 2}, F_{n}, F_{n 3}, Y_{p}, Z_{m e} \text { and } Z_{m d}\right) \text {, }
$$

where $\varepsilon_{x}$ is the own-price supply elasticity of input $x$.

## A2.4 Utility Functions and Exogenous Product Demands

Now examine the consumer demands for the final products of the beef industry, which are assumed exogenous to the model. Consumer theory and the relationship between the indirect utility function, expenditure function and the derived Marshallian and Hicksian demand functions can be found in many economics textbooks (eg. Varian 1992) and will not be discussed in detail here.

In brief, the indirect utility function for given income is defined as

$$
v(p, m)=\max _{x}\{u(x): p x \leq m\}
$$

where $x$ is the commodity vector, $p$ is the price vector, $m$ is income and $u(x)$ is the utility function. Using Roy's identity, the Marshallian demand functions are derived as

$$
x_{i}(p, m)=-\frac{\frac{\partial v(p, m)}{\partial p_{i}}}{\frac{\partial v(p, m)}{\partial m}} \quad(i=1, \ldots, n)
$$

The inverse of the indirect utility function is the expenditure function, or equivalently, the expenditure function for a given utility level is given by

$$
e(p, u)=\min _{x}\{p x: u(x) \geq u\} .
$$

The Hicksian demand functions can be derived as

$$
h_{i}(p, u)=\frac{\partial e(p, u)}{\partial p_{i}} \quad(i=1, \ldots, n) .
$$

There are a set of relationships relating $v(p, m) x(p, m), e(p, u)$ and $h(p, u)$ (eg. see Varian 1992, p106). In particular,

$$
x_{i}(p, m)=h_{i}(p, v(p, m)),
$$

ie. Marshallian demand at income $m$ is Hicksian demand at utility $v(p, m)$.

As the expenditure function for a given utility level is completely analogous to a cost function for a given output level, the properties of $e(p, u)$ are similar to those of the cost function discussed earlier and will not be repeated (sce for example, Varian 1992, p104). Thus, analogous to the comparative static properties for the conditional factor demand in production theory, the properties for Hicksian demand are that (i) $h(p, u)=\nabla_{p} e(p, u)$ are $\operatorname{HD}(0)$ in $p$; and (ii) Hessian matrix $\nabla_{p} h(p, u)=\nabla_{p p} e(p, u)$ is symmetric and NSD.

However, unlike the case of the cost function where output is observable, Hicksian demand is not observable because utility is not directly observable. The relationship that links the derivatives of the unobservable Hicksian and the observable Marshallian demand functions is the Slutsky equation:
(A.2.11) $\quad \frac{\partial h_{i}(p, u)}{\partial p_{j}}=\frac{\partial x_{i}(p, m)}{\partial p_{j}}+\frac{\partial x_{i}(p, m)}{\partial m} x_{j}(p, m) \quad(i, j=1, \ldots, n)$.

Using the Slutsky equation, the above comparative static properties in terms of the Marshallian demand functions are that (i) $x(p, m)$ are $\operatorname{HD}(0)$ in $(p, m)$; and (ii) the Slutsky matrix with elements in Equation (A.2.11) is symmetric and NSD. In terms of demand elasticities, the homogeneity condition becomes

$$
\begin{equation*}
\sum_{j=1}^{n} \eta_{i j}+\eta_{i m}=0 \quad(i=1, \ldots, n) \quad \text { (homogeneity), and } \tag{A.2.12}
\end{equation*}
$$

the symmetry condition becomes

$$
\begin{equation*}
\eta_{i j}=\frac{s_{j}}{s_{i}} \eta_{j i}+s_{j}\left(\eta_{j m}-\eta_{i m}\right) \quad(i, j=1, \ldots, n) \quad(\text { symmetry }), \tag{A.2.13}
\end{equation*}
$$

where $s_{i}(i=1, \ldots, n)$ is the expenditure share of the $i$ th commodity. A NSD Slutsky Hessian matrix implies that

$$
\begin{equation*}
\mathrm{H}_{\eta}=\left(\eta_{\mathrm{ij}} \frac{\mathrm{x}_{\mathrm{i}}}{\mathrm{p}_{\mathrm{j}}}+\eta_{\mathrm{im}} \frac{\mathrm{x}_{\mathrm{i}} \mathrm{x}_{\mathrm{j}}}{\mathrm{~m}}\right)_{\mathrm{n} \times n} \quad \text { is NSD } \quad \text { (concavity). } \tag{A.2.14}
\end{equation*}
$$

Derivation is straightforward and thus omitted.

Now come back to the implications of these conditions to specification of the final beef demand functions in the model. As discussed in Chapter 2 (Section 2.5.3), the Marshallian economic surplus areas will be used as measures of welfare, which implies that the marginal utility of income is constant and the income effect will be ignored. Under this assumption, a symmetric and NSD Marshallian substitution matrix is equivalent to a symmetric and NSD Slutsky matrix. As the expenditure on beef is only a small proportion in the consumers' budget,
this should not introduce a significant error by using the economic surplus as a welfare measure (Willig 1976).

The two types of beef are assumed non-substitutable in the export market. Thus, no integrability restrictions with the rest of the model are required for each of these export demands, except that the own-price demand elasticities are negative to be consistent with the NSD Hessian, that is
(A.2.15)

$$
\eta_{(\mathrm{Qie}, \text { pie })} \leq 0
$$

$$
(i=n, s)
$$

As for the domestic demand, the two types of berf are modelled as substitutes and relate to the utility maximization of the same domestic consumer. Because other competing commodities in the consumer's budget, as well as income, are assumed exogenously constant and do not appear in the model, the integrability conditions in the context of this model relate only to the $2 \times 2$ sub-Hessian matrix of the two domestic beef products (denoted by $x_{1}$ and $x_{2}$ for convenience).

In particular, symmetry implies that

$$
\frac{\partial \mathrm{x}_{1}}{\partial \mathrm{p}_{2}}=\frac{\partial \mathrm{x}_{2}}{\partial \mathrm{p}_{1}} \quad \text { (symmetry), or }
$$

$$
\begin{equation*}
\left.\eta_{12}=\left(\frac{\lambda_{2}}{\lambda_{1}}\right) \eta_{21} \quad \text { (symmetry }\right) \tag{A.2.16}
\end{equation*}
$$

where $\left(\lambda_{2} / \lambda_{1}\right)$ is the relative budget shares of the two commodities, and concavity implies that

$$
\eta_{11} \leq 0 \quad \text { and } \quad\left|\begin{array}{ll}
\eta_{11} & \eta_{12}  \tag{A.2.17}\\
\eta_{21} & \eta_{22}
\end{array}\right| \geq 0 \quad \text { (concavity). }
$$

The $2 \times 2$ sub-Hessian matrix and the $2 \times 2$ demand elasticity matrix are not necessarily singular now. For normal situations where $\eta_{\mathrm{ii}}<0, \eta_{\mathrm{ij}} \geq 0$. and $\left|\eta_{\mathrm{ii}}\right|>\left|\eta_{\mathrm{ij}}\right|$, Equation (A.2.17) are naturally satisfied. In these situations, homogeneily also means that

$$
\begin{equation*}
\eta_{11}+\eta_{12}=-\sum_{j=3}^{n} \eta_{i j}-\eta_{1 m} \leq 0 \text { and } \eta_{21}+\eta_{22}=-\sum_{j=3}^{n} \eta_{\mathrm{ij}}-\eta_{1 \mathrm{l}} \leq 0, \tag{A.2.18}
\end{equation*}
$$

which are naturally satisfied. Complete discussion on the integrability problem and the integrability conditions for the "incomplete demand system" can be found in Epstein (1982), LaFrance and Hanemann (1989) and LaFrance(1991).

# Appendix 3. Specification of Equilibrium Prices and Quantities 

In this appendix, the specification of equilibrium prices and quantities for all inputs and outputs of all industry sectors are detailed. The original data sources, assumptions made regarding the price and quantity relationships among different levels and the derivation of the unavailable data are given. Refer to Table 4.3 or Figure 4.1 in Chapter 4 for variable definitions. The resulting set of average prices and quantities for 1992-1997 are summarised in Table 5.1.

## A3.1 Quantities

The annual quantities of the four types of cattle/heef products at all production and marketing stages are required for the period of 1992 to 1997.

