CHAPTER 5
The Longitudinal Study
COMPARING FEBRUARY JUNE AND NOVEMBER

In this chapter, a comparison of results for the three interviews is made in the areas of Addition and Subtraction; Reversibility; Place Value. The areas of *more and less* and *one-to-one association* were not the main focus of the longitudinal study and are only discussed with regard to their relationship to the three areas above. This chapter outlines and discusses the method of collating and coding the data collected at the interviews into levels of responses. The levels set out in this chapter developed as a result of the observations made in the February interviews; the subsequent June and November interviews provided further support and detail for these levels. A numbering system for the levels was decided upon with level 1 representing the lowest level of operation. Other relevant details observed during the study are also discussed for each task under the section, Features of the Levels. This chapter aims to discuss the patterns of performance that developed over the year, how the children developed, and any apparent trends in this development. An analysis of the poor, middle and top performers throughout the year is also presented in this chapter. The discussion that occurs in this chapter should, be considered with regard to the small size of the group involved in the study. Though it should be noted that the 21 children involved provided a wide variety of data over the period of the interviews.

ADDITION AND SUBTRACTION

Addition and Subtraction was considered under six task areas: Adding with Counters, Subtracting with Counters, Adding without Counters, Subtracting without Counters, Written Addition and Written Subtraction.

Addition With Counters.

The Year One syllabus is focused on the use of concrete objects to develop children's understanding of number and much time is spent counting, grouping, and sorting objects. The Addition with Counters task focused on this aspect of the syllabus, and involved the children adding two groups of counters.

Counters were place in front of the children to represent the following two additions:

4 + 2 = 5 + 3 =

The children were asked

1) How many counters in each group?

2) How many altogether?

The identification of these levels was possible by grouping the processes and responses received at the first interviews. The levels observed for this task, are shown in Table 5.1, which includes the number of children performing at each level for each task.

Table 5.1

Observed Levels for Addition with Counters over three Sessions

Observed Levels	**Number of students / Interview**		
	1	**2**	**3**
1.The child displayed no understanding of what was required.	1	0	0
2.The child attempts to count but mistakes are made.	7	3	1
3.The child can count the objects from one correctly.	10	13	12
4.The child can count on from the first amount to find the total.	2	2	3
5.The child answers from their own base knowledge of numbers.	1	3	5

The features of each level are discussed below:

Level 1

When the task was given the child was unable to answer or to understand what was required. They may give an answer to the question just for the sake of giving an answer but it is totally unrelated and indicates a lack of understanding about addition, such as the answer is smaller than the two parts in the question. For example, one child when considering the counters 4 + 2 answered 3.

Level 2

A typical response at this level involved the child moving the two sets of objects together and trying to count them but being unable to do so correctly. Alternatively, the child may attempt to answer by guessing but displays in the guess an understanding that the answer should be greater than either number in the task.

Level 3

At this level the correct answer is produced by the child counting all objects put together. There is a need to touch the counters in some way, this usually involves the child pushing the two groups of counters together. Some children did not do this, instead they pointed or touched the counters as they counted out loud from one.

Level 4

At this level some touching was still involved but a child displayed the ability to "count on" to get the correct answer. They could hold one number in their head while adding on the next group.

Level 5

Children performing at this level did not touch the counters and were able to subitize to assess the value of each group of counters then use their number base knowledge to get the answer. At this level no working out was audible and the children had to be asked how they got their answer. A typical response to the question: How did you get that so quick? was "everyone knows 2 and 3 make 5" or "3 and 3 are 6 so 3 and 5 are 2 more, that's 8". Here children drew on and related a number of pieces of information to reach the answer.

Table 5.1 shows that only one student at the first interview performed at level 1. At the initial interview seven students performed at level 2 while throughout the year this number declined. The largest group of the children performed at level 3 at the beginning and throughout the year. Level 4 consistently had two or three students performing at this level. Initially only one student was at level 5 but the number of students who were able to perform at this level increased to five by the third interview.

Observations made in June indicated that by this second observation six out of 21 students had increased their level of performance in addition with counters while two children decreased their level of performance and the rest of children remained stable.

By the end of Year One, 12 out of 21 children were operating at a higher level than they had done in February, seven were stable and two had declined. Two children displayed an unusual pattern of level development over the three observations. Their performance can be summarised by the levels 2-5-3 and 3-5-3. Both indicated a growth in levels of development of understanding then a decline. These children appeared to use less sophisticated strategies. This may have been due to some development, other than instruction, that occurred between February and June when addition was not discussed in class. Then between June and November the introduction of addition may have caused some confusion for these children with the knowledge they had already developed. All students functioning at level 1 (with no idea of what to do) or level 2 (trying to count but making mistakes) improved over the year. Only four of the 13 children functioning initially at level 3 and above, improved; these four children gave level 3 answers at the first interview.

The results tend to indicate a 'push' in class instruction to using the method of "counting from 1" (as 12 out of 21 students finished at level 3) and no instruction or activities which encourage the development of subitizing. School activities appear to have encouraged development of skills at the lower end of the levels.

Subtraction With Counters

The Year One syllabus does not discuss subtraction and leaves subtraction to be dealt with in Year Two. Consequently, the children in the study did not encounter subtraction in the classroom throughout the year. Although the children from the composite class may have been exposed indirectly to formal subtraction while their Year Two classmates were being taught. Subtraction with counters involved the children shown a group of counters placed in front them. Each child was asked:

1) How many are there?

2) If I want to have "x" amount how many do I need to take away?

The counters illustrated the questions

5 - ? = 3 8 - ? = 5

The levels identified for this task are shown in Table 5.2. Included this table are the number of students at each level for each of the three interviews:

Table 5.2

Observed levels for Subtraction with Counters over three sessions

Observed levels	**Number of students / interview** 1	2	3
1.The child displayed no understanding of what was required.	4	0	0
2.The child physically separates the counters making minor mistakes or recognises a difference of one.	3	3	3
3.The child uses the ability to separate the counters and achieves the correct response. Counting or subitizing is used.	13	16	16
4.The child counts backwards to answer.	1	1	1
5.The child uses number knowledge to answer correctly.	0	1	1

The features of each level are discussed below:

Level 1

At this level the child exhibited no understanding of the task and could not begin, or gave a totally unrelated answer.

Level 2

Here children assessed the total amount of counters by counting and then attempted to answer the task by moving away the counters not needed. They did not arrive at the correct answer as they seemed to become confused at this point or had some trouble counting correctly. These children were able to recognise a difference of one only.

Level 3

At this level children could either physically or visually separate the number of counters not required and then either count or subitize the remaining counters. By watching the eye movement throughout this task it was possible to see the eye movement of separation and counting, often the children moved their head with small nods as they did this.

Level 4

A child at this level was able to assess either by subitizing or counting the initial number of counters and then give the answer without returning to the counters. Counting backwards was used.

Level 5

The children were able to subitize the two groups of counters in front of them, and apply their number knowledge to work out the answer.

It was increasingly apparent throughout the interviews of the importance of "subitizing" - the ability to apprehend numerosities. Children exhibited the ability to subitize the amount of counters while working with addition and subtraction with counters. The ability to subitize appeared closely linked to the children's success and speed. Subitizing was frequently used by the children who answered questions using known number facts.

At the first interview, four children were unable to attempt the task but at interview two and three none of the children performed at level 1. Three children at each of the interviews performed at level 2. The largest proportion of the students performed at level 3 throughout the year. Initially only one child performed at the higher levels but as the year progressed two children performed at level 4 and 5.

At the first interview 14 out of 21 of the children were able to answer correctly all the tasks. After the second and third interviews the result was similar, with 18 children able to answer the task correctly and 13 children remaining stable over the year in performance. Six out of seven of the children who increased their level were originally at level 1, with no idea of what to do, or level 2 separating the objects either visually or physically, but making minor mistakes. No major increase in the level of understanding appears to have occurred throughout the year with the bulk of the children showing little or no gains for the year that they had spent at school. One child, who had shown the ability to answer subtraction from a base of knowledge, failed to do so at the third interview and another, who, at the February and June interviews, had been able to get the task correct, had difficulty in the November interview.

The results indicate that some children who began school with higher levels of understanding of subtraction, over the year at school lost some of this ability. Other students, who at the beginning of the year performed at level 1 or 2, made progress even though no tuition in class had been received. This tends to indicate a readiness at this age to work with subtraction even though the syllabus did not prescribe the teaching of subtraction to Year One.

Addition Without Counters

The Year One Syllabus places much initial emphasis on addition work with counters and only towards the end of the year does abstract addition appear. All this work is done using numbers of ten or less.

Addition without Counters was tested by asking each child to verbally address the following questions

3 + 1 is ?	4 + 2 is ?
6 + 3 is ?	10 + 3 is ?
5 + 6 is ?	

The levels identified for this task are summarised in Table 5.3.

A detailed look at the levels follows:

<u>Level 1</u>

At this level the child often responded "I can't do that" or gave a unrelated answer that indicated a lack of understanding, such as "4 and 2 make 1".

<u>Level 2</u>

Here a child displayed the ability to add on one more but could not add on any more numbers. The child may have made an attempt to answer other additions but had difficulty. Often answers given were not obviously unreasonable and may only have been out by one such as "4 and 2 make 7".

<u>Level 3</u>

At this level a child had the ability to add to a base of five and often used their fingers to achieve answers.

Level 4

Operating in level 4 indicated a child could either add comfortably to ten by ones or used a base of five. When using a base of five the children showed an understanding that consistently a full hand of fingers represented five and did not need to be counted. An example of the use of a base of five by one child was for the question 4 plus 3, first 4 fingers were raised then a further three. Instead of counting all the fingers the child recognised the answer is seven because the display of fingers had taken on a position value. All fingers on one hand and two on the other instantly meant seven. It appears important for children to do this before they can move to the next step. The child had begun to accept representation for numerical amounts.

Level 5

This is a transition level where children began to count on. This is a development of the previous stage and it does not involve counting out the second part of the addition. Hence, 4 plus 3 involved 4 fingers being shown then counting 5, 6, 7 as the next three fingers were raised. It does not involve counting out three then working out the total as done in earlier levels.

Level 6

Counting on with out fingers did occur with some children, and children were not limited to working with numbers less than ten. At this level a child could retain the memory of one lot of ten already counted on their hands and then begin to use the hands again, or use unseen toes (covered with shoes) to help work out an answer.

Level 7

Children performing at level 7 were using a number base knowledge that they had built up to answer the questions. When given the question they used no body movement but sat still and worked out the answer. If asked "how did you get that?" they usually responded with a number fact such "I know 5 plus 5 is 10 so 5 plus 6 must be 11" and so could justify their answer.

At the beginning of the year, seven children were performing at levels 1 and 2. By the end of the year none of the children were performing at these levels. The largest number of students centred at the first interview around level 3, at the second interview around level 4 and at the third interview around levels 5 and 6. This indicates that the group were generally progressing upwards through the levels.

After two observations all children (except one) who were performing at level 4 and above, either declined in level (3 out of 7) or made no improvement (3 out of 7). The one child in this group that did improve, did so marginally. Largely the Table

Table 5.3

Observed levels for Addition without Counters over three sessions

Observed levels	Number of Students / interview 1	2	3
1.The child displayed no idea or understanding of what was required.	3	2	0
2.The child can add only 1 or makes lots of mistakes with each attempt.	4	4	0
3.The child displays the ability to add up to a small base and cannot add beyond five.	7	1	3
4.The child uses fingers to facilitate addition to ten by either: *counting from one or *has a base five.	2	8	2
5.The child begins to count on.	1	3	5
6.The child begins with the combination of fingers and straight answers given. Also fingers and toes may be used to give answers over ten.	2	1	8
7.The child answers without a finger display and the child demonstrates a base of knowledge.	2	2	3

improvement seen over the whole group was by children who initially performed at the lower levels (levels 1 and 2).

By the third observation none of the children showed a decline in performance, and those who had declined in the second interview returned to their previous level. Between the second and third interview very simple addition with counters had been introduced in class. Children, who in the first interview had performed at level 3 and below, made the most significant improvement over the year while students who had performed above level 3 had made no improvement or very minor improvement (2 out of 7 students).

By the end of the year the range of performance and levels in the group had decreased and the children were now performing closer to the same level. The lower level performers had caught up to the upper level performers who had made less gains over the year.

Subtraction Without Counters

Subtraction is not dealt with specifically in the Year One Syllabus, except briefly at the end of the year where it is mentioned as a reverse operation for addition. During Year One ,children would not normally encounter subtraction with concrete support. *Subtraction without Counters* was tested by verbally asking each child the following questions

3 - 2 is ?	5 - 4 is ?	4 - 2 is ?
8 - 5 is ?	12 - 4 is ?	

The levels identified for this task, along with the number of students performing at each level for the three interviews are included in Table 5.4.

A detailed look at the levels follows:

<u>Level 1</u>

Children at this level were unable to answer the problem. They would either state that they could not proceed or give an unrelated answer just to escape the situation.

<u>Level 2</u>

At level 2, a child usually was able to attempt the question and produce an answer that was not unrealistic and appeared reasonable. Here, subtracting a difference of one was completed correctly and the child displayed an understanding of subtraction but lacked the number skills to answer questions with larger differences.

<u>Level 3</u>

A child at this level was able to use their fingers to work out the difference or could work with small values in the question less than 5 and find the correct answer from their own number experiences, an example is 4 take away 2. Many children seemed familiar with the sentence 2 and 2 make 4 and were able to use this to answer 4 take away 2.

Level 4

Children performing at level 4 could work with numbers up to ten and used their fingers to find the difference. At this level children had began to count backwards with their fingers to support this backward counting. This required more skill for numbers over ten as the child had to be able to use their fingers twice or some how keep track of the ten. Children who could not do this were categorised as 4a. as they displayed all the other skills of th s level and may have just entered this level of operation.

