Chapter 1
INTRODUCTION

"All children know far more than they get credit for."
Heidi Mills (1993)

"What does education do?
It makes a straight-cut ditch of a free, meandering brook."
Henry David Thoreau (1850)

Introduction

The period of initial year entry into school for young children is a crucial time of development, both academically and emotionally. Today, maybe more than before, many children enter school with a wider range of experiences and knowledge that reflect the influences of the media and technology on their everyday lives. What mathematics knowledge do young children possess?, is an important question for teachers and curriculum developers.

This chapter outlines the background of this study under three main sections. The first section, Number Theory, considers the historical background to theories of number development in young children, and in particular, the development of constructivism as a mathematical education theory. The second section discusses Models of Number Development. The final section considers current research on young children's abilities in mathematics as they begin school.

NUMBER THEORY

This section is divided into two parts. The first part deals with theories of learning. Traditional and constructivist points of view are discussed. The second part, Theory and Number, considers the application of constructivist approaches to the learning of mathematics.

Theories of Learning

Traditionally, children have been seen as a 'blank slate' upon entering school. The typical role of the teacher has been to impart knowledge to students by filling up this empty space. This method can be referred to as the absorption method (Baroody 1987, p.6). Gergen (1992, p.18) described traditional education theory as based on the following two principles: exegenic, in which the world holds or exists as the primary knowledge and therefore is representative, and endogenic in which that knowledge is built up through reasoning in the mind and "dispassionate observation" (1992, p.25). This view sees knowledge as a truth to be learnt.

In recent years educators have proposed that children have already developed knowledge which they bring to school, and that children continue to build knowledge from their experiences. This idea has become known as constructivism because of the notion that children construct their own knowledge. Early constructivist ideas were present in the writings of Kant (1780) and Vico (1710), cited in von Glasersfeld (1992), but it was Piaget who spent some "sixty years establishing the basis for a dynamic constructivist theory of knowing" (von Glasersfeld, 1992, p.6). Piaget stressed that knowing is an adaptive activity and not a stimulus - response phenomena.

"As a cognitive position, constructivism holds that all knowledge is constructed and that the instruments of construction include cognitive structures that are themselves products of developmental construction" (Piaget, 1953, p.61). Piaget's ideas "place the individual as primary" (Lerman, 1995, p.133). A basic tenet of constructivism is that "knowledge is not passively received but is actively built up by the cognizing subject... (and) ...the function of cognition is adaptive and serves in the organisation of the experiential world rather than in the discovery of ontological reality" (von Glasersfeld, 1989, p.162). Constructivist ideas hold that children, existing in an informal environment, are constantly learning and trying to make sense out of their environment. The children do this by constructing and gathering their own ideas of how things work. Children make links with experiences and events that occur around them, and devise their own set of rules for various situations. In this way, children enter school with a vast range of experiences and knowledge from their normal life that they can use in a variety of situations. Within such a context, the role of the teacher is to facilitate the use and development of this knowledge in a formal situation. The children "do not learn by being told" rather, "they assess and reassess their thinking to encompass new knowledge that they gain from their experiences and in this process, expand their conceptual understanding" (Anderson, 1996, p.35).

In general, constructivists believe in the basic premise that "all knowledge is constructed and that the instruments of construction in the cognitive structure are either innate or themselves a product of developmental constructivism" (Noddings, 1990, p.10). Two main types of constructivism are referred to in the literature. They are radical constructivism, based strongly upon the ideas of Piaget, and social constructivism, which has grown "out of an attempt to incorporate an explanation for intersubjectivity into an overall constructivist position" (Lerman, 1995, p.134). Radical constructivism is centred on a series of stages that a child passes through relevant to a particular age, whereas social constructivism takes into account the social interactions of the child and the experiences of the child along with the developmental stage of the child. Social constructivism is more flexible than radical constructivism because social constructivism focuses more than radical constructivism on the environmental factors to influence the child's development.

The Development of Number Concepts

The impact of constructivism on teaching has important implications for the early development of mathematics. This is especially true in a world where young children are surrounded increasingly by the use of numbers in the home. In particular, the ever increasing number of electrical appliances, such as microwaves, digital clocks and video players which use number displays and combinations, plays an important role.

Constructivism as already discussed, was initially a theory considered and discussed by psychologists to explain how the evolution of knowledge occurs within a person. Baroody (1987), in line with the constructivist view, felt that mathematical development in children parallels the history of the development of number in that "it grows out of practical concern and concrete experience" (Baroody, 1987, p.26). This differs from non-constructivist models which "treat mathematics as a ready made product that schooling must help children to absorb" (Romberg, 1984, p.38).

The foundation for much of our early childhood mathematics, which has been taught in schools for the last thirty years, is grounded in the work of Jean Piaget, *The Child's Conception of Number* (1952). This work of Piaget led to the development of radical constructivism which "is currently a major, if not dominant, theoretical orientation in the mathematics education community, in relation to children's learning" (Cobb, Wood, & Yackel, 1991, p.162). Piaget emphasised the importance of conservation of quantities, one-to-one correspondence, grouping and configurations for grouping numbers of objects, which he estimated, occurred around the ages of five-to-six years. He believed numerical correspondence and order develop after these skills are refined. Piaget's view proposes that "before reaching the age of reason (seven years or so) children are incapable of understanding number and arithmetic" (Baroody, 1987, p.104).

While constructivists believe in the basic premise that all knowledge is constructed, many have conceptual differences. Recently some developmental researchers (Fuson, 1988, Young-Loveridge, 1989) criticised Piaget's approach to early stages of number development, particularly, his use of language in experiments, his lack of recognition of the central role of language in connection with cultural inheritance in construction of knowledge, the validity of some of his results and the emphasis on conservation for number success. Some current research (Fuson, 1988; Urbanska, 1993; Wright, 1991, 1994) has indicated that children of approximately five years of age have a wide variety of number experiences and are aware of many number concepts. Hence, they cannot all be classified as operating at the one level.

Steffe and D'Ambrosio (1995) argued that constructivism "does not stipulate a particular model of teaching mathematics", but sets out basic premises and the "goal is now to formulate models of teaching" (p.148). A number of researchers (e.g., Baroody & Ginsburg, 1982; Brush, 1978; Fuson, 1988; Urbanska, 1993) have sought to do this and found that children do not come to school without knowledge of number and counting, rather children bring mathematical notions which they have developed as they have needed them to operate in their own world. This may involve the ability to count out numbers and to add and subtract with small numbers, such as the situation of getting two biscuits from a jar for themselves and a friend. This requires the child to add two and two to calculate the total biscuits required. Brush (1978) found that pre-schoolers displayed this type of intuitive mathematics and recognised the effect of adding. Gelman and Gallistel (1978) were critical of researchers in the field of cognitive development who only consider young children in light of what they cannot do on tasks of conservation and believed that an "analysis of counting errors made provide evidence that young children can and do apply principles when counting" (1978, p.204). They (1978, p.237) disagreed with Piaget's (1952) view that inability to conserve indicates a lack of concept of number, and observed that though the children lacked the ability to reason about numerical relationships, they possessed knowledge about specific numerosities.

According to constructivism principles, intuitive, informal mathematics that children develop forms the foundation for formal mathematics and, as such, should be used to develop formal mathematics in school. It is clear that "children develop their own (informal) mathematics even before they receive any formal training in school" (Baroody & Ginsburg, 1990, p.51). The role of the teacher is then "to find ways to facilitate and build on their students' ideas to encourage the construction of increasingly powerful conceptual operations. This requires listening to children's explanations and "developing an understanding of the underlying conceptual operations that underscore children's thinking" (Cobb, Wood, & Yackel, 1991, p.67). Therefore "constructivist teaching involves an understanding of each child's thinking in order to plan further learning opportunities that will take that child to higher, more inclusive levels of understanding" (Anderson, 1996, p.36).

Recent studies have focused on identifying models of constructivism in the teaching and learning of mathematics to identify what processes, concept and skills children use. The basic tenet of the constructivist approach underlies the development of current research. Piaget's ideas remain a strong influence on many early childhood curriculums, such as the Queensland early primary school mathematics syllabus.

Overview

Over the last thirty years, mathematics education concerns have become increasingly focused on the student. Much of the reason for this approach can be traced to the work of Piaget. Previously, traditional education theory had been concerned with children obtaining knowledge as a truth to be learnt. Hence, getting the correct answer was a central concern and not how the end result was achieved. Constructivist theory, which developed from the work of Piaget, has children constructing their own knowledge rather than passively receiving knowledge from the teacher. Within this framework of constructivism, a range of thoughts and ideas have been proposed in the area of mathematical development of young children.

MODELS OF NUMBER DEVELOPMENT

This section deals with models developed by a number of prominent mathematical researchers to describe number development and the central ideas involved in the acquisition of number understanding. The work of Carpenter and Moser (1983), Fuson (1988) and Steffe and Cobb (1988), offer developmental frameworks, and are discussed in this section. Finally, the role of subitizing (apprehending the number value of a group visually) and recognition of patterns is identified as a key to number development. Current research in this area is outlined under the heading Subitizing and Pattern Recognition.

Research by Fuson

Fuson, focused her work extensively in the area of mathematical development. In particular, she investigated young children's use of counting procedures and the way children begin to count. Her (1988) book *Children's Counting and Concepts of Number* discussed the research work she carried out. Her work has been adapted by other researchers (Wright, 1994, Young-Loveridge, 1989) in developmental framework for their studies.

Fuson's results support the principle that children, giving last word responses when counting, do seem to be following a last word rule (Fuson 1988, p.243). Last word response occurs when the child counts out five objects "1, 2, 3, 4, 5" and begins to realise that the last amount/word that they say is the number of objects and therefore establishes the last word response rule to a question of how many. Fuson set out hypothesis sequences in the developmental progression of the three conceptual relationships between count meanings and cardinality. These ideas are briefly summarised below.

The three main relationships identified by Fuson (1988) are:

1. connecting count and cardinality meaning;
2. representation of solution procedures for addition and subtraction procedures;
3. representation of addition and subtraction word problems.

Fuson found five levels of concept development in these areas. They are:

level 1. The meaning of number words begins forming in single set situations and children display the ability to answer the "how many? " question.

level 2. Count-to-cardinal transition. The child begins to count on. Perceptual unit items simultaneously represent addend within the sum. The child must "keep track" of what has been added while continuing to count on. From this level onwards "keeping track" is important to success.

level 3. Cardinal-to-count transition. This gives the child the ability to solve simple addition and subtraction.

level 4. More difficult addition and subtraction that require a combination of concepts is possible.
Representational units, now the cardinal numbers themselves, represent both addends and sums.
Representation of numbers has become increasingly general.

level 5. The child can develop solution procedures for base ten numbers and multi digit algorithms. Reversibility is possible and the child can relate separate addition and subtraction within specific problem types.

(Fuson, 1988, pp.248-298)

The levels set out here are a brief summary of the complex levels that Fuson

(1988, pp.248-298) presented. Fuson's diagram of her levels and relationships summarised here is reproduced in appendix A. In levels 1 and 2 "earliest relationships between count meanings and cardinality meanings are formed within single set situations" (Fuson, 1988, p.296) and then these ideas develop progressively in later levels. "Children eventually come to use number words themselves to represent addition and subtraction" (Fuson, 1988, p.297). From this point, children can represent abstract situations, and develop counting on and counting down strategies. Higher levels allow children to be able to decide whether to add or subtract for a situation. A cardinal number to children at higher levels has become a concrete perceptual unit which allows children to manipulate number knowledge to suit new situations.

Fuson (1988) did not reflect in depth on the results of her findings for the classroom situation. However, it is clear from her results that many children, some as young as three years old, are able to add and subtract with small numbers, and that children beginning school often have already developed many number skills and concepts. School text books do not adequately cater for these children according to Fuson. Texts do not expose children to a variety of experiences of types of addition and subtractions that would encourage progress through the levels of development that she identified. She found "first graders already have the conceptual knowledge" (Fuson, 1988, p.271) that school mathematics proposes to develop in the first year at school. Furthermore, some of her results and conclusions are critical of the theories of Piaget. Fuson found that one-to-one correspondence was not essential to the development of numerosity in children and she was also critical of Piaget's lack of provision for language effect on his results.

Research carried out by Fuson (1988) provides a framework, describing the development of counting and number concepts in children. Her work set out three main conceptual relationships between counting and cardinality, and five levels of concepts and strategies used to form these three conceptual relationships. She found that children as young as three were already beginning to use these strategies to form conceptual relationships with counting.

