Chapter 3. Functional Forms, Types of Exogenous Shifts and Economic Surplus Changes using EDM

3.1 Introduction

In this chapter[1], the relationship between the assumptions about the types of exogenous shifts and about the functional forms of demand and supply curves and the estimated economic surplus changes using EDM is examined.

As reviewed in Sections 2.5.1 and 2.5.2 in Chapter 2, in EDM applications, impacts of technology, promotion and government policies have been modelled as exogenous shifts in the relevant supply or demand curves, and these shifts have been assumed to be parallel or proportional. It has been recognised in the literature that the assumption about the nature of the exogenous shift is a source of error (Lindner and Jarrett 1980; Miller, Rosenblatt and Hushak 1988, Chung and Kaiser 1999; Wohlgenant 1999). Functional form of the supply and demand curves is another issue. Some have assumed explicit functional forms such as linear and constant elasticity, and others have followed Muth (1964) in applying comparative statics to general functional forms. It has been understood that linear approximation is implied by such operation. Despite the efforts of Alston and Wohlgenant (1990), what has not been fully appreciated are the conditions under which the EDM results are exact, and the extent of errors when these conditions are not met. In particular, two questions arise that are of theoretical and empirical importance: (a) For an assumed parallel or proportional shift, what functional form is required of the demand and supply curves to make the EDM measures of both price and quantity changes and surplus changes exact? (b) When the true demand and supply curves are not of this functional form, how accurate are the EDM results and what determines the sizes of the errors?

Regarding the first question, the main point of confusion concerns whether linear approximation of demand and supply functions based on point estimates of demand and supply elasticities necessarily requires the global imposition of either a linear or constant elasticity functional form. Alston and Wohlgenant (1990) showed that EDM results are exact when the true demand and supply functions are linear and the research-induced shift is parallel. Hurd[2]

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[1] The content of this chapter has been published in Zhao, Mullen and Griffith (1997).
[2] However Hurd (1996) disregarded the nature of the research-induced supply shift which is a vital assumption for the estimation of surplus changes.

(1996) argued that the required functional form depends on whether the percentage changes in prices and quantities, $E(.)$, are proportional, $\Delta(.)/(.)$, or log-differenced, $\Delta \ln(.)$, where $(.)$ refers to a price or quantity variable and Δ implies a finite change of the variable. With respect to the second question, Alston and Wohlgenant (1990) provided empirical evidence that, when the true demand and supply curves are of constant elasticity rather than linear form, and when a parallel shift is assumed, the errors in the EDM results are small as long as the size of the exogenous shift is small.

In this chapter, through Taylor expansion and graphical illustration of a single-market model, the results of Alston and Wohlgenant (1990) and Hurd (1996) are summarised and extended to define the conditions under which EDM measures are exact. These conditions relate to how percentage changes in prices and quantities are defined, the functional form of supply and demand curves and the nature of the exogenous shift. Analytical expressions for the errors when these conditions are not satisfied are derived so that determinants of the sizes and directions of the errors can be identified. Two scenarios are considered, relating to the case of a parallel shift and the case of a proportional shift.

In 3.2 and 3.3, the relationship between the assumptions about the functional form of demand and supply and about the types of exogenous shift and the EDM estimates, of both the price and quantity changes and the economics surplus changes respectively, is examined. Analytical expressions for the errors are derived for true demand and supply functions of any form. The conclusions from this mathematical exercise and their implications for empirical applications are given in 3.4. All mathematical proofs for the results are given in Appendix 1.

3.2   Estimating Price and Quantity Changes

Consider the single-market model presented in Section 2.3, Chapter 2. Assume that the true demand and supply curves for the commodity are not known but can be represented in general form as

(3.1) $S_1 : Q = S(P)$ *initial supply curve*

(3.2) $D_1 : Q = D(P)$ *initial demand curve*

As shown in Figure 3.1, the intersection of the above curves, $E_1(Q_1, P_1)$, is the initial equilibrium point. Assume that a new technology will cause the supply curve to shift down in the price direction such that

(3.3) $S_2 : Q = S(P - K)$ *new supply curve*

where $K = K(P)$ specifies the amount and type of an exogenous supply shift. In empirical applications, the per unit cost change at Q_1 is often expressed as a percentage of P_1 such that $K(P_1) = \lambda P_1$, where $K < 0$ and $\lambda < 0$ for a downward supply shift. The new equilibrium point is the intersection of D_1 and S_2, denoted as $E_2(Q_2, P_2)$ in Figure 3.1.

To estimate the impact of this exogenous shift in supply, EDM employs knowledge about the current equilibrium price and quantity (P_1 and Q_1), the demand and supply elasticity values at E_1 (η and ε) and the percentage supply shift at E_1 (λ), to approximate the changes in price and quantity and in economic surplus associated with displacement to E_2.

Price and quantity changes are given by totally differentiating the logarithms of Equations (3.2) and (3.3) at point E_1 to give:

(3.4) $dQ/Q = \eta(dP/P)$ or $d \ln Q = \eta(d \ln P)$

(3.5) $d Q/Q = \varepsilon(d P/P - \lambda)$ or $d \ln Q = \varepsilon(d \ln P - \lambda)$

Solving (3.4) and (3.5) jointly gives:

(3.6) $d P/P = d \ln P = \lambda \varepsilon/(\varepsilon - \eta)$ and $d Q/Q = d \ln Q = \lambda \varepsilon \eta/(\varepsilon - \eta)$

Equation (3.6) gives the exact solutions to the percentage changes at point E_1 in infinitesimal terms. Note that in (3.6) there are two equivalent definitions of percentage changes in infinitesimal terms which give rise to two ways of approximating *finite* percentage changes (which are only equal in the limit), and whether the supply shift is parallel or proportional has not been defined.
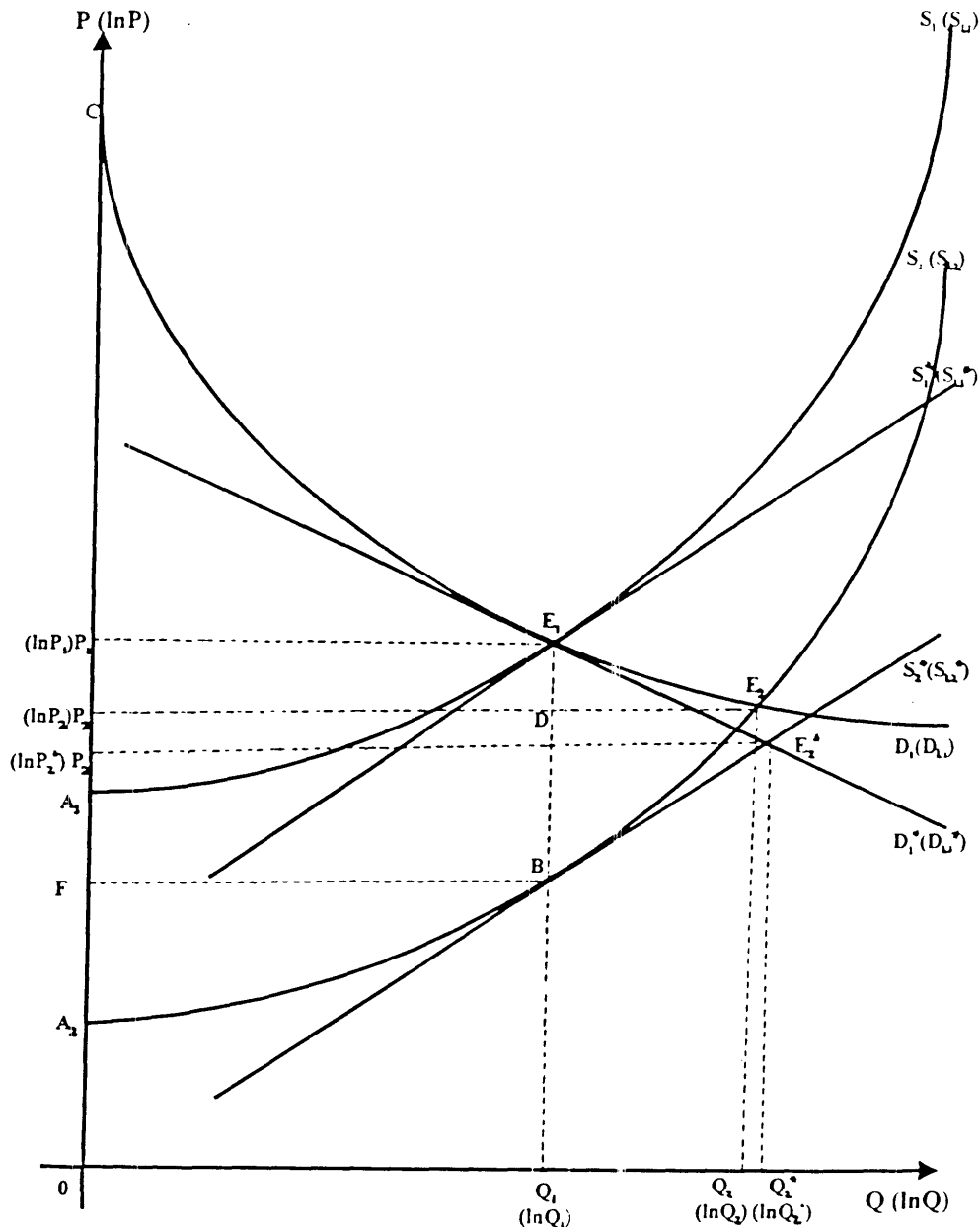
(Q,P) Plane: parallel shift and linear approximation

S_1 and D_1:	true supply and demand curves of any functional form.
$S_1{}^*$ and $D_1{}^*$:	linear approximations of S_1 and D_1, respectively.
S_2 and $S_2{}^*$:	parallel shifts of S_1 and $S_1{}^*$, respectively.

(lnQ,lnP) Plane: proportional shift and log-linear approximation

S_{L1} and D_{L1}:	true supply and demand curves of any functional form, expressed on (lnQ,lnP) plane.
$S_{L1}{}^*$ and $D_{L1}{}^*$:	linear approximations of S_{L1} and D_{L1}, respectively, on (lnQ,lnP) plane, representing log-linear approximations on the (Q,P) plane.
S_{L2} and $S_{L2}{}^*$:	parallel shifts of S_{L1} and $S_{L1}{}^*$, respectively, on (lnQ, lnP) plane, representing proportional shifts on (Q,P) plane.

Figure 3.1 Parallel Shift and Linear Approximation on (Q, P) Plane and, for Relabelled Axes, on (lnQ, lnP) Plane

In applying EDM, most often percentage changes have been approximated linearly as $\Delta(.)/(.)$ and a parallel shift in supply has been assumed. An alternative approach is to approximate percentage changes as $\Delta \ln(.)$ and to assume a proportional shift in supply. In the following it is demonstrated that the former approach is exact for linear demand and supply curves and the latter is exact for constant elasticity demand and supply curves. The expressions for errors when the true demand and supply curves do not take either of these forms are then derived.

Parallel Supply Shift and Linear Approximation of Price and Quantity Changes

A common assumption has been that new technology results in a constant per unit reduction in costs for all levels of production, and hence the shift in supply is parallel. The exact price and quantity changes, $d(.)/(.)$ in (3.6), are approximated by $\Delta(.)/(.)$ at initial equilibrium, which implies a local linear approximation to the demand and supply curves around E_1. Analytically, if we define

$$(3.7) \qquad K \equiv \lambda P_1 \qquad \text{such that } K \equiv \text{constant for all } Q > 0$$

$$(3.8) \qquad EP = (P_2 - P_1)/P_1, \qquad EQ := (Q_2 - Q_1)/Q_1$$

$$(3.9) \qquad EP^* = (P_2^* - P_1)/P_1 = \lambda \varepsilon/(\varepsilon - \eta) \quad \text{and} \quad EQ^* = (Q_2^* - Q_1)/Q_1 = \lambda \varepsilon \eta/(\varepsilon - \eta),$$

where $E(.)$'s are the true relative changes and $E(.)^*$'s are the EDM estimates, through a Taylor expansion of the demand and supply functions, it can be shown that

$$(3.10) \quad EP - EP^* = [2Q_1(\eta - \varepsilon)]^{-1} P_1^2 [S^{(2)}(c_2)(EP - \lambda)^2 - D^{(2)}(c_1)(EP)^2] = \mathbf{O}(\lambda^2) \quad (\lambda \to 0)$$

and

$$(3.11) \quad EQ - EQ^* = [2Q_1(\eta - \varepsilon)]^{-1} P_1^2 [\eta S^{(2)}(c_2)(EP - \lambda)^2 - \varepsilon D^{(2)}(c_1)(EP)^2] = \mathbf{O}(\lambda^2) \quad (\lambda \to 0)$$

where $D^{(2)}(.)$ and $S^{(2)}(.)$ are the second order derivatives of the demand and supply functions and $P_2 \le c_i \le P_1$ ($i = 1, 2$). Derivations of Equations (3.10) and (3.11) are given in Proposition 2, Appendix 1.

An immediate result from these expressions is that $EP = EP^*$ and $EQ = EQ^*$ when $S^{(2)}(c_2) = D^{(2)}(c_1) = 0$. In other words, the EDM estimates of price and quantity changes in (3.9) are exact when the demand and supply curves are strictly linear around the local area of the current equilibrium point. Equations (3.10)-(3.11) also show that, for nonlinear demand and supply curves, the approximation errors are small as long as the percentage shift λ is small (with infinitesimal order of $O(\lambda^2)$ when $\lambda \to 0$).

Determinants of the sizes and directions of the errors are apparent from these expressions. For example, if it is assumed that the supply curve is increasing and concave and the demand curve is decreasing and concave in the vicinity of the equilibrium point, that is,

(3.12) $\varepsilon > 0$, $S^{(2)}(P) < 0$,   and $\eta < 0$, $D^{(2)}(P) > 0$ $(P \in (P_2, P_1))$,

then it can be shown that the more inelastic (ε| and $|\eta|$, smaller) and curved ($| D^{(2)}(c_1)|$ and $|S^{(2)}(c_2)|$, larger) are the demand and supply functions, the larger are the errors in EP^*. Additionally, under the assumptions in (3.12), it can be shown that $EP \geq EP^*$ (Remark 3 of Proposition 2, Appendix 1). In other words, the size of a price decrease (when $EP < 0$) is always overestimated and the size of a price increase (when $EP > 0$) is always underestimated. This analytical result confirms the empirical result of Alston and Wohlgenant (1990) but also demonstrates that their finding could be conditional on the nature of the curvature of the demand and supply functions. Further, the direction of error in estimating EQ depends on the relative sizes of the demand and supply elasticities and curvatures. It can be positive or negative (Remark 4 of Proposition 2, Appendix 1). The empirical result from Alston and Wohlgenant (1990), that quantity change is always overestimated, does not hold generally.

Upper bounds for the errors in the estimates of price and quantity changes can also be derived based on the derived error expressions. They are given in Remark 2 of Proposition 2, Appendix 1. The sizes of errors can be estimated using these bounds if knowledge of the sizes of first and second order derivatives of demand and supply is available.

Figure 3.1 illustrates how EDM uses local linear approximation to estimate the new equilibrium point E_2 when a parallel shift is assumed. The true supply curve S_1 is shifted

down parallel to S_2 and the intersection of S_2 with the true demand curve D_1 is the new equilibrium E_2. Under the specification in Equations (3.7)-(3.9) for the parallel shift case, when using EDM the straight line that is tangent to D_1 at E_1, denoted by D_1^*, is used to locally approximate D_1. Similarly, the tangent to the initial supply curve S_1 at point E_1, denoted by S_1^*, is used to locally approximate S_1. S_1^* is then shifted down by a constant K in the price direction to obtain S_2^*, which is the tangent to S_2 at point $B(Q_1, P_1 + K)$. S_2^* is used as a local approximation to the new supply curve S_2. The point E_2^* is used to approximate E_2.

It can be observed in Figure 3.1, as has been shown analytically, E_2^* is close to E_2 as long as the shift K is small. They coincide when D_1 and S_i ($i=1,2$) are locally linear. Also, EP is overestimated for the case of a downward shift in supply, but EQ can be over or under estimated depending on the relative distance from S_2 to S_2^* and from D_1 to D_1^*.

Proportional Supply Shift and Log-linear Approximation of Price and Quantity Changes

Another approach to approximation presumes that the research-induced supply shift is proportional and that the demand and supply curves are characterized by constant elasticities (log-linear) rather than by constant slopes (linear). This is equivalent to approximating the infinitesimal percentage change $d\ln(.)$ in equation (3.6) with the finite change $\Delta\ln(.)$. If we assume

(3.13) $K = \lambda P$ where $\lambda \equiv$ constant for all $Q > 0$

(3.14) $EP = \ln P_2 - \ln P_1$, $EQ = \ln Q_2 - \ln Q_1$

(3.15) $EP^* = \ln P_2^* - \ln P_1 = \lambda \varepsilon/(\varepsilon-\eta)$ and $EQ^* = \ln Q_2^* - \ln Q_1 = \lambda \varepsilon \eta/(\varepsilon-\eta)$

and apply a Taylor expansion to the logarithm of the demand and supply functions, it can be shown that

(3.16) $EP - EP^* = [2(\eta-\varepsilon)]^{-1}[S_L^{(2)}(k_2)(EP-\lambda)^2 - D_L^{(2)}(k_1)(EP)^2]$

                 $= O(\lambda^2)$ $(\lambda \rightarrow 0)$ and

(3.17) $EQ - EQ^* = [2(\eta-\varepsilon)]^{-1}[\eta S_L{}^{(2)}(k_2)(EP-\lambda)^2 - \varepsilon D_L{}^{(2)}(k_1)(EP)^2]$

$$= O(\lambda^2) \ (\lambda \to 0),$$

where $D_L(.)$ and $S_L(.)$ are the demand and supply functions $D(.)$ and $S(.)$ expressed on the $(\ln Q, \ln P)$ plane, $D_L{}^{(2)}(.)$ and $S_L{}^{(2)}(.)$ are the second order derivatives of $D_L(.)$ and $S_L(.)$ and $\ln P_2 \leq k_i \leq \ln P_1$ $(i=1,2)$. Derivation of expressions in (3.16) and (3.17) is given in the proof of Proposition 7, Appendix 1.

Again, $EP=EP^*$ and $EQ=EQ^*$ when $D_L{}^{(2)}(k_1)=S_L{}^{(2)}(k_2)=0$, which imply exact measures of price and quantity change for local log-linear demand and supply. For any unknown demand and supply functions, the "order of magnitude" $O(\lambda^2)$ guarantees small errors in the EDM estimates of price and quantity changes as long as λ is small. Similar empirical implications for the directions and determinants of these errors can be drawn as for the linear approximation case. Also, upper bounds for these errors are given in Remark 2 of Proposition 7, Appendix 1, which can be used to estimate the sizes of errors given information on the first (i.e. elasticities) and second order derivatives of the supply and demand curves around the base equilibrium.