Level 5

At this level children used a bank of knowledge of addition and subtraction experience to work out the answer. Occasionally some would fall back into the practices of level 4 when the question was more difficult. They did this to check their calculations and then returned to level 5 reassured by the result.

Table 5.4

Observed levels for Subtraction without Counters over three sessions

Observed Level	**Number of Students / Interview** 1	2	3
1.The child displayed no idea or understanding of what was required.	8	8	3
2.The child attempts with some indication of understanding or only answers a difference of one.	7	5	3
3.The child used fingers to answer or was able to work with small familiar values.	4	1	4
4a.The child answered only values below ten correct.	1	3	2
4.The child combined use of fingers and counting backwards	0	4	6
5.The child appeared to have knowledge of number facts which is used to "work out" the answer mentally.	1	0	3

At both the first and second interviews a large number of the students (eight) performed at level 1. At the first interview only two students were able to perform at level 4 and above. Over the year, particularly between the second and third interviews, an increase in the number of students at the higher levels became obvious.

Subtraction is not covered in Year One, and initial observations indicated the difficulty of subtraction as opposed to addition for children of this age. After the June observation, a large group (9 out of 21 children) had made no movement in their level while six out of 21 children by June were moving from levels 1-3 to level 4. The remaining students showed a decrease in performance level which may have been due to lack of familiarity with subtraction in the classroom.

After the November observation 13 out of 21 students displayed an increase in their performance level of subtraction without counters. Many of these increases covered a number of levels even though no work on subtraction had been carried out in class. By the last interview 11 out of 21 children were performing at level 4 and higher, indicating a readiness, not addressed in the curriculum, to work with subtraction.

Written Addition

Written questions are not introduced until the very end of Year One or the beginning of Year Two. When written questions are introduced pictures and concrete material are used to support the written question.

This task involved each child working on their own piece of paper on a series of additions and subtractions in the following form:

3 + 2 =	5 + 1 =
4 + 3 =	5 + 4 =

The children would not have encountered this style of question in class. The levels used to trace the progress of the children in this task and the number of children at each level for the three interviews is provided in Table 5.5.

A detailed look at the levels follows:

<u>Level 1</u>

The child could not read the questions. At this level the children could not attempt the question and were unable to read with understanding the mathematical symbols and numbers. They recognised that the symbols held some sort of message about number, though they were unsure what to do.

<u>Level 2</u>

The child has begun to recognise that the number symbols represented number amounts. The child may be able to read the number with understanding of its value, but cannot read the addition sign or the equal sign. Often a child at this level would ask about the equal sign "What does this mean?". When told to do what they thought, the children sometimes chose to ignore the symbol.

Level 3

The child could read and answer the questions correctly. A child at this level recognised both the numbers and the mathematical symbols and was able to put them together as they appeared and make sense out of the message. This is a complex process that requires understanding of language and the ability to apply it to personal knowledge then convert it back tc written language. Some children were able to answer:

a) only the simple questions. (This was affected by the child's ability in addition and not mathematical reading.);

b) all questions.

Table 5.5

Observed levels for Written Addition over three sessions

Observed Levels	**Number of Students / Interviews** 1	2	3
1.The child recognises that the questions have some message but could not read the questicn.	7	1	0
2.The child begins to understand.	3	4	1
3.The child can read and answer correctly.	11	16	20

The students at the first interview were spread over the three levels observed with the majority (n=11) at level 3. At the June interview the knowledge the children had brought with them did not seem to be affected adversely by the classroom program. Over this period (February to June) more children (five) were now able to read and answer written questions. By June a total of 16 out of 21 children could read and answer written questions in this topic area which will not enter the curriculum until the beginning of the Year Two. At the third interview all but one child could read and answer correctly the written questions. The knowledge displayed by the children of written numbers was not being developed in class. Only in the second part of the year addition was to be briefly mentioned and not in written form.

The November interviews seemed to indicate that the capable students, though not extended by the school curriculum, did not lose their skills while the improvement was seen largely in those children who began the year operating at levels 1 or 2. These results raise the question: Was the year spent bringing those students who initially performed at a low level up to the level of the other students?

Written Subtraction

As previously mentioned, subtraction in any form is not dealt with in Year One. *Written Subtraction* is not expected until the middle of Year Two.

This Written Subtraction task involved each child, on their own piece of paper, attempting a series of subtractions in the following form:

4 - 2 =   5 - 3 =

7 - 1 =   9 - 3 =

The levels observed for this task and the number of students performing at each level for the three tasks is shown in Table 5.6

Table 5.6

Observed levels for Written Subtraction over three sessions

Observed Levels	**Number of Students / Interview**		
	1	**2**	**3**
1.The child can not read the question.	12	3	0
2.The child begins to understand.	1	9	11
3.The child reads the minus sign as a addition sign.	1	4	2
4.The child can read and answer the questions correctly.	7	5	8

A detailed look at the levels is provided:

<u>Level 1</u>

The child could not read the questions. At this level children could not attempt the question and were unable to read the mathematical symbols and numbers. However, they did recognise that the symbols held some sort of message about number though they were unsure which numbers.

<u>Level 2</u>

The child begins to recognise. The child may be able to read the number but cannot read the minus sign or the equal sign.

Level 3

The child reads the minus sign as a addition sign and answers correctly for this condition. The child has begun to recognise that numbers and symbols together give a message and instruction, which in this case needs to be completed, but sees all symbols between two numbers as representing that the two numbers should be added together. These children were often unsure of the equals sign and would say "I do not really know what that means" pointing to the sign, but when told to do what they thought, they proceeded to write an answer.

Level 4

The child could read and correctly answer

a) only the simple questions;

b) all questions.

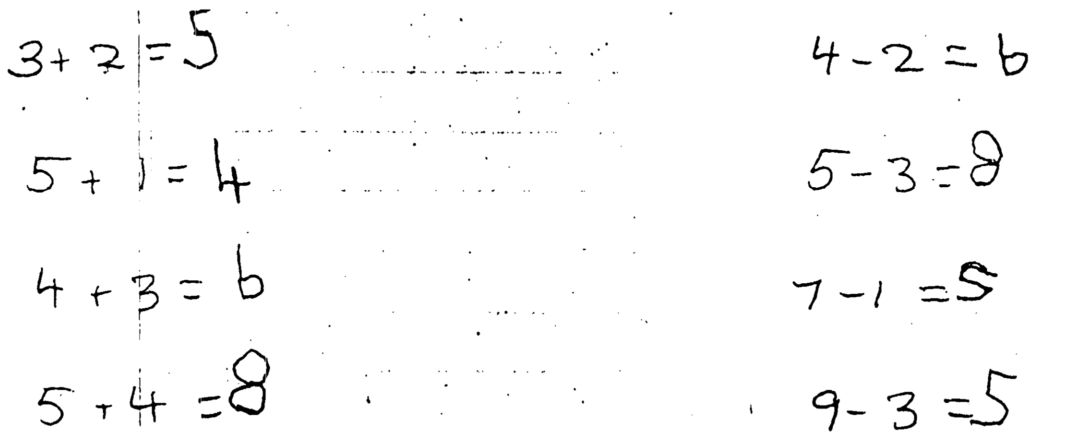
This level is the same as level 3 of the written addition tasks and involves a complex series of thoughts for the young child.

More than half of the students (12 out of 21 children) at the first interview performed at level 1. The main group of the rest of the students performed at level 4 with only a small group at levels 2 and 3. By the second interview, the students who had performed at level 1 were more evenly spread throughout the levels. The third interview showed eleven students performing at level 2 and eight students at level 4.

At the June interviews a significant increase in the performance of these students at level 1 was apparent. Five out of eight of the children, who had performed at levels 3 and 4 in February showed signs of confusion, making mistakes and decreased in their level. By November four out of seven of these students who had been able to complete the task at level 4 could not, and 11 out of 21 children could not tell the difference between addition and subtraction symbols. It is possible that classroom experiences with addition had caused this confusion. Only eight students in November could answer written subtraction questions correctly yet twelve students at the same interviews were able to answer correctly all verbal subtractions. This indicates that if the children had have been taught the use of the written minus symbol in class, more of them may have answered the written questions correctly also.

Examples of the children's written addition and subtraction are provided in Figure 5.1.

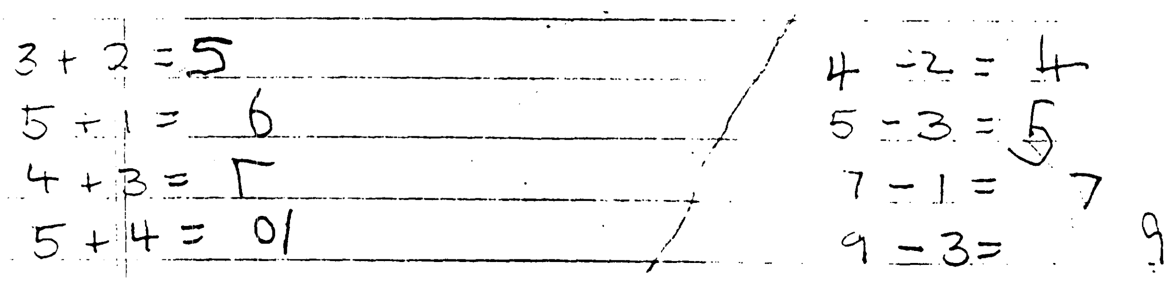
Student 5 June interview.



Written Addition level 2

Written Subtraction level 2

Student 7 June interviews



Written Addition level 3

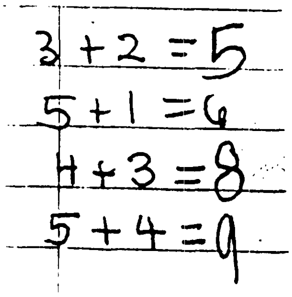
Written Subtraction level 1

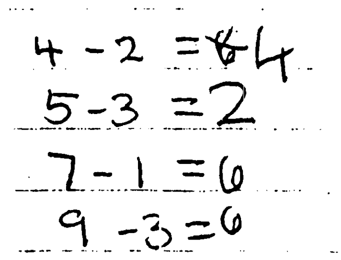
Figure 5.1: Written Addition and Subtraction Examples

The work of two students identified as student 5 and student 7 is shown here at the June interviews on the written addition and subtraction tasks. Student 5 indicates some grasp of written addition and subtraction, and may be bordering on level 3 for written subtraction, confusing the addition and subtraction symbols. Student 7, although writing numbers back to front indicates a sound grasp of written addition, but becomes confused by the subtraction symbol.

Figure 5.2 provides an example of written addition and subtraction by a student at the June and the November interviews.

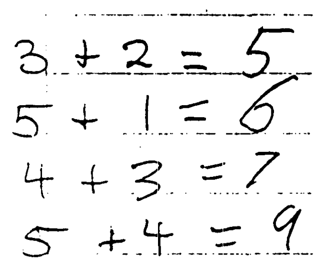
June Interview

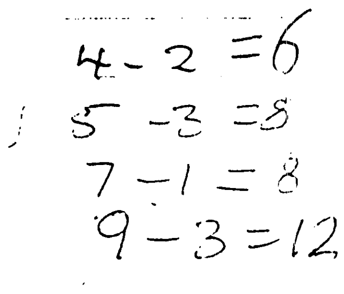




level 3 level 4

November Interviews





level 3 level 3

Figure 5.2: A comparison of one child's written addition and subtraction at the June and November Interviews.

In Figure 5.2 at the June interview, the student shows a sound grasp of written addition and subtraction. However, by November this student is confused by the subtraction symbol and answers all the written questions as additions.

The results for written subtraction indicate that the students did not benefit from a year at school in the development of this area. The improvement that was noted was concerned with higher levels of reading numbers rather than recognition of the subtraction symbol.

REVERSIBILITY

Reversibility is only considered briefly in the Year One Syllabus. This occurs at the end of the year when subtraction is talked about as the reverse of addition. Otherwise this type of task is considered too difficult. It is important to remember when considering the results of this task that addition is not introduced in a written form until the very end of Year One or the beginning of Year Two, and that reversibility appears in the primary curriculum in Year Three.

For the reversibility task each child, on their own piece of paper, was asked to complete the missing numbers to make the addition true.

4 + ? = 6

3 + ? = 7

? + ? = 8

The levels identified for Reversibility and the number of students performing at each level for the three tasks is given in Table 5.7.are:

Table 5.7

Observed levels for Reversibility over three sessions

Observed Levels	Number of Students / Interviews		
	1	**2**	**3**
1.No attempt made of the question.	12	4	3
2.The child tries to complete the blank spaces by making a pattern.	1	4	8
3.The answers indicate some understanding of the task but the numbers and signs are mixed without regard to their written order.	1	5	3
4.Children at this level were able to work with reversibility to varying degrees.	7	7	7

A detailed look at the levels follows:

Level 1

At this level either no attempt was made or the answer given was totally unrelated to the task. This level was characterised by children who stated "I can't do that" or "I don't understand". Many of these children were unable to read the task so were precluded from attempting it. These were the children who performed at level one in the written addition and subtraction tasks.

Level 2

The answer given at level 2 was an attempt to form a pattern. The children ignored the sign of = and + and looked only at the numbers. They would then offer 6 as the answer to 5 + ? = 7. Surprisingly some of these children had been successful when answering the written tasks for addition and so had at least some understanding of = and + signs but chose to ignore them in this case. Perhaps the complexity of what was required meant that the signs were too much to deal with so the children ignored them.

Level 3

Symbols and numbers were considered at this level. Children tried to take the symbols and the numbers into account but had difficulty identifying that the missing number was part of the addition. Instead these children added together the two numbers in the question without regard to the numbers and symbols position. They would offer the answer 10 to the question 3 + ? = 7.

Level 4

At this level the children were able to work with reversibility to varying degrees. Three substages were identified, namely:

A) Here children could complete the task correctly although for question 3, for which there were two blank spaces and a number of possible solutions, they could only give one solution.

B) Children at this level were able to give two or three answers to question three. They were able to recognise that a number of combinations could result in the same answer and were able to identify at least two or more of these combinations.