Step 1. $Q_{e}, Q_{n e}, Q_{s e}$ and $Z_{e} \Rightarrow Z_{n e}, Z_{s e}, Y_{n e, y} Y_{s e}$ and $Y_{e}$
$Q_{e}, Q_{n e}$ and $Q_{s e}$, quantities of total export beef, grainfed export beef and grassfed export beef, respectively, measured in kilotons shipped weight. are obtained from Agriculture, Fisheries and Forestry, Australia (K. Wade, AFFA, per. comm 1998). A data spreadsheet is obtained from the Quota Administration and Statistics Unit, AFFA that lists the annual quantities of Australian beef exports, in separate grainfed and grassfed quantities, to more than 100 countries for the period of 1992-1998. The grainfed figures in these data are based on the exporter specified grainfed amounts as indicated on the He.llth Certificate applications.
$Z_{e}$, the total Australian export beef in kilotons carcass weight, is taken from Table 150, Australian Commodity Statistics (ABARE 1998).

The saleable yield for converting the export carcass weight to the export shipped weight is obtained as the ratio of $Q_{e}$ to $Z_{e}$. The average of this ratio for 1992-1997 is about $68 \%$. This same yield percentage is used to derive the carcass weights for both export grainfed and export grassfed beef; that is $Z_{n e}=Q_{n e} / 0.68$ and $Z_{s e}=Q_{s e} / 0.68$.

A commonly used conversion factor of 0.55 (Griffith, Green and Duff 1991) is applied to all
four beef categories to convert the cattle live weights to beef carcass weights. In particular, $Y_{n e}=Z_{n e} / 0.55, Y_{s e}=Z_{s e} / 0.55$ and $Y_{e}=Y_{n e}+Y_{s e}$.

## Step 2. $Z_{d} \Rightarrow \boldsymbol{Y}_{d}$ and $\boldsymbol{Y}$

$Z_{d}$, the total domestic beef consumption in kilotons carcass weight, is obtained from Table 150, Australian Commodity Statistics (ABARE 1998). Live weight total domestic beef quantity $Y_{d}$ is derived using the 0.55 conversion percentage, ie. $Y_{d}=Z_{d} / 0.55$. The total cattle live weight is calculated as $Y=Y_{d}+Y_{e}$.

## Step 3. Derivation of Average Slaughtering Weights $\operatorname{WPH}\left(Y_{n e}\right)$ and $\operatorname{WPH}\left(Y_{n}\right)$

The total domestic beef quantity is given in Step 2. However there is no published information available on the separate quantities for grainfed and grassfed domestic consumption. The only information is an estimated figure of domestic qrainfed quantity for 1994 in a MRC research report (MRC 1995).

In this MRC commissioned study, the total domestic grainfed production was derived from the information on numbers of grainfed cattle slaughtered by major retailers and factored up by their estimated market share in comparison with the butchers. Then, the domestic feedlotfinished cattle was assumed to equal the throughput of major feedlots servicing the domestic grainfed market and the residual assumed to be grain supplemented. As a result, from their discussions with the national beef retail managers of Woolworths and Coles and with a major Sydney retailer, and based on the information from the AMLC commissioned Nielsen Survey and from LMAQ and NSW saleyard reports, they estimated that the total number of domestic grainfed cattle is in the order of 1.2 million head, of which 390,000 head are fed in major feedlots and the residual of 811,000 head grain supplemented. Based on this information, they estimated the number of cattle slaughtered and the total carcass weight for each market segment for 1994 (Chart 3.1 and 3.2, MRC 1995).

In the current study, information on separate domestic grainfed and grassfed cattle for the period of 1992-1997 is required. The slaughtering weight per head for the major feedlot finished cattle in 1994 from the MRC study is taken and assumed unchanged for other years. This, together with the cattle turn-off number in major feedlots described in Step 4 below, is
used to derive the domestic grainfed (major feedlot finished for the purpose of this study) cattle quantity.

Specifically, in Table A3.1, the number of cattle slaughtered, the total carcass weight and the average carcass weight per head for the various market segments for 1994 are assembled based on the information in the above mentioned MRC report (MRC 1995). These market segments include domestic feedlot finished $\left(y_{n d}(f f e d l o t)\right.$ ), domestic grain supplemented ( $y_{s d}($ suppl.)), domestic grassfed ( $y_{s d}($ grass $)$ ), export grainfed ( $y_{n e}$ ) and export grassfed ( $y_{s e}$ ) cattle. For the purpose of this study, the domestic grainfed $\mathrm{Y}_{\text {nd }}$ only includes $y_{n d}($ feedlot $)$, ie. $Y_{n d}=y_{n d}($ feedlot $)$ and $Y_{s d}=y_{s d}($ grass $)+y_{s d}$ (suppl.). The slaughtering numbers and carcass weights marked with (.)* are taken from MRC (Chart 3.1 and 3.2, 1995). An assumption is made that the average slaughtering weight for total domestic grainfed cittle $\left(y_{n d}(\right.$ feedlot $)+y_{s d}($ suppl. $\left.)\right)$ derived from the MRC figures is the same for its two components ( $y_{n d}$ (feedlot) and $y_{s d}($ suppl.)). Based on this assumption, the figures in (.) ${ }^{* * *}$ are derived. In particular, the average carcass weight for $y_{n e}$ is 332 kg and for $y_{n}$ (feedlot) is 292 kg per head. In live weight, $\operatorname{WPH}\left(Y_{n e}\right)=332 / 0.55=604 \mathrm{~kg}$ (l.w.) and $\mathrm{WPH}\left(\mathrm{Y}_{\mathrm{n}}\right)=292 / 0.55=531 \mathrm{~kg}(\mathrm{l} . \mathrm{w}$.). These figures are used in the derivation of other data below.

Step 4. $N, N_{n}$ and $W P H\left(Y_{n}\right) \Rightarrow Y_{n}$ and $Y_{s}$

The total cattle slaughtering number in thousands of heads, N , for each year of 1992-1997 is taken from the Australian Commodity Statistics (Table 150, ABARE 1998). The cattle 'offfeed' numbers from the major feedlots are obtained from feedlot survey information (C. Toyne, ABARE, per. comm. 1998). Using the average weight per head for feedlot finished cattle in 1994 for all other years, the total feedlot finished cattle live weight can be derived as $Y_{n}=\left(N_{n}\right)\left(\mathrm{WPH}\left(Y_{n}\right)\right)$. The total grassfed cattle (including grain-supplemented) live weights can be obtained accordingly as $Y_{s}=Y-Y_{n}$.

Step 5. $Y_{n}, Y_{s}, Y_{n e}$ and $Y_{s e} \Rightarrow Y_{n d}, Y_{s d}, Z_{n d}$ and $Z_{s d}$

Domestic grainfed and grassfed quantities can be calculated as $Y_{n d}=Y_{n}-Y_{n e}$ and $Y_{s d}=Y_{s}-Y_{s e}$. Using the conversion factor of 0.55 as discussed in Step 1, the carcass weights for the two domestic products is calculated as $Z_{n d}=0.55 Y_{n d}$ and $Z_{s d}=0.55 Y_{s d}$.

Table A3.1 Derivation of Average Slaughtering Weights for Various Market Segments

|  | $\begin{gathered} \text { Cattle } \\ \text { Number } \\ \text { (‘000 heads) } \\ \hline \end{gathered}$ | Total Carcass Weights (k.t c.w.) | WPH(Weight Per Head) (kg c.w.) | WPH(Weight Per Head) (kg l.w.) |
| :---: | :---: | :---: | :---: | :---: |
| $\mathbf{y}_{\text {nd }}($ feedlot) | $390{ }^{*}$ | $81.9{ }^{* * *}$ | $210^{* *}$ | $382^{* * *}$ |
| $\mathbf{y ~}_{\text {sd }}$ (suppl.) | $811^{*}$ | $170.3^{* *}$ | $210^{* *}$ | $382^{* * *}$ |
| $\begin{aligned} & \mathbf{y}_{\text {nd }} \text { (feedlot) } \\ & +\mathbf{y}_{\mathrm{sd}}(\text { suppl. }) \end{aligned}$ | 1201* | 252.2" | $210^{* * *}$ | $382^{* * *}$ |
| $\mathbf{y}_{\text {sd }}$ (grass) | $2419{ }^{*}$ | $425^{*}$ | $176 * *$ | $320 * *$ |
| $\mathbf{y}_{\text {ne }}$ | $782^{*}$ | $260{ }^{*}$ | $332^{* * *}$ | $604{ }^{* * *}$ |
| $\mathrm{y}_{\text {se }}$ | $3842^{*}$ | 881.5* | $229 * * *$ | $416^{* * *}$ |
| y | $8244{ }^{*}$ | $1818{ }^{*}$ | $221^{* * *}$ | $402{ }^{* * *}$ |
| $\begin{aligned} & \mathbf{Y}_{\mathrm{s}}=\mathbf{y}_{\mathrm{sd}}(\text { grass }) \\ & \\ & +\mathrm{y}_{\mathrm{yd}}(\text { suppl. }) \\ & \\ & +\mathrm{y}_{\mathrm{se}} \end{aligned}$ | $7072^{* * *}$ | $1476.8^{* * *}$ | $209 * *$ | $380^{* * *}$ |
| $\begin{aligned} Y_{n}= & y_{n d}(\text { feedlot }) \\ & +y_{n e} \end{aligned}$ | $1172^{* * *}$ | $341.9^{* * *}$ | $292 * *$ | $531 * *$ |

* MRC (1995);
** Assuming that the feedlot finished cattle for domestic mirket have the same average slaughtering weight as the grain supplemented cattle for domestic market;
*** Derived from data in (.)* and (.)**.