Since a proportional shift and log-linear demand and supply curves on the (Q, P) plane are equivalent to a parallel shift and linear demand and supply curves on the $(\ln Q, \ln P)$ plane, the axes in Figure 3.1 can be relabelled as $\ln P$ and $\ln Q$. and the same geometric interpretation of the EDM approach can be made to the log-linear case as for the linear approach. Now the linear approximation is made in the $(\ln Q, \ln P)$ plane instead of the (Q, P) plane.

Alternatively, approximation of $E_2{}^*$ for E_2 can be demonstrated in the (Q, P) plane as in Figure 3.2. Under the specification in (3.13)-(3.15) for the proportional shift case, EDM uses the log-linear curves $D_1{}^*$, $S_1{}^*$ and $S_2{}^*$, which are tangent to the true demand and supply curves D_1, S_1 and S_2 at E_1, to locally approximate D_1, S_1 and S_2. It is obvious from both Figure 3.1 and 3.2 that, as shown mathematically above, the difference between the true new equilibrium point E_2 and the EDM estimate $E_2{}^*$ is small as long as the amount of shift is small and turns to zero when the demand and supply become exactly log-linear locally.
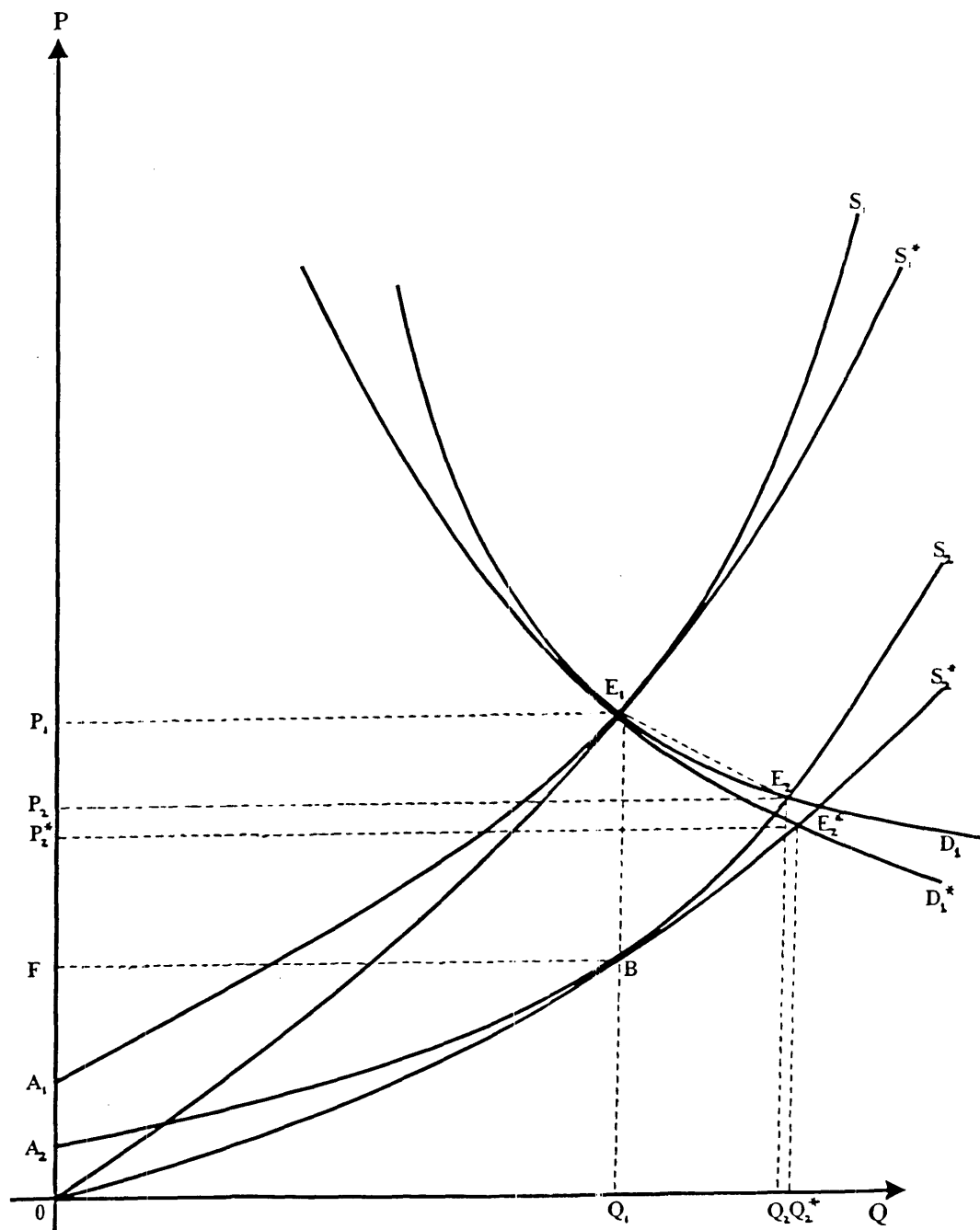
S₁ and D₁: true supply and demand curves with any functional form.

S_1^* and D_1^*: log-linear approximations of S_1 and D_1, respectively.

S_2 and S_2^*: proportional shifts of S_1 and S_1^*, respectively.

Figure 3.2 Proportional Shift and Log-Linear Approximation on (Q, P) Plane

3.3   Estimating Economic Surplus Changes

The displacement from E_1 to E_2 will cause changes in producer, consumer and total surpluses. As for the case of price and quantity changes, the way in which these surplus changes are approximated depends on assumptions made about the nature of the supply shift and the definition of percentage change $E(.)$, or equivalently, whether linear or log-linear functions are used to approximate the true functional forms.

Note that the discussion in this chapter concerns the measure of economic surplus. The additional approximation error in using consumer surplus rather than the compensating or equivalent variation is not considered. As mentioned in 2.5.3 of Chapter 2, changes in economic surpluses are used as measures of changes in welfare.

Parallel Supply Shift and Linear Approximation of Price and Quantity Changes

If a parallel supply shift is assumed and the price and quantity changes are estimated by Equation (3.9), surplus change areas as illustrated in Figure 3.1 can be approximated with EDM as (Alston 1991):

(3.18) $\Delta CS^* = \text{Area}(P_2^* E_2^* E_1 P_1) = -P_1 Q_1 \, EP^{'} (1+0.5EQ^*)$

(3.19) $\Delta PS^* = \text{Area}(FBE_2^* P_2^*) = P_1 Q_1 \, (EP^* - \lambda)(1+0.5EQ^*)$

(3.20) $\Delta TS^* = \text{Area}(FBE_2^* E_1 P_1) = -\lambda P_1 Q_1 (1+0.5EQ^*)$

where $\lambda < 0$ for a downward supply shift. However the 'true' surplus change areas under a parallel shift are given by

(3.21) $\Delta CS = \text{Area}(P_2 E_2 E_1 P_1) = \int\limits_{P_2}^{P_1} D(P)dP$

(3.22) $\Delta PS = \text{Area}(FBE_2 P_2) = \int\limits_{P_1+K}^{P_2} S(P - K)dP$

$$(3.23) \qquad \Delta TS = \mathrm{Area}(FBE_2E_1P_1) = \int_{P_2}^{P_1} D(P)dP + \int_{P_1+K}^{P_2} S(P-K)dP$$

Figure 3.1 illustrates the errors in using (3.18)-(3.20) to approximate (3.21)-(3.23). Since $D_1{}^*$ and $S_2{}^*$ are used to local linearly approximate D_1 and S_2, the true surplus changes and their EDM estimates will be the same when the true demand and supply curves are exactly linear around E_1. When the true demand and supply curves D_1 and S_i (i=1, 2) are not linear, the errors in all surplus measures are insignificant for a small shift, depending on the magnitude of the error in EP^*, with ΔTS^* being particularly accurate (triangle $BE_2{}^*E_1$ approximating BE_2E_1).

Analytically, it can be shown that (Propositions 3, 4 and 5, Appendix 1)

$$(3.24) \qquad \Delta CS - \Delta CS^* = - [2(\eta-\varepsilon)]^{-1}P_1{}^3[S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2]$$

$$- [2(\eta-\varepsilon)]^{-1}\eta P_1{}^3[S^{(2)}(c_2)(EP-\lambda)^2 EP^* \cdot D^{(2)}(c_1)(EP)^2 EP^*]-(1/6)P_1{}^3\, D^{(2)}(c_1)\, (EP)^3$$

$$-[8Q_1(\eta-\varepsilon)^2]^{-1}\eta P_1{}^5[S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2]^2 - (1/6)P_1{}^3\, D^{(2)}(c_1)(EP)^3$$

$$= O(\lambda^2)\ (\lambda \to 0),$$

$$(3.25) \qquad \Delta PS - \Delta PS^* = [2(\eta-\varepsilon)]^{-1}P_1{}^3[S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2]$$

$$+[2(\eta-\varepsilon)]^{-1}\varepsilon P_1{}^3(EP^*-\lambda)[S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2] + (1/6)P_1{}^3\, S^{(2)}(c_2)(EP-\lambda)^3$$

$$+ [8Q_1(\eta-\varepsilon)^2]^{-1}\varepsilon P_1{}^5[S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2]^2$$

$$= O(\lambda^2)\ (\lambda \to 0) \ \text{and}$$

$$(3.26) \qquad \Delta TS - \Delta TS^* = - [4(\eta-\varepsilon)]^{-1}\eta P_1{}^3 EP[S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2]$$

$$+ (1/6)P_1{}^3\, [S^{(2)}(c_2)(EP-\lambda)^3 - D^{(2)}(c_1)(EP)^3]$$

$$+ [4(\eta-\varepsilon)]^{-1}\varepsilon P_1{}^3(EP-\lambda)[S^{(2)}(c_2)(EP-\lambda)^2 - D^{(2)}(c_1)(EP)^2]$$

$$= O(\lambda^3)\ (\lambda \to 0).$$

It is clear from these expressions that the errors are zero for local linear demand and supply and ignorable for nonlinear functions. The higher order $O(\lambda^3)$ in (3.26) also provides a mathematical explanation for the "striking" small error in ΔTS^* observed by Alston and Wohlgenant (1990).

Under the assumptions in (3.12), it can also be shown to be almost always true that (Remark 3's of Propositions 3 and 4)

(3.27) $\Delta CS \leq \Delta CS^*$

(3.28) $\Delta PS \geq \Delta PS^*$

This implies that for a downward supply shift, ΔCS will almost always be overestimated and ΔPS underestimated. These results are consistent with the empirical evidence from Alston and Wohlgenant (1990). Upper bounds for these errors are given in Remarks 2's of Propositions 3, 4 and 5 in order to estimate the sizes of errors if information is available about the ranges of elasticities and curvatures of demand and supply in the local area.

Another interesting result is that the errors in measuring ΔPS and ΔCS are largely due to the errors in estimating the price and quantity changes (especially EP). It can be shown that if the new equilibrium point is known exactly, that is, $E_2^* = E_2$, the errors in measuring ΔPS and ΔCS, which now only arise from assuming that the curves joining points E_1 and E_2 and joining points B and E_2 are linear, are trivial comparing to the errors caused by not accurately locating the new equilibrium point. This result is shown analytically in Proposition 6 of Appendix 1, where the differences are of the order of $O(\lambda^3)$ if there are no errors in changes of price and quantity.

The results in this section are for a downward supply shift. Similar results can be shown to hold for an upward supply shift and a demand shift.

Proportional Supply Shift and Log-linear Approximation of Price and Quantity Changes

Referring to Figure 3.2, when the research-induced supply shift is of a proportional nature, the economic surplus changes associated with log-linear curves S_1^*, S_2^* and D_1^*, as used in EDM, can be derived as (Proposition 8, Appendix 1)

(3.29) $\Delta CS^{**} = \mathrm{Area}(P_1 E_1 E_2^* P_2^*) = \int_{P_2^*}^{P_1} D_1^*(P)\,dP = P_1 Q_1 (\eta+1)^{-1}(1 - e^{(\eta+1)EP^*})$

(3.30) $\Delta PS^{**} = \text{Area}(OE_2{}^*P_2{}^*)\text{-Area}(OE_1P_1) = \int\limits_0^{P_2{}^*} S_2{}^*(P)\ dP - \int\limits_0^{P_1} S_1{}^*(P)\ dP$

$= P_1 Q_1(\varepsilon+1)^{-1}(e^{(\eta+1)EP^x} - 1)$ and

(3.31) $\Delta TS^{**} = P_1 Q_1((\eta+1)^{-1}-(\varepsilon+1)^{-1})(1 - e^{(\eta+1)EP^*})$

Thus, (3.29)-(3.31) are the exact surplus measures if the true demand and supply are log-linear. When a proportional supply shift is assumed and the true demand and supply are of any functional form, the exact surplus changes as illustrated in Figure 3.2 are

(3.32) $\Delta CS = \text{Area}(P_2 E_2 E_1 P_1) = \int\limits_{P_2}^{P_1} D_1(P)\ dP$

(3.33) $\Delta PS = \text{Area}(A_2 E_2 P_2)\text{-Area}(A_1 E_1 P_1) = \int\limits_{A_2}^{P_2} S_2(P)\ dP - \int\limits_{A_1}^{P_1} S_1(P)\ dP$ and

(3.34) $\Delta TS = \text{Area}(A_2 E_2 P_2)\text{-Area}(A_1 E_1 P_1) + \text{Area}(P_1 E_1 E_2 P_2)$

$= \int\limits_{A_2}^{P_2} S_2(P)\ dP - \int\limits_{A_1}^{P_1} S_1(P)\ dP + \int\limits_{P_2}^{P_1} D_1(P)\ dP$

It can be shown mathematically that[3]

(3.35) $|\Delta CS^{**}-\Delta CS| = O(\lambda^2)\ (\lambda\rightarrow 0)$, but

(3.36) $|\Delta PS^{**}-\Delta PS| = O(\lambda)\ (\lambda\rightarrow 0)$ and

(3.37) $|\Delta TS^{**}-\Delta TS| = O(\lambda))$ $(\lambda\rightarrow 0)$.

Because the surplus changes themselves (ΔCS, ΔPS and ΔTS) are of the order $O(\lambda)$, results in (3.35)-(3.37) imply that, when a proportional shift is assumed and the true demand and supply are not of constant elasticity, using (3.29)-(3.31) to log-linearly approximate (3.32)-(3.34) will

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[3] The mathematical proof of these results is rather long and thus not included in Appendix 1.

be likely to cause large errors in measures for ΔPS and ΔTS^4, even though ΔCS will still be quite accurate.

3.4   Summary and Implications for EDM Applications

There have been concerns about the assumptions required for EDM results and the resulting economic surplus changes to be exactly correct and, when these assumptions are not met in empirical applications, the extent of approximation errors. In this chapter, the issues of functional form and nature of the exogenous shift in EDM applications are reexamined and clarified through an analytical approach. The results proved can be summarized as follows:

i. When demand and supply curves are *locally* linear and there is a *parallel* exogenous shift in demand or supply, the EDM estimates of both price and quantity changes and economic surplus changes are exact if percentage change is defined as $E(.)=\Delta(.)/(.)$;

ii. When demand and supply curves are *locally* log-linear (constant elasticity) and there is a *proportional* exogenous shift (or a parallel shift on the (lnQ,lnP) plane), the price, quantity and surplus changes[5] estimated using the EDM procedure are exact if percentage change is defined as $E(.)=\Delta ln(.)$[6];

iii. In empirical applications, if a parallel shift is assumed, the EDM errors in estimates of both price and quantity changes and economic surplus changes are small as long as the exogenous shift is small (with order $O(\lambda^3)$ for total surplus change and $O(\lambda^2)$ for others when $\lambda \rightarrow 0$), whatever the form of the true demand and supply curves;

iv. If a proportional shift is assumed and the true functional form is not of constant elasticity, the errors in price and quantity changes are small for a small exogenous shift (with order $O(\lambda^2)$ when $\lambda \rightarrow 0$), but the welfare measures can involve significant error even when measured using formulae appropriate for constant elasticity models (order $O(\lambda)$ for producer and total surplus even though order $O(\lambda^2)$ for consumer surplus when $\lambda \rightarrow 0$); and

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[4] Errors are also shown to be large if equations (3.18)-(3.20) for the linear-parallel case are used rather than (3.29)-(3.31). Proof is not included in Appendix 1 to save space.

[5] Global log-linearity is required for the surplus changes to be exact.

[6] Provided surplus changes are estimated by integration of the log-linear functions.

v. The exact expressions and upper bounds of the EDM approximation errors for these two cases are derived to identify the determinants and directions of the errors.

Three contributions are made in this chapter. First, analytical expressions of the approximation errors in measuring surplus changes relate the sizes and signs of the errors to the underlying demand and supply parameters. For example, it can be seen that the more inelastic and curved the demand and supply curves, the larger the errors. General conditions for overestimation and underestimation can also be easily recognised. This enables more general conditions for Alston and Wohlgenant's (1990) empirical findings to be identified. Some of their results on the signs of the errors are shown to be specific to their constant elasticity function. Second, while parallel-shift linear-approximation and proportional-shift log-linear-approximation are two commonly used approaches in EDM applications, it is shown that significant errors in surplus changes are possible when a proportional shift is assumed. Third, since only local rather than global linearity is required for the parallel shift, the restriction that supply has to be elastic in order to have a positive intercept (for example, Kim, *et al.* 1987; Godyn, Brennan and Johnston 1987; Voon and Edwards 1991c; Piggott, Piggott and Wright 1995; Hill, Piggott and Griffith 1996) is shown to be unnecessary.

Finally, in the analysis, the industry is assumed to consist of identical marginal firms. Wohlgenant (1997) has shown that when there are inframarginal firms, the shape of the supply curve and the nature of the shift from technical change for the industry may be different from those applying to the individual firm. In this situation the conventional measures of producer surplus are likely to be inaccurate. Additional data such as the distribution of firms by cost structure and how technical change affects these different firms are needed to accurately calculate producer surplus changes.

Chapter 4. The Australian Beef Industry Disaggregation and Model Specification

4.1 Introduction

In this chapter, the structure of the Australian beef industry is reviewed and an equilibrium displacement model for the industry is specified.

The horizontal and vertical structure of the beef industry is examined in 4.2. Horizontally, shares of market segments and the associated product specifications are discussed. The industry is considered as producing four types of beef depending on whether it is grain-finished or grass-finished and whether it is for the export or domestic market. Vertically, beef production and marketing is disaggregated into sectors of breeding, backgrounding, grass/grain finishing, processing, marketing and final consumption. Accordingly, the structure of the model is defined.