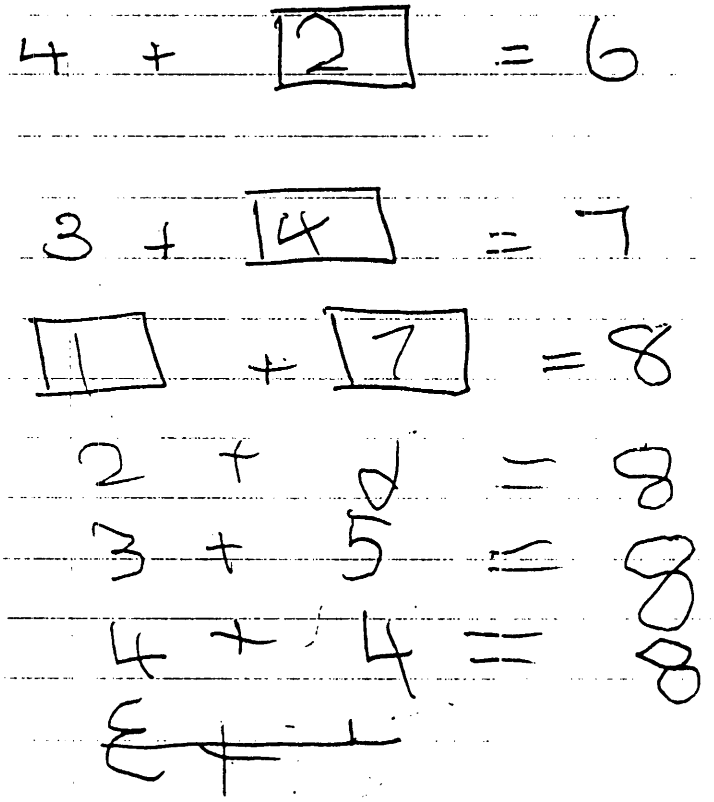
C) At this point the children were able to give four or more possible solutions to question 3. Some children were even able to say that 3 + 5 is the same as 5 + 3 and to use 0 + 8 indicating a higher level of understanding of addition principles and addition by zero.

Throughout the year, a third of the children observed was operating at level four. They were able to answer at least two out of three of the reversibility questions, although, in terms of the syllabus, they would not require this until Year Three. Figure 5.3 shows examples of these responses.

Both students shown in Figure 5.3 are able to operate confidently with reversibility. The number of other possible combinations provided by both students to answer the third question indicates a high level of understanding of addition concepts.

The results over the year may have been influenced by tuition in addition of numbers and patterns. The emphasis by a number of children, eight by the end of the year, who wanted to form patterns was most likely a result of the great amount of pattern work carried out in Year One. At the beginning of the year only one child had given a level 2 response (patterns). However, by June, four children were operating in this level. These children were willing to ignore the mathematical symbols in order to find a pattern. This is illustrated in Figure 5.4.

Student 12 November interview



Student 21 November interview

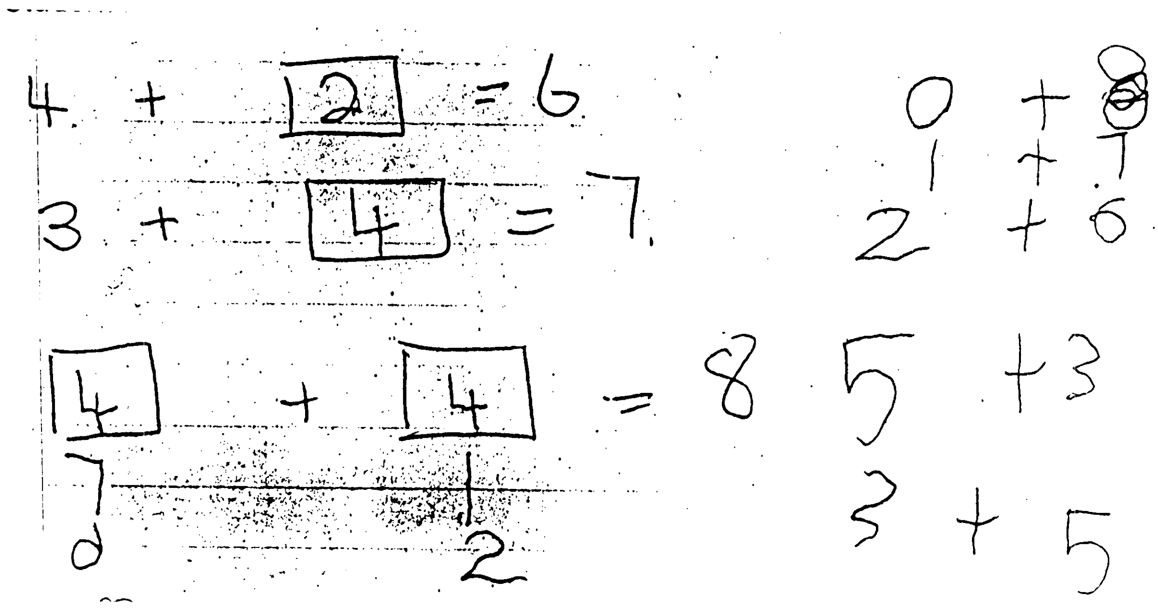


Figure 5.3: Reversibility level 4 responses

Figure 5.4 shows the work of two students identified as student 18 and student 20 at the November interviews. Student 18 has tried to complete the task by filling the blank boxes with numbers that will complete a pattern. This student has ignored the mathematical symbols. Student 20 has considered the mathematical symbols but is confused by the arrangement of the blank boxes and chooses to ignore this aspect for questions one and two, by adding the addend and the answer to find the missing number. This will not work for question 3 so student 20 chooses to leave this question.

Student 18 November interview

$$4 + \boxed{5} = 6$$

$$3 + \boxed{4} = 7$$

$$\boxed{6} + \boxed{7} = 8$$

Reversibility level 2

Student 20 November interview

$$4 + \boxed{10} = 6$$

$$3 + \boxed{10} = 7$$

$$\boxed{} + \boxed{} = 8$$

Reversibility level 3

Figure 5.4: Examples of Reversibility at the November interviews

Over the year, 11 out of 21 of the children had increased the level at which they were performing. This result is surprising considering this style of question has not been shown in the classroom and indicates a development of knowledge of mathematics occurring outside of the classroom curriculum. Throughout the year three children had shown a decline in their understanding in June then return to their previous level by November. A further three children who were unable to do this task in the first interview, performed well at the June interviews indicating understanding of the task but by November had fallen back a level and showed a keenness to produce patterns. It is difficult to know if this pattern is a general one or if it arises as a consequence of certain learning experiences by a few students.

PLACE VALUE

The development of place value is listed as an aim of the Year Two syllabus and is not mentioned as suitable objective for Year One in the Year One Source Book (1989). This is possibly due to the concentration in Year One on the numbers one to ten.

The Place Value task involved three activities to gain a picture of the children's understanding of *Place Value*.

A) The child was given a piece of paper with five numbers written on it and asked to read these numbers:

30 35 90 14 41.

B) Verbally each child was asked to answer the following:

10 + 10 is ?

10 + 5 is ?

20 + 10 is ?

10 + 13 is ?

C) Each child had six numbers written on a page. The child was asked to put the numbers in order from smallest to biggest The numbers appeared as follows:

7 2 4 10 8 12.

To facilitate the analysis a separate set of levels was identified for each activity then blended to produce an overall set of levels for the task. Each of the tasks own levels are discussed below then the combined levels for the whole Place Value Task are presented.

Reading Two Digit Numbers

Level 1. No attempt was made to read the numbers.

Level 2. The child could read the numbers but only single units e.g., 14 is read as 1 and 4.

Level 3. The child began to read tens and units in numbers but has difficulty differentiating 14 and 41.

Level 4. The child could read all numbers correctly. Here the responses showed that both the number value and its position were being considered.

Adding Numbers Greater than 10

Level 1. The child could not answer the question at all and may have said "I can't do that".

Level 2. The child displayed rote learnt knowledge such as "10 plus 10 that's 20", but was unable to work out answers to unfamiliar combinations.

Level 3. The child was able to use fingers and toes to work out answers and retained the value of one ten in the memory while doing so. This displayed an ability to retain one fact while working on another.

Level 4. The child was able to add the tens and the ones separately in the head by using acquired knowledge and an understanding of the place value of the number. This method of response involved co-ordinating previous knowledge to apply to a new situation.

Ordering Numbers

Level 1. The child either rewrote the numbers in the same order or could not attempt the task.

Level 2. The child often wrote all the counting numbers down or ordered the numbers but saw numbers such as 12 as 1 and 2 separately. This responses displayed a single unit approach and place value was irrelevant.

Level 3. The child could correctly order the numbers. This type of response considered both the number value and its position.

The three questions used for place value gave an overall position of the level of thinking of each child and were therefore considered together to assess the child's level. This was done because as the year progressed it became clear that the levels within each of the tasks were related. For example, a child operating at the overall level 1 in place value exhibited level two in reading two digit numbers (reading only single units such as 14 is read as 1 and 4); level two in adding numbers greater than ten (unable to work out addition unless rote learnt); and level two in ordering numbers (could only order single digit numbers). As a result, an overall set of levels was developed. The overall levels identified for this task and the number of students performing at each level for the three tasks are featured in Table 5.8.

Table 5.8
Observed levels for Place Value over three sessions

Observed Levels	Number of Students / Interview 1	2	3
1. The child can not order numbers and is unable to read two digit numbers.	11	5	2
2. The child begins to read tens and units in numbers but is easily confused and need to use fingers and toes to add numbers greater than ten. The child can also order numbers correctly.	8	6	8
3. The child can read two digit numbers consistently and add numbers greater than ten using knowledge of place value without concrete support.	2	10	11

Over the three interviews 16 out of 21 of the students displayed an increased understanding of place value. By November, eleven students were displaying a good understanding of place value, performing at the top levels. This apparent improvement may have been a result of maturation rather than instruction as the children only dealt with written numbers up to ten in class. None of the children declined in their initial level over the year, and they were clearly not adversely affected by what had gone on in the classroom. The extensive work in the classroom with concrete objects and grouping may have helped this progress indirectly. Five children remained stable in their understanding. Two of these remained at the lowest level, showing no understanding of place value at all. Three children who had performed at the top levels in the June interviews did not do so in the last interviews in November. Two of these children finished at a higher level than they had been at in February and one returned to same level as February. (The patterns of levels for theses children were 1-3-2; 1-3-2 and 4-3-4). These children appeared to have lost some of their understanding of place value.

The results indicate that by the end of Year One many children have or are developing a sound grasp of place value. These children have begun to read tens and ones in numbers and order two digit numbers correctly. The increased number of children performing at level 3 by the November interviews indicates a readiness in these children to work with numbers greater than ten.

TRENDS OVER THE YEAR

The number of children whose level of performance declined, remained stable (level of performance the same in November as in February) or improved throughout the year was monitored to assess an overall trend in any direction. The following results in Table 5.9 show the number of children that either declined, remained stable or improved from February to November for each task.

Table 5.9: Movement of children in levels from February to November

TASK	NO. DECLINED	NO. STABLE	NO.IMPROVED
addition with counters	2	7	12
subtraction with counters	1	13	7
add without counters	1	6	14
sub without counters	2	7	12
written addition	1	9	11
written subtraction	4	2	15
reversibility	1	4	16
place value	0	5	16

Throughout the year there is a obvious overall improvement trend in the group. Subtraction with counters indicated the task with least improvement with 13 children remaining stable in their performance throughout the year. Subtraction appeared to suffer the smallest progression of all the tasks over the year with written subtraction resulting in the largest number of decline in performances over the year. In this latter case four children who had, at the February interview, been able to perform at the highest level and complete the task successfully could not do so in November. This result could not be accounted for by the lack of written tuition in the Year One Syllabus as in the case of written addition only one child declined and a large group of nine students remained stable over the year.

In the area of the six addition and subtraction tasks, while on average more than 50% of the students consistently improved, much of the improvement was minor. A stronger trend of improvement was seen in the areas of Reversibility and Place Value, although these were not dealt with in the syllabus. This may have been a result of maturation or indirect number work done in class. The students who did not improve in these areas were already performing at the top levels.

A number of children in each task did not make any improvement but either remained stable in their performance or declined. The children in this group who did not improve were the same one or two children who did not improve from level 1 for many of the tasks or the 'top performers' in the group who maintained their initial knowledge base throughout the year.

POOR, MIDDLE AND TOP PERFORMERS

Throughout this study three main groups emerged. These could be classified as the poor, middle and top performers. The poor performers were those students (n=2) who were noticeably performing consistently at the lower levels in each task. The middle performers (n=16), which involved the bulk of the children, performed consistently around the middle levels. The top performers (n=3) began the year at the upper levels for each task and maintained this throughout the year.

The children who stood out as poor performers began the year at the February interviews at the lower end of the levels for each task and failed over the year to make progress while others moved up levels. Two children, in particular, had patterns of levels for *Subtraction with counters* of level 1 for each interview. These same two children remained at level 1 for each interview on the *Place Value* task and *Subtraction without Counters* task also. In *Place Value* they showed an improvement of level of performance in June but by November their responses had returned to level 1. In other tasks minor improvement was made over the year to the next level, but these two children failed to mprove their level of performance as much as other students who had begun the year at the same level.

Most of the children in this study fell into the group labelled middle performers. These children began the year either at the lower end of the levels or in the middle of the levels. Throughout the year they made noticeable progress in most task areas improving at times two or more levels over the year. This group began the year spread out in their apparent understanding of Mathematics but by the end of the year were even more homogeneous and appeared to have caught up to the top performers.

The top performers stood out as displaying a potential to undertake more advanced Mathematics than was on offer in the Syllabus. These children, were consistently performing in the top levels in each of the tasks. Over the year their progress appeared less obvious as more students were able to perform at the higher levels for each task. The tasks may have produced a ceiling effect.