Research by Steffe and Cobb

The work of Steffe and Cobb (1988) has been extensive in the area of counting development. They used Piaget's constructivism as a theoretical basis. Their book, *Construction of arithmetical meanings and strategies* (1988), presented the results of a teaching experiment carried out between 1980 and 1982. The experiment aimed at building a model to describe the cognitive changes in children's early formation of number and their conceptualisations. Earlier work (Steffe, von Glasersfeld, Richards, & Cobb, 1983) had identified five counting types which Steffe and Cobb continued to refine. In 1988, Steffe and Cobb specified a progression of five stages for development of counting. A brief outline of the five types is:

1. Perceptual Unit Items- Children can count only the items that they can see.
2. Figural Unit Items- Children can count items that are not seen.
3. Motor Unit Items- Motor acts or movements become countable.
4. Verbal Unit Items- Words become a substitute for a countable item such as counting on.
5. Abstract Unit Items- The child develops an abstract collection of number where the number itself is a countable unit.

(Steffe & Cobb, 1988, pp.8-220)

Each level incorporates earlier levels and represents a significant growth in concept development.

The teaching experiment conducted by Steffe and Cobb (1988) involved interviewing six children entering First Grade. These sessions which were video taped, ran bi-weekly over two years. It was found that these children were working in the first three levels of counting types. The results yielded an in-depth view of the counting processes of young children entering school, and indicated that reform of some current practices with this age group at school was necessary. As von Glasersfeld (1982) proposed, the importance of the language dimension of number as vital to children's understanding was clear in the results. Hence, Steffe and Cobb were concerned that children were asked to learn a written number system with little reference to their verbal number sequence (1988, p.321). They advocated that written work be abandoned and be replaced with counting up and down schemes, which emphasise problem solving.

Steffe and Cobb were interested in how children adapted their counting scheme. Their results supported Gallistel and Gelman (1978) ideas that assimilation of new instances occur in the cognitive structure and that children are constantly assimilating new experiences with their knowledge. The view of counting as a scheme is also consistent with the ideas of Piaget. Although Piaget's ideas form a strong basis for the work of Steffe and Cobb, they also believe that new concepts are formed under the influence of adult guidance. Vygotsky's "zone of proximal development" (1965) was important in predicting the progress of the children even though his view of the teachers role as a "direct guide towards the adult's conceptual construction was not accepted" (Steffe & Cobb, 1988, p.v).

Research by Carpenter and Moser

The research of Carpenter and Moser has also been extensive in the area of number development. Moser (1988) outlined five elements which formed the basis for the work as important to the development of number concepts: classification, seriation, conservation, subitizing and counting (1988, p.114). He also supported Gelman and Gallistel's (1978) five counting principles.

Moser and Carpenter (1984) conducted a longitudinal study where they found children displayed the ability to perform a variety of techniques, such as addition and subtraction, before they entered school. The children displayed "a reasonably well developed conception of addition and subtraction even though the children had not learned the formal terminology associated with the operations" (Moser & Carpenter, 1984, p.11).

Carpenter and Moser (1983, p.19) identified three basic levels that apply to the development of addition and subtraction. These are summarised as follows:

<u>Level 1</u>. Using fingers or concrete objects to count all objects by either physically joining the two sets or counting the total without physically joining the sets.

<u>Level 2</u>. Use of counting sequences. This may be done in one of three ways;
a) counting all without using physical objects. It involves mental counting which is often hard to describe how the child proceeds but may involve double count such as 6 is 1, 7 is 2, 8 is 3 .....
b) counting on from the first addend in the problem .
c) counting on from the largest addend in the problem.

<u>Level 3</u>. Recalling number facts. Children use learned number facts to solve problems such as when

given 6+7 they proceed with 6+6 is 12 so 6+7
is 12+1.

Furthermore Carpenter and Moser (1983) also identified five strategies used by children to reach each of these levels. These strategies were used in both addition and subtraction situations. They are:

Strategy 1. *Separating From.* Concrete Objects are used and removed.

Strategy 2.

a) *Separating To.* Elements are removed until the number of objects remaining is equal to the small number given in the problem.

b) *Counting Down To.* This involves a backward counting sequence.

Strategy 3. *Additive Action.*

a) *Adding on.* The child sets out the smaller number of objects and adds on more objects until the larger amount is reached.

b) *Counting Up*. This involves forward counting.

Strategy 4. *Matching.* Concrete objects are necessary and are matched and the unmatched objects are then counted.

Strategy 5. *Choice.* A combination of counting backward and forward is used.

(Carpenter & Moser, 1983, pp.19-21)

The strategies 1, 2a), 3, and 4 are ones used when operating in Moser and Carpenter's(1983) level 1 for the development of addition and subtraction. Strategy 2b), 3b), and 5, are strategies used for level two development of addition and subtraction. The highest strategy and level identified is number recall, which is level 3.

The levels outlined by Moser and Carpenter (1983) are not independent but, like Fuson's (1988) levels, they suggest that counting strategies "build upon the direct modeling strategies of the previous level" (Carpenter & Moser, 1983, p.31).

Subitizing and Pattern Recognition.

Fuson (1988) researched the practice of subitizing and found that it occurred in many children but was not universal. Further, "subitizing sets of 2, 3 and 4 clearly does not precede last word responding" (Fuson, 1988, p.221). Fuson found different children subitize different size sets and that subitizing experience is not essential for counting development. These results supported the findings of Gelman and Gallistel (1978). Steffe, von Glasersfeld, Richards and Cobb (1983), however, disagreed and believed that subitizing plays an indispensable role in the development of arithmetic operations and last word responding (von Glasersfeld, 1982, p.192).

Von Glasersfeld emphasised the importance of subitizing more than Fuson in the development of arithmetic, and also saw it as a representational mode arguing "that perception of composite figural patterns plays an even more fundamental role as a building block in the genesis of number" (von Glasersfeld, 1982, p.192). He suggested that in subitizing, a child associates figural patterns with number words by a semantic connection. Subitizing is not counting or numerical but rather it refers to pattern recognition - a neural mechanism. This is an important first step. Fuson agreed with Von Glasersfeld that number word responses are learned or discovered from subitizing and that this discovery is the first entry into numerosity - to seeing number consists of a unit of ones. The second step is where number is abstract. In other words the number word takes on the value of the set of objects itself. The word five is associated in the child's mind with five objects. This is an extension of Gelman and Gallistel's view that subitizing should be thought of as a way "to group elements together so as to enhance counting" (1978, p.244).

Overview

A number of models based on constructivist theory have been developed to describe and trace the growth of number concepts in young children. Prominent researchers in this area have included Carpenter and Moser (1983), Fuson (1988) and Steffe and Cobb (1978). Although each has developed his/her own set of levels to trace the development of number and counting in young children, all agree that young children, aged five and six years, move through a series of levels of counting. There is a consensus that the earlier levels are incorporated within higher levels. The onset of a child beginning to count on and count down has been identified by these researchers as significant in number development. Carpenter and Moser incorporate their levels into three main stages whereas Fuson, and Steffe and Cobb both have five stages. The ability to recall and use abstract number knowledge is

recognised by all three as the highest level of development.
Furthermore, the importance of subitizing in the development of counting and number understanding has been proposed by Steffe, von Glasersfeld, Richards and Cobb (1982) as vital to the process. Fuson (1988) disagreed with this and found that it was not essential to number development.

RELATED RESEARCH IN THE AREA OF CHILDREN BEGINNING SCHOOL

This section discusses the current research on the specific area of children beginning their first year at school. A number of research projects have considered the interaction of informal knowledge that children bring with them to school and formal classroom mathematics. They have done this in the light of their various national and state designed curriculum documents with interesting results. The work discussed in this section includes that of Aubrey (1993), Sophian and McCorgay (1994), Urbanska (1993), Wright (1991,1994), and Young-Loveridge (1989).

A New Zealand study, conducted by Young-Loveridge, University of Waikato, considered the counting ability that children bring to their first year of school and is outlined in Young-Loveridge's (1989) article "The Development of Children's Number Concepts: the first year of school". The study was initiated because of growing evidence that New Zealand children were not achieving in mathematics to the same extent as other western countries, and to assess the success of the recent introduction of a new school curriculum in primary mathematics.

The study was conducted with a sample of 81 students, typical of the socio-economic make up of New Zealand society. The children were interviewed at the beginning of their first year of school and again one year later. The tasks involved rote counting, completing forward and backward number sequences, enumeration, understanding the cardinality rule, pattern recognition for small numbers, ordinal numbers, numerical difference, producing a set of a given size, addition and subtraction with concrete objects and addition and subtraction with imaginary objects. A teacher judgement questionnaire was developed for teachers.

"The study showed that the majority of children entered school with considerable understanding of number concepts and skills" (Young-Loveridge, 1989, p.47). The results indicated "children who entered school with relatively little knowledge about number concepts made greater learning gains on the Number Task Interviews than did their peers" (Young-Loveridge, 1989, p.55). The higher achievers made less gains. More than half the children in the study were able to subtract small values with concrete objects on entering school. This supports the findings of Gelman and Gallistel (1978). It was found that certain skills were learnt by the children throughout the year even though they had not been taught them specifically. Further, teachers tended to underestimate the mathematical knowledge of students who achieved high scores. After one year at school all children were found to have made considerable gains but there still existed a high degree of variability between the knowledge and skills of the high and low achievers in the group. It was discovered that in the first year of school, many children were taught concepts of number that they already knew.

Young-Loveridge (1989) disagreed with the Piagetian approach to teaching mathematics which emphasised sorting, matching, comparing, ordering and classifying with relatively little work in number. Instead, she supported the view of Gelman and Gallistel (1978, p.202) that failure on Piagetian tasks of conservation does not mean children at this age are lacking cognitive capacity in the number area. The study indicated the importance of children learning at their own level and not spending time redoing concepts that they have already mastered.

Urbanska (1993) conducted a number of experiments in Poland which "provided evidence of a considerable degree of numerical competence exhibited by six year olds" (p.265). She interviewed children on aspects related to five topic areas, namely, counting, equipotency of sets, dividing, adding and comparing. All but one of the children tested showed counting ability to ten, and approximately half the children were able to count to one hundred - a skill not required until the end of the First Grade. Urbanska concluded that "numerical competence of children just beginning their O-class includes a considerable amount of knowledge and skills that come into the syllabus of that as well as subsequent higher grades" (1993, p.273). She found that the children attempted to use this knowledge in the classroom situation but that often the teacher failed to refer to, or make use of, the children's "spontaneous mathematics" during lessons. It was concluded that this was mainly due to a lack of awareness by teachers, their own poor mathematical knowledge, and the traditional authority structure of the classroom.

A similar study titled *A Study of the Numerical Development of 5-year-olds and 6-year-olds* was conducted in Australia in New South Wales primary schools by Wright (1994). The study dealt with 41 children interviewed at three stages throughout the school year. The work of Steffe *et al* (1983) and Fuson (1988) were used as a developmental framework, and the results compared to Young-Loveridge's study, discussed earlier. The Wright study involved the use of five models; counting stages, levels of forward number word sequences, levels of backward number word sequences, numerical recognition and recognition of spatial patterns. Wright (1994, p.39) found that "there was significantly less variability in the numerical knowledge among the beginning Year One children than in the beginning

Kindergarten children". (Kindergarten is the first year of schooling in the New South Wales system aimed at children aged five years). He found a wide range of abilities between beginning kindergarten students, but also found that all children improved throughout the year. By the time the children had begun Year One (their second year of school) the range of ability was not as wide. There was a suggestion that there is narrowing of the range of knowledge as children progressed through their first year of school. The findings of this study supported the view that "mathematics programs typical of the first year of school significantly underestimate 5-year-olds prior knowledge" (Wright, 1994, p.41). Wright (1994) suggested that the current curriculum in New South Wales for early childhood needs review.

In an earlier study, outlined in his article "What number knowledge is possessed by children beginning the Kindergarten year of school?" (1991), Wright found that the Kindergarten year mathematics curriculum is most suited to the least advanced students. Wright proposed that variation in mathematical ability is wide even at Kindergarten level. "One child before the age of four, has acquired number knowledge that the other child will not acquire before the age of seven" (Wright, 1991, p.1). In this study Wright used a five stage model of the levels of counting that he developed based on the work of Steffe and Cobb (1988) and his own investigations. The five stages are:

1. Perceptual
2. Figurative
3. Initial Number Sequence(sequential integration)
4. Tacitly-Nested Number Sequence (progressive integration's)
5. Explicitly-Nested Number Sequence (part - whole operation)

(Wright 1991, p.2)

This study also tested forward number word sequence and backward number word sequence, although this differed from the methods used by Fuson (1988). Forty-five children were involved and interviewed in the study. Children's responses were coded and a level determined for each child. Wright (1991), consistent with Gelman and Gallistel's (1978) view that "researchers who keep their eyes open will find still more unexpected ability in young children" (1978 p.242), emphasised that there was a wide range of abilities among the children, which was not well catered for by the curriculum. He reflected that teachers and educators should take account of the prior knowledge of the children and adjust the curriculum to incorporate current research findings.