In 4.3, production functions and decision-making functions are specified for all industry sectors in general functional forms. From these, the demand and supply relationships among prices and quantities of all sectors are derived in 4.4. These are then used to derive the equilibrium displacement model. Integrability conditions underlying the model specification are examined in 4.5. Constraints among market parameters implied by these integrability conditions are derived. The final model, with integrability conditions imposed at the current equilibrium points, is presented in 4.6, and the chapter is summarised in 4.7.

4.2 Industry Review and Model Structure

4.2.1 Horizontal Market Segments and Product Specifications

Based on information from various sources (ABARE 1998, MRC 1995, AFFA 1998), the various market segments of the Australian beef industry, the associated product specifications and the average percentage shares of various market segments for 1992-1997 are summarised in Table 4.1. Calculation of these market shares is detailed in Chapter 5. As stated in Chapter 5, the model simulates the average equilibrium situation over the period of 1992-1997 to abstract

from any climatic impacts (such as drought in 1994) or abnormal events (such as 'mad cow' disease in 1996 and the Asian crisis in 1998) that occurred in an individual year.

Export Market

As shown in Table 4.1, during 1992-97, 62% of Australian-produced beef was sold overseas. On average, 14% of exported beef is grain finished and 86% are grass finished. The dominant destination of Australian grainfed beef is Japan, which accounts for over 90% of export grain-finished beef. The second significant market is South Korea, accounting for the majority of the rest of the export grainfed segment. The Japanese grainfed market primarily consists of four product categories (B3, B2, B1 and Grainfed Yearling). Each has a different specification in terms of days on feed, age and slaughtering weight. The percentage break-downs among the four components is based on information from the Japanese middle market, into which about 70% of Australian export to Japan is destined (MRC 1995). There are two major product specifications for the South Korean market.

The two biggest markets for Australian grassfed beef are US and Japan. Australian beef to the US is predominately lower quality manufacturing beef, while grassfed beef to Japan is mostly yearling grassfed and high quality grassfed (MRC 1995, p47).

Domestic Market

Competition from chicken and pork and an increasing requirement for consistency in meat quality by the major supermarket chains have resulted in an increase in the amount of grainfed beef in the domestic market. In Table 4.1, the domestic grainfed segment is disaggregated into two categories: cattle that are fed in major commercial feedlots and cattle that are grain-supplemented on pasture or in small opportunistic feedlots (with capacity of less than 500 head). As data on grain-supplemented cattle and opportunistic feedlots are unavailable, in this thesis, cattle grain-supplemented outside the major feedlots are modelled as part of the grass-finishing sector. This treatment accommodates the study of grain-finishing technologies that are specific for cattle backgrounding and feedlots. According to information from an AMLC/ALFA feedlot survey (Toyne, ABARE, per. comm. 1998), the cattle turn-off from the surveyed major feedlots has almost doubled during 1992-1997.

Australian consumers have a preference for yearling beef. As can be seen from Table 4.1, there are two differences between the domestic grainfed yearling and the Japanese grainfed yearling. Firstly, heifers are acceptable in Australia. Secondly, the Australian slaughtering weight is slightly lower than that for the Japanese market for this category. Domestic grassfed are mostly yearlings, which are lighter and younger in comparison to export cattle.

4.2.2 Vertical Structure of Beef Production and Marketing

Production of final consumable beef involves various stages that separate the industry into different sectors. Typical grassfed beef production system can be stylised as follows. The calves are bred and produced in the cattle breeding sector. They are weaned from cows at around 9 months to become weaners. Weaners are sold for restock to the grass-finishing sector. They stay on pasture, and sometimes are supplemented with grain (especially during drought years), until they reach a certain age and weight. They are then sold as finished live cattle in the saleyard to go to abattoirs. They are slaughtered and processed in the abattoirs and then sold as beef carcasses to domestic retailers (major supermarket chains and butchers) and exporters. Domestic retailers cut and trim the carcasses into saleable retail beef cuts, and pack them as ready-to-sell packs on the shelf for final consumers. Similarly, exporters, although in reality they are often not separated physically from abattoirs, convert beef carcass into the export cuts as required by overseas destinations.

A similar process applies to grainfed beef production in terms of the breeding, processing and marketing phases. In addition, grain finishing cattle also involves backgrounding and feedlot-finishing. The backgrounding phase is critical to the achievement of age and weight requirements for feedlot entry, especially for certain Japanese grainfed categories. It is often done on pasture by cattle producers, sometimes contracted by large feedlots. The cattle are introduced to grain and additives in this phase. The backgrounded cattle then enter the feedlot for a strictly controlled nutritional program for fixed numbers of days, in order to reach the specifications of particular markets. In Table 4.2, the age and weight requirements for each stage of weaning, backgrounding, lot-finishing and processing for the various grainfed market segments, as reviewed by a MRC research report (MRC 1995) are reproduced. It provides an indication of the timing and requirements of various phases of grainfed cattle production.

Table 4.1 Australian Beef Industry Disaggregation and Product Specifications

Market Segments				Product Specifications			
				Carcass weight(kg)	Age (mths)	Sex	Days on Feed (days)
Export (62%)	**Grainfed (14%)**	Japan: (92%)	JP B3 (18%)	380-420	24-28	steers	230-300
			JP B2 (37%)	340-380	24-28	steers	150
			JP B1 (34%)	330-360	26-30	steers	100
			JP Grainfed Yearling (11%)	240-260	16-18	steers	100
		Korea: (7%)	K1	220-320	30-36	steers & heifers	100
			Fullset	280-350	24-36	mainly steers	100
		Others: (1%)	Taiwan, EU, US, Canada,etc.				
	Grassfed (86%)	US (37%) Japan (28%) Korea (9%) Canada (7%) Taiwan (5%) Others (14%)		Mainly lower quality manufacturing beef for the US market and high quality fullset and yearlings for the Japanese market. Quality to other countries are mixed.			
Domestic (38%)	**Grainfed (18%)**			Carcass weight(kg)	Age (mths)	Sex	Days on Feed (days)
		Commercial feedlot finished (18%)		200-260	16-20	steers & heifers	70
	Grassfed (82 %)	Grain supplemented on pasture or fed in opportunistic feedlots		Mostly yearling beef.			

Sources: ABARE (1998), MRC (1995) and AFFA (1998)

Industry Disaggregation and Model Specification

Table 4.2 Grainfed Cattle Requirements at Different Phases

|		JP B3	JP B2	JP B1	JP Grainfed Yearling	Korean K1	Korean Fullsets	Domestic Grainfed
Abattoir	**Output:** weight age saleable yield	380-420 kg 24-28 mths 67-69%	360 kg 24-28 mths 69-70%	330-360 kg 26-30 mths 70% plus	240-260 kg 16-20 mths 70%	220-320 kg 24-36 mths 70% plus	280-350 kg 24-36 mths 70%	240-260 kg 16-20 mths 70% plus
Feedlot	**Output:** weight age days on feed	680-720 kg 24-28 mths 230 300days	680 kg 24-28 mths 150 days	600-660 kg 26-30 mths 100 days	420-470 kg 18-20 mths 100 days	400-580 kg max 36 mths 100 days	500-650 kg 24-36 mths 100 days	420-450 kg 20 mths 70 days
Back-ground	**Output:** weight age days on feed	380-420 kg 16-18 mths 6-12 mths	400-500 kg 20-22 mths 10-12 mths	400-500 kg 22-26 mths 10-12 mths	290-350 kg 15-17 mths 10-12 mths	250-430 kg 24-32 mths 14-22 mths	330-470 kg 22-32 mths 12-22 mths	330-350 kg 16-18 mths 6-8 mths
Cow-Calf Operator	**Output:** weight age	250-280 kg 9-10 mths	180-240 kg 7-10 mths	180-240 kg 7-10 mths	170-220 kg 7-10 mths	150-170 kg 7-10 mths	160-190 kg 7-10 mths	170-220 kg 7-10 mths

Source: MRC (1995)
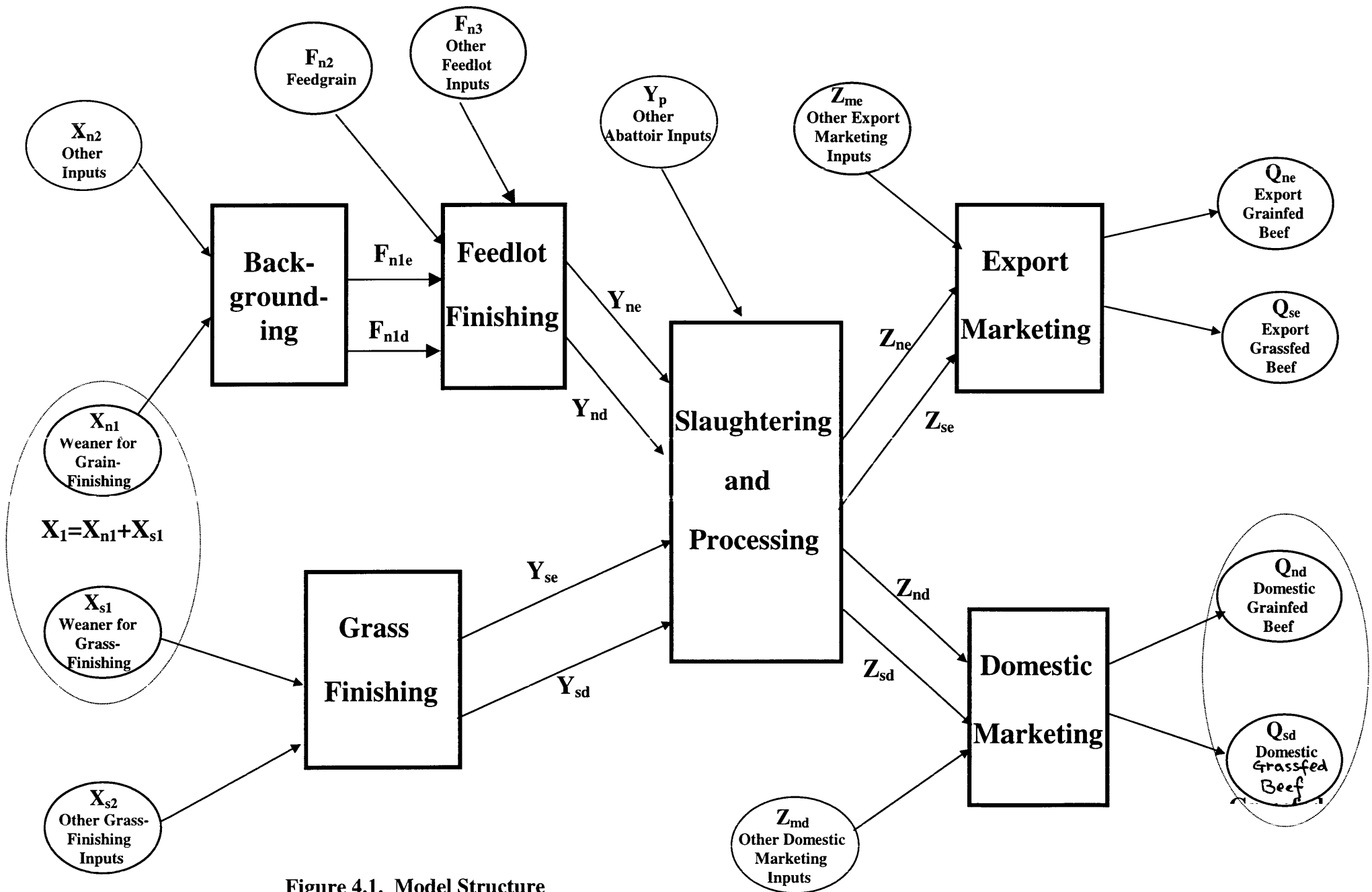
Figure 4.1. Model Structure

4.2.3 Structure of the Model

As pointed out in Chapter 1, a model disaggregated along both vertical and horizontal directions is required in order to study the returns of new technologies and promotion campaigns that occur in various industry sectors and markets, as well as the benefit distribution among different industry groups. Based on the above review of the industry structure, the structure of the model is specified in Figure 4.1, where each rectangle represents a production function, each arrowed straight line represents a market of a product, with the non-arrowed end being the supply of the product and the arrowed end being the demand of the product, and each oval represents a supply or demand schedule where an exogenous shift occurs.

Horizontally, the industry is modelled as producing four products along most parts of the vertical chain, based on whether it is grain or grass finished and whether it is for domestic or export market. Inputs other than the cattle input and feedgrain (in feedlot sector) are combined as one 'other inputs' in all sectors. As shown in Table 4.1, beef is not a homogenous product, and different market segments have different product specifications. The product specifications are controlled along most stages of the production chain and differentiated in prices. Note that the supply of weaners (X_1) for all four product categories is assumed homogenous in quality. There are some differences in breeds for suitable grain and grass finishing. However, there are no observable price differences at this level. Weaner prices fluctuate more with changes of weather or season than with destinations (Gaden, NSW Agriculture, per. comm. 1999).

Vertically, the industry is disaggregated into breeding, backgrounding-feedlot-finishing/grass finishing, processing, marketing and consumption. This enables separate analyses of various technologies in traditional farm production, feedlot nutrition, meat processing and meat marketing, as well as beef promotion.

4.3 Specification of Production Functions and Decision-Making Problems

4.3.1 Cost and Revenue Functions and Derived Demand and Supply Schedules for the Six Industry Sectors

As can be seen from Figure 4.1, there are six industry sectors (in the six rectangles) whose production functions and decision-making problems can be specified completely within the model. All are characterised by multi-output technologies.

Assume that (1) all sectors in the model are profit maximizers; (2) all multioutput production functions are separable in inputs and outputs; and (3) all production functions are characterised by constant returns to scale.

Consider first the specification of a general multioutput technology represented by a twice-continuously differentiable product transformation function

(4.3.1) $F(x, y)=0$

that uses k inputs $x=(x_1, x_2, ..., x_k)'$ to produce n outputs $y=(y_1, y_2, ..., y_n)'$. The output separability assumption ensures that there exists a scalar output index $g=g(y)$ such that Equation (4.3.1) can be written as[1] (Chambers 1991, p286)

(4.3.2) $g(y) = f(x)$.

The assumption of profit maximization implies that the industry's allocation problem can be considered in two parts. The first is cost minimization for a given level of the output vector. The cost function can be specified as

(4.3.3) $C(w, y) = \min\{w'x: y\}$

where $w=(w_1, w_2, ..., w_k)'$ are input prices for x. When the technology is assumed to be output separable, the multi-output cost function can be simplified to a single-output cost function as (Chambers 1988)

(4.3.4) $C(w, y) = \min_{x}\{w'x: y\}= \min_{x}\{w'x: g=g(y)\} = \hat{C}(w, g)$

where $\hat{C}(w, g)$ is the cost function for single-output technology $g=f(x)$.

---

[1] In this instance, the assumption of input separability, that ensures the existence of an input index $f(x)$ such that $f(x)=g(y)$, is equivalent to the assumption of output separability.

When constant returns to scale is also assumed, which implies in the case of output and input separable technology that $f(\lambda x) = \lambda g$ and $g(\lambda y) = \lambda f$ for any $\lambda > 0$, the cost function can be written as

(4.3.5) $\hat{C}(w, g) = \min\limits_{x} \{w'x: f(x) = g\}$

$= \min\limits_{x} \{w'x: f(x/g) = 1\}$ (use $\lambda = 1/g$)

$= g \min\limits_{x} \{w'(x/g): f(x/g) = 1\} = g\hat{C}(w, 1) = g\hat{c}(w)$

where $\hat{c}(w)$ is the unit cost function associated with the minimum cost for producing one unit of g.

Assume $\hat{c}(w)$ is differentiable in w. Applying Shephard's lemma (Chambers, 1991, p262) to the above cost function gives the output-constrained input demand functions

(4.3.6) $x_i = \dfrac{\partial}{\partial w_i} \hat{C}(w, g) = g\hat{c}_i'(w)$ $(i = 1, 2, ..., k)$

where $\hat{c}_i'(w)$ $(i = 1, 2, ..., k)$ are partial derivatives of the unit cost function $\hat{c}(w)$.

The second part of the profit maximization is to maximize revenue for a given input mix; that is, the revenue function can be written as

(4.3.7) $R(p, x) = \max\limits_{y} \{p'y: x\}$

where $p = (p_1, p_2, ..., p_n)'$ are output prices. Similarly, the input separability and constant returns to scale assumptions imply that

(4.3.8) $R(p, x) = \max\limits_{y} \{p'y: x\} = \max\limits_{y} \{p'y: f = f(x)\}$

$= \hat{R}(p, f) = \max\limits_{y} \{p'y: g(y) = f\} = f \max\limits_{y} \{p'(y/f): g(y/f) = 1\}$

$$= f \hat{R}(p, 1) = f \hat{r}(p)$$

where $\hat{R}(p, f)$ is the revenue function for single-input technology $g(y)=f$ and $\hat{r}(p)$ is the unit revenue function associated with maximum revenue from one unit of input index f. If $\hat{r}(p)$ is differentiable in p, the input-constrained output supply functions can be derived using Samulson-McFadden Lemma (Chambers, 1991, p264):

(4.3.9) $\qquad y_j = \dfrac{\partial R(p,x)}{\partial p_j} = f \ \hat{r}_j{}'(p) \qquad (j = 1, 2, ..., n)$

where $\hat{r}_j{}'(p)$ (j = 1, 2, ..., n) are partial derivatives of the unit revenue function $\hat{r}(p)$.

Based on these general results for any multi-output technology, and under the three assumptions made at the beginning of this section, the product transformation functions for the six industry sectors in the model can be written as

(4.3.10) $\qquad F_{n1}(F_{n1e}, F_{n1d}) = X_n(X_{n1}, X_{n2})$ $\qquad\qquad$ *backgrounding*

(4.3.11) $\qquad Y_n(Y_{ne}, Y_{nd}) = F_n(F_{n1e}, F_{n1d}, F_{n2}, F_{n3})$ $\qquad$ *feedlot finishing*

(4.3.12) $\qquad Y_s(Y_{se}, Y_{sd}) = X_s(X_{s1}, X_{s2})$ $\qquad\qquad$ *grass-finishing*

(4.3.13) $\qquad Z(Z_{se}, Z_{sd}, Z_{ne}, Z_{nd}) = Y(Y_{se}, Y_{sd}, Y_{ne}, Y_{nd}, Y_p)$ $\qquad$ *processing*

(4.3.14) $\qquad Q_d(Q_{nd}, Q_{sd}) = Z_d(Z_{nd}, Z_{sd}, Z_{md})$ $\qquad\qquad$ *domestic marketing*

(4.3.15) $\qquad Q_e(Q_{ne}, Q_{se}) = Z_e(Z_{ne}, Z_{se}, Z_{me})$ $\qquad\qquad$ *export marketing*

All symbols are defined in Table 4.3. The variables on the left sides of the equations are outputs for the relevant sectors; those on the right sides are inputs. Given the vertical structure of the sectors, outputs from an earlier sector become inputs for later sectors. In general, variables subscripted with "n" and "s" are related to grainfed and grassfed respectively, and those related to domestic and export markets carry a "d" and "e" respectively.