At the beginning of the study for the purpose of collation each child had been given a number. These numbers are referred to in this section when discussing progress through the year. Table 5.10 below lists the students (numbered) performing at the top levels for each task (level in brackets) at each of the three interviews. An example of the type of responses received from a top performers is shown in Figure 5.3 (provided by student coded 21). This response shows that the child has an advanced grasp of *Reversibility* and an understanding of the addition of zero.

Table 5.10: Students Performing at the Top Levels

task	Feb.	June	Nov.
add with counters (level 4/5)	19, 20, 13	8, 13 19, 21	3, 8, 10, 13, 14, 15 18, 19
subtract with counters (level 4/5)	21	8, 21	6, 19
add without counters (level 5/6)	3, 11, 12, 19, 21	3, 6, 19, 20, 21	1, 3, 6, 7, 8, 9, 11, 12 14,15,16,18, 19,20,21
subtract without counters (level 4/5)	19, 21	6, 8, 14, 19, 20	3, 6, 8, 9, 10, 11, 14, 18, 19, 20, 21

task	Feb.	June	Nov.
written addition (level 3)	3, 6, 8, 9, 10, 11, 12, 13, 16, 19, 20, 21	1, 3, 6, 7, 7, 8, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21	all students except student 2
written subtraction (level 4)	3, 8, 10, 11, 19, 21	3, 8, 10, 11, 19	3, 7, 8, 9 10, 21, 21
reversibility (level 4)	2, 3, 8, 12, 13, 19, 21	1, 3, 6, 8, 12, 19, 21	2, 3, 11, 12, 13, 19, 21
place value	3, 19, 21	3, 6, 8, 11, 14, 15, 16, 18, 19, 21	2, 3, 6, 8, 10, 11, 12, 14, 19, 20, 21

From the February interviews, two students, in particular performed consistently at a high level on the eight tasks (addition with counters, subtraction with counters, addition without counters, subtraction without counters, written addition, written subtraction, reversibility and place value). Students 19 and 21 performed in the top levels for seven out of eight of these tasks with student 19 missing only subtraction with counters while student 21 missed addition with counters. The next most successful student was student 3 who performed at high levels for 5 out of 8 of the tasks missing only addition and subtraction with counters and subtraction without counters.

As the year progressed the size of the group of students performing at the top levels grew. By November many of the middle performers had appeared to catch the top performers in the tasks. Initially, this group was dominated by males but by November more girls were represented in this group though the proportion was still not representative of the proportion of females in the study. Over the year, more girls than boys, proportional to the whole group, had improved to join the higher levels. This tends to indicate that the females may have been getting more from the school environment than the males, but further investigation would be necessary to establish this.

CONCLUSION

This study followed the progress of twenty-one students over their first year of school in three main areas namely addition and subtraction, reversibility and place value. The same tasks were given to each child at three interview points throughout the year. After the February observations of each of the twenty-one children involved in this study it was possible to distinguish levels of operating in each task which could be used to trace the longitudinal development of the children over the year. These levels, developed after the first interviews in February, were further enhanced by the interviews in June and November which gave a greater detail to each level and helped to support and expand the levels proposed after the February interviews. The levels were coded using numbers 1, 2..., where level 1 represented the lowest operating level in the task. Each level was identified in this study because it was seen to be a significant step of progress from the previous level towards understanding the concept and task.

The results of the study indicate that throughout the year most children either improved as a result of their first year at school or did not lose much of the informal knowledge that they had acquired before entering school. The fact that the number of students performing at the top levels increased substantially over the year is a good indicator of this.

Surprisingly, improvement in performance by many children occurred in areas that were not dealt with in the classroom, such as reversibility, written addition and place value. Throughout the study the children exhibited an increased ability to read and understand the value of numbers, and the place value of tens and units. This growth in performance may have been due to personal development of the children through maturation though undoubtedly the rich environment of number stimulus in the classroom would also have indirectly influenced growth in number understanding. The patterns of development for children were not always marked by consistent improvement. Some children's levels showed a different path by declining in levels of performance in June then returning to higher levels in November. This occurred in tasks associated with addition with counters, and reversibility. The written addition and subtraction tasks also showed more improvement between February and June which then dropped off and in some cases regressed by November.

Formal school mathematics in the first year appeared to affect a number of areas in the children's responses. Subtraction, in particular did not improve as much as the other tasks, and more decline in the level of responses was obvious with the subtraction tasks. The written addition and subtraction tasks provided information showing that some children who in February and June were able to read and work with addition and subtraction symbols had trouble differentiating between the two by November. The fact that the addition symbol was introduced towards the end of the year and that subtraction was not talked about seems to have confused some children. The initial readiness exhibited by many of the children throughout the year raises the question: Why is the subtraction symbol not being taught? Another area of interest was concerned with the reversibility task. At the first interview only one child tended to lean towards filling the blank space to make a pattern, however, by the end of the year eight children were clearly trying to produce a pattern and chose to ignore the mathematical symbols. This emphasis on patterns appeared to be a by-product of the first year at school.

Clearly, over the year the middle and lower performers at the February interviews were the students who by November had benefited the most for one year at school. For some children, the benefit of informal Mathematics that they had brought with them to school had been lost by the end of the year as it appears that the year had been spent bring other children up to their level. The students at the extremes of the levels of understanding had by November become clearly identifiable with two students who began with level one performances in most tasks making only minor progress over the year and, in some tasks, such as place value, written subtraction and reversibility making no progress at all. The two top performers were also clear as they had begun the year performing at high levels and continued to do so throughout the year without regressing. By November these two students were bunched with many other children. These results must be interpreted within the context of ceiling effects. For 90% of the students, their performance levels were by November very close to one another, contrary to the range that had been evident at the beginning of the year, and only the two weaker students were now markedly apart from the group. One year at school had brought the ability to perform the tasks in this study for most of the group closer.

CHAPTER 6
ANALYSIS

In Chapter 5 the results of the interviews were discussed and the responses identified into levels. This Chapter considers the levels found in this study in light of the SOLO Taxonomy. This chapter deals with each task separately and then in the final section considers the longitudinal SOLO implications. Initially, each task is considered individually where SOLO is applied with the use of a mapping technique. The mapping technique was adapted from Watson and Mulligan (1990) to represent cues, concepts and processes used in the task and relates directly to the SOLO levels described for each task. The appendix includes maps for each exit point for each level of the individual tasks. For some tasks the levels found in this study corresponded to those identified by other researchers such as Carpenter and Moser (1983), and Steffe and Cobb (1988) described in Chapter One. When this occurs, for a task, a brief discussion of the relationship is given. At the end of each task a summary table is provided comparing the levels found in this study with SOLO levels and other researchers levels where applicable. At times, some levels have been identified as operating in the first learning cycle of the concrete symbolic mode and this is indicated by the use of a 1 after the level.

The chapter is divided into six sections. The first five relate to the tasks: More and Less, One-to-One Association, Addition and Subtraction, Reversibility, and Place Value. The final section of the chapter discusses overall changes and progression throughout the school year in terms of SOLO under the heading of Longitudinal SOLO Implications.

MORE AND LESS

The task area of *More and Less* was considered by the children viewing pairs of flashcards with dots on the cards and indicating which card had more or less. Also, the children were read pairs of numbers and were asked to decide which number was more or less out of the pair provided. All students involved in the study showed the ability to recognise more and less from flashcards, though some children distinguished themselves from the group by using counting skills when the difference was minimal and hard to recognise. The influence of the end number (the unit value) in two digit numbers for the child when making a decision of more or less emerged as an important unexpected factor, particularly in the later interviews throughout this task. Three levels of thinking were identified when children were asked to decide between the number which was more or less in pairs of numbers.

These three observed levels are shown in Table 6.1, which is based on Table 4.2.

Table 6 1

Observed levels for More and Less

Observed Levels
1.A) The child could only answer correctly with very small numbers under 5. B) The child could only answer correctly with numbers less than 10.
2. The child gave correct answers for all except those over 20 or 30 and seemed to favour the units in the number to make a decision i.e. 29 is greater than 34 because the 9 is bigger than the 4.
3. The child completed the task correctly.

Applying SOLO

The tasks for *More and Less* and the responses received fitted into the SOLO modes of ikonic and concrete symbolic. The three observed levels of responses were identified as the SOLO levels in this stem as corresponding to level 1 operating in the ikonic mode with level 1B) transitional, level 2 unistructural and multistructural in the first cycle of the concrete symbolic mode, and level 3 relational in the first cycle of the concrete symbolic mode.

Level 1 was identified as ikonic as the children at this level were able to work with units only, though they could compare their value. Level 1B) is transitional as at this level children are incorporating more values to their knowledge of number understanding. Level 2 was identified as unistructural 1 and multistructural 1 as the children at this level were beginning to work with tens and units but at times became confused. Finally, level 3 was identified as relational 1 in the concrete symbolic mode because children at this level appeared to have mastered the number system with two digits numbers and were comfortable with the numerosity of two digit numbers. The use of a mapping procedure adapted from Watson and Mulligan (1990) helped to relate the levels identified in this study to the SOLO levels. Figure 6.1 is a map outline displaying cues, concepts and processes and possible responses for a multistructural response.

The map shown in Figure 6.1 below lists the key concepts/process and response for this task. It is the concepts which primarily determine a child's level. The three main concepts are first the recognition of more and less and an attempt to identify which is more and which is less. This was usually done by visual appearance of a group and only with numbers less than ten. Second, the child has a range of numbers that can be answered but with values over twenty the child reverts to the unit value dominating their perceived size of the whole number. Here the concept of more and less has developed but the numerousity of the numbers is only understood for a limited group of familiar numbers. Finally, child has a base of number values knowledge that can be adjusted by the child to work out the numerousity of a number and if it is larger or smaller than another number.

CUES	CONCEPTS/PROCESS	RESPONSE
Which is more or less?	recognises more and less and attempts to identify.	recognises task
		answers for values less than ten correctly
number pairs	has a range of numbers that can be answered but over twenty reverts to the unit value dominating the size	mostly correct but larger numbers wrong.
	has a grasp of number values	answers all correct

Figure 6.1: More and Less Mapping.

Figure 6.2 is completed as a map outline displaying cues, concepts and processes and possible responses for a multistructural 1 response. A map for each SOLO level is included in appendix B.

Table 6.2 shows the observed levels and matched SOLO levels.

Table 6.2

A summary of levels for More and Less

Observed Levels	SOLO
1.A) The child could only answer correctly with very small numbers under 5.	relational (lk)
B) The child could only answer correctly with numbers less than 10.	transitional
2. The child gave correct answers for all except those over 20 or 30 and seemed to favour the units in the number to make a decision i.e. 29 is greater than 34 because the 9 is bigger than the 4.	unistructural / multistructural 1 (C.S)
3. The child completed the task correctly	relational 1 (C.S)

ONE-TO-ONE ASSOCIATION

The one-to-one association task involved the children matching one group of blocks to a set of blocks set-up in a straight line by the interviewer. This task was based on the work of Piaget (1953) which set out the importance of one-to-one association. This area is strongly emphasised in the Queensland Primary School Syllabus. The children involved in this study were observed to be operating at levels which paralleled SOLO levels see Table 6.3 (based on Table 4.3), with the support of the mapping procedure outlined below in Figure 6.2.

Table 6 3

Levels for one-to-one association

Observed Levels
1. The child attempts the task but has no plan.
2. The child is not be able to correctly complete the task because they matched one too many or too few.
3. The child was able to correct error after some time.
4. The child is able to complete the task with ease.

Some children recognised that they had matched incorrectly and took a number of attempts to complete this task. The way the children began the task may have to some degree influenced the rate of success. Those children who piled the new blocks one on top of each original block always achieved success while the children who put the new block beside the original block may have begun initially adding to the right side of the block but by the end also had blocks on the left side and consequently ended up with one more block than required.

Applying SOLO to One-to-One Association

The responses for this task are level 1, 2 and 3 in the ikonic mode and level 4 as a unistructural level in the first cycle of the concrete symbolic mode. The observed level 1 was identified as the SOLO level of unistructural due to the inability of children at this level to keep track of the blocks or to plan their attempt at matching the blocks. Level 2 was identified as multistructural as children at this level were able to add new blocks while also being aware of how they began the task for example, adding to the left only. Level 3 for this task fits the SOLO relational level in the ikonic mode as visual appearance is important but counting may be used to check the result. Level 4 is the unistructural level in the first cycle in the concrete symbolic mode as the students have demonstrated consistent conservation of the number of blocks.

The application of SOLO levels to the result was done with the help of relating the concepts, processes and responses as shown in the map outline set out in Figure 6.2. Four key concepts and processes were identified that the children used to reach one of the three final responses. The concepts outlined in the Map match up with the features of the observed levels. The first concept matches level 1, the second concept matches level 2 and the third and fourth concepts are features of level 3. Only concepts three and four can lead to a correct response.

CUES	CONCEPTS/PROCESS	RESPONSE
row of blocks	• attempts to match the blocks with no plan	• incorrect
can you put the same amount of blocks out?	• cannot keep track of matching to left or right.	• one too many or too less
	• counts to check	• matches blocks
	• places blocks in the front of the first group.	

Figure 6.2: One-to-One Association Mapping

Figure 6.2 has been completed to show a relational ikonic response map. Other levels of response maps are in appendix B Table 6.4 summarises the observed levels and relevant SOLO levels for one-to one association.

Table 6.4

A summary of levels for one-to-one association

Observed Levels	**SOLO**
1. The child attempts the task but has no plan.	unistructural (Ik)
2. The child is not be able to correctly complete the task because they matched one too many or too few.	multistructural (Ik)
3. The child was able to correct error after some time.	relational (Ik)
4. The child is able to complete the task with ease.	unistructural 1 (C.S)

ADDITION AND SUBTRACTION

Six different addition and subtraction tasks were posed throughout the study. The tasks were: Adding with counters, Subtracting with Counters, Adding without Counters, Subtracting without Counters, Written Addition and Written Subtraction. These were each considered separately to find if the responses to the tasks could be identified into SOLO levels. These levels are, where suitable, considered in the light of the current research discussed in Chapter One, particularly, research conducted by Carpenter and Moser (1983) and Steffe and Cobb (1988).