## Step 6. $\boldsymbol{R}_{(Z d / Q d)}-$ - Domestic Saleable Yield Percentage

The quantities for domestic retail cuts are not reported in published sources. A saleable yield percentage $R_{(Z d / Q d)}$ is used to convert the carcass weights to the weights of saleable cuts. $R_{(Z d / Q d)}$ need to be consistent with the various retail cuts that are included in the measurements of the prices and quantities. In the model, the major cuts that comprise a beef carcass are included in the calculation of domestic retail beef quantities and prices. They include rump steak, sirloin, topside, chuck, blade and mince. Based on a study by Griffith, Green and Duff (1991), these cuts comprise $72 \%$ of the weight of a beef carcass. Thus a yield percentage $R_{(Z d / Q d)}=0.72$ is specified. The derivation of the associated retail price $P_{d}$ is given in Section A3.2 in this Appendix.

Step 7. $Z_{n d}$ and $Z_{s d} \Rightarrow Q_{n d}$ and $Q_{s d}$

Based on the discussion above in Step 6, the domestic retail beef quantities are calculated as
$Q_{n d}=0.72 Z_{n d}$ and $Q_{s d}=0.72 Z_{s d}$.

Step 8. $\operatorname{WPH}\left(Y_{n e}\right), Y_{n e}$ and $N_{n} \Rightarrow N_{n e}$ and $N_{n d}$

Using the average slaughtering weight per head ( $\mathrm{WPH}\left(Y_{n e}\right)$ ) and the total slaughtering weight for export grainfed cattle $\left(Y_{n e}\right)$, the cattle numbers for the export grainfed beef are derived as $N_{n e}=Y_{n e} / \mathrm{WPH}\left(Y_{n e}\right)$. The domestic feedlot-finished cattle number is then calculated as $N_{n d}=N_{n}$ $N_{n e}$.

## Step 9. Derivation of WPH $\left(F_{n I e}\right), \mathrm{WPH}\left(F_{n I d}\right), \mathrm{WPH}\left(X_{n e}\right)$ and WPH $\left(X_{n d}\right)$

Data on cattle quantities at feeder and weaner levels are not available from published sources. They are derived from information on the average per head weights of export quality feeders $\left(\mathrm{WPH}\left(F_{n l e}\right)\right)$ and weaners $\left(\mathrm{WPH}\left(X_{n e}\right)\right)$ and domestic quality feeders $\left(\mathrm{WPH}\left(F_{n I d}\right)\right)$ and weaners (WPH $\left(X_{n d}\right)$ ). As summarised in Table 4.2, he cattle weight requirements at different production stages for the various Japanese, Kore.in and domestic grainfed market segments are provided in MRC (1995, p93-102). These include feeder weights after backgrounding and weaner weights before backgrounding. Market slares for the four Japanese grainfed categories are also given in MRC (1995, p57). Percentages of Japanese and Korean components in the total export grainfed beef are from the data provided by AFFA (K. Wade, AFFA, per. comm. 1998). Based on this information, a spreadsheet is established to derive the average per head weights for grainfed export and domestic cattle at feeder and weaner levels for each year of 1992-1997. The derivation for the 1992-1997 average figures is outlined in Table A3.2. The mid-points of the weight ranges for various market segments are weighted by their market shares to derive the average feeder and weaner weights per head for export and domestic markets.

Step 10. WPH $\left(F_{n l e}\right), \mathbf{W P H}\left(F_{n l d}\right), N_{n e}$ and $N_{n d}: \Rightarrow F_{n l e}$ and $F_{n l d}$

The feeder quantities for export and domestic m.rkets are calculated as $F_{n / e}=\left(\mathrm{WPH}\left(F_{n t e}\right)\right)\left(N_{n e}\right)$ and $F_{n l d}=\left(\mathrm{WPH}\left(F_{n l d}\right)\right)\left(N_{n d}\right)$.

Table A3.2 Derivation of Average Feeder and Weaner Weights Per Head


* MRC (1995)
** AFFA (1998)
*** Derived from figures in (.)* and (.) $)^{* *}$.

Step 11. $\mathrm{WPH}\left(X_{n e}\right), \mathrm{WPH}\left(X_{n d}\right), N_{n e}, N_{n d}$ and $N_{:} \Rightarrow X_{n I}$ and $X_{s I}$

Total weaner quantities for feedlot finishing are derived as $X_{n I}=\left(\mathrm{WPH}\left(X_{n e}\right)\right)\left(N_{n e}\right)$ $+\mathrm{WPH}\left(X_{n d}\right)\left(N_{n d}\right)$. As discussed in Chapter 4, it is assumed in this study that the weaner cattle are not differentiated in quality regardless of whether they are for grain or grass finishing. Thus, the average weaner weight per animal for grass-finishing is assumed as the same as that for grain-finishing. The average weight for weaners for grain-finishing is calculated as $\operatorname{WPH}\left(X_{n I}\right)=X_{n I} / N_{n}$, and the quantity for weaners for grass-finishing is derived as $X_{s l}=\mathrm{WPH}\left(X_{n I}\right)\left(N_{s}\right)$.

## Step 12. Derivation of $\boldsymbol{F}_{\boldsymbol{n} 2}$

Feedgrain consumption $F_{n 2}$ is estimated from the "per kilogram liveweight gain feedgrain consumption" calculated from the data in a feedlot case study of the Beef CRC (Meppem 1995). In this study, the feed cost per kilogram liveweight gain for cattle on feed 150 days is $\$ 1.02$. The feed composition is $88 \%$ of feedgrain, $10 \%$ of roughage and $2 \%$ of
additives, and the prices of the three components are $\$ 150, \$ 110$ and $\$ 1000$ per tonne respectively. From these data, the feedgrain consumption per kilogram liveweight gain is calculated as 5.51 kilograms. The annual feedgıain consumption is calculated by multiplying this amount by the total liveweight gain each year; that is, $F_{n 2}=5.51^{*}\left(Y_{n e}+Y_{n d}-F_{n / e^{-}}-F_{n: l d}\right)$. Details of the derivation is in Zhao and Griffith (1999).

## A3.2 Prices

The prices for the four types of cattle/beef at all production and marketing stages and the prices for feedgrain for the period of 1992 to 1997 are given in Zhao and Griffith (1999). The original data sources, assumptions made regarding the price relationships and the derivation of all prices are given below. The resulting average prices for all inputs and outputs for 1992-1997 are listed in Table 5.1.

## Step 1. $v_{d}, v_{e-s t r}, v_{e-h f r}$ and $v_{e-c o w}$

The Australian Commodity Statistics (ABARI: 1998) publishes annual saleyard prices for domestic yearling, export ox (301-350kg, c.w.) and export cow (201-260kg, c.w.). Another source for finished live cattle prices is The I.and newspaper (NLRS 1998), which reports weekly 'over the hook' (OTH) prices for v.rious local and export cattle categories. In particular, during 92-96, it reports the OTH prices for two domestic yearling/steer grades (140180 and 180-220), two export steer grades (240-320 and 320-400), two export heifer grades (180-250 and 250-320) and three export cow griades (150-180, 180-220 and 220+). Since 1997, even more categories for both domestic and export markets are reported. For example, prices on cattle to specific countries such as Japan, Korea, EU and US are reported from 1997. The annual averages of these weekly prices are obtained from the National Livestock Reporting Services (A. Galea, NLRS, per. comm. 1998).

However, before November 1997, the prices reןorted do not differentiate between grainfed and grassfed cattle. Only since November 1,97 are separate grassfed and grainfed cattle prices are reported for various domestic, Korean and Japanese categories.

In order to specify the four finished cattle prices according to export/domestic and grassfed/grainfed for 1992 to 1997, the aggreg.ted domestic prices ( $v_{d}$ ), export ox/steer prices
( $v_{e-s t r}$ ), export heifer prices ( $v_{e-h f r}$ ) and export cow prices ( $v_{e-c o w}$ ) are obtained from the above described sources. Then using the weekly information on separate grainfed and grassfed prices since November 1997, a grainfed price premium for each category is obtained through regressing over the weekly observations. The four required prices $v_{n e}, v_{s e}, v_{n d}$ and $v_{s d}$ for the period of 1992-1997 are derived using these price premium results.