Table 4.3 Definition of Variables and Parameters in the Model

Endogenous Variables:

X_{n1}, X_{n2}:	Quantities of weaner cattle for lot-finishing and other inputs to the backgrounding sector, respectively.
X_n:	Aggregated input index for the feedlot finishing sector.
w_{n2}:	Price of other inputs to the backgrounding sector.
F_{n1e}, F_{n1d}, F_{n2}, F_{n3}:	Quantities of backgrounded cattle for export and domestic markets, feedgrain and other feedlot inputs, respectively.
F_{n1}:	Aggregated output index of the backgrounding sector.
F_n:	Aggregated input index of the feedlot sector.
s_{n1e}, s_{n1d}, s_{n2}, s_{n3}:	Prices of F_{n1e}, F_{n1d}, F_{n2}, F_{n3}.
Y_{ne}, Y_{nd}:	Quantities of feedlot-finished live cattle for export and domestic markets, respectively.
Y_n:	Aggregated output index of feedlot sector.
v_{ne}, v_{nd}:	Prices of grain-finished live cattle for export and domestic markets, respectively.
X_{s1}, X_{s2}:	Quantities of weaner cattle and other inputs to the grass finishing sector, respectively.
X_s:	Aggregated input index for the grass finishing sector.
X_1:	Quantity of total weaners, $X_1 = X_{n1} + X_{s1}$
w_1:	Price of weaners.
w_{s2}:	Price of other inputs to the grass finishing sector.
Y_{se}, Y_{sd}:	Quantities of grass-finished live cattle for export and domestic markets, respectively.
Y_s:	Aggregated output index for the grass finishing sector;
v_{se}, v_{sd}:	Prices of grass-finished live cattle for export and domestic markets, respectively.
Y_p:	Quantity of other inputs used in the processing sector.
v_p:	Price of other inputs used in the processing sector.
Y:	Aggregated input index for the processing sector.
Z:	Aggregated output index for the processing sector.
Z_{ne}, Z_{nd}:	Quantities of processed grain-fed beef carcass for export and domestic markets, respectively.
u_{ne}, u_{nd}:	Prices of processed grain-fed beef carcass for export and domestic markets, respectively.
Z_{se}, Z_{sd}:	Quantities of processed grass-fed beef carcass for export and domestic markets, respectively.
u_{ne}, u_{nd}:	Prices of processed grass-fed beef carcass for export and domestic markets, respectively.
Z_{me}, Z_{md}:	Quantities of other marketing inputs to export marketing and domestic marketing sectors, respectively.
u_{me}, u_{md}:	Prices of other marketing inputs to export marketing and domestic marketing sectors, respectively.
Z_e, Z_d:	Aggregated input indices to export marketing and domestic marketing sectors, respectively.
Q_e, Q_d:	Aggregated output indices for export marketing and domestic marketing sectors, respectively.
Q_{ne}, Q_{se}:	Quantities of export grain-fed and grass-fed beef, respectively.
p_{ne}, p_{se}:	Prices of export grain-fed and grass-fed beef, respectively.
Q_{nd}, Q_{sd}:	Quantities of domestic grain-fed and grass-fed retail beef cuts, respectively.
p_{nd}, p_{sd}:	Prices of domestic grain-fed and grass-fed retail beef cuts, respectively.

Exogenous Variables:

T_x: Supply shifter shifting down supply curve of x vertically due to cost reduction in production of x ($x = X_1$, X_{n2}, X_{s2}, F_{n2}, F_{n3}, Y_p, Z_{md}, Z_{me}).

t_x: Amount of shift T_x as a percentage of price of x ($x = X_1$, X_{n2}, X_{s2}, F_{n2}, F_{n3}, Y_p, Z_{md}, Z_{me}).

N_x: Demand shifter shifting up demand curve of x vertically due to promotion or taste changes that increase the demand in x ($x = Q_{se}$, Q_{ne}, Q_{sd}, Q_{nd}).

n_x: Amount of shift N_x as a percentage of price of x ($x = Q_{sd}$, Q_{se}, Q_{nd}, Q_{ne}).

Parameters:

$\eta_{(x, y)}$: Demand elasticity of variable x with respect to change in price y.

$\varepsilon_{(x, y)}$: Supply elasticity of variable x with respect to change in price y.

$\tilde{\eta}_{(x, y)}$: Constant-output input demand elasticity of input x with respect to change in input price y.

$\tilde{\varepsilon}_{(x, y)}$: Constant-input output supply elasticity of output x with respect to change in output price y.

$\sigma_{(x, y)}$: Allen's elasticity of input substitution between input x and input y.

$\tau_{(x, y)}$: Allen's elasticity of product transformation between output x and output y.

κ_x: Cost share of input x ($x = X_{n1}$, X_{n2}, X_{s1}, X_{s2}, F_{n1e}, F_{n1d}, F_{n2}, F_{n3}, Y_{ne}, Y_{nd}, Y_{se}, Y_{sd}, Y_p, Z_{nd}, Z_{sd},

Z_{md}, Z_{ne}, Z_{se} and Z_{me}), where $\sum_{i=1}^{2} \kappa_{Xni} = 1$, $\sum_{i=1}^{2} \kappa_{Xsi} = 1$, $\sum_{i=1e,1d,2,3} \kappa_{Fni} = 1$, $\sum_{i=ne,nd,se,sd,p} \kappa_{Yi} = 1$,

$\sum_{i=n,s,m} \kappa_{Zid} = 1$, $\sum_{i=n,s,m} \kappa_{Zie} = 1$.

γ_y: Revenue share of output y ($y = F_{n1e}$, F_{n1d}, Y_{ne}, Y_{nd} Y_{se}, Y_{sd}, Z_{ne}, Z_{nd}, Z_{se}, Z_{sd}, Q_{ne}, Q_{se} Q_{nd} and

Q_{sd}), where $\sum_{i=e,d} \gamma_{Fni} = 1$, $\sum_{i=e,d} \gamma_{Yni} = 1$, $\sum_{i=e,d} \gamma_{Ysi} = 1$, $\sum_{i=ne,nd,se,sd} \gamma_{Zi} = 1$,

$\sum_{i=n,s} \gamma_{Qie} = 1$, $\sum_{i=n,s} \gamma_{Qid} = 1$.

ρ_{Xn1}, ρ_{Xs1}: Quantity shares of X_{n1} and X_{s1}, ie. $\rho_{Xn1} = X_{n1}/(X_{n1} + X_{s1})$, $\rho_{Xs1} = X_{s1}/(X_{n1} + X_{s1})$.

As shown in Equation (4.3.5), the total cost functions related to these production functions are also separable for given output levels and can be written as

$$(4.3.16) \qquad C_{Fn1} = F_{n1} * c_{Fn1}(w_1, w_{n2}) \qquad\qquad \textit{backgrounding}$$

$$(4.3.17) \qquad C_{Yn} = Y_n * c_{Yn}(s_{n1e}, s_{n1d}, s_{n2}, s_{n3}) \qquad\qquad \textit{feedlot-finishing}$$

$$(4.3.18) \qquad C_{Ys} = Y_s * c_{Ys}(w_1, w_{s2}) \qquad\qquad \textit{grass-finishing}$$

$$(4.3.19) \qquad C_Z = Z * c_Z(v_{se}, v_{sd}, v_{ne}, v_{nd}, v_p) \qquad\qquad \textit{processing}$$

$$(4.3.20) \qquad C_{Qd} = Q_d * c_{Qd}(u_{nd}, u_{sd}, u_{md}) \qquad\qquad \textit{domestic marketing}$$

$$(4.3.21) \qquad C_{Qe} = Q_e * c_{Qe}(u_{ne}, u_{se}, u_{me}) \qquad\qquad \textit{export marketing}$$

where C_y represents the total cost of producing output index level y and $c_y(.)$ represents the unit cost function ($y = F_{n1}, Y_n, Y_s, Z, Q_d$ and Q_e). Quantity are represented by capital letters and prices by lower-case variables.

Following Equation (4.3.8), the revenue functions subject to given input levels for the six multi-output sectors can be represented as

$$(4.3.22) \qquad R_{Xn} = X_n * r_{Xn}(s_{n1e}, s_{n1d}) \qquad\qquad \textit{backgrounding}$$

$$(4.3.23) \qquad R_{Fn} = F_n * r_{Fn}(v_{ne}, v_{nd}) \qquad\qquad \textit{feedlot-finishing}$$

$$(4.3.24) \qquad R_{Xs} = X_s * r_{Xs}(v_{se}, v_{sd}) \qquad\qquad \textit{grass-finishing}$$

$$(4.3.25) \qquad R_Y = Y * r_Y(u_{nd}, u_{sd}, u_{ne}, u_{se}) \qquad\qquad \textit{processing}$$

$$(4.3.26) \qquad R_{Zd} = Z_d * r_{Zd}(p_{nd}, p_{sd}) \qquad\qquad \textit{domestic marketing}$$

$$(4.3.27) \qquad R_{Ze} = Z_e * r_{Ze}(p_{ne}, p_{se}) \qquad\qquad \textit{export marketing}$$

where R_x represents total revenue produced from the fixed input index level x and $r_x(.)$ represents the unit revenue function associated with one unit of input index x (x = X_n, F_{nl}, X_s, Y, Z_d and Z_e).

Following Equations (4.3.6) and (4.3.9), the demand and supply functions for all endogenous input and output variables in the model can be derived accordingly.

4.3.2 Profit Functions and Exogenous Supplies of Factors

Supply schedules for factors X_1, X_{n2}, X_{s2}, F_{n2}, F_{n3}, Y_p, Z_{me} and Z_{md} and demand schedules for beef products Q_i (i = ne, se, nd and sd) (refer to definitions of variables in Table 4.3) are exogenous to the model. Decision making problems from which the supplies of these factors and the demands of these products are derived can not be completely specified within the context of the model, because only some of the decision variables for these decision makers are included in the model.

Consider first the specification of the exogenous factor supply. Let x be any exogenous input to the model, ie. x = X_1, X_{n2}, X_{s2}, F_{n2}, F_{n3}, Y_p, Z_{me} or Z_{md}. Suppose the production function for the producer of x is

$$F(x, O) = 0$$

where O is the vector of all inputs and other outputs of the production function. The profit function can be specified as

$$(4.3.28) \quad \pi = \max_{x,O} \{ w_x x + W'O : F(x, O) = 0 \} = \pi(w_x, W)$$

where w_x is the price for *x* and *W* is the price vector for O. Following Varian (1992, p25), each element in O is set negative if it is an input and positive if it is an output. The supply of x can be derived using Hotelling's Lemma as

$$(4.3.29) \quad x = \frac{\partial}{\partial w_x} \pi(w_x, W) = \pi_{wx}'(w_x, W) = x(w_x, W)$$

where $\pi_{wx}'(.)$ is the partial derivative of $\pi(w_x, W)$ with respect to w_x. Supply of X_1, X_{n2}, X_{s2}, F_{n2}, F_{n3}, Y_p, Z_{me} or Z_{md} can be derived accordingly.

Note that, as shown in Figure 4.1, as we have assumed that the weaners supplied to both grain-finishing and grass-finishing sectors are homogenous, X_{n1} and X_{s1} have a joint supply schedule (the supply of $X_1=X_{n1}+X_{s1}$) and receive the same price. They do have separate demands, the sum of which being the demand for X_1.

4.3.3 Utility Functions and Exogenous Demand for Beef Products

Demands for the final beef products are exogenous to the model. Consider first the export demand. As reviewed in 4.2, export grainfed and grassfed beef (Q_{ne} and Q_{se}) are of very different quality and are mostly exported to markets in different countries. For example, during 1992-97 around 92% of total Australian export grainfed beef went to Japan, while only 28% of export grass-fed beef were sold in Japan (Table 4.1). The majority of export grass-fed beef is sold in North America and other Asian countries, where almost no Australian grain-fed beef were present. For this reason, the demand for Q_{ne} and Q_{se} are assumed to be independent and to relate to different consumers.

Suppose that the indirect utility function for a given income level m for the consumer of Q_{ie} (i = n, s) can be specified as (Varian 1992, p99)

$$(4.3.30) \qquad v(p_{ie}, P, m) = \max_{Qie,Q} \{u(Q_{ie}, Q): p_{ie}Q_{ie} + P'Q = m\}$$

where p_{ie} is the price of Q_{ie} (i = n, s), Q is the vector of all other commodities that the consumer of Q_{ie} also consumes, P is the price vector of Q, and u(.) is the consumer's utility function. The Marshallian demand equations can be derived using Roy's identity (Varian 1992, p106) as:

$$(4.3.31) \qquad Q_{ie}(p_{ie}, P, m) = - \frac{\dfrac{\partial\, v(p_{ie}, P, m)}{\partial\, p_{ie}}}{\dfrac{\partial\, v(p_{ie}, P, m)}{\partial\, m}} \qquad (i = n,\ s)$$

That is, the demand for each of Q_{ne} and Q_{se} is only related to own price and is independent of the rest of the model. In particular, the demand for Q_{ne} is not affected by the price of Q_{se}, and *vise versa*.

However, for the domestic retail market, grainfed and grassfed beef (Q_{nd} and Q_{sd}) are substitutes for consumers. Grainfed beef often appears on the shelf as high quality gourmet brands, while the cheaper brands (like 'Savings' or 'Farmland' in the major supermarkets) are often grassfed. The consumers' demands for the two beef products respond to the relative prices of the two products as well as prices of other competing meat products such as lamb, pork and chicken. In this case, there are two variables (Q_{nd} and Q_{sd}) in the domestic consumers' decision making problem that are from within the model. Assume that the indirect utility function for given income m for domestic consumers is (Varian 1992, p99):

$$(4.3.32) \qquad v(p_{nd}, p_{sd}, P, m) = \max_{Q_{nd}, Q_{sd}, Q} \{u(Q_{nd}, Q_{sd}, Q): p_{nd}Q_{nd} + p_{sd}Q_{sd} + P'Q = m\}$$

where p_{nd} and p_{sd} are prices for Q_{nd} and Q_{sd}, P and Q are price and quantity vectors of all other commodities. The Marshallian demand equations can be derived using Roy's identity as:

$$(4.3.33) \qquad Q_{nd}(p_{nd}, p_{sd}, P, m) = - \frac{\dfrac{\partial v(p_{nd}, p_{sd}, P, m)}{\partial p_{nd}}}{\dfrac{\partial v(p_{nd}, p_{sd}, P, m)}{\partial m}}$$

$$(4.5.34) \qquad Q_{sd}(p_{nd}, p_{sd}, P, m) = - \frac{\dfrac{\partial v(p_{nd}, p_{sd}, P, m)}{\partial p_{sd}}}{\dfrac{\partial v(p_{nd}, p_{sd}, P, m)}{\partial m}}$$

4.4 The Equilibrium Model and its Displacement Form

4.4.1 Structural Model

As shown above, the structural model describing the demand and supply relationships among all variables in the model can be derived as partial derivatives from the decision-making problems specified in Equations (4.3.16)-(4.3.27), (4.3.28), (4.3.30) and (4.3.32). Comparative

statics will then be applied to the structural model to derive the relationships among the *changes* in all variables, that is, the equilibrium displacement model. The changes in all prices and quantities due to a new technology or promotion will then be estimated in order to measure the welfare implications in later chapters.

At this stage, general functional forms for all decision-making functions as well as for all demand and supply functions are assumed. Also assume that exogenous changes result in parallel shifts in the relevant demand or supply curves. Incorporating the exogenous shifters that represent impacts of various new technologies and promotions in the demand or supply functions, the structural model system that describes the equilibrium of the Australian beef industry is given as follows. Variables outside the partial system are assumed unaffected by the displacements and thus kept constant. As a result, without losing generality, they are not included explicitly in the model. Again, definitions of all endogenous and exogenous variables in the general form model below are given in Table 4.3.

Input Supply to Backgrounding and Grass-Finishing Sectors:

(4.4.1) $\quad X_1 = X_1(w_1, T_{X1})$ $\qquad$ *weaner supply*

(4.4.2) $\quad X_1 = X_{n1} + X_{s1}$ $\qquad$ *weaner supply equality*

(4.4.3) $\quad X_{n2} = X_{n2}(w_{n2}, T_{Xn2})$ $\qquad$ *supply of other backgrounding inputs*

(4.4.4) $\quad X_{s2} = X_{s2}(w_{s2}, T_{Xs2})$ $\qquad$ *supply of other grass-finishing inputs*

Following Equation (4.3.29), Equations (4.4.1) and (4.4.3)-(4.4.4) are supply functions of weaners and other inputs to the backgrounding and grass-finishing sectors, derived from their individual profit functions in (4.3.28). Other prices in W in Equation (4.3.29) are assumed exogenously constant and thus are not included in the equations. The supply for X_{n1} and X_{s1} are restricted by the same supply schedule for X_1 in (4.4.1) through the identity in (4.4.2).

T_{Xi} is the supply shifter shifting down the supply curve of X_i due to technologies that reduce the production cost of X_i (i = 1, n2 and s2). In particular, T_{X1} represents exogenous changes such as breeding and farm technologies in weaner production, T_{Xn2} represents backgrounding

technologies in areas such as nutrition and management, and T_{Xs2} represents, for example, farm technologies and improved farm management in cattle grass-finishing.

Output-Constrained Input Demand of Backgrounding and Grass-Finishing Sectors:

(4.4.5) $X_{n1} = F_{n1} \, c'_{Fn1,1}(w_1, w_{n2})$ *demand for weaners for backgrounding*

(4.4.6) $X_{n2} = F_{n1} \, c'_{Fn1,n2}(w_1, w_{n2})$ *demand for other backgrounding inputs*

(4.4.7) $X_{s1} = Y_s \, c'_{Ys,1}(w_1, w_{s2})$ *demand for weaners for grass-finishing*

(4.4.8) $X_{s2} = Y_s \, c'_{Ys,s2}(w_1, w_{s2})$ *demand for other grass-finishing inputs*

Following (4.3.6), Equations (4.3.6)-(4.3.8) are derived as partial derivatives of the cost functions in Equations (4.3.16) and (4.3.18) using Shephard's Lemma. $c'_{Fn1,j}(w_1, w_{n2})$ (j = 1 and n2) and $c'_{Ys,j}(w_1, w_{s2})$ (j = 1 and s2) are partial derivatives of the unit cost functions $c_{Yn}(w_1, w_{n2})$ and $c_{Ys}(w_1, w_{s2})$ respectively.