Levels of Adding with Counters.

Adding with counters involved observing the children answer addition questions with the help of counters placed to represent numbers. During this study five levels of development were observed in this task and identified in terms of SOLO in the ikonic and concrete symbolic modes. These levels were compared to those described by Carpenter and Moser (1983). The levels found in this study are summarised in the Table 6.5 which is based on Table 5.1.

Table 6 5

Observed levels for Addition with Counters

Observed levels
1.The child displays no understanding of what was required.
2.The child attempts to count but mistakes are made.
3.The child can a) count the objects from one correctly. b)subitize the two groups then counts from one to answer.
4.The child can count on from the first amount to find the total.
5.The children answers from his/her own knowledge base of numbers.

Applying SOLO to Addition with Counters

The SOLO Taxonomy corresponded well to the observed levels. The observed levels were in the SOLO ikonic and concrete symbolic modes. Level 1 observed was not operating in the stem of addition with counters and not classified in SOLO. The observed level 2 involved children focusing on the single process of counting objects and putting the two groups together which is multistructural. Level 3 has mastered combining the two groups to get one answer so is relational in the ikonic mode. Children who subitized the two groups of counters but still counted from one to get the answer were unistructural in the first cycle of the concrete symbolic mode. At the observed level 4, children were able to retain the first amount and count on to get the total more quickly. These children were using two or more methods and pieces of information to achieve their answer and so operating multistructurally in the first cycle of the concrete symbolic mode. The observed level 5 involved children using a number knowledge base that they already had and applying it to this situation. These children were relating their knowledge and using it to answer new questions. Observed level 5 equates to the SOLO relational level in the first cycle of the concrete symbolic mode. The application of mapping to the observed levels shows the path of the level and the concepts and processes for each of the observed levels. Figure 6.3 is an Addition with Counters map.

CUES	CONCEPTS/PROCESS	RESULTS
• counters	• counts from one	• incorrect answer
	• can not time finger movement and words.	
• How many counters in each group? How many altogether?	• counts on from the first number uses base of knowledge	• correct answer
	• checks by recounting	

Figure 6.3: Addition with Counters Mapping

In the Map for Addition with Counters four concepts/process were identified that lead to either an incorrect or correct response. The concepts reflect the child's level. Counting from one involves the child counting all the counters every time and beginning at one. The child tries to match a verbal number to a counter and sometimes does not achieve this as they cannot keep their fingers and words synchronised. At the concept where the child begins to count on from the first number, multistructural 1 thinking occurs.

Figure 6.3 is completed as a Addition with Counters Map for a child operating at observed level 4 and SOLO multistructural 1 in the concrete symbolic mode. Other maps for responses are in appendix B.

Comparison with other studies

The levels found in this study were then considered in regard to the three basic levels of addition identified by Carpenter and Moser (1983) and discussed in Chapter One. These levels were

LEVEL 1. Using fingers and concrete objects to count all objects.

LEVEL 2. Use of counting sequences.

LEVEL 3. Recalling number facts.

The five observed levels related to the levels of Carpenter and Moser with the observed level 1 not operating in Carpenter and Moser's levels. The observed levels 2 and 3 were both consistent with Carpenter and Moser's level 1 as concrete objects were involved. The observed level 4 involved the use of counting sequences and corresponded to Carpenter and Moser's level 2. Finally level 5 observed used a number knowledge base or recall of number facts which is Carpenter and Moser's level 3. A summary of the observed levels and their relationship to the three levels of Carpenter and Moser and the SOLO levels is given in Table 6.6.

Table 6.6

A summary of levels for Addition with Counters

Observed Levels	Carpenter & Moser	SOLO
1.The child displayed no understanding of what was required.		not operating in this stem.
2.The child attempts to count but mistakes are made.	1	multistructural(Ik)
3.The child counts a)the objects from one correctly.	1	relational(Ik)
b)subitize the two groups then counts from one to answer.		unistructural 1 (C.S)
4.The child counts on from the first amount to find the total.	2	multistructural 1 (C.S)
5.The child answers using a number fact.	3	relational 1 (C.S)

Levels of Subtraction with Counters.

In this task the children were shown a number of counters and asked how many they would need to take away to end up with 'x' amount. Observations made throughout the study identified four levels which were related to the SOLO levels in the ikonic mode. They were also considered in light of the work of Carpenter and Moser (1983) on strategies of subtraction as discussed in Chapter One. The four levels observed are summarised in Table 6.7. which is equivalent to Table 5.2.

Table 6.7

Observed Levels for Subtraction with Counters

Observed levels
1.The child displayed no understanding of what was required.
2.The child physically separates, making minor mistakes or recognises a difference of one.
3.The child uses the ability to separate the counters and achieves the correct response a) by counting b) by subitizing.
4.The child counts backwards to answer.
5.The child answers using a number fact

Applying SOLO to Subtraction with Counters

The observed levels in this stem also match well to the SOLO levels. The observed level 1 was not operating in this stem. Level 2 involved only simple actions - counting out each amount, physically separating, and then counting out what was left - and fitted the SOLO multistructural level. The observed level 3 involved the ability to subitize amounts and count new groups of counters in the head while remembering what amount needs to remain. Children at this level were operating at a relational level in the ikonic mode, as visual appearance is important when subitizing, while also unistructurally in the concrete symbolic mode as they needed to remember some details while doing another action when counting on. The children who used the subitizing with the counting at this level appeared to be operating multistructurally in the first concrete symbolic learning cycle. Level 4 was relational in the first cycle of the concrete symbolic mode as at this level children were able to apply acquired knowledge to a new situation, and, in some cases, modify that knowledge to help to solve the problem. Figure 6.4 shows a map for subtraction with counters.

CUES	CONCEPTS/PROCESS	RESULTS
	• counts from one	
• counters	• subitizes first amount	• incorrect
• how many counter?	• physically separates by counting or subitizing	• answers some correctly
• how many do I need to take away to have 'x' left?	• uses eyes to separate 'x' from the first group and counts the remainder	• answers correctly
	• uses a base of knowledge.	
	• checks by recounting	

Figure 6.4: Subtraction with Counters Mapping

The map outline for this task identifies 5 concepts/processes that lead to either a correct or incorrect result. The first concept involves always counting from one to get the answer. The child may use subitizing to assess smaller amounts. In the next two concepts a child uses physical or visual separation to work out the answer, and counts or subitizes the remaining amount. The fourth concept involves a child using his/her own base of knowledge about numbers and addition and subtraction facts to answer the problem mentally. Lastly, some children use the process of checking their answer by recounting. This is a relational 1 process.

Figure 6.4 is completed as map for a multistructural ikonic mode response and observed level 2 exit. Other maps of level 3 and level 4 are included in appendix B.

Comparison with other studies

This task resulted in data that correlated directly with Carpenter and Moser's (1983) three levels of subtraction. Level 1 identified in this study was failure to operate in the task. Level 2, 3 and 4 matched respectively with Carpenter and Moser's levels 1, 2 and 3. Carpenter and Moser's levels for subtraction were the same as their levels for addition:

LEVEL 1. Using fingers and concrete objects to count all objects.

LEVEL 2. Use of counting sequences.

LEVEL 3. Recalling number facts.

The observed level 2 and 3 in this study involved children physically separating objects as described by Carpenter and Moser's level 1. Level 4 observed in this study matched Carpenter and Moser's level 2 as it involved counting sequences. Finally, observed level 5, as with Carpenter and Moser's level 3 involved children who displayed the ability to use a base of number facts to answer the question.

A summary of the observed levels and their relationship to the three levels of Carpenter and Moser and the SOLO levels is given in Table 6.8.

Levels of Adding Without Counters

Adding Without Counters involved children answering simple addition questions delivered verbally. From analysis of the data, initially six levels were identified in the children's responses. These levels continued to be refined throughout the year of observations and were found to relate well to SOLO levels, indicating a learning cycle within the addition stem of the ikonic and concrete symbolic modes. The levels were also found to relate well to Steffe and Cobb's (1988) five counting types. The levels observed are described in Table 6.9 which is based on Table 5.3.

Table 6 8

A summary of levels for Subtraction with Counters

Observed levels	**Carpenter & Moser.**	**SOLO**
1.The child displayed no understanding of what was required.		not operating in this stem.
2.A child can physically separate making minor mistakes or recognises a difference of one.	1	multistructural(Ik)
3.The child uses the ability to separate the counters and achieves the correct response. a) by counting b) by subitizing		unistructural 1 (C.S)
4.The child counts backwards from the first amount.	2	multistructural 1 (C.S)
5.The child answers using a number fact.	3	relational 1 (C.S)

Table 6.9

Observed levels for Addition without Counters

Observed levels
1. The child displayed no idea or understanding of what was required.
2. The child can add on 1 only or makes lots of mistakes with each attempt.
3. The child displays the ability to add up to a small base and cannot add beyond five.
4. The child uses fingers to facilitate addition to ten by either * counting from one or * uses a base five.
5. The child begins to count on.
6. The child begins with the combination of fingers and straight answers given. Also fingers and toes may be used to give answers over ten.
7. The child provides answers without a finger display and the child demonstrates number facts knowledge.

Applying SOLO to Adding without Counters

In terms of SOLO, the observed level 1 was not operating in the stem of addition without counters. Level 2 involved children only able to focus on one set of counting out of numbers at a time and addition of one more only. This level was identified as unistructural in the ikonic mode as it featured only very simple addition. Children at level 3 were still in the ikonic mode using their fingers but were operating multistructurally as they were able to work with a number of addition combinations to the value of five. The observed level 4 indicated a transition to the concrete symbolic mode where children have refined their use of finger assisted counting and are developing a base of five knowledge to work with. At level 5 a child is operating unistructurally in the first cycle of the concrete symbolic mode with words replacing fingers. Level 6 is multistructural in the first cycle as at this level a child begins to combine fingers, counting on and acquired knowledge. Level 7 was identified as relational 1 in the concrete symbolic mode as at this level children had a base of knowledge they could use and manipulate to calculate a variety of addition combinations. Figure 6.5 is an Addition Without Counters map.

CUES	CONCEPTS/PROCESS	RESULTS
'A'+'B'=?	counting knowledge of the next number is used	adds only one more
	uses finger and counts from one	
	shows a small base of knowledge for results less than five	can only answer values less than five
	counts on with fingers fingers take on a place value e.g. means six	can only answer values less than ten
	without counting fingers and toes are used	all correct
	counts on without fingers	
	A knowledge of number facts used	

Figure 6.5: Adding without Counters Mapping

The mapping process identified seven concepts/process that can lead to one of four possible results. The eight concepts relate to the observed levels 2 to 7. For some levels two concepts are features of a particular observed level.

Figure 6.5 is completed as a Addition Without Counters map for a concrete symbolic multistructural 1 exit. Other maps for Adding Without Counters are included in appendix B.

Comparison with other studies

These levels largely follow the five counting types of Steffe and Cobb (1988) discussed in Chapter One. Steffe and Cobb (1988) identified five counting types as follows

1. Perceptual Unit Items.
2. Figural Units Items.
3. Motor Unit Items.
4. Verbal Unit Items.
5. Abstract Unit Items.

The observed levels in this study related well to the counting types identified by Steffe and Cobb. Children in the observed level 1 could not operate in this task and related well with Steffe and Cobb's level 1 where children can count only what they can see. At observed level 2, minor mistakes were made and a child had clearly moved to Steffe and Cobb's level 2 where they can count items that are not seen. The observed level 3 involved a child displaying the ability to add up to a small base of number, such as five. This fits in with both levels 2 and 3 of Steffe and Cobb as the child used the motor action of fingers to help them to answer the task. The observed level 4 was a further development of using motor acts or movements and therefore matched Steffe and Cobb's level 3 Level 5 indicated a transition between Steffe and Cobb's levels 3 and 4 with counting on techniques in conjunction with figures used. The observed level 6 matched Steffe and Cobb's level 4 with the use of words substituting for countable items. Finally, the observed level 7 equalled Steffe and Cobb's level 5, where the child at this level demonstrates a knowledge of number facts.

Below the observed levels found in this study are displayed in Table 6.10 with the relevant counting type of Steffe and Cobb and the SOLO learning cycle level for each responses type.

Table 6.10

Summary of levels for Addition without Counters

Observed levels	Steffe & Cobb.	SOLO
1.The child displays no idea or understanding of what was required.	1	not operating in this stem
2.The child adds on 1 only or makes lots of mistakes with each attempt.	2	unistructural (Ik)
3.The child displays the ability to add up to a small base and cannot add beyond five.	2/3	multistructural (Ik)
4.The child uses fingers to facilitate addition to ten by either a) counting from one or b) uses a base five.	3	relational (Ik) transition to concrete symbolic mode
5.The child begins to count on.		unistructural 1 (C.S.)
6. The child uses a combination of fingers and straight answers. Also fingers and toes may be used to give answers over ten.	3/4	multistructural 1 (C.S)
7. The child provides answers without a finger display, and the child demonstrates a base of knowledge.	5	relational 1 (C.S.)

Subtraction without Counters

Five levels of subtraction without counters were observed during the study. In this task, children were asked simply subtractions verbally. These levels largely follow the five counting types of Steffe and Cobb (1988) discussed in Chapter One. The levels found are displayed in Table 6.11 which is drawn from table 5.4.

Table 6.11

Observed Levels for Subtraction without Counters

Observed levels
1.The child displays no idea or understanding of what was required.
2.The child attempts with some indication of understanding or can only answer with a difference of one.
3.The child uses fingers to answer or was able to work with small familiar values.
4a.The child answers only values below ten correctly. 4.The child combines use of fingers and counting backwards
5.The child appears to have a bank of knowedge of number facts which is used to "work out" the answer mentally.