As there are inconsistencies in the NLRS reported categories for 1992-1996 and 1996-1997, the domestic finished cattle prices, $v_{d}$, are taken from the saleyard yearling prices on ABARE (Table 143, 1998). The domestic yearling saleyald prices published by ABARE (1998) are very similar to the two yearling/steer prices by NLRS (A. Galea, NLRS, per. comm. 1998).

Similarly, for consistency purposes, $v_{e-s t r}$ and $v_{e-\text { ow }}$ are taken from the export quality ox (301350 kg ) and cow ( $201-260 \mathrm{~kg}$ ) prices inn ABARE (Table 143, 1998). They are very similar to the prices in the relevant categories reported by NLRS (1998). $v_{e-h f r}$ are taken from the export heifer ( $180-250 \mathrm{~kg}$ ) prices for 1992-1996 and export heifer (170-230kg) price for 1997 from NLRS (A. Galea, NLRS, per. comm. 1998i).

## Step 2. Saleyard Grainfed Price Premiums for Domestic ( $\boldsymbol{r}_{Y(d o m)}$ ) and Japanese ( $r_{Y(J P)}$ ) Markets

Twenty weekly OTH price observations during 1998 for grainfed and grassfed cattle for two domestic categories and two Japan categories are collected from The Land newspaper (NLRS 1998a). The grainfed prices are regressed on the relevant grassfed prices in each of the above four categories. When the intercept is allowed to be non-zero in the regressions, three out of the four regressions show statistically insignificant intercepts, but all four categories show very strong significance of the grassfed price vari.ble. Thus a proportional price premium is assumed, and the regressions are run again without intercepts. Remarkably, the two local catergories show the same price parameters of 1.11 ( $t$-values are 118 and 121 respectively) and the two Japanese grades have price parameters of 1.082 and 1.085 ( $t$-values are 125 and 82) respectively. Therefore, $11 \%$ and $8 \%$ are assumed as the grainfed price premiums for the domestic and Japanese markets, ie. $\mathrm{r}_{\mathrm{Y}(\mathrm{dom})}=11 \%$ and $\mathrm{r}_{\mathrm{Y}(\mathrm{JP})}=8 \%$.

Step 3. $v_{e-s t r}, v_{e-h f r}, v_{e-c o w}$ and $r_{Y(J P)} \Rightarrow v_{n e}$ and $v_{s e}$

As can be seen in Table 4.1, during 1992-1997, on average $14 \%$ of the Australian export beef are grainfed. In the export grassfed category, more than one-third are to the US market. The majority of the beef to the US is low quality manufacturing beef such as cows. For example, almost all beef to the US during 1996 and 1997 was low quality frozen meat rather than chilled (AMLC 1996/97). $v_{\text {e-cow }}$ is assumed to account for the US's share of the total grassfed export price. An average of $v_{e-s t r}$ and $v_{e-h f r}$, denoted $v_{e \text { noncow }}$, is taken as the price for export cattle excluding cows. Then using the $8 \%$ grainfed premium for the cattle to Japan, separate grainfed and grassfed export prices for export cattle excluding cows are derived from the aggregated prices $v_{e \text {-noncow }}$. That is, $v_{e \text {-noncow-grassfed }}:=\mathrm{v}_{\mathrm{e} \text {-noicow }} /\left(1+r_{Y(J P)} * \rho_{(n / n o n c o w)}\right)$ and $v_{\text {e-noncow-grainfed }}=$ $\left(1+r_{Y(J P)}\right)\left(\nu_{e-n o n c o w-g r a s s e d}\right)$, where $\rho_{(n / n o n c o w)}$ is the proportion of grainfed cattle in the total export cattle excluding the US segment, and the grainfed premium $\mathrm{r}_{\mathrm{Y}(\mathrm{JP})}=0.08$. Note that Japan accounts for $92 \%$ of the total grainfed export for the modelled time period. Finally, the export grassfed price for finished cattle is the weighted average of $v_{\text {e-noncow-grassed }}$ and $v_{\text {e-cow }}$, ie. $v_{\text {se }}=$ $\rho_{(U S / s e)} \nu_{e(c o w)}+\left(1-\rho_{(U S / s e)}\right) \nu_{e-n o n c o w-g r a s s e d}$, where $\rho_{(U S / s e)}$ is the proportion of US component in the total grassfed export cattle obtained from AFFA (K. Wade, AFFA, per. comm., 1998). As cows are not part of the grainfed segment, $v_{n e}=v_{\text {e.nonc: } \boldsymbol{w} \text {-grainfed }}$. Details of the derivation are in Zhao and Griffith (1999).

## Step 4. $\mathbf{v}_{\mathbf{d}}$ and $\mathbf{r}_{\mathbf{Y}(\text { dom })} \Rightarrow \mathbf{v}_{\mathbf{n d}}$ and $\mathbf{v}_{\mathbf{s d}}$

Using the grainfed price premium specified in Step 2, the domestic grassfed and grainfed prices for finished cattle are calculated from the aggregated domestic price $v_{d}$ as $v_{s d}=v_{d} /\left(1+r_{Y(d o m)} *\right.$ $\left.\rho_{(n d / d)}\right)$ and $v_{n d}=\left(1+r_{Y(d o m)}\right) v_{s d}$, where $\left.\rho_{(n d / d)}\right)=Y_{n d} / Y_{d}$ is the proportion of feedlot finished cattle in the domestic market and $r_{Y(d o m)}=0.11$ is the domestic grainfed cattle premium.

## Step 5. $u_{d}, v_{d}$ and $\Delta_{u d-v d} \Rightarrow u_{n d}$ and $u_{s d}$

The domestic processed beef carcass prices $\left(u_{c}\right)$ are taken from the monthly averages of the wholesale price survey data inn The Australian Meat Industry Bulletin (Nielson Marketing Research 1997a). There is no information published on separate grainfed and grassfed wholesale prices. Assume that the costs of slaughtering and processing per kilogram cattle into beef carcass are the same for both grainfed and grassfed. This implies that the two domestic
categories have the same price mark-up as the observed aggregated price difference $\Delta_{u d-v d}=u_{d}-$ $v_{d} / 0.55$ where $\Delta_{u d-v d}$ is measured as per kilogram carcass weight. That is, the domestic wholesale price grainfed carcass is $u_{n d}=v_{n} / / 0.55+\Delta_{u d-v d}$ and for grassfed carcass is $u_{s d}=\nu_{s d} / 0.55+\Delta_{u d-v d}$.

## Step 6. $v_{e}$ and $\Delta_{u d d v d} \Rightarrow u_{e}, u_{n e}$ and $u_{s e}$

Information on export beef carcasses is not often reported. Unlike the domestic market where the cattle are slaughtered in abattoirs and then cut and packed into retail cuts in supermarkets and local butchers, the cattle for export are often slaughtered, cut, boned, trimmed, processed and then packed into boxes within abattoirs. Physically the export marketing sector may be simply the boning and packing rooms in abattoirs. The model is structured with a separate export marketing sector in order to be consistent with the domestic market for the joint processing sector. As the carcass quantities for export beef are converted from live weights in the same way as domestic beef (ie. with a $55 \%$ conversion factor), it is reasonable to assume that the cost for slaughtering and processing per kilogram of export cattle to beef carcass is the same as that of domestic cattle. Under this assumption, the export carcass prices are calculated as $u_{n e}=v_{n e} / 0.55+\Delta_{u d-v d}, u_{s e}=v_{s e} / 0.55+\Delta_{u d-v d}$ and $u_{\epsilon}=v_{e} / 0.55+\Delta_{u d-v d}$.

## Step 7. $p_{e}$ and $\Delta_{p n e-p s e} \Rightarrow p_{n e}$ and $p_{s e}$

The prices for shipped weight export beef are obtained from the unit values of Australian export beef and veal in ABARE (Table 146, 1998). The prices are reported in financial years. The calendar year prices for $p_{e}$ are estimated as the averages of prices of each two adjacent financial years.