Backgrounding and Grass-Finishing Sectors Equilibrium:

(4.4.9) $X_n(X_{n1}, X_{n2}) = F_{n1}(F_{n1e}, F_{n1d})$ *backgrounding quantity equilibrium*

(4.4.10) $c_{Fn1}(w_1, w_{n2}) = r_{Xn}(S_{n1e}, S_{n1d})$ *backgrounding value equilibrium*

(4.4.11) $X_s(X_{s1}, X_{s2}) = Y_s(Y_{se}, Y_{sd})$ *grass-finishing quantity equilibrium*

(4.4.12) $c_{Ys}(w_1, w_{s2}) = r_{Xs}(v_{se}, v_{sd})$ *grass-finishing value equilibrium*

Equations (4.4.9) and (4.4.11) are the multi-output product transformation functions of the two sectors, imposing aggregated inputs equal to aggregated outputs in quantity. Equations (4.4.10) and (4.4.12) set the unit costs (c_{Fn1} and c_{Ys}) from producing per unit of aggregated outputs (F_{n1} and Y_s) equal to the unit revenue (r_{Xn} and r_{Xs}) earned from per unit of aggregated input (X_n and X_s). They are derived from the industry equilibrium condition that total cost equals total revenue and equalities in (4.4.9) and (4.4.11).

<u>*Input-Constrained Output Supply of Backgrounding and Grass-Finishing Sectors:*</u>

(4.4.13) $F_{n1e} = X_n \, r'_{Xn,n1e}(\, S_{n1e}, S_{n1d}\,)$ *export-backgrounded-feeder supply*

(4.4.14) $F_{n1d} = X_n \, r'_{Xn,n1d}(\, S_{n1e}, S_{n1d}\,)$ *domestic-backgrounded-feeder supply*

(4.4.15) $Y_{se} = X_s \, r'_{Xs,se}(\, V_{se}, V_{sd}\,)$ *export-grass-finished cattle supply*

(4.4.16) $Y_{sd} = X_s \, r'_{Xs,sd}(\, V_{se}, V_{sd}\,)$ *domestic-grass-finished cattle supply*

Following Equation (4.3.9), Equations (4.4.13)-(4.4.16) are derived as partial derivatives of the revenue functions in Equations (4.3.22) and (4.3.24) using the Samulson-McFadden Lemma (Chambers, 1991, p264). $r'_{Xn,j}(S_{n1e}, V_{n1d})$ ($j = n1e$ and $n1d$) and $r'_{Xs,j}(V_{se}, V_{sd})$ ($j = se$ and sd) are partial derivatives of $r_{Xn}(S_{n1e}, S_{n1d})$ and $r_{Xs}(V_{se}, V_{sd})$, respectively.

<u>*Other Input Supply to Feedlot Sector*</u>

(4.4.17) $F_{n2} = F_{n2}(\, S_{n2}, T_{Fn2})$ *feedgrain supply*

(4.4.18) $F_{n3} = F_{n3}(\, S_{n3}, T_{Fn3})$ *supply of other feedlot inputs*

Equations (4.4.17) and (4.4.18) are the supplies of feedgrain and other inputs to the feedlot sector, following (4.3.29). T_{Fn2} represents feedgrain industry technologies that shift down the feedgrain supply curve. T_{Fn3} represents feedlot technologies due to, for example, feed nutrition research and improved feedlot management.

<u>*Output-Constrained Input Demand of Feedlot Sector.*</u>

(4.4.19) $F_{n1e} = Y_n \, c'_{Yn,n1e}(S_{n1e}, S_{n1d}, S_{n2}, S_{n3})$ *export-feeder demand*

(4.4.20) $F_{n1d} = Y_n \, c'_{Yn,n1d}(S_{n1e}, S_{n1d}, S_{n2}, S_{n3})$ *domestic-feeder demand*

(4.4.21) $F_{n2} = Y_n \, c'_{Yn,n2}(S_{n1e}, S_{n1d}, S_{n2}, S_{n3})$ *feedgrain demand*

(4.4.22) $F_{n3} = Y_n \, c'_{Yn,n3}(S_{nle}, S_{nld}, S_{n2}, S_{n3})$ *other feedlot input demand*

Following (4.3.6), Equations (4.4.6)-(4.4.8) are derived from the cost function in Equation (4.3.17) using Shephard's Lemma. $c'_{Yn,j}(.)$ (j = nle, nld, n2 and n3) are partial derivatives of the unit cost functions $c_{Yn}(.)$.

Feedlot Sector Equilibrium:

(4.4.23) $F_n(F_{nle}, F_{nld}, F_{n2}, F_{n3}) = Y_n(Y_{ne}, Y_{nd})$ *quantity equilibrium*

(4.4.24) $c_{Yn}(S_{nle}, S_{nld}, S_{n2}, S_{n3}) = r_{Fn}(V_{ne}, V_{nd})$ *value equilibrium*

As explained for the backgrounding and grass-finishing equilibrium in (4.4.9)-(4.4.12), Equations (4.4.23) and (4.4.24) are the quantity and value equilibrium for the feedlot sector.

Input-Constrained Output Supply of Feedlot Sector:

(4.4.25) $Y_{ne} = F_n \, r'_{Fn,ne}(V_{ne}, V_{nd})$ *export-grain-finished cattle supply*

(4.4.26) $Y_{nd} = F_n \, r'_{Fn,nd}(V_{ne}, V_{nd})$ *domestic-grain-finished cattle supply*

Following Equation (4.3.9), Equations (4.4.25)-(4.4.26) are derived from the revenue function in (4.3.23) using Samulson-McFadden Lemma. $r'_{Fn,j}(.)$ (j = ne and nd) are the partial derivatives of $r_{Fn}(.)$.

Other Input Supply to Processing Sector:

(4.4.27) $Y_p = Y_p(v_p, T_{Yp})$ *supply of other processing inputs*

Equation (4.4.27) is the supply of other inputs to the processing sector, derived as in Equation (4.3.29). T_{Yp} is the exogenous supply shifter representing processing technologies in abattoirs due to research and improved management.

Output-Constrained Input Demand of Processing Sector:

(4.4.28) $Y_{se} = Z\,c'_{Z,se}(v_{se},\,v_{sd},\,v_{ne},\,v_{nd},\,v_p)$ *export-grass-fed cattle demand*

(4.4.29) $Y_{sd} = Z\,c'_{Z,sd}(v_{se},\,v_{sd},\,v_{ne},\,v_{nd},\,v_p)$ *domestic-grass-fed cattle demand*

(4.4.30) $Y_{ne} = Z\,c'_{Z,ne}(v_{se},\,v_{sd},\,v_{ne},\,v_{nd},\,v_p)$ *export-grain-fed cattle demand*

(4.4.31) $Y_{nd} = Z\,c'_{Z,nd}(v_{se},\,v_{sd},\,v_{ne},\,v_{nd},\,v_p)$ *domestic-grain-fed cattle demand*

(4.4.32) $Y_p = Z\,c'_{Z,p}(v_{se},\,v_{sd},\,v_{ne},\,v_{nd},\,v_p)$ *other processing input demand*

Following Equation (4.3.6), the above five equations are derived from the cost function of the processing sector in Equation (4.3.19) using Shephard's Lemma, where $c'_{Z,j}(v_{se},\,v_{sd},\,v_{ne},\,v_{nd},\,v_p)$ ($j = se,\,sd,\,ne,\,nd,\,p$) are partial derivatives of the unit cost function $c_Z(v_{se},\,v_{sd},\,v_{ne},\,v_{nd},\,v_p)$.

Processing Sector Equilibrium:

(4.4.33) $Y(Y_{se},\,Y_{sd},\,Y_{ne},\,Y_{nd},\,Y_p) = Z(\,Z_{se},\,Z_{sd},\,Z_{ne},\,Z_{nd}\,)$ *quantity equilibrium*

(4.4.34) $c_Z(v_{se},\,v_{se},\,v_{ne},\,v_{nd},\,v_p) = r_Y(\,u_{se},\,u_{sd},\,u_{ne},\,u_{nd}\,)$ *value equilibrium*

Equation (4.4.33) is the product transformation function for the processing sector in (4.3.13) that equalizes the aggregated input index Y with the aggregated output index Z. Equation (4.4.34) sets the unit cost c_Z of producing a unit of aggregated output Z equal to the unit revenue r_Y earned per unit of aggregated input Y.

Input-Constrained Output Supply of Processing Sector:

(4.4.35) $Z_{se} = Y\,r'_{Y,se}\,(\,u_{se},\,u_{sd},\,u_{ne},\,u_{nd}\,)$ *export-grassfed beef carcass supply*

(4.4.36) $Z_{sd} = Y\,r'_{Y,sd}(\,u_{se},\,u_{sd},\,u_{ne},\,u_{nd}\,)$ *domestic-grassfed beef carcass supply*

(4.4.37) $Z_{ne} = Y \, r'_{Y,ne} \, (\, u_{se}, u_{sd}, u_{ne}, u_{nd} \,)$ *export-grainfed beef carcass supply*

(4.4.38) $Z_{nd} = Y \, r'_{Y,nd} (\, u_{se}, u_{sd}, u_{ne}, u_{nd} \,)$ *domestic-grainfed beef carcass supply*

Following Equation (4.3.9), Equations (4.4.35)-(4.4.38) are derived as partial derivatives of the processing revenue function in Equation (4.3.25) using the Samulson-McFadden Lemma. $r'_{Y,j}(u_{se}, u_{sd}, u_{ne}, u_{nd})$ ($j = se, sd, ne$ and nd) are partial derivatives of the unit revenue function $r_Y(u_{se}, u_{sd}, u_{ne}, u_{nd})$.

Other Input Supply to Marketing Sectors:

(4.4.39) $Z_{md} = Z_{md}(u_{md}, T_{Zmd})$ *supply of other domestic marketing inputs*

(4.4.40) $Z_{me} = Z_{me}(u_{me}, T_{Zme})$ *supply of other export marketing inputs*

Equations (4.4.39) and (4.4.40) are the supplies of other inputs to the domestic and export marketing sectors respectively, following Equation (4.3.29). T_{Zmd} represents technologies (in boning, packing, distributing, etc.) and more efficient management in domestic marketing sector (such as major supermarket chains). T_{Zmd} represents technologies in boning, packing, etc. and improved management in export marketing sector.

Output-Constrained Input Demand of Marketing Sectors:

(4.4.41) $Z_{sd} = Q_d \, c'_{Qd,sd} (\, u_{sd}, u_{nd}, u_{md} \,)$ *domestic-grass-fed beef carcass demand*

(4.4.42) $Z_{nd} = Q_d \, c'_{Qd,nd} (\, u_{sd}, u_{nd}, u_{md} \,)$ *domestic-grain-fed beef carcass demand*

(4.4.43) $Z_{md} = Q_d \, c'_{Qd,md} (\, u_{sd}, u_{nd}, u_{md} \,)$ *other domestic marketing input demand*

(4.4.44) $Z_{se} = Q_e \, c'_{Qe,se} (\, u_{se}, u_{ne}, u_{me} \,)$ *export-grass-fed beef carcass demand*

(4.4.45) $Z_{ne} = Q_e \, c'_{Qe,ne} (\, u_{se}, u_{ne}, u_{me} \,)$ *export-grain-fed beef carcass demand*

(4.4.46) $Z_{me} = Q_e \, c'_{Qe,me} (\, u_{se}, u_{ne}, u_{me} \,)$ *other export marketing input demand*

Again following Equation (4.3.6), Equations (4.4.41)-(4.4.46) are derived from the cost functions of the marketing sectors in Equations (4.3 20) and (4.3.21) using Shephard's Lemma. $c'_{Qd,j}(u_{sd}, u_{nd}, u_{md})$ ($j = sd$, nd, md) and $c'_{Qe,j}(u_{se}, u_{ne}, u_{me})$ ($j = se$, ne, me) are partial derivatives of the unit cost functions $c_{Qd}(u_{sd}, u_{nd}, u_{md})$ and $c_{Qe}(u_{s_i}, u_{ne}, u_{me})$, respectively.

Domestic Marketing Sector Equilibrium:

(4.4.47) $Z_d(Z_{sd}, Z_{nd}, Z_{md}) = Q_d(Q_{sd}, Q_{rd})$ *quantity equilibrium*

(4.4.48) $c_{Qd}(u_{sd}, u_{nd}, u_{md}) = r_{Zd}(p_{sd}, p_{nd})$ *value equilibrium*

Export Marketing Sector Equilibrium:

(4.4.49) $Z_e(Z_{se}, Z_{ne}, Z_{me}) = Q_e(Q_{se}, Q_{ne})$ *quantity equilibrium*

(4.4.50) $c_{Qe}(u_{se}, u_{ne}, u_{me}) = r_{Ze}(p_{se}, p_{ne})$ *value equilibrium*

Equations (4.4.47) and (4.4.49) are the product transformation functions for the domestic and export marketing sectors respectively, and Equations (4.4.48) and (4.4.50) impose value equilibrium between unit costs and unit revenues of the two marketing sectors.

Input-Constrained Output Supply of Marketing Sectors:

(4.4.51) $Q_{sd} = Z_d\, r'_{Zd,sd} (p_{sd}, p_{nd})$ *domestic-retail-grass-fed beef supply*

(4.4.52) $Q_{nd} = Z_d\, r'_{Zd,nd} (p_{sd}, p_{nd})$ *domestic-retail-grain-fed beef supply*

(4.4.53) $Q_{se} = Z_e\, r'_{Ze,sd} (p_{se}, p_{ne})$ *export-grass-fed beef supply*

(4.4.54) $Q_{ne} = Z_e\, r'_{Ze,ne} (p_{se}, p_{ne})$ *export-grain-fed beef supply*

Following Equation (4.3.9), Equations (4.4.51)-(4.4.54) are derived as partial derivatives of the revenue functions in Equations (4.3.26) and (4.3.27) using the Samulson-McFadden Lemma.

$r'_{zd,j}(p_{sd}, p_{nd})$ (j = sd, nd) and $r'_{ze,j}(p_{se}, p_{ne})$ (j = se, ne) are partial derivatives of the unit revenue functions $r_{zd}(p_{sd}, p_{nd})$ and $r_{ze}(p_{se}, p_{ne})$, respectively.

Domestic Retail Beef Demand:

(4.4.55) $Q_{sd} = Q_{sd}(p_{sd}, p_{nd}, N_{Qsd}, N_{Qnd})$ *domestic grassfed beef demand*

(4.4.56) $Q_{nd} = Q_{nd}(p_{sd}, p_{nd}, N_{Qsd}, N_{Qnd})$ *domestic grainfed beef demand*

Following Equations (4.3.33) and (4.3.34), Equations (4.4.55) and (4.4.56) are the demand equations for domestic grassfed and grainfed beef. Income is assumed exogenously constant during the modelled small displacements and thus omitted in the demand equations. N_{Qsd} and N_{Qnd} are domestic demand shifters representing changes in demand for grass-fed and grain-fed beef, respectively, due to promotion or taste changes in the domestic market.

Export Demand for Australian Beef:

(4.4.57) $Q_{se} = Q_{se}(p_{se}, N_{Qse})$ *export grassfed beef demand*

(4.4.58) $Q_{ne} = Q_{ne}(p_{ne}, N_{Qne})$ *export grainfed beef demand*

Following the derivation of Equation (4.3.31), Equations (4.4.57) and (4.4.58) are own-price-dependent demand functions for Australian grassfed and grainfed beef. As discussed in Section 4.3.3, Australian grassfed and grainfed beef are assumed non-substitutable due to their very different quality, end uses and countries of consumption. Also, income is assumed constant during the small shift and impacts from other competing meat prices at overseas markets are also not included explicitly. N_{Qse} is a demand shifter representing changes in demand for grain-fed Australian beef in Japanese or Korean markets due to promotion or taste changes. N_{Qne} represents promotion or demand changes for grass-fed Australian beef in overseas markets.

Equations (4.4.1)-(4.4.58) represent the structural equilibrium model of the Australian beef industry in general functional form. As can be seen from Figure 4.1, there are 23 factor or product markets that involve 46 price and quantity variables. There are also 12 aggregated input and output index variables for the six multi-output sectors. This amounts to 58 endogenous variables for the 58 equations in the system. The exogenous variables are the 12 shifters (ie. T_{xi}

(i = 1, n2, s2), T_{Fni} (i = 2, 3), T_{Yp}, T_{Zi} (i = md, me), and N_{Qi} (i = se, ne, sd, nd)) representing impacts of new technologies in individual sectors and promotion in domestic and overseas markets. The ultimate objective is to estimate the resulting changes in all prices and quantities in order to estimate the welfare implications of these exogenous shifts.

4.4.2 The Model in Equilibrium Displacement Form

The model system given by Equations (4.4.1)-(4.4.58) defines an equilibrium status in all markets involved. When a new technology or promotion disturbs the system through an exogenous shifter, a displacement from the base equilibrium results. As reviewed in Chapter 2, the relationships among changes in all the endogenous price and quantity variables and the exogenous shifters can be derived by totally differentiating the system of equations at the initial equilibrium points. The model in equilibrium displacement form is given by Equations (4.4.1)'-(4.4.58)' as follows. $E(.)=\Delta(.)/(.)$ represents a small finite relative change of variable (.). All market parameters refer to elasticity values at the initial equilibrium points. As shown in Chapter 3, local linear approximation is implied while totally differentiating the model and approximating the finite changes, and the approximation errors in the resulting relative changes of all variables are small as long as the initial exogenous shifts are small. Definitions of all parameters are also given in Table 4.3.