In a similar study in England, conducted by Aubrey (1993), the mathematical knowledge of sixteen children in a reception class (children in their first year of school) was investigated. Aubrey's (1993) results were outlined in her article "An investigation of the mathematical knowledge and competencies which young children bring into school". The aim of the study was linked to the introduction of the National Curriculum, and tasks were designed to link with key aims set out in the National Curriculum to assess success. Consistent with other findings elsewhere, Aubrey found that reception children clearly possessed skills of sorting, matching, counting and ordering, recognising and writing numbers and simple mathematical relationships on entering school. She warned that children should be encouraged to extend their knowledge, not be retaught a single method to gain knowledge they already have. Similar to Young-Loveridge, Aubrey emphasised that teachers and educators should be aware of children's early inventive strategies, and adjust programs to help development on a more individual continuum.

Sophian and McCorgay (1994) conducted two experiments examining part-whole knowledge (where whole numbers are combined to form other numbers) and early arithmetic problem solving. Work in the area of the part-whole knowledge closely relates to Piaget's class inclusion and the work of Fuson (1988) in addition. The experiments were designed primarily to achieve statistical results rather than observing the methods used by the children. The major findings of the part-whole experiment were that children as young as five years "were sensitive to the part-whole structure" (Sophian & McCorgay, 1994, p.17). In problem solving, a marked improvement in accuracy of answers was noted in children aged five-to-six years which was not apparent in four year olds. Sophian and McCorgay (1994) attributed this to instruction received in First Grade. Other studies (Fuson, 1988; Urbanska, 1993; Young-Loveridge, 1989) found similar results in children of the same age who are not yet attending school. Fuson's (1988) results question the reasoning of Sophian and McCorgay. Sophian and McCorgay (1994) proposed their results form an intermediary point between Piaget and his critics, supporting Piaget's view of the importance of class inclusion and that children as young as five "showed very limited knowledge of the number combinations needed to generate precisely correct responses" (1994, p.30); yet provide evidence of inclusion reasoning in young children. For the classroom this means "instruction must explicitly show children how that knowledge applies to the new material they are learning in school" (Sophian & McCorgay, 1994, p.32). Sophian and McCorgay agreed with Fuson (1988) and Wright (1991,1994) that school texts do not adequately provide a variety of experiences.

Overview

Research on young children aged four-to-six years has been conducted by a number of researchers including Aubrey (1993), Sophian and McCorgay (1994), Urbanska (1993), Wright (1991,1994), and Young-Loveridge (1989). Aubrey (1993), Urbanska (1993), Wright (1991,1994), and Young-Loveridge (1989) have all found that children begin school with a well developed base of numerical knowledge and skills. Both Young-Loveridge (1989) and Wright (1994) found that after one year at school, children did make improvements in their number understanding. Furthermore, mathematics in the first year of school appears to be more suited to the less capable students. In contrast Sophian and McCorgay (1994) found that instruction received in the first year of school contributed significantly to the problem solving skills of children.

CONCLUSION

Theories of education which saw the teacher as the sole provider of knowledge, have dominated education in the past. In recent times these views have been largely replaced by notions which imply that learning people build upon or modify their existing knowledge base. The name given to this model of learning is constructivism and has its modern roots in the writings and research of Piaget. Piaget's work on the development of number and counting has for many years formed the basis of educational thinking in this area, however there is an increasing number of researchers who question some of the applications of Piagetian theory to education. The emphasis on what these children can do, highlighted by Gallistel and Gelman (1978), as opposed to the things that they cannot do, has become another important feature of recent research.

It is evident from current research that young children entering their first year of school are moving through a series of levels of counting. Different researchers have devised their own set of levels of counting but agree that different levels do exist and each level is dependent on the previous level (Carpenter & Moser, 1983; Fuson, 1988; Steffe & Cobb, 1988; Wright, 1991,1994). Subitizing is clearly an important part in the development of counting and the development of the last word response (Fuson, 1988; Gallistel & Gelman, 1978; Steffe & Cobb, 1988), while the ability to conserve may not be essential in numerosity development.

The results of much of the current research indicate that many children enter school with an already established knowledge of number and counting beyond what is expected of them in their first year at school (Aubrey, 1993; Urbanska, 1993;

Wright, 1992, 1994; Young-Loveridge, 1989). It is also apparent that there is a wide range of knowledge and skills possessed by these children which, in general, is not catered for within the current curriculum. As a result, the less successful students tend to make larger gains in their first year of school than other students, and by the end of the first year this range of knowledge is less evident (Wright, 1994; Young - Loveridge, 1989).

In Queensland there is little, if any, research work available concerning mathematics on children entering their first year of school. The current curriculum in this state is heavily based on the Piagetian school of mathematics education, emphasising sorting and grouping with numbers between 1 and 10 only. Addition is only briefly introduced at the end of the first year.

As a result of this discussion of current research, two main research questions seem worthy of investigation:

1. What knowledge of mathematics, particularly that concerning number, do children, entering Year One in Queensland, bring to school?

2. What progress do children make in their first year of schooling in Queensland in their development of number understanding?

A number of other subsidiary questions also arise which seem appropriate and are connected with the two main research questions stated above. They are:

* What sorts of abilities do children have on entering school associated with recognising and counting objects with numbers?
* Do children develop a concept of numerosity of a number and can they work with numbers without concrete objects in their first year of school?
* Is subitizing obvious in the development of the children's understanding of number?
* Is one-to-one association necessary for successful addition and subtraction?
* Do children enter school with the ability to read number and mathematical symbols?
* When does a sense of place value develop? Are grade one children ready to work with numbers greater than 10?
* Is there sufficient scope for all children in the Queensland syllabus?
* What are the implications of the results of recent research discussed in this chapter, in terms of the Queensland mathematics education System ?

It is these questions that form the basis of the investigation carried out in this study. The next chapter discusses the SOLO framework within which the study will be conducted.

CHAPTER 2
SOLO

INTRODUCTION

In the previous chapter several models were developed to explain growth in number understanding in young children. These included the five levels of Fuson (1988), five counting types of Steffe and Cobb (1988), and three levels and five strategies of Carpenter and Moser (1988).

One framework that has not been used in this context, but which appears to offer potential is that of the SOLO Taxonomy (Biggs & Collis, 1982, 1991). It represents a model, used extensively in categorising responses of the upper primary and secondary level, that can allow the description of early intuitive understanding as well as subsequent development.

This chapter looks at the SOLO model in detail with the view that this model will assist in analysing results from this study This chapter outlines the background of SOLO, beginning with its basis in Piaget's ideas, followed by a discussion of the development of SOLO which includes a description of the SOLO model. Finally, a discussion of how SOLO relates to this research project is provided.

BACKGROUND: Piaget

Piaget hypothesised a series of stages of development which he further divided into substages to trace human development. The main stages (Flavall, 1963, p.86) are as follows:

1.Sensory Motor Intelligence (0 - 2 years),
2.Pre-operational Representations (2 - 7 years),
3.Concrete Operations (7 - 11 years),
4.Formal Operations (11 - 15 years).

Of interest to this study are the stages of pre-operational representation and concrete operations. This study investigates the interface as children move from the pre-operational stage to the concrete operations stage. The two stages are described below in detail.

The pre-operational period is "characterised by imaginative play through exploring and experimenting, combined with questioning, talking and listening" (Isaacs, 1965, p.13). Children begin to develop a sense of space relations and time relations. They develop an ability for representation, imitation and language, and can use symbolism as transductive, and thinking as syncretic where many unrelated things are linked and a reason can always be found. "By the age of four there is the first inkling of length, distance, and number" (Isaacs, 1965, p.15).

The intuitive period, in the later part of the pre-operational stage, is characterised by an egocentric view. Children display no ability for reversibility, class inclusion, conservation or seriation. In this stage, children are extremely reliant on perception and intuition, "the upper limit cognitions are inner symbolic manipulations of reality" (Flavell, 1963, p.148). They can not understand relational terms and think in absolute terms, and are unable to form true concepts.

The concrete operations stage, which begins at approximately seven years of age, is characterised by the consideration of a number of factors in making a judgement. The size or amount of a group is not determined only by the "look" of the group, but by comparison with other groups or by comparison using counting techniques. Children operating at the concrete operations stage are able to use symbols in various forms of writing to carry messages. In terms of mathematics, numbers and number symbols are used to represent amounts, and children can appreciate the abstract value of numbers. An important feature of this stage is the ability of children to relate qualities and attributes of objects. Children in the concrete operations stage are able to conserve quantities, complete one-to-one correspondence and complete series.

Most people operate on a daily basis in the concrete operations stage and the focus of most of school curriculum is the development of thinking associated with concrete operations.

THE SOLO TAXONOMY

In this section the development of the SOLO Taxonomy is detailed along with SOLO's main features, including modes and levels. Further, development in multimodal functioning and learning cycles identified in the SOLO Taxonomy are also discussed. Two research studies concerning number and using SOLO are outlined.

The development of SOLO

The SOLO Taxonomy was first outlined by Biggs and Collis (1982) and "describes the growth in complexity of performance for many learning tasks, from the earliest engagement in the task to expertise" (Biggs & Collis, 1989, p.151). It offers an important tool for evaluation and assessment across different areas. The SOLO Taxonomy builds upon the ideas of Piaget as its theoretical base and is similar to other neo-Piagetian formulations provided by researchers such as Case (1982), Fischer and Knight (1990), and Demetriou (1988). However, SOLO, unlike Piaget's theory, sets out to classify responses not a person. In doing so it considers two aspects, namely, the mode of functioning and the level of response within a mode. Studies carried out using SOLO have suggested that responses do tend to fall into age patterns, but children at the same age may be operating at a variety of modes and levels within modes.

The Modes

The SOLO Model adopted five modes which are similar to Piaget's stages. A major difference between SOLO and Piaget's model is the placement of Piaget's early formal stage into concrete operations. This arose because Biggs and Collis (1982) identified children under 16 as still limited by their own concrete experience.

Biggs and Collis (1989) described modes as "levels of abstraction, progressing from concrete actions to abstract concepts and principles". Although ages are suggested for each mode this "does not imply that the way students perform in different situations is typical of their stage of cognitive development, nor that this is necessarily related to their age" (Pegg, in press). The five modes and their characteristics are:

Sensori-motor: This mode is concerned with the use of motor skills. Sensori-motor is the most basic mode and is seen from birth where an infant can react by giving a motor response to a stimulus, such as waving hands or crawling. These skills are developed into high level performances in professional sportspeople, dancers and gymnasts.

Ikonic: This mode develops from around the age of 18 months and draws on actions with meaning and is frequently affect laden. It deals with intuitive knowledge and mythical ideas. Internalisation of language makes the naming of objects possible and words and images can stand for an object. Egocentricity limits the child's point of view to their own world and needs. Only one emotion can be recognised at a time. Finally the ikonic mode has a mythic aspect which is obvious through fantasy, belief in fairy stories and make believe. The arts and drama are high level developments of the ikonic mode.

Concrete Symbolic: This mode develops from six years of age and is a profound step which results in the ability to read and write. As the name of this mode suggests the central feature of this mode is the ability to use and interpret written symbols to represent ideas and thoughts. It involves the ability to process symbols in a disembedded context, cold with little paralinguistic support (Donaldson, 1978). The symbols of the written language are important and used in writing, reading and mathematics. Mastery of these symbols, and effective use of the rules and the systems in which they exist, is the aim of education. Successful operation in the concrete symbolic mode is therefore a central aim of schools.

Formal 1: This type of thinking appears around the age of 16 and is the mode of operation expected for a university education. This is a higher level of thinking beyond the relationships of concrete symbolic and involves relationships in the abstract context. At this level a person can theorise about possible outcomes in a variety of circumstances and generate alternative hypotheses about ordering the world.

Post Formal: Post Formal, when evident, occurs from about the age of 20 onwards. This is the highest mode of operation and is the most contentious. Thinking in this mode questions the conventional bounds of knowledge and involves an exceptional knowledge and understanding of the functioning of a discipline. Most people will never reach this mode of operation and very little research has occurred to address its features.

The three modes ikonic, concrete symbolic and formal are the basis of most school education and curriculum development. Children begin school with a well developed understanding in the ikonic mode and school proceeds to instruct children primarily in the concrete symbolic mode with the aim of operating in the formal mode by the time students reach Years 11 and 12.

The Levels

The other significant feature of the SOLO model are the levels of responses that were identified by Biggs and Collis (1982) within each mode. The five levels of response are:

Prestructural: A response at this level is not operating in the particular stem of knowledge for the mode.