Information on separate grainfed and grassfed export shipped weight prices is not available. A price premium is estimated based on the prices on principal overseas markets reported on Table 145 in ABARE (1998). If the 'Japan boneless chilled' price in this table is taken as an exportgrainfed price indicator, 'Japan boneless frozer' price as a good-quality or non-cow grassfed export price, and 'US boneless frozen as a US cow/manufacturing beef price, the price differences between the grainfed and the two gr.tssfed types are $\$ 2.8$ and $\$ 2.6$ per kg (f.o.b.) on average for 1992-1997. As the information only serves as a rough guide, the difference between the f.a.s. (free alongside ship) and f.o.b. (free on board) prices, ie. the loading charges, is
ignored. Roughly one-third of export-grassfed beef are US manufacturing beef. In light of all these, a $\$ 2.6$ price premium is assumed for the export market, ie. $\Delta_{\text {pne-pse }}=\$ 2.6 / \mathrm{kg}$ (shipped weight). The grainfed and grassfed prices are then calculated as $p_{s e}=p_{e}-2.6 * \rho_{(n e / e)}$ and $p_{n e}=$ $p_{s e}+2.6$, where $\rho_{(n e / e)}=Y_{n e} / Y_{e}$ is the proportion of grainfed beef in total export. As the same quantity conversion percentages (ie. $55 \%$ and $68 \%$ ) are used for both grainfed and grassfed, the grainfed proportion is the same at live, carcass and shipped weight levels.

Step 8. $\mathbf{p}_{\mathrm{d}}$ and $\Delta_{\text {pnd-psd }} \Rightarrow \mathbf{p}_{\text {nd }}$ and $\mathbf{p}_{\text {sd }}$

The retail beef prices reported by ABARE (Table 144, 1998) are price averages of selected beef cuts that do not include some lower quality cut s . As discussed in Step 6 of 5.2.1, six major retail cuts are included in the domestic retail cuantity and price calculation, which gives a domestic saleable yield percentage $\mathrm{R}_{(\mathrm{Z} / / \mathrm{Qd})}=72 \%$. The weights for the six cuts are taken from Griffith, Green and Duff (1991); they are: $9.4 \%$ for rump steak, $15.3 \%$ for sirloin, $13.5 \%$ for top side, $19 \%$ for chuck, $15.7 \%$ for blade, and $2^{\prime \prime} .1 \%$ for beef mince. The prices for these cuts are taken from the national averages of the monthly retail selling prices published on the Australian Meat Industry Bulletin (Nielsen Marketing Research 1997b). The domestic retail price $\mathrm{p}_{\mathrm{d}}$ is calculated as the weighted average price of the six cuts.

A grainfed premium is needed in order to derive the grainfed and grassfed prices from the aggregated price $p_{d}$. Australia has no domestic grading system that could provide the information on quantity or price of grainfed beef that is sold through the retail outlets. Based on talks with people from the industry (for example, B. Gaden, NSW Agriculture, per. comm. 1998), a grainfed premium of $\$ 2.5$ per kilograin is assumed. That is $\Delta_{\mathrm{pnd}} . p_{s d}=\$ 2.5 / \mathrm{kg}$ (retail cuts).

The domestic retail grainfed and grassfed beef prices for the quantities in Step 7 of 5.2.1 are calculated as $p_{s d}=p_{e}-2.5^{*} \rho_{(n d / d)}$ and $p_{n d}=p_{s d}+2.5$.

## Step 9. $s_{n l d}$ and $s_{n l e}$

The Land newspaper (NLRS 1998b) reports the weekly feeder cattle prices in three categories: domestic feeder steers under 320 kg , domestic feeder heifers under 320 kg and export feeder steers over 400 kg . The annual averages of these prices are obtained from NLRS (A. Galea,

NLRS, per. comm. 1998). The average of the two domestic feeder prices is used as domestic feeder price $s_{n l d}$, and the over 400 kg price as export feeder price $s_{n l e}$.

## Step 10. $\mathbf{w}_{1}$

The Land newspaper (NLRS 1998c) also reports weekly the weaner prices in the CALM system in terms of location, weight range, average weight, whether it is for restock and the price in $\mathrm{c} / \mathrm{kg}$ live weight and $\$ /$ head. As described in Step 11 of A3.1 in this Appendix, the average weight for all the weaners is at the level of 208 kg live weight. Around 100 weaner price observations from the CALM report, that are for restock and have reasonable weight ranges, are chosen and entered to a spreadsheet. Average weaner prices ( $w_{l}$ ) are calculated for each year and then for the whole 1992-1997 period.

## Step 11. $s_{n 2}$

The feed barley prices reported by ABARE (Talle 45, 1998) are used as the feedgrain prices for the cattle feedlots. Barley is the preferred feed in cattle feedlots. The financial year prices of every adjacent two years are averaged to approxinate the calendar year figures.

## Appendix 4. SHAZAM Code for Sensitivity Simulation

```
*Shazam program for sensitivity simulation in Chapter 8.
set nowarn
set nodoecho
set ranseed=66
file 11 x1psfm.o
file 12 x1psot.o
file 13 x1.cs.o
file 14 xlrpsfm.o
file 15 x1rpsot.o
file 16 x1rcs.o
file 21 zit.o
file 22 zsigcc1.o
file 23 zsigcc2.o
file 24 zsigcol.o
file 25 zsigco2.o
file 26 zsigco3.o
file 27 zsigco4.o
file 28 zips1.o
file 29 zips2.o
file 30 ztaul.o
file 31 ztau2.o
file 32 ztau3.o
gen1 n=2000
sample 1 1
*set exog shifter values
gen1 tx1=-0.01
gen1 txn2=-0
gen1 txs2=-0
gen1 tfn2=-0
gen1 tfn3=-0
gen1 typ=-0
gen1 tzme=-0
gen1 tzmd=-0
gen1 nqne=0
gen1 nqse=0
gen1 nqnd=0
gen1 nqsd=0
*set price and quantity values and calculat: quantity, cost and revenue shares:
gen1 qsd=404
gen1 qnd=92
gen1 psd=7.81
gen1 pnd=10.31
gen1 tvzqd=qnd*pnd+qsd*psd
gen1 rqsd=(qsd*psd)/tvzqd
```

```
gen1 rqnd=1-rgsd
gen1 qse=665
gen1 qne=110
gen1 pse=3.06
gen1 pne=5.66
gen1 tvzqe=qse*pse+qne*pne
gen1 rqse=(qse*pse)/tvzqe
gen1 rqne=1-rqse
gen1 znd=128
gen1 zsd=561
gen1 und=2.70
gen1 usd=2.45
gen1 kzsd=(zsd**usd)/tvzqd
gen1 kznd=(znd*und)/tvzqd
gen1 kzmd=1-kzsd-kznd
gen1 zse=974
gen1 zne=161
gen1 use=2.13
gen1 une=2.45
gen1 kzse=(zse*use)/tvzqe
gen1 kzne=(zne*une)/tvzqe
gen1 kzme=1-kzse-kzne
gen1 tvyz=zse*use+zsd*usd+zne*une+znd*und
gen1 rzse=zse*use/tvyz
gen1 rzsd=zsd*usd/tvyz
gen1 rzne=zne*une/tvyz
gen1 rznd=znd*und/tvyz
gen1 yse=1772
gen1 ysd=1019
gen1 yne=293
gen1 ynd=232
gen1 vse=1.03
gen1 vsd=1.21
gen1 vne=1.20
gen1 vnd=1.34
gen1 kyse=(yse*vse)/tvyz
gen1 kysd=(ysd*vsd)/tvyz
gen1 kyne=(yne*vne)/tvyz
gen1 kynd=(ynd*vnd)/tvyz
gen1 kyp=1-kyse-kysd-kyne-kynd
gen1 tvfyn=yne*vne+ynd*vnd
gen1 ryne=(yne*vne)/tvfyn
gen1 rynd=1-ryne
gen1 fn1e=205
gen1 fn1d=172
gen1 fn2=819
gen1 sn1e==1.12
gen1 sn1d=1.02
gen1 sn2=0.176
```

```
gen1 kfn1e=fnle*sn1e/tvfyn
gen1 kfn1d=fn1d*sn1d/tvfyn
gen1 kfn2=fn2*sn2/tvfyn
gen1 kfn3=1-kfn1e-kfn1d-kfn2
gen1 tvxys=yse*vse+ysd*vsd
gen1 ryse=yse*vse/tvxys
gen1 rysd=1-ryse
gen1 tvxfr:=fn1e*sn1e+fn1d*sn1d
gen1 rfn1e=fn1e*sn1e/tvxfn
gen1 rfn1d=1-rfn1e
gen1 xn1=206
gen1 xs1=1542
gen1 x1=xn1+xs1
gen1 w1=1.12
gen1 kxn1=(xn1*w1)/tvxfn
gen1 kxn2=1-kxn1
gen1 kxs1=(xs1*w1)/tvxys
gen1 kxs2=1-kxs1
gen1 rhoxn1=xn1/x1
gen1 rhoxs1=1-rhoxn1
?do %=1,n
*draw elasticity values:
* ipx1~N(0.9,0.2**2 |>0)
?do $=1,100
gen1 z=nor(1)
gen1 ipx1=0.9+0.2*z
?endif(ipx1.gt.0)
*gen1 dripx1=$
*print dripx1
endo
* itd: itss~N(-1.1,0.2**2|<0), itnn~N(1.5*it;s,0.1**2|itnrı<itss),
* itsn~N(0.3,0.1**2|itsn>0, itsn<-itss, itns:3.3itsn<-itnn)
?do $=1,100
gen1 z=nor(1)
gen1 itqsdsd=-1.1+0.2*z
?endif(itqsdsd.lt.0)
endo
?do $=1,100
gen1 z=nor(1)
gen1 itqndnd=1.5*itqsdsd+0.1*z
?endif(itqndnd.lt.itqsdsd)
endo
?do $=1,100
gen1 z=nor(1)
gen1 itqsdnd=0.3+0.1*z
```

gen1 itqndsd=(rqsd/rqnd)*itqsdnd
?endif((itqsdnd.ge.0).and.(itqsdnd.1t.(-itqsdsd)).and.(itqndsd.lt.(-itqndnd)))
endo

```
*ite: itse~N(-5,1.2**2|itse<0,|itse|>|itsdsö|),
* itne~N(-2.5,1**2| itne<0, |itne|>|.itndnci |,|itne|<|itse|)
```