Input Supply to Backgrounding and Grass-Finishing Sectors:

(4.4.1)' $EX_1 = \varepsilon_{(X1, w1)}(Ew_1 - t_{X1})$

(4.4.2)' $EX_1 = \rho_{Xn1}EX_{n1} + \rho_{Xs1}EX_{s1}$

(4.4.3)' $EX_{n2} = \varepsilon_{(Xn2, wn2)}(Ew_{n2} - t_{Xn2})$

(4.4.4)' $EX_{s2} = \varepsilon_{(Xs2, ws2)}(Ew_{s2} - t_{Xs2})$

Output-Constrained Input Demand of Backgrounding and Grass-Finishing Sectors:

(4.4.5)' $EX_{n1} = \tilde{\eta}_{(Xn1, w1)}Ew_{n1} + \tilde{\eta}_{(Xn1, wn2)}Ew_{n2} + EF_{n1}$

$(4.4.6)'$ $\qquad$ $EX_{n2} = \widetilde{\eta}_{(Xn2,\,w1)}Ew_{n1} + \widetilde{\eta}_{(Xn2,\,wn2)}Ew_{n2} + EF_{n1}$

$(4.4.7)'$ $\qquad$ $EX_{s1} = \widetilde{\eta}_{(Xs1,\,w1)}Ew_{s1} + \widetilde{\eta}_{(Xs1,\,ws2)}Ew_{s2} + EY_{s}$

$(4.4.8)'$ $\qquad$ $EX_{s2} = \widetilde{\eta}_{(Xs2,\,w1)}Ew_{s1} + \widetilde{\eta}_{(Xs2,\,ws2)}Ew_{s2} + EY_{s}$

Backgrounding and Grass-Finishing Sectors Equilibrium:

$(4.4.9)'$ $\qquad$ $\kappa_{Xn1}EX_{n1} + \kappa_{Xn2}EX_{n2} = \gamma_{Fn1e}EF_{n1e} + \gamma_{Fn1d}EF_{n1d}$

$(4.4.10)'$ $\qquad$ $\kappa_{Xn1}Ew_{1} + \kappa_{Xn2}Ew_{n2} = \gamma_{Fn1e}Es_{n1e} + \gamma_{Fn1d}Es_{n1d}$

$(4.4.11)'$ $\qquad$ $\kappa_{Xs1}EX_{s1} + \kappa_{Xs2}EX_{s2} = \gamma_{Yse}EY_{se} + \gamma_{Ysd}EY_{sd}$

$(4.4.12)'$ $\qquad$ $\kappa_{Xs1}Ew_{1} + \kappa_{Xs2}Ew_{s2} = \gamma_{Yse}Ev_{se} + \gamma_{Ysd}Ev_{sd}$

Input-Constrained Output Supply of Backgrounding and Grass-Finishing Sectors:

$(4.4.13)'$ $\qquad$ $EF_{n1e} = \widetilde{\varepsilon}_{(Fn1e,\,sn1e)}Es_{n1e} + \widetilde{\varepsilon}_{(Fn1e,\,sn1d)}Es_{n1d} + EX_{n}$

$(4.4.14)'$ $\qquad$ $EF_{n1d} = \widetilde{\varepsilon}_{(Fn1d,\,sn1e)}Es_{n1e} + \widetilde{\varepsilon}_{(Fn1e,\,sn1d)}Es_{n1d} + EX_{n}$

$(4.4.15)'$ $\qquad$ $EY_{se} = \widetilde{\varepsilon}_{(Yse,\,vse)}Ev_{se} + \widetilde{\varepsilon}_{(Yse,\,vsd)}Ev_{sd} + EX_{s}$

$(4.4.16)'$ $\qquad$ $EY_{sd} = \widetilde{\varepsilon}_{(Ysd,\,vse)}Ev_{se} + \widetilde{\varepsilon}_{(Ysd,\,vsd)}Ev_{sd} + EX_{s}$

Other Input Supply to Feedlot Sector

$(4.4.17)'$ $\qquad$ $EF_{n2} = \varepsilon_{(Fn2,\,sn2)}(Es_{n2} - t_{Fr2})$

$(4.4.18)'$ $\qquad$ $EF_{n3} = \varepsilon_{(Fn3,\,sn3)}(Es_{n3} - t_{Fn3})$

Output-Constrained Input Demand of Feedlot Sector:

(4.4.19)' $EF_{nle} = \tilde{\eta}_{(Fnle, snle)}Es_{nle} + \tilde{\eta}_{(Fnle, snld)})Es_{nld} + \tilde{\eta}_{(Fnle, sn2)})Es_{n2} + \tilde{\eta}_{(Fnle, sn3)}Es_{n3} + EY_n$

(4.4.20)' $EF_{nld} = \tilde{\eta}_{(Fnld, snle)}Es_{nle} + \tilde{\eta}_{(Fnld, snld)})Es_{nld} + \tilde{\eta}_{(Fnld, sn2)})Es_{n2} + \tilde{\eta}_{(Fnld, sn3)}Es_{n3} + EY_n$

(4.4.21)' $EF_{n2} = \tilde{\eta}_{(Fn2, snle)})Es_{nle} + \tilde{\eta}_{(Fn2, snld)})Es_{nld} + \tilde{\eta}_{(Fn2, sn2)})Es_{n2} + \tilde{\eta}_{(Fn2, sn3)}Es_{n3} + EY_n$

(4.4.22)' $EF_{n3} = \tilde{\eta}_{(Fn3, snle)})Es_{nle} + \tilde{\eta}_{(Fn3, snld)})Es_{nld} + \tilde{\eta}_{(Fn3, sn2)})Es_{n2} + \tilde{\eta}_{(Fn3, sn3)}Es_{n3} + EY_n$

Feedlot Sector Equilibrium:

(4.4.23)' $\kappa_{Fnle}EF_{nle} + \kappa_{Fnld}EF_{nld} + \kappa_{Fn2}EF_{n2} + \kappa_{Fn3}EF_{n3} = \gamma_{Yne}EY_{ne} + \gamma_{Ynd}EY_{nd}$

(4.4.24)' $\kappa_{Fnle}Es_{nle} + \kappa_{Fnld}Es_{nld} + \kappa_{Fn2}Es_{n2} + \kappa_{Fn3}Es_{n3} = \gamma_{Yne}Ev_{ne} + \gamma_{Ynd}Ev_{nd}$

Input-Constrained Output Supply of Feedlot Sector:

(4.4.25)' $EY_{ne} = \tilde{\varepsilon}_{(Yne, vne)}Ev_{ne} + \tilde{\varepsilon}_{(Yne, vnd)}Ev_{nd} + EF_n$

(4.4.26)' $EY_{nd} = \tilde{\varepsilon}_{(Ynd, vne)}Ev_{ne} + \tilde{\varepsilon}_{(Ynd, vnd)}Ev_{nd} + EF_n$

Other Input Supply to Processing Sector

(4.4.27)' $EY_p = \varepsilon_{(Yp, vp)}(Ev_p - t_{Yp})$

Output-Constrained Input Demand of Processing Sector:

(4.4.28)' $EY_{se} = \tilde{\eta}_{(Yse, vse)})Ev_{se} + \tilde{\eta}_{(Yse, vsd)})Ev_{sd} + \tilde{\eta}_{Yse, vne)}Ev_{ne}$

$+ \tilde{\eta}_{(Yse, vnd)})Ev_{nd} + \tilde{\eta}_{(Yse, vp)}Ev_p + EZ$

(4.4.29)' $EY_{sd} = \tilde{\eta}_{(Ysd, vse)})Ev_{se} + \tilde{\eta}_{(Ysd, vsd)})Ev_{sd} + \tilde{\eta}_{(Ysd, vne)})Ev_{ne}$

$+ \tilde{\eta}_{(Ysd, vnd)})Ev_{nd} + \tilde{\eta}_{(Ysd, vp)}Ev_p + EZ$

$(4.4.30)'$ $\quad EY_{ne} = \tilde{\eta}_{(Yne, vse)})Ev_{se} + \tilde{\eta}_{(Yne, vsd)})Ev_{sd} + \tilde{\eta}_{(Yne, vne)})Ev_{ne}$

$$+ \tilde{\eta}_{(Yne, vnd)})Ev_{nd} + \tilde{\eta}_{(Yne, vp)}Ev_p + EZ$$

$(4.4.31)'$ $\quad EY_{nd} = \tilde{\eta}_{(Ynd, vse)})Ev_{se} + \tilde{\eta}_{(Ynd, vsd)})Ev_{sd} + \tilde{\eta}_{(Ynd, vne)})Ev_{ne}$

$$+ \tilde{\eta}_{(Ynd, vnd)})Ev_{nd} + \tilde{\eta}_{(Ynd, vp)}Ev_p + EZ$$

$(4.4.32)'$ $\quad EY_p = \tilde{\eta}_{(Yp, vse)})Ev_{se} + \tilde{\eta}_{(Yp, vsd)})Ev_{sd} + \tilde{\eta}_{(Yp, vne)})Ev_{ne}$

$$+ \tilde{\eta}_{(Yp, vnd)})Ev_{nd} + \tilde{\eta}_{(Yp, vp)}Ev_p + EZ$$

Processing Sector Equilibrium:

$(4.4.33)'$ $\quad \kappa_{Yse}EY_{se}+\kappa_{Ysd}EY_{sd}+\kappa_{Yne}EY_{ne}+\kappa_{Ynd}EY_{nd}+\kappa_{Yp}EY_p$

$$=\gamma_{Zse}EZ_{se}+\gamma_{Zsd}EZ_{sd}+\gamma_{Zne}EZ_{ne}+\gamma_{Znd}EZ_{nd}$$

$(4.4.34)'$ $\quad \kappa_{Yse}Ev_{se}+\kappa_{Ysd}Ev_{sd}+\kappa_{Yne}Ev_{ne}+\kappa_{Ynd}Ev_{nd}+\kappa_{Yp}Ev_p$

$$=\gamma_{Zse}Eu_{se}+\gamma_{Zsd}Eu_{sd}+\gamma_{Zne}Eu_{ne}+\gamma_{Zrd}Eu_{nd}$$

Input-Constrained Output Supply of Processing Sector:

$(4.4.35)'$ $\quad EZ_{se} = \tilde{\varepsilon}_{(Zse, use)}Eu_{se} + \tilde{\varepsilon}_{(Zse, usd)}Eu_{sd} + \tilde{\varepsilon}_{(Zse, une)}Eu_{ne}+ \tilde{\varepsilon}_{(Zse, und)}Eu_{nd} + EY$

$(4.4.36)'$ $\quad EZ_{sd} = \tilde{\varepsilon}_{(Zsd, use)}Eu_{se} + \tilde{\varepsilon}_{(Zsd, usd)}Eu_{sd} + \tilde{\varepsilon}_{(Zsd, une)}Eu_{ne}+ \tilde{\varepsilon}_{(Zsd, und)}Eu_{nd} + EY$

$(4.4.37)'$ $\quad EZ_{ne} = \tilde{\varepsilon}_{(Zne, use)}Eu_{se} + \tilde{\varepsilon}_{(Zne, usd)}Eu_{sd} + \tilde{\varepsilon}_{(Zne, une)}Eu_{ne}+ \tilde{\varepsilon}_{(Zne, und)}Eu_{nd} + EY$

$(4.4.38)'$ $\quad EZ_{nd} = \tilde{\varepsilon}_{(Znd, use)}Eu_{se} + \tilde{\varepsilon}_{(Znd, usd)}Eu_{sd} + \tilde{\varepsilon}_{(Znd, une)}Eu_{ne}+ \tilde{\varepsilon}_{(Znd, und)}Eu_{nd} + EY$

Other Input Supply to Marketing Sectors:

$(4.4.39)'$ $\quad EZ_{md} = \varepsilon_{(Zmd, umd)}(Eu_{md} - t_{Zmd})$

$(4.4.40)'$ $\quad EZ_{me} = \varepsilon_{(Zme, ume)}(Eu_{me} - t_{Zme})$

Output-Constrained Input Demand of Marketing Sectors:

(4.4.41)' $EZ_{sd} = \tilde{\eta}_{(Zsd, usd)}Eu_{sd} + \tilde{\eta}_{(Zsd, und)}Eu_{nd} + \tilde{\eta}_{(Zsd, umd)}Eu_{md} + EQ_d$

(4.4.42)' $EZ_{nd} = \tilde{\eta}_{(Znd, usd)}Eu_{sd} + \tilde{\eta}_{(Znd, und)}Eu_{nd} + \tilde{\eta}_{(Znd, umd)}Eu_{md} + EQ_d$

(4.4.43)' $EZ_{md} = \tilde{\eta}_{(Zmd, usd)}Eu_{sd} + \tilde{\eta}_{(Zmd, und)}Eu_{nd} + \tilde{\eta}_{(Zmd, umd)}Eu_{md} + EQ_d$

(4.4.44)' $EZ_{se} = \tilde{\eta}_{(Zse, use)}Eu_{se} + \tilde{\eta}_{(Zse, une)}Eu_{ne} + \tilde{\eta}_{(Zse, ume)}Eu_{me} + EQ_e$

(4.4.45)' $EZ_{ne} = \tilde{\eta}_{(Zne, use)}Eu_{se} + \tilde{\eta}_{(Zne, une)}Eu_{ne} + \tilde{\eta}_{(Zne, ume)}Eu_{me} + EQ_e$

(4.4.46)' $EZ_{me} = \tilde{\eta}_{(Zme, use)}Eu_{se} + \tilde{\eta}_{(Zme, une)}Eu_{ne} + \tilde{\eta}_{(Zme, ume)}Eu_{me} + EQ_e$

Domestic Marketing Sector Equilibrium:

(4.4.47)' $\kappa_{Zsd}EZ_{sd} + \kappa_{Znd}EZ_{nd} + \kappa_{Zmd}EZ_{md} = \gamma_{Qsd}EQ_{sd} + \gamma_{Qnd}EQ_{nd}$

(4.4.48)' $\kappa_{Zsd}Eu_{sd} + \kappa_{Znd}Eu_{nd} + \kappa_{Zmd}Eu_{md} = \gamma_{Qsd}Ep_{sd} + \gamma_{Qnd}Ep_{nd}$

Export Marketing Sector Equilibrium:

(4.4.49)' $\kappa_{Zse}EZ_{se} + \kappa_{Zne}EZ_{ne} + \kappa_{Zme}EZ_{me} = \gamma_{Qse}EQ_{se} + \gamma_{Qne}EQ_{ne}$

(4.4.50)' $\kappa_{Zse}Eu_{se} + \kappa_{Zne}Eu_{ne} + \kappa_{Zme}Eu_{me} = \gamma_{Qse}Ep_{se} + \gamma_{Qne}Ep_{ne}$

Input-Constrained Output Supply of Marketing Sectors:

(4.4.51)' $EQ_{sd} = \tilde{\varepsilon}_{(Qsd, psd)}Ep_{sd} + \tilde{\varepsilon}_{(Qsd, pnd)}Ep_{nd} + EZ_d$

(4.4.52)' $EQ_{nd} = \tilde{\varepsilon}_{(Qnd, psd)}Ep_{sd} + \tilde{\varepsilon}_{(Qnd, pr d)}Ep_{nd} + EZ_d$

(4.4.53)' $EQ_{se} = \tilde{\varepsilon}_{(Qse, pse)}Ep_{se} + \tilde{\varepsilon}_{(Qse, pne)}Ep_{ne} + EZ_e$

(4.4.54)' $EQ_{ne} = \tilde{\varepsilon}_{(Qne, pse)}Ep_{se} + \tilde{\varepsilon}_{(Qne, pne)}Ep_{n..} + EZ_e$

Domestic Retail Beef Demand:

(4.4.55)' $EQ_{sd} = \eta_{(Qsd, psd)}(Ep_{sd} - n_{Qsd}) + \eta_{(Qsd, pnd)}(Ep_{nd} - n_{Qnd})$

(4.4.56)' $EQ_{nd} = \eta_{(Qnd, psd)}(Ep_{sd} - n_{Qsd}) + \eta_{(Qnd, pnd)}(Ep_{nd} - n_{Qnd})$

Export Demand for Australian Beef:

(4.4.57)' $EQ_{se} = \eta_{(Qse, pse)}(Ep_{se} - n_{Qse})$

(4.4.58)' $EQ_{ne} = \eta_{(Qne, pne)}(Ep_{ne} - n_{Qne})$

Derivation of Equations (4.4.1)'-(4.4.58)' from the general form model in Equations (4.4.1)-(4.4.58) is tedious, but straightforward. Details are not presented.

4.5 Integrability Conditions

4.5.1 The Integrability Problem

So far, the demand and supply equations for all inputs and outputs in the model have been derived conceptually from the underlying decision-making specifications in equations (4.3.16)-(4.3.27), (4.3.28), (4.3.30) and (4.3.32) without specific functional forms. Using local linear approximation of all demand and supply functions. linear relationships among small finite relative changes of all variables have been derived. If all the market-related parameters are known in the displacement model in Equations (4.4.1)'-(4.4.58)', the changes in all 58 price and quantity variables resulting from any one of the 12 exogenous shifts can be solved. However, there is a crucial question for a multi-market model like this one which is not always addressed in EDM applications: how can it be ensured that all the parameters and specifications of demand and supply equations are consistent in the sense that *(a)* there exists a set of underlying decision-making preference functions in Equations (4.3.16)-(4.3.27), (4.3.28), (4.3.30) and (4.3.32) that can be recovered from the demand and supply functions in Equations

(4.4.1)'-(4.4.58)' (*mathematical integrability*); and *(b)* the preference functions satisfy the regularity conditions to be *bona fide* cost, revenue, profit and utility functions (*economic integrability*)? In short, the demand and supply specification needs to satisfy the integrability conditions.

The integrability problem is especially relevant when the purpose of the model is to measure economic welfare and its distribution. As reviewed in Section 2.5.4, Just, Hueth and Schmitz (1982) have shown that the distribution of the total benefits from an exogenous shift in a market can be measured as economic surplus change areas off the partial (*ceteris paribus*) supply and demand curves in various markets, and that they add up to the total benefits, which can also be measured as surplus area changes off the general equilibrium (*mutatis mutandis*) supply and demand curves *in any single market*. However, empirically, the above results will not be exactly true if all the demand and supply functions are specified in an *ad hoc* fashion and the parameter values in different equations are estimated or chosen independently. That is, if the integrability conditions do not hold, total welfare change can be different from different ways of measuring it (Just, Hueth and Schmitz 1982, Appendices A.5, B.13 and D.4). Thus, integrability is a necessary and sufficient condition for the existence of exact welfare measures (LaFrance 1991, p1496).

In the context of this model, for the six sectors whose input and output decisions are completely determined within the model, all input demand and output supply functions need to be integrable with the relevant underlying cost and revenue functions. Also, as the two types of beef in the domestic market (Q_{nd} and Q_{sd}) are assumed substitutes for the same consumer group, the demand for Q_{nd} and Q_{sd} needs to be consistent with the consumers' utility function.

4.5.2 Integrability Conditions in Terms of Market Parameters

In Appendix 2, the required properties of cost, revenue, profit and utility functions are examined to derive the required properties for the demand and supply functions. The implied constraints in terms of market-related parameters are summarised below.

Output-Constrained Input Demand

Integrability relating the output-constrained input demand in Equation (4.3.6) to the cost function in (4.3.4) will be satisfied if the following homogeneity, symmetry and concavity

conditions hold. The notation is the same as that in Section 4.3 except that the output index g is represented by y here for convenience.

Homogeneity is given by

$$(4.5.1) \qquad \sum_{j=1}^{k} \tilde{\eta}_{ij}(w, y) = 0 \qquad (i = 1, ..., k) \qquad \textit{(homogeneity)},$$

where $\tilde{\eta}_{ij}(w, y)$ is the constant-output input demand elasticity of x_i with respect to a change in input price w_j $(i, j = 1, ..., k)$.

The symmetry condition requires that

$$(4.5.2) \qquad s_i(w, y)\tilde{\eta}_{ij}(w, y) = s_j(w, y)\tilde{\eta}_{ji}(w, y) \qquad (i, j = 1, ..., k)\,\textit{(symmetry)},$$

where $s_i(.) = (w_i x_i / C)$ is the cost share of the ith input in total cost $(i = 1, ..., k)$.