Unistructural: A unistructural response represents a single idea, skill or data element that is used by itself. Often unistructural responses are inconsistent as a result of a high need for closure. Students giving a "unistructural response tend to seize upon the first relevant dimension that comes to mind" (Biggs & Collis, 1982).

Multistructural: A number of skills or data elements are used in a sequence or individually to give a response. This represents a distinct increase in complexity over a unistructural response. Responses at the multistructural level may still be inconsistent, but closure is only achieved after a number of pieces of information is considered.

Relational: A relational response uses information and skills in a co-ordinated relationship to arrive at a response. Relational responses can use information in a variety of ways with the individual displaying the ability to choose a correct combination of relationships to achieve a response. Closure is achieved when all factors are considered and interrelated.

Extended Abstract: A response at this level indicates higher level thinking and movement to the next learning cycle in a new mode.

The three levels of most relevance to this study are the unistructural, multistructural and relational levels. The prestructural level is a response not yet operating in the mode and extended abstract level indicates operating in the unistructural level of the next mode.

The periods of movement between unistructural, multistructural and relational levels all involve an increase in complexity of thinking. Collis and Biggs (1991, pp.193-198) identify four major transitions in levels important throughout school in the target mode of concrete symbolic. (The term target mode is often used for the mode desired for a lesson.)

1) From relational ikonic to unistructural (concrete symbolic). In early primary school the aim is to have children move from their sensori-motor and ikonic knowledge to the use of concrete symbolic symbols and language.

2) From unistructural to multistructural (concrete symbolic). The consolidation of and automation of the unistructural knowledge and skills occur. A process that is the aim of primary education.

3) From multistructural to relational. This involves the ability to relate the multistructural knowledge into a coherent system and is the aim of high school education.

4) From relational (concrete symbolic) to unistructural (formal). This involves a shift in thinking to more formal thought processes and should be achieved in the final years of school for the more able students.

Figure 2.1 gives an overview of the SOLO model (for simplicity the prestructural and extended abstract levels have been omitted) and the movement through modes and levels. This figure shows a course of optimal development through the modes where the path of learning is unidimensional and earlier modes are subsumed. There are other paths of development also possible through multimodal functioning.

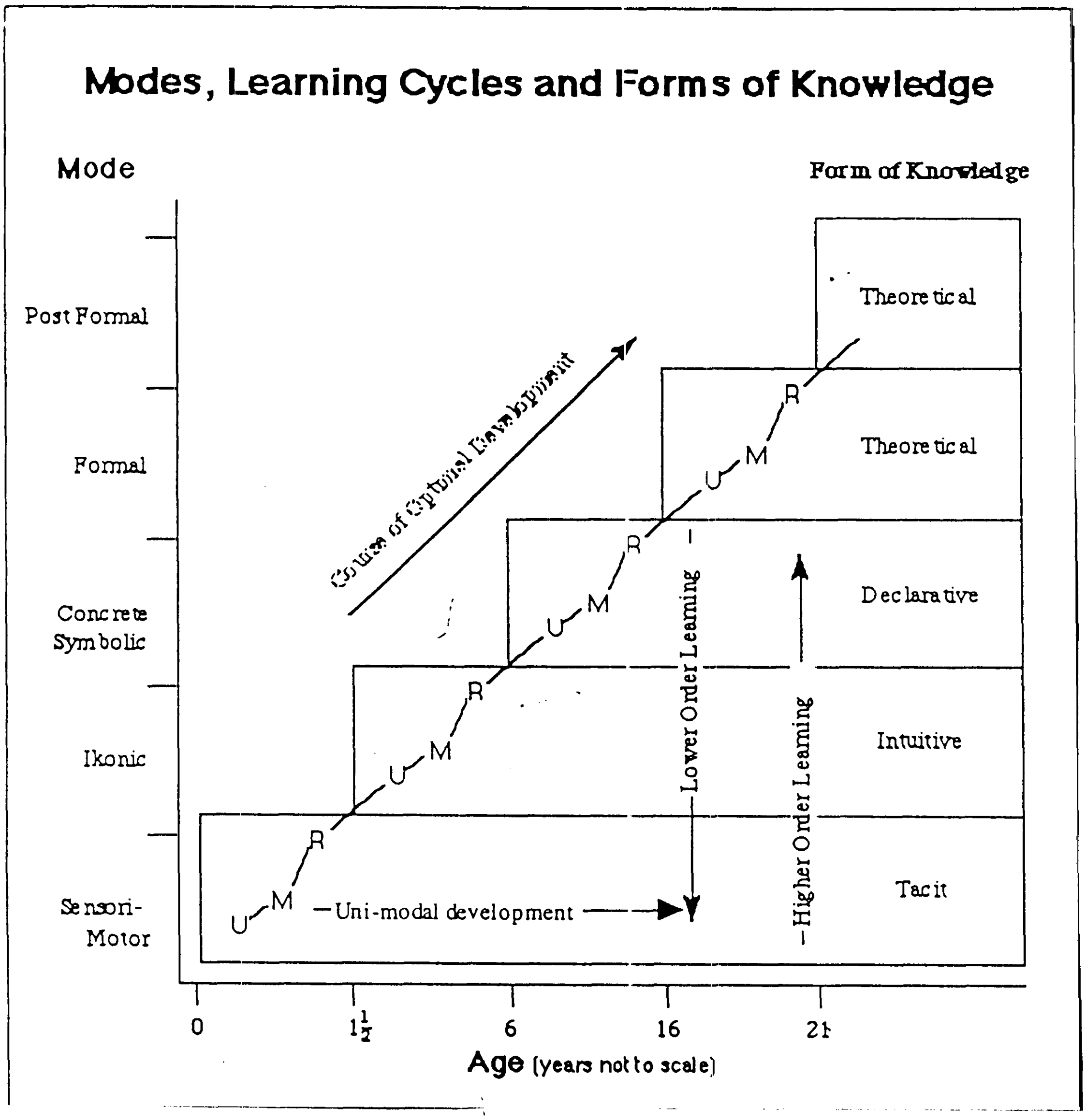


Figure 2.1 Modes Learning Cycles and Forms of Knowledge (Biggs & Collis 1991, p.66)

Multimodal Functioning

Over the last ten years the SOLO model has been developed further by Biggs and Collis. In particular they proposed that a person may operate in any mode or more than one mode at a given time. In fact a person may respond to a task one day in a particular mode and the next day in a different mode. Once a person has reached a particular mode, that mode remains available for use throughout life and is not consumed by the higher modes.

The structure and form of the SOLO model links development in one mode with development and growth in other modes. Development in the concrete symbolic mode can be facilitated by development in the ikonic mode. In this way SOLO is different from the single path development advocated by Piaget. SOLO, instead, allows for both optimal path development (as seen in Figure 2.1) and multimodal functioning, and suggests four main teaching strategies to develop and facilitate growth in a mode. They are:

1) Unimodal.
2) Top down.
3) Bottom up.
4) Two way.

A brief description of each of these methods follows.

Unimodal: This is where learning occurs within only one mode. The most obvious time is during infancy when a baby has only one mode (i.e., the sensori motor) available.

Top down: Top down is where the target learning mode is an earlier developing mode and a later mode can be used to help facilitate learning. An example of this occurs when a adult learns a new sport. Here the techniques and skills of games can be taught in the concrete symbolic mode to help facilitate development in the ikonic and sensori motor modes.

Bottom up: This method of learning utilises experiences in earlier modes to help the development of a higher mode. Teaching strategies are multimodal where experiences in the sensori-motor and ikonic modes are used to develop skills and knowledge associated with the concrete symbolic mode.

Two way instruction: This method is most common where the target mode is the ikonic mode. In this approach both the sensori motor and the concrete symbolic modes are employed to develop ideas in the ikonic mode.

Learning Cycles

A significant development of the SOLO model was the identification of cycles of levels which repeat themselves (i) within modes and (ii) across modes. Most importantly the same level names can be used in both cases; the difference being the nature of the element used in the particular cycle / mode. The three levels most commonly discussed are unistructural, multistructural, and relational which operate in each mode. Early work on SOLO (1982) had concentrated on these three levels within the modes but more recent developments in SOLO indicate that levels of unistructural, multistructural, and relational exist within a spiralling relationship. A number of researchers (Pegg, 1991; Watson, Collis, & Campbell, 1992) have identified learning cycles and cycles of levels within the concrete symbolic mode. At present current research has "identified only two cycles," although "there are probably a number of cycles within a given mode" (Pegg, in press, p.17). "Within each mode a child (person) moves through cycles of levels of responses" (Pegg, 1991, p.21). These levels in the cycles are:

* Unistructural- a single response or reason.
* Multistructural- multiple reasons or actions not linked.
* Relational- linking reasons or actions, defining relationships. This level is closely related to the unistructural response in the next cycle of levels.

The cycles are linked in a spiralling chain and appear and relate as follows in Figure 2.2: Spiralling Levels within a mode.

U U U
M M M
R R R

Figure 2.2: Spiralling Levels within a mode

"In summary then, a feature of this model is the marriage between the cyclical nature of learning and the hierarchical nature of cognitive development" (Collis, 1988, p.2).

Number Research

The SOLO Taxonomy has been applied frequently to high school subjects (Biggs & Collis, 1982) but less research is available in the area of number development in young children aged 5 and 6 years. Two such studies are briefly described here.

Collis outlined his investigations on research published on addition and subtraction in 4 and 5 year olds and how it relates to SOLO (1988). He discussed the importance of a multimodal model as many children at this age operate in both the ikonic and concrete symbolic modes and notes the importance of the educator in utilising all modes of operation.

Watson and Mulligan (1990) identified unistructural, multistructural, relational, and extended abstract responses in the ikonic mode in early multiplication worded problems. Consideration of these levels helps to illuminate the levels that exist in modes. The levels identified in the ikonic mode by Watson and Mulligan (1990) were as follows:

unistructural - a child gave an answer but failed to show evidence of any processing of information.

multistructural - physical processing of the questions occurred and concrete models

were set-up (e.g. their fingers) to gain an answer.
relational - attempts were made to integrate several aspects of the problem into a whole.
extended abstract - consistently correct responses were achieved at this level and conservation was a feature. This level was seen as a transition to the concrete symbolic mode.

The study identified two levels in the concrete symbolic mode:
unistructural - internalisation of the question has occurred and the child can answer without concrete support in one step.
multistructural - more complex questions involving two steps could be done by monitoring the process.

The study by Watson and Mulligan (1990) highlighted the interaction and transition between the ikonic and concrete symbolic modes in young children.

RELATING TO SOLO

The period of interest in the SOLO model for the purposes of this study is the ikonic and concrete symbolic modes. Children operating in the ikonic mode display characteristics of those mentioned previously in Piaget's pre-operational stage. In this mode children generalise, with the help of language and their qualitative judgement improves. Children try to make sense, and come to terms with, their environment, through the use of images. "The qualitative method of thinking will be driven by the 'look' of things rather than attempt to make a quantitative judgement " (Collis, 1988, p13). In this mode thinking is largely intuitive. The ikonic mode development "lays a sound basis for much of future development in both cognitive and non cognitive areas" (Collis & Romberg, 1990, p.9). Once children pass the age of six years, the ikonic mode continues to be used and developed in some cases to a very high degree.

The concrete symbolic mode involves the use of symbols and writing. It differs from the ikonic mode as visual cues are not needed and a person can operate through symbolic systems and it "involves a significant shift in abstraction ... to written second order symbol systems that apply to the experienced world" (Collis & Romberg, 1991, p.96). People operating in this mode display the concept of conservation and they are able to see the results of actions. The ability to reason systematically is a feature of this mode. "The elements develop from mere signifiers to concepts which are manipulated using a logic of classes and equivilances; both elements and manipulations being directly tied to the empirical world" (Collis, 1988, p.1). Instruction in school is largely based on the process of induction into the concrete symbolic mode of operation and "the primary years bring students firmly into the concrete symbolic mode" (Biggs & Collis, 1989, p.161).

Although the ikonic mode is valuable in problem solving, mathematics is impossible without functioning in the concrete symbolic mode. The concrete symbolic mode enables the manipulation of symbols and a rearrangement of information. During the first year of school, the role of education is to facilitate the development of the concrete symbolic mode in young children using their ikonic experiences.

In this study the interaction of informal and formal knowledge is considered. This mirrors in SOLO the interaction of the ikonic mode with the concrete symbolic. When a child initially arrives at school they are assumed to be functioning primarily in the ikonic mode, and the teacher proceeds to instruct the child to work in the concrete symbolic mode. The ikonic mode is very important although it often seems ignored in school. "There seems to be little conscious attention given to earlier developed skills in the haste to push forward into 'real' mathematics which can be seen to begin in the concrete symbolic mode" (Collis, 1988, p.7). Alternatively, given the rich experiences available for children to learn mathematics before they enter school, it is possible that many children are already performing in the concrete symbolic mode. For these children it would seem that continued work in the ikonic mode would be excessive.

