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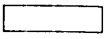
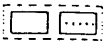
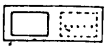
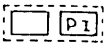
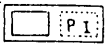
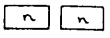
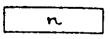
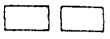
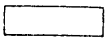
APPENDIX A
Fuson's levels (1988)

Table 8-3. Developmental Levels of Solution Procedures

Level	Cardinal conceptual operation	Conceptual units	Cardinal conceptual structure[a]	Additive (forward) solutions	Subtractive (backward) solutions
				addend + addend = [*s*]	sum − addend = [*a*]
				addend + [*a*] = sum	sum − [*a*] = addend
I	Cardinal integration	Perceptual unit items single representation addend *or* sum		Count all	Take-away *a*
				Add on up to *s*	Separate to *a*
II	Embedded integration	Perceptual unit items simultaneous representation addend *within* sum		Count on *a* with objects	Count down *a* with objects
				Count up to *s* with objects	Count down to *a* with objects
III	Embedded integration	Sequence unit items simultaneous representation addend *within* sum		Sequence count on *a*	Sequence count down *a*
				Sequence count up to *s*	Sequence count down to *a*
IV	Numerical equivalence	Cardinal numbers can be decomposed into ideal unit items[b]		Add (also thinking strategy[c])	Subtract (also thinking strategy)
				Subtract (also thinking strategy)	Subtract (also thinking strategy)
V	Base-ten integration	Base-ten numbers Perceptual ten-unit, hundred-unit, thousand-unit items		Multidigit addition	Multidigit subtraction
				Multidigit subtraction	Multidigit subtraction

Note. Brackets enclose what is unknown. PI is paired integrations: the pairing of the embedded integration of the second addend with the integration of the keeping-track method. In the cardinal conceptual structures for levels II and III, a dotted line represents the unknown set and a solid line represents a known set; the top sketch in each level is the conceptual structure for the solution procedures in the top row and the bottom sketch is the structure for the solution procedures in the bottom row.

[a] See Table 8-4 for more details of the cardinal conceptual structures.

[b] At this level a child may solve a problem using an object or a sequence procedure, but the procedure will not necessarily directly model the problem representation. Children at this level also can use derived facts and strategies such as the over-ten method.

[c] Thinking strategies operate on cardinal numbers and relate the sum of two numbers to the sum of two related numbers.

APPENDIX B
Sample Maps

More and Less Mapping- relational/transitional Ik.

CUES	CONCEPTS/PROCESS	RESPONSE
Which is more or less?	recognises more and less and attempts to identify.	recognises task
		answers for values less than ten correctly
number pairs	has a range of numbers that can be process but over twenty reverts to the unit value dominating the size	mostly correct but larger numbers wrong.
	has a grasp of number values	answers all correct

More and Less Mapping- relational 1 C.S.

CUES	CONCEPTS/PROCESS	RESPONSE
Which is more or less?	recognises more and less and attempts to identify.	recognises task
		answers for values less than ten correctly
number pairs	has a range of numbers that can be process but over twenty reverts to the unit value dominating the size	mostly correct but larger numbers wrong.
	has a grasp of number values	answers all correct

One to One Association Mapping - unistructural Ik

CUES	CONCEPTS/PROCESS	RESPONSE
row of blocks	attempts to match the blocks with no plan	incorrect
can you put the same amount of blocks out?	cannot keep track of matching to left or right.	one too many or too less
	counts to check	matches blocks
	places blocks in the front of the first group.	

One to One Association Mapping - multistructural Ik

CUES	CONCEPTS/PROCESS	RESPONSE
row of blocks	attempts to match the blocks with no plan	incorrect
can you put the same amount of blocks out?	cannot keep track of matching to left or right.	one too many or too less
	counts to check	matches blocks
	places blocks in the front of the first group.	

One to One Association Mapping - unistructural 1 C.S.