Applying SOLO to Subtraction without Counters

The levels observed were also considered in the light of the SOLO Taxonomy. The observed levels were seen to belong to the ikonic and concrete symbolic modes. Level 1 was identified as not operating in the subtraction without counters stem. Level 2 fitted the unistructural response in the ikonic mode of SOLO, as children were only able to work with simple differences of one. At level 3 a child was able to use fingers to answer a range of questions. This was seen to be in the ikonic mode though operating multistructurally. On reaching level 4 a child showed signs of making the transition from ikonic to early concrete symbolic with the combined use of fingers and counting backward technique to answer questions. This level was seen to incorporate unistructural and multistructural levels in the first cycle of the concrete symbolic mode. Level 5 observed corresponds to the relational level in the first cycle of the concrete symbolic mode as it involved children relating information they knew to new situations. A map for Subtracting without Counters is shown in Figure 6.6. This is a map for a multistructural ikonic response.

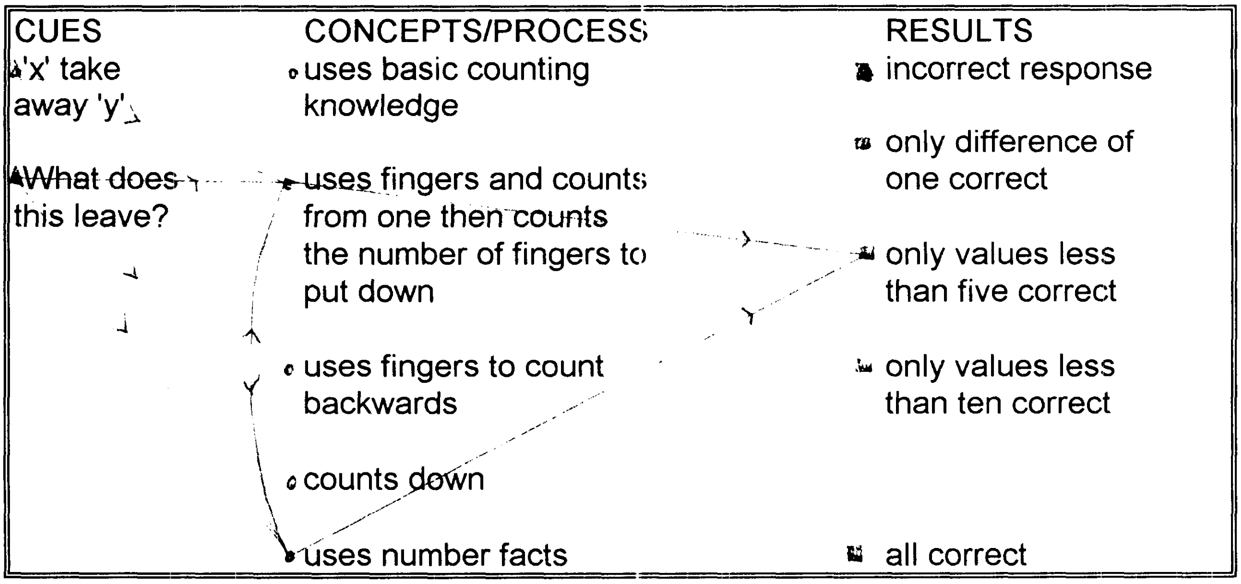


Figure 6.6: Subtraction without Counters Mapping

The mapping procedure identified five concepts/processes that lead to one of four results. The concepts match the key features of the observed levels 2 to 5 including level 4a. Applying the map to each response allows the application of SOLO to the task.

Comparisons with other Studies

All observations made supported the work of Steffe and Cobb (1988). The levels found in this study matched exactly those identified by Steffe and Cobb. Steffe and Cobb identify their levels as follows;

> 1.Perceptual Unit Items- Children count only the items that they can see.
> 2.Figural Unit Items- Children count items that are not seen.
> 3.Motor Unit Items- Motor acts or movements become countable.
> 4.Verbal Unit Items- Words become a substitute for a countable item such as counting on.
> 5.Abstract Unit Items- The child develops an abstract collection of number where the number itself is a countable unit.
>
> (Steffe & Cobb, 1988 p.8-220)

Both the observed results, and the levels of Steffe and Cobb, indicate that children operating at the highest level had developed their own base of number facts and mathematical knowledge which they are able to manipulate to produce solutions to a variety of questions. The development of this base of knowledge is essential for mathematical achievement, and obviously should be encouraged in all children even at this young age.

A summary of the observed levels matched with the relevant levels of Steffe and Cobb(1988) and the SOLO levels is shown in Table 6.12.

Table 6.12

A Summary of the levels for Subtraction without Counters

Observed levels	**Steffe & Cobb.**	**SOLO**
1.The child displays no idea or understanding of what was required.	1	not operating in this stem
2.The child attempts with some indication of understanding or can only answer with a difference of one.	2	unistructural (Ik)
3.The child uses fingers to answer or was able to work with small familiar values.	3	multistructural (Ik)
4a.The child answers values below ten correct.	4	relational (Ik)
4.The child combines the use of fingers and counting backwards		unistructural 1 / multistructural 1 (C.S)
5.The child appears to have a bank of knowledge of number facts which is used to "work out" the answer.	5	relational 1 (C.S)

Written Addition Questions

Here the questions were observed with the intention to investigate the children's ability to read and understand mathematical symbols and language. The SOLO levels that matched refer to levels of operation within the concrete symbolic mode of the stem of basic recognition of numbers and mathematical symbols. The levels found here did not link significantly with other research discussed in Chapter One. Three levels were differentiated and are shown in Table 6.13 which is drawn from Table 5.5.

Table 6.13

Observed levels for Written Addition

Observed Levels
1.The child recognises that the written questions have some message but could not read the questions.
2.The child has begun to recognise the meaning of number and mathematical symbols.
3.The child reads and answers correctly the questions.

Applying SOLO to Written Addition

The observed levels were also considered in terms of SOLO. The observed level 1 was classified as SOLO unistructural level in the stem of reading mathematics in the concrete symbolic mode. Level 1 was classified as unistructural due to it simple characteristics where the child recognised that the numbers held a message but could not put the message together. Level 2 was identified as the SOLO multistructural level within this stem due to features that children at this level could recognise and their attempts to interpret the question. Finally, level 3 represented SOLO multistructural and relational level within the first cycle of this stem. At level 3, children to varying degrees, were able to interpret a number of combinations of mathematical symbols and number symbols then calculate suitable responses. These children were able to relate different areas of knowledge - reading and mathematical calculation - to arrive at an answer.

CUES	CONCEPTS/PROCESS	RESULTS
written questions	tries but cannot recognise symbols	incorrect answer
pen and paper	reads numbers not symbols	
can you answer these?	reads the numbers and symbols	Reasonable or correct answer
	uses various methods of fingers and base of knowledge to solve	

Figure 6.7: Written Addition Mapping

SOLO Mapping was used to support the relationship between SOLO and the observed levels. Figure 6.7 identifies three key cues and four concepts that lead to one of two results. The third and fourth concepts together result in a correct level 3 response. Figure 6.7 shows a map for a multistructural response. Other maps for unistructural and relational responses are included in Appendix B.

Table 6.14 shows a summary of the observed levels and matched SOLO levels.

Table 6.14

A summary of levels for Written Addition

Observed Levels	**SOLO**
1.The child recognises that the written questions have some message but could not read the questions.	unistructural 1 (C.S.)
2.The child has begun to recognise.	multistructural 1 (C.S.)
3.The child reads and answers correctly the questions.	relational 1 (C.S.)

Written Subtraction Questions

As with the written addition questions, the ability of a child to read the questions was observed. The SOLO levels refer to levels of operation within the concrete symbolic mode of the stem of basic recognition of numbers and mathematical symbols. The levels found here did not link significantly with other research discussed in Chapter One. Four levels were differentiated and are shown in Table 6.15, which is drawn from Table 5.6.

Table 6.15

Observed levels for Written Subtraction

Observed Levels
1.The child recognises that the written questions have some message but could not read the questions.
2. The child has begun to recognise written number and mathematical symbols.
3. The child read the minus sign as a addition sign.
4. The child could read and correctly answer.

Applying SOLO to Written Subtraction

Level 1 was classified as SOLO unistructural level in the stem of reading mathematics in the first cycle of the concrete symbolic mode, due to the simple recognition that the symbols had a message but the inability to interpret what the message was. Both levels 2 and 3 were identified as the SOLO multistructural 1 level within this stem. At these levels a child is beginning to read the numbers and symbols and realise they relate to spoken and counted number but is not sure of the relationship. Level 4 was identified as the relational level in the first cycle within the stem of reading mathematics in terms of SOLO. This level is characterised by children who had the ability to read the questions and relate the information to their own knowledge of number, and then convert their answer to a written response.

SOLO Mapping was used to support the relationship between SOLO and the observed levels. A map for Written Subtraction is shown in figure 6.8.

CUES	CONCEPTS/PROCESS	RESULT
Written Questions	tried but unable to interpret symbols	incorrect response
Pen and paper	reads number not symbols	answers as a addition
	reads minus as a plus	
Can you answer these ?	reads numbers and symbols	
	uses a variety of methods to work out the answer	answer correctly or minor errors

Figure 6.8: Written Subtraction Mapping

The concepts/processes that are identified in the mapping outline related directly to the observed levels identified with both the fourth and fifth concepts features of level 4. The five concepts identified lead to one of three possible results. Figure 6.8 shows a multistructural 1 response. Others maps for unistructural and relational responses are included in Appendix B.

Table 6.16 shows a summary of the observed levels and SOLO levels.

Table 6.16

A summary of levels for Written Subtraction

Observed Levels	SOLO
1.The child recognises that the written question have some message but could not read the questions.	unistructural 1 (C.S.)
2. The child has begun to recognise.	multistructural 1 (C.S.)
3. The child read the minus sign as a addition sign.	multistructural 1 (C.S.)
4. The child could read and correctly answer.	relational 1 (C.S.)

REVERSIBILITY

Four levels of reversibility were recognised after observation of the children's responses. These levels were also related to the SOLO levels within this stem of knowledge. This tasked required that children be operating in the concrete symbolic mode. The levels that emerged are shown Table 6.17.

Table 6.17

Observed levels for Reversibility

Observed Levels
1. The child made no attempt.
2. The child's answer was an attempt to form a number pattern.
3. The child considered symbols and numbers but did not answer correctly.
4. The child was able to work with reversibility to varying degrees a) only for questions 1 and 2 b) for all questions with two or three solutions for question 3 c) four or more solutions for question three indicating an understanding of adding zero and addition associative law.

Applying SOLO to Reversibility

The application of SOLO to this task was achieved readily. Level 1 was seen to fit the category of not operating in this stem of reversibility thinking in the concrete symbolic mode. A child at level 1 would still be functioning in the ikonic mode for this task. A level 2 response was recognised as unistructural 1 in this stem due to the responses being only a single way of looking at the numbers as forming patterns. At level 3, the combination the children displayed took into account mathematical symbols and numbers to give answer showed a multistructural 1 approach which lacked the ability to relate the symbols and numbers correctly. Finally at level 4 children had displayed the ability to relate the numbers, the symbols and their relevant position to calculate the missing digits. The children who responded at level 4a) were operating in the relational 1 in the concrete symbolic model. Responses that gave more than two possible solutions to part three - level 4b) - showed a higher level of thinking indicating a transition to the second cycle in the concrete symbolic mode. Responses at level 4c) indicated a concise overview of the question and could be interpreted as operating at the unistructural level of the second cycle in the concrete symbolic mode.

SOLO Mapping was used to support the relationship between SOLO and the observed levels. Figure 6.9 shows a map outline for Reversibility.

CUES	CONCEPTS/PROCESS	RESULTS
'a'+? ='b'	looked at the numbers only	formed a pattern
	considered numbers and symbols	answered incorrectly
? + ? = 8	added the answer and the other number	answered only single blanks
	filled in one missing part	answered all blanks
	could group numbers to get an answer	only one solution to Q3
	could see a number of possible solutions	correct multiple solutions to Q3

Figure 6.9: Reversibility Mapping

The Reversibility map outline identifies six concepts/processes that relate to the observed levels 2 to 4. Five different results were identified that also fitted within the observed levels. Figure 5.9 shows a map for a multistructural 1 response. Others maps for unistructural 1, relational 1 and unistructural 2 responses are included in Appendix B. Table 6.18 shows a summary of the observed levels and the matching SOLO levels.

Table 6.18

A Summary of levels for Reversibility

Observed Levels	SOLO
1. The child made no attempt.	ikonic
2. The child's answer was an attempt to form a pattern.	unistructural 1
3. The child considered symbols and numbers but could not answer correctly.	multistructural 1
4.The child was able to work with reversibility to varying degrees	
a) only for question 1 and 2	relational 1
b) all question with two or three solution for question 3	transition
c) four or more solution for question three indicating an understanding of adding zero and addition.	unistructural 2

PLACE VALUE

Place Value was considered by the use of three observation tasks, namely, reading two digit numbers; adding numbers greater than 10; and putting numbers out of order, back into correct counting order. In Chapter Five the integration of the levels in the three tasks into one set of levels was discussed in detail. These levels are shown in Table 6.19 drawn from Table 5.8.

Table 6.19

Levels for Place Value

Observed Levels
1. The child cannot order number and is unable to read two digit numbers.
2. The child begins to read tens and units in numbers but is easily confused and need to use fingers and toes to add numbers greater than ten. The child can order numbers correctly.
3. The child can read two digit numbers consistently and adds numbers greater than ten using their knowledge of place value without concrete support.