CONCLUSION

The SOLO Taxonomy provides a general theoretical model for cognitive development within which the research conducted in this study could be considered, while still working comfortably with constructivist theory and current research in number development. The model also provides means of assessing and comparing data collected to evaluate movement and change over the study. SOLO raises a number of additional research questions.

1. What are the implications of applying SOLO to cognitive development of number in children beginning their first year at school?
2. Can cycles of development be identified in the area of early number development?
3. If so can these levels provide a means of interpreting student cognitive development, and how do they link with other levels identified by researchers into early number development?

CHAPTER 3
EXPERIMENTAL DESIGN

Introduction

In Chapter One it was established that a number of different researchers have found that young children move through levels of development of concepts and counting (Fuson, 1988; Steffe & Cobb, 1988; Wright, 1991, 1994; Carpenter & Moser, 1983). Furthermore, the results of current research indicate that children enter school with some knowledge of number and counting already constructed from their informal learning experiences (Urbanska, 1993; Young-Loveridge, 1989; Aubrey, 1993; Wright, 1992, 1994). As a result of the discussion in Chapter One, two main research questions and several smaller questions arose which are the focus of this study. The two main questions are:

1. What knowledge of mathematics, particularly number, do children, entering Year One in Queensland bring to school?

2. What progress do children in Queensland make in their first year of school in their development of number understanding?

This chapter outlines the design and features of the study formulated to answer the questions raised in Chapter One. The design developed to address the questions posed necessitated two studies; first, a study at the beginning of the school year with children entering Year One, and, second, a longitudinal study over the first year at school. The organisation and arrangement of these two studies is discussed in this chapter, under five sections. They are: Context which discusses the Queensland Primary School Mathematics Syllabus; Sample where the schools and children involved in the investigation are described; Design, which describes the method of research adopted; Instrument, which details the nature of the interviews used in the research; and, finally, Data Analysis Plan. The Data Analysis Plan uses the SOLO taxonomy, discussed in Chapter Two, as theoretical framework for the analysis of the observations.

CONTEXT

The study was carried out in Queensland primary schools where young children begin school the year that they turn six years of age. The first year of school is Year One and children attend school for 12 years, one less than the other states of Australia. This first year is considered approximately equivalent to the Kindergarten and Year One in New South Wales.

The Queensland Syllabus for primary mathematics (Year One included) has three main aspects - process, affects, and concepts - with an emphasis on the use of concrete materials. Non-government schools in Queensland are not required to follow the syllabus. The Queensland syllabus for mathematics in Year One includes Number, Statistics and Geometry. Statistics involves collecting information and interpreting and recording data. Basic picture graphs and bar graphs are also included. Geometry, required by the syllabus, involves plane shapes, such as triangles, quadrilaterals, circles; and 3-D shapes, such as prisms, cylinders, spheres, cones, and pyramids. The concept of length area and volume and associated language are to be discussed although only informal units of measurement are used.

In the syllabus, the concept of number is developed as well as the associated language. Children are to be introduced to the numbers 1 to 10. Basic addition with the use of the + symbol is introduced and addition with calculators is permitted. The concept of subtraction, the opposite of addition, is discussed but operations with subtraction do not occur. Number operations are an important component and the concepts of *more and less* are featured heavily. Other number properties and principles expected to be covered include basic properties of zero and one, and associative and distributive properties. Number is also developed in the curriculum in the area of money, with coins to the value of twenty cents, and also in units of measurement, especially time.

Education policy in Queensland is currently experiencing some major new initiatives especially in the area of lower primary. The Government has recently, introduced the state-wide Year Two Net testing program where all students in Year Two are tested internally to determine if they are performing at a prescribed level. The state-wide Year Two assessment for reading and writing commenced in 1996 and in 1997 numeracy also became part of the compulsory assessment. Trial version validation tests and number development continuums have been published displaying expected level of performance indicators for children to have achieved by May of Year Two.

The inclusion of numeracy and the performance indicators are sure to affect the mathematics taught in Year One. Five phases of development are set out in the numeracy development continuum. These phases are labelled from A to E. The early phases A and B are suggested as a suitable standard for Year One and require significantly less developed skills. Phase C, focuses on the use of objects, pictures and numbers less than 10. Phase C, the level where Year Two children are expected to show the key indicators, features many skills. These include:

1. counts forward and backwards
2. demonstrates an understanding of one to one correspondence
3. demonstrates the ability to count rationally
4. describes and creates auditory movement or visual patterns involving two elements
5. compares and classifies numbers to 10
6. represents numbers to 10
7. demonstrates an understanding of the links between all forms of representation of numbers
8. identifies and uses ordinal numbers to tenth
9. demonstrates an understanding of the concept of addition
10. calculates and records addition situations to 10
11. recalls particular addition facts to 10
12. demonstrates an understanding of the links between all forms of representation of addition.
13. uses appropriate language in mathematical situations
14. solves problems using a variety of strategies
15. applies number concepts to situations involving money.

(The Year 2 Diagnostic Net
Number Development Continuum
Trial Version, The State of
Queensland
Department of Education, 1995, p.14.)

These performance standards are designed not to override the syllabus but to reinforce the syllabus standards.

SAMPLE

Three co-educational primary schools on the central Gold Coast of Queensland were used to select a sample of 21 Year One students. Two of the schools were Catholic and the third school was a local state school. All the children involved in the study were beginning Year One for the first time. Children repeating were not eligible. The teachers did not know these children before selecting the children from a class list in the first week of school. Three children involved in the study were specifically asked for as they had been involved in an earlier pilot study.

The three schools are within a radius of 15 kilometres of the central part of the Gold Coast in South-East Queensland. This area is middle class with more than 12% of households with an income in excess of $50,000 and a below average number of low income earners (Australian Bureau of Statistics, 1991). One Catholic school was a small school for the region with approximately 400 students. At this school, Year One has two streams with approximately 30 students in each class. When the Principal of the school was approached for permission to undertake the study, he agreed and letters were sent to the parents of only one nominated Year One class seeking permission for their children to be involved.

The second school involved was another Catholic school with approximately 600 students and three Year One classes. The Year One class sizes were approximately 22 students. Direct contact was made with three parents who had children in a particular Year One class at this school requesting their children be involved. This was done so the children could be interviewed outside school time.

The State Primary school had approximately 700 students. The Principal and the State Education Department were approached about the study, after official application and a proposal for the study was submitted and approved, letters were sent out by the school for parental permission. None of the parents refused to grant permission. A specific class and teacher were nominated for the study to ensure minimal disruption to school. The class involved was a composite Year One and Year Two class containing 50 students- half of which were Year One- in one room. Two teachers worked with this class at all times.

DESIGN

This study adopted the approach with an emphasis on observing and the individual, and using an interpretative style of thought where data and observation are seen in light of meaning. The idea that "researchers must find ways to discover intention and to cultivate latent meaning" (Wilson, 1977, p.63) was adopted. This approach looks for understanding and searches to shed light on the subject.

The initial study was conducted within the first three weeks of school commencing. All children involved in the study were interviewed individually in an isolated area and asked to complete a number of mathematical tasks to ascertain what knowledge they had brought with them to school.

The second study was carried out as a longitudinal study, with a series of interviews over the school year. The initial study (interviews during the first three weeks of school commencing) formed the first stage of the longitudinal study. The second interviews took place in the last two weeks in June and the last interviews were in November, towards the end of the school year. Similar to the first interviews, the later interviews were conducted in an isolated area with the children on an individual basis. All interviews took approximately twenty-to-thirty minutes. Three main forms of data were collected. They were:

1) Verbal interaction between the researcher and each student were noted by the interviewer.

2) Written responses to questions were collected.
3) Patterns of action and behaviour such as body movement were noted by the researcher.

After undertaking a small pilot project, in which young children were observed in several mathematical activities, the decision was taken not to audio tape the students. While there are many advantages to audio or video taping, the pilot study indicated that the information required could be successfully identified and recorded by the interviewer. The children spoke slowly and extensive notes including direct quotes were able to be taken. Notes on hand, body and eye movement were also possible. In addition, children wrote throughout the interview and all written work during the interviews was collected and analysed. One advantage of this approach was that the schools and parents were much more willing to have their students/children involved. This resulted in a broader sample upon which to base the research than would otherwise have been the case.

Recording of the interview was achieved by giving the children their own piece of paper and pen at the beginning of each interview. The children were asked to write their name at the top of the page. This helped to establish the child's writing skill. This paper was then used throughout the interview by the children, while a separate set of notes were taken of verbal and physical responses to the activities. As will be seen when the activities involved are discussed, this provided a suitable and adequate record of the interviews. A decision was made that prompting, such as "if you move the blocks it might help", would not be used. Probing was used, such as "how did you get that answer?" or "can you show me what you did?".

INSTRUMENT

The interviews involved the students in five activities. These were referred to as: More and Less, One to One Association, Addition and Subtraction, Reversibility, and Place Value. The initial study used all tasks while the second and third interviews focused on the last three tasks. The tasks are discussed in detail below.

More and Less.

The ability of children to discriminate between *more and less* is important in the development of number understanding. It involves the ability to recognise larger groups of objects in a set or smaller amounts. *More and Less* questions feature strongly in the Year One syllabus and appeared often in the work the children were given throughout the year. The Year One syllabus outlines activities for comparing the size of groups and suggests that teachers should introduce and model terms, such as *more and less* and *most and least.* These activities focused primarily on the visual appearance of groups and later in the year moving towards using number names for comparison.

Hence, it was important to establish the knowledge the children possess in identifying *more and less* on beginning school. The introduction of numbers between 1 and 10 including counting and recognition of symbols was also a large part of the mathematics in Year One and these activities helped to establish the children's understanding of these numbers. The three sections to this activity referred to as 1A, 1B and 1C are outlined below.

1A) A series of six flashcards with bright yellow dots in pairs were shown to the children. Children were then asked to point to which card in the pair had more dots and which had less dots.

This activity aimed to establish whether the children could recognise *more and less* concepts in a purely visual sense and whether children used counting skills to back up their initial instinct.

1B) Initially, children were asked to listen to the following number combinations and to identify which number is more. The numbers used were:

9, 7; 2, 5; 11, 12; 20, 10; 14, 41; 25, 32; 41,39.

Following this, the children were asked to listen to the following number combinations and identify which number is less. The numbers used were:

6, 8; 4, 3; 9, 15; 2, 7; 15, 25; 24, 21; 31,29.

This activity aimed to establish whether the children had a concept of the number itself and its numerosity, i.e., 5 represents 5 units and is not merely something recited, such as 1, 2, 3 .... , and it can be said to be larger than 2 or smaller than 7.

1C) Each child had six numbers written on a page. The child was asked to put the numbers in order from smallest to biggest. The numbers appeared in the following order:

7 2 4 10 8 12.

This activity aimed to establish whether each child had a concept of a number's value in relation to other numbers.

One-to-One Association

One-to-One Association refers to the ability of a person or child to match two groups in a one-for-one arrangement. It may involve matching objects or words to objects. The Year One Syllabus emphasises the importance of *One-to-One Association* and outlines activities to develop this skill. The Queensland Syllabus for Year One notes that "one-to-one correspondence is an essential prerequisite to understanding number" (1989, p.142). The syllabus suggests that Year One children be provided with a variety of situations where they are required to match the elements of two groups, such as linking up a line of boys with girls.

One-to-One Association was tested with the use of blocks. Six blue blocks were placed in a straight line and each child was asked to select one yellow block for each blue block which was present. (The child was given a container of blocks.) This activity was included as it features strongly in the work of Piaget and activities used in Year One in Queensland. It was also included to compare results of each child with addition and subtraction results. This enabled data to be collected to assess if *one-to-one association* was important in achieving success with addition and subtraction.

Addition and Subtraction

The Grade One Syllabus for Queensland places much emphasis on the importance of number development. The aims of the syllabus include:

a) Identify number by sight alone,
b) Identify numbers to 10,
c) Construct new numbers,
d) Recognise numerals to 10,
e) Matching numerals to amounts,
f) Writing numerals,
g) Exploring number combinations,
h) Grouping Items,
i) Games consolidating number ideas to ten.

Aims for the development of counting skills are
A) Recalling number names from one to 10
B) Identifying the correct sequence of
C) Estimating and counting numbers of
D) Counting from a selected number
E) Investigating counting patterns.

Finally activities for calculating with numbers focus on
1) Number stories,
2) Investigating situations which involve addition,
3) Practising addition examples (with concrete materials),
4) Investigating recall strategies such as counting on,
5) Exploring subtraction as the inverse of addition.
(Year 1 Mathematics Source Book 1989, pp.135-189)

It is important to note that only a few subtraction concepts are introduced in Year One and that the main focus for subtraction does not occur until Year Two. Similarly, numbers with values greater than ten do not become the central focus until Year Two. The initial interviews were designed with these syllabus aims in mind.