Concavity requires that $H_{\eta} = \left(\tilde{\eta}_{ij}(w, y)\right)_{k \times k}$ is negative semidefinite, or specifically

$$(4.5.3) \qquad (-1)^m H_{\eta m} = (-1)^m \begin{vmatrix} \tilde{\eta}_{11} & \tilde{\eta}_{12} & \cdots \tilde{\eta}_{1m} \\ \tilde{\eta}_{21} & \tilde{\eta}_{22} & \cdots \tilde{\eta}_{2m} \\ \vdots & \vdots & \vdots \\ \tilde{\eta}_{m1} & \tilde{\eta}_{m2} & \cdots & \tilde{\eta}_{mm} \end{vmatrix} \geq 0 \quad (m = 1, ..., k) \quad \textit{(concavity)},$$

where $H_{\eta m} = \left|\left(\tilde{\eta}_{ij}\right)_{m \times m}\right| = \left|\left(\tilde{\eta}_{ij}(w, y)\right)_{m \times m}\right|$ (m = 1, ..., k) is the mth principal minor of H_{η}. That is, the principal minors of the input demand elasticity matrix H_{η} alternate in sign between nonpositive (when k is odd) and nonnegative (when k is even). In fact, it can be shown that under the homogeneity condition in (4.5.1), H_{η} is singular and thus $H_{\eta k} \equiv 0$.

Using Allen-Uzawa's definition of elasticity of input substitution (McFadden 1978, p79-80)

$$(4.5.4) \qquad \tilde{\eta}_{ij}(w, y) = s_j(w, y)\sigma_{ij}(w, y) \qquad (i, j = 1, ..., k),$$

where $\sigma_{ij}(w, y)$ is the Allen-Uzawa elasticity of substitution between the ith and jth inputs (i, j = 1, ..., k), the homogeneity condition can also be written as

(4.5.1)' $$\sum_{j=1}^{k} s_j(w, y)\sigma_{ij}(w, y) = 0 \qquad i = 1, ..., k) \qquad (homogeneity).$$

The symmetry condition becomes

(4.5.2)' $$\sigma_{ij}(w, y) = \sigma_{ji}(w, y) \qquad (i, j = 1, ..., k) \qquad (symmetry).$$

In other words, in terms of input substitution, the symmetry condition simply means that the Allen-Uzawa substitution elasticity is symmetric.

The concavity condition in terms of the input substitution parameter implies $H_\sigma = (\sigma_{ij}(w, y))_{k \times k}$ is negative semidefinite, or,

(4.5.3)' $(-1)^m H_{\sigma m} = (-1)^m \begin{vmatrix} \sigma_{11} & \sigma_{12} & \cdots & \sigma_{1m} \\ \sigma_{21} & \sigma_{22} & \cdots & \sigma_{2m} \\ \vdots & \vdots & & \vdots \\ \sigma_{m1} & \sigma_{m2} & \cdots & \sigma_{mm} \end{vmatrix} \geq 0 \quad (m = 1, ..., k) \quad (concavity).$

where $H_{\sigma m} = \left| (\sigma_{ij})_{m \times m} \right| = \left| (\sigma_{ij}(w, y))_{m \times m} \right|$ (m = 1, ..., k) is the mth principal minor of H_σ. That is, the principal minors of the input substitution elasticity matrix H_σ alternate signs. In addition, it can be shown that under the homogeneity condition in (4.5.1), H_σ is singular and thus $H_{\sigma k} = 0$. In other words, the condition in Equation (4.5.3)' only needs to be checked for m = 1, ..., k-1.

In summary, the output-constrained input demand functions in Equations (4.4.5)-(4.4.8), (4.4.19)-(4.4.22), (4.4.28)-(4.4.32) and (4.4.41)-(4.4.46) in the model need to satisfy conditions in Equations (4.5.1)-(4.5.3), or equivalently (4.5.1)'-(4.5.3)', in order to be integrable.

Input-Constrained Output Supply

The input-constrained output supply functions in the model are derived following general results in Equations (4.3.8) and (4.3.9). In order to be integrable relative to the revenue function in (4.3.8), the output supply in (4.3.9) needs to satisfy the following *homogeneity*, *symmetry* and *convexity* conditions. The derivation is in Appendix 2.

Using Allen-Uzawa's elasticity of product transformation (McFadden 1978, p79-80), i.e.

(4.5.5) $\tilde{\varepsilon}_{ij}(p,x) = \gamma_j(p,x)\tau_{ij}(p,x)$,

the homogeneity condition is given by

(4.5.6) $\sum_{j=1}^{n}\tilde{\varepsilon}_{ij}(p,x) = 0$ $(i = 1, ..., n)$ *(homogeneity)*, or

(4.5.6)' $\sum_{j=1}^{n}\gamma_j(p,x)\tau_{ij}(p,x) = 0$ $(i = 1, ..., n)$ *(homogeneity)*,

where $\tilde{\varepsilon}_{ij}(p,x)$ is the input-constrained output supply elasticity of y_i with respect to a change in output price p_j, $\gamma_j(.) = (p_j y_j/R)$ is the share of the *j*th output in total revenue, and $\tau_{ij}(p, x)$ is the Allen-Uzawa elasticity of product transformation between the *i*th and *j*th outputs $(i, j = 1, ..., n)$.

The symmetry condition is given by

(4.5.7) $\gamma_i(p,x)\tilde{\varepsilon}_{ij}(p,x) = \gamma_j(p,x)\tilde{\varepsilon}_{ji}(p,x)$ $(i, j = 1, ..., n)$ *(symmetry)*, or

(4.5.7)' $\tau_{ij}(w,y) = \tau_{ji}(w,y)$ $(i, j = 1, ..., n)$ *(symmetry)*.

In other words, the symmetry condition simply implies symmetry of the Allen-Uzawa product transformation elasticity.

The convexity condition requires that $H_\varepsilon = \left(\tilde{\varepsilon}_{ij}(p,x) \right)_{n \times n}$ or $H_\tau = \left(\tau_{ij}(p,x) \right)_{n \times n}$ is positive semidefinite. Thus, in terms of principal minors of these matrices, the convexity condition is equivalent to

$$(4.5.8) \qquad H_{\varepsilon m} = \begin{vmatrix} \tilde{\varepsilon}_{11} & \tilde{\varepsilon}_{12} & \cdots & \tilde{\varepsilon}_{1m} \\ \tilde{\varepsilon}_{21} & \tilde{\varepsilon}_{22} & \cdots & \tilde{\varepsilon}_{2m} \\ \vdots & \vdots & & \vdots \\ \tilde{\varepsilon}_{m1} & \tilde{\varepsilon}_{m2} & \cdots & \tilde{\varepsilon}_{mm} \end{vmatrix} \geq 0 \quad (m = 1, ..., n) \quad (convexity), \text{ or}$$

$$(4.5.8)' \qquad H_{\tau m} = \begin{vmatrix} \tau_{11} & \tau_{12} & \cdots & \tau_{1m} \\ \tau_{21} & \tau_{22} & \cdots & \tau_{2m} \\ \vdots & \vdots & & \vdots \\ \tau_{m1} & \tau_{m2} & \cdots & \tau_{mm} \end{vmatrix} \geq 0 \quad (m = 1, .. , n) \quad (convexity).$$

That is, all principal minors of H_ε and H_τ are non-negative. Again, both matrices are singular under the homogeneity condition, so the condition in (4.5.8) and (4.5.8)' is always true for m=n.

In summary, the integrability conditions for the output supplies in Equations (4.4.13)-(4.4.16), (4.4.25)-(4.4.26), (4.4.35)-(4.4.38) and (4.4.51)-(4.4.54) are given by the *homogeneity, symmetry* and *convexity* conditions in Equations (4 5.6)-(4.5.8), or their equivalent forms in (4.5.6)'-(4.5.8)'. These conditions will ensure the recovery of the "proper" underlying revenue functions (Equations (4.3.22)-(4.3.27)).

Exogenous Input Supply

For the exogenous supply of inputs X_1, X_{n2}, $X_{,2}$, F_{n2}, F_{n3}, Y_p, Z_{me} and Z_{md} in Equations (4.4.1), (4.4.3)-(4.4.4), (4.4.17)-(4.4.18), (4.4.27), and (4.4.3'))-(4.4.40), the decision-making problem is given in (4.3.28)-(4.3.29). As each of these eight inputs is the only decision variable from the model that appears in each relevant profit function, the three conditions as derived in Appendix 2 become very simple for this case. In fact, ensuring that the own-price supply elasticity is non-negative, ie.

(4.5.9) $\varepsilon_x \geq 0$ $(x = X_1, X_{n2}, X_{s2}, F_{n2}, F_{n3}, Y_p, Z_{me}$ and $Z_{md})$,

is the only requirement for the model for the recovery of the 'proper' profit functions, where ε_x is the own-price supply elasticity of input x ($x = X_1, X_{n2}, X_{s2}, F_{n2}, F_{n3}, Y_p, Z_{me}$ and Z_{md}).

Exogenous Output Demand

The integrability conditions for the group of demand equations in the model are discussed in Appendix 2. For the exogenous demand for Q_{ne} and Q_{se} in the export market, because the two types of beef are assumed to be consumed by different consumers and thus non-substitutable, the demand for each of Q_{ne} and Q_{se} needs to be integrable with the demand for other commodities that are not in the model. As a result, the only requirement within the model system necessary for the recovery of a 'proper' utility function in Equation (4.3.30) is that the own-price demand elasticities in Equations (4.4.57) and (4.4.58) are non-positive, ie.

(4.5.10) $\eta_{(Qie, pie)} \leq 0$ $(i = n, s)$.

For the domestic demand for Q_{nd} and Q_{se}, the two types of beef are modelled as substitutes and relate to the utility maximization of the same domestic consumer in Equation (4.3.32). As a result, the demand for Q_{nd} and Q_{se} need to relate integrably with each other, as well as with demands of other commodities in the domestic consumer's budget. As discussed in Chapter 2 (Section 2.5.3), the Marshallian economic surplus areas will be used as measures of welfare, which implies that the marginal utility of income is constant and the income effect will be ignored. Under this restrictive assumption, the integrability conditions of a symmetric and negative semidefinite Slutsky matrix means a symmetric and negative semidefinite Marshallian substitution matrix (Appendix 2). In particular, symmetry implies

(4.5.11) $\eta_{ij} = (\lambda_j/\lambda_i)\eta_{ji}$ $(i, j = nd, sd)$

where λ_j/λ_i is the relative budget shares of the two commodities. As shown in Appendix 2, homogeneity and concavity conditions will not be violated when

(4.5.12) $\eta_{ii} \leq 0$, $\eta_{ij} \geq 0$ and $|\eta_{ii}| \geq |\eta_{ij}|$ $(i, j = nd, sd)$,

which will always be satisfied by choosing sensible values of demand elasticities.

4.5.3 Integrability Considerations for EDM

As shown in Chapter 3, with the EDM approach, linear-in-price functions for all the demand and supply functions described in Equations (4.4.1)-(4.4.58) are in effect assumed around the local areas of the initial equilibrium points in all markets involved. This implies that the underlying preference functions in Equations (4.3.16)-(4.3.27), (4.3.28), (4.3.30) and (4.3.32) are of quadratic-in-price form locally. As discussed in Appendix 2, the homogeneity condition requires the constrained demand and supply functions to be homogeneous of degree zero (HD(0)) in prices. An immediate problem for satisfying the integrability conditions is that local linear demand and supply functions are locally HD(1) in prices by default rather HD(0). That is to say, the homogeneity condition can not be imposed on a linear function beyond a single point. In other words, to be globally integrable, the demand and supply functions can not be of an ordinary linear form.

However, the linear functions implicitly assumed in the derivation of the displacement model in Equations (4.4.1)'-(4.4.58)' are only local linear approximations of the true demand and supply functions in Equations (4.4.1)-(4.4.58), which are not necessarily of a linear functional form and which *can* satisfy the integrability conditions locally or even globally. For example, a normalised quadratic cost function (that is, a quadratic function with all prices divided by one input price) is globally HD(1) and the derived normalised linear input demands are globally HD(0). Thus, a symmetric and semidefinite parameter matrix for this functional form will give a global integrable specification. In other words, imposing the integrability conditions at a single point for this functional form implies satisfaction of integrability globally.

In the empirical specification of the model below, integrability conditions are imposed at the *initial equilibrium point*. These conditions are assumed to also hold locally for the true demand and supply functions in Equations (4.4.1)-(4.4.58) (for example if the true functional form is normalised linear). The displacement equations in (4.4.1)'-(4.4.58)' are viewed as a local linear approximation to the integrable model in Equations (4.4.1)-(4.4.58). Equivalently, the preference functions underlying Equations (4 4.1)'-(4.4.58)' are local second-order approximations to the true integrable preference functions underlying Equations (4.4.1)-(4.4.58) at around the initial equilibrium point. As only small displacements from the initial equilibrium point (resulting from 1% exogenous shifts) are considered in the study and the

model is integrable at the initial equilibrium point, the errors in the welfare measures will be small when parallel exogenous shifts are assumed (refer to results Chapter 3).

The argument of second-order-approximation was suggested by Burt and Brewer (1971, p816) and explained by LaFrance (1991) in the case of the integrability of an incomplete demand system. LaFrance (1991) pointed out that, when the integrability conditions are imposed at a single point, a quadratic preference function based on the integrable values of first and second order derivatives at the base point "allows us to approximate the exact compensating variation of a price change from the base point to second order" (p1496).

Another justification for the small-error argument in this model is based on the empirical results by LaFrance (1991), who examined the integrability problem and its effects on consumer welfare measures in the context of an incomplete demand system. He compared four ways of imposing integrability conditions in the econometric estimation of a demand system. The first three approaches involve linear demand functions: the first one imposing symmetry of cross-price derivatives to ensure a unique welfare measure; the second one imposing Slutsky symmetry at a single point (sample mean); and the third one restricting the cross-price effect matrix to be symmetric, negative semidefinite. The fourth approach involved nonlinear demand functions satisfying so-called "weak integrability" (LaFrance and Hanemann 1989) for the incomplete demand system, which is claimed to enable the estimation of "exact welfare measures" (LaFrance and Hanemann 1989, p263). In his empirical example of a price policy, the estimates for the trapezoid welfare changes from all four approaches were very similar. However, when the triangular "deadweight loss" is the measure of interest, the first two approaches exhibited significant errors while the third approach was still a reasonably good approximation (15% error). While the model in this thesis deals with a different empirical problem, some insights can still be drawn from LaFrance's (1991) results. In the current study, it is the whole trapezoid welfare change rather than the triangular "deadweight loss' that is of interest. Thus, the errors in using a linear demand and supply system satisfying integrability conditions at the base equilibrium are expected to be small for the small displacements considered.

4.6 Displacement Model with Point Integrability Conditions

Using the definitions of the elasticities of input substitution and product transformation in Equations (4.5.4) and (4.5.5) and imposing equality restrictions of homogeneity and symmetry

in Equations (4.5.1)'-(4.5.2)', (4.5.6)'-(4.5.7)' and (4.5.11), the displacement model in Equations (4.4.1)'-(4.4.58)' is transformed to Equations (4.6.1)-(4.6.58) below. Inequality constraints required by concavity and convexity in Equations (4.5.3)', (4.5.8)', (4.5.9), (4.5.10) and (4.5.12) will be ensured when setting the parameter values in Chapter 5.

Input Supply to Backgrounding and Grass-Finishing Sectors:

(4.6.1) $EX_1 = \varepsilon_{(X1,\,w1)}(Ew_1 - t_{X1})$

(4.6.2) $EX_1 = \rho_{Xn1}EX_{n1} + \rho_{Xs1}EX_{s1}$

(4.6.3) $EX_{n2} = \varepsilon_{(Xn2,\,wn2)}(Ew_{n2} - t_{Xn2})$

(4.6.4) $EX_{s2} = \varepsilon_{(Xs2,\,ws2)}(Ew_{s2} - t_{Xs2})$

Output-Constrained Input Demand of Backgrounding and Grass-Finishing Sectors:

(4.6.5) $EX_{n1} = -\kappa_{Xn2}\sigma_{(Xn1,\,Xn2)}Ew_1 + \kappa_{Xn2}\sigma_{(Xn1,\,Xn2)}Ew_{n2} + EF_{n1}$

(4.6.6) $EX_{n2} = \kappa_{Xn1}\sigma_{(Xn1,\,Xn2)}Ew_1 - \kappa_{Xn1}\sigma_{(Xn1,\,Xn2)}Ew_{n2} + EF_{n1}$

(4.6.7) $EX_{s1} = -\kappa_{Xs2}\sigma_{(Xs1,\,Xs2)}Ew_1 + \kappa_{Xs2}\sigma_{(Xs1,\,Xs2)}Ew_{s2} + EY_s$

(4.6.6) $EX_{s2} = \kappa_{Xs1}\sigma_{(Xs1,\,Xs2)}Ew_1 - \kappa_{Xs1}\sigma_{(Xs1,\,X.2)}Ew_{s2} + EY_s$

Backgrounding and Grass-Finishing Sectors Equilibrium:

(4.6.9) $\kappa_{Xn1}EX_{n1} + \kappa_{Xn2}EX_{n2} = \gamma_{Fn1e}EF_{n1e} + \gamma_{Fn1d}EF_{n1d}$

(4.6.10) $\kappa_{Xn1}Ew_1 + \kappa_{Xn2}Ew_{n2} = \gamma_{Fn1e}Es_{n1e} + \gamma_{Fn1d}Es_{n1d}$

(4.6.11) $\kappa_{Xs1}EX_{s1} + \kappa_{Xs2}EX_{s2} = \gamma_{Yse}EY_{se} + \gamma_{Ysd}EY_{sd}$

(4.6.12) $\kappa_{Xs1}Ew_1 + \kappa_{Xs2}Ew_{s2} = \gamma_{Yse}Ev_{se} + \gamma_{Ysd}Ev_{sd}$

Input-Constrained Output Supply of Backgrounding and Grass-Finishing Sectors:

$$(4.6.13) \qquad EF_{nie} = -\gamma_{Fn1d}\tau_{(Fn1e, Fn1d)}Es_{n1e} + \gamma_{Fn1d}\tau_{(1n1e, Fn1d)}Es_{n1d} + EX_n$$

$$(4.6.14) \qquad EF_{n1d} = \gamma_{Fn1e}\tau_{(Fn1e, Fn1d)}Es_{n1e} - \gamma_{Fn1e}\tau_{(Fn1e, Fn1d)}Es_{n1d} + EX_n$$

$$(4.6.15) \qquad EY_{se} = -\gamma_{Ysd}\tau_{(Yse, Ysd)}Ev_{se} + \gamma_{Ysc}\tau_{(Yse, Ysd)}Ev_{sd} + EX_s$$

$$(4.6.16) \qquad EY_{sd} = \gamma_{Yse}\tau_{(Yse, Ysd)}Ev_{se} - \gamma_{Yse}\tau_{(Yse, Ysd)}Ev_{sd} + EX_s$$