CUES	CONCEPTS/PROCESS	RESPONSE
row of blocks	attempts to match the blocks with no plan	incorrect
can you put the same amount of blocks out?	cannot keep track of matching to left or right.	one too many or too less
	counts to check	matches blocks
	places blocks in the front of the first group	

Addition with Counters Mapping - multistructural Ik

CUES	CONCEPTS/PROCESS	RESULTS
counters	counts from one	incorrect answer
	can not time finger movement and words.	
How many counters in	counts on from the first number	correct answer
each group?	uses base of knowledge	
How many altogether?	checks by recounting	

Addition with Counters Mapping - relational Ik

CUES	CONCEPTS/PROCESS	RESULTS
counters	counts from one	incorrect answer
	can not time finger movement and words.	
How many counters in	counts on from the first number	correct answer
each group?	uses base of knowledge	
How many altogether?	checks by recounting	

Addition with Counters Mapping - unistructural 1 C.S.

CUES	CONCEPTS/PROCESS	RESULTS
counters	counts from one	incorrect answer
	can not time finger movement and words.	
How many counters in	counts on from the first number	correct answer
each group?	uses base of knowledge	
How many altogether?	checks by recounting	

Addition with Counters Mapping - relational 1 C.S.

CUES	CONCEPTS/PROCESS	RESULTS
counters	counts from one	incorrect answer
	can not time finger movement and words.	
How many counters in	counts on from the first number	correct answer
each group?	uses base of knowledge	
How many altogether?	checks by recounting	

Subtraction with Counters Mapping - unistructural 1 C.S.

CUES	CONCEPTS/PROCESS	RESULTS
	• counts from one	
• counters	• subitizes first amount physically separates by counting or subitizing	• incorrect
• how many counters?		• answers some correctly
• How many do I need to take away to have "x" left	• uses eyes to separate 'x' from the first group and counts the remainder	• answers correctly
	• uses a base of knowledge.	
	• checks by recounting	

Subtraction without Counters Mapping - multistructural 1 C.S.

CUES	CONCEPTS/PROCESS	RESULTS
	• counts from one	
• counters	• subitizes first amount physically separates by counting or subitizing	• incorrect
• how many counters?		• answers some correctly
• How many do I need to take away to have "x" left?	• uses eyes to separate 'x' from the first group and counts the remainder	• answers correctly
	• uses a base of knowledge.	
	• checks by recounting	

Subtraction without Counters Mapping - relation 1 C.S.

CUES	CONCEPTS/PROCESS	RESULTS
	• counts from one	
• counters	• subitizes first amount	• incorrect
• how many counter?	physically separates by counting or subitizing	
• How many do I need to take away to have "x" left?	• uses eyes to separate x' from the first group and counts the remainder	• answers correctly
	• uses a base of knowledge	
	• checks by recounting	

Adding without Counters Mapping - unistuctural lk

CUES	CONCEPTS/PROCESS	RESULTS
'A'+'B'=?	counting knowledge of the next number is used	adds only one more.
	uses finger and counts from one	
	shows a small base of knowledge for results less than five	can only answer values less than five
	counts on with fingers fingers take on a place value e.g. means six	can only answer values less than ten
	without counting fingers and toes are used	all correct
	counts on without fingers	
	A base of knowledge is used	

Adding without Counters Mapping - multistructural Ik

CUES	CONCEPTS/PROCESS	RESULTS
'A'+'B'=?	counting knowledge of the next number is used	adds only one more.
	uses fingers and counts from one	can only answer values less than five
	shows a small base of knowledge for results less than five	
	counts on with fingers fingers take on a place value e.g. means six	can only answer values less than ten
	without counting fingers and toes are used	all correct
	counts on without fingers	
	A base of knowledge is used	

Adding without Counters Mapping - relational Ik

CUES	CONCEPTS/PROCESS	RESULTS
'A'+'B'=?	counting knowledge of the next number is used	adds only one more.
	uses fingers and counts from one	can only answer
	shows a small base of knowledge for results less than five	values less than five
	counts on with fingers fingers take on a place value e.g. means six	can only answer values less than ten
	without counting fingers and toes are used	all correct
	counts on without fingers	
	A base of knowledge is used	

Adding without Counters Mapping - relational 1 C.S.