The area of addition and subtraction was considered under three parts referred to as 3A), 3B) and 3C).

3A) Addition and Subtraction with Counters

Counters were place in front of the child to represent the additions:

4 + 2 = 5 + 3 =

Each child was asked

1) How many counters in each group?
2) How many altogether?

The children were allowed to touch and move the counters if they wished.

A group of counters was placed in front the child. The child was asked

1) How many are there?
2) If I want to have "3" how many do I need to take away?

The counters were placed to represent the following questions:

5 - ? = 3 8 - ? = 5

These two activities were used to study the ability of the child to count objects (is subitizing used, how often, and to what level?), the methods used by the child to add and subtract, and to establish if children entering school could complete this activity.

3B) Addition and Subtraction without Counters

Each child was verbally asked the following questions

3 + 1 is ?	3 - 2 is ?
4 + 2 is ?	4 - 2 is ?
6 + 3 is ?	5 - 4 is ?
10 + 3 is ?	8 - 5 is ?
5 + 6 is ?	12 - 4 is?

This activity was designed to investigate how children worked with addition and subtraction using abstract number words and no concrete materials.

3C) Written Addition and Subtraction

On their own piece of paper each child was given a series of additions and subtractions in the following form:

3 + 2 =	4 - 2 =
5 + 1 =	5 - 3 =
4 + 3 =	7 - 1 =
5 + 4 =	9 - 3 =

Here, the aim was to discover whether the children were able to recognise and work with the written language including symbols of numbers and mathematical symbols.

Reversibility

Reversibility is defined here as the ability of a child to use the answer and one part of the question to work backwards to find out the original question. Reversibility is dealt with only briefly in the Year One syllabus, towards the end of the school year. In the section on exploring subtraction, the concept of subtraction is introduced as the inverse of addition. No actual calculations are made but the concept is discussed with concrete support.

For this task, on their own piece of paper, each child was asked to complete the missing number to make the addition true. The three examples used were:

4 + ? = 6

3 + ? = 7

? + ? = 8

As outlined by Vygotsky (1965) reversibility is an important sign of mathematical understanding and Fuson (1988) included reversibility as a feature of level 5 in her levels of concept development. Therefore this task was included to investigate the children's understanding of various digit combinations.

Place Value

Place Value is the development of the concept that digits take on unit values, tens values and so on according to their order of recording. The development of place value is listed as an aim of the Year Two syllabus and is not mentioned as suitable for Year One in the Year One Source Book (1989). This is due to the concentration in Year One on the numbers one to ten. For the purposes of this study the place value task involved three parts, 5A, 5B and 5C.

5A) The child was given a piece of paper with five numerals written on it and asked to read these numerals. The numerals used were:
30 35 90 14 41

5B) Each child was asked to respond verbally to the following four questions:
10 + 10 is ?
10 + 5 is ?
20 + 10 is ?
10 + 13 is ?

5C) Ordering of the numbers described earlier in the *More and Less* section activity also formed part of this task. Children were given six numbers written on a piece of paper and asked to put the numbers in order from smallest to biggest. The numbers appeared in the following order:
7 2 4 10 8 12.

These activities were designed to assess whether any of the children in the study had some understanding of place value by being able to recognise the 4 in 14 has a different value to the 4 in 41, and whether they were able to work with numbers beyond 10.

The five activities listed here were carried out by each child at each of the three interviews. It took approximately 20-30 minutes for the children to complete these tasks. A large amount of information was collected throughout the interviews which was then analysed.

DATA ANALYSIS PLAN

Data were collected for each child in the three interview periods namely, February, June and November. This information was then collated to facilitate the identification of patterns or trends. The data were considered in light of the responses received for each section in the first interview. Firstly, all responses received for each question were looked at separately. These responses were categorised into phases of development of the concept. The responses were not only categorised according to the answer given but the method used to attain the answer.

The phases were based on Fuson's (1988) levels of concept development in number and Steffe & Cobb's (1988) five counting types. In particular types 3 to 5 (see Chapter 1 for details) were relevant for this study. The work of Carpenter and Moser (1983), in their three levels and five strategies for addition and subtraction, was considered when classifying and observing responses in this study. At an intermediary stage, the observed levels in each section in this study were labelled 1 to 5. The results of the second two interviews in June and November were the also categorised into observed levels. As this process occurred, substages and details of the phases began to become clearer and more detailed. It was then possible to compare what observed level a child had been at in February to the observed level in June and November for each child in the group. This also gave a means for looking at an overall growth, or change, and trend in each different area for the whole group by comparing the observed level patterns of the children, e.g., in the area of written number addition or reversibility. Not only was an overall picture available for the whole school year but also for shorter periods, e.g., June to November, and for a particular set of interviews.

The observed levels and their labels, which had been designated temporally, were then considered in light of the SOLO Taxonomy to find if these matched SOLO levels. The observed levels were examined to assess if they clearly formed the levels of SOLO and whether spiralling within levels was evident. The use of SOLO, in analysis of results and tracing patterns of development were used to enhance conclusions. To increase the validity of SOLO, application SOLO maps would be developed as used by Mulligan and Watson (1990) to trace levels for each task.

Once this was achieved, analysis was conducted in the following areas;

1. Each child for each task was placed at a level for the February interview, then June and, finally, November.
2. Each child's pattern of development for each task was then mapped out, for example, student "X" 's pattern for addition with concrete objects may appear as

low M - high U - low M   or
level 3-level 2-level 3
(Feb.) (Jun.) (Nov.)

3. Each task was then examined separately to see how the group had performed at

each interview.

4. A total analysis of performance over the period for all tasks was made by comparing the number of higher and lower level responses in the first interviews to the subsequent interviews.

SUMMARY

The experimental design for this project involved setting up two studies with twenty-one students. The initial study aimed to assess what knowledge of mathematics, and in particular number, children entering Year One in Queensland bring to school. Second, a longitudinal study was developed to investigate what progress children make in their first year of school in their development of number understanding.

Children were selected from three different schools on the Gold Coast in the south-east corner of Queensland. At the interviews, students worked on five main task areas, described as: More and Less, One to One Association, Addition and Subtraction, Reversibility, and Place Value.

All task areas formed the basis for the initial study while only the areas of Addition and Subtraction, Reversibility, and Place value were addressed in the longitudinal study. The same tasks were given at each of the interviews to each child to allow for comparison. The tasks featured involvement by the children and were recorded through interview notes and the collection of written work produced by each child during the interview. The task were based around current research from Fuson (1988), Steffe and Cobb (1988), Moser and Carpenter (1983), Young-Loveridge (1989), Wright (1991,1994) and the work of Piaget (1952).

A data analysis plan was designed to collected the data from the initial interview and assess groups and patterns in the responses. This work was then to be compared with the SOLO Taxonomy and with the current research work (listed above and discussed in detail in Chapter One). The levels identified were then used to trace the progress of the twenty-one children over the year in the three-interview series.

CHAPTER 4
INITIAL RESULTS

Introduction

An initial aim of this study was to assess the knowledge that children bring with them to school. This chapter summarises the results of the February interviews to discover the depth of this knowledge. This study reports on the first of the three-interview series. Throughout this chapter the results are discussed under the headings of the five task sections of the interviews as set out in Chapter 3, namely, More and Less, One-to-One Association, Addition and Subtraction, Reversibility, and Place Value.

MORE AND LESS

The *More and Less* tasks involved each child being shown a series of flash card pairs with dots randomly placed on them. The children were then asked to identify (i) which card had more (with the first three pairs), and (ii) which had less (with the second three pairs).

All children involved in this study displayed a sound understanding of the words and concepts 'more and less' by correctly answering these questions. Two main responses to the task were identified and are shown in Table 4.1.

Table 4.1

Summary of responses to More and Less Flashcard tasks in Interview 1

RESPONSE	NUMBER OF CHILDREN
more-and-less decisions were made on a visual appearance basis.	18
counting was used in difficult situations to support the answer given.	3

The first response was adequate for most of the tasks but in the case where the number of dots on each card were either 9 or 10, answers became less accurate. Most children continued to make their more-or-less decisions based on a visual estimate as they had done for easier card differences, such as 3 and 6. However, some children, recognising that this required a more precise technique, opted to count individual dots. This technique indicated that these children recognised that counting would give them a more accurate answer and indicated that these children were comfortable using counting as a tool. The children displayed no clear preference or ease with the individual concepts of more or less.

After the flashcards, a series of number pairs was read out and the children were again asked to identify which number in the pair was either more or less. The number pairs used were as follows:

Which is more?

9, 7; 2, 5; 11, 12; 20, 10; 14, 41; 25, 32; 41,39.

Which is less?

6, 8; 4,3; 9, 15; 2, 7; 15, 25; 24, 21; 31, 29.

The responses to this task fell into the five groups which are summarised in Table 4.2.

Table 4.2

Summary of responses to More and Less Number Pairs tasks in Interview 1

RESPONSE	NUMBER OF CHILDREN
problems with task and could only work with 2 and 5 and 4 and 3	2
only numbers less than ten correct	3
only numbers less than twenty correct	5
all correct but difficulty with 14 and 41	2
all correct- no apparent difficulties	9

Two children had difficulty with this task but achieved some success with very small numbers. A further three children had difficulty working with numbers over ten, displaying comfort only with the numerosity of the numbers 1 to 10. Five children were able to answer successfully all questions where the numbers involved were less than twenty. The numbers 14 and 41 when paired together caused the only point of difficulty encountered by two of the children. Nine children had no difficulty with this task and displayed a good understanding of the value of numbers up to fifty. Overall, this task provided a clear breakdown of the number values that the children in this group felt comfortable using.

Overview of *More and Less*

The responses given by the children for these two tasks indicated that the children had already developed a concept of More and Less before receiving any

Year One formal education. Visual appearance of a group of dots was clearly important to the children in making assessment of *More and Less*. Many children, who were not yet using counting to help them to assess the size of the group, were able to appreciate the number value represented by a number name when asked, for example, which is more 9 or 7. A distinct ease and familiarity was apparent when the children worked with numbers less than 10 with only nine of the children comfortable with the full range of numbers used in the activities. The most confusion occurred with the first task on the cards, when a larger numbers of dots were used and the question required more than a visual judgement to obtain the correct answer.

ONE-TO-ONE ASSOCIATION

This task involved matching one set of blocks to another set of blocks in a one-to-one relationship. Six blue blocks were place in a straight line and each child was asked to put out one yellow block for each blue block. (The children were given a container of blocks.)

The four groups of responses to the one-to-one association task are shown in Table 4.3.

Table 4.3

Summary of responses to One-to-One Association in Interview 1

RESPONSE	NUMBER OF CHILDREN
unable to match the blocks	1
incorrectly matched the blocks	3
initially incorrect but used counting to check and correct	3
matched blocks correctly	14

Four children failed to match the b ocks correctly. These children all attempted the task but became confused. One of the four, was unable to match the blocks in any form while the other three children became confused by beginning to match the blocks side by side, starting for example, on the left side but on reaching the end had blocks on both the left and right side of the original blocks. Hence, these children ended up with one too many blocks and when asked by the interviewer to check their work were happy with what they had done and could not see their mistake.

A further three children displayed a loose concept of one-to-one association by matching the blocks incorrectly. These children were able to recognise their mistake by visually noticing that something was not correct or by checking and counting that they did not have the same number of blocks for each set. In some cases it took three attempts to get the correct result. This process involved counting and moving blocks and rechecking until the child was satisfied that they had the task correct. All of these children, who rechecked their answers, were able eventually to match the one-to-one association correctly. Out of 21 children involved in the study, 14 were able to display good one-to-one association, matching the number of different coloured blocks with ease.

Overview of *One-to-One Association*

One-to-One Association in this activity was generally successfully undertaken by most of the children. The placement of the blocks by the children had a large effect on the success or speed of success for each child on the task. Only four children in the group were unable to complete this task correctly. This indicates that most children could already fulfil the syllabus requirement for the end of Year One, and that these children displayed an ability to match elements of two groups competently.

ADDITION AND SUBTRACTION

Addition and subtraction concepts formed an important part of the initial interview and are discussed here. For the purposes of this study the area of *Addition and Subtraction* was considered under six activities, Adding with Counters, Subtracting with Counters, Adding without Counters, Subtracting without Counters, Written Addition, and Written Subtraction.

These topic areas and the responses received for each activity are discussed below.

Adding with Counters

This task involved the children adding groups of counters together to solve two addition questions. Counters were place in front of the children to represent the following additions:

4 + 2 = 5 + 3 =

The children were asked

1) How many counters in each group?
2) How many altogether?