Other Input Supply to Feedlot Sector

$$(4.6.17) \qquad EF_{n2} = \varepsilon_{(Fn2, sn2)}(Es_{n2} - t_{Fn2})$$

$$(4.6.18) \qquad EF_{n3} = \varepsilon_{(Fn3, sn3)}(Es_{n3} - t_{Fn3})$$

Output-Constrained Input Demand of Feedlot Sector:

$$(4.6.19) \qquad EF_{nie} = -(\kappa_{Fn1d}\sigma_{(Fn1e, Fn1d)}+\kappa_{Fn2}\sigma_{(Fn1e, Fn2)}+\kappa_{Fn3}\sigma_{(Fn1e, Fn3)})Es_{n1e}$$
$$+ \kappa_{Fn1d}\sigma_{(Fn1e, Fn1d)}Es_{n1d} + \kappa_{Fn2}\sigma_{(Fn1e, Fn2)}Es_{n2}+\kappa_{Fn3}\sigma_{(Fn1e, Fn3)}Es_{n3} + EY_n$$

$$(4.6.20) \qquad EF_{n1d} = \kappa_{Fn1e}\sigma_{(Fn1e, Fn1d)}Es_{n1e} + \kappa_{Fn2}\sigma_{(Fn1d, Fn2)}Es_{n2} + \kappa_{Fn3}\sigma_{(Fn1d, Fn3)}Es_{n3}$$
$$-(\kappa_{Fn1e}\sigma_{(Fn1e, Fn1d)}+\kappa_{Fn2}\sigma_{(Fn1d, Fn2)}+\kappa_{Fn3}c_{(Fn1d, Fn3)})Es_{n1d} + EY_n$$

$$(4.6.21) \qquad EF_{n2} = \kappa_{Fn1e}\sigma_{(Fn1e, Fn2)}Es_{n1e} + \kappa_{Fn1d}c_{(Fn1d, Fn2)}Es_{n1d} + \kappa_{Fn3}\sigma_{(Fn2, Fn3)}Es_{n3}$$
$$-(\kappa_{Fn1e}\sigma_{(Fn1e, Fn2)}+\kappa_{Fn1d}\sigma_{(Fn1d, Fn2)}+\kappa_{Fn3}c_{(Fn2, Fn3)})Es_{n2} + EY_n$$

$$(4.6.22) \qquad EF_{n3} = \kappa_{Fn1e}\sigma_{(Fn1e, Fn3)}Es_{n1e} + \kappa_{Fn1d}\sigma_{(Fn1d, Fn3)}Es_{n1d} + \kappa_{Fn2}\sigma_{(Fn2, Fn3)}Es_{n2}$$
$$-(\kappa_{Fn1e}\sigma_{(Fn1e, Fn3)}+\kappa_{Fn1d}\sigma_{(Fn1d, Fn3)}+\kappa_{Fn2}c_{(Fn2, Fn3)})Es_{n3} + EY_n$$

Feedlot Sector Equilibrium:

$$(4.6.23) \qquad \kappa_{Fn1e}EF_{n1e} +\kappa_{Fn1d}EF_{n1d} +\kappa_{Fn2}EF_{n2} +\kappa_{Fn3}EF_{n3} = \gamma_{Yne}EY_{ne} +\gamma_{Ynd}EY_{nd}$$

(4.6.24) $\kappa_{Fn1e}ES_{n1e} + \kappa_{Fn1d}ES_{n1d} + \kappa_{Fn2}ES_{n2} + \kappa_{Fn3}ES_{n3} = \gamma_{Yne}EV_{ne} + \gamma_{Ynd}EV_{nd}$

Input-Constrained Output Supply of Feedlot Sector:

(4.6.25) $EY_{ne} = -\gamma_{Ynd}\tau_{(Yne,\ Ynd)}EV_{ne} + \gamma_{Ynd}\tau_{(Yne,\ Ynd)}EV_{nd} + EF_n$

(4.6.26) $EY_{nd} = \gamma_{Yne}\tau_{(Yne,\ Ynd)}EV_{ne} - \gamma_{Yne}\tau_{(Yne,\ Ynd)}EV_{nd} + EF_n$

Other Input Supply to Processing Sector

(4.6.27) $EY_p = \varepsilon_{(Yp,\ vp)}(Ev_p - t_{yp})$

Output-Constrained Input Demand of Processing Sector:

(4.6.28) $EY_{se} = -\ (\kappa_{Ysd}\sigma_{(Yse,\ Ysd)} + \kappa_{Yne}\sigma_{(Yse,Yne)} + \kappa_{Ynd}\sigma_{(Yse,\ Ynd)} + \kappa_{Yp}\sigma_{(Yse,\ Yp)})EV_{se}$

$+\kappa_{Ysd}\sigma_{(Yse,Ysd)}EV_{sd} + \kappa_{Yne}\sigma_{(Yse,Yne)}EV_{ne} + \kappa_{Ynd}\sigma_{(Yse,Ynd)}EV_{nd} + \kappa_{Yp}\sigma_{(Yse,Yp)}EV_p + EZ$

(4.6.29) $EY_{sd} = \kappa_{Yse}\sigma_{(Yse,\ Ysd)}EV_{se} - (\kappa_{Yse}\sigma_{(Yse,Ysd)} + \kappa_{Yne}\sigma_{(Ysd,\ Yne)} + \kappa_{Ynd}\sigma_{(Ysd,\ Ynd)}$

$+\kappa_{Yp}\sigma_{(Ysd,Yp)})EV_{sd} + \kappa_{Yne}\sigma_{(Ysd,Yne)}EV_{ne} + \kappa_{Ynd}\sigma_{(Ysd,Ynd)}EV_{nd} + \kappa_{Yp}\sigma_{(Ysd,Yp)}EV_p + EZ$

(4.6.30) $EY_{ne} = \kappa_{Yse}\sigma_{(Yse,Yne)}EV_{se} + \kappa_{Ysd}\sigma_{(Ysd,Yne)}EV_{sd} - (\kappa_{Yse}\sigma_{(Yse,Yne)} + \kappa_{Ysd}\sigma_{(Ysd,Yne)}$

$+\kappa_{Ynd}\sigma_{(Yne,Ynd)} + \kappa_{Yp}\sigma_{(Yne,Yp)})EV_{ne} + \kappa_{Ynd}\sigma_{(Yne,Ynd)}EV_{nd} + \kappa_{Yp}\sigma_{(Yne,Yp)}EV_p + EZ$

(4.6.31) $EY_{nd} = \kappa_{Yse}\sigma_{(Yse,\ Ynd)}EV_{se} + \kappa_{Ysd}\sigma_{(Ysd,\ Ynd)}EV_{sd} + \kappa_{Yne}\sigma_{(Yne,\ Ynd)}EV_{ne}$

$-(\kappa_{Yse}\sigma_{(Yse,Ynd)} + \kappa_{Ysd}\sigma_{(Ysd,Ynd)} + \kappa_{Yne}\sigma_{(Yne,Ynd)} + \kappa_{Yp}\sigma_{(Ynd,Yp)})EV_{nd} + \kappa_{Yp}\sigma_{(Ynd,Yp)}EV_p + EZ$

(4.6.32) $EY_p = \kappa_{Yse}\sigma_{(Yse,\ Yp)}EV_{se} + \kappa_{Ysd}\sigma_{(Ysd,\ Yp)}EV_{sd} + \kappa_{Yne}\sigma_{(Yne,\ Yp)}EV_{ne}$

$+\kappa_{Ynd}\sigma_{(Ynd,\ Yp)}EV_{nd} - (\kappa_{Yse}\sigma_{(Yse,Yp)} + \kappa_{Ysd}\sigma_{(Ysd,Yp)} + \kappa_{Yne}\sigma_{(Yne,Yp)} + \kappa_{Ynd}\sigma_{(Ynd,Yp)})EV_p + EZ$

Processing Sector Equilibrium:

(4.6.33) $\kappa_{Yse}EY_{se} + \kappa_{Ysd}EY_{sd} + \kappa_{Yne}EY_{ne} + \kappa_{Ynd}EY_{nd} + \kappa_{Yp}EY_p$

$= \gamma_{Zse}EZ_{se} + \gamma_{Zsd}EZ_{sd} + \gamma_{Zne}EZ_{ne} + \gamma_{Znd}EZ_{n1}$

$$(4.6.34) \quad \kappa_{Yse}Ev_{se}+\kappa_{Ysd}Ev_{sd}+\kappa_{Yne}Ev_{ne}+\kappa_{Ynd}Ev_{nd}+\kappa_{Yp}Ev_{p}$$

$$=\gamma_{Zse}Eu_{se}+\gamma_{Zsd}Eu_{sd}+\gamma_{Zne}Eu_{ne}+\gamma_{Znd}Eu_{nd}$$

Input-Constrained output supply of Processing Sector:

$$(4.6.35) \quad EZ_{se} = -(\gamma_{Zsd}\tau_{(Zse,\ Zsd)}+ \gamma_{Zne}\tau_{(Zse,\ Zne)}+ \gamma_{Znd}\tau_{(Zse,\ Znd)})Eu_{se}$$

$$+\gamma_{Zsd}\tau_{(Zse,\ Zsd)}Eu_{sd} + \gamma_{Zne}\tau_{(Zse,\ Zne)}Eu_{ne}+ \gamma_{Znd}\tau_{(Zse,\ Znd)}Eu_{nd} + EY$$

$$(4.6.36) \quad EZ_{sd} = \gamma_{Zse}\tau_{(Zse,\ Zsd)}Eu_{se} + \gamma_{Zne}\tau_{(Zsd,\ Zne)}Eu_{ne}+ \gamma_{Znd}\tau_{(Zsd,\ Znd)}Eu_{nd}$$

$$-(\gamma_{Zse}\tau_{(Zse,\ Zsd)}+ \gamma_{Zne}\tau_{(Zsd,\ Zne)}+ \gamma_{Znd}\tau_{(Zsd,\ Znd)})Eu_{sd} + EY$$

$$(4.6.37) \quad EZ_{ne} = \gamma_{Zse}\tau_{(Zse,\ Zne)}Eu_{se} + \gamma_{Zsd}\tau_{(Zsd,\ Zne)}Eu_{sd} + \gamma_{Znd}\tau_{(Zne,\ Znd)}Eu_{nd}$$

$$-(\gamma_{Zse}\tau_{(Zse,\ Zne)}+ \gamma_{Zsd}\tau_{(Zsd,\ Zne)}+ \gamma_{Znd}\tau_{(Zne,\ Znd)})Eu_{ne} + EY$$

$$(4.6.38) \quad EZ_{nd} = \gamma_{Zse}\tau_{(Zse,\ Znd)}Eu_{se} + \gamma_{Zsd}\tau_{(Zsd,\ Znd)}Eu_{sd} + \gamma_{Zne}\tau_{(Zne,\ Znd)}Eu_{ne}$$

$$-(\gamma_{Zse}\tau_{(Zse,\ Znd)}+ \gamma_{Zsd}\tau_{(Zsd,\ Znd)}+ \gamma_{Zne}\tau_{(Zne,\ Znd)})Eu_{nd} + EY$$

Other Input Supply to Marketing Sectors:

$$(4.6.39) \quad EZ_{md} = \varepsilon_{(Zmd,\ umd)}(Eu_{md} - t_{Zmd})$$

$$(4.6.40) \quad EZ_{me} = \varepsilon_{(Zme,\ ume)}(Eu_{me} - t_{Zme})$$

Output-Constrained Input Demand of Marketing Sectors:

$$(4.6.41) \quad EZ_{sd} = - (\kappa_{Znd}\sigma_{(Zsd,\ Znd)}+ \kappa_{Zmd}\sigma_{(Zsd,\ Zmd)})Eu_{sd}$$

$$+ \kappa_{Znd}\sigma_{(Zsd,\ Znd)}Eu_{nd} + \kappa_{Zmd}\sigma_{(Zsd,\ Zmd)}Eu_{md} + EQ_{d}$$

$$(4.6.42) \quad EZ_{nd} = \kappa_{Zsd}\sigma_{(Zsd,\ Znd)}Eu_{sd} + \kappa_{Zmd}\sigma_{(Znd,\ Zmd)}Eu_{md}$$

$$- (\kappa_{Zsd}\sigma_{(Zsd,\ Znd)}+ \kappa_{Zmd}\sigma_{(Znd,\ Zmd)})Eu_{nd} + EQ_{d}$$

$$(4.6.43) \quad EZ_{md} = \kappa_{Zsd}\sigma_{(Zsd,\ Zmd)}Eu_{sd} + \kappa_{Znd}\sigma_{(Znd,\ Zmd)}Eu_{nd}$$

$$- (\kappa_{Zsd}\sigma_{(Zsd,\ Zmd)}+ \kappa_{Znd}\sigma_{(Znd,\ Zmd)})Eu_{md} + EQ_{d}$$

(4.6.44) $EZ_{se} = - (\kappa_{Zne}\sigma_{(Zse, Zne)} + \kappa_{Zme}\sigma_{(Zse, Zme)})Eu_{se}$

$+ \kappa_{Zne}\sigma_{(Zse, Zne)}Eu_{ne} + \kappa_{Zme}\sigma_{(Zse, Zme)}Eu_{me} + EQ_e$

(4.6.45) $EZ_{ne} = \kappa_{Zse}\sigma_{(Zse, Zne)}Eu_{se} + \kappa_{Zme}\sigma_{(Zne, Zme)}Eu_{me}$

$- (\kappa_{Zse}\sigma_{(Zse, Zne)} + \kappa_{Zme}\sigma_{(Zne, Zme)})Eu_{ne} + EQ_e$

(4.6.46) $EZ_{me} = \kappa_{Zse}\sigma_{(Zse, Zme)}Eu_{se} + \kappa_{Zne}\sigma_{(Zne, Zme)}Eu_{ne}$

$- (\kappa_{Zse}\sigma_{(Zse, Zme)} + \kappa_{Zne}\sigma_{(Zne, Zme)})Eu_{me} + EQ_e$

Domestic Marketing Sector Equilibrium:

(4.6.47) $\kappa_{Zsd}EZ_{sd} + \kappa_{Znd}EZ_{nd} + \kappa_{Zmd}EZ_{md} = \gamma_{Qsd}EQ_{sd} + \gamma_{Qnd}EQ_{nd}$

(4.6.48) $\kappa_{Zsd}Eu_{sd} + \kappa_{Znd}Eu_{nd} + \kappa_{Zmd}Eu_{md} = \gamma_{Qsd}Ep_{sd} + \gamma_{Qnd}Ep_{nd}$

Export Marketing Sector Equilibrium:

(4.6.49) $\kappa_{Zse}EZ_{se} + \kappa_{Zne}EZ_{ne} + \kappa_{Zme}EZ_{me} = \gamma_{Qse}EQ_{se} + \gamma_{Qne}EQ_{ne}$

(4.6.50) $\kappa_{Zse}Eu_{se} + \kappa_{Zne}Eu_{ne} + \kappa_{Zme}Eu_{me} = \gamma_{Qse}Ep_{se} + \gamma_{Qne}Ep_{ne}$

Input-Constrained Output Supply of Marketing Sectors:

(4.6.51) $EQ_{sd} = -\gamma_{Qnd}\tau_{(Qsd, Qnd)}Ep_{sd} + \gamma_{Qnd}\tau_{(Qsd, Qnd)}Ep_{nd} + EZ_d$

(4.6.52) $EQ_{nd} = \gamma_{Qsd}\tau_{(Qsd, Qnd)}Ep_{sd} - \gamma_{Qsd}\tau_{(Qsd, Qnd)}Ep_{nd} + EZ_d$

(4.6.53) $EQ_{se} = -\gamma_{Qne}\tau_{(Qse, Qne)}Ep_{se} + \gamma_{Qne}\tau_{(Qse, Qne)}Ep_{ne} + EZ_e$

(4.6.54) $EQ_{ne} = \gamma_{Qse}\tau_{(Qse, Qne)}Ep_{se} - \gamma_{Qse}\tau_{(Qse, Qne)}Ep_{ne} + EZ_e$

Domestic Retail Beef Demand:

(4.6.55) $EQ_{sd} = \eta_{(Qsd, psd)}(Ep_{sd} - n_{Qsd}) + \eta_{(Qsd, pnd)}(Ep_{nd} - n_{Qnd})$

(4.6.56) $EQ_{nd} = \eta_{(Qnd, psd)}(Ep_{sd} - n_{Qsd}) + \eta_{(Qnd, pnd)}(Ep_{nd} - n_{Qnd})$

where $\eta_{(Qnd,psd)} = \eta_{(Qsd,pnd)}(\dfrac{p_{sd}^{(1)} Q_{sd}^{(1)}}{p_{nd}^{(1)} Q_{nd}^{(1)}})$, and $p_i^{(1)}$ and $Q_i^{(1)}$ (i=sd and nd) are the initial price

and quantity, respectively, for the two domestic beef products.

Export Demand for Australian Beef:

(4.6.57) $EQ_{se} = \eta_{(Qse, pse)}(Ep_{se} - n_{Qse})$

(4.6.58) $EQ_{ne} = \eta_{(Qne, pne)}(Ep_{ne} - n_{Qne})$

4.7 Summary

In this chapter, the horizontal and vertical structure of the Australian beef industry was reviewed and an equilibrium displacement model was specified, involving 58 endogenous variables of prices and quantities and 12 exogenous variables representing various research and promotion scenarios.

Because the objective of this study is to examine the returns of alternative investment scenarios to various individual industry groups, a disaggregated model along both horizontal and vertical directions of the industry is required. Due to the expansion of grainfed beef exports to Asian markets and the increasing quality requirements in the domestic market, the feedlot industry and R&D in grain-finishing cattle have received greater attention in recent years. Product specifications for domestic consumption and export beef are also differentiated. As a result, four cattle or beef products were identified in the model depending on whether the cattle is grain or grass finished and whether it is for domestic or export consumption. Vertically, the beef industry was disaggregated into sectors of breeding, backgrounding, grass/grain finishing, processing, marketing and final consumption. This enables identification of benefits to individual industry groups from investments in different sectors.

The demand and supply relationships among prices and quantities of all sectors were specified in general functional forms, derived from the underlying decision-making functions. The displacement model (in Equations (4.6.1)-(4.6.58)) was derived through comparative static analysis, which linearly related the percentage changes of all prices and quantities of all sectors with the exogenous shifter variables and a set of market elasticities. Integrability restrictions among all market elasticities were examined and imposed at the base equilibrium point. The model can be solved to obtain the impacts of an investment to the prices and quantities, once values of the market parameters and the exogenous variables are specified. These will be specified in Chapter 5.