Applying SOLO to Place Value

The levels identified in the place value tasks matched well the first cycle in the concrete symbolic mode of SOLO. Level 1 was identified as unistructural 1 in the concrete symbolic mode. At this level responses showed little understanding of place value. These children tried to read the number but were only able to read single digits. The position of the number meant nothing at this level. In some cases responses at this level recognised the number 10 but only due to rote memory not due to an understanding of the tens position value. At level 2 recognition of tens and units begins indicating multistructural 1 level. Responses at this level display the ability to order numbers and to add two digit numbers which require that one fact is retained while another is worked on. Often ikonic techniques are used to gain a satisfactory answer, such as counting on toes and fingers, when adding two digit numbers. Finally, level 3 is relational in the first cycle of the concrete symbolic mode due the ability displayed at this level to understand position value of numbers and to add number in the same positional value immediately without counting out all the numbers. The application of place value knowledge to answer addition may indicate a unistructural 2 response.

The mapping developed for place value represents this combination of the three tasks to give an overall view of the combined concepts and processes of each level of place value thinking. Figure 6.10 shows a map for the overall levels and features of the place value tasks.

CUES	CONCEPTS/PROCESS	RESULTS
Reading numbers	sees 14 as 1 and 4 - single units	reads values less than ten
Adding numbers greater than ten	recognises that tens and units exist but unsure of what it means	reads all values
		orders correctly
Ordering numbers	rote learnt recognition of ten or number facts	calculates addition correctly
	uses fingers to answer addition	
	adds tens and ones separately	

Fig 6.10: Place Value Mapping

In the map, five concepts/processes were identified which are features of the observed levels. These lead to one or more of the four results. A child performing level 3 would achieve the second, third, and fourth result on the map. Figure 6.10 shows a relational 1 (C.S) level for the place value tasks. Other level maps are included in Appendix B.

A summary of the observed levels matched with SOLO levels is shown in Table 6.20.

Table 6.20

A Summary of levels for Place Value

Observed Level	SOLO
1. The child cannot order number and is unable to read two digit numbers.	unistructural 1 (C.S.)
2. The child begins to read tens and units in numbers but is easily confused and need to use fingers and toes to add numbers greater than ten. The child can also correctly order numbers.	multistructural 1 (C.S.)
3. The child reads two digit numbers consistently and adds numbers greater than ten.	relational 1 (C.S.)
The child uses knowledge of place value to answer without concrete support.	unistructural 2 (C.S.)

LONGITUDINAL DEVELOPMENT IN TERMS OF SOLO

This section considers the longitudinal development of the 21 children in the study in terms of SOLO. Therefore the task areas of *More and Less* and *One- to-One Association* are not included as they were not involved in the longitudinal study over the school year. This section focuses on the tasks of *Addition and Subtraction*, *Reversibility*, and *Place Value.*

At the beginning of the year, the children were functioning in a wide range of SOLO levels, both in the ikonic and concrete symbolic mode. The Queensland Syllabus indicated an emphasis on the development of relational level operation in the ikonic mode with the strong leaning towards concrete materials, counting, sorting and matching objects in Year One. Over the year the main growth in levels of performance for Addition and Subtraction tasks in terms of SOLO was seen by students that initially performed at the unistructural level in the first cycle in concrete symbolic mode. By the end of the year the bulk of the group were performing at multistructural 1 in the concrete symbolic mode.

The interviews in February indicated, particularly for Addition and Subtraction with Counters, that many of the children were already operating in the relational level in the ikonic mode or unistructurally in the first cycle of the concrete symbolic mode. Throughout the year, the largest proportion of the group stayed performing at these levels. However, there was, by the end of the year no students performing unistructurally and only one at the multistructural level in the ikonic mode. At the first interview the Addition without Counters task received seven responses from children operating in the ikonic mode. In June six children were still in the ikonic mode but by November all children were operating in the first stem of the concrete symbolic mode. In the Subtraction without Counters task, over the school year, eight children moved from ikonic thinking to responses coded within the concrete symbolic mode. Although there was an obvious trend of movement through the levels over the year, three children made no progress and were still not operating in this stem at the November interviews. This was very interesting and requires further research to ascertain why these children failed to progress.

As with other areas of Addition and Subtraction, the most significant areas of improvement in levels of performance for Written Addition and Subtraction were seen with the students who began the year operating at a unistructural level in the concrete symbolic mode. These children appeared to have more room for growth than children performing at higher levels who made little or no improvement. In the area of written subtraction four out of seven children who had performed initially at the relational 1 level in the ikonic mode, were operating multistructurally in the first cycle of the concrete symbolic mode at the end of the year. Some factor over the period of the study had caused the level of these children's performance to regress.

In Reversibility, twelve children began the year unable to undertake the tasks given. By the end of the year all but three chi dren in the group were able to achieve some success. Significantly, one third of the children throughout the year performed at relational 1 in the concrete symbolic mode, in this task, displaying a sound grasp of reversibility. In November there still existed a large range of SOLO levels of performance within the group.

At the February interviews on the Place Value Tasks, approximately half the children were performing at the unistructural 1 level in the concrete symbolic mode, with only two children at the highest relational 1 level in the concrete symbolic mode. By the end of the year this situation had reversed. In this task over the year, a total of 16 children improved their level of performance in SOLO levels.

In terms of SOLO the poor performers often failed to operate within the stem of the tasks considered or remained in the ikonic mode or at a unistructural level in the concrete symbolic mode. The middle performers made improvements between SOLO modes or within SOLO modes. Some of these children made the transition to the higher SOLO levels but in general remained at the multistructural level responses in the concrete symbolic mode. The top performers were consistently at the relational level in the concrete symbolic mode for most tasks over the year.

The overall improvement of the children involved in the study is clear when the responses are considered in light of SOLO. The improvement shown is largely from children performing initially at unistructurally levels in the concrete symbolic mode or in the ikonic mode to multistructural levels in early concrete symbolic thinking. Children performing at higher SOLO levels in the concrete symbolic mode tended to remain stable in their performance over the year in all areas except written subtraction.

CONCLUSION

This study observed the initial knowledge that young children brought with them in their first year at school in the area of number, and traced the development of this knowledge over the first year of school. A series of interviews at three intervals over the year lead to the discovery of levels of development of number understanding in the areas of More and Less, One-to-One Association, Addition and Subtraction, Reversibility, and Place Value. The distinguishing of levels was achieved as a result of grouping responses of the children during interviews. This chapter has applied the SOLO Taxonomy to the levels that emerged in this study with relative ease. Clearly, the SOLO Taxonomy provides a useful means for assessment of performance and interpreting cognitive development of children over the year.

Furthermore, the levels identified were considered with regard to work of other researchers in this area such as Fuson(1988), Steffe and Cobb (1988) and Carpenter and Moser (1983). Many of the levels that became apparent throughout the study were closely related to those used by these researchers. In some cases the levels that emerged throughout this study matched one-for-one with those found by other researchers and the levels of SOLO, in particular for addition and subtraction with counters, while at other times some of the levels found did not correspond. In More and Less, and Adding without Counters the levels did not match, and in Adding without Counters the results of this study found a number of levels that were incorporated into single levels identified by Steffe and Cobb (1988). However, the SOLO Taxonomy provided a hierarchy of levels that explained the responses that had been observed and identified as being significant steps in the progress of understanding.

In some tasks a mix of ikonic and concrete symbolic responses could be observed which typically reflected the developmental stage of these children. Not all levels of SOLO were seen distinctively in all the tasks observed in the study such as Subtraction without Counters where the observed level 4 matched the SOLO level of unistructural 1 and multistructural 1 in the concrete symbolic mode. The observed levels were able to be identified as learning cycles in the SOLO Taxonomy. All the tasks were able to identify the first learning cycle in the concrete symbolic mode and the reversibility and place value tasks identified both the first and the second cycle in the concrete symbolic mode supporting research on the existence of learning cycles. These results indicate that other learning cycles may exist in the ikonic mode and early development of the concrete symbolic mode.

Finally, the levels set out here represent the steps to higher levels of understanding and thinking in each of the specified task areas that were observed in this study. The level found in this study correspond well with the SOLO Taxonomy. The levels found in this study support the work of Steffe and Cobb (1988) and Carpenter and Moser (1983) that there are levels of development in understanding number, and in some cases matched the levels identified by these researchers. The finding of levels that fit the SOLO theoretical framework, although based on a small sample, provides educators with a series of levels of number development that could be useful in setting curriculum guidelines on the development of number in children in their first year of school. In particular, the longitudinal development of the sample group over their first year at school indicates that, for Queensland, the curriculum may need to consider the development of the top performers more and the introduction of subtraction in Year One. The levels discussed here provide a basis for such considerations.

CHAPTER 7
CONCLUSION

This study was designed to investigate two broad primary questions that were raised after consideration of the current literature. A number of more specific questions on mathematics education were also raised along with the suitability of application of SOLO levels to the development of number and counting in young children beginning their first year of school. The results to these questions are discussed in this chapter.

This chapter discusses the overall result of the two areas investigated and in a summary of results, each of the minor questions posed. Furthermore, as well as evaluating the design, the chapter considers the practical and theoretical implications of the results of this study for Year One mathematics education in Queensland. Suggestions for Year One mathematics education are made as well as possible areas of further research in this topic.

EVALUATION OF THE DESIGN

The study used three different primary schools in an attempt to attain a more accurate picture of children beginning and progressing through Year One in Queensland. The three schools available were all within the one region of the central Gold Coast and the sample of students involved was relatively small, consisting of 21 students. A variable in the classroom situation occurred with the state school, the nominated class for the interviews was a composite class consisting of Year One and Year Two students, 25 of each. Hence the learning situation was different here and may have influenced the progress of the student throughout the year as the Year One students may have been exposed to Year Two work indirectly in the same classroom. The teachers in this class often divided the class in two, one teacher taking a group experiencing difficulty with simple counting and addition, the other taking the more capable students to do more advanced addition. The groups were split according to the children's ability not their year level so that some Year One students may have been exposed to Year Two mathematical requirements.

A more purposive sampling may have been desirable in choosing the children involved in the study. This was not possible due to the beginning of the interviews early in the year. This meant children had to be chosen and permission notes sent home immediately so that interviews could be done in the first weeks of school. There was also the need to fit in with different teachers' schedules.

"Fitting in" with schools and teachers caused some difficulty as arranged interview times were often changed at the last minute for other school events, such as guest speakers, extended assemblies or extra curricula classes. Some teachers proved more helpful than others and some interview times were difficult to attain. It was decided that interviews would not infringe on children's play time, therefore interviews could not be started when there was less than forty minutes to recess or lunch. This way children did not feel rushed. In one school, a withdrawal room was often available to be used for the interviews but this had to be vacated for catechist teachers at times. The situation was not as conducive in the other schools as there was not a specific room available for use, and interviews moved between the library, office and resource room. This environment was not always consistent or comfortable. A third group of children were interviewed in their own home, which was often more peaceful and offered less distraction and easier accessibility to the children. Although, there were problems as it was outside of the school and the learning environment.

A video with an operator was seen to be the best method of recording the interviews. However, a video was unavailable, as was an assistant to operate a video. It was thought that a video on a stand would not have captured the hand feet and other body movements observed. A operator would have been necessary for best results. However, in the aim of maintaining as naturalistic and comfortable environment as possible under these type of conditions, taping or video taping did not occur. Thus, it is possible some reaction to questions may have been missed, though this is unlikely due to the extensive notes taken and the nature of the tasks which required written work from the children.

The length of the interviews was approximately thirty minutes. This may have been too long for some children, though all children were very enthusiastic about the interviews and were very keen to have their turn. Children encountered at other times would ask "when are you going to do that question thing with me again?". The individual interview instrument used was excellent for this style of data collection as it allowed for detailed observation of each child as they solved the tasks and allowed the interviewer to develop a rapport with each child. The longitude nature of the study and the young age of the participates raised some validity concern over the maturation of the participants and this effect on the results, but this was unavoidable when working with young children over a period of time. Concern over maturation of the students was outweighed by the gains made in observations over the extended period.

The tasks used for the study proved to give an abundance of data to be considered and analysed. The structure of tasks and form of data collection made it possible for the study to be replicated. Area three- *Addition and Subtraction* could have been expanded to include other types of addition and subtraction but the length of time of interviews was a constant constraint on the tasks. Overall, the design of the study proved effective in gathering information to use in considering the questions posed in Chapter One and gaining insight into the number ability of Year One students in Queensland.

OVERALL RESULT

This study was design to investigate two focus areas. The first involved a study of 21 children in February at the beginning of their first year at school. The aim was to investigate what knowledge these children had brought with them to school in the area of Mathematics, particularly number. The second research focus was a longitudinal study, conducted over the year, whereby the February interviews were followed up with interviews in June and November. This longitudinal study aimed to trace the development of the 21 children over the year in three main areas -addition and subtraction, reversibility, and place value. The overall results of these two investigations are discussed below under their respective headings.

1. What knowledge of mathematics, particularly that concerning number, do children, entering Year One in Queensland, bring to school?

Children entering Year One in Queensland schools bring with them a varied knowledge and experience of Mathematics. This knowledge may be the result of the technological age in which we live where numbers are used on the remote control, on the television, on the microwave and in computers. All of which are now common in the home. This study set out to assess this knowledge and discovered that the range in ability displayed by a small sample of students beginning Year One was large. This supports the findings of Wright (1994). The range of knowledge displayed included children who found it difficult to count beyond five and displayed no understanding of the numerosity or number value of a number amount. These children had trouble ordering numbers correctly and counting out given amounts of counters. Other children in the group could add and subtract confidently with counters and some without counter support. Some children were also able to read and write numbers and work with number sentences to a varying degree. Some children displayed advanced skills of reversibility, and a number knowledge which enabled them to manipulate numbers to answer questions set in new situations.
These more competent children displayed a good understanding of numerosity of numbers and of some basic number laws, such as the effect of adding zero and the associative law of addition. They also demonstrated the ability to recognise that a number of different combinations could produce the one solution. Overall a

significant range of ability was identified. A small group of children were clearly able to deal with Year Three mathematics while others were only ready to work with the very basics of Year One mathematics.

2. What progress do children make in their first year of schooling in Queensland in their development of number understanding?