The children were allowed to touch and move the counters if they wished. The responses the children gave fell into the following four groupings as detailed in Table 4.4.

Table 4.4

Summary of responses to Addition with Counters task in Interview 1

RESPONSE	NUMBER OF CHILDREN
used counting but had trouble matching words and finger movement	8
counted all objects successfully from one	10
subitized and counted on	2
used a base knowledge to add the two groups	1

None of the children involved in this study failed to attempt to complete this task. All displayed an understanding of addition, albeit, to varying degrees. Of the 21 children involved in the study, eight children used counting to add, but they either moved their fingers too fast, spoke too fast or too slowly for their finger movement, or had trouble matching the numbers with the objects. Interestingly, four of these same children had no difficulty on the *One-to-One association* task. The difference appeared to be that in the case of addition they were less focused on the task of matching their words with their counting finger movements. Two children, who had problems on the *One-to-One association* task, counted from one successfully here to get the correct answer. Two children 'counted on' to get the answer, subitizing the first set of counters, such as three, then saying "four, five" to add the next two counters. One child used knowledge of base number facts to give the answer straight away.

Throughout the task, some children at each response level used varying degrees of subitizing. Five out of eight of the children who used counting, but had trouble matching word and finger movements, were able to subitize amounts of two and three. Seven out of ten of the children, who counted all the objects successfully from one, used the ability to subitize the addends up to the value of five, then counted the addends together from one to answer the task.

Subtracting with Counters

For this activity a group of counters was placed in front the children. The child was asked:

1) How many are there?

2) If I want to have 'x' amount how many do I need to take away?

The number of counters provided initially were:
(i) five, then the child was asked "if I want to have three how many do I need to take away ?".
(ii) eight, then the child was asked "if I want to have five how many do I need to take away ?".
The number of students in each response category given for this task is detailed in Table 4.5.

Table 4.5
Summary of responses to Subtracting with Counters tasks in Interview 1

RESPONSE	NUMBER OF CHILDREN
could not understand or attempt the task	4
minor mistakes and used physical separation	3
all correct using subitizing or counting	13
correct response using subitizing and number facts knowledge base	1

The interviews indicated that subtracting was more difficult than addition for some children. Four children were unable to undertake the task. Three children attempted the task but could not correctly answer. The other 14 children found no difficulty and completed the task successfully.

Nine of the successful children used a visual subitizing style to obtain their answers and did not touch or move the counters. These children would look at the initial group of counters, then they could be observed moving their eyes across the group of counters separating out the number they required in the end. Finally, these students would count the counters remaining. This was done quietly with the only indication of counting taking place being the movement of the head in a nodding fashion, or silent movement of the lips, before pronouncing the amount that should be taken away. Throughout the process the children would not touch the counters at all or point to the counters. One child, after counting the first amount said "That's eight so if you want five you need to take away three". This child had established a number knowledge base to work with and did not need to check his answer by counting or subitizing.

Adding without Counters

The task *Adding without Counters* involved the children answering a number of questions which were verbally given to them, such as 4 plus 2.

Each child was verbally asked the following questions:

3 + 1 is ?   4 + 2 is ?

6 + 3 is ?   10 + 3 is ?

5 + 6 is ?

The children were instructed that they could use any means to find the answer including their fingers if required. They responded with their answer to the question verbally. The responses given by the children could be categorised into five groups as detailed in Table 4.6.

Table 4.6

Summary of responses to Adding without Counters Tasks in Interview 1

RESPONSE	NUMBER OF CHILDREN
unable to understand or attempt questions	3
able to add the number one	4
able to work with numbers that totalled five or less	6
uses fingers and toes to get the answer	6
able to use number facts	2

Children entering their first year of school, involved with this study displayed a good understanding of addition without counters. These children had received no formal tuition on addition, and so the knowledge they displayed reflected the informal knowledge they had developed themselves.

Only three children were unable to make any attempt at all on the questions. These children shook their heads or stated "I can not do it". Four students were able to add the number one or, despite making many mistakes, displayed an understanding that addition meant the first number increased in value. Their answers were unrealistic but verfied a consistent response that was greater than either one of the two numbers in the addition, indicating an understanding that addition increases the value of the number in the sum. A further six children could only work with numbers that totaled five and less.

Six children were able to use their fingers and toes to work the answer to the last question. These children did one of two actions: (i) they counted out the first and second numbers on their fingers then counted all the fingers put up together, or (ii) they held up the first amount of fingers then counted on the second amount, e.g., 5 + 3 would mean five fingers would be raised then the child would say "six, seven, eight" while raising the next three fingers. Two children displayed a knowledge of number facts to arrive at the correct answer. One of these students when asked how he got the answer to 5 + 6 said "Well I know 5 + 5 is 10 and 6 is 5 + 1 so the answer must be 10 + 1 which is 11".

Subtraction without Counters

This task involved the children answering subtraction questions without concrete support. Each child was asked, in words, the following questions

3 - 2 is ? 8 - 5 is ? 4 - 2 is

5 - 4 is ? 12 - 4 is ?

The responses the children gave are detailed in Table 4.7.

Table 4.7

Summary of responses to Subtraction without Counters tasks in Interview 1

RESPONSE	NUMBER OF CHILDREN
unable to understand or attempt task	8
able to subtract a difference of one	7
able to use fingers with values of ten or less	5
able to apply number facts	1

These data were consistent with the performance of the students with the addition and subtraction tasks involving counters. Seven children could only work with a difference of one, and could not subtract numbers of two or greater. These children exhibited an understanding of subtraction indicating that they realised that the answer must be less than the first number in the question. Only one child was able to use number facts to work out the answers. The remainder of the children used their fingers but were only able to work with numbers less than 10. Children using their fingers encountered some difficulty with numbers greater than ten as they did not display the ability to use the same fingers twice, thus holding the ten value. These children counted out the first amount on their fingers then put down the amount of fingers to be subtracted, and finally counted the remaining finger to get the answer.

Written Addition

The task referred to as *Written Addition* involved the children reading and answering on their own piece of paper, written questions that used numbers and mathematical symbols in the following form:

3 + 2 = 5 + 1 =

4 + 3 = 5 + 4 =

The responses to this task fell into the three groups described in Table 4.8.

Table 4.8

Summary of responses to Written Addition Task in Interview 1

RESPONSE	NUMBER OF CHILDREN
unable to read the question	8
able to begin to read-trouble interpreting the symbols	2
able to read and answer questions correctly	11

The inclusion of this task was to assess the readiness and ability of the children to read numbers and mathematical symbols. The ability to read and answer written questions is a skill not expected of children entering school. Eight children fitted this expectation and were unable to read the questions. However 11 out of 21 children in the study were able to read and answer written questions correctly. They had clearly reached the stage of development where the number symbol takes on a countable value. Furthermore two children showed signs of beginning to understand by verbally reading out the questions, although they had trouble interpreting the equality sign. These 13 children could all read the numbers and were able to make recognisable written numbers with their own writing to give the answer. Frequently, the numbers were written backwards, and while the number formation was not precise, the numbers were identifiable. An example can be seen in Figure 4.1.

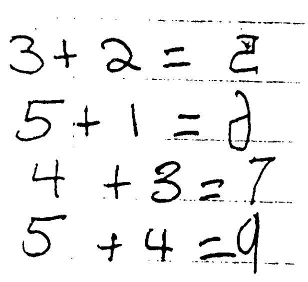


Figure 4.1: A student's response to written addition questions.

The response provided in Figure 4.1 shows this child has a sound grasp of addition, and the numbers formed to answer the question are easily readable.

This response was typical of a number of responses given by the children, while the answers written were readable some numbers are formed backwards. In this case both the 5 and 6 are written backwards while 7 and 9 are correct.

Seven children who were able to read the questions and write numbers correctly were not able to write their own names or form letters correctly indicating a greater exposure to numbers than letters in the informal learning environments. The children, who were unable to complete the task, made statements such as "I don't know that" and made no further attempt.

Written Subtraction

This task, similar to the written addition task, involved the children working on their own sheet of paper. They were expected to read the question and then answer the subtraction questions using number and mathematical symbols. The questions asked were:

4 - 2 = 5 - 3 =

7 - 1 = 9 - 3 =

As with the addition task, this task concentrated on the ability to read and interpret numbers and symbols. The groups of responses identified are summarised in Table 4.9.

Table 4.9

Summary of results to Written Subtraction tasks in Interview 1

RESPONSE	NUMBER OF CHILDREN
unable to read the question	12
able to begin to read - trouble with symbols	2
able to read and answer the questions correctly	7

The minus sign was not as well recognised by the children as the addition sign, with a total of 12 children unable to read the question. Nevertheless, seven children were able to complete the task successfully, and a further two children showed signs of beginning to understand the symbol system. One of these children tried to read the minus sign as an addition sign and answered the question correctly for an addition. The 12 children, who made no attempt, or gave unrelated answers to complete the task, appeared confused by the - and = signs. An example of the different understanding of addition and subtraction can be seen in Figure 4.2

student 6

student 2

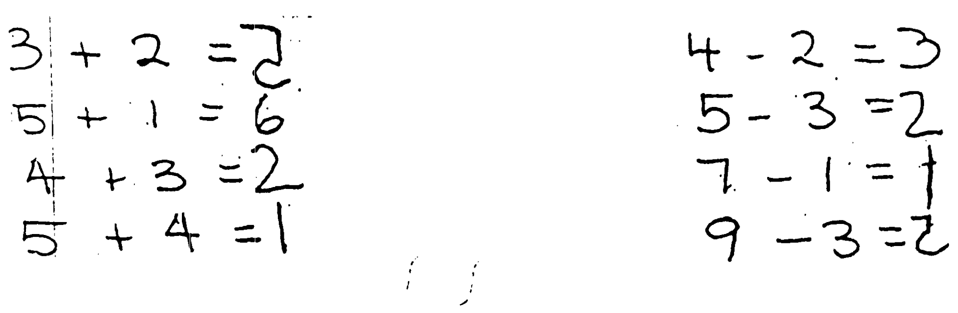


Figure 4.2: A comparison of written addition and subtraction answers.

Figure 4.2 shows two students (identified by the numbers 2 and 6) on written addition and subtraction tasks. It can be seen that student 6 has a sound knowledge of addition and is able to read and interpret the numbers and written symbols successfully. However, on the subtraction task it appears the student has first added, then subtracted. It is clear the child recognises the difference between the addition and subtraction symbols but is unsure what the minus sign requires of him/her. Student 2 does not have as good a grasp of addition as student 6, but indicates a level of comfort working with numbers less than five in both the addition and subtraction tasks. This student displays some grasp of subtraction, indicating in the answers provided that the answer to a subtraction is less than the first number of the question.

Overview of Addition and Subtraction

Observation of the tasks set out here gave an abundance of data on the knowledge of children entering their first year of school in the area of addition and subtraction of numbers. When working with counters, more than 50% of students were able to add together two number amounts correctly and all the children displayed some understanding that the task required putting the two amounts together. Subtraction with counters proved to give very similar results with 14 out of 21 students able to answer the task correctly. The ability of children in the group to count and use counting techniques to help answer the tasks was evident, showing that many of the group already felt comfortable with counting and number use on entering Year One.

When the addition task was presented, without the use of counters, three children were unable to understand the task. This task gave a more in-depth picture of the children's comfort zones with numbers indicating that although some had been able to work with values to ten with counters, without counters they were less comfortable and could only deal with values to five. This was also evident with subtraction without counters where some children, who had worked well with values to ten on the subtraction with counters tasks, were now only able to subtract differences of one. In both these tasks it became clear that there was a small group of two students who were able to answer tasks by using a base of knowledge that they had of addition and subtraction facts, rather than counting techniques. Generally, the children performed at a lower level in the area of subtraction, compared to addition, with eight children unable to even begin the task and displaying no understanding of subtraction.

This trend continued to be apparent with the written addition and subtraction tasks. Here, eight children were unable to read the addition questions and twelve were unable to read the subtraction questions. However, the number of children who were able to read and complete the task indicated a sound understanding of number and mathematical symbols even though they had received no formal education in this area. The responses given to these tasks, indicated that the children could also reproduce recognisable written numbers to answer the task.

Overall, the range of ability among the children in the interviews in the area of addition and subtraction was large. All children displayed some understanding of number concepts and calculations. For some children this was the ability to count amounts or to see that two groups needed to be put together. Other children were able to read and interpret mathematical and number symbols fluently, and work with these in an abstract context. A small group of students showed signs of already developing a sound base of number facts that could be used and manipulated to answer a variety of situations. It was clear that not all the children possessed the same level of understanding. Nevertheless, many were already able to satisfy syllabus requirements for Year One and, in the area of subtraction, they were able to meet the criteria for Year Two.