CUES	CONCEPTS/PROCESS	RESULTS
'A'+'B'=?	counting knowledge of the next number is used	adds only one more.
	uses fingers and counts from one	can only answer
	shows a small base of knowledge for results less than five	values less than five
	counts on with fingers fingers take on a place value e.g. means six	can only answer values less than ten
	without counting fingers and toes are used	all correct
	counts on without fingers	
	a base of knowledge is used	

Subtraction without Counters Mapping - unistructural Ik

CUES	CONCEPTS/PROCESS	RESULTS
'x' take away 'y'	uses basic counting knowledge	incorrect response
What does this leave?	uses fingers and counts from one then counts the number of fingers to put down	only difference of one correct
	uses fingers to count backwards	only values less than five correct
	counts down	only values less than ten correct
	uses number facts	all correct

Subtraction without Counters Mapping - relational Ik

CUES	CONCEPTS/PROCESS	RESULTS
'x' take away 'y'	uses basic counting knowledge	incorrect response
What does this leave?	uses fingers and counts from one then counts the number of fingers to put down	only difference of one correct
	uses fingers to count backwards	only values less than five correct
	counts down	only values less than ten correct
	uses number facts	all correct

Subtraction without Counters Mapping - multistructural 1 C.S.

CUES	CONCEPTS/PROCESS	RESULTS
'x' take away 'y'	uses basic counting knowledge	incorrect response
What does this leave?	uses fingers and counts from one then counts the number of fingers to put down	only difference of one correct
	uses fingers to count backwards	only values less than five correct
	counts down	only values less than ten correct
	uses number facts	all correct

Subtraction without Counters Mapping - relational 1 C.S.

CUES	CONCEPTS/PROCESS	RESULTS
'x' take away 'y'	uses basic counting knowledge	incorrect response
What does this leave?	uses fingers and counts from one then counts the number of fingers to put down	only difference of one correct
	uses fingers to count backwards	only values less than five correct
	counts down	only values less than ten correct
	uses number facts	all correct

Written Addition Mapping - unistructural 1 C.S.

CUES	CONCEPTS/PROCESS	RESULTS
written questions	tries but can not recognise symbols	incorrect answer
pen and paper	reads numbers not symbols	
	reads the numbers and symbols	Reasonable or correct answer
can you answer these?	uses various methods of fingers and base of knowledge to solve	

Written Addition Mapping - relational 1 C.S.

CUES	CONCEPTS/PROCESS	RESULTS
written questions	tries but can not recognise symbols	incorrect answer
pen and paper	reads numbers not symbols	
	reads the numbers and symbols	Reasonable or correct answer
can you answer these?	uses various methods of fingers and base of knowledge to solve	

Written Subtraction Mapping - unistructural C.S.

CUES	CONCEPTS/PROCESS	RESULT
Written Questions	tried but unable to interpret symbols	incorrect response
Pen and paper	reads numbers not symbols	answers as a plus
	reads minus as a plus	
	reads numbers and symbols	
Can you answer these?	uses a variety of methods to work out the answer	answers correctly or minor errors

Written Subtraction Mapping - relational 1 C.S.

CUES	CONCEPTS/PROCESS	RESULT
Written Questions	tried but unable to interpret symbols	incorrect response
Pen and paper	reads numbers not symbols	answers as a plus
	reads minus as a plus	
	reads numbers and symbols	
Can you answer these?	uses a variety of methods to work out the answer	answers correctly or minor errors

Reversibility Mapping - unistructural 1 C.S.