?do $\$=1,100$
gen1 $z=$ nor (1)
gen1 itqsese $=-5+1.5^{*} z$
?endif((itqsese.le.0).and.(itqsese.lt.itqsdsa))
endo
?do $\$=1,100$
gen1 $z=$ nor (1)
gen1 itqnene=--2.5+0.7*z
?endif((itqnene.le.0).and.(itqnene.lt.itqndrd).and.(itqnene.gt.itqsese))
endo
*ipfn2~N(0.8,0.1**2|>0)
?do $\$=1,100$
gen1 $z=$ nor (1)
gen1 ipfn2=0.8+0.1*z
?endif(ipfn2.gt.0)
?endo
*all 6 of ips (others) center at the mixed dist'n:

* $50 \% \mathrm{~N}(5,1.7 * * 2)$ and $50 \% 4.5+\exp (0.2$ )
?do $\$=1,100$
gen1 $z=$ nor (1)
gen1 ipxn2=5+1.7*z
?endif(ipxn2.ge.0)
?endo
gen1 lam=0.2
gen1 p=uni(1)
gen1 u=uni(1)
gen1 $x x=4.5+(-\log (u)) / l a m$
if(p.gt.0.5) ipxn2=xx
?do $\$=1,100$
gen1 ipxs2=ipxn2+nor(0.5)
?endif(ipxs2.gt.0)
endo
?do $\$=1,100$
gen1 ipyp=ipxn2+nor(0.5)
?endif(ipyp.gt.0)
endo
?do $\$=1,100$
gen1 ipzme=ipxn2+nor(0.5)
?endif(ipzme.gt.0)
endo
?do $\$=1,100$
gen1 ipfn3=ipxn2+nor (0.5)
?endif(ipfn3.gt.0)
endo

```
?do $=1,100
gen1 ipzmd=ipxn2+nor(0.5)
?endif(ipzmd.gt.0)
endo
*tau(Fn1e,fn1d) and tau(yse,ysd): N(-2,0.5**2 | < ) . Note, higher chance of >2?)
?do $=1,100
gen1 z=nor(1)
gen1 taufed=-2+0.5*z
?endif(taufed.1t.0)
endo
?do $=1,100
gen1 z=nor(1)
gen1 tauysed=--2+0.5*z
?endif(tauysed.1t.0)
endo
*sig(cattle,cattle): all 9 of them around N(0.05,0.1**2|>0)
?do $=1,500
gen1 z=nor(1)
gen1 sigysesd=0.05+0.1*z
?endif(sigysesd.ge.0)
endo
?do $=1,100
gen1 sigfed=sigysesd+nor(0.025)
?endif(sigfed.ge.0)
endo
?do $=1,100
gen1 sigysene=sigysesd+nor(0.025)
?endif(sigysene.ge.0)
endo
?do $=1,100
gen1 sigysend=sigysesd+nor(0.025)
?endif(sigysend.ge.0)
endo
?do $=1,100
gen1 sigysdne=sigysesd+nor(0.025)
?endif(sigysdne.ge.0)
endo
?do $=1,100
gen1 sigysdnd=sigysesd+nor(0.025)
?endif(sigysdnd.ge.0)
endo
?do $=1,100
gen1 sigynend=sigysesd+nor(0.025)
?endif(sigynend.ge.0)
endo
?do $=1,100
gen1 sigzsdnd=sigysesd+nor(0.025)
?endif(sigzsdnd.ge.0)
endo
?do $=1,100
gen1 z=nor(1)
gen1 sigzsene=sigysesd+0.025*z
?endif(sigzsene.ge.0)
```

```
endo
*sig(cattle,others)all around mixed 70%N(0.065,0.1**2)plus 30% 0.08+exp(4.1).
?do $=1,500
gen1 sigxs12=0.065+nor(0.1)
?endif(sigxs12.ge.0)
endo
gen1 lam=4.1
gen1 p=uni(1)
gen1 u=uni(1)
gen1 zz=0.08+(-log(u))/lam
if(p.1t.0.3) sigxs12=zz
?do $=1,500
gen1 sigxn12=sigxs12+nor(0.05)
?endif(sigxn12.ge.0)
endo
?do $=1,500
gen1 sigfe2=si.gxs12+nor(0.05)
?endif(sigfe2.ge.0)
endo
?do $=1,500
gen1 sigfe3=sj.gxs12+nor(0.05)
?endif(sigfe3.ge.0)
endo
?do $=1,500
gen1 sigfd2=sj.gxs12+nor(0.05)
?endif(sigfd2.ge.0)
endo
?do $=1,500
gen1 sigfd3=si.gxs12+nor(0.05)
?endif(sigfd3.ge.0)
endo
?do $=1,500
gen1 sigf23=sigxs12+nor(0.05)
?endif(sigf23.ge.0)
endo
?do $=1,500
gen1 sigysep=sigxs12+nor(0.05)
?endif(sigysep.ge.0)
endo
?do $=1,500
gen1 sigysdp ==sigxs12+nor(0.05)
?endif(sigysdp.ge.0)
endo
?do $=1,500
gen1 sigynep=sigxs12+nor(0.05)
?endif(sigynep.ge.0)
endo
?do $=1,500
```

```
gen1 sigyndp=sigxs12+nor(0.05)
?endif(sigyndp.ge.0)
endo
?do $=1,500
gen1 sigzsdmd=sigxs12+nor(0.05)
?endif(sigzsdmd.ge.0)
endo
?do $=1,500
gen1 sigzndmd=:sigxs12+nor(0.05)
?endif(sigzndmd.ge.0)
endo
?do $=1,500
gen1 sigzseme=sigxs12+nor(0.05)
?endif(sigzseme.ge.0)
endo
?do $=1,500
gen1 sigzneme=:sigxs12+nor(0.05)
?endif(sigzneme.ge.0)
endo
*tau(cattle,cattle) around N(-0.05, 0.1)
?do $=1,500
gen1 z=nor(1)
gen1 tauyned=--0.05+0.1*z
?endif(tauyned.le.-0.01)
endo
?do $=1,500
gen1 tauzsesd==tauyned+nor(0.025)
?endif(tauzsesd.le.0)
endo
?do $=1,500
gen1 tauzsene=tauyned+nor(0.025)
?endif(tauzsene.le.0)
endo
?do $=1,500
gen1 tauzsend=tauyned+nor(0.025)
?endif(tauzsend.le.0)
endo
?do $=1,500
gen1 tauzsdne:=tauyned+nor(0.025)
?endif(tauzsdne.le.0)
endo
?do $=1,500
gen1 tauzsdnd:=tauyned+nor(0.025)
?endif(tauzsdnd.le.0)
endo
?do $=1,500
gen1 tauznend=tauyned+nor(0.025)
?endif(tauznend.le.0)
endo
```

```
?do $=1,500
gen1 tauqsene:= tauyned+nor(0.025)
?endif(tauqsene.le.0)
endo
?do $=1,500
gen1 tauqsdnd== tauyned+nor(0.025)
?endif(tauqsdnd.le.0)
endo
write(21) itqsdsd itqndnd itqsdnd itqndsd itqsese itqnene
write(22) sigysesd sigysene sigysend s:.gysdne sigysdnd sigynend
write(23) sigfed sigzsdnd sigzsene
write(24) sigxs12 sigxn12
write(25) sigfe2 sigfe3 sigfd2 sigfd3 sigf23
write(26) sigysep sigysdp sigynep sigyndp
write(27) sigzsdmd sigzndmd sigzseme sj.gzneme
write(28) ipx1 ipfn2
write(29) ipxr2 ipxs2 ipfn3 ipyp ipzme ipzmd
write(30) taufed tauysed
write(31) tauyned tauqsene tauqsdnd
write(32) tauzsesd tauzsene tauzsend táuzsdn\geqslant tauzsdnd tauznend
sample 1 1
?n1 58 / ncoef=58 solve coef=epq numeric it?r=15000
*Equation numbers are as in 'model-int\ing'-9/3/99.
*1-4
?eq ex1-ipx1*(ew1-tx1)
?eq ex1-rhoxn1*exn1-rhoxs1*exs1
?eq exn2-ipxn2*(ewn2-txn2)
?eq exs2-ipxs2*(ews2-txs2)
*5-8
?eq exn1+kxn2*sigxn12*(ew1-ewn2)-efn1
?eq exn2-kxn1*sigxn12*(ew1-ewn2)-efn1
?eq exs1+kxs2*sigxs12*(ew1-ews2)-eys
?eq exs2-kxs1*sigxs12*(ew1-ews2)-eys
* 9-12
?eq kxn1*exn1+kxn2*exn2-rfn1e*efn1e-rfn1d*efnid
?eq kxn1*ew1+kxn2*ewn2-rfn1e*esn1e-rfn1d*esn1d
?eq kxs1*exs1+kxs2*exs2-ryse*eyse-rysd*eysd
?eq kxs1*ew1+kxs2*ews2-ryse*evse-rysd*evsd
*13-18
?eq efn1e+rfn1d*taufed*(esn1e-esn1d)-exn
?eq efn1d-rfn1e*taufed*(esn1e-esn1d)-exn
?eq eyse+rysd*tauysed*(evse-evsd)-exs
?eq eysd-ryse*tauysed*(evse-evsd)-exs
?eq efn2-ipfn2*(esn2-tfn2)
?eq efn3-ipfn3*(esn3-tfn3)
*19-22
?eq efn1e+(kfnld*sigfed+kfn2*sigfe2+kfn3*sigfe3)*esn1e-kfn1d*sigfed*esn1d&