REVERSIBILITY

Reversibility is dealt with only briefly in the Year One syllabus and this is meant to occur towards the end of the year. In the section on exploring subtraction, the concept of subtraction is introduced as the inverse of addition. No actual calculations are made, but the concept is discussed with concrete support. The reversibility task set up for this study involved three questions in which one or both numbers in the addition question were missing, but the answers were given. For two questions, the child was required to fill in one missing number to satisfy the number sentence and the third question had two blank spaces allowing the child to come up with as many number combinations as he/she could and still give the same answer. The questions asked were:

3 + ? = 7

4 + ? = 6

? + ? = 8

The large range of responses received on this task indicates that Reversibility is a difficult concept that requires a higher level of understanding of addition and number sentences to complete. The response groupings are described in Table 4.10.

Table 4.10

Summary of responses to Reversibility Tasks in Interview 1

RESPONSE	NUMBER OF CHILDREN
unable understand or attempt the task	13
add addend and answer -ignore the position of the signs	1
answer the first two questions and one solution only for question three	4
answered all questions with two or more solutions for question three	3

Thirteen children were unable to recognise what the task required and made no attempt at all. From their earlier work on the addition and subtraction tasks, particularly the written tasks, it was obvious that this task was beyond them at the time. Of these thirteen, one child put n numbers, (that in later interviews would prove to be common with others), trying to make a pattern, for example 4 + ? = 6 became 4 + 5 = 6. This is illustrated in Figure 4.3 below. Here the child leaves the second two questions without any attempt. The first question is attempted by completing a pattern of stepping by one.

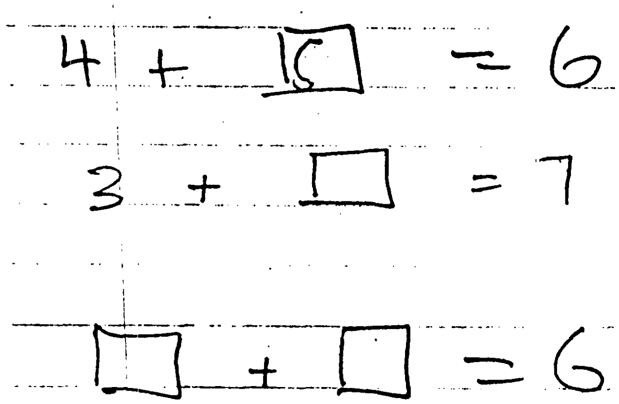


Figure 4.3: An attempt at a Reversibility question.

One child tried to interpret the symbols and numbers but ignored the position of the information and answered 3 + ? = 7 by inserting 10 giving the number sentence 3 + 10 = 7. Seven children in the study were able to answer at least two out of three of the questions. The two questions that required only one missing number to be inserted were attempted with more ease. Within these, successful children demonstrated a range of understanding. Some children were only able to complete the questions with one missing square while two children were able to complete question three with two blank squares. Of the children who completed the third question with two blank squares, one child could only give one solution and the other three were able to give two possible sets of solutions. This is illustrated in Figure 4.4.

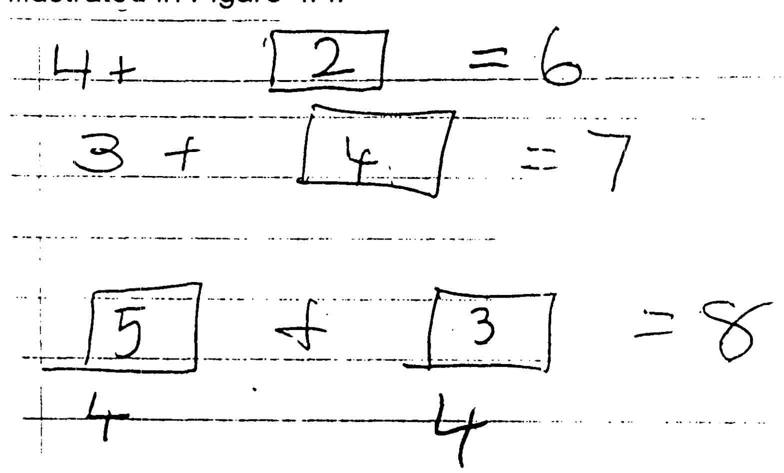


Figure 4.4: Student response to the reversibility task.

Figure 4.4 shows the response of a child exhibiting a sound grasp of addition and number facts. This child is able to correctly answer the reverse combination for all three questions and to recognise and supply a second combination for the third part of the task.

None of the children, at this point, in the study was able to answer more than two different sets of solutions for part three of the task.

Overview of Reversibility

This task was markedly more difficult for the majority of the children than the other tasks in the interviews, as only seven students were able to answer the questions. Within these seven children's attempts a large range of understanding of number sentences and reversibility was displayed. The small group who performed the best on the task exhibited ability well beyond the expectations of the Year One syllabus.

PLACE VALUE

The area of place value knowledge possessed by Year One children was investigated in this study with the use of three broad tasks. Each child was asked: (i) to read a series of two-digit numbers, (ii) answer questions adding numbers greater than ten, and (iii) to place, in ascending order, a jumbled set of numbers. These tasks were set out as follows:

A) Each child was given a piece of paper with five numbers written on it and asked to read these numbers. The numbers were:

30 35 90 14 41

B) Each child was asked verbally to answer in words the following questions:

10 + 10 is ?

10 + 5 is ?

20 + 10 is ?

10 + 13 is ?

C) Each child was provided with six numbers written on a page. The child was asked to put the numbers in order from smallest to biggest. The numbers appeared as follows:

7 2 4 10 8 12

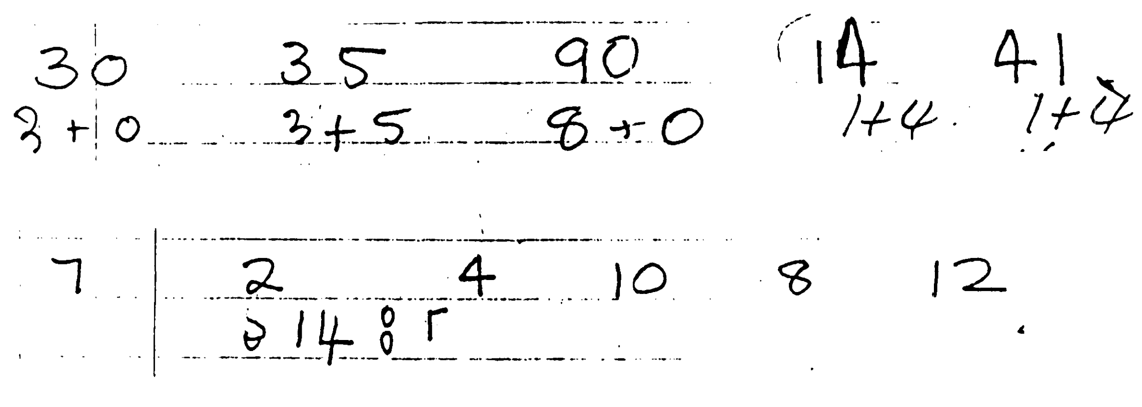
These three tasks were considered together to give an overall view of each child's understanding of place value. The responses provided fell into the three groups summarised in Table 4.11.

Table 4.11

Summary of responses to Place Value Tasks in Interview 1

RESPONSE	NUMBER OF CHILDREN
unable to read two digit numbers or add numbers over the value of ten	11
able to show some understanding of place value	7
able to read and add two digit numbers	3

During the interviews 11 out of 21 of the children showed no concept of place value when reading numbers from the page and adding numbers over the value of ten in oral questions. These children read 35 as 3 and 5 and they were also unable to order numbers correctly as shown in Figure 4.5. They also displayed a lack of grasp of the number symbol representing an amount. Number symbols had not yet taken on numerousity values such as "I think you say seven before eight" spoken by one child. The fact that eight is a larger amount than seven did not seem to be involved in this child's thinking. When attempting to reorder number they perceived 12 as 1 and 2 and showed little understanding that a number symbol represented an amount, for example they were unable to identify which was greater seven or eight.



line 1: Written numbers given to the child.
line 2: The child's verbal response to reading the numbers.
line 3: Written numbers given to the child to place in ascending order.
line 4: The child's own written reordering of numbers.

Figure 4.5: Interview notes on a child's response to (i) reading numbers and (ii) ordering numbers.

Figure 4.5 shows in line 2 that this child reads each number as a single digit number and had trouble recognising the number nine. When the same child ordered numbers as shown in Figure 4.5 line 4, the child was unable to order the numbers correctly. This child indicated little or no understanding of the numerosity of the numbers. Numbers had not yet taken on values for this child.

Seven children were able to begin to read two digit numbers correctly. They showed some indication, by saying that 14 (fourteen) and 41 (forty one) are different but could not read the number. When probed on what is the difference the children answered that the position of the digit four had changed its value. These children could order the smaller numbers correctly.

Three children had a good understanding of place value and were able to read all the numbers. These children used a variety of methods to add numbers with two digits, such as a combination of recall of basic number facts with which the child was familiar, and fingers to support their thinking. These three children used recall of number facts without their fingers in the simpler addition tasks completed earlier in the interview. However, in this less familiar situation they seemed to need to use this more basic technique of finger support, to backup their thinking.

Overview of Place Value

The responses given to the tasks concerning *Place Value* indicate that many of these children (around 50%) are only comfortable working with single digit numbers. There is some indication that an understanding of place value has begun in the other children, and once again a small group distinguished themselves as able to work with ease with all tasks. These children would not be stretched in the area of place value in Year One.

OVERVIEW

The results of the initial series of interviews, held in February of the children's first year at school in Queensland indicated that many of the children possess a good base of mathematical knowledge, not expected of them by the Queensland Primary School syllabus. This was witnessed in most pupils by several features. They exhibited a firm grasp of *more and less*, achieved success with *one-to-one association*, and displayed number understanding and calculation concepts evident in the *addition and subtraction* tasks. Furthermore, the group of children who were able to deal with complex *reversibility* tasks confirmed a strong development of basic Mathematics, as did the developing understanding of place value shown by almost half of the children in the sample. All children displayed the ability to differentiate visually between *more and less* with ease, and many of the children displayed the ability to work comfortably with numbers up to fifty. Subitizing was observed to be used frequently by a number of the children when counters were involved and, in some cases, children were able to subitize groups of numbers as large as five. Subitizing was also observed to be an important skill for the children who demonstrated most success on the tasks.

The majority of the children worked well with addition. They drew on their real-life experiences to provide answers. One child indicated understanding of addition of large numbers was easy because 'I can use the microwave. To get 20 seconds you push 10 and 10 again, to get 15 seconds you push 10 and then 5". Finally, many of the children were unable to form letters correctly to write their own name at the top of their page, however, they were able to write reasonable answers to mathematical questions.

However, despite this overall positive summation, it is not to say that all children in the sample indicated an ability beyond what the syllabus describes. In fact the range of ability in the children tested was very large. Clearly, some children operated at a level of number understanding that was well beyond that expected in Year One and perhaps only required in Year Two. They indicated understanding of place value, and a variety of number combinations that could be used to give a solution. Others had little or no experience with number and were unable to read and recognise numbers. These children were still counting all groups from one when adding new members, and they were unable to operate without concrete objects.

Clearly, on entering school, children bring with them a range of experiences from a variety of backgrounds. Some children already have a well developed informal knowledge of mathematics, while for others their background may not be so rich in mathematics. The data collected here provided an insight into this diversity, within the constraints of the small sample of students entering Year One. Nevertheless, these data do imply that it may not be appropriate to simply accept that all students will have the same mathematical skills and hence should receive the same diet of mathematics.

In summary, for the sample tested the following results were identified:

1. Nearly all children could differentiate between the concepts *more and less* with a group of items.
2. Most children appreciated the number value of a verbal number and differentiate between *more and less* with two numbers.
3. Approximately 50% of children could match items correctly in *one-to-one*

association.

4. Approximately 60% of children could add and subtract with counters successfully.
5. Most children were able to add one more without counters. At least one third of the children could add together numbers between one and ten.
6. A substantial number of children demonstrated some concept of subtraction (75%), that is the answer is smaller than the original amount. At least one quarter could subtract with values less than ten.
7. Approximately a third of the students could read numbers and mathematical symbols and respond with written answers.
8. While reversibility was identified as a difficult concept for this age group, approximately one third of the children in the study were able to attempt reversibility type questions to some degree.
9. Approximately 50% of children demonstrated signs of developing some understanding of place value.

These data indicate that many children have already begun to construct a complex network of mathematical understanding when they enter formal education. In some cases the knowledge that the children appear to have is beyond the expectation of the Year One syllabus, which implies that the syllabus may not be instep with many new Year One students. The progress that these students make over the year is addressed in Chapters Five and Six.