CUES	CONCEPTS/PROCESS	RESULTS
'a'+? ='b'	looked at the numbers only	
? + ? = 8	considered numbers and symbols	formed a pattern
	added the answer and the other number	answered incorrectly
	filled in one missing part	answered only single blanks
	could group numbers to get an answer	answered all blanks
		only one solution to Q3
	could see a number of possible solutions	answered with correct multiple solutions

Reversibility Mapping - relational 1 C.S.

CUES	CONCEPTS/PROCESS	RESULTS
'a'+? ='b'	looked at the numbers only	
? + ? = 8	considered numbers and symbols	formed a pattern
	added the answer and the other number	answered incorrectly
	filled in one missing part	answered only single blanks
	could group numbers to get an answer	answered all blanks
		only one solution to Q3
	could see a number of possible solutions	answered with correct multiple solutions

Reversibility Mapping - unistructural 2 C.S.

CUES	CONCEPTS/PROCESS	RESULTS
'a'+? ='b'	looked at the numbers only	
2 + ? = 8	considered numbers and symbols	formed a pattern
	added the answer and the other number	answered incorrectly
	filled in one missing part	answered only single blanks
	could group numbers to get an answer	answered all blanks
		only one solution to Q3
	could see a number of possible solutions	answered with correct multiple solutions

Place Value Mapping - multistructural 1 C.S.

CUES	CONCEPTS/PROCESS	RESULTS
Reading numbers	sees 14 as 1 and 4 - single units	reads values less than ten
Adding numbers greater than ten	recognises that tens and units exist but unsure of what it means	reads all values
		orders correctly
ordering numbers	rote learnt recognition of ten or number facts	calculates addition
	uses fingers to answer addition	
	adds tens and ones separately	

Place Value Mapping - unistructural 2 C.S.

CUES	CONCEPTS/PROCESS	RESULTS
Reading numbers	sees 14 as 1 and 4 - single units	reads values less than ten
Adding numbers greater than ten	recognises that tens and units exist but unsure of what it means	reads all values
		orders correctly
ordering numbers	rote learnt recognition of ten or number facts	calculates addition
	uses fingers to answer addition	
	adds tens and ones separately	

APPENDIX C

Approval to carry out the study and sample letter sent to Parents

Department
of Education

Education House
30 Mary Street, Brisbane
Queensland, Australia

Refer to:
Telephone:
Our ref: 7/95

Mrs Margaret Lesley Gorrie
44 Martinique Way
Clear Island Waters Q 4226

Dear Mrs Gorrie

I refer to your application seeking approval to conduct a research study in Queensland state schools.

Approval is granted for the principals of the selected state schools to be approached with a view to securing their cooperation in the proposed research project. The attached outline of the research lists the schools selected, describes briefly the nature and requirements of the study and specifies any conditions which we wish to impose.

A memorandum to principals advising them of this approval is attached. You should provide copies of this memorandum and the attached outline to each principal approached. Although approval has been granted by this department, it should be noted that the schools are under no obligation to participate in the study.

This approval is subject to the condition that, on completion of your study, a copy of your research report be forwarded to the department. This should be accompanied by an abstract and summary of the study where appropriate.

Should you have any queries relating to this study I can be contacted on telephone: (07) 237 0770.

Yours sincerely

JAN GILLIES
Acting Director
Quality Assurance and School Review Directorate

- 9 FEB 1995

Dear

My name is Margaret Gorrie, I am a registered teacher and I am researching the development of children in the area of Mathematics in the first year of school, Year One.

I would like to inter view and observe your child as part of my research work. This would involve 3 sessions over the school year. These sessions would be conducted during school as part of your child's mathematics program.

Involvement in this study would be beneficial to your child and help improve understanding and knowledge of educators of this period of mathematics development. The principal Mr Paul Allen is supportive of this study in the school. Your child's name will remain confidential throughout the research. Please return the form below , by the 7th of February , to give permission for your child to be involved.

Yours sincerely

Margaret Gorrie

I give my permission for my child _____ to participate in this study.

Parent signature_____date_____