-kfn2*sigfe2*esn2-kfn3*sigfe3*esn3-eyn
```

```
?eq efn1d-kfn1e*sigfed*esn1e-kfn2*sigfd2*esn2-kfn3*sigfd3*esn3&
+(kfn1e*sigfed+kfn2*sigfd2+kfn3*sigfd3)*esn1&-eyn
?eq efn2-kfn1e*sigfe2*esn1e-kfn1d*sigfd2*esnLd-kfn3*sigf23*esn3&
+(kfn1e*sigfe2+kfn1d*sigfd2+kfn3*sigf25)*esn2-eyn
?eq efn3-k.fn1e*sigfe3*esn1e-kfn1d*sigfd3*esnld-kfn2*sigf23*esrn2&
+(kfn1e*sigfe3+kfn1d*sigfd3+kfn2*sigf2\Xi)*esn3-eyn
*23-27
?eq kfn1e*efn1e+kfn1d*efn1d+kfn2*efn2+kfn3*efn3-ryne*eyne-rynd*eynd
?eq kfn1e*esn1e+kfn1d*esn1d+kfn2*esn2+kfn3*esn3-ryne*evne-rynd*evnd
?eq eyne+rynd*tauyned* (evne-evnd)-efn
?eq eynd-ryne*tauyned*(evne-evnd)-efn
?eq eyp-ipyp*(evp-typ)
*28-32
?eq eyse+(kysd*sigysesd+kyne*sigysene+k.ynd*s i.gysend+kyp*si.gysep)*evse&
-kysd*sigysesd*evsd-kyne*sigysene*evne-kynd*;igysend*evnd--kyp*sigysep*evp-ez
?eq eysd-kyse*sigysesd*evse+(kyse*sigysesd+k,rne*sigysdne+kynd*sigysdnd&
+kyp*sigysdp)*evsd-kyne*sigysdne*evne-k.ynd*si.gysdnd*evnd-kyp*sigysdp*evp-ez
?eq eyne-kyse*sigysene*evse-kysd*sigyscine*ev;d+(kyse*sigysene+kysd*sigysdne&
+kynd*sigynend+kyp*sigynep)*evne-kynd*sigyne rd*evnd-kyp*sigynep*evp-ez
?eq eynd-kyse*sigysend*evse-kysd*sigyscind*ev;d-kyne*sigynend*evne+(kyse&
*sigysend+kysd*sigysdnd+kyne*sigynend+k.yp*si|fyndp)*evnd-kyp*sigyndp*evp-ez
?eq eyp-kyse*sigysep*evse-kysd*sigysdp*evsd-cyne*sigynep*evne-kynd*sigyndp*evnd&
+(kyse*sigysep+kysd*sigysdp+kyne*sigyn\inp+kyn`**sigyndp)*evp-ez
*33-34
?eq kyse*eyse+kysd*eysd+kyne*eyne+kynd*eynd+kyp*eyp&
-rzse*ezse-rzsd*ezsd-rzne*ezne-rznd*eznd
?eq kyse*evse+kysd*evsd+kyne*evne+kynd*evnd+:cyp*evp&
-rzse*euse-rzsd*eusd-rzne*eune-rznd*eund
*35-38
?eq ezse+(rzsd*tauzsesd+rzne*tauzsene+rznd*tizuzsend)*euse&
-rzsd*tauzsesd*eusd-rzne*tauzsene*eune-rznd**auzsend*eund-ey
?eq ezsd-rzse*tauzsesd*euse-rzne*tauzsdne*eur:e-rznd*tauzsdnd*eund&
+(rzse*tauzsesd+rzne*tauzsdne+rznd*tauzsdnd)*eusd-ey
?eq ezne-rzse*tauzsene*euse-rzsd*tauzsdne*eu::d-rznd*tauznend*eund&
+(rzse*tauzsene+rzsd*tauzsdne+rznd*tauznend)*eune-ey
?eq eznd-rzse*tauzsend*euse-rzsd*tauzsdnd*eu::d-rzne*tauznend*eune&
+(rzse*tauzsend+rzsd*tauzsdnd+rzne*tauznend) eund-ey
*39-40
?eq ezmd-ipzmd*(eumd-tzmd)
?eq ezme-ipzme*(eume-tzme)
*41-46
?eq ezsd+(kznd*sigzsdnd+kzmd*sigzsdmd)*eu:sd&
-kznd*sigzsdnd*eund-kzmd*sigzsdmd*eumd-eqd
?eq eznd-kzsd*sigzsdnd*eusd-kzmd*sigzndmd*eunid&
```

```
+(kzsd*sigzsdnd+kzmd*sigzndmd)*eund-eqd
?eq ezmd-kzsd*sigzsdmd*eusd-kznd*sigzndmd*eund&
+(kzsd*sigzsdmd+kznd*sigzndmd)*eumd-eqd
?eq ezse+(kzne*sigzsene+kzme*sigzseme)*euse&
-kzne*sigzsene*eune-kzme*sigzseme*eume--eqe
?eq ezne-kzse*sigzsene*euse-kzme*sigzneme*eume&
+(kzse*sigzsene+kzme*sigzneme)*eune-eqe
?eq ezme-kzse*sigzseme*euse-kzne*sigzneme*eune&
+(kzse*sigzseme+kzne*sigzneme)*eume-eqe
*47-50
?eq kzsd*ezsd+kznd*eznd+kzmd*ezmd-rqsd*eqsd-rqnd*eqnd
?eq kzsd*eusd+kznd*eund+kzmd*eumd-rqsd*epsd-rqnd*epnd
?eq kzse*ezse+kzne*ezne+kzme*ezme-rqse*eqse-rqne*eqne
?eq kzse*euse+kzne*eune+kzme*eume-rqse*epse-rqne*epne
*51-54
?eq eqsd+rqnd*tauqsdnd*(epsd-epnd)-ezd
?eq eqnd-rqsd*tauqsdnd*(epsd-epnd)-ezd
?eq eqse+rqne*tauqsene*(epse-epne)-eze
?eq eqne-rqse*tauqsene*(epse-epne)-eze
*55-58. Note: need to impose constraint to 5;-56.
?eq eqsd-itqsdsd*(epsd-nqsd)-itqsdnd*(\epsilonpnd-n qnd)
?eq eqnd-itqndsd*(epsd-nqsd)-itqndnd*( (fpnd-n mnd)
?eq eqse-itqsese*(epse-nqse)
?eq eqne-itqnene*(epne-nqne)
*?coef ey 0 ep 0 ew1 0 ew2 0 ex1 0 ex2 0
?end
=gen1 ex1t=epq:1
gen1 ew1t=epq:2
gen1 exn1t=epq:3
gen1 exs1t=epq:4
gen1 exn2t=epq:5
gen1 ewn2t=epq:6
gen1 exs2t=epq:7
gen1 ews2t=epq:8
gen1 efn1t=epq:9
gen1 eyst=epq:10
gen1 efn1et=epq:11
gen1 efn1dt=epq:12
gen1 esn1et=epq:13
gen1 esn1dt=epq:14
gen1 eyset=epq:15
gen1 eysdt=epq:16
gen1 evset=epq:17
gen1 evsdt=epq:18
gen1 exnt=epq:19
gen1 exst=epq:20
gen1 efn2t=epq:21
gen1 esn2t=epq:22
gen1 efn3t=epq:23
```

```
gen1 esn3t=epq:24
gen1 eynt=epq:25
gen1 eynet=epq:26
gen1 eyndt=epq:27
gen1 evnet=epq:28
gen1 evndt=epq:29
gen1 efnt==epq:30
gen1 eypt=epq:31
gen1 evpt=epq:32
gen1 ezt=epq:33
gen1 ezset=epq:34
gen1 ezsdt=epq:35
gen1 eznet=epq:36
gen1 ezndt=epq!:37
gen1 euset=epq!:38
gen1 eusdt:=epct:39
gen1 eunet:=epci:40
gen1 eundt:=epci:41
gen1 eyt=epq:42
gen1 ezmdt=epq!:43
gen1 eumdt=epq:44
gen1 ezmet=epq:45
gen1 eumet=epor:46
gen1 eqdt=epq:47
gen1 eqet=epq:48
gen1 eqsdt=epq:49
gen1 eqndt=epc:50
gen1 epsdt=epq:51
gen1 epndt=epq:52
gen1 eqset=epq:53
gen1 eqnet=epq:54
gen1 epset=epq:55
gen1 epnet=epq:56
gen1 ezdt=epq:57
gen1 ezet=epq:58
*values and surplus are measured in millions of Australian dollars ($(A)m)
gen1 dpsx1 = w1*x1 * (ew1t - tx1) * (1+0.5*e:1t)
gen1 dpsxn2 = kxn2*tvxfn * (ewn2t - txr.2) * (1+0.5*exn2t)
gen1 dpsxs2 = kxs2*tvxys * (ews2t - txs2) * (1+0.5*exs2t)
gen1 dpsfm=dpsx1+dpsxn2+dpsxs2
gen1 dpsfn2=kfn2*tvfyn*(esn2t-tfn2)*(1+0.5*e&n2t)
gen1 dpsfn3=kfn3*tvfyn*(esn3t-tfn3)*(1+0.5*efn3t)
gen1 dpsyp=kyp*tvyz*(evpt-typ)*(1+0.5*\inypt)
gen1 dpszme=kzme*tvzqe*(eumet-tzme)*(1+0.5*e zmet)
gen1 dpszmd=kzmd*tvzqd*(eumdt-tzmd)* (1+0.5*e:mdt)
gen1 dcsqne = rqne*tvzqe * (nqne - epn\epsilont) * (1+0.5*eqnet)
gen1 dcsqse = rqse*tvzqe * (nqse - eps\int) * (1+0.5*eqset)
gen1 dcsqe = dcsqne+dcsqse
gen1 dcsqnd = rqnd*tvzqd * (nqnd - epncit) * (1+0.5*eqndt)
gen1 dcsqsd = rqsd*tvzqd * (nqsd - epscit) * (1+0.5*eqsdt)
gen1 dcsqd=dcsqnd+dcsqsd
```