After three interview periods spread over the year and analysis of the data collected, it was apparent that many of the children possess knowledge of mathematics and number that is not expected of them. This knowledge does not appear to be put to use in the classroom situation. The first half of the school year is spent working on mathematical activities that are often too simple for many of the students. A great amount of mathematics time is spent: grouping objects; matching two objects with the number two and the like; working with amounts less than five; drawing more or less of the objects shown; and, learning to recognise the written English number words for the numbers one to ten. For many of these children this work is covering skills in which they have already demonstrated competence. While the correct spelling of number words is a skill that many do not possess, it should develop as their reading ability for English does and should not be seen as an explicit part of number development.

The Year One experience for the children with fewer skills provided a better beginning. This was indicated by the improvement these children made over the year. These children received a good base of concrete experience, and a helping hand to develop an understanding of numbers that they had not yet constructed for themselves. The work facilitated their knowledge development. Though for some children who experienced real difficulty constructing mathematical knowledge, Year One experiences seemed to offer few solutions, and they remained operating at the lower levels for many tasks.

For the children who displayed a good development of informal mathematics, Year One provided a reinforcement of much of what they already knew and perhaps helped to polish their writing and recognition of individual numbers more than anything. However, Year One mathematics for these children failed to develop their addition and subtraction skills or encourage refinement of skills, such as counting on and subitizing. The results indicating an emphasis on counting from one, which may be suitable for less capable children but represented a regression for many of these children.

The few children who displayed real understanding of mathematical concepts did not benefit by the Year One experience, working on many tasks that were at a lower level than the level at which they were currently able to operate. These children were given activities to count out groups of three and four objects to match the numbers when they already had developed a complex network of mathematical knowledge of addition and subtraction combinations.

The syllabus appears to be aimed at the children with the weakest background knowledge. It is focused on the traditional idea of education that children are a blank slate, though it does suggest it is up to the individual teacher to extend and challenge each student. Over the year, the less able students made significant progress, while children who had already developed a strong base of informal mathematical knowledge frequently made no progress. By the end of the year the range of ability had narrowed. This pattern supports the findings of Young-Loveridge (1989) and Wright (1991). This is a disturbing trend for educators as the aim of education is to educate and improve all members of the group. This seems not to be promoted in this circumstance. Here the children experiencing difficulty are gaining suitable tuition while the more capable are not receiving suitable work to encourage further development (although the very weak student showed no large development). Extension for the truly "clever" is not obvious from the development of these children over the year. There appears to be no advantage for those children who have already developed a rich knowledge of number before going to school as at school they stagnate waiting for the other children to 'catch up'.

SUMMARY OF RESULTS

As a result of the investigation of the two main areas of this study, a number of other questions arose and observations were made. In this section these additional results are discussed to summarise the in-depth presentation that occurred in chapter four, five and six.

The results of this study indicated that on entering school children do have the ability to recognise and count objects with numbers and are generally ready to move on to higher-level tasks. This result supports the findings of Young-Loveridge (1989) and Gelman and Gallistel (1978). Approximately 60% of the students in this study had, at the February interview, developed a concept of numerosity of a number, and could work with numbers without concrete support. Many of these students (25%) were able to go further, and work with adding numbers that gave answers over the value of 10. In their first year of school, these children did not learn anything new and in fact did not utilise the knowledge they had brought with them. While the areas of addition, one-to-one association, and grouping are strongly emphasised in the first year, the results of this study did not support the idea that addition and subtraction success is completely dependent on one-to-one association. This view was also expressed by Gelman and Gallistel (1978, p.202).

Rather, a number of children who were successful at addition and subtraction, had difficulty with one-to-one association. These two skills may be independent of each other. The results of this study indicate that the role of subitizing in the development of a child's number understanding is important. Although not all children who were able to successfully add and subtract displayed the ability to subitize, all the children who were able to recall number facts used subitizing when working with concrete materials. They then used their knowledge to work out the relevant answer. These children did not need to touch concrete material at all or to count from one.

In this study, children displayed the ability to subitize different amounts; some could only subitize up to three, while other children could subitize as many as five. The children performing at a higher level used subitizing more than the less successful children. The importance of subitizing emphasised in this study supports the view of Fuson (1988) and von Glasersfeld (1982) that subitizing plays an indispensable role in the development of arithmetic operations.

Many of the children involved in the study came to the February interviews unable to write the letters of the alphabet and in some cases could not even write their own name. Yet these same children were able to write number symbols to a recognisable standard, and all were able to read numbers, at least to five. More than half the children entered school with the ability to read number and mathematical symbols to the point where they could read and answer number sentences with + and = signs. The minus sign was not as well recognised, which may be a result of its less general use in society than the addition sign; although one in three of the children were still able to read and work with it.

The results of the study indicate that *Place Value* is an important area of development during the first year at school. Most children at the February interviews showed no understanding, or only early development of understanding, of place value yet by the end of the year approximately 80% of the children had improved their understanding of place value. This is interesting as numbers over ten were not dealt with in their first year of school so it appears this development is largely due to maturation and indicates that tuition and activities related to place value may enhance the development that is occurring. These results indicate that Year One children are ready to work with numbers greater than ten, and the Year One syllabus should include some work with written numbers greater than ten.

The current mathematics syllabus for Queensland Primary Schools does not provide enough scope for many of the more capable children. The syllabus appears to assume that the children entering Year One are operating in the ikonic mode for mathematical number development. This is indicated by the strong emphasis on the use of concrete materials and related activities, such as drawing groups of three trees or six dogs, which feature in the mathematics tasks. It is clear from these results that most of the children are operating in the concrete symbolic mode or are at least doing so by June in their first year.

IMPLICATIONS FOR EDUCATION

The results of this study indicate that the following areas could be developed within the syllabus and the Year One classroom:

1. A more individual type of approach to learning mathematics similar to that used in reading in Year One where children take home readers that suit their own level of reading. This would allow more capable students to continue to develop their mathematical knowledge and not spend time redoing work they already understand.
2. The use of written mathematics in Year One would seem to be a logical extension of the knowledge that the study has indicated that many children may bring with them to school in the area of reading and writing number, and mathematical symbols.
3. Activities that encourage the use of subitizing rather than counting from one should feature in the syllabus and children who already subitize should not be forced to count from one to find answers to questions concerning how many.
4. Addition, without concrete objects, should be given to those children ready to work with abstract numbers. This readiness should be assessed from the beginning of Year One and continue throughout the year as more students develop appropriate skills.
5. Patterns are an important element of mathematics but should not be taught to the detriment of other mathematical relationships. The results here indicated confusion about addition and subtraction symbols as a result of pattern instruction during Year One. Therefore, teaching patterns could be handled more sensitively in Year One so that the children have a better understanding of their place.
6. Mathematics should involve problem-solving activities drawing on the children's real-life encounters with mathematics. Many of the tasks and activities suggested in the syllabus seem to be over simplified or tedious, such as grouping or drawing amounts of objects over and over. This is not to say all tasks should be 'fun' but they should at least be stimulating and take advantage of the enormous enthusiasm children at this age have for learning.
7. Extension activities and rapid progression programs should be available for the more gifted students. Mathematics, for a child who is gifted, should be more than sitting doing basic tasks without being challenged. Interestingly, all the children without exception approached the tasks in this study with enthusiasm and their enjoyment was obvious. We, as educators, must work to provide school

mathematics that maintains this enjoyment and desire to learn. One particular child involved in the study, who displayed a good understanding of number in the tasks, was questioned at the interviews about school mathematics and answered "all we do is count little groups or draw pictures of numbers. I hate drawing the pictures". By drawing the pictures of numbers the child meant tasks where they were required to draw five trees or three cats. Obviously, a negative association has already been set up in Year One of mathematics for this child, on a task that is not relevant to his needs. It is not adequate for the syllabus to note that teachers should extend all children, rather programs should be available.

8. Teachers must strive to be more aware of the range of experience and knowledge of mathematics that exists within Year One children. Teachers must aim to utilise this knowledge more successfully. While it is difficult in large classes to offer extension activities, the large proportion of children in this study who were operating above the expectation of the syllabus highlights the need for a differentiated approach in the classroom. This suggests that a better means of catering for the variety of skills and experience needs to be found.

9. The two children who were less able and failed to progress indicate that the current method of mathematics teaching may not be suitable for all children and could be leaving a proportion of students behind. A variety of methods needs to be employed.

IMPLICATIONS TO THEORETICAL DEVELOPMENT

Clearly the strategies used by children in the tasks given throughout this study can be separated into levels of development. These levels have been outlined in depth in Chapter 5 and Chapter 6. The levels highlight and support evidence of the wide range of abilities in Year One students in the area of number understanding. The identification of the levels of development that children use to solve tasks is useful in teaching and developing new tasks that suitably encourage students to progress to the next level and help to improve understanding of a child's progress. Tasks could be design to foster and allow for children working at a variety of levels.

A number of levels have emerged from the data collected throughout this study that parallel those developed by Carpenter and Moser (1983), in particular. Furthermore, both Fuson (1988) and Carpenter and Moser (1983) found that children had a well developed concept of addition and subtraction even though they had not received formal education. This study, in the initial February interviews, made a similar finding.

Subitizing also emerged out of this study as a key process in the development of number understanding, reinforcing the importance of subitizing as emphasised by Glasersfeld, Richards and Cobb (1983). Although it is not obvious in this study whether last word responses were learned or discovered from subitizing.

The work of Young-Loveridge (1989) found that child with less knowledge made greater gains over the year. The result of the longitudinal study conducted here indicate the same trend. While this is clear, the results also support the work of Wright (1991, 1994), namely, that all students made some progress over the year. Wright stated that "mathematics programs typical of the first year of school significantly underestimate 5-year-olds prior knowledge" (1994, p.41). The interviews in February uphold this statement with many children displaying a sound base knowledge beyond or equivalent to the Year One syllabus. This study extended the observations made of the children's base knowledge showing that many children entering the first year of school were able to work comfortably with written numbers and mathematical symbols, without concrete support. This contrasts with the view of Steffe and Cobb (1988, p.321) that written work should be abandoned in the first year of school.

The application of SOLO to the tasks, highlighted the fact that many of the children began Year One already operating in the concrete symbolic mode of thinking, whereas the syllabus indicates an expectation of operation only in the ikonic mode. The levels found in this study, when considered in terms of SOLO, indicate the existence of learning cycles in the concrete symbolic mode in number development in young children. This work represents an initial, tentative step at exploring an early cycle in the concrete symbolic mode. It also showed SOLO can offer an overall framework within which research from the investigation can be considered and compared.

FURTHER RESEARCH

This study opens up a number of areas for further research. Here the knowledge that children bring with them to the first year of school has been investigated to discover a network of information and informal knowledge that these young children have when entering school. Yet, we are uncertain as to what exact factors have contributed to this knowledge. Further research into the backgrounds of children who show high levels of understanding of mathematics would help to distinguish more specifically the role of the socio-economic background, parental involvement, experience in the home, older siblings influence, television programs, such as Sesame Street, and other influences in the development and acquisition of this informal mathematics.

In the second phase of this study, the progress of twenty-one children has been followed throughout Year One in three areas: Addition and Subtraction; Reversibility and Place Value. Eight tasks have been used to study the progress of the children in these areas. These tasks have offered insight into the children's progress but further study could investigate a broader variety of addition and subtraction type tasks that require other number manipulations.

Another area, which appears to warrant further study, is the performance of children identified as being top performers, and comparing their success in school and their teacher's perception of these children. It would be valuable to know whether the teachers also perceived these children as top performers or placed no value on their intuitive knowledge. In addition a study observing in the classroom, as carried out by Urbanska (1993), to assess what use the teacher made in the classroom of the children's informal mathematics in Queensland would be useful. Both the initial study and the longitudinal study conducted here indicate that informal knowledge may not be utilised in the classroom but this utilisation has not been observed from within the classroom, so no firm conclusions can be drawn in this area.

Finally, a study to investigate earlier number development in pre-school children could consider the possibility of SOLO learning cycles in the ikonic mode. This could provide a basis upon which to explore the number knowledge of these children who performed at the lower levels in the study and did not seem to show improvement over the year.

SUMMARY

This study has achieved the aims to discover:

1. What knowledge of mathematics, particularly that concerning number, do children, entering Year One in Queensland, bring to school?
2. What progress do children make in their first year of schooling in Queensland in their development of number understanding?

The initial study found that children bring a good base of mathematical knowledge in the area of number. Many children were able to work confidently with number symbols without the support of concrete materials. Exposure to informal mathematics outside of the school environment had allowed many children to develop and construct their own informal set of mathematical concepts and rules.

Over the year nearly all children in the group made some progress, but analysis of data indicated that some of the average performers in the first interviews made significantly more progress than the children who had begun the year with an already well developed knowledge base. The more capable children appeared to be put in a situation where they had to wait for the other children to 'catch up'.

A number of areas have been identified as needing further consideration in the Year One mathematics education areas as a result of this study. Clearly, educators must strive to capture the knowledge that young children bring to school to foster further development within the classroom situation, while also helping the less developed students. All student should make substantial gains in their first year of school.

This study has supported the work of current researchers in the field(Fuson 1988, Carpenter & Moser 1983, Young-Loveridge 1989, Wright 1991, 1994) and identified areas for further research. The results of the longitudinal study verify that a series of levels exist in the development of number and counting in young children aged five and six years. These levels are able to be categorised within the SOLO Taxonomy. Furthermore, the levels found in this study indicate the usefulness of SOLO in assessing children's progress in early number development and verify the existence of learning cycles in the concrete symbolic mode.

The first year of formal education is vital in the development of each child. It is clearly important that children are considered in the light of the knowledge that they bring with them to school, and the school mathematics program designed around the development and use of this knowledge. The levels of number development identified in this study provide a theoretical basis from which the development of a curriculum that enhances each child, relevant to their appropriate level, and helps to encourage progress to higher levels rather than regression and stagnation could occur.