gen 1 dts $=d p s x 1+d p s x n 2+d p s x s 2+d p s f n 2+d p s f n 3 s$ $+d p s y p+d p s z m e+d p s z m d+d c s q n e+d c s q s e+d c s q r i d+d c s q s d$
write(11) dpsx1 dpsxn2 dpsxs2 dpsfm write(12) dpsf.n2 dpsfn3 dpsyp dpszme dpszmd write(13) dcsqne dcsqse dcsqe dcsqd dts
gen1 $\mathrm{rpsx} 1=\mathrm{dpssx} / \mathrm{dts}$
gen1 rpsxn2=dpsxn2/dts
gen1 rpsxs2=dpsxs2/dts
gen1 rpsfm=dpsfm/dts
gen1 rpsfn2=dpsfn2/dts
gen1 rpsfn $3=$ dpsfn $3 / d t s$
gen1 rpsyp=dpsyp/dts
gen1 rpszme=dpszme/dts
gen1 rpszmd=dpszmd/dts
gen1 rcsqne=dc:sqne/dts
gen1 rcsqse=dc:sqse/dts
gen1 rcsqe=dcsqe/dts
gen1 $\mathrm{rcsqd}=\mathrm{dcsq} q / \mathrm{dts}$
write(14) rpsxi1 rpsxn2 rpsxs2 rpsfm
write(15) rpsfin2 rpsfn3 rpsyp rpszme rpszmd
write(16) rcsqne rcsqse rcsqe rcsqd
?endo
sample 1 n
do $\%=21,32$
rewind \%
endo
read(21) iqsdsdv iqndndv iqsdndv iqndsclv iqsesev iqnenev read(22) sysesdv sysenev sysendv sysdnev sys Indv synendv
read(23) sfedv szsdndv szsenev
read(24) sxs 12 v sxn12v
read (25) sfe2v sfe3v sfd2v sfd3v sf23v
read(26) sysepv sysdpv synepv syndpv
read(27) szsdmdv szndmdv szsemev sznemev
read(28) ipx1v ipfn2v
read(29) ipxn2v ipxs2v ipfn3v ipypv ipzmev iozmdv
read(30) tfedv tysedv
read(31) tynedv tqsenev tqsdndv
read (32) tzsesdv tzsenev tzsendv tzsdneev tzs lndv tznendv
stat iqsdsdv iqndndv iqsdndv iqndsdv içsesev iqnenev
stat sysesdv sysenev sysendv sysdnev sy'sdndv synendv
stat sfedv szsdndv szsenev
stat sxs12v sxn12v
stat sfe2v sfe3v sfd2v sfd3v sf23v
stat sysepv sysdpv synepv syndpv
stat szsdmdv szndmdv szsemev sznemev
stat ipx1v ipfn2v
stat ipxn2v ipxs2v ipfn3v ipypv ipzmev ipzmd ${ }_{T}$
stat tfedv tysedv
stat tynedv tqsenev tqsdndv
stat tzsesdv tzsenev tzsendv tzsdnev tzsdndv tznendv
do $\%=11,16$

```
rewind %
endo
read(11) dpsx1v dpsxn2v dpsxs2v dpsfmv
read(12) dpsfn2v dpsfn3v dpsypv dpszmers dpszndv
read(13) dcsqnev dcsqsev dcsqev dcsqdv dt.sv
read(14) rpsx1.v rpsxn2v rpsxs2v rpsfmv
read(15) rpsfn2v rpsfn3v rpsypv rpszmev rpszmdv
read(16) rcsqnev rcsqsev rcsqev rcsqdv
stat dpsx1v dpsxn2v dpsxs2v dpsfmv
stat dpsfn2v dpsfn3v dpsypv dpszmev dp:szmdv
stat dcsqnev dcsqsev dcsqev dcsqdv dtsv
stat rpsx1v rpsxn2v rpsxs2v rpsfmv
stat rpsfn2v rpsfn3v rpsypv rpszmev rpszmdv
stat rcsqnev rcsqsev rcsqev rcsqdv
stop
```


[^0]:    ${ }^{1}$ If percentage change is defined as $\mathrm{E}()=.\Delta() /.($.$) , a parallel slift on the ( \mathrm{U}, \mathrm{V}$ ) plane by a constant $\lambda$ along the V direction is equivalent to a proportional shift by a constant percentage ( $1-\mathrm{e}^{-\lambda}$ ), instead of $\lambda$. The new supply curve on the $(\mathrm{Q}, \mathrm{P})$ plane will be $\mathrm{S}_{2}: \mathrm{Q}=\mathrm{S}\left(\mathrm{P}-\mathrm{P}\left(1-\mathrm{e}^{-\lambda}\right)\right.$ ), instead of $\mathrm{Q}=\mathrm{S}(\mathrm{P}-\lambda \mathrm{P})$. The difference between $\lambda$ and $\left(1-\mathrm{e}^{-\lambda}\right)$ is a higher order infinitesimal $\mathrm{O}\left(\lambda^{2}\right)$ which is caused by the different definitions of percentage change $\Delta \ln ($.$) and$ $\Delta() /.($.$) . The difference of the two percentage changes is O\left(\Delta^{\prime}().\right